

THE STRENGTH OF THE LUNAR INTERIOR: APPLICATION OF A THICK SHELL MODEL TO THE MOON

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We come from stardust,
One and all compelled to strive,
To understand the universe
Or a grain of sand...
Our consciousness expands,
We're in its thrall
Slaves of our quest to know it all,
Fail constantly,
Yet grow beyond myopic expectations
Surprised by wondrous revelations
Totally unforeseen...

- *P. R. Reindler*

ABSTRACT

A self-gravitating, elastic, spherical thick shell model is used to derive the present state of the lateral variations of density and stress within the lunar interior. The model is allowed to deform under the load of an initial surface topography, and internal density perturbations such that the resulting deformed body gives rise to the observed surface topography and gravity specified by the spherical harmonics of degree and order up to $n = 70$. Effects of model parameters on the density and stress distribution within the Moon are reviewed and optimal choices are made concerning these parameters, with the chosen model displaying density perturbations and stresses strongly concentrated in the upper 300 km. Density perturbations are generally $\pm 200 \text{ kg/m}^3$ within the crustal layers, with peaks of up to $\pm 400 \text{ kg/m}^3$. These perturbations diminish quickly with depth, becoming less than $\pm 80 \text{ kg/m}^3$ by 150 km depth, and generally less than $\pm 5 \text{ kg/m}^3$ by 300 km depth. The major stress differences associated with the mascon basins are found in the upper mantle reaching maximums of 6-8 MPa at about 150 km depth. The introduction of a partially molten layer in the upper mantle increases the stresses in the overlying strata significantly. The stresses for this type of model reach values above 10 MPa beneath the major mascon basins, between the base of the crust to the top of the partially molten layer, at 180 km depth. This model could be interpreted as representing the lunar interior soon after mascon formation, some 3 Gya. A moderate correlation between the soft layer model stress distribution and shallow moonquake epicenters exists, with the majority of the moonquake epicenters being in proximity to regions of 5-10 MPa in the models. This correlation suggests that the lunar lithosphere has been strong enough to support and sustain large remnant stresses over the majority of lunar history.

RÉSUMÉ

Ce mémoire présente un modèle de la Lune sous la forme d'un corps sphérique, élastique et gravitationnel. Le modèle est utilisé pour déterminer l'état actuel des variations de densité et de pression à l'intérieur de la Lune. Le modèle peut se déformer, sous l'effet d'une topographie initiale et d'une variation interne de densité, de telle sorte que le résultat peut expliquer la topographie et gravité observées sous forme d'harmoniques sphériques jusqu'à l'ordre $n = 70$. L'effet des paramètres du modèle sur la densité et la variation de pression à l'intérieur de la Lune est examiné et le choix des valeurs optimales de ces paramètres est justifié. Le modèle choisi montre que les variations de densité et de pression sont fortement concentrées dans les premiers 300 km sous la surface lunaire. Les variations de densité sont généralement de $\pm 200 \text{ kg/m}^3$ à l'intérieur de la croûte avec des maxima pouvant atteindre $\pm 400 \text{ kg/m}^3$. Ces variations diminuent cependant rapidement avec la profondeur. Elles atteignent $\pm 80 \text{ kg/m}^3$ à 150 km de profondeur et sont généralement inférieures à $\pm 5 \text{ kg/m}^3$ à une profondeur de 300 km. Les variations importantes de pression associées aux mascons lunaires se trouvent dans le manteau supérieur et peuvent atteindre 6-8 MPa entre 150 et 300 km de profondeur. L'introduction d'une couche à rigidité réduite dans le manteau supérieur augmente de façon significative les contraintes sur les couches situées immédiatement au-dessus. Les contraintes générées par ce type de modèle peuvent atteindre des valeurs supérieures à 10 MPa sous les mascons lunaires, entre la base de la croûte et la couche à rigidité réduite, à 180 km de profondeur. Ce modèle peut être interprété comme représentant l'intérieur de la Lune peu de temps après la formation des mascons, soit il y a environ trois milliards d'années. Il existe une faible corrélation entre les pressions associées au modèle avec couche à rigidité réduite et l'emplacement des tremblements

de Lune. La majorité des tremblements de Lune sont en effet situés à proximité des régions où les contraintes sont élevées, soit de l'ordre de 5 à 10 MPa. Cette corrélation suggère que la lithosphère lunaire a été suffisamment forte pour supporter des contraintes élevées durant une grande partie de l'histoire géologique de la Lune.

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1. INTRODUCTION

One of the great accomplishments of the Apollo missions was the implementation of a seismic network on the Moon. The seismic stations established a rudimentary seismic network and demonstrated that seismic activity is still plentiful on the Moon, indicating that the lunar interior is presently not in a state of complete isostatic equilibrium, but rather is still supporting and releasing anomalous stresses. Deep moonquakes at depths of 800-1000 km were discovered and found to be the most frequent of lunar seismic events (Goins *et al.* 1981b; Goins *et al.* 1981c; Lammlein *et al.* 1974; Meissner *et al.* 1973). These moonquakes were found to have a cyclical periodicity, and have since been directly related to tidal forces in the Earth-Moon system (Cheng and Toksoz 1978; Lammlein *et al.* 1974; Minshull and Goultry 1988; Toksoz *et al.* 1977). It was the discovery of much shallower moonquakes that first suggested the presence of significantly larger stresses than those of the deep events, within the upper regions of the Moon. These shallow moonquakes, or High Frequency Teleseismic (HFT) events (Nakamura *et al.* 1974) are less frequent, but conversely more energetic, dominating the seismic energy release of the Moon. The stress drops associated with these events vary significantly with estimates in the range of 1-10 MPa, and several events being potentially much larger (>100 MPa) (Binder and Oberst 1985; Goins *et al.* 1981b). In comparison, stress drops of about 0.01 MPa are associated with the deeper, tidally induced events (Goins *et al.* 1981b). The origin of the shallow moonquakes has not been determined, however, they are far too energetic to be associated with the same tidal forces which generate the deeper moonquakes (Lammlein 1977). These events are

concentrated in the upper mantle (Lammlein 1977; Nakamura *et al.* 1974, 1977), and may be evidence of internal alteration or response due to a multitude of large impacts which initially bombarded the lunar surface. The HFT events attest to the fact that there may still be anomalous density anomalies and stresses in the upper mantle from early lunar history due to such events as surface bombardment, magmatic activity, and external volcanic activity in the form of surface loading of mare basalts.

The long term support of density perturbations, and resulting stresses within the lunar interior are not surprising since the interior cooled quickly early in its history (Spohn *et al.* 2001). Surface impacts caused a wide range of alteration and, unlike the Earth's crust, the Moon's crust has never been generally resurfaced, or experienced any global scale tectonic activity. However, large impacts on the Moon have undergone a variety of changes; flood basalts have filled the largest near side basins, smaller craters show signs of isostatic compensation through central crater uplift, doming, and cracking, and yet other craters show evidence of major degradation and alteration due to subsequent impacts (Baldwin 1971; Reindler and Arkani-Hamed 2001; Wieczorek and Phillips 1999). The mare basalt filled near side basins have long been known to have large mass concentrations, or *mascons* (Muller and Sjogren 1968), associated with them. For several mascon basins the excess mass concentration possibly consists of a super-isostatic load, hydrodynamically emplaced after impact. The crust and mantle were fluidized by the initial impact shock wave, after which the mantle over-rebounded beyond a state of full compensation and may have undergone rapid cooling, essentially freezing the mantle plug overshoot in place (Neumann *et al.* 1996; Spudis 1993; Wieczorek and Phillips 1999). Mare basalts were subsequently introduced at the

surface, giving rise to the presently observed mascon signature. In comparison, the even larger South Pole-Aitken basin (hereafter referred to as SPA basin) has been shown to be in a state of near complete isostatic compensation (Arkani-Hamed, 1998; Neumann *et al.* 1996; Zuber *et al.* 1994). In the present context, the term basin will be reserved for the mascon basins and the SPA basin. Other non-filled craters on the Moon have been shown to have a wide range of compensation states, demonstrating that no single global compensation mechanism was present (Reindler and Arkani-Hamed, 2001; Smith *et al.* 1997). These different features suggest a variety of impacts on the Moon and a complex nature of response by the lunar interior to the impacts, implying that many density perturbations, deformations, and stresses have been internally supported since the initial formation of these surface features.

Using the latest topography and gravity data, a new global evaluation of stress within the Moon is now possible to a previously unattainable degree. A new analysis of density perturbations and stresses within the Moon can provide a better understanding of lunar internal processes, of the Moon's internal structure, of the stresses associated with the shallow moonquakes, and of their concentration and distribution within the upper mantle. To begin, a historical review of past work on the problem of surface loading, support mechanisms, and stresses within the lunar lithosphere in terms of thin and thick shell modeling will be presented. Based on this, the application of thick shell theory is justified for the Moon and a preferred model is presented. The effects of many variable parameters within the thick shell model are discussed and applied, demonstrating the strengths and weaknesses of the model. Based on these results, stresses in the upper mantle, possibly associated with moonquakes, are investigated both in terms of global

location and magnitude. Finally, the overall results are discussed along with their implications and the inherent setbacks of this type of modeling.

2. PAST WORK ON LITHOSPHERIC MODELING

Stresses due to the surface loading of a planetary body, and specifically the Moon, have been extensively modeled (e.g. Arkani-Hamed 1973a,b, 1998; Banerdt *et al.* 1982; Banerdt 1986; Comer *et al.* 1985; Conel and Holstrom 1968; Janes and Melosh 1990; Kuckes 1977; Longman 1962, 1963; Melosh 1978; Phillips and Lambeck 1980; Pullan and Lambeck 1981; Schultz and Zuber 1994; Solomon and Head 1979, 1980; Turcotte *et al.* 1981; Willeman and Turcotte 1982). Pioneering work on surface loading of the Moon associated with the newly discovered mascon basins began with the application of initial data provided by the first lunar orbiters (e.g. Conel and Holstrom 1968; O'Keefe 1968; Sjogren 1977). Basic elastic half-space models were presented and described the mascons as high density circular discs, approximating the basaltic load within the basins overlying a crustal layer of lower density. These models were able to describe the anomalous gravity signatures observed from the orbiters. From such models, stresses of 3-5 MPa were found to be associated with the mascon basins, over depths of several hundred kilometers. These results suggested that less than 10 MPa deviatoric stress was required to support the mascon basins (Conel and Holstrom 1968; O'Keefe 1968). Later attempts at calculating internal stresses proved to be more refined due to both the application of new data, as well as the implementation of more advanced and dynamical models. The most common applications of surface loading of the lunar

lithosphere have been the thin shell and thick shell models, used to determine the stresses associated with the major basins as well as the elastic lithospheric thickness required to support them. The elastic lithosphere is the part of the crust and upper mantle which is capable of elastically supporting surface loads, without viscously deforming over geological time.

2.1 Flexure Models (Thin plate / Thin shell)

The most frequent applications for the mascon-loading problem on the Moon have been flexure models of a thin elastic lithosphere. In these models, the lithosphere is approximated as a plate or shell, supporting an applied surface load through flexure, while overlying an inviscid fluid interior (Brotchie 1971; Kraus 1967). These type of models are based on four initial postulates, namely that 1) the plate or shell is thin in relation to the radius of curvature, or horizontal dimensions of the load, 2) the deflections of the plate or shell are small, 3) the transverse normal stress to the surface is negligible, and 4) the normals to the surface are preserved and undergo no change in length during deformation (Kraus 1967). The first assumption is the most essential to thin plate or shell theory, with all other assumptions only being applicable given the validity of this first assumption. To be sufficiently thin, the shell must generally be less than one tenth of the planetary radius (Janes and Melosh 1990; Kraus, 1967). The application of the second assumption results in a model which is linear and elastic (i.e., a Hookean solid), and the third assumption, that the normal stress is zero, is reasonable in accordance with the first assumption; namely, that the load considered is large enough

and not too highly concentrated in comparison to the shell thickness. Finally, the fourth assumption states that all strain components in the direction of the normal to the reference surface vanish, and thus following Hooke's law so do all the associated shearing stress components as well (Kraus 1967).

The majority of models for the problem of lithospheric stresses and strength on the Moon are based on flexure theory. The first models of the lunar lithosphere using this theory were of an elastic thin plate overlying a non-viscous interior (e.g. Kuckes 1977; Solomon and Head 1979, 1980). These thin plate models suggested an elastic lithosphere of 25 to 100 km thick, and corresponding stresses of 40-100 MPa, with the largest stresses associated with the thinnest lithosphere. The elastic plate model of Solomon and Head (1980) gave relatively thin lithospheric thicknesses of 25-75 km soon after major basin formation, 3.6-3.8 b.y. ago, increasing to about 100 km during the period of 3.6-3.0 b.y. ago. More elaborate models, employing a viscoelastic plate overlying a Newtonian fluid substratum, were also used to study surface loading (Pullan and Lambeck 1981). Their model produced stress differences of 20-60 MPa for surface loads approximating the mascons over lithospheric shells of 25 km, 50 km, and 100 km thickness, with the largest stresses once again being associated with the thinnest lithosphere. However, Pullan and Lambeck (1981) showed that any estimation of lithospheric thickness using the thin plate method must be considered as an effective elastic thickness, and that the actual lithospheric thickness would actually be larger. As well, they found that models with variable thickness plates lead to further underestimation of the lithospheric thickness, leading them to believe that the lithospheric thicknesses previously suggested (e.g. Solomon and Head 1980) were

underestimated. Thus, thin plate theory may not be a good representation of deformation or of stress, in the case of the Moon, since the lithosphere may have been too thick for the thin plate model assumptions to be applicable, even during major mascon basin volcanism, around 3.6-3.8 Ga. The results from these thin plate models may have inherent problems with the assumptions, when applied to the Moon, resulting in an underestimation of the lithospheric thickness. This implies that the lunar lithospheric thickness, at the end of mascon formation, may have been larger than 100 km in thickness.

Thin shell models were also developed for the application to planetary surface loading. Support of planetary topography through membrane stresses compared to flexural stresses within planetary lithospheres was also reviewed (Janes and Melosh 1990; Turcotte *et al.* 1981; Willemann and Turcotte 1981). These models assumed thin planetary lithospheres with long wavelength surface loads which could be supported by membrane stresses. The results demonstrated that for smaller wavelength loads the surface loads were supported completely by flexure of the thin shell. Applying the model to the Earth, Mars and Moon, Turcotte *et al.* (1981) obtained lithospheric thicknesses of 1000, 175, and 19 km, respectively, leading the authors to conclude that membrane stresses are relevant in the support of stresses on the surface of the Moon and Mars, but not for the Earth. For the Moon, they showed that membrane stresses within such a thin lithosphere can support the surface topography spherical harmonics up to $n = 17$, thus suggesting that long wavelength features are not supported through flexure, for smaller planetary bodies. Membrane support of long wavelength features increases with decreased planetary radius and increased lithospheric thickness, thus justifying the

lithospheric support mechanisms for the Moon and Mars, but discounting it as a major support on the Earth. These results further question the use of thin shell modeling in the case of the Moon, since, as stated, long wavelength loads are not flexurally supported and short wavelength loads do not satisfy the postulates for thin shell modeling, being too small in comparison to the thickness of the shell. More recent applications of the classical thin shell model to the Moon, using the most recent data, have been undertaken (e.g. Arkani-Hamed, 1998). However, despite the superior data available at present, the general results of both elastic and viscoelastic models still suggest an elastic lithosphere of only 20 to 50 km, for sufficient support of the mascon basins (Arkani-Hamed, 1998).

Alternative models to thin plate and thin shell modeling have been proposed, due to the drawbacks of their initial assumptions and applicability on the Moon. One alternative model involves an axisymmetric gaussian surface load which is applied to a thick elastic plate overlying an inviscid fluid (Janes and Melosh 1990; Melosh 1978; Schultz and Zuber 1994). These models focus on the surface tectonic expressions of lithospheric stresses as a means of justifying the modeled lithosphere. The models suggest that tectonic features are essentially a function of lithospheric thickness; loads on thin lithospheres are supported by buoyancy forces from beneath, and loads on thick lithospheres are supported by the distribution of stress throughout the layer (Melosh 1978; Schultz and Zuber 1994). Thus, the lack of major tectonic features on the Moon strongly supports the idea of a thicker lithosphere of over 50 km, particularly at the time of mare emplacement. Melosh (1978) also noted that in the case of the Moon, his model results were generally in good agreement with those of the thin plate theory. Janes and Melosh (1990) further demonstrated distinct tectonic features for narrow loads on thick

lithospheres, and wide loads on thin lithospheres using another alternative model to thin shell plate theory, and under appropriate conditions their results reproduced similar results to thin shell and thin plate models. However, model results based on the use of tectonic features to constrain the lithospheric thickness should only be taken into consideration with other supporting evidence, due to the lack of evident tectonic features on the lunar surface.

2.2 Thick Shell Models

Thick shell theory has been successfully applied in many various formulations to the Earth (Arkani-Hamed 1970; Kaula 1963; Longman 1962, 1963), and subsequently was applied to the Moon (Arkani-Hamed 1973a,b). Despite the success of the Earth models, relatively few thick shell models have been applied to the Moon. Arkani-Hamed (1973a,b) first applied thick shell theory, investigating the distribution of density and stress within the Moon. In the first paper (Arkani-Hamed 1973a), the density perturbations giving rise to the observed gravity field were fully constrained to within the top crustal layer (of 50 km). He considered an entirely rigid Moon as well as one with a "soft" layer of reduced rigidity in the upper mantle, representing a region of partial melt. These variations were introduced to represent two end-member accretionary models of the Moon, either an entirely solid or partially molten Moon after accretion. The results showed larger stresses associated with the soft layer models; in some cases over 10 MPa beneath several of the mascon basins and below the crust. These stresses were concluded as too high, thus leading the author to suggest that the

entire Moon had been solid since the time of the formation of the mascon basins, supporting the cold accretionary theory for the origin of the Moon. In a second paper (1973b) Arkani-Hamed used the same model and allowed for density variations throughout the entire Moon, as well as applying newer gravity data. The results were relatively consistent with the first: 1) the lunar interior contains lateral density variations of roughly $\pm 250 \text{ kg/m}^3$ within the crust, diminishing quickly with depth, and becoming negligible below 300 km depth. 2) Stresses of 7-10 MPa have been supported in the upper 400 km of the Moon ever since the end of mare volcanism. Neither study (Arkani-Hamed 1973a,b) applied the effect of topography, which was not available at the time. This was addressed in the second paper by approximating the topographic effects by a suggested factor of 1.3 for the density variations. The author thus concluded that the lunar interior has most likely supported up to 10 MPa of stress over the past 3.3 billion years, and has not had a molten layer within its upper mantle since this time.

Several aspects of this model must be noted. The first is that the thick shell model presumes that the Moon is no longer dynamically active; i.e., it is essentially a "dead" Moon model. Therefore, the results can be interpreted as being representative, to a first order approximation, of the Moon soon after the period of major internal and surface alteration, roughly 3 billion years ago. Following the period of peak surface mare volcanism (3.8-3.6 Ga), the Moon has only been moderately volcanically active with significantly reduced volcanism during the Eratosthenian (3.2-1.1. Ga) epoch (Wilhelms 1987), ending by the beginning of the Copernican epoch (1.1 Ga) (Wilhelms 1987), by which point it appears that the Moon became entirely inactive (Manga and Arkani-Hamed 1991; Schultz and Spudis 1983; Spohn *et al.* 2001; Taylor 1982;

Wilhelms 1987). Returning to the thick shell model of Arkani-Hamed, the Moon was essentially approximated as a crust overlying a homogeneous mantle, with average density and average seismic parameters used throughout the interior and no application of a core, due to the lack of any evidence of specific internal structure. While it is known that the lunar interior is very uniform due to its high moment of inertia factor of 0.3932 (Konopliv *et al.* 1998), distinct mantle regions as well as a small core have been proposed (Hood 1986; Hood *et al.* 1999; Khan *et al.* 2000; Konopliv *et al.* 1998; Nakamura 1983). The gravitational potential model available at the time was also very unrefined, significantly overestimating the density perturbations and stresses in the model (Kermani and Arkani-Hamed 1974; Phillips and Lambeck 1980). The lack of topographic effects further resulted in a somewhat incomplete representation of the stresses within the Moon. Despite these drawbacks, the results demonstrated the applicability of such models for first order approximations of densities and stresses within the lunar interior, and their depths. However, the overall conclusion of Arkani-Hamed (1973a), that the Moon has been completely solid since 3.3 Ga, may not be warranted. This conclusion was arrived at under the assumption that stresses of over 10 MPa within the lunar lithosphere, associated with the partially molten mantle models, were too large to be supported over geological time. The seismic energy release associated with the shallow moonquakes (Binder and Oberst 1985; Goins *et al.* 1981b; Nakamura *et al.* 1979) has since suggested that large remnant stresses within the upper 300 km of the Moon may, in fact, still be present.

2.3 Summary of the Previous Models

Thus, several comparisons between the thin and thick shell models can be made. A major difference between the two models is that since the thin shell model cannot internally deform or contain perturbations, it solely responds by flexure. The fundamental assumption that there is no deformation within the shell, and that only the lithospheric shell can support the load, is in direct contrast with thick shell modeling, which allows for perturbations and deformations throughout all parts of the model. On the Moon, the thin shell model is generally applied to the surface loading of a basaltic load on a thin elastic layer, before any significant growth of the lithosphere. In this manner, thin shell modeling is applied to approximate stresses and displacements due to the support of the largest mascon basins, and their surface basaltic loads, at the time of, and soon after emplacement. This is different from thick shell theory, which models the cold, dead Moon, long after the period of lunar volcanism and activity ceased. Therefore, thin and thick shell modeling can be regarded as two end members of lithospheric support models, when applied to the Moon.

The evolution and growth of the Moon's lithosphere was likely an accelerated one when compared to the other terrestrial planets, due to the Moon's smaller size. Lunar volcanism likely began very early in its history, as seen by older regions displaying pre-Imbrian volcanism (Wilhelms 1987; Yingst and Head 1997; Yingst and Head 1999), distinct from the later volcanic events which filled the major basins. Cryptomare, mare material that has been later covered by ejecta but is still at the surface, is believed to be present in many craters (Head 1993; Head and Wilson 1992). It is

presumed that the majority of lunar volcanism, particularly flows associated with the major mascon basins, primarily occurred during the Lower and Upper Imbrian epochs, spanning some 650 million years, from 3.85-3.2 billion years ago, and that since this time, volcanic activity has been greatly reduced, and the Moon has been completely inactive over the past billion years (Manga and Arkani-Hamed 1991; Schultz and Spudis 1983; Spohn *et al.* 2001; Taylor 1982; Wilhelms 1987). A recent thermal evolution model (Spohn *et al.* 2001) produced an early thermal lithosphere that grew quickly, and may have been as thick as 250 km after only a billion years of evolution, with an elastic component of up to 150 km thick. The thermal lithosphere represents all of the crust and mantle that is too cold and stiff to engage in mantle convection. Other results further support a thick elastic lithosphere, being up to 200-300 km by the end of mascon formation some 3 Ga, along with associated stresses of roughly 10 MPa (Phillips and Lambeck 1980). Stresses as high as 100 MPa are generated by flexure models (Kuckes 1977) associated with very thin lithospheres (25 km) and are unnecessarily large, suggesting that the use of a thicker lithosphere is a more appropriate approximation.

Therefore, models using thin elastic plates and shells for support of surface loads on the Moon generally suggest elastic layers which are thinner than the actual lunar elastic lithosphere. Thin shell results suggest elastic layers on the order of 20-100 km, for the period of mascon formation and mare emplacement (3.2-3.8 Ga). Due to the smaller size of the Moon and the effects of relatively large surface features, thin shell modeling may not be appropriate for a current stress analysis due to the inherent model assumptions and as a result may be greatly underestimating the lithospheric thickness (Kraus 1967; Pullan and Lambeck 1981; Zhong and Zuber 2000). Deformation, stress,

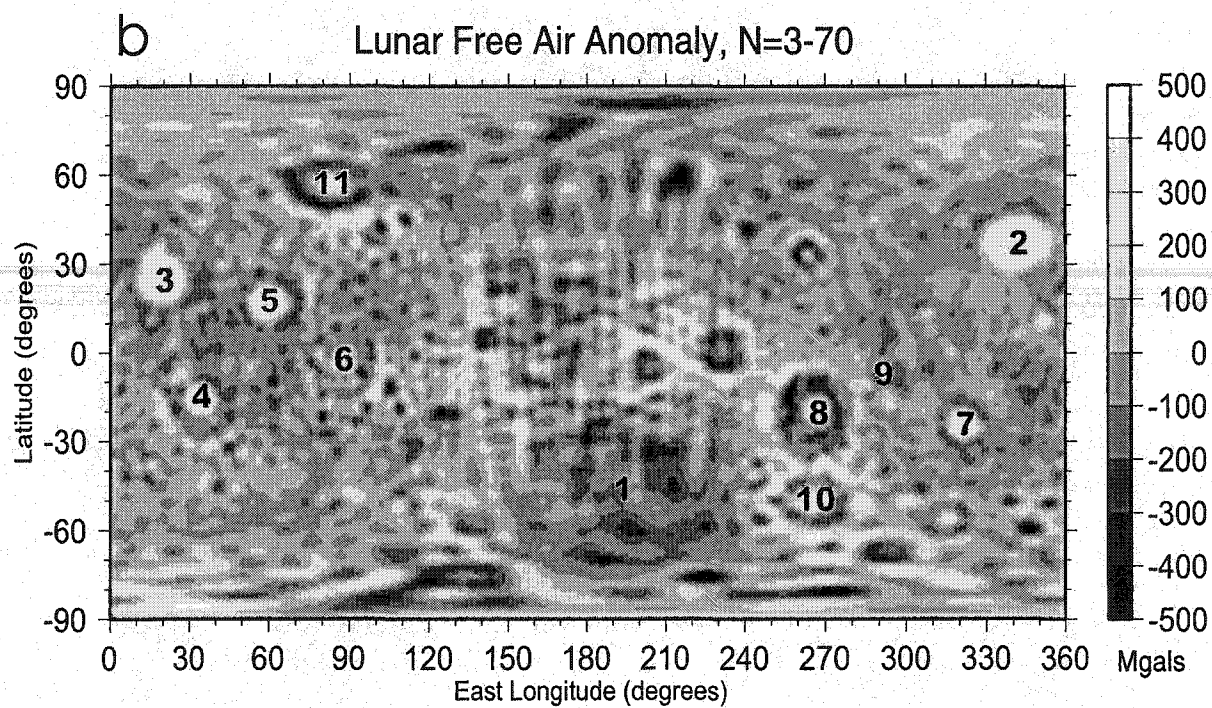
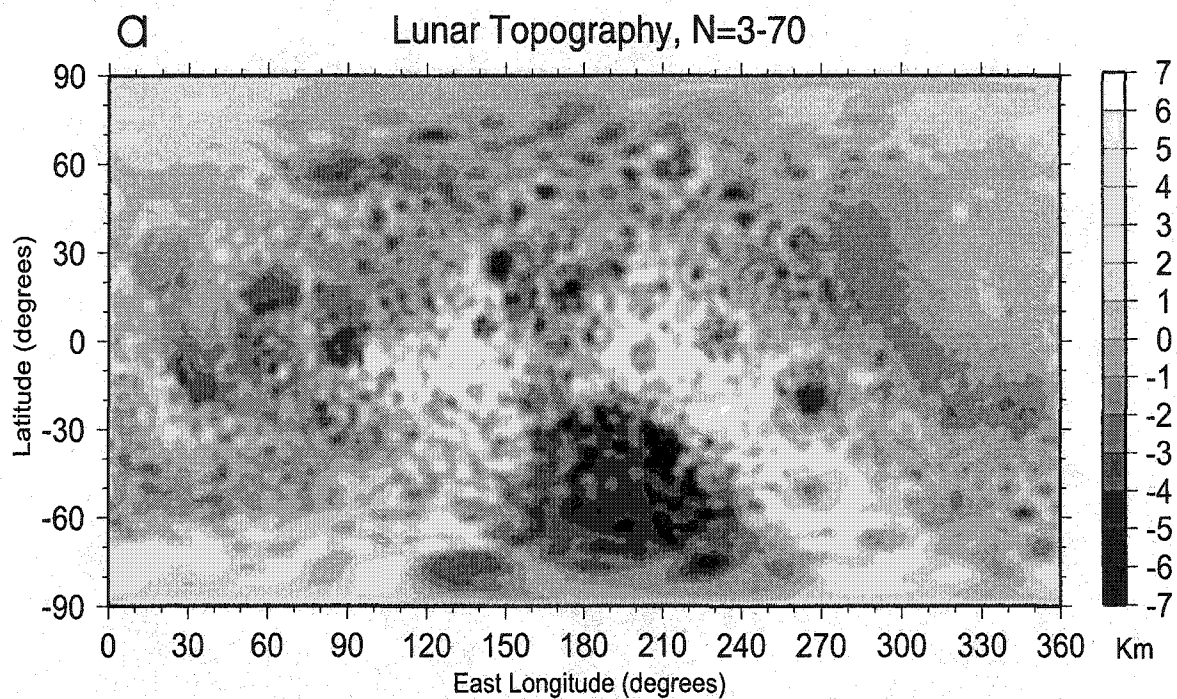
or density variations deep within the interior of the Moon are also not considered in elastic shell theory. Thus, using a thick shell model, I determine the present density and stress distribution for a solid Moon model. I also consider the introduction of a soft layer below roughly 200 km depth, as a first order approximation of the distribution of density and stress soon after the completion of extensive volcanic activity, at 3 Ga. Hence, a re-evaluation of the internal density distribution and stress state of the Moon is undertaken in this thesis using thick shell theory, adopting the latest seismic, gravity, and topography data. Based on the above historical data, the effect of lithospheric thickness will also be reviewed, and put into context with the viability of the different models.

3. RECENT DATA

The acquisition of new topography and gravity data from the Clementine (Lemoine *et al.* 1997; Nozette *et al.* 1994; Smith *et al.* 1997; Zuber *et al.* 1994) and Lunar Prospector (Konopliv *et al.* 1998, 2001) missions in the 1990's has made a new assessment of lunar models both feasible and more accurate than had been previously possible. As well, modeling of the lunar interior derived from the complete seismic data set has provided newer models of internal structure (Khan *et al.* 2000; Khan and Mosegaard 2001; Nakamura 1983). Previously, the coverage of the gravitational potential and topography data was sparse, limited to the tracks of the Apollo mission orbiters on the lunar near side. Apollo era topography measurements were concentrated in an equatorial band of roughly ± 26 degrees latitude, and were characterized by an absolute radial accuracy of only 400 meters (Kaula *et al.* 1974). The Clementine

mission mapped the topography of the Moon to a previously unattainable precision, with a radial error of less than 100-130 meters, with almost uniform coverage over ± 80 degrees latitude and an inter-track spacing of roughly 60 km (Smith *et al.* 1997). The corresponding topographic spherical harmonic representation has been expanded up to and including $n = 90$ (Neumann, personal communication). The gravitational potential of the Moon has also been drastically refined by line-of-sight tracking of both the Clementine and Lunar Prospector missions and is a vast improvement over the earlier potential models (see Konopliv *et al.* 2001). However, although the gravity model for the near side and limb regions has been refined and improved, there are still no direct gravitational measurements for the lunar far side, leaving the far side region with the lowest resolution in the global gravity field model. This zone of lower gravity resolution is centered on the far side, with a region of roughly 60 degrees in radius (Konopliv *et al.* 2001). Given these constraints, the most recent lunar gravitational potential spherical harmonic model has been expanded up to and including degree $n = 150$ (Konopliv, personal communication). Although the gravity data are not globally uniform, these are the most accurate and extensive data for any planetary body other than Earth. The spherical harmonic model of the lunar topography is described by real data up to $n = 66$ (Neumann, personal communication), beyond which the model is extrapolated. This, along with the reduced gravity field for the lunar far side, has lead us to adopt the maximum degree of $n = 70$ for the topography and gravity spherical harmonics. The spherical harmonic expansions of the topography and free air anomaly maps used in this study, for $n = 3-70$, are shown in Fig.1. The expansions exclude the coefficients up to $n = 3$, because the gravitational potential is defined with respect to the center of mass and

FIG. 1. Global maps of the spherical harmonic expansions of a) the topography, and b) the free air anomaly. The expansions have been made for degrees up to and including, $n = 70$. The free air anomaly map over the central far side shows the effects of lower resolution in this region. The South Pole-Aitken basin (1) is the largest crater on the Moon, but has a subdued free air anomaly due to it being almost completely compensated. The most predominant free air anomaly highs are associated with the major mascon basins: Imbrium (2), Serenitatis (3), Nectaris (4), Crisium (5), Smythii (6), Humorum (7), Orientale (8), Grimaldi (9). As well, other craters show mascon-like negative rings of low free air anomaly, with central highs, such as Mendel-Rydberg (10) and Humboldtianum (11).



does not have the $n = 1$ coefficients, while the lunar rotation and possible motion of its rotation axis may contribute to the $n = 2$ coefficients (Lambeck and Pullan 1980; Melosh 1975).

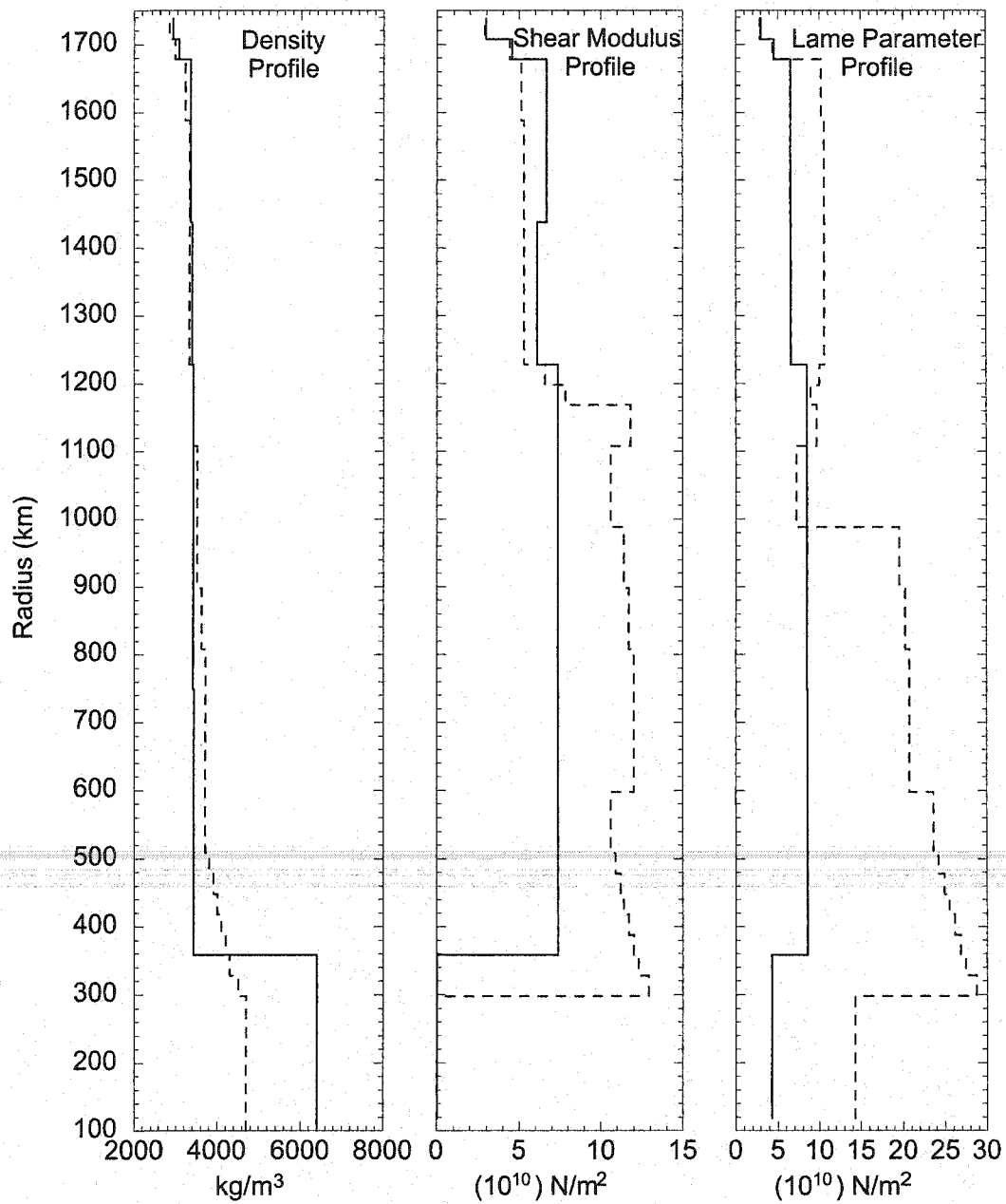
A recent reevaluation of the complete seismic data from the Apollo seismic network has also provided a new seismic velocity model for the lunar interior (Khan *et al.* 2000; Khan and Mosegaard 2001). Although it is based on the same data set as the earlier Nakamura seismic model (1983) (hereafter referred to as the Nakamura model), it provides a different seismic velocity profile for the Moon. Models previous to the Nakamura model did not incorporate the full set of seismic data from the Moon (e.g. Goins *et al.* 1981c; Nakamura *et al.* 1976; Toksoz *et al.* 1974). Only the recent Khan *et al.* (2000) model, and the Nakamura (1983) model make use of the entire seismic data set. Khan *et al.* (2000) proposed a new seismic profile based on a Monte Carlo probability model, whereas Khan and Mosegaard (2001) proposed a shear wave velocity profile for the deeper lunar interior based on a Monte Carlo inversion of lunar spheroidal free oscillations. This study also provided a rough density profile for the lunar interior. Combining the seismic models of Khan *et al.* (2000) and Khan and Mosegaard (2001) generates a new, complete seismic model (hereafter referred to as the Khan model). The discrepancies between the Nakamura and Khan models are quite apparent. A thinner crust of 45 km is proposed in the Khan model, and is on the order of 10-15 km thinner than has previously been suggested (Goins *et al.* 1981c; Nakamura 1983; Neumann *et al.* 1996). The Khan model also has noticeably higher velocities for both compressional and shear waves in the deeper lunar interior, below 500 km depth, resulting in a significant variation of the rigidity and Lamé parameter, specifically in the deeper

interior (Fig. 2). The effects of this different model on the density perturbations and stresses within the Moon will be discussed in section 5.6. I used the Nakamura model for the initial, nominal model runs, although a later comparison of the two seismic models is made. Therefore, we review both seismic models that make use of the complete seismic data sets available from the lunar seismic network.

4. APPLYING THE THICK SHELL MODEL

The arguments, in section 2, for the use of thick shell modeling as opposed to thin shell modeling in the particular case of the Moon lead to the following conclusions. Thin shell models approximate the lunar lithosphere as a very thin shell overlying an inviscid interior. This layer is assumed to be homogeneous and internally non-deformable. The layer can support surface loading by membrane stresses if the load is large enough, or by flexure for smaller loads. In cases where the lithospheric shell thickness is not thin, i.e. greater than approximately one tenth of the planetary radius or one tenth of the horizontal load dimensions, the thin shell model cannot be accurately applied (Banerdt *et al.* 1982; Brochie 1971; Kraus 1967; Zhong and Zuber 2000). The normal stresses, assumed to be zero for thin shell models, may also be very important since deformations and stresses associated with isostatic compensation within the lunar interior must also be considered and thus modeling flexure of the lithosphere alone is inadequate. Thin shell models do not allow for stresses extending into the deeper regions of the Moon, due to the assumption of a non-viscous fluid below the lithosphere. In the present study the entire range of surface features is investigated and the full

FIG. 2. Seismic model comparison between the nominal Nakamura (solid) model and the Khan model (dashed) for the a) density b) shear modulus and c) Lamé parameter. The two models are similar at shallow depths, but become quite different below 500 km depth. This is due to the poorly constrained inversion of seismic data at depth for the Moon. The core of the Khan model has a very small density contrast at the core/mantle boundary, leading to more displacements and stresses at depth. The larger rigidity of the Khan model over the Nakamura model in the deeper interior (roughly double) is capable of supporting more stress for long wavelength features overlying it.



spectrum of spherical harmonics, $n = 3-70$, is used within the formulation of the thick shell model to determine the density perturbations and stresses throughout the entire Moon. Therefore, the present day distribution of density and stress can be determined as well as a first order approximation of the density variations and stresses within the Moon at the end of the period of major lunar volcanism, at 3 Ga, assuming that the lunar topography and gravity have not significantly changed since.

4.1 The Thick Shell Model

Similar to models which were first successfully applied to the Earth (e.g. Arkani-Hamed 1970; Kaula 1963; Longman 1962, 1963) and later to the Moon and other planets (e.g. Arkani-Hamed 1973a,b; Banerdt *et al.* 1982) we use a thick shell model to derive the deformations, density anomalies, and stresses within a spherically symmetric Moon. The thick shell model makes several initial assumptions: 1) the Moon is a spherically symmetric and self-gravitating body, 2) no rotational motion is considered, 3) the model contains a linearly elastic, isotropic interior, 4) the elastic moduli only vary radially, 5) the stresses are regarded as purely hydrostatic, with no thermal stresses considered, and 6) the model has density perturbations introduced into it and is allowed to deform by means of dilatation and displacement to a state of equilibrium. The density perturbations are determined using the gravity and topography that are known on the (presently observed) deformed surface. The resultant model is regarded as a dead Moon model representing the internal distribution of density and stress within the Moon at a time when all surface and internal activity has ceased. In this manner, the model can be

applied for the present state of the Moon, as well as approximating the Moon soon after the cessation of major volcanic activity since which time it is assumed that no significant alteration of the surface representation of gravity or topography has occurred.

The equilibrium equations and their incorporation into the matrix equation governing the deformations within the Moon are described in detail by Arkani-Hamed (1973b) and will not be reviewed here so as not to burden the text. The derived equation is a first order ordinary differential matrix equation dependent on the spherical harmonic degree, n , and order, m , the radius, r , as well as other physical parameters such as the Lamé parameter, λ , the shear modulus, μ , and the density, ρ , and is represented in concise form by the introduction of dependent variables comprising the $\bar{\mathbf{Y}}$ vector components (Alterman 1959) as:

$$\frac{d}{dr} \bar{\mathbf{Y}}_{nm}(r) = \bar{\mathbf{M}}_n(r) \cdot \bar{\mathbf{Y}}_{nm}(r) + \bar{\mathbf{D}}_{nm}(r) \quad (1)$$

where the $\bar{\mathbf{Y}}$ vector is composed of the physically relevant variables y_1 (radial displacement), y_2 (radial stress), y_3 (tangential displacement), y_4 (tangential stress), y_5 (gravitational perturbation), and y_6 (gradient of potential minus contribution of radial displacement). The $\bar{\mathbf{M}}$ matrix is a 6 x 6 matrix dependant on the physical properties of the initially undeformed lunar model, and the $\bar{\mathbf{D}}$ vector contains the density perturbations within the model, which are to be determined. Equation (1) is solved numerically with the application of the relevant boundary and model conditions, as described below, giving rise to the solution vector, $\bar{\mathbf{Y}}$, and the density perturbations contained in $\bar{\mathbf{D}}$.

4.2 Boundary and Model Conditions

Equation (1) is subject to several boundary conditions, but the boundary conditions alone are insufficient to fully solve for the density perturbations within the model. Without any further constraints, the non-unique solution of the density perturbations within the lunar interior giving rise to the surface gravity field leaves the problem underdetermined. Therefore, two other model conditions are introduced. Imposing these two extra constraints allows the density perturbations to be uniquely determined, and the distribution of stress within the model to be solved. The four relevant boundary conditions are presented, followed by the two added model constraints.

The boundary conditions which are relevant to the current problem are: 1) The regularity of the solution at the center of the Moon, 2) the stress free condition at the surface of the deformed Moon, 3) the Dirichlet condition for the gravitational potential at the surface, and 4) the Neumann condition for the gravitational potential at the surface. The first condition applies to the matrix $\overline{\overline{\mathbf{M}}}$, which has dependencies of $1/r$ and $1/r^2$, and thus is undefined near the center of the Moon. To avoid this problem the solution vector, $\overline{\mathbf{Y}}$, is expanded by a Taylor series at a small distance, r_o , from the center of the Moon (called the central sphere), as is described by Arkani-Hamed (1973a). The second condition makes use of the topography data and leads to the following conditions for y_2 and y_4 in the $\overline{\mathbf{Y}}$ vector, on the initially undeformed surface ($r = a$):

$$y_2 = -\rho_c g_0 h_{nm} \quad (2)$$

$$y_4 = 0 \quad (3)$$

where ρ_c is the average crustal density, g_0 is the observed surface gravity, and h_{nm} is the spherical harmonic coefficient of topography for degree, n , and order, m . Equation (2) states that the radial stress at the surface ($r=a$) is equal to the negative stress associated with the observed topography, so that after deformation the total resultant stress at the deformed surface will be zero, while equation (3) states that the tangential stress on the surface, before and after deformation, is negligible, since it is assumed that the major stress component is the radial one. The third boundary condition applies to y_5 , and requires that the variations in the gravitational potential must lead to a model which satisfies the observed gravitational potential data,

$$y_5 = \delta \Phi_n \quad (4)$$

where $\delta \Phi_n$ are the perturbations in the observed gravitational potential for different harmonic degrees, n . The last boundary condition is essentially the continuity of the gravitational potential derivative, the gravity. In terms of y_5 and y_6 , this condition can be written as

$$y_6 = -\frac{(n+1)}{a} y_5 \quad (5)$$

where, as stated before, y_6 is the gradient of the potential minus the contribution of radial displacement.

However, as mentioned above, these conditions alone are not sufficient to solve the equations and obtain the density perturbations and stresses; as is, this model is underdetermined. Therefore, two other constraints are introduced into the model; the minimization of the shear-strain energy and the minimization of the mass perturbations within the entire volume of the Moon, as has previously been done (Arkani-Hamed

1970, 1973b; Kaula 1963). The first constraint arises from the assumption that the Moon as a whole evolves toward a state of minimum stress; the resulting density perturbations and stresses represent a lower bound on possible model solutions. In terms of vectors and matrices, the shear-strain equation can be written as (Arkani-Hamed 1970):

$$E_{sh} = 4\pi \int_0^a \mu r^2 dr \bar{\mathbf{Y}}'^T \cdot \bar{\mathbf{P}} \cdot \bar{\mathbf{Y}}' \quad (6)$$

where μ is the rigidity, a is the surface radius. The vector $\bar{\mathbf{Y}}'$ is similar to $\bar{\mathbf{Y}}$ but without the y_5 and y_6 terms. The matrix $\bar{\mathbf{P}}$ is a 4 x 4 matrix, similar to $\bar{\mathbf{M}}$ and is also dependant on the physical properties of the lunar model, as is described in detail by Arkani-Hamed (1970, 1973b). However, the density perturbations in the resulting model have an oscillatory nature, oscillating back and forth from positive to negative values or vice versa along the radius. To reduce the oscillatory nature of the density perturbations, the amplitude of the mass perturbations, γ_i , defined as (Arkani-Hamed 1973b)

$$A_{mass} = \sum_i |\gamma_i|^2 = \sum_i |h_i \cdot \Delta\rho_{i,nm}|^2 \quad (7)$$

is also minimized, where h_i is the thickness of the i^{th} layer and $\Delta\rho_{i,nm}$ are the density perturbation coefficients within that layer. However, as a result of applying this constraint, the overall shear-strain energy of the model increases; when the shear-strain energy is fully minimized the mass oscillations are larger, and when the mass oscillations are fully minimized the total shear-strain energy is greatly increased. Therefore, a critical balance between the two constraints is required. To obtain a proper balance between the constraints a weighted proportion of the mass oscillation

minimization is applied. The weighting factor, W_m , is applied to reduce the effect of the mass oscillation minimization in relation to the more important shear-strain minimization. The effect of varying this weighting factor is reviewed in section 5.2, and an appropriate balance between the two constraints is presented.

4.3 The Internal Model of the Moon

For a numerical solution, the interior of the initially undeformed Moon is divided into a finite number of spherically symmetric layers within each of which the seismic parameters and initial densities are considered constant. The nominal internal Moon model contains 54 equal-thickness layers, each being 30 km thick, extending from the central sphere radius, $r_0 = 118$ km, to the surface, $a = 1738$ km. The justification of the layering choices will come with the discussion of layering effects on the model, in section 5.3. The seismic profile of Nakamura (1983) is used for determining the Lamé constant and rigidity associated with each layer. The density profile for the Moon is taken from the thermodynamic models of Kuskov (1997) and Kuskov and Kronrod (1998) which are also based on Nakamura's seismic model. The specific characteristics of the nominal internal Moon model are shown in table 1. The model contains a core of 358 km, in close agreement with the generally proposed radius of a lunar core (Hood 1986; Hood *et al.* 1999; Konopliv *et al.* 1998; Kuskov and Kronrod 1998). The core is modeled as a pseudo-liquid by reducing the rigidity of the core by a factor of 10^3 and the Lamé constant by 50% compared to the overlying mantle (see table 1). The average crustal thickness is taken to be 60 km (Goins *et al.* 1981c; Nakamura 1983; Neumann *et*

TABLE 1. The nominal model used for representing the internal structure of the undeformed Moon. This model contains a core of 358 km in radius, a 390 km lower mantle, a 480 km middle mantle, and a 450 km upper mantle. The crust is comprised of an upper less dense layer, and a lower more dense layer, and is 60 km in thickness.

Table 1 - Internal Moon Model

Region	Radius (km)	Density (kg/m ³)	Lamé (10 ¹⁰ kg/m s ²)	Shear (10 ¹⁰ kg/m s ²)	Gravity (m/s ²)
	118				0.2111
Core		6400	4.27	7.39 x 10 ⁻³	
	358				0.6404
Lower Mantle		3420	8.54	7.39	
	748				0.7833
Middle Mantle		3400	8.49	7.35	
	1228				1.1939
Upper Mantle		3370	6.58	6.09	
	1438				1.3816
Upper Mantle		3320	6.50	6.69	
	1678				1.5918
Lower Crust		3050	4.52	4.54	
	1708				1.6117
Upper Crust		2900	2.99	2.97	
	1738				1.6283

al. 1996; Wieczorek and Phillips 1998), and is divided into two 30 km layers, representing a mid-crustal discontinuity in the range of 20 to 30 km (Simmons *et al.* 1973; Wieczorek and Phillips 1997, 1998). The major internal discontinuities within the model are within 10 km of the proposed seismic discontinuities of the Nakamura (1983) model. Although Nakamura (1983) stated that the discontinuities may, in fact, only represent gradual changes instead of sharp discontinuities, for simplicity the discontinuities are left as proposed by the initial model. The gravity is calculated at the boundary between each layer and is linearly extrapolated throughout the layer during computation. The model layers need to be thin enough to validate the assumption of constant seismic parameters and linear gravity within each layer, yet thick enough to avoid a large concentration of density perturbations among the uppermost layers (see section 5.3).

4.4 The Numerical Solution

A full solution of the matrix equation, with the application of all of the relevant boundary and model constraints is given by Arkani-Hamed (1973b). Solving the ordinary differential matrix equation (1) is done by applying the method of matricants (Gantmacher 1959), resulting in the equation,

$$\bar{\mathbf{Y}}_i = \bar{\bar{\Omega}}(r_0; r_i) \cdot \bar{\mathbf{Y}}(r_0) + \bar{\bar{\Gamma}}(r_j; r_i) \cdot \bar{\mathbf{X}}(r_j) \quad (8)$$

where $\bar{\bar{\Omega}}(r_0; r_i)$ and $\bar{\bar{\Gamma}}(r_j; r_i)$ are the matricant propagators for the boundary effects and the internal source effects of the j^{th} layer on layer, i , of the internal Moon model, and are described in detail in the appendix. The vector $\bar{\mathbf{X}}(r_j)$ contains the density perturbations

for the j^{th} layer, and the vector $\bar{\mathbf{Y}}(r_0)$ is the expansion of the solution vector over the central sphere, at radius r_0 (Arkani-Hamed, 1973a). The Moon model contains layers with constant internal parameters of the initial density, Lamé parameter, and shear modulus. The radius and gravity at the base and top of each layer are inputted, and are then interpolated throughout the layer for calculation purposes. The spherical harmonic coefficients of the density perturbations throughout each layer are determined by solving the above matricant equation, by the method of generalized least squares with the introduction of Lagrangian multipliers (see Arkani-Hamed, 1973b). From the solution of the density perturbations, the solution for all of the $\bar{\mathbf{Y}}$ vector components are determined.

The method for calculating maximum stress differences follows that defined by Arkani-Hamed (1973a,b). Briefly, the maximum stress difference (maximum shear stress) associated with a point is given by (Sokolnikoff 1956):

$$S = |\tau_1 - \tau_2|/2 \quad (9)$$

where τ_1 and τ_2 are the maximum and minimum principal stresses, which are roots of the cubic equation:

$$\tau^3 - \theta_1 \tau^2 + \theta_2 \tau - \theta_3 = 0 \quad (10)$$

where the coefficients θ_1 , θ_2 , and θ_3 are stress invariants, defined by Arkani-Hamed (1973a,b). These stress invariants are related to the $\bar{\mathbf{Y}}$ vector through

$$\bar{\mathbf{T}} = \bar{\mathbf{R}} \cdot \bar{\mathbf{Y}}' \quad (11)$$

where $\bar{\mathbf{T}}$ is the vector of elements comprising the stress invariants, and $\bar{\mathbf{R}}$ is dependent on the physical properties of the lunar model (see Arkani-Hamed 1973a). The stresses

are always positive, since they are calculated as the absolute difference between the maximum and minimum principal stresses, and are determined at the base of each model layer, with the top of the surface layer having no stress due to the non-stress boundary condition.

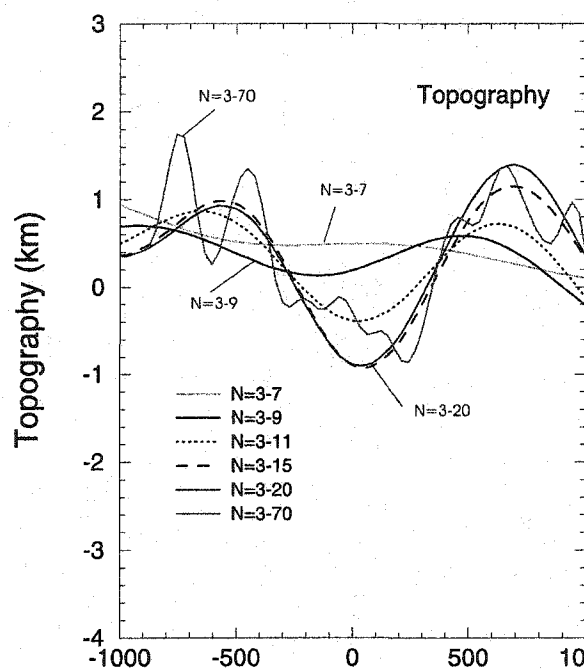
5. MODEL RESULTS

5.1 Nominal Model Results

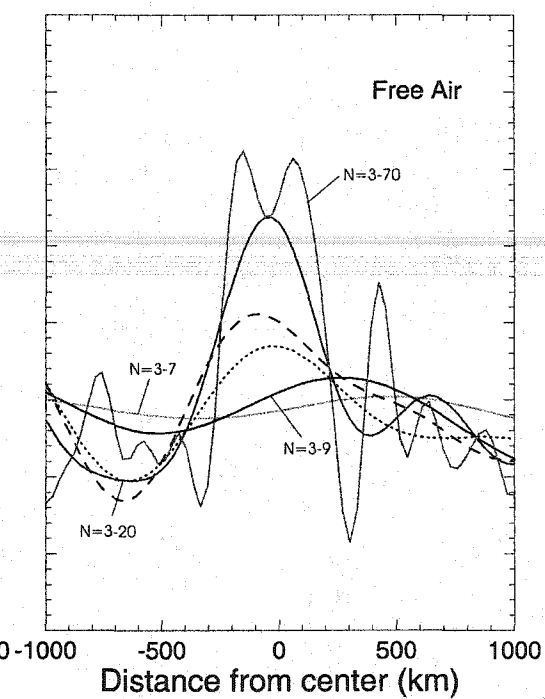
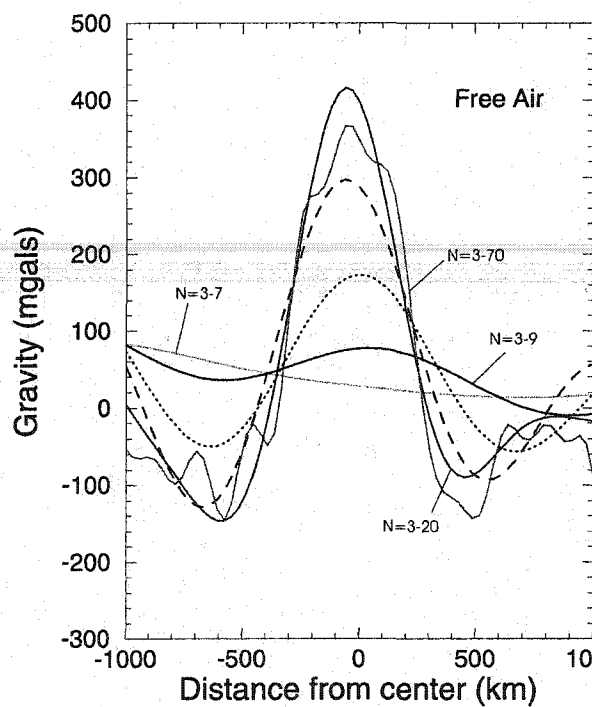
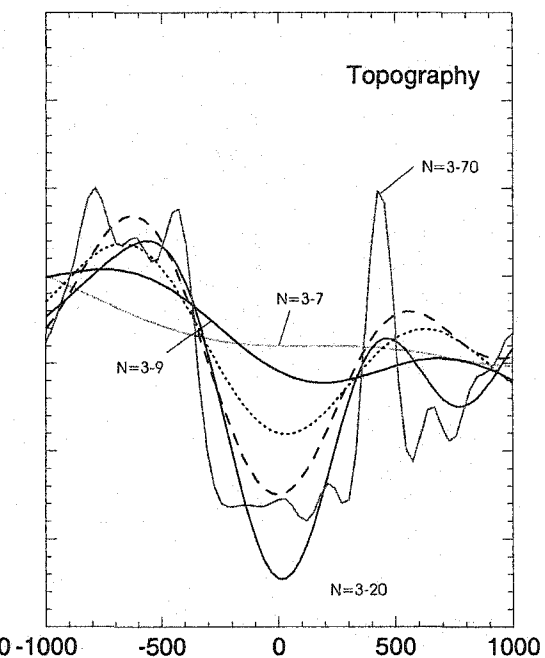
First, the results associated with the nominal model are presented, followed by results for a wide range of varied parameters including the effect of the weighting factor, W_m , the effect of variable layering, the effect of the core characteristics, the effect of a soft layer within the mantle, and the effect of a different seismic model. In all models, the penetration of different density perturbation coefficients is limited to distinct depths and regions within the Moon. This is done to avoid fictitious density perturbations at depth due to shallower features, and to isolate the effects of particular lunar features. For example, the harmonics dominated by the mascon basins, $n = 7-20$ (see Fig. 3), will not penetrate to the core mantle boundary, since the source region for the mascons does not extend to those depths. The deepest source region, and thus the deepest zone of effect for density perturbations associated with the mascon basins, is a depth of roughly 500 km (Alley and Parmentier 1998; Heiken *et al.* 1991; Manga and Arkani-Hamed 1991; Taylor 1982; Wicczorek and Phillips 2000). Therefore, the harmonics are grouped for three different ranges; 1) the low harmonics of $n = 3-7$, extending through

FIG. 3. Profile of topography and free air anomaly for two representative mascon basins (Serenitatis and Crisium). The mascon basins dominate the spherical harmonics starting at $n = 3-9$ to $n = 3-20$, after which the major characteristics of the basins have generally been established. Green lines denote the representation for $n = 3-7$, for which little of the mascon characteristics are present. Blue lines denote the representation for $n = 3-20$, at which point the majority of the mascon characteristics are present. Red lines denote the use of the full spectrum of coefficients ($n = 3-70$).

Serenitatis



Crisium



the entire mantle down to the core/mantle boundary, 2) the major basin harmonics, of $n = 8-20$, extending to a maximum depth of 510 km, and 3) the high harmonics, of $n = 21-70$, extending to a maximum depth of 210 km. For the nominal model, a specific weighting factor for each harmonic range has been designated, and is listed in table 2. The justification of these choices will come with further discussion of the effect of weighting factor in section 5.3.

The distribution of density perturbations generated within the nominal model is presented for the near surface crustal layers (Fig. 4), and in the upper mantle at depths of 150 km and 300 km (Fig.5). Figures 4 and 5 demonstrate how the density perturbations responsible for the observed gravity field are strongly concentrated in the upper 150 km. Note the different scales used in these figures. The largest density variations are located within the upper crustal layer (0-30 km depth), with the majority of the variations being less than $\pm 200 \text{ kg/m}^3$. Several specific areas have density perturbations of up to $\pm 400 \text{ kg/m}^3$, namely the mascon basins, the SPA basin, and the far side highlands. The variations in the lower crust (30-60 km depth) are more subdued, with almost all of them being less than $\pm 100 \text{ kg/m}^3$, with the same high density regions showing peaks of generally $\pm 200 \text{ kg/m}^3$. Several of the mascon basins were probably in a state of over compensation before the time of mare fill, and still have not relaxed the overcompensating mass (Neumann *et al.* 1996; Spudis 1993; Wieczorek and Phillips 1999). Similarly, the SPA basin and the central far side highlands have been shown to have undergone substantial compensation, demonstrated by prominent mantle upwelling beneath the SPA basin and a crustal root descending into the mantle beneath the highlands (Arkani-Hamed 1998; Neumann *et al.* 1996; Zuber *et al.* 1994). Thus, the

TABLE 2. The choice of weighting factor for the nominal model over the different representative spherical harmonic groups. The large weighting factor over the higher harmonics allows for more reduced oscillations of mass perturbations, while keeping the total shear-strain energy at a near minimum.

Table 2 - Optimal Weighting Factors

Spherical Harmonics	Weighting Factor, W_m
$n = 3 - 7$	1.0×10^{-3}
$n = 8 - 20$	5.0×10^{-2}
$n = 21 - 70$	1.0×10^{-2}

FIG. 4. The density perturbations generated by the nominal model within the a) upper and b) lower crust. The density perturbations range from generally $\pm 200 \text{ kg/m}^3$ for the majority of the lunar upper crust, with the SPA basin (1), the far side highlands (2), and several of the mascon craters (3) showing perturbations as high as $\pm 400 \text{ kg/m}^3$. Several mascon-like craters show similar high density plugs, such as Mendel-Rydberg (4), Humboldtianum (5), Moscoviense (6), and Freundlich-Sharonov (7). These large density variations are due to the compensating mantle plug, or crustal root beneath these larger features. The density perturbations are reduced in the lower crust, with strong peaks still associated with the largest features.

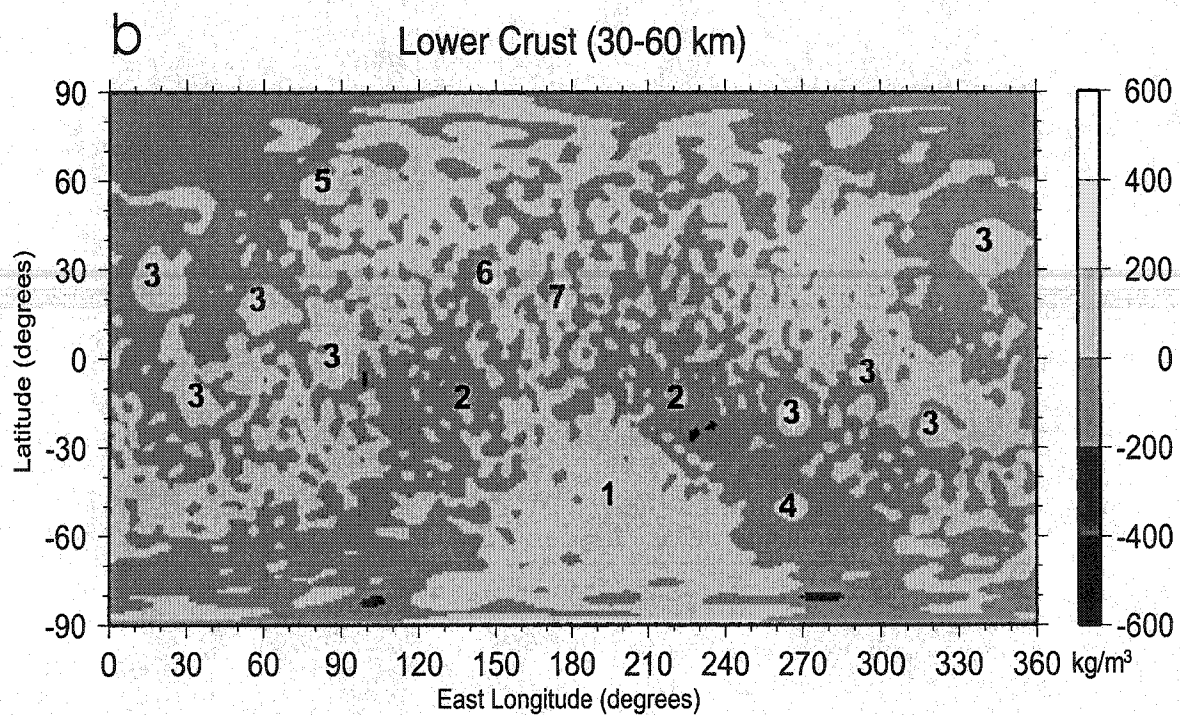
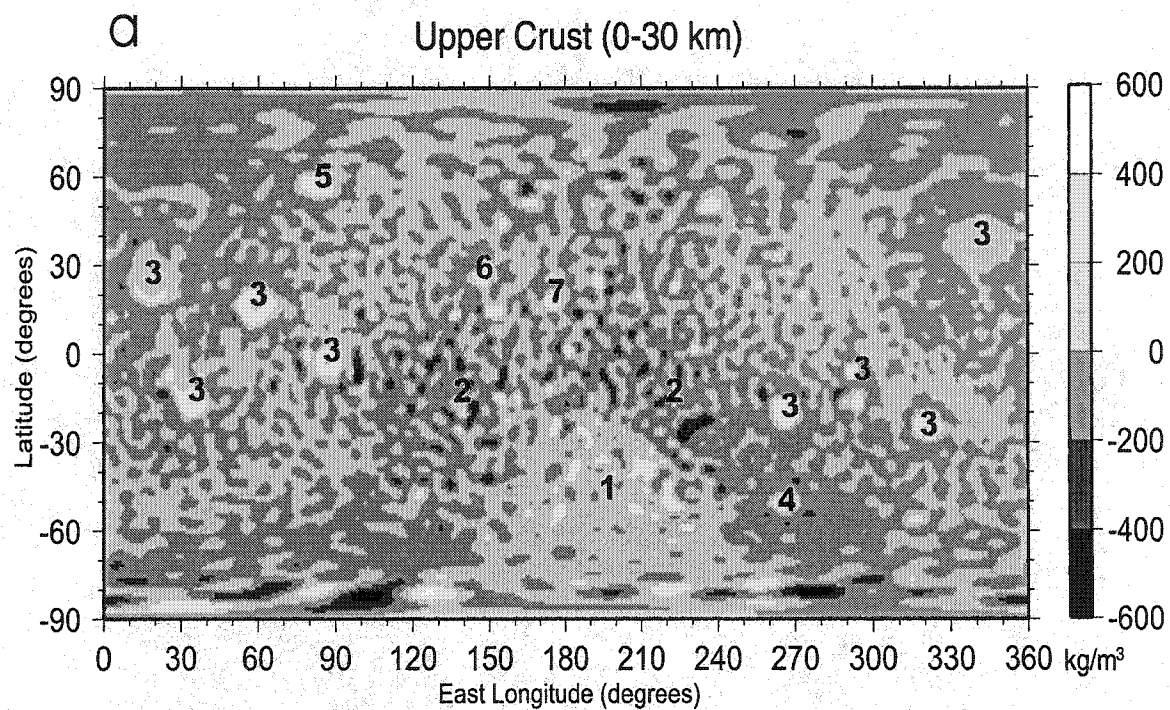
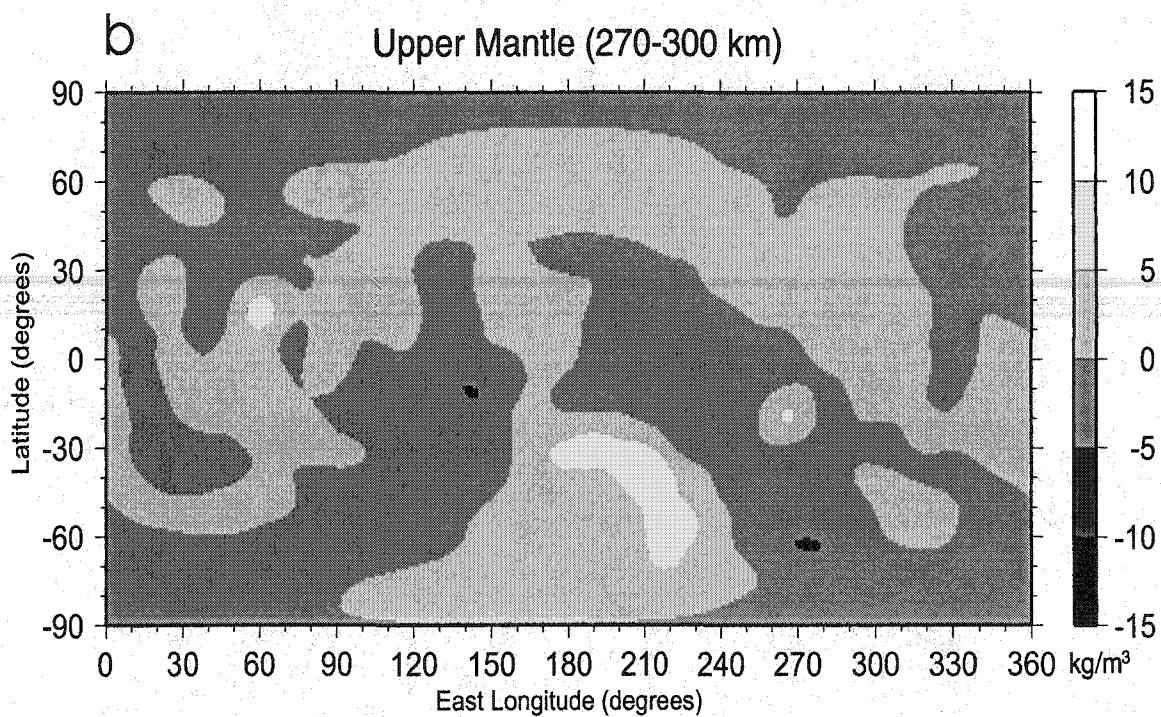
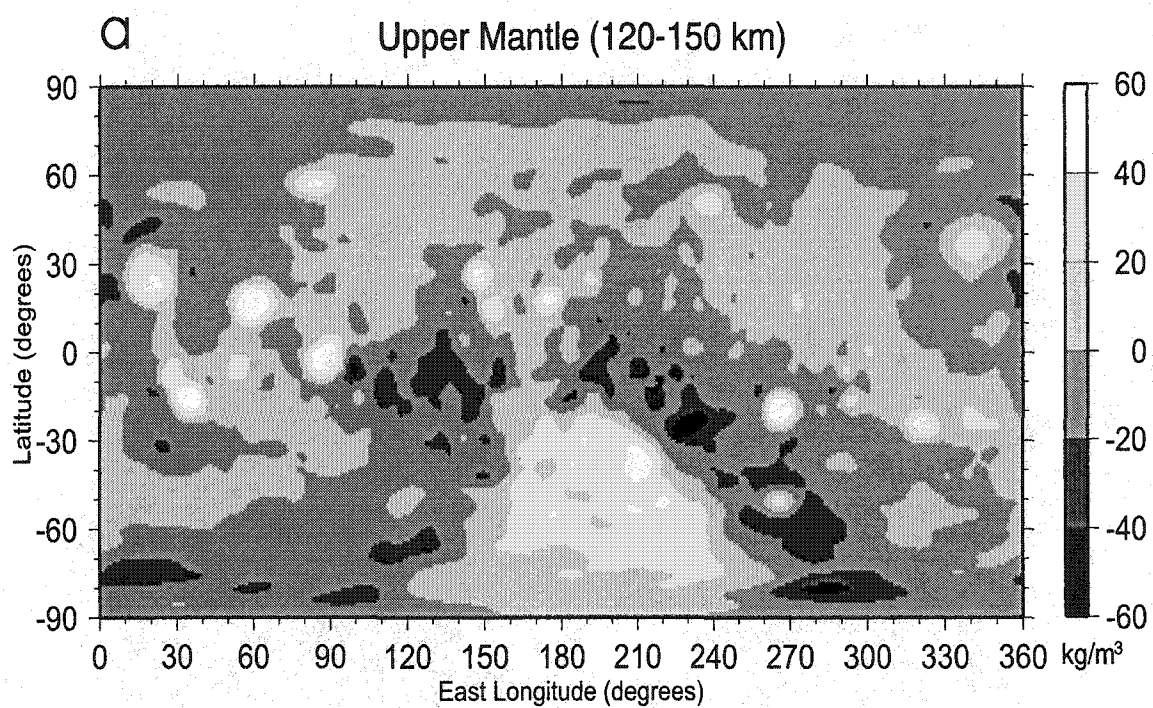


FIG. 5. The density perturbations generated by the nominal model within the upper mantle at depths of a) 120-150 km and b) 270-300 km. The density perturbations are less than $\pm 80 \text{ kg/m}^3$ by 150 km depth, with the major peaks being associated with the SPA basin, far side highlands, and mascon basins, as noted in Fig. 4. At 300 km depth the density perturbations have reduced to less than $\pm 5 \text{ kg/m}^3$ everywhere, except for a few areas associated with the large features mentioned above. The transition of density anomalies associated with these features, from positive to negative for the SPA and mascon basins, and from negative to positive beneath the far side highlands, demonstrates the oscillations seen in the density perturbation solution, even under the influence of the mass oscillation minimization condition.



mantle response beneath the major mascon basins and the SPA basin is being represented by the model as a strong positive anomalous density in the crustal layers, whereas for the far side highlands the model represents the crustal root as a large, negative, anomalous density variation within the crustal layers. Therefore, other large positive density anomalies in the crust may be representative of regions of major mantle upwelling, and significant compensation. This is evident for the mascon basins, but can be seen as well with some of the smaller, recently proposed (e.g. Arkani-Hamed 1998; Konopliv *et al.* 2001; Reindler and Arkani-Hamed, 2001; Zuber *et al.* 1994) mascon-like craters such as Mendel-Rydberg, Humboldtianum, and Moscoviense (see Fig. 4). At 120-150 km depth all density variations have reduced to less than $\pm 80 \text{ kg/m}^3$ and by 300 km depth the Moon has negligible density perturbations, being less than $\pm 5 \text{ kg/m}^3$. Several of the mascon basins, as well as the SPA basin, have negative density anomalies of as much as -5 kg/m^3 associated with them at depths of 300-500 km, being directly due to the oscillatory nature of the modeled density perturbations. In a similar manner, positive anomalies of $+5 \text{ kg/m}^3$ are associated with the far side highlands surrounding the SPA basin, further revealing oscillatory effects of the density anomalies within the model for the longer wavelength features. The constraint of minimizing the mass perturbation oscillations has reduced this consequence but, as is seen, still leaves some remnant effects, but these are relatively negligible. The weighting factor used for this minimization, and the effect it has on the model, is further discussed in the next section. The density perturbations further decay with depth, and essentially disappear by the upper/middle mantle boundary, at 510 km depth. It must also be noted that most of the larger density perturbations are located on the far side, in the area of poorest gravity

coverage (Konopliv *et al.* 2001). Lesser gravity coverage can lead to a serious overestimation of density perturbations and stresses (Kermani and Arkani-Hamed 1974; Phillips and Lambeck 1980). Considering that the continuity of the potential data is an essential boundary condition of the model solution, this region may be expected to display anomalous features. Overall, the majority of the crustal areas show variations of less than $\pm 200 \text{ kg/m}^3$, representing the vast range of compensation which occurs within the crust, at the upper/lower crustal boundary, and more importantly at the boundary between crust and mantle.

The internal stress distribution within this nominal model shows large stresses within the crust associated with the high density perturbations. However, stresses within this model extend to much greater depths, generally corresponding with the locations of the major mascon basins. Fig. 6 displays the global stress distribution within the crustal layers. The largest stresses in the crust are associated with the SPA basin and the surrounding far side highlands, with certain areas reaching stresses of 6-8 MPa. The near side mascon basins are slightly more subdued, but distinct rings of stress outline them, generally with stresses of 4-6 MPa. Several distinct craters (Korolev, Hertzprung, Mendel-Rydberg, and Humboldtianum) show this same distinctly mascon-like ring stress configuration. At the base of the crust the stresses are significantly reduced, and by 150 km depth (Fig. 7) the large stresses associated with the SPA basin and the far side highlands are essentially gone. At this depth the strong representations of the major mascon basins along with several mascon-like craters begin to dominate. The stresses are 6-8 MPa beneath all of the mascon basins and evident mascon-like craters at 150 km depth, reducing to 4-6 MPa at 300 km for only the largest mascon

FIG. 6. The stress distribution generated by the nominal model at the base of the a) upper and b) lower crust. In the upper crust stresses reach 6-8 MPa in specific areas associated with the SPA basin (1) and the lunar highlands (2). Stresses outside these areas do not generally exceed 4 MPa in the upper crust. In the lower crust, the peak stresses associated with the SPA basin and far side highlands reduce significantly down to below 6 MPa, and stresses in ring structures associated with several of the mascon basins and a few mascon-like craters (such as Mendel-Rydberg and Humboldtianum) become more prominent, being on the order of 4-6 MPa.

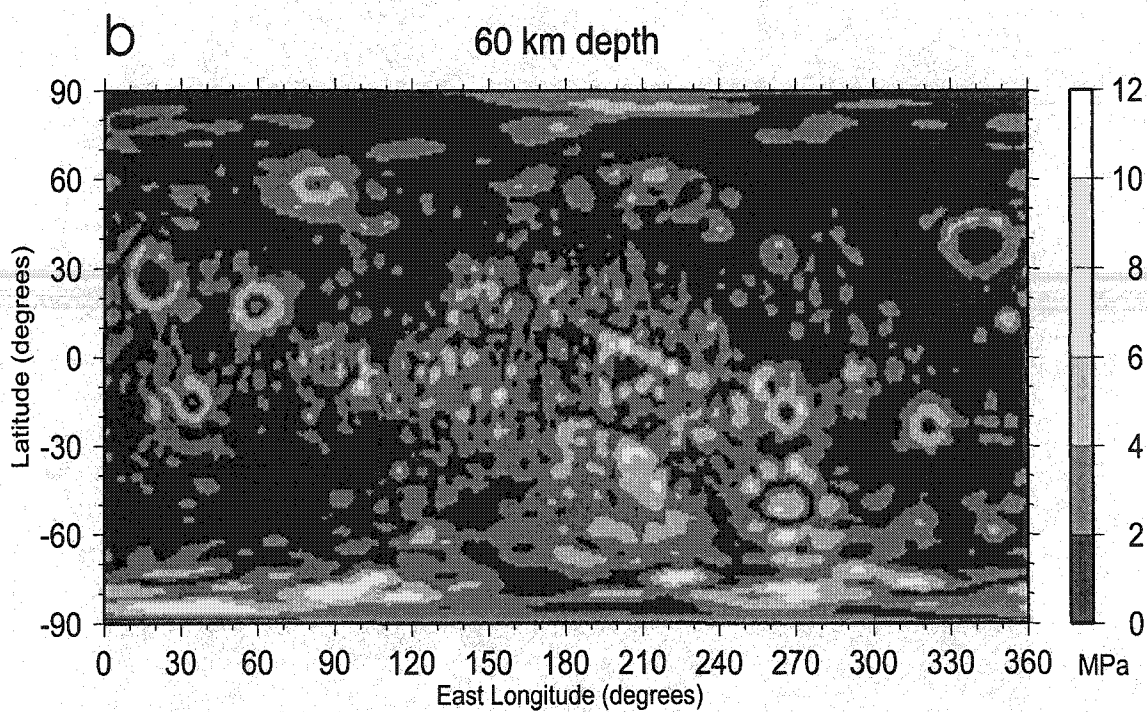
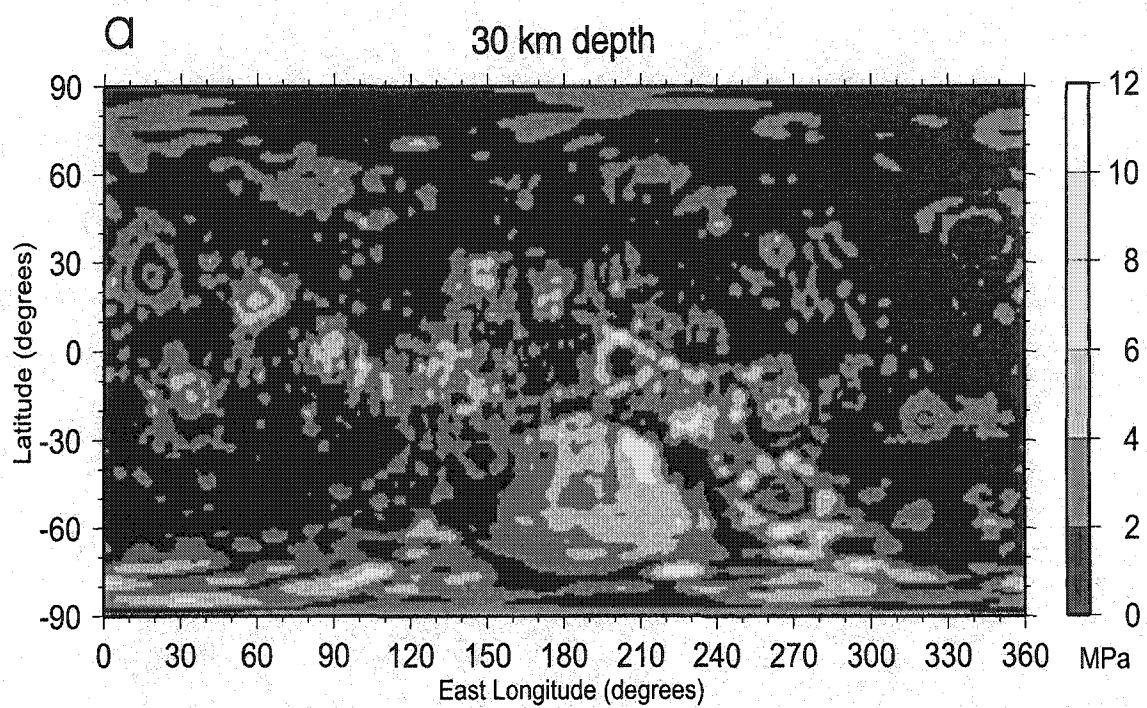
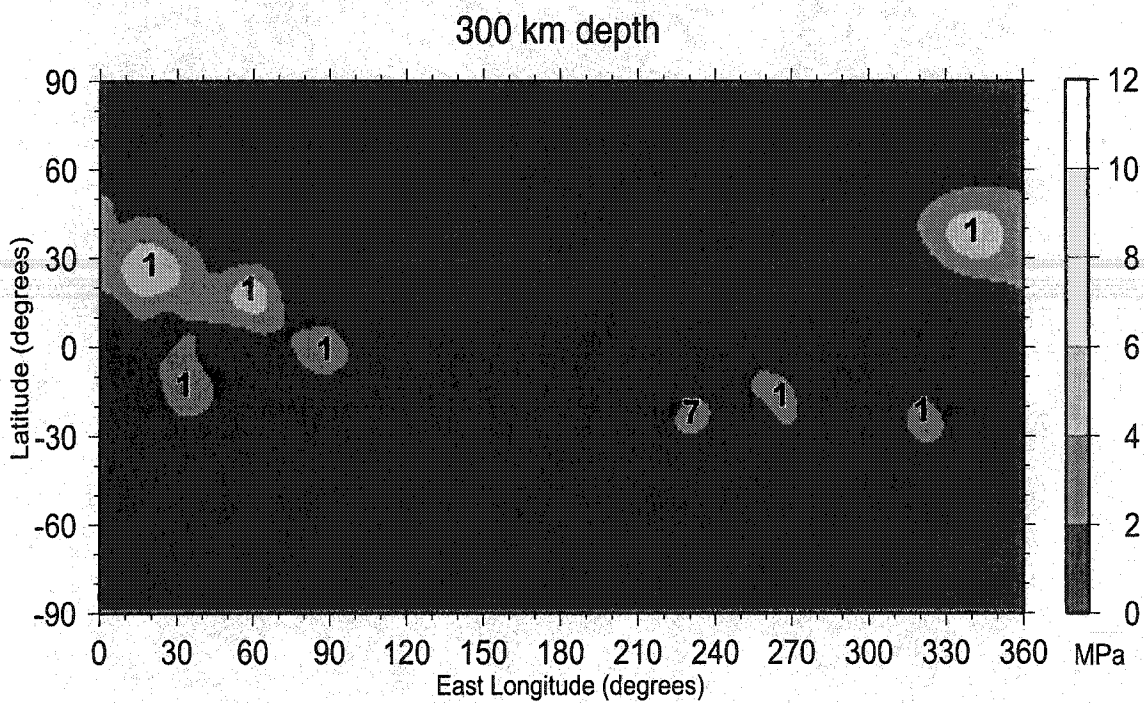
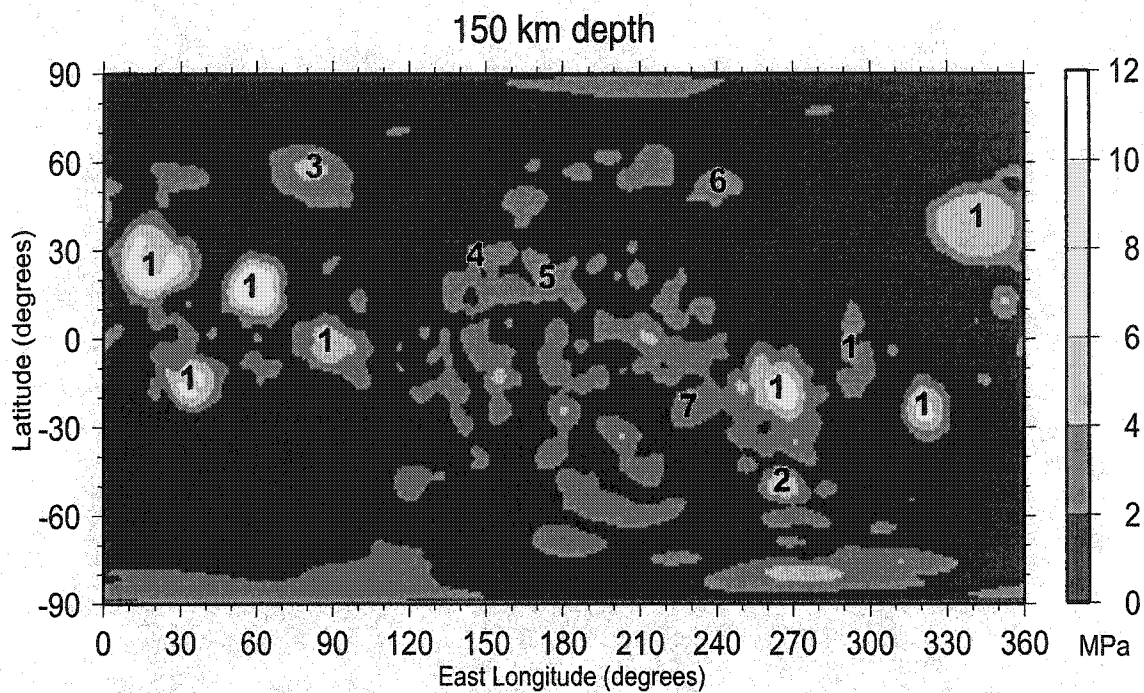


FIG. 7. The stress distribution generated by the nominal model in the upper mantle at depths of a) 150 km and b) 300 km. At 150 km depth the strong stresses associated with the SPA basin and the far side highlands has disappeared. From 150 km to 300 km depth the stress distribution is dominated by the mascon basins (1), with stresses of 6-8 MPa at 150 km depth and 4-6 MPa by 300 km depth. Several mascon-like craters such as Mendel-Rydberg (2), Humboltianum (3), Moscoviense (4), Freundlich-Sharonov (5), Coulomb-Sarton (6), and Hertzprung (7) have large stresses beneath them at 150 km depth, but by 300 km the stress has significantly reduced.



basins. Below this depth the stresses diminish quickly, and by 500 km depth only small remnant stresses of less than 3 MPa associated with Imbrium and Serenitatis remain. Individual stress profiles beneath several of the well known mascon basins, as well as for some of the largest recently discovered mascon-like craters (Konopliv *et al.* 2001; Reindler and Arkani-Hamed, 2001) are shown in Fig. 8 and 9, respectively. These mascon craters/basins represent the major non-isostatic features on the Moon, and hence the main loads on the lunar interior. The observed mass concentrations are generally a combination of emplaced mare material, and the super-isostatic response of the mantle to the initial basin forming event. In some cases, hydrodynamical uplift of the mantle occurred almost instantaneously, due to the shock waves and fluidization of the crust at impact. The subsequent rapid cooling of the crust probably "froze" the super-isostatic configuration of the crust and mantle leaving a large excess mass (Neumann *et al.* 1996; Spudis 1993; Taylor 1982; Wieczorek and Phillips 1999). The results presented here suggest that for a fully elastic lunar interior, capable of rigidly supporting the non-isostatic loads of the mascon basins, large (> 5 MPa) stresses do not extend beyond 500 km depth. Serenitatis, Crisium, Humorum, and Orientale have the largest magnitude stresses among the mascon basins, reaching highs of 6-8 MPa in this solid Moon model, at depths of less than 200 km.

Many different parameters have been considered, and optimal choices regarding these parameters have been made to produce this nominal model. This model provides the best possible present day representation of the Moon, however, many variations exist for the model parameters. The modeled density perturbations and stresses vary under the influence of these parameters, which in some cases may greatly affect the resulting

FIG. 8. The stress distribution with depth beneath Serenitatis (27° , 20°), Smythii (-2° , 87°), Humorum (-24° , 321°), and Orientale (-19° , 266°). The profiles are taken along a longitudinal west-to-east line, crossing the basin center. The latitudinal center of each basin is kept constant for the profiles. It is clearly shown that for even the largest of the mascon basins, the anomalous stresses are concentrated in the upper 300 km, and do not extend beyond 500 km depth.

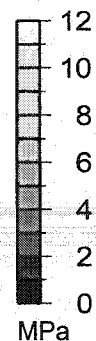
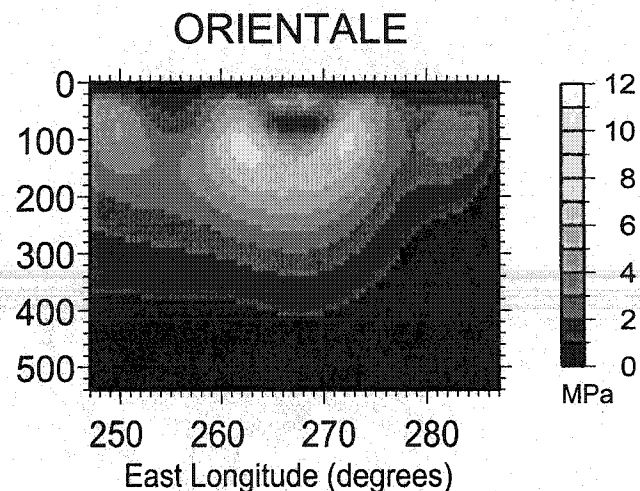
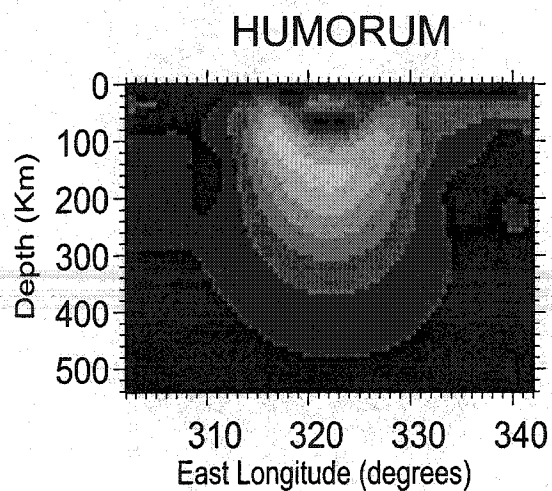
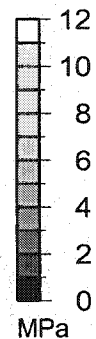
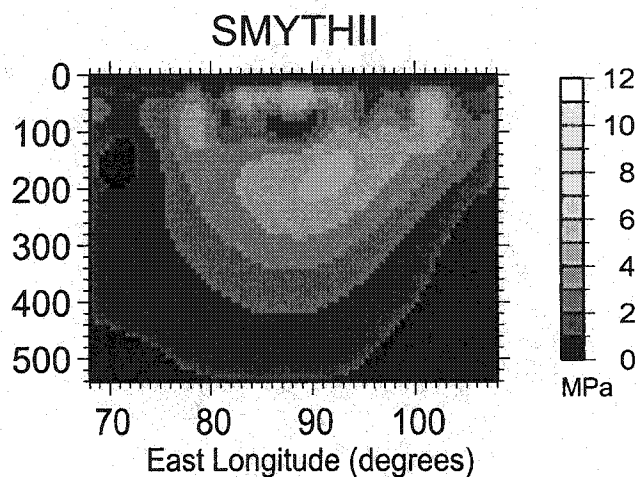
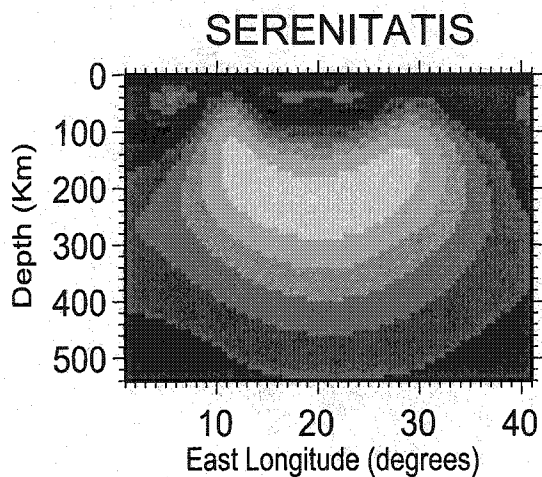
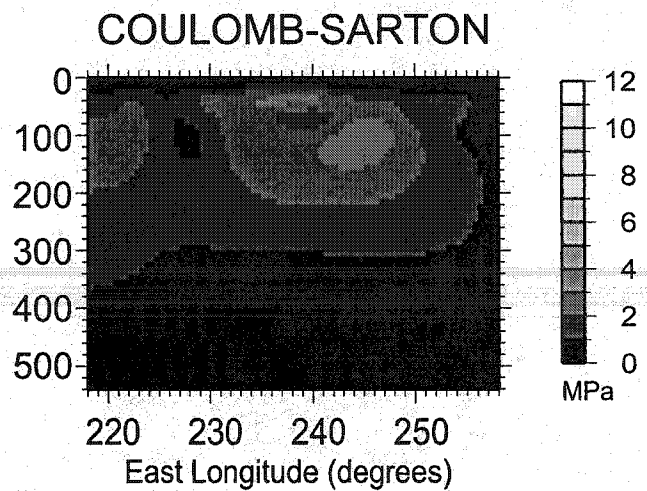
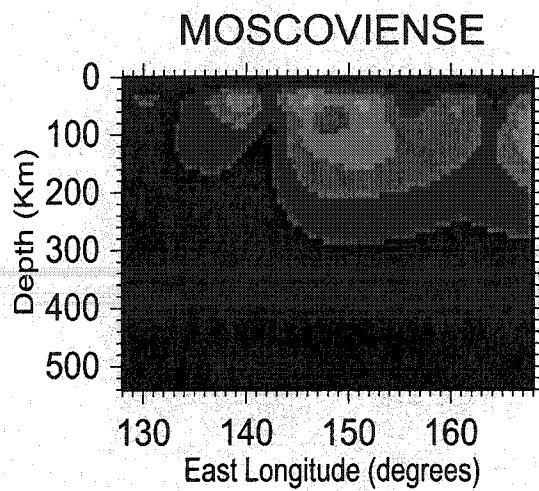
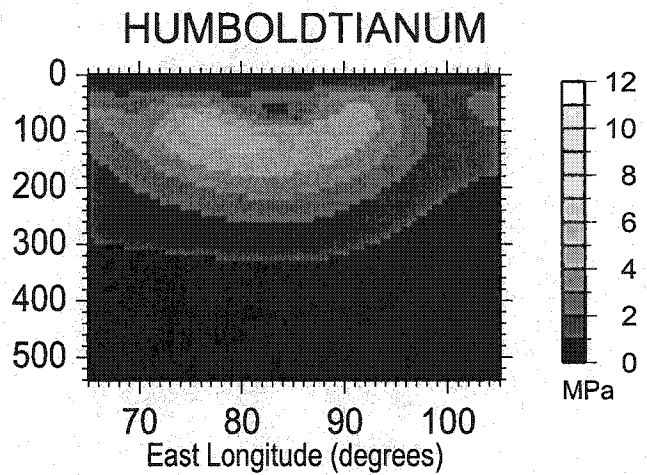
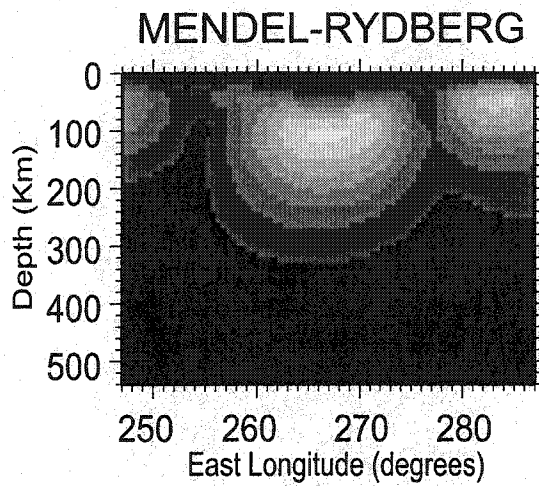


FIG. 9. The stress distribution with depth in the upper 500 km beneath several mascon-like craters: Mendel-Rydberg (-50° , 266°), Humboldtianum (58° , 84°), Moscoviense (26° , 147°), and Coulomb-Sarton (52° , 237°). The profiles are taken along a longitudinal west-to-east line, crossing the crater center. The latitudinal center of each basin is kept constant for the profiles. These craters have strong stresses beneath supporting their surface loads. Their stresses generally do not extend below 300 km depth.



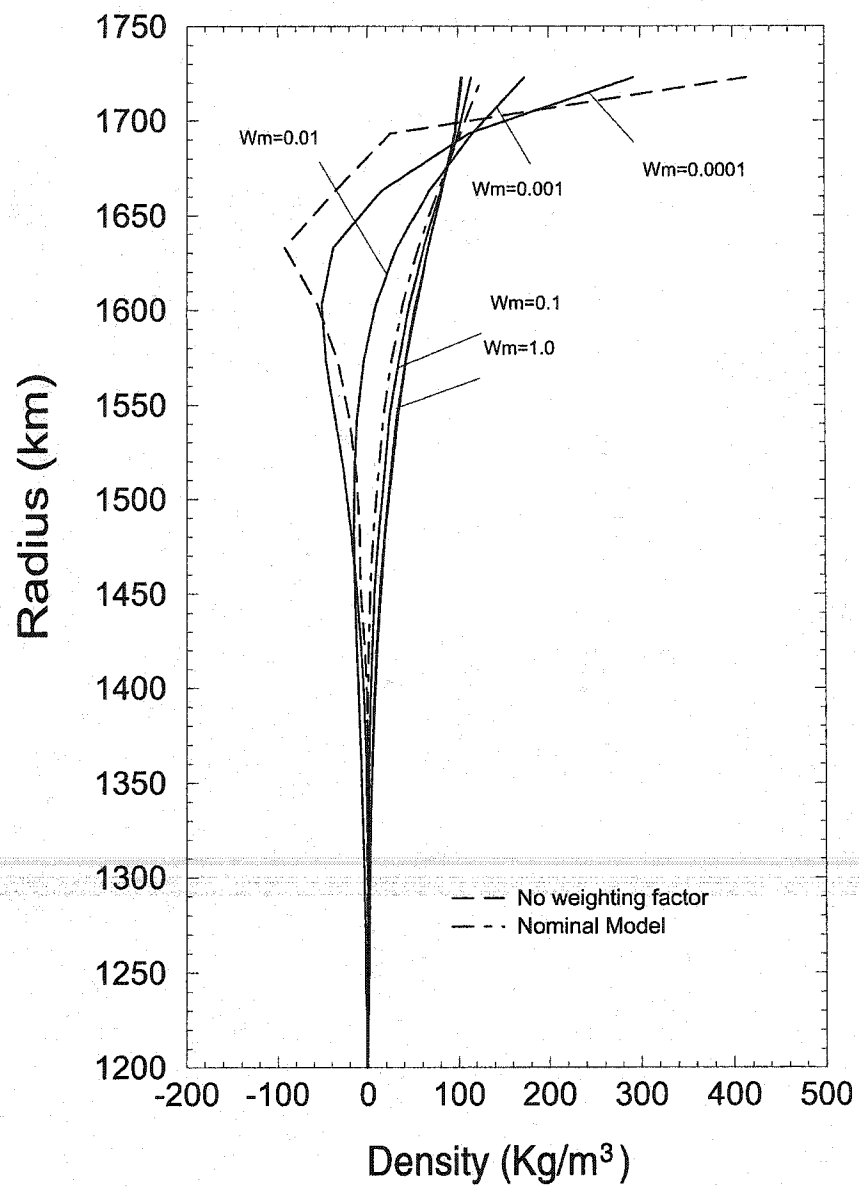
distribution of density and stress. The next sections present further investigations on the response of the thick shell model, and the effects of a wide range of parameter variations.

5.2 Effect of the Weighting Factor

The choice of weighting factor is crucial for the overall model solution to maintain a proper balance between the two minimization constraints; namely, the minimization of total shear-strain energy and the minimization of radial oscillations of the mass perturbations. As mentioned, the shear-strain minimization is applied because of the non-unique solution of density perturbations within the Moon which give rise to the observed gravity field. The minimization assumes that the entire lunar body evolves toward a state of minimum total energy. However, as has been previously noted (Arkani-Hamed 1973b), the density perturbations generated in such a model tend to oscillate at depth within the Moon model. This was shown to be the case (in the last section) beneath the mascon basins, the SPA basin, and the far side highlands, for the nominal choice of weighting factor. Fig. 10 demonstrates the effect of varying the mass oscillation minimization weighting factor on the density perturbations directly beneath the Serentitatis basin. The minimization of the mass perturbation oscillations reduces the oscillatory characteristic of the density perturbations. However, as a consequence of this minimization, the total shear-strain energy for the model increases. The optimal solution lies in an adequate weighting of the minimization conditions. In the present model the minimization of the shear-strain energy is held constant, isolating the balance

FIG. 10. The effect of increased weighting factor, W_m , on the density distribution below Serenitatis basin. The dashed line denotes the results for a model with no minimization of mass perturbation oscillations ($W_m=0.0$). All density perturbations for these basins are concentrated in the upper 500 km. The oscillation of the density perturbations from positive to negative are significantly reduced with the increase in weighting factor. The effect of increasing the weighting factor is seen to reduce after a value of $W_m = 0.01$, and by $W_m = 0.1$, the maximum reduction has occurred.

Serenitatis



of mass oscillations and total energy to the weighting factor, W_m , associated with the minimization of the mass oscillations. The objective is to obtain the largest possible W_m for which the total shear-strain energy remains a near minimum.

To determine the largest possible weighting factor, W_m , the relative energy associated with increasing spherical harmonic is compared. Fig. 11 shows the total shear-strain energy, as a function of W_m , for different ranges of degree, n , starting at $n = 3$, and going up to $n = 3-70$. The shear-strain energy for each degree is normalized by the $W_m=0$ energy for the lowest harmonic considered ($n=3$). The region of shear-strain energy increase begins at a weighting factor of 10^{-3} , and by 10^{-1} the energy has already essentially reached a maximum within the model. Therefore, the range of 10^{-3} to 10^{-1} is the best range of W_m for appropriately maintaining an overall near minimum shear-strain energy as well as obtaining a relatively large minimization of the mass perturbation oscillations. However, the shear-strain energy associated with the Moon is dominated by the low harmonics, with more energy gained over the harmonics of degree, $n = 3-10$, than over the degrees, $n = 10-70$, implying that the minimization of shear-strain energy becomes less important over the higher degree harmonics. Thus, a lower W_m is used for the low harmonics, $n = 3-7$, while W_m is increased for higher harmonics, thus allowing an overall reduction of shear-strain energy, while further minimizing the mass oscillations in the upper lunar regions. The chosen weighting factors are listed in table 2, and are associated with the previously determined groupings for the major harmonic divisions. The variation of weighting factor over harmonic intervals provides a different overall energy profile. Fig. 12 displays the total shear-strain energy, again normalized to the $W_m=0$ energy for the lowest harmonic considered ($n=3$), for different constant weighting

FIG. 11. The effect on the total normalized shear-strain energy associated with cumulative harmonic degrees for variations of the weighting factor, W_m . The total energies have been normalized to the energy for $n=3$, and $W_m = 0$. The total shear-strain energy remains a minimum for $W_m < 0.001$. For $0.001 < W_m < 0.1$ the energy is in a state of transition, increasing rapidly for increased W_m , and by $W_m = 1.0$, the total shear-strain energy has reached a maximum. The majority of the total energy for W_m is associated with the low harmonics ($n < 20$).

Shear-Strain Energy vs W_m

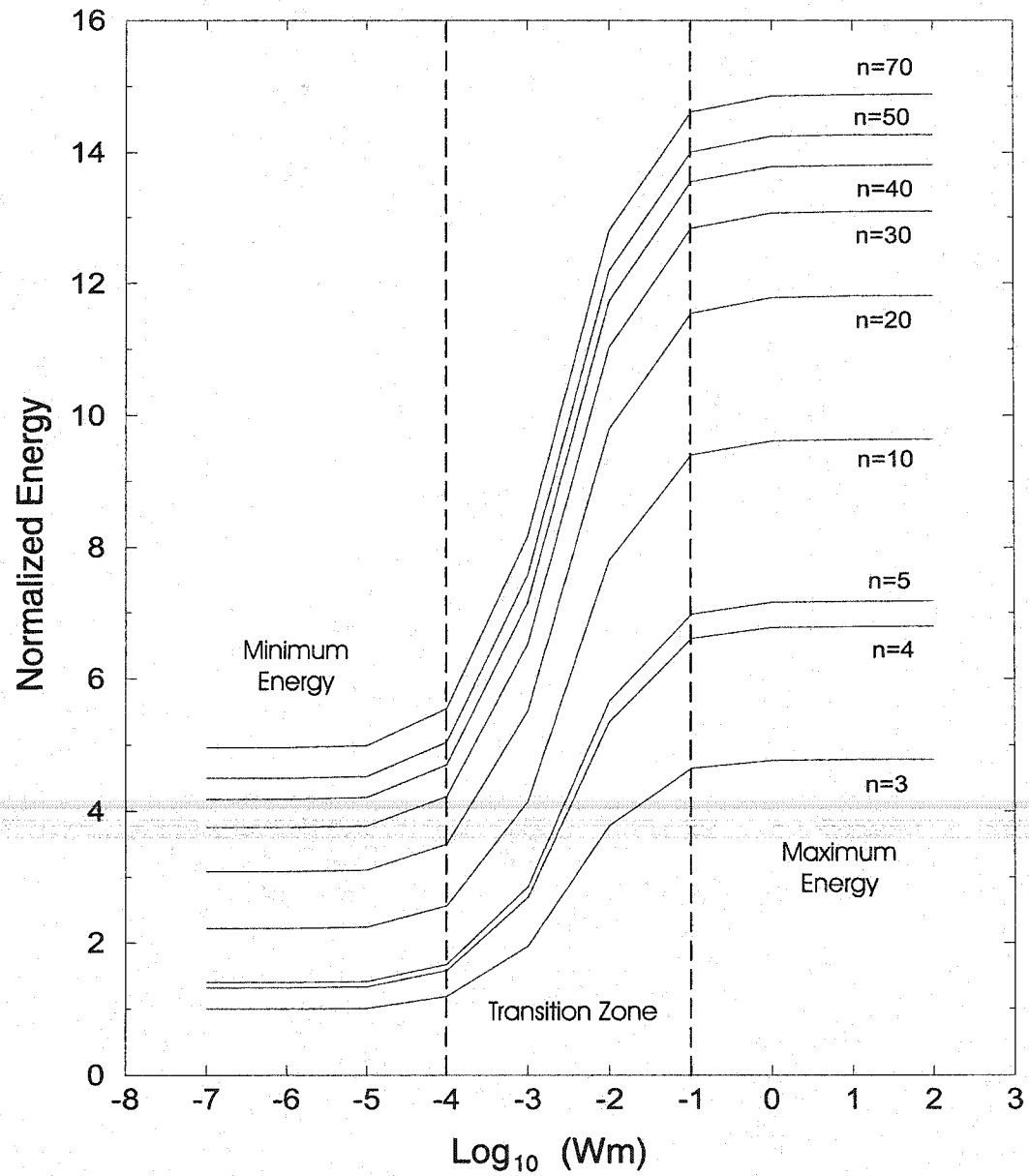
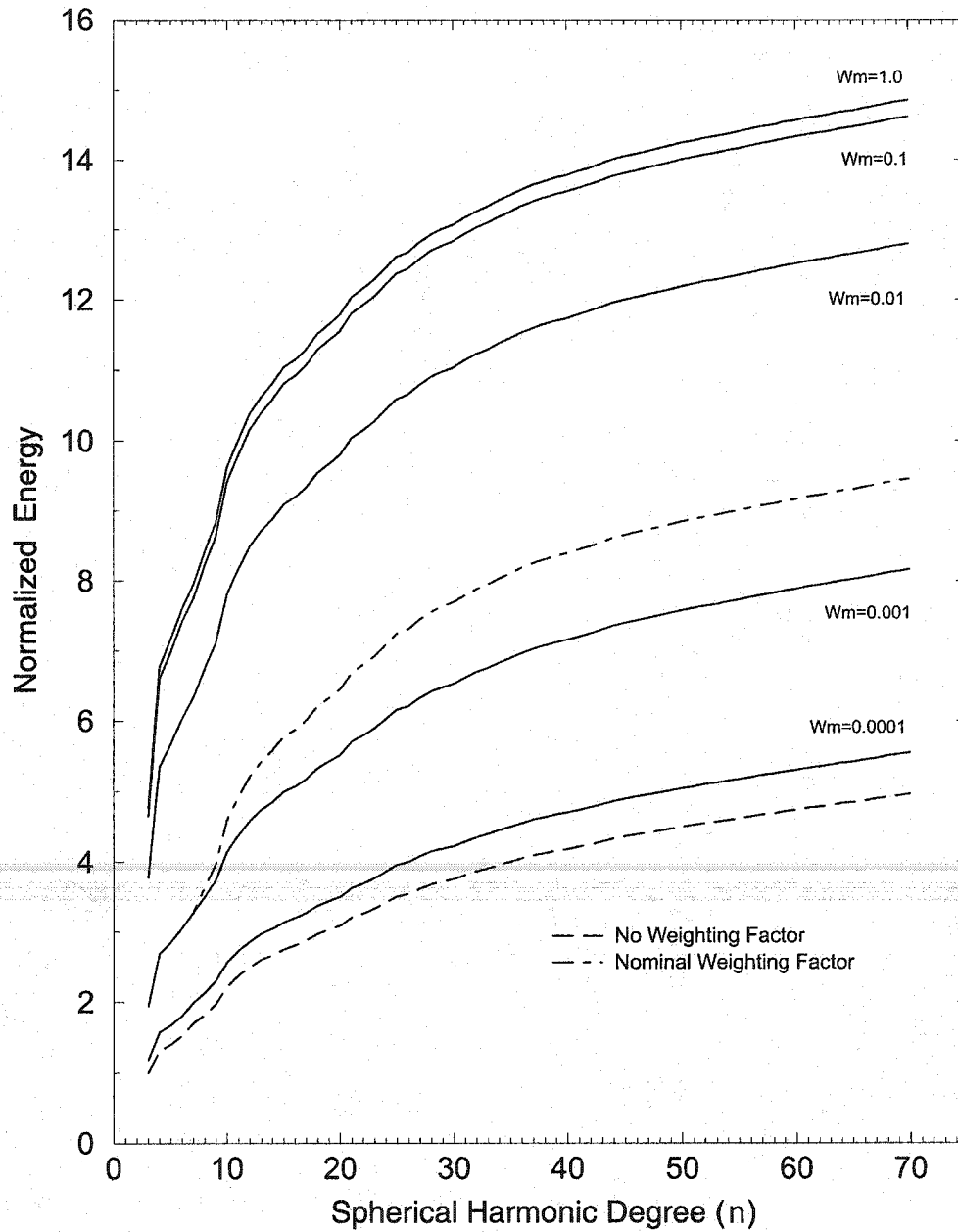


FIG. 12. The total normalized shear-strain energy associated with different constant W_m values, and the proposed nominal model. The total energies have been normalized to the energy for $n=3$, and $W_m = 0$. The model with $W_m = 0.0001$ displays almost no energy gain over the $W_m = 0.0$ model. A small relative increase is also seen with the $W_m = 0.001$ model. However, the use of $W_m = 0.01$ generates a large increase of total shear-strain energy. The nominal model generates only slightly larger total energy, and allows for the use of $W_m = 0.01$ over the higher harmonics ($n > 20$).

Shear-Strain Energy vs Harmonic Degree



factors. The solid lines denote constant weighting factors, the $W_m=0$ line shows the minimum shear-strain energy possible for the present model, and the dashed line shows the energy for the choice of optimal weighting factors over the different harmonic divisions. Although the energy associated with the choice of weighting factors is slightly higher than that for the constant weighting factor of 1×10^{-3} , the weighting factor is in fact 5 times larger over the mascon basin harmonics ($n = 8-20$), and 10 times larger over the higher harmonics ($n = 21-70$). Thus, this choice leads to a greater overall reduction in mass oscillations, at the sacrifice of only a small relative increase in the total shear-strain energy. Therefore, applying a variable weighting factor allows for a significantly lower total energy, while more efficiently reducing the mass oscillations, particularly in the upper 500 km where the mass oscillations are most concentrated (see Fig. 10).

5.3 Effect of layer variations

The preferred model contains 54 layers of 30 km thickness. The 30 km layers were chosen from a wide range of considered models, including models with layers of 20, 25, 30, 50 and 60 km. Due to the boundary condition imposing the continuity of the modeled gravity with the observed gravity field, density perturbations in the model tend to be concentrated in the uppermost layers. As a result, thinner layers (hence more total layers) generate stronger density perturbations in the uppermost regions of the model. Conversely, thicker layers result in lower density perturbations, since the perturbations are basically dispersed throughout the entire thickness of the layers. For example, a

model with layers chosen to be larger than 30 km thick, say twice as thick, will result in smaller density perturbations over a single crustal layer (of 60 km thickness), in comparison to the perturbations found in a thinner upper crustal layer, but will be larger than the density perturbations within a thinner lower crustal layer. Conversely, for the validity of the assumption of constant layer parameters (Lamé parameter, shear modulus, and density), sufficiently small layers must be used.

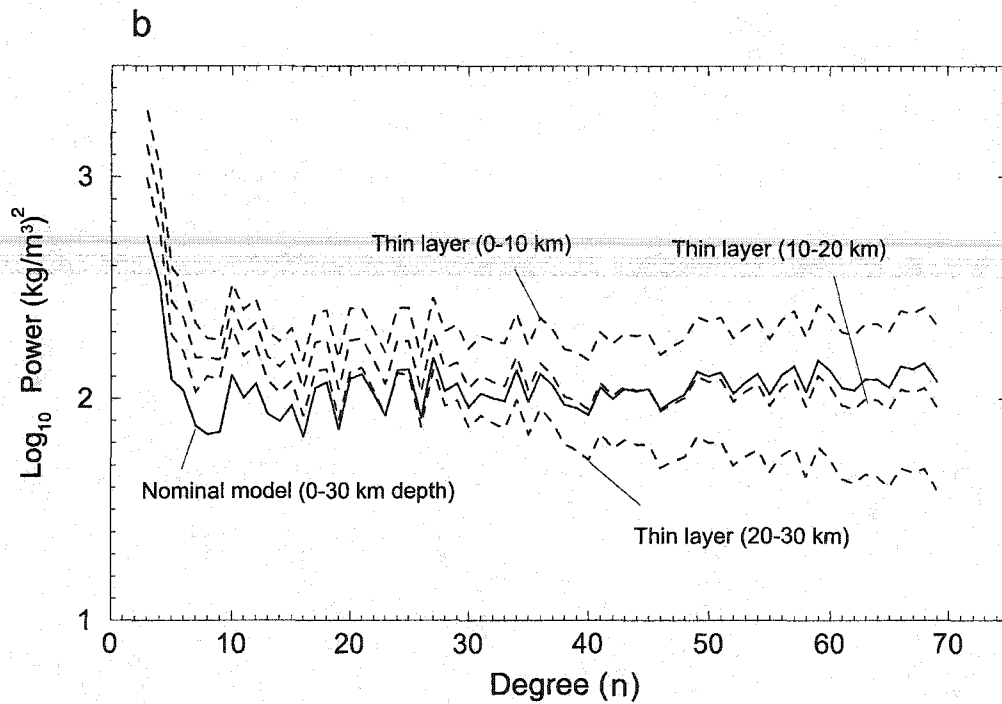
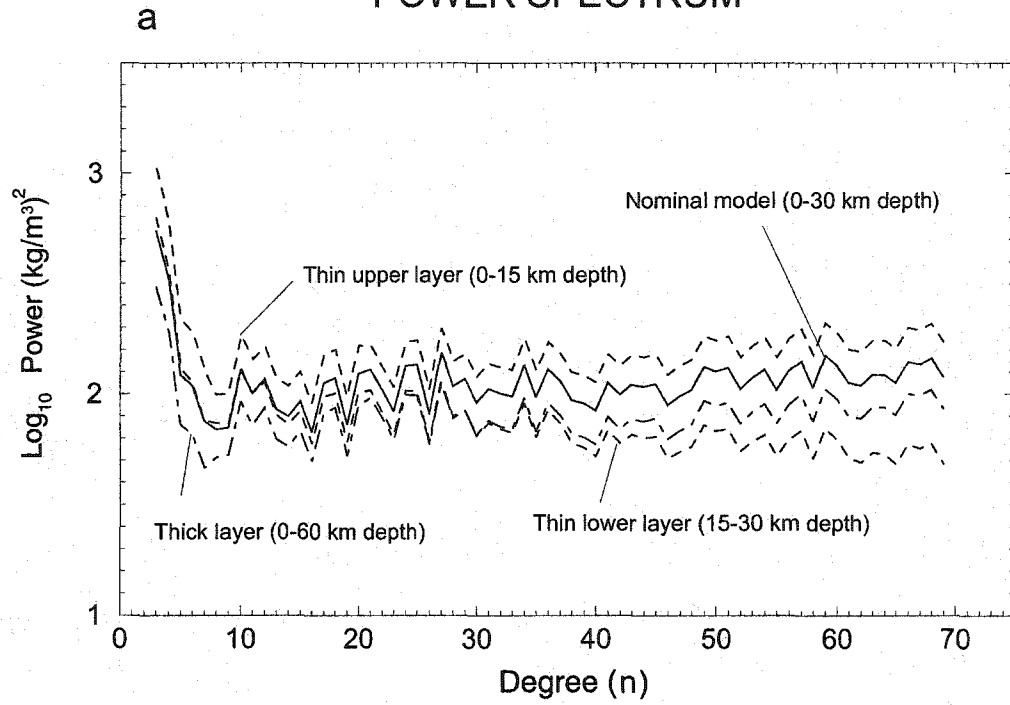
Another way to see the effects of layering is in a spectral domain representation. Fig. 13 displays the power spectrum of the density perturbation coefficients within the uppermost layers, for different variations of layering near the surface of the Moon. The power spectrum, ϵ_n , displays the power associated with a set of spherical harmonic coefficients for independent harmonic degrees, n , and is determined by the equation

$$\epsilon_n = \sum_{m=0}^n (\Delta\rho_{nm})^2 \quad (12)$$

In Fig. 13a the lower degree coefficients display the strong effect of layering variations, while the power spectra shows a white noise characteristic for the higher degree coefficients. The low degree density perturbation coefficients for the thin layer model have larger power, while the thick crustal model has lower power associated with its spectrum. The thin layer model has reduced amplitudes for its density coefficients in the lower crustal layers, balancing the larger perturbations in the thin near surface layers. Layer variations can have a major influence on the density and stress distribution, particularly within the near surface layers. Another model with layers decreasing in thickness, starting from 150 km depth and getting thinner toward the surface was also applied (Fig. 13b). The layers were reduced by 5 km increments every second layer, starting from 30 km layers at 150 km depth, and resulting in 10 km layers at the surface.

FIG. 13. The power spectrum for the density perturbation coefficients for models with a) thicker and thinner even layers, and b) uneven thin layering. The spectrum in (a) displays how the power is evenly distributed for all even layering models. However, for the thin layer model the higher power of the coefficients results in larger density perturbations in the upper 15 km layer than is seen in the 30 km layer, or the 60 km layer. As well, it can be seen that the power distribution takes on a white noise characteristic after about $n = 15-20$. Alternating layer thickness (b) results in increased power, and density perturbations, over the harmonics of up to $n = 25-30$. The power associated with the thinnest layers is very large in the surface layer (upper 10 km), but is less than the 30 km model by the base of the upper crustal layer (20-30 km). Therefore, constant layering is preferred over variable layer thicknesses.

POWER SPECTRUM



The density perturbations for the reducing-layer model have more power within the thinner surface layer of 10 km, as compared to the nominal model surface layer of 30 km. However, looking at the average power over all 3 upper crustal layers for the incremental model (upper 30 km), the density perturbations for degrees $n < 20$ have more power compared to the nominal 30 km layer model (Fig. 13b). Therefore, variable layering seems to generate artificially increased density distributions, particularly among the lower harmonics. To avoid this artificial increase in density perturbations, the nominal model has constant thickness layering.

Finding a model with acceptable layering for the Moon is highly arbitrary; however, the nominal choice represents the best determinable balance of all potential layering models. At present, layers of less than 30 km have no structural basis within the Moon, particularly in the upper 150 km, where the majority of the density perturbations are concentrated (see section 5.1). The crust does appear to have a distinct seismic boundary at 20-30 km depth, but only a minor density boundary (Kuskov and Kronrod 1998; Simmons *et al.* 1973; Nakamura 1983; Wieczorek and Phillips 1997, 1998). Choosing 30 km layers also results in a model which has layer interfaces within 10 km of all proposed structural boundaries (e.g. Nakamura 1983). Variable thickness layering, as in the reducing thickness model, can further distort the magnitudes of the density anomalies, and is to be avoided. Therefore, constant layering was adopted throughout the Moon model. The 30 km layers are thick enough to avoid over-stacking of density perturbations and stresses within the upper crust, yet thin enough to allow for a more refined and precise view of the distributions within the Moon. Thus, 30 km

layers are the best compromise for layering, given the present knowledge of the lunar interior.

5.4 Effect of Core Parameters

Another parameter of the lunar model that can vary is the representation of the lunar core. The presence of a lunar core has not been definitively confirmed, however there is enough evidence to strongly suggest its existence (e.g. Hood 1986; Hood *et al.* 1999; Konopliv *et al.* 1998; Kuskov and Kronrod 1998). The size of a core is limited by geophysical evidence limiting the end-member models to consist of either a slightly larger iron-sulfur core of roughly 400-500 km radius or a smaller, purely iron core of around 300 km radius (Khan and Mosegaard 2001; Konopliv *et al.* 1998; Kuskov and Kronrod 1998). As well, dynamical evidence from laser ranging data suggests a fluid metallic core of 300-375 km (Williams *et al.* 1999). A 358 km liquid core of intermediate composition between pure iron (8100 kg/m³) and iron-sulfur (4700 kg/m³) was applied for the nominal model, based on the best estimate for a core radius and composition based on compositional models of the Moon (Hood *et al.* 1999; Konopliv *et al.* 1998; Kuskov and Kronrod 1998). A liquid core on the Moon may be the consequence of a rapid thermal evolution of the upper parts of the Moon, characterized by the growth of a large insulating thermal lithosphere early in its evolution (Spohn *et al.* 2001). As a result heat transport to the surface was greatly reduced, and the interior of the Moon could not cool efficiently. The rate of heat dissipation from the Moon's core became very small, and almost constant, after only about one billion years of lunar

evolution, due to the insulating effect of the large lithosphere (Spohn *et al.* 2001). This implies that the core is still very hot and most likely liquid, particularly for the case of an iron-sulfur core, which would stay molten longer, due to its lower liquidus temperature than that of a pure iron core.

To remain consistent with the geophysical constraints a slightly smaller core for the pure Fe model (298 km), and a slightly larger core for the Fe-S model (418 km) are applied. The density perturbations associated with these two end-member liquid core models were determined, and displayed only very marginal differences from the nominal model. Only the largest, long wavelength features ($n=3-7$) are allowed to penetrate down to the core/mantle boundary, and even these features show no significant effects due to different core models at the surface. This is due to the fact that the maximum range of the density perturbations is only about 500 km, as was seen for the nominal model. Hence, core effects beyond 1000 km depth have essentially no surface ramifications. As well, the density contrast between the core compositions and the overlying mantle are quite large ($\sim 1300-4700 \text{ kg/m}^3$), not allowing for much displacement and compensation at this boundary. The differences in the stress distributions between the same compositional core models and the nominal model are equally minute. One other characteristic of the lunar core to be considered is the possibility of an entirely solid core. Due to the higher solidus of iron, a solid core model for the Moon will be applied with a pure Fe or high Fe composition. The nominal model was applied with a solid core of the same intermediate composition, with the Lamé parameter and shear modulus (rigidity) of the core set to be equal to that of the lower mantle. Yet again, the results for this core variation showed no significant change in the

density perturbations and stress distribution throughout the upper mantle and crust. One difference between the solid and liquid core models is expected in the region directly overlying the core/mantle boundary. The stresses for the solid core model should be reduced in this area, since the underlying solid core can support more of the stress. This is, in fact, the case, however the stress differences are so small at this depth within the Moon (<1 MPa) that the overall stress variations in this region are negligible, regardless of a solid or liquid core.

The major characteristics of the core, whether it is pure Fe or Fe-S in composition, or whether it is solid or liquid, appear to have no significant effect on the density and stress and distributions within the outer regions of the Moon. The stresses associated with the two compositional models vary by less than ± 0.1 MPa from the nominal model within the crust and become essentially indistinguishable by 300 km. The possibility of an entirely solid core has an equally minor effect on the overall results of the model, and is not considered a major model variable. The small effect on the modeled density and stress distribution of different core models is a direct result of the relatively small core in comparison to the large lunar mantle. The core comprises only 1-4 % of the total mass and comprises less than 2% of the total volume of the Moon, and hence is too small to be a major factor in the support and compensation of even the longest wavelength surface features.

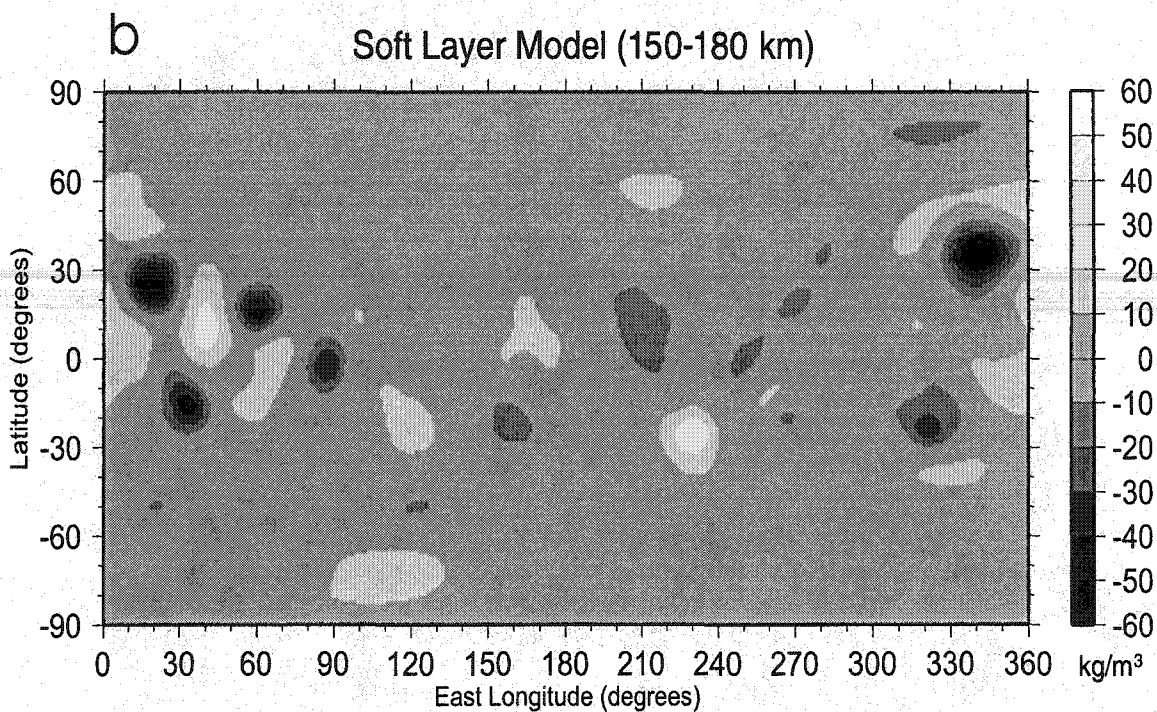
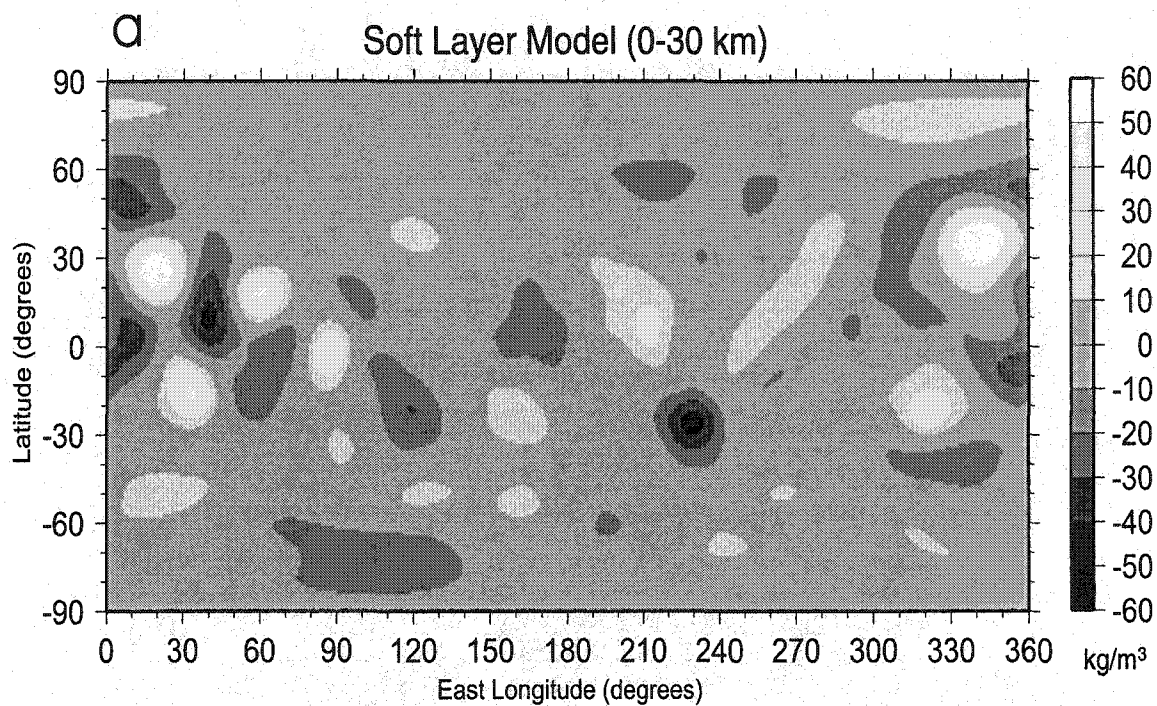
5.5 Effect of a Soft Mantle

The nominal model assumes an entirely solid mantle, and does not take any partial melting within the mantle into account. This was done to obtain an initial first order interpretation of the present distribution of density and stress within the Moon. Another model parameter to be introduced which may have a significant effect on the density and stress distribution is the incorporation of a soft region in the upper mantle. The incorporation of such a soft layer can be used to approximate the distribution of density and stress within the Moon for a time when partial melting within the upper mantle played a major role in planetary processes, and in the support of surface loads and stresses within the upper regions of the Moon. The upper mantle below the elastic lithosphere of the Moon was still a major region of partial melting, even after the period of extensive lunar volcanism had ended, roughly 3.0-3.2 billion years ago (Spohn *et al.* 2001; Taylor 1982; Wieczorek and Phillips 2000). As modeled here, the partially molten or soft region is not capable of supporting stress over geological time, and hence the overlying shell of such a soft region will have to support more stress, with no support coming from beneath (Arkani-Hamed 1973a,b).

As discussed in section 2, the lunar elastic lithosphere has been modeled extensively, and has been suggested by thin shell modeling to have been about 100 km thick following the period of the major mare forming events (Kuckes 1977; Pullan and Lambeck 1981; Solomon and Head 1980), while other models (Phillips and Lambeck 1980; Spohn *et al.* 2001) have suggested an even larger (200-300 km) lithosphere. In the current model, an elastic thickness of 210 km is examined, with the soft, partial melting

layer extending beneath the lithosphere to 510 km depth. The thickness of the model lithosphere was chosen to approximate the thickness of the lunar lithosphere after extensive lithospheric thickening following the strongest period of lunar volcanism, at roughly 3 billion years ago. The nominal model is applied with this partially molten region, that is approximated by reducing its rigidity by a factor of 10^3 for the specified depth range. The variations of density perturbations from the nominal model for a soft layer model, within the upper crust and above the partial melt layer at 150-180 km depth, are shown in Fig. 14. The general density variations in the crust are essentially the same as the density variations observed with the nominal model (Fig. 4), being generally less than $\pm 20 \text{ kg/m}^3$ with peaks reaching up to about $\pm 60 \text{ kg/m}^3$. In the upper mantle, however, the variations do not diminish rapidly, remaining as high as $\pm 60 \text{ kg/m}^3$ from the nominal model. The density anomalies in association with the mascon basins, have strong negative anomalies of up to -50 kg/m^3 . This is another representation of the oscillations within the density perturbations. Instead of being distributed more slowly and evenly through the deeper parts of the Moon, the soft layer has the effect of concentrating the negative density perturbations above the soft layer, beneath the major mascon basins. The variations of stress for the soft model from the nominal model within the middle of the crust, and at 180 km depth are shown in Fig. 15. The stresses in the crust are only moderately affected by the soft mantle layer, showing differences of at most $\pm 1\text{-}2 \text{ MPa}$ from those seen in the nominal model (Fig. 6). The largest stresses in the crust are once again associated with the SPA basin, and the surrounding far side highlands. The most significant effect of the soft layer is seen in the stress at 180 km depth. The stresses are strongly increased by as much as $4\text{-}6 \text{ MPa}$, in comparison to the

FIG. 14. The variations of density perturbations from the nominal model for the a) crust (0-30 km) and b) upper mantle (180-210), due to the introduction of a soft layer in the upper mantle extending from 210 to 510 km depth. These maps are created by subtracting the nominal map from the soft layer maps. The density variations in the upper crust are relatively small, being generally no more than $\pm 20 \text{ kg/m}^3$ with peaks as high as $\pm 60 \text{ kg/m}^3$. The density anomalies remain strong at 180-210 km depth. The large negative anomalous density perturbations below the major basins at this depth represent the oscillations in the density perturbations, which have been concentrated just above the soft layer boundary.



nominal model, in the regions beneath the mascon basins (Fig. 7). The stresses in the soft layer model reach highs of 10-12 MPa near the Serenitatis and Crisium basins. Large areas of increased stress beyond 4-6 MPa greater than in the nominal model are apparent, extending between several of the near side mascon basins (Imbrium, Serenitatis, Nectaris, Crisium, and Smythii). The stresses previously seen extending to beyond 200 km depth for the nominal model are now concentrated in the layers directly overlying the beginning of the soft layer, resulting in the higher stresses displayed in Fig. 15. This effect appears to be generally limited to the mascon basins, with several of the larger mascon-like craters (Humboldtianum, Mendel-Rydberg, Moscoviense, Freundlich-Sharonov) not showing any noticeably concentration of stresses at 180 km depth. Comparing the stress profiles for the mascon basins (Fig. 16) and several of the mascon-like craters (Fig. 17) demonstrates the effect of the soft layer on different types of craters. The major super-isostatic load of the mascon basins is supported by stresses concentrated just above the boundary between the lithosphere and the soft layer. In the case of the slightly smaller mascon-like craters, the surface loads are more easily supported within the lithosphere itself, thus avoiding a concentration of stress at the base of the lithosphere.

Thus, a soft layer has been applied based on the most reliable model of lunar evolution and the most likely scenario of mascon basin support. The model incorporates a 300 km thick soft layer extending from the upper mantle, at 210 km, to the beginning of the middle mantle at 510 km. The introduction of such a layer appears to have only a minor effect on the density and stress distribution within the crust, but has a larger effect near the boundary between lithosphere and soft layer. The presence of such a layer

FIG. 15. The variations of stress from the nominal model generated in the a) crust (30 km) and b) upper mantle (180 km) by a model with a soft layer at 210-510 km depth. These maps are created by subtracting the nominal map from the soft layer maps. The stresses in the crust are less affected by the introduction of a soft layer, being generally less than ± 2 MPa. Just above the soft layer, however, the stresses greatly increase from the nominal model, with increases of 2-8 MPa associated with the major mascon basin regions. The mascon basins are the major features affected by the introduction of the soft layer, with comparatively small stress increases associated with the SPA basin and the far side highland region.

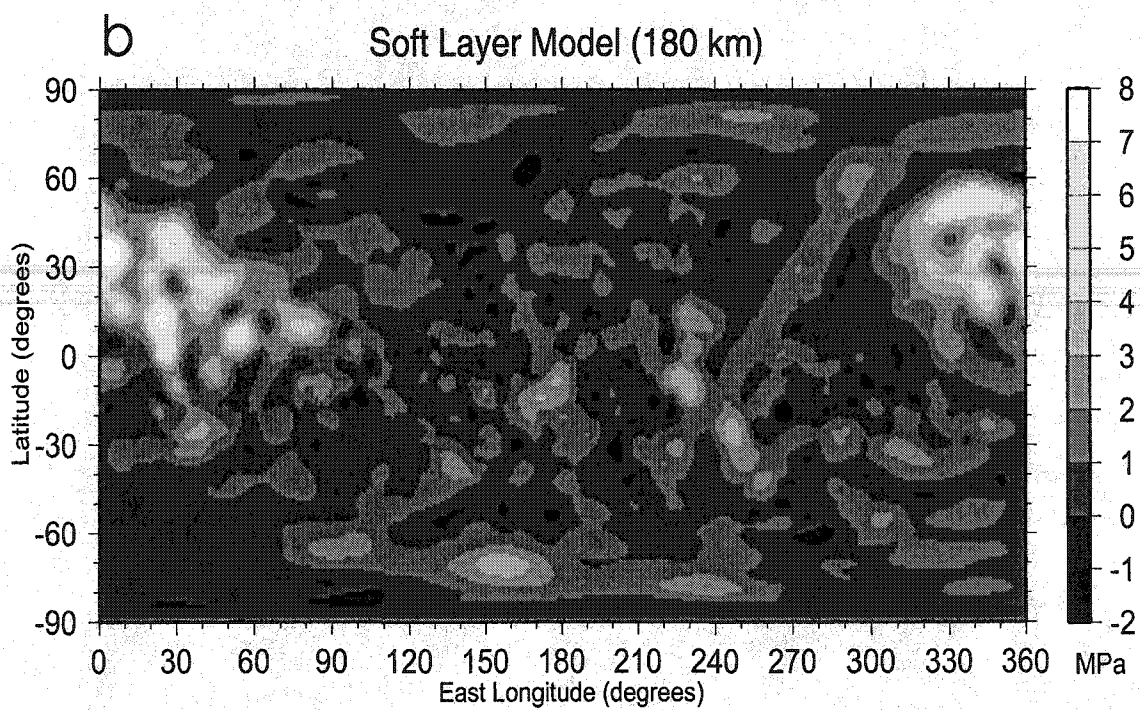
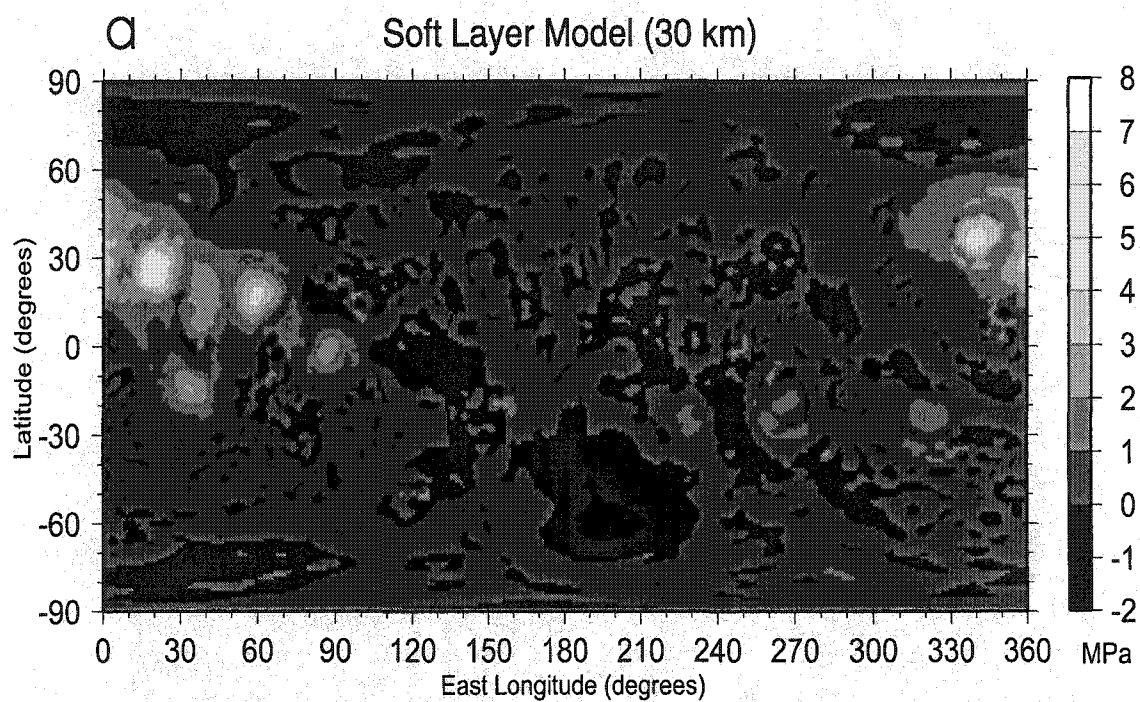


FIG. 16. The stress distribution with depth beneath Serenitatis (27° , 20°), Smythii (-2° , 87°), Humorum (-24° , 321°), and Orientale (-19° , 266°) for the soft layer model. The profiles are taken along a longitudinal west-to-east line, crossing the basin center. The latitudinal center of each basin is kept constant for the profiles. The stresses associated with the largest basins are concentrated and loaded just above the soft layer, reaching highs of 9-13 MPa. The soft layer comprises the whole region below 210 km depth, and supports no stress.

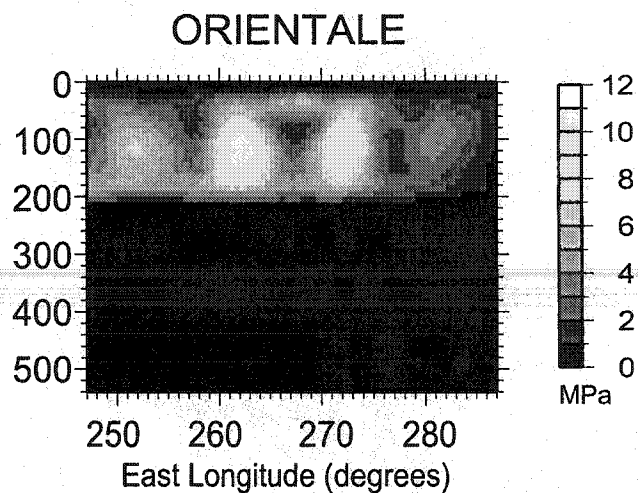
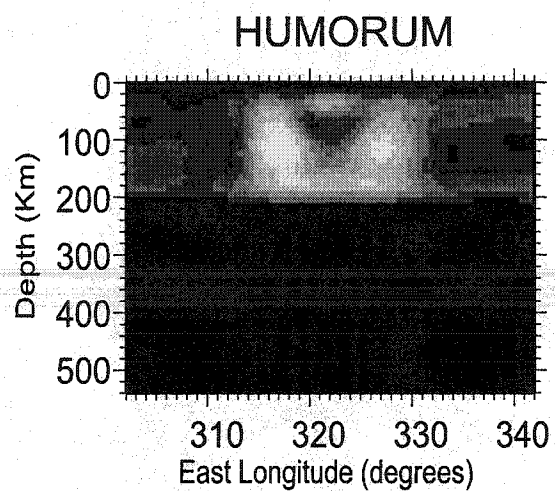
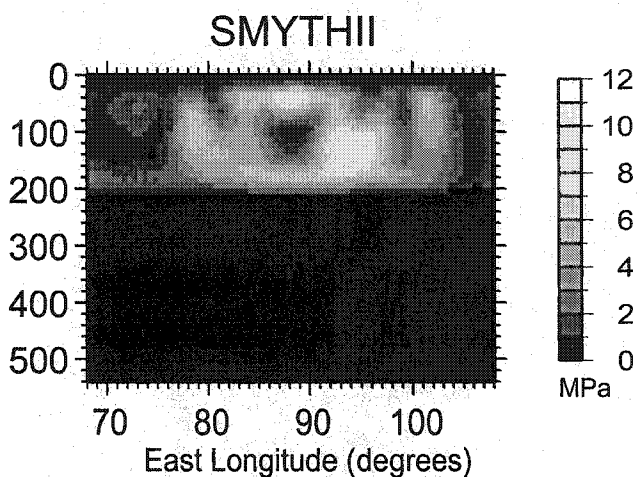
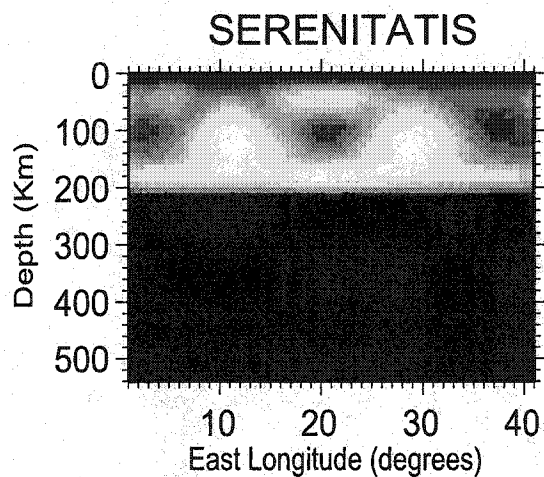
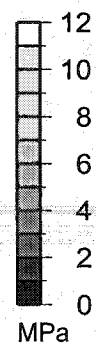
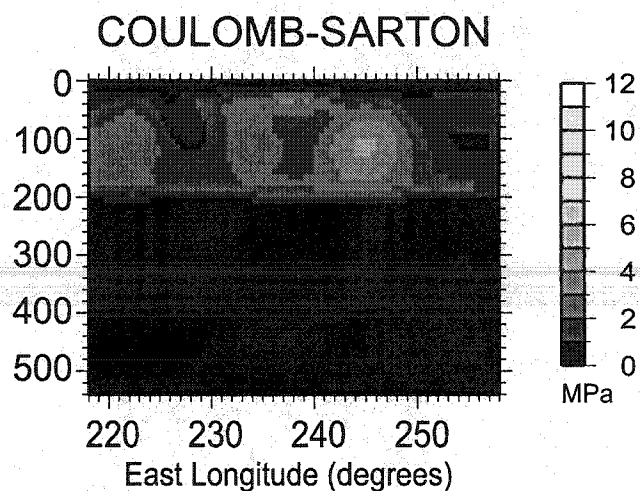
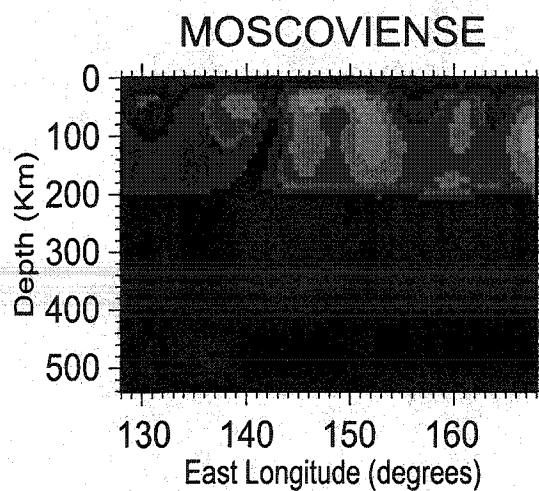
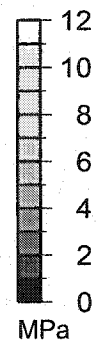
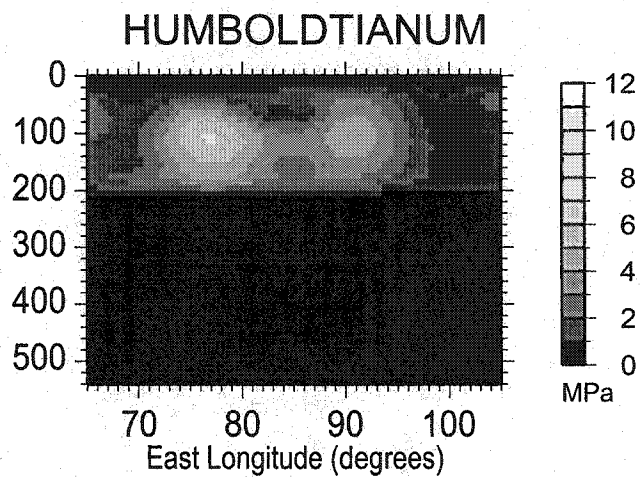
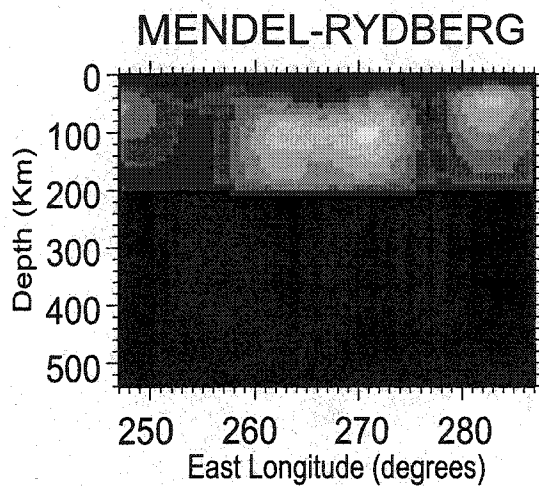


FIG. 17. The stress distribution with depth in the upper 500 km for the soft layer model beneath several mascon-like craters: Mendel-Rydberg (-50° , 266°), Humboldtianum (58° , 84°), Moscoviense (26° , 147°), and Coulomb-Sarton (52° , 237°). The profiles are taken along a longitudinal west-to-east line, crossing the crater center. The latitudinal center of each basin is kept constant for the profiles. The stresses associated with these craters are well preserved in the upper mantle, and do not extend below 210 km. The introduction of a soft layer has little effect on the stress distribution for the mascon-like craters.



appears to most significantly influence the mascon basins, suggesting that the lunar lithosphere was able to support stresses of 10-12 MPa, soon after the completion of mascon formation, roughly 3 billion years ago. The increase in stress in the lithosphere is directly due to the reduced rigidity in the soft layer region, removing any support of overlying density variations within these layers, thus leaving the overlying shell to support all surface features and loads. The lack of stresses at 180 km depth beneath the SPA basin and the lunar highlands suggests that these features may be almost completely compensated within the upper regions of this Moon model. The modeled stresses near the mascon basins at 180 km depth are the largest stresses associated with any of the variable parameter models, and are on the order and depth of those associated with the shallow moonquakes (Goins *et al.* 1981b). It is proposed that a stress distribution such as the one displayed here may have been "frozen" into the Moon after the time of decreased volcanic activity. In this way, these remnant stresses within the lunar interior may be currently playing a key role in the seismic activity on the Moon. A correlation between the shallow moonquake epicenters and the maximum lithospheric stresses will be reviewed in the next section.

It has also been proposed that, at present, there may currently exist a small amount of partial melt and near liquidus temperatures below 1000 km depth, thus explaining the high attenuation zone at such depths and providing a pooling region for stresses released by tidal forces in the form of deep focus moonquakes (Goins *et al.* 1981c; Nakamura 1983; Nakamura *et al.* 1974; Khan *et al.* 2000). A model with a partial melt layer at 1000 km depth was run, and generated only minor increases in stress in the overlying rigid middle mantle, being less than 1 MPa. However, the stress drops

associated with the deep focus moonquakes are very small, estimated at only about 0.01MPa. Stresses this small could be generated by tidal forces in a region of slightly larger stresses overlying a soft layer at depth, but they could also be generated by other smaller internal compositional or structural variations, and the implementation of a soft layer at depth is not essential for the proper representation of a stress field energetic enough to generate the deep focus moonquakes.

5.6 Effect of a Different Seismic Model

As mentioned in section 3, a new model of the internal seismic velocity structure of the Moon was recently presented. The results of the two most recent seismic models of the Moon (Khan *et al.* 2000; Khan and Mosegaard, 2001) have been used to generate an alternative seismic profile, designated the Khan model, to the Nakamura (1983) model. The higher wave velocities of the Khan model at depth increases the Lamé parameter and shear modulus (rigidity) within the Moon significantly (see Fig. 2). The increased rigidity, particularly in the deeper regions of the Moon, results in more support of long wavelength features at depth, resulting in lower density perturbations in the upper parts of the Moon. However, due to the depth of the major variations between the seismic models, the density variations within the crust of the Khan model are only moderately subdued, being generally $\pm 20 \text{ kg/m}^3$ less than the nominal model. The density perturbations diminish more slowly with depth as compared to the nominal model, leading to model variations of less than $\pm 5 \text{ kg/m}^3$ by 120-150 km depth. Similar to the Nakamura model, beyond 300 km depth the perturbations are essentially

negligible. The stress differences between the two seismic models within the crust and upper mantle are also very consistent, differing by less than ± 0.5 MPa. , both models having highs of 6-8 MPa in the crust below the SPA basin and at 150 km depth below the major mascon basins. The stresses at 150 depth are smaller for the Khan model beneath all of the mascon basins, while the stresses associated with the SPA basin are moderately larger than those of the nominal model (Fig. 7). The reduced stresses associated with the mascon basins in the Khan model are a direct result of the reduced density perturbations due to the more rigid deeper interior of this model. Smaller density perturbations result in less overlying load, in turn generating smaller overall stresses at depth.

For a full comparison of the Nakamura and Khan seismic models we also applied a soft layer in the upper mantle of the Khan model. The resulting Khan soft layer model obtains larger stresses associated with the mascon basins, with stresses being larger by 1-2 MPa. Besides the mascon basins, the lack of other stress variations between the Khan model and the nominal model is due to the relatively well constrained and consistent upper mantle structure of the Moon. The significant differences between the two seismic models are their representation of the deep interior, which is only a factor for the largest surface features. The Khan model obtains larger stresses in the deeper regions as well, particularly in the layers near the core/mantle boundary. Stress differences in this area reach up to about 1 MPa. This is in comparison to the much lower stresses (~ 0.3 MPa) for the nominal Nakamura model in the same region. The moderate density contrast at the core/mantle boundary in the Khan model, with a core of only 200 kg/m^3 difference from the overlying mantle layer, as well as the gradual increase of density in

the lower mantle, allows for more displacements and hence more stress to accumulate in the layers of the lower mantle.

The application of the Khan seismic model has been reviewed, and is shown to produce a different distribution of density and stress in comparison to the Nakamura model. The Khan model obtains smaller density variations, particularly within the crustal layers, in the case of an entirely rigid body model. This is due to the much higher rigidity of the Khan model as well as the relatively small contrast between core and mantle densities, which further facilitates compensation of long wavelength features at the core/mantle boundary. Stresses in the rigid mantle Khan model are also more subdued being about 0.5 MPa less than the nominal model beneath the mascon basin regions, due to the reduced overlying density perturbations. In the application of a partially molten layer, the variations between the two seismic models is only moderately significant with respect to the regions associated with the mascon basins. The soft layer removes the deeper support of these features, and the lower rigidity associated with the Khan model in the upper 500 km generates larger stresses for the mascon basins. Therefore, applying a new seismic model is influential on the overall distribution of density perturbations and stresses within the Moon, particularly for the largest surface features, due to the inconsistencies of the seismic models over their deeper interiors, beyond 500 km depth.

6. SHALLOW MOONQUAKES

The results from the previous section demonstrate several different scenarios for the distribution of stress within the Moon. The nominal Moon model displays stress differences of generally 4-8 MPa associated with the mascon basins in the upper mantle, above 300 km. Parameters such as model layering and core parameters can have a moderate effect on the stress distribution, but overall the stresses within the uppermost parts of the Moon remain in the same range. The soft layer model increases stresses in the layers directly overlying the reduced rigidity region, with stresses in certain areas being over 10 MPa. This model may be representative of stresses built up in the lunar lithosphere after the cessation of major lunar volcanism and at a time when the Moon became relatively cool and dead. Current day shallow seismic activity is believed to generally involve stress drops on the order of 0.2-16 MPa, with several distinct high energy shallow events having stress drops of over 20-100 MPa (Binder and Oberst 1985; Goins *et al.* 1981b). In this section, a further investigation into a possible spatial correlation between the shallow moonquakes and high stress regions within my models will be undertaken.

Shallow moonquakes have been surprising to seismologists ever since their discovery in the early 1970's (Nakamura *et al.* 1974) due to their large magnitudes of up to 5 on the Richter scale (Goins *et al.* 1981c) and their concentration in the upper 300 km of the Moon, but below the crust (Goins *et al.* 1981c; Lammlein 1977; Nakamura *et al.* 1974, 1979). These events are intriguing since essentially all large seismic events on the Earth are associated with plate tectonic activity. A study by Shirley (1986)

suggested a possible correlation between shallow moonquakes and shallow Earthquakes. Although the author was unable to propose a direct origin for the shallow moonquakes, it was suggested that the correlation between Earth and Moon quakes could demonstrate a triggering tidal effect between the two bodies. One problem with correlating shallow moonquake epicenters and modeled stresses is that the moonquake epicenters are not well constrained, and may be off by several degrees (Nakamura *et al.* 1979; Shirley 1986). The distribution of shallow moonquake epicenters and Apollo seismic stations overlying the stress distribution for the soft layer seismic models at 180 km depth is shown in Fig. 18. The majority of the shallow moonquake activity is in a region of elevated stress, displaying a moderate consistency between the stress models and the observed seismic events. The maximum stresses within three degrees of the moonquake epicenters and their respective depths for the Nakamura and Khan models are listed in table 3. Also listed in the table are the 26 positively located shallow moonquakes (of 28 total events), their location, estimated stress drops and estimated magnitudes.

For both the Nakamura and Khan models, 7 of 26 moonquake epicenters have stresses in their surrounding regions of 5 MPa or more. Of the depths associated with the maximum stresses for the moonquake epicenters, 8 are consistent with 13 source depths estimated by Khan *et al.* (2000). The modeled stress differences in this study are generally consistent, within 2-3 MPa, of the minimum stress drops estimated by Binder and Oberst (1985), with 18 of the 26 epicenters for the (nominal) Nakamura model, and 17 of 26 for the Khan model showing consistent stresses. The regions of high stress associated with the shallow moonquakes may represent source regions, which have been supported and have prevailed in the lunar lithosphere for over 3 billion years. The

FIG. 18. Global distribution of measured moonquake epicenters (white circles with black dots) overlying both the nominal soft layer model as well as the Khan soft layer model at 180 km depth. The distribution demonstrates the general proximity of shallow moonquakes to the high stress areas, near Imbrium and Serenitatis. Some shallow moonquakes are completely isolated from high model stresses. The lack of measured moonquakes on the lunar far side is possibly due to the close, near side location of all the seismic stations. The proximity of all the seismic stations effects the inversion of seismic data, and reduces the precision of the determination of moonquake epicenters as well.

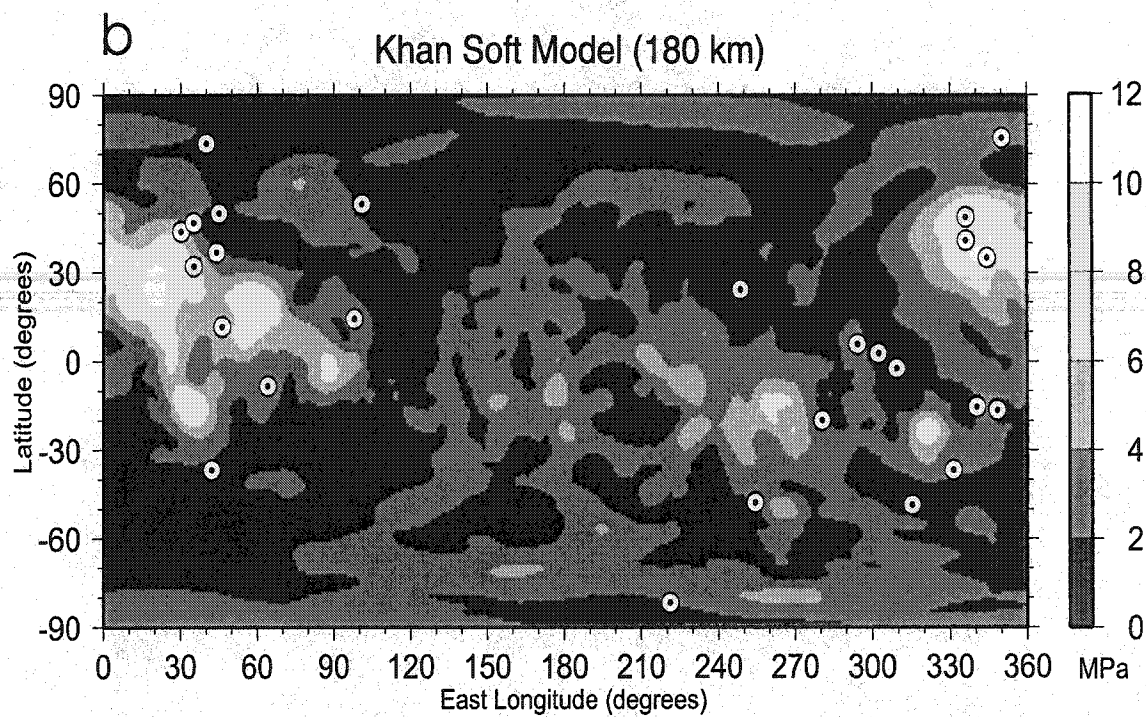
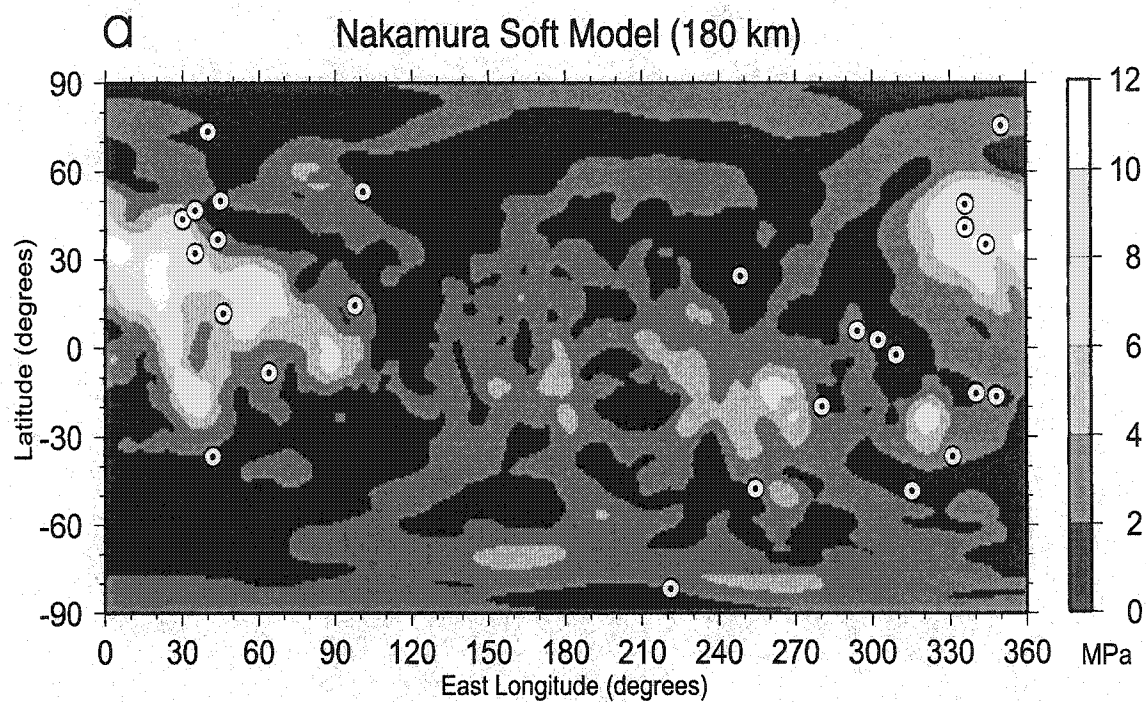


TABLE 3. Dates, locations, magnitudes, depths, proposed stress drops, and modeled stresses and depths associated with the 26 positively located shallow moonquake events. The magnitudes (M) are approximately a value of 1 lower than Richter scale magnitudes (Nakamura *et al.* 1979). The first stress and depth columns are associated with the Nakamura model (Nak), and the second two columns with the Khan model (Khan). Both the Nakamura and Khan models generate stresses which are greater than those estimated for the moonquake stress drops for 16 of the 26 shallow moonquakes. The depths of maximum modeled stress are only moderately consistent with those proposed by Khan *et al.* (2000), with only 6 of 13 depths being in relative agreement.

TABLE 3 - Shallow Moonquake Stresses

#	Date ⁽¹⁾	Lat ^(2,3)	Long ^(2,3)	M ⁽²⁾	Depth ⁽⁴⁾	Stress Drop ⁽²⁾	(Nak) Stress	Depth	(Khan) Stress	Depth
1	04-17-71	48.0	35.0	2.8	75±21	>100	3.8	60	3.6	90
2	05-20-71	42.0	336.0	2.0	48±46	>10	9.9	120	9.3	120
3	01-01-72	54.0	101.0	1.9	173±55	>6	2.7	120	2.6	120
4	09-17-72	12.0	46.0	1.0	-	>1	6.1	150	5.6	150
5	12-06-72	51.0	45.0	1.4	-	>1	2.8	90	2.7	90
6	12-09-72	-20.0	280.0	1.2	-	>0.5	4.7	90	4.5	90
7	02-08-73	33.0	35.0	0.8	-	>0.2	8.0	180	7.0	180
8	03-13-73	-83.1	221.0	3.2	49±35	>110	3.0	60	2.9	60
9	06-20-73	6.0	294.5	2.2	115±23	>16	4.0	90	3.7	90
10	10-01-73	-37.0	331.0	1.1	-	>0.8	3.4	180	3.2	180
11	02-23-74	36.0	344.0	0.7	-	>0.2	10.1	180	10.2	180
12	03-27-74	-48.0	254.0	1.6	-	>0.7	5.3	30	5.3	30
13	04-19-74	-37.0	42.0	0.9	-	>0.5	2.3	60	2.2	60
14	07-11-74	14.8	98.0	2.7	133±41	>40	3.9	90	3.6	90
15	01-03-75	24.9	247.6	3.2	75±16	>200	2.7	60	2.4	30
16	01-12-75	75.0	40.0	1.7	102±26	>5	2.4	30	2.4	30
17	01-13-75	-2.0	309.0	1.1	-	>2	2.9	90	2.7	90
18	02-13-75	-15.2	339.6	1.4	130±70	>3	2.8	120	2.6	120
19	05-07-75	-49.0	315.0	1.3	-	>2	3.5	60	3.3	60
20	05-27-75	3.0	302.0	1.4	-	>2	2.6	60	2.5	30
21	11-10-75	-8.0	64.0	1.8	73±19	>7	3.7	30	3.7	30
22	01-04-76	45.3	30.4	1.8	88±39	>4	6.4	180	5.7	180
23	01-12-76	38.0	44.0	1.1	-	>1	3.4	180	3.0	180
24	03-06-76	48.9	335.9	2.3	74±27	>20	9.6	180	8.6	180
25	03-08-76	-16.1	348.4	1.8	120±24	>9	2.7	90	2.5	30
26	05-16-76	77.0	350.0	1.5	-	>3	2.9	180	2.9	180

¹ Binder and OBerst (1985)

² Nakamura *et al.* (1979)

³ Goins *et al.* (1981c)

⁴ Khan *et al.* (2000)

largest modeled stress drops of 20-100 MPa suggested for the most energetic shallow moonquakes (Binder and Oberst 1985; Goins *et al.* 1981b) may represent anomalous stresses within the lunar lithosphere. They may have been supported in the lunar lithosphere at a time when it was thinner, evidenced by the relatively shallow depths for these higher energy moonquakes, all of them being estimated to be shallower than 133 km depth (Khan *et al.* 2000).

The seismic energy released by the Moon is dominated by the shallow moonquakes, which are much more energetic than the deep focus moonquakes (Goins *et al.* 1981b). Although the presented models, in general, do not generate stresses far greater than 10 MPa, the majority of the shallow moonquakes are well represented. The estimations of stress release for the lunar moonquakes are relatively consistent with the results presented here, with the exception of several large stress drop (> 20 MPa) shallow moonquakes. The soft layer models presented in this study are generally consistent with present day seismic observations, further supporting the validity of the soft layer assumptions, and suggesting that the present day seismic events are associated with stresses which have been rigidly supported by lunar lithosphere, and interior for the past 3 billion years. As well, it should be noted that the use of the shear-strain minimization in the present models results in proposed stress distributions which represent the lower bound on stresses within the Moon, and thus the actual stresses could be significantly larger.

7. SUMMARY AND CONCLUSIONS

A thick shell model has been adopted to represent the internal distribution of density and stress variations within the Moon. This model is more accurate than previous thick shell models (Arkani-Hamed 1973a,b) due to the incorporation of the topography, as well as the further refinement of the topography and gravity models (Konopliv *et al.* 1998; Smith *et al.* 1997). The model presented here also utilizes a more accurately modeled lunar interior, as well as applying thinner layering. The thick shell model of Arkani-Hamed (1973b) resulted in density perturbations of roughly $\pm 250 \text{ kg/m}^3$ in a 50 km thick crustal layer as a result of using only the low degree harmonics, of up to $n=13$. The density variations in the current model are generally less than $\pm 200 \text{ kg/m}^3$, with certain areas peaking to as high as $\pm 400 \text{ kg/m}^3$, in the upper 30 km of the nominal solid Moon model. The anomalies reduce to less than $\pm 100 \text{ kg/m}^3$ in the lower crust (30-60 km depth), with peaks of generally $\pm 200 \text{ kg/m}^3$. The peaks associated with SPA basin and the far side highlands are representative of compensation of these features both at the upper/lower crustal boundary, as well as at the more predominant crust/mantle boundary. The reduced resolution of the gravity data for the central far side (Konopliv *et al.* 2001) may also have a considerable effect on the strong far side anomalies. The density perturbations are concentrated in the upper 150 km, with variations of less than $\pm 5 \text{ kg/m}^3$ prevailing at 300 km depth. High stress differences within the crust generally correspond with the areas of large density anomalies, and reach highs of 6-8 MPa. These stresses are similar to the maximum stresses of 7-10 MPa determined by Arkani-Hamed (1973b). In the deeper regions stresses due to surface loads of the mascon basins

are the only remaining high stress regions, again reaching 6-8 MPa at 150 km, and 4-6 MPa at 300 km depth. This nominal model represents a simple state of the Moon; one where the entire lunar mantle is capable of supporting stress. This may be good approximation of the present state of the Moon. A soft layer was introduced into the upper mantle representing, to a first order, the distribution of density and stress after the period of major lunar volcanism. It is presumed that the surface representation of the topography and gravity has not significantly changed since this time, allowing for this representation. The associated stress field for the soft layer model is more consistent with the location and stress drops associated with the present day shallow moonquakes than the entirely rigid Moon model. The stresses associated with the mascon basins in the soft layer model reach highs of 10-12 MPa at 150-180 km depth. Thus, it is proposed that the stresses associated with the soft layer model represent a distribution of stress which was essentially frozen into the upper mantle and is being released by the triggering effect of the present day Earth-Moon tidal forces (Shirley 1986). The significantly higher viscosity of the Moon (Lefftz and Legros 1993) also supports the idea that the interior may have supported such anomalous stresses for several billion years.

The effect of other model parameters on the density and stress distribution within the Moon has been reviewed and shown to also have significant effects. The near surface density dependence on the choice of weighting factor, W_m , can have a significant effect on the modeled distributions of density and stress. The nominal choice of weighting factor varies over three distinct spherical harmonic intervals. This was done to retain a minimum total shear-strain energy, while more effectively reducing

oscillations of the mass perturbations in the upper regions of the Moon. However, due to the relationship of the density perturbations and the shear-strain energy, any choice of weighting factor has significant tradeoffs. In the current application the shear-strain minimization is regarded as the stronger constraint, and when stringently applied, the shear-strain energy constraint results in a model representing the lower bounds of density and stress within the Moon. The effect of layering can also greatly effect the calculated distribution of density. The use of overly thin layers results in the concentration of density perturbations in the uppermost layers, due to the surface gravity boundary condition. To avoid these effects, even thickness layers of 30 km were chosen based on physical evidence of boundaries within current seismic structure models of the Moon. The choice of core parameters proves to be less of an influence on the overall model results. Although there is a small difference between the end-member compositional models for the core, the effect is negligible in comparison to the effects of the weighting factor and layer variations. The application of a solid or liquid core model has equally negligible effect on the distribution of density and stress than do the different compositional core models. The lack of major effects of core parameters on density perturbations in the outer regions of the Moon results from the lunar interior which is predominately composed of mantle, with the core comprising only a few percent of the overall volume of the Moon. The longest wavelength features ($n = 3-7$) are allowed to penetrate to the core/mantle boundary, but the huge thickness of the mantle along with the large density contrast between mantle and core ($\Delta\rho \approx 1300-4700 \text{ kg/m}^3$) result in very little compensation of the long wavelengths at this depth. The features most likely

to show affects of core variations, surface features such as the SPA basin and Mare Procellarum, show very minor responses.

A different seismic model was applied and directly compared with the nominal model. The newer Khan model has several distinctly different characteristics over the Nakamura model, highlighted by being much more rigid at depths beyond 500 km, allowing this model to support larger features at depth. However, the results show only moderately smaller density perturbations and stresses within the lunar crust for the Khan model, as a result of the more rigid deep interior. The stresses for the Khan model with a soft layer become larger at 180 km depth, than in the Nakamura model by up to 2 MPa. The inclusion of the soft layer removes the support of the rigid deeper interior for the Khan model, since no shear stresses can be supported through the soft layer. However, the overall distribution of density and stress within each model is comparable.

Shallow moonquake epicenters seem to be roughly concentrated near the major lunar basins. This is in agreement with our stress distribution, which shows the strongest stresses within the upper mantle being associated with the largest lunar basins. For most of the near side, a moderate correlation exists between the moonquake epicenters and high stresses within the presented soft layer models. No shallow moonquakes have been observed on the near side, southeast region, or on the central far side region in association with either the far side highlands or the SPA basin. It has been proposed that shallow moonquakes occur on the far side, but have not been detected by the seismic network due to the limitations of the close, near side implementation of the seismic stations (Goins *et al.* 1981b). The lack of moonquakes on the near side, southeast area is more unclear; it may be due to tectonics, propagation effects, or perhaps just chance

(Goins *et al.* 1981b). The results presented here suggest that below the crust the stresses on the far side and in the southeast region of the near side are significantly smaller than near the larger, mare filled regions. The near side has a region of concentrated stress near Imbrium, extending to mare Smythii, and is in close proximity to the majority of the shallow moonquake events. The smaller stresses associated with the central far side may be sufficient to generate smaller far side quakes, gauging from the stress drops associated with the smaller shallow moonquakes (Binder and Oberst 1985; Goins *et al.* 1981b), but direct evidence of such activity is not available. A better correlation between the modeled stresses and the shallow moonquakes may not be evident due to inherent model assumptions regarding the lunar interior, the uncertainties associated with the moonquake epicenters, or to the uncertainties associated with the calculation of stress drops for the moonquakes. It appears from my results that stresses within the Moon, at present or since the end of mascon formation, have certainly not exceeded 12-14 MPa in magnitude. Even if this is taken as a lower limit for the internal stresses, due to the shear-strain energy minimization, stress drops on the order of those calculated for the largest shallow moonquakes, about 20-100 MPa, cannot be resolved by my models. Due to the shallow nature of these large moonquakes (Khan *et al.* 2000; Nakamura *et al.* 1979) it is possible that these stress drops represent remnant stresses built up at a time when the lunar lithosphere was thinner, and the stresses were greater, such as modeled by thin shell models (Kuckes 1977; Solomon and Head 1979, 1980).

Future work on lunar seismicity could greatly improve the internal modeling of the Moon, and in turn greatly improve results of modeling similar to mine. The results presented for the upper 210 km of a soft layer model, strongly support the idea of a

thick elastic lithosphere at the end of mascon forming events, suggesting that such a rapidly growing lithosphere sustained and essentially froze remnant stresses. These stresses, supported by the inherent rigidity of such a cold and thick lithosphere as well as the high viscosity interior, may have been sustained for billions of years, being released gradually by tidal triggering within the uppermost regions of the Moon. The rapidly cooling and growing lithosphere played an integral part in insulating the interior and hampering heat transfer from the core (Spohn *et al.* 2001). Thus, for all models considered, stresses did not exceed 12 MPa, suggesting that at present, this may represent the current threshold strength of the lunar interior.

8. APPENDIX

Solving the first order ordinary differential matrix equation (1), involves the use of the method of matricants (Gantmacher 1959). We start with the original equation in a slightly different form than on page 19:

$$\frac{d}{dr} \bar{\mathbf{Y}} - \bar{\mathbf{M}} \cdot \bar{\mathbf{Y}} = \bar{\mathbf{D}} \quad (\text{A-1})$$

To begin, the solution of an ordinary differential equation such as equation (A-1) requires a matrix, $\bar{\mathbf{U}}$, such that:

$$\bar{\mathbf{U}} \cdot \left[\frac{d}{dr} \bar{\mathbf{Y}} - \bar{\mathbf{M}} \cdot \bar{\mathbf{Y}} \right] = \frac{d}{dr} (\bar{\mathbf{U}} \cdot \bar{\mathbf{Y}}) \quad (\text{A-2})$$

Trying a typical form for $\bar{\mathbf{U}}$:

$$\bar{\mathbf{U}} = e^{\bar{\mathbf{A}}} \quad (\text{A-3})$$

And plugging this back into (A-2) gives

$$\begin{aligned} e^{\bar{\mathbf{A}}} \cdot \left[\frac{d}{dr} \bar{\mathbf{Y}} - \bar{\mathbf{M}} \cdot \bar{\mathbf{Y}} \right] &= \frac{d}{dr} (e^{\bar{\mathbf{A}}} \cdot \bar{\mathbf{Y}}) \\ \frac{d}{dr} (e^{\bar{\mathbf{A}}} \cdot \bar{\mathbf{Y}}) &= e^{\bar{\mathbf{A}}} \cdot \frac{d}{dr} \bar{\mathbf{Y}} + \left(e^{\bar{\mathbf{A}}} \cdot \frac{d}{dr} \bar{\mathbf{A}} \right) \cdot \bar{\mathbf{Y}} \end{aligned}$$

and therefore,

$$\frac{d}{dr} \bar{\mathbf{A}} = -\bar{\mathbf{M}} \quad \Rightarrow \quad \bar{\mathbf{A}} = -\int_{r_0}^r \bar{\mathbf{M}}(t) dt \quad (\text{A-4})$$

Thus going back to (A-1),

$$\frac{d}{dr}(\bar{\mathbf{U}} \cdot \bar{\mathbf{Y}}) = \bar{\mathbf{U}} \cdot \bar{\mathbf{D}} \quad (\text{A-5})$$

and applying the solution to this standard differential equation gives:

$$\bar{\mathbf{Y}}(r) = \bar{\mathbf{U}}^{-1}(r) \cdot \left[\int_{r_o}^r \bar{\mathbf{U}}(t) \cdot \bar{\mathbf{D}}(t) dt + \bar{\mathbf{C}} \right] \quad (\text{A-6})$$

Now, noting that: a) $\bar{\mathbf{U}}(r_o) = \bar{\mathbf{I}}$
b) $\bar{\mathbf{Y}}_o = \bar{\mathbf{Y}}(r_o)$

Where the radius, r_o , represents the radius of the central sphere. The implementation of these substitutions gives:

$$\bar{\mathbf{Y}}(r) = \bar{\mathbf{U}}^{-1}(r) \cdot \bar{\mathbf{Y}}(r_o) + \bar{\mathbf{U}}^{-1}(r) \cdot \int_{r_o}^r \bar{\mathbf{U}}(t) \cdot \bar{\mathbf{D}}(t) dt \quad (\text{A-7})$$

Now, plugging back in $\bar{\mathbf{U}}(r) = e^{\int_{r_o}^r \bar{\mathbf{M}}(t) dt}$ finally gives:

$$\bar{\mathbf{Y}}(r) = e^{\int_{r_o}^r \bar{\mathbf{M}}(s) ds} \cdot \bar{\mathbf{Y}}(r_o) + e^{\int_{r_o}^r \bar{\mathbf{M}}(s) ds} \cdot \int_{r_o}^r e^{-\int_{r_o}^t \bar{\mathbf{M}}(s) ds} \cdot \bar{\mathbf{D}}(t) dt$$

$$\text{OR} \quad \bar{\mathbf{Y}}(r) = e^{\int_{r_o}^r \bar{\mathbf{M}}(s) ds} \cdot \bar{\mathbf{Y}}(r_o) + \int_{r_o}^r e^{\int_{r_o}^t \bar{\mathbf{M}}(s) ds} \cdot \bar{\mathbf{D}}(t) dt \quad (\text{A-8})$$

Now, defining the **Matricants** as:

$$\text{For boundary effects} \quad \overline{\overline{\Omega}}(r_o; r) = e^{\int_{r_o}^r \overline{\overline{\mathbf{M}}}(s) ds}$$

$$\text{For source effects} \quad \overline{\overline{\Gamma}}(t; r) = e^{\int_t^r \overline{\overline{\mathbf{M}}}(s) ds}$$

Therefore, the final result is,

$$\overline{\mathbf{Y}}(r) = \overline{\overline{\Omega}}(r_o; r) \cdot \overline{\mathbf{Y}}(r_o) + \int_{r_o}^r \overline{\overline{\Gamma}}(t; r) \cdot \overline{\mathbf{D}}(t) dt \quad (\text{A-9})$$

This is the matricant solution of the matrix equation (A-1).

Now, if the solution is required for a domain of N layers such that $\mathbf{M}$ and $\mathbf{D}$ can be assumed constant within each layer, i , the following results:

$$\int_{r_o}^r \overline{\overline{\mathbf{M}}}(s) ds = \sum_{j=1}^N \overline{\overline{\mathbf{M}}}_j \delta_j \quad ; \quad \delta_j = r_j - r_{j-1}$$

and

$$\overline{\overline{\Omega}}(r_o; r) = e^{\sum_{j=1}^N \overline{\overline{\mathbf{M}}}_j \delta_j} = \prod_{j=1}^N e^{\overline{\overline{\mathbf{M}}}_j \delta_j}$$

$$\overline{\overline{\Gamma}}(t; r) = e^{\sum_{j=k}^N \overline{\overline{\mathbf{M}}}_j \delta_j} = \prod_{j=k}^N e^{\overline{\overline{\mathbf{M}}}_j \delta_j}$$

In a multi-layered media, with layers from $j=1$ to N , the final equation for the i^{th} layer is:

$$\overline{\mathbf{Y}}_i = \prod_{j=1}^N e^{\overline{\overline{\mathbf{M}}}_j \delta_j} \cdot \overline{\mathbf{Y}}(r_o) + \sum_{k=1}^N \left(\prod_{j=k}^N e^{\overline{\overline{\mathbf{M}}}_j \delta_j} \right) \cdot \overline{\mathbf{D}}_k \delta_k \quad (\text{A-10})$$

where the first matricant propagates the boundary effects of each layer, and the second matricant propagates the source effects for each layer.

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