ROBUST FAULT DIAGNOSIS IN LINEAR AND NONLINEAR SYSTEMS BASED ON UNKNOWN INPUT, AND SLIDING MODE FUNCTIONAL OBSERVER METHODOLOGIES

by

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Abstract

The field of observer based fault diagnosis for complex control systems has become an important topic of research in the control community over the last three decades. Recently, special attention has been paid to the problem of robust fault diagnosis for linear and nonlinear uncertain systems. Many proposed fault diagnosis approaches are based on robust state observer techniques, which can provide the right estimation of system states under the existence of a large class of model uncertainties and disturbances, known as unknown inputs. It is noted that robust state estimation requires strong restrictive existence conditions, which confines its practical application. On the other hand, it is unnecessary to estimate all states for the objective of fault diagnosis. This thesis is an attempt to accomplish robust fault diagnosis under weaker existence conditions through the development of the unknown input, and sliding mode functional observer theory. The proposed functional observers can estimate a function of system states by decoupling the effect of the unknown inputs.

The necessary and sufficient conditions for the existence of unknown input functional observer (UIFO) for linear systems are obtained with the aid of Loop Transfer Recovery (LTR) technique. A constructive design procedure is given. The problem of estimating the unknown input is also addressed. Two kinds of reduced-order unknown input estimators using only measured outputs are presented. They extend full-order input estimators design in the existing literature and have advantage of working for certain class of non-minimum phase systems.

Under a UIFO framework, the unknown input decoupled residual generator is developed, and the remaining freedom for fault diagnosis observer design, after unknown input decoupling, is completely revealed. A fault diagnosis algorithm is proposed,

which combines unknown input decoupling theory and the Beard-Jones detection filter, or input estimator. This algorithm offers maximum residual dimension and is therefore more applicable than existing robust fault diagnosis schemes which are based on unknown input observer. Representation of a sensor fault, as a mathematical equivalent of an actuator fault, is further developed. The structured properties of the augmented system for sensor fault detection are provided.

The results for linear systems are extended to bilinear systems with unknown inputs. For bilinear systems, a robust fault diagnosis observer with linear estimation error dynamic can be derived under special structured conditions. A robust fault diagnosis observer with bilinear estimation error dynamic which improves the fault isolation capability of the system is proposed under less conservative conditions. For a class of bilinear systems with bounded control inputs, the existence conditions for a robust fault diagnosis observer are relaxed further.

A robust functional observer design, using the sliding mode principle, is studied in depth for linear systems and for a class of nonlinear systems, which are subject to bounded unknown inputs. The connections between the unknown input observer and the sliding mode observer methodology are investigated. It is shown that a sliding mode functional observer (SMFO) can be designed under weaker conditions than those for UIFO. Finally, the potential, advantages and disadvantages of fault diagnosis using SMFO are discussed extensively.

Numerical examples are presented throughout the thesis to illustrate the applicability of the proposed estimation and fault diagnosis methods. Many of these cannot be handled by the existing methods in the literature.

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Dedication

To my wife Liyan, my son Ze-Kun Victor and my parents.

Table of Acronyms

BJDF:	Beard-Jones Detection Filter
DOF:	Degree of Freedom
ELTR:	Exact Loop Transfer Recovery
FDIA:	Fault Detection, Isolation and Accommodation
FDI:	Fault Detection and Isolation
GLRT:	Generalized Likelihood Ratio Test
GUIO:	Generalized Unknown Input Observer
LTR:	Loop Transfer Recovery
LMI:	Linear Matrix Inequality
PIO:	Proportional Integral Observer
SCB:	Special Coordinate Basis
SII:	Structural Invariant Index
SMO:	Sliding Mode Observer
SMFO:	Sliding Mode Functional Observer
SMOFO:	Sliding Mode Output Functional Observers
SMOO:	Sliding Mode Output Observer
SPD:	Symmetric Positive Definite
UIE:	Unknown Input Estimator
UIFO:	Unknown Input Functional Observer
UIFDO:	Unknonw Input Fault Diagnosis Observer
UIO:	Unknown Input Observer
UIRG:	Unknown Input Residual Generator

Contents

List of Figures

Chapter 1

Introduction

The increasing complexity and risk of modern control systems and the growing demand for quality, cost efficiency, reliability and safety have led to a increasing demand for on-line automatic fault detection, isolation and accommodation (FDIA) capabilities in automatic control systems. Component failure can have disastrous effects on the operation of any system, and the consequences can be extremely serious in terms of massive property damage and loss of life. For example, aircraft accidents claim the lives of many people and many accidents are the result of instrument failure [45]. The accident in Three-Mile Island-2 nuclear power plants in 1979 resulted in almost total destruction of the reactor core. Today, FDIA requirement extends beyond systems normally accepted as safety-critical, e.g., nuclear reactors, aircraft and many chemical processes, to systems such as autonomous vehicles and some process control systems where the system availability is vital. Since 1998, the new California Air Resource Board (CARB) and the Environmental Protection Agency (EPA) regulations have required that On Board Diagnostics Generation II (OBDII) should be rolled into all light duty vehicles sold in the North American fleet since 1998. OBDII requires fault detection capability for all components whose failure can result in emission levels beyond a certain level. With so many driving forces, the techniques of detecting and isolating system faults have been of considerable interest to the civil and military industries as well as to the university researchers during the last three decades ([29], [36], [85], [88], [113], [122]). The goal of this research effort is to develop more effective solutions for fault detection and isolation in automatic control systems.

1.1 Basic Concept of Fault Diagnosis

Faults in an automated system can occur in both hardware and software of the plant and the control units. In this thesis, we concentrate upon the hardware faults. The term *fault* is used to mean an unexpected change in the functional units that tends to degrade overall system performance and leads to undesirable but still tolerable behavior of the system. The term *failure* suggests complete breakdown of the system. Typical examples of faults are:

1. Construction defects such as cracks, ruptures, fractures, leaks and loose parts.

2. Actuator faults such as damage in the bearings, deficiencies in force or momentum, defects in the gears, and aging effects.

3. Sensor faults such as scaling errors, hysteresis, drift, dead zone, short cuts, and contact failures.

4. Abnormal parameter variations in the system.

5. External obstacles such as collisions and clogging of outflows.

From the viewpoint of fault diagnosis, faults are divided into three categories: actuator faults, component faults (faults in the framework of the process), and sensor faults, as shown in Figure 1.1. The faults can commonly be described as input signals. In addition, there is always modelling uncertainty due to unmodelled disturbances, noise and model mismatch. This may not be critical for the system operation but may obscure the fault detection by raising false alarms. The modelling uncertainty is taken into consideration by vectors of *unknown inputs*.

The purpose of fault detection is to determine the occurrence of faults in the functional units of the plant. After detection, fault isolation procedures are used to determine the location, type and magnitude of the fault. These two steps constitute the concept of fault diagnosis, or fault detection and isolation (FDI). The objective

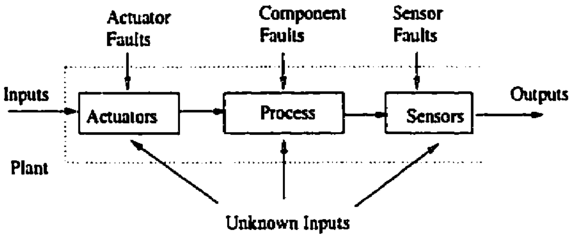

Figure 1.1: Definition of faults in the plant of the process

of fault accommodation is to reconfigure a system, in particular the controls, so that performance degradation is minimized. Fault accommodation techniques are beyond the scope of this thesis, and can be found in [53].

Many fault diagnosis methods have been developed and can be divided into three categories:

1. Hardware redundancy

   One of the easiest ways to detect and identify a fault is to use a replication of hardware in what is known as a hardware redundant system in which outputs from identical components are compared for consistency. Massive sensor and control actuator redundancy is often required in advanced flight control systems and in nuclear reactor control systems in order to provide sufficient safety margins against failure. The hardware redundancy for system monitoring or diagnostic facilities needs more space, consumes more power, and can be very expensive. Sometimes it may cause complex problems when incorporated with other redundant devices.

2. Signal-based approach

   Another well established method in practice is the signal-based approach. In this case, one extracts proper signals or symptoms from the system, which carry as much information as possible about the faults of interest. Typical symptoms are the magnitude of the measured signals, arithmetic or quadratic mean values, statistical moments of amplitude distribution or envelop, spectral power densities, correlation coefficients, covariance, etc. Clearly, the signal-based methods are limited in their efficiency, in particular for early fault detection and for the detection of faults that occur in dynamic systems during their transient operation.

3. Model-based approach

   More powerful than the signal-based approach is the model-based approach. A general scheme of model-based fault diagnosis is depicted in Figure 1.2. The main tasks can be subdivided into fault detection by residual generation, and fault isolation by residual evaluation.

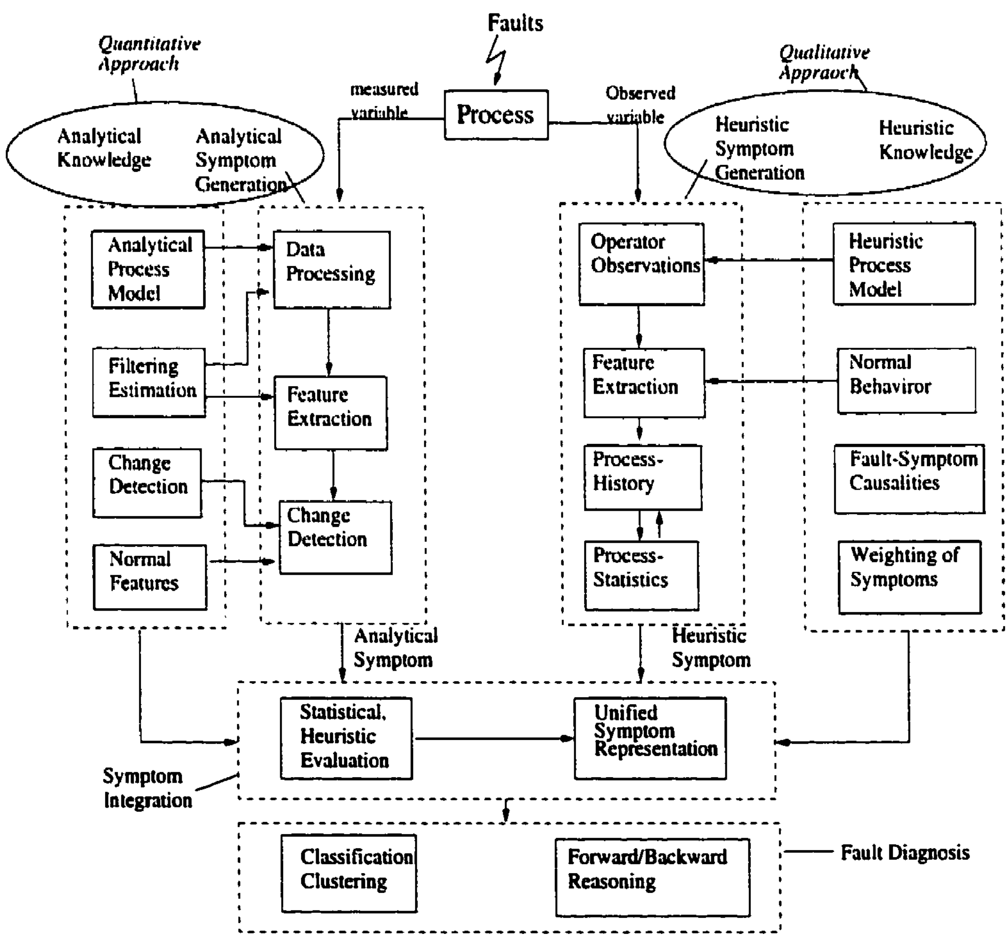

Figure 1.2: Scheme of model-based fault diagnosis

Figure 1.2 illustrates the two main research fields in model-based fault diagnosis,

i) **Analytical model-based FDI**

In this approach, the analytical knowledge of the process is used to produce quantifiable, analytical residual information. To do this, first, data processing has to be performed based on measured process variables to generate characteristic values. In some cases, the special features can be extracted from these characteristic values, e.g. physically defined process coefficients or special filtered or transformed residuals. These features are then compared with the normal features of the non-faulty process to detect and isolate the faults. There are three analytical model-based fault diagnosis schemes:

1. Parity Space Approach

   The parity space approach is based on the test of consistency of parity equations by using the sensor outputs and applied inputs. From the inconsistency of the parity equations one can detect the faults. Chow and Willsky [15] considered parity space generalizations from the state space model of the system. Further contributions focusing on transfer function relations are due to Gertler and his co-workers [37].

2. Observer-based Approach

   The basic idea of observer-based methods consists of the reconstruction of the outputs of the system of interest, with the aid of observers or Kalman filters, and the use of the estimation error (or innovation, respectively) as the residual. The observer feedback gain enters the calculation of the residual generator and the gain design problem provides freedom for achieving the required performance (see [2],[24],[97]). The parity space approach is considered to be an open-loop strategy for residual generation, and the observer-based approach is a closed-loop strategy. The closed-loop strategy offers greatly improved potential, in terms of robustness, over the open-loop methods.

3. Parameter Estimation and Identification Approach

   This approach is based on the assumptions that the faults are reflected in the physical system parameters. It uses well-known parameter estimation methods to identify the parameters on-line. The results are compared with parameters of the reference model obtained initially under fault-free conditions. A change in the process is indicated if substantial discrepancy exists [40],[54]. The detection logic should be intelligent enough to distinguish the real cause of the parameter change which may be due to faults or disturbance.

Reference [38] compares similarities and differences among above three approaches.

ii) **Qualitative model-based (or knowledge-based) FDI**

Heuristic symptoms can be produced by using qualitative information provided by human operators, or based on a qualitative model. The qualitative model is derived in terms of facts and rules from the description of the system structure and behavior (first principle). Both the dynamic behaviors of the process and heuristic symptoms are characterized by a small number of symbols, or by qualitative values like on, or off, or by limit values. Many kinds of diagnostic reasoning strategies have been developed [85],[113], where artificial intelligence, expert systems and fuzzy logic techniques play key roles. Reference [29] provided an excellent survey of qualitative model-based residual generation methods.

The purpose of fault diagnosis is to detect the faults of interest and their causes as early as possible, so that failure of the overall system can be avoided. Of particular importance is the detection of incipient (soft) faults. There are several reasons for incipient fault detection: avoidance of dangerous operating conditions, long distance diagnosis, increases in productivity, and automatic quality assurance. For those large-scale complex systems for which mathematical models are very difficult to build, qualitative model-based methods may be the only choice. However, analytical model-based methods are the most capable of detecting incipient faults, since the mathematical model of the system represents the deepest and most concise knowledge of the process. In practice, mathematical models of many systems can be built approximately. The problem is that an accurate model of the plant is very difficult to obtain in many practical situations. The task of robust FDI is to design residuals which are highly sensitive to instrument faults, but insensitive to process model uncertainties.

This is clearly a very tough task. The approaches based on state observer are one of the most important and favored methods in robust model-based FDI because it offers more design freedom, compared with other approaches.

In section 1.2, an overview of past and current work on robust observer-based FDI techniques is provided, which is the main topic in this dissertation.

1.2 Robust Observer-Based Fault Diagnosis: An Overview

An observer provides an estimation of variables for a dynamic system by using the known system's inputs and outputs measured by sensors, and mathematical model of the system. In standard control theory, the aim of an observer is to estimate precisely the running value of the state of system. In most controlled systems, the dimension of output vector is less than that of the state vector for several reasons (e.g. technical, cost, etc.). On the other hand, many control laws derived from state space concept often require the knowledge of the state. Therefore, state observers are often necessary for actual implementation of the controller. Linear observer theory has been well established since the work of Luenberger in 1964 [69] and Kalman and Bucy in 1961 [60]. However, special attention has to be paid when applying observer theory for fault detection and isolation. The basic configuration of observer-based fault diagnosis is shown in Figure 1.3.

The development of the observer-based FDI method began in the early 1970s. Beard [2] and Jones [58] introduced the so-called Beard-Jones detection filter (BJDF). BJDF is a generalized observer designed in time domain such that the effect of different faults map into different directions or planes in the residual vector space. In 1986, Massounia [72] reformulated the BJDF problem and solved it in a geometrical framework. White and Speyer [124] improved the design procedure using a spectral approach, and more recently Park and Rizzoni [82] developed a closed-form expression of BJDF using eigenstructure assignment. In a stochastic setting, Willsky and Jones [122] proposed the Generalized Likelihood Ratio Test (GLRT). The GLRT is a

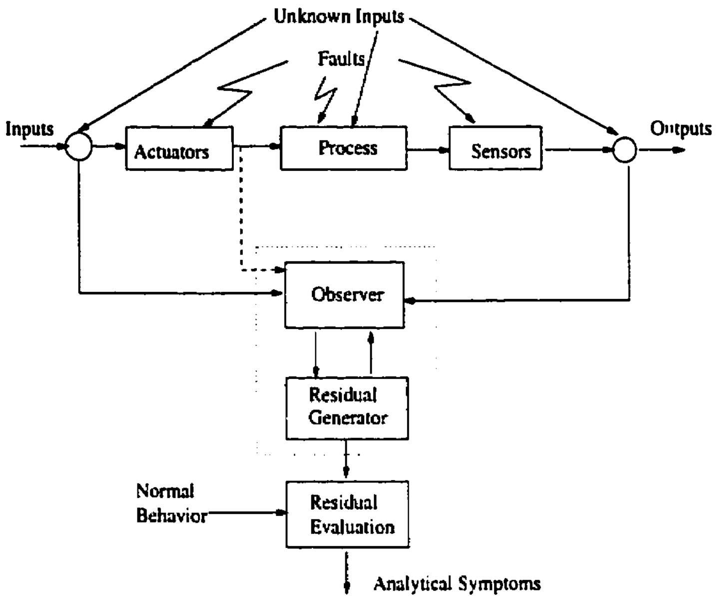

Figure 1.3: Basic configuration of observer-based fault diagnosis

statistical test that looks for a change in the statistical properties of the innovation; a Kalman filter is used for generation of innovation.

These early works were based on the assumption that the system under consideration is linear and that a sufficiently accurate mathematical model of the system is available. False alarm may happen frequently when these approaches are used to linear systems subject to unknown inputs, because the fault signal and unknown input are very likely to produce a similar residual signal. A straightforward method to create robustness with respect to unknown inputs is to generate unknown input decoupled residuals. If conditions for a perfect decoupling are not fulfilled, one can determine an optimal approximation in the sense of compromise between sensitivity with respect to faults, and robustness with respect to unknown inputs. The problem of perfect decoupling of faults from the unknown inputs has been attacked in both frequency and time domain. A frequency domain solution was given by Frank and Ding in 1994 [28]. There are three kinds of time domain solutions. The first one is known as the *unknown input observer (UIO)* based approach (see Chang and Hsu [11], Chen *et al.* [13], Frank *et al.* [24], Saif and Guan [97]). The second is the eigenstructure assignment approach (Patton and Chen [86], Wang and Daley [118]).

These two approaches do not make assumptions on the size and the time functions or on the frequency characteristics of the faults or of the unknown inputs. The third is the sliding mode FDI observer (Hermans and Zarrop [47]), where unknown inputs are assumed to be bounded. The practical importance of the decoupling approach lies in the fact that it allows small faults to be detected, even if there are large modeling errors. Of course, certain conditions have to be satisfied in order for decoupling between faults and unknown inputs to be made possible.

Recently, some methods to improve the robustness of BJDF have been proposed by Dougles in 1993 [19] and Chung in 1997 [16] respectively. The GLRT was generalized by Mangoubi in 1995 [71] to make it insensitive to a large class of noise and plant model uncertainties. These works were based on the newly developed H_∞ and differential game theory. and are much more complicated than unknown input decoupled residual design.

As a first step. all the investigations mentioned above considered linear multivariable uncertain systems with a nominally linear time-invariant part and an unknown input part. Many industrial processes are of nonlinear nature. and consequently. have a nonlinear mathematical model. The use of linear robust approaches is limited if the system to be monitored is strongly nonlinear. Linear uncertain system models can cover a small class of nonlinear systems by representing nonlinear parts as unknown inputs. However. they will introduce too many unknown inputs which will make perfect or approximate decoupling difficult. Therefore, the study of nonlinear observer-based FDI has received considerable attention in the past few years. With the application of nonlinear observer theory, some nonlinear system FDI approaches have been obtained, principally in the detection, and also with some restrictions, for isolation of faults.

Based on a Thau-type nonlinear observer, the BJDF method is generalized by Garg and Hedrick in 1995 [32] for a class of Lipschitz nonlinear systems, with the linearity assumption being made on the output vector. A nonlinear observer, which was constructed by sliding mode design techniques, was used for fault diagnosis of control affine nonlinear systems by Krishnaswasi and Rizzoni in 1995 [62]. For state affine systems, Hammouri *et al.* used Kalman-like time-varying observers to build the residual for FDI [44]. The fault diagnosis based on UIO for linear systems (in different versions) is generalized to bilinear systems by Yang and Saif [136], Yu and Shields [140]. Seliger and Frank [100], Yang and Saif [136] extended linear UIO to a more general class of nonlinear systems by applying a nonlinear state transformation, and applied their proposed nonlinear UIO to fault diagnosis for nonlinear uncertain systems.

1.3 Thesis Outline

Although there is a wealth of research in robust observer-based FDI methods, there are still many unsolved problems, even for linear systems. In the remaining chapters, we seek to build improved robust observer-based FDI schemes. Many of the existing robust FDI approaches are based on the solution of UIO. However, due to restrictive conditions for UIO design, the possibility of direct use of the related methods in various applications is limited. UIO obtains estimation of all state variables under existence of unknown inputs. In many FDI applications, it is rarely the case that an estimation of all states is really required. In the other word, UIO provides redundant information for FDI application. In this thesis, the robust FDI schemes are based on robust estimation of a function of states. Two approaches to achieve robust estimation are studied, namely unknown input functional observer (UIFO) and sliding mode functional observer (SMFO). These observers can generally be solved under weaker conditions than those for UIO, and extend the possibility for designing a robust FDI scheme.

Generally, the robust FDI problem is a multiobjective optimization problem. Given the measurements, faults and unknown input distribution matrices of a system, the objective is to build a residual vector with maximum dimension, where the effects of unknown inputs should be minimized. In the meantime, the residual sensitivity corresponding to faults should be maximized and different faults should result in different residual modes. This is by far more complicated than the task of getting robust estimation of states or a function of states. The No Free Lunch (NFL) theorem [70] tells us that *without any structural assumptions on an optimization problem,* *no algorithm can perform better on average than blind search.* Though far less celebrated than other famous impossibility theorems, the NFL theorem does remind us that structural information and analysis is very important for optimization problems. Throughout this thesis, a tool called Special Coordinated Basis (SCB) transform [94] is used for both the analysis and design of the robust FDI observer for linear systems. The SCB decomposes the state, input and output spaces of a system into several distinct parts. In doing so, SCB displays explicitly the structure of a given system much more clearly than any other existing tools, although it may seem very complicated at the glance. SCB has been a successful tool in robust control investigations [94, 95], and its application for robust FDI has not been reported before. Nonlinear coordinate transformation [56] is used when nonlinear systems are considered. With complete knowledge of the inner structure of a given system, with both unknown inputs and faults, several new theorems on the conditions for solvability of unknown input functional observer, robust sliding mode functional observer and robust FDI observers are derived. Moreover, all of these conditions are interpreted in terms of certain well-known structural properties of linear multivariable systems. These results are not only significant in theory, but also very useful in practical applications.

A preview of each chapter is given as follows. Chapter 2 addresses the problem of estimating linearly independent functions of state variables of a linear system that is driven by both known and unknown inputs. The necessary and sufficient conditions for the existence of the observer, referred to as a UIFO, are provided. Furthermore, a constructive design approach, based on the Loop Transfer Recovery (LTR) theory, is presented.

Compared with state estimation, very little research has been carried out on the estimation of unknown inputs. The early methods of estimating unknown inputs require differentiation of the measured output, which is generally undesirable due to measurement noise. Chapter 3 examines the problem of unknown input estimation using only measured outputs for linear time-invariant systems. Two kinds of reduced-order input estimators are developed. The first is an extension of the state/input estimator proposed by Corless and Tu [17]. The second is based on an adaptive observer technique [119]. Except for the different format, both of them are designed using the same algorithm. The main achievement of this extension is that the proposed approach, as opposed to previous studies, works for certain non-minimum phase systems.

The results in Chapter 2 and 3 are used in Chapter 4 to derive a systematic design approach for an unknown inputs fault detection observer(UIFDO). The proposed design approach offers computational simplicity and unifies previous methods using the eigenstructure assignment and the classical unknown input observer theory. It is worth noting that our novel design method will result in the maximum dimension of the residual vector, which was not previously reported. Special attention has been paid also to the following three problems: a) generalization of the Beard-Jones Detection Filter (BJDF) to linear uncertain systems; b) direct estimation of the faults from the unknown input decoupled residual, using the input estimator in Chapter 3; c) exploiting the special properties of sensor fault detection and isolation, which is different with actuator or component fault FDI.

Chapter 5 discusses the design of robust fault diagnosis observers for a class of nonlinear systems, namely bilinear systems. It is generally true that observability and therefore observer existence for nonlinear systems (including bilinear systems) is input dependent. Bilinear observers, whose existence is input independent, generally may exist under certain restrictive conditions. In this chapter, we present two alternatives for the design of such observers, along with existence conditions for each of the observers. The first proposed observer is based on extension of the results in Chapter 4. This observer is attractive due to simplicity in its design, where observer error dynamics is made linear. The second proposed observer has a bilinear error dynamics. Its design is more complicated than the first observer. However, the advantage offered by this observer is improved capability for fault isolation. Finally, for bilinear uncertain systems with bounded inputs, an unknown input decoupled fault diagnosis observer is proposed. It exists under less restrictive conditions than those for the input independent observers.

Chapter 6 is devoted to the development of SMFO for linear and nonlinear systems with unknown inputs. Through analysis on the difference of two famous sliding mode observers (SMO) design, namely Utkin SMO and Walcott-Zak SMO, a fundamentally new SMO is proposed which can handle more unknown inputs than existing SMO. It proves also that the Walcott-Zak SMO is equivalent to UIO in terms of existence conditions. It provides the conditions under which the proposed SMO will not be able to estimate all states, and suggests a SMFO design approaches. These results are extended to a general class of nonlinear uncertain systems. In addition, we shed light on some of the earlier works on nonlinear unknown inputs observer in this chapter.

In Chapter 7, the robust FDI schemes using SMFO are discussed extensively. It is shown that SMFO based FDI method can work under certain conditions where the UIFO based FDI method is inapplicable.

Chapter 8 summarizes the contribution and states the conclusion of this thesis. It also gives suggestions for areas of future research.

Chapter 2

Unknown Input Functional Observers for Linear Systems

The problem of estimating the state of a linear time-invariant multivariable system, subject to arbitrary unknown inputs, has received considerable attention over the last two decades [42, 49, 63, 117]. The unknown inputs may represent unknown external excitation, an unmodelled or time-varying part of a system that has a nominal and known linear part. The problem is significant since, in practice, there are many situations where there are unknown inputs present and where a conventional observer cannot provide the correct estimation of the system's state. Besides being of theoretical importance, the unknown input observer (UIO) technique provides guidelines for treating various problems. Its application in the design of fault diagnostic observers and decentralized observers are well known [11, 13, 96, 97].

On the other hand, however, the restrictive conditions for the UIO design limit its practical application. In many applications it is unnecessary to estimate all states. If observing a function of the states is amenable for the underlying tasks, the observer problem can generally be solved under weaker conditions. This situation corresponds to the problem of unknown input functional observers (UIFO).

This chapter examines the UIFO problem from the viewpoint of the exact loop transfer recovery (ELTR) theory. We will show that there is a close relationship between the UIFO and the ELTR problems. Based on this relationship and on the existing result of ELTR theory, we derive necessary and sufficient conditions for the existence of UIFO. It will be shown that as opposed to the existence conditions for UIFO given by Tsui in [110], our conditions are weaker, and the proposed design procedure is explicit and simple. It also proves that some results given by Tsui in [110] are incorrect. The application of the UIFO theory for fault diagnostic observer design will be discussed in Chapter 4.

2.1 Problem Formulation

Consider a linear time-invariant system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Gd(t) \\ y(t) &= Cx(t) \end{aligned} \tag{2.1}$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^m, d \in \mathbb{R}^q$ and $y \in \mathbb{R}^p$ are respectively the state vector, the known input vector, the unknown input vector and the output vector of the system. Matrices A, B, C and G are nominal system matrices with compatible dimensions. Without loss of generality, it is assumed that rank $(G) = q$ and rank $(C) = p$. It should be noted that this model can represent a class of linear systems with uncertain parameters. It can also represent a class of nonlinear, time-varying and time-delay systems with a nominal linear part, because $d(t)$ can be any signal. The way to represent different kinds of system uncertainties as unknown inputs term Gd in the above formulation is given explicitly by Saif and Guan in [97]. Because distribution matrix of unknown input, namely G in (2.1), is known and constant, the unknown input is considered as the structured uncertainties. On the other hand, a system with unstructured uncertainties can be described by

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A)x(t) + (B + \Delta B)u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{2.2}$$

It is assumed that $\|\Delta A\| \leq \alpha$ and $\|\Delta B\| \leq \beta$, where α and β are finite real scalars and are known *a priori*. However, no assumption is made on the structure of the matrices ΔA and ΔB. In this work, we shall only concentrate on systems that can be described by (2.1).

The objective of the UIFO problem is to find a Luenberger-type observer of the form

$$\begin{aligned} \dot{z}(t) &= Fz(t) + Ly(t) + TBu(t) \\ w(t) &= Nz(t) + My(t) \end{aligned} \tag{2.3}$$

such that the estimation error, defined as $e(t) = z(t) - Tx(t) \to 0$ as $t \to \infty$ for arbitrary $z(0)$ and $x(0)$. In addition it is desired to have $e(t)$ independent of the unknown inputs $d(t)$. In the above, T is a full row rank constant matrix which is not specified *a priori*. To obtain as much information about the states as possible in the observer design, matrices F, L, M and N should be so determined such that T has as many linearly independent rows as possible.

It is easy to show that the observation error is governed by the differential equation

$$\dot{e}(t) = Fe(t) + (FT + LC - TA)x(t) - TGd(t).$$

Immediately we know that a solution to the UIFO problem exists if and only if matrices F, L and T satisfy that

$$F \text{ is Hurwitz,} \tag{2.4}$$

$$TA - FT = LC, \tag{2.5}$$

$$TG = 0. \tag{2.6}$$

Depending on the situation under consideration, we may require T to satisfy additional properties. It would be of interest to identify these situations. As an example, in order to use the observer (2.3) for estimating all states of the system, T must satisfy

$$T_c = \begin{bmatrix} T \\ C \end{bmatrix}_{n \times n} \text{ is invertible.} \tag{2.7}$$

In such a case, the classical unknown input observer (UIO) is obtained, where by setting

$$[N \quad M] = \begin{bmatrix} T \\ C \end{bmatrix}^{-1}$$

we can achieve $w(t) \to x(t)$. On the other hand, if

$$rank(C) < rank(T_c) < n, \tag{2.8}$$

only part of the state variables can be estimated. However, dimensionwise, a larger function of states can be estimated than the outputs. The observer (2.3), with F, L and T satisfying conditions (2.4)-(2.6) and (2.8), is called a generalized unknown input observer (GUIO).

2.2 Connection between the LTR and UIFO

Almost all previous research on UIFO problem has concentrated on solving matrix equations (2.4)-(2.6) and (2.7) or (2.8) by various techniques. Here, we shall use a fundamentally different method. To do this, we start from the review and understanding of the well known loop transfer recovery (LTR) theory from the viewpoint of unknown input observer design.

The LTR concept was first introduced by Doyle and Stein in [20, 21], where it was shown that an observer based closed loop control system may suffer from loss of robustness. The loop transfer function determines the sensitivity and robustness of its corresponding feedback system. In standard LTR theory, it is believed that because the controller contains feedback from the plant inputs (since the observer always contains control inputs), the loop transfer function of the corresponding observer based feedback system is not the same as that of the full-state feedback system. Therefore, controllers realized by state observers have different robustness property from the controllers using ideal full-state feedback. To remedy this problem using LTR techniques, the target loop is "recovered" by suitable feedback control gain and observer gain design.

The plant model considered in LTR is

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t). \end{aligned} \tag{2.9}$$

Assume a full-order observer is used, with T and K being the state feedback and the observer gains respectively. The observer is of the form

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K(y(t) - \hat{y}(t)). \tag{2.10}$$

Lemma 2.2.1 *[94] The target loop transfer function (based on full state feedback controller) and that achieved by the observer based controller, are the same, i.e. exact LTR is achieved, if and only if*

$$A_c = A - KC \quad \textit{is Hurwitz} \tag{2.11}$$

and

$$M_d(s) = T(sI - A + KC)^{-1}B = 0 \tag{2.12}$$

In theory, LTR is not a pure observer design problem because the controller gain T may be the part of LTR's design parameters. However, because the freedom in choosing the controller gain is often used to satisfy closed loop system specifications, almost no extra freedom remains within the controller gain design so that it may be used further to recover the target loop. As a result, the observer is the main tool in achieving the desired goals.

When we consider the recovery of loop transfer function from input, we have implicitly assumed that the system has unknown inputs, whose input matrix is the same as B. In such a case, the system model can be expressed as

$$\dot{x}(t) = Ax(t) + Bu(t) + Bv(t). \tag{2.13}$$

For system (2.13), if the observer in (2.10) is used, an estimation error would exist. The error dynamics in this case is given by

$$\dot{e}(t) = (A - KC)e(t) + Bv(t). \tag{2.14}$$

If $\hat{x}$ is directly used for feedback control, the robust property of the full state feedback system will be kept, if and only if

$$u(t) = T\hat{x}(t) = T(x(t) - e(t)) \to Tx(t) \text{ or } Te(t) \to 0. \tag{2.15}$$

Clearly it is impossible to estimate the state of the system with no error by using the observer in (2.10) if there are disturbances in the system, unless rank $C = n$. Therefore, the following will hold

$$Te(t) = Te^{A_c t}e(0) \to 0$$

if and only if $T(sI - A + KC)^{-1}B = 0$ and A_c is stable, where $e(0)$ is the initial estimation error. On that basis, (2.15) is equivalent to the ELTR condition (2.12). In other words, ELTR is equivalent with the requirement of $Tx(t)$ being estimated under existence of unknown inputs $Bv(t)$. It should be noted that the procedure for deriving Lemma 2.2.1 in LTR literature is much more complicated.

Going back to the UIFO design probelm, a mathematically compatible problem is to make $T\hat{x}(t)$ approximate $Tx(t)$, where $x(t)$ is the state for a linear system (2.1). which is subject to general unknown inputs $Gd(t)$. This can be considered as a full-order UIFO problem, which is solvable if and only if T and K satisfy (2.11) and

$$M(s) = T(sI - A + KC)^{-1}G = 0. \tag{2.16}$$

Then results in LTR theory can be used directly to solve the full-order UIFO problem. The close relationship between the full-order and reduced-order observer is used to extend the result for the general UIFO problem defined by Section 2.1. In LTR design, T is independent with requirement of $Te = 0$ due to its role as controller gain. Therefore the only way to achieve ELTR with a full-order observer is to make e locate in $Ker(T)$. On the contrary, T is an important design parameter for UIFO. It is this difference that will be exploited here to derive the complete solution for the UIFO problem.

2.2.1 LTR and Special Coordinate Basis

The LTR theory has been well established after more than ten years of study. It turns out that the structural properties of a given system play a dominant role in the LTR (see [94]). To that end, we would like to recall a Special Coordinate Basis (SCB) transform of a linear time invariant system. The SCB has a distinct feature of explicitly displaying many important properties of a given system.

Theorem 2.1 *[94] For any given system Σ characterized by the triplet (A, G, C), there exists:*

- *coordinate free non-negative integers $n_a^-, n_a^+, n_b, n_c, n_d$ and $m_d \leq q, q_i, i = 1, \ldots, m_d$,*
- *non-singular state, output, and input transform matrices $\Gamma_1, \Gamma_2, \Gamma_3$ which take the given Σ into a special coordinate basis that displays explicitly both the finite and infinite zero structures of Σ. The SCB can be described by the following compact form:*

$$\bar{x} = \Gamma_1^{-1} x, \bar{y} = \Gamma_2^{-1} y, \bar{d} = \Gamma_3^{-1} d:$$

$$\bar{x} = [(x_a^-)', (x_a^+)', x_b', x_c', x_d']'$$

$$\bar{y} = [y_d', y_b']', \bar{d} = [d_d', d_c']'$$

$$\bar{\Sigma} : \begin{cases} \dot{\bar{x}} = & \bar{A}\bar{x} + \overline{Gd} \\ \bar{y} = & \bar{C}\bar{x} \end{cases}$$

where $\bar{A} = \Gamma_1^{-1} A \Gamma_1, \bar{G} = \Gamma_1^{-1} G \Gamma_3$ *and* $\bar{C} = \Gamma_2^{-1} C \Gamma_1$ *are given by*

$$\bar{A} = \left[\begin{array}{cc|c|c|c} A_{aa}^- & 0 & L_{ab}^- C_b & 0 & L_{ad}^- C_d \\ 0 & A_{aa}^+ & L_{ab}^+ C_b & 0 & L_{ad}^+ C_d \\ \hline 0 & 0 & A_b & 0 & L_{bd} C_d \\ \hline G_c E_{ca}^- & G_c E_{ca}^+ & L_{cb} C_b & A_{cc} & L_{cd} C_d \\ \hline G_d E_a^- & G_d E_a^+ & G_d E_b & G_d E_c & A_d \end{array}\right],$$

$$\bar{G} = \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \\ \hline 0 & 0 \\ \hline 0 & G_c \\ \hline G_d & 0 \end{array}\right], \bar{C} = \left[\begin{array}{cc|c|c|c} 0 & 0 & 0 & 0 & C_d \\ 0 & 0 & C_b & 0 & 0 \end{array}\right].$$

Structural properties of linear multivariable systems that remain invariant under any static-state feedback and output injection play an important role in many control problems [77, 94, 95]. In his seminar paper in 1973, Morse studied the structural invariants of the triplet $\Sigma = (A, G, C)$ under various transformation groups [77]. Similar to the classical Kalman decomposition, the main result in Reference [77] was to show that under an appropriate transformation group, the state space for the above triplet can be decomposed into four independent subspaces with speical properties. A canonical form was therefore proposed, and its was shown that for a system Σ this canonical form was uniquely determined by a list of monic polynominals I_1 and three lists of positive integers, I_2, I_3 and I_4. For details of that work, the interested reader should consult [77]. SCB has close relationship with the theory on structural invariants. The number of elements in the lists I_1, I_2, I_3 and I_4, which is called the structure invariant index (SII) of the system Σ, are equal to the integers $n_a = n_a^- + n_a^+, n_b, n_c$ and n_d in the SCB theorem. For the sake of brevity only several properties of SII that are essential to the discussions in this thesis and have been proved by SCB theory are outlined in the following. Again, for an in-depth discusion of these topics, Reference [77, 94, 95] should be consulted.

Property 1. The system Σ is right invertible if and only if x_b and hence y_b do not exist (i.e. $n_b = 0$), and left invertible if and only if x_c and hence u_c do not exist (i.e. $n_c = 0$).

Property 2. The number of stable and unstable zeros of the system are given by n_a^-, n_a^+ respectively . Eigenvalues of A_{aa}^- and A_{aa}^+ are stable and unstable zeros of the system respectively.

Property 3. n_d is the number of infinite zeros, and $n_d = \sum_{i=1}^{m_d} q_i$, where $q_i, i = 1, ..., m_d$ is the number of infinite zeros of order of i, m_d is the highest order of an infinite zero. Further, $m_d = rank(C_d) = rank(G_d)$.

Property 4. (A_b, C_b) and (A_d, C_d) form observable pairs. Unobservability will arise only in the variables x_a and x_c. The given system (A, C) is observable (detectable) if and only if (A_{ob}, C_{ob}) is an observable (detectable) pair, where

$$A_{ob} = \begin{bmatrix} A_{aa} & 0 \\ G_c E_{ca} & A_{cc} \end{bmatrix}, A_{aa} = \begin{bmatrix} A_{aa}^- & 0 \\ 0 & A_{aa}^+ \end{bmatrix}$$

$$C_{ob} = \left[\begin{array}{cc} E_a & E_{ca} \end{array} \right], E_a = [E_a^-, E_a^+], E_{ca} = [E_{ca}^-, E_{ca}^+].$$

Remark 2.2.1 In the context of G being distribution matrix of unknown input, SCB transforms the original system (7.8) into four interconnected subsystems: 1) A subsystem without output and unknown input (x_a); 2) A subsystem with output and without unknown input (x_b); 3) A subsystem without output and with unknown input (x_c) and 4) A subsystem with both output and unknown input (x_d). It should be noted that not all four subsystems will exist in many practical situations. Properties 1-4 tell us the connection between certain system properties and the existence of certain subsystems.

Lemma 2.2.2 *Let $(\overline{A}, \overline{G}, \overline{C})$ be the SCB transform matrices of (A, G, C). Matrices $\overline{T}, \overline{K}$ satisfy conditions (2.11) and (2.16) for system $(\overline{A}, \overline{G}, \overline{C})$ if and only if $T = \overline{T}\Gamma_1^{-1}, K = \Gamma_1\overline{K}\Gamma_2^{-1}$ satisfy conditions (2.11) and (2.16) for system (A, G, C). Further, $\overline{F}, \overline{T}, \overline{L}$ satisfy conditions (2.4)-(2.6) for system $(\overline{A}, \overline{G}, \overline{C})$ if and only if $F = \overline{F}, T = \overline{T}\Gamma_1^{-1}, L = \overline{L}\Gamma_2^{-1}$ satisfy conditions (2.4)-(2.6) for system (A, G, C).*

Proof. Because $A = \Gamma_1\overline{A}\Gamma_1^{1}, C = \Gamma_2\overline{C}\Gamma_1^{-1}$,

$$A - KC = \Gamma_1\overline{A}\Gamma_1^{1} - \Gamma_1\overline{K}\Gamma_2^{-1}\Gamma_2\overline{C}\Gamma_1^{-1} = \Gamma_1(\overline{A} - \overline{KC})\Gamma_1^{-1}.$$

Thus,

$$sI - A + KC = \Gamma_1(sI - \overline{A} + \overline{KC})\Gamma_1^{-1}.$$

and because $G = \Gamma_1\overline{G}\Gamma_3^{-1}$,

$$T(sI - A + KC)^{-1}G = \overline{T}\Gamma_1^{-1}\Gamma_1(sI - \overline{A} + \overline{KC})^{-1}\Gamma_1^{-1}\Gamma_1\overline{G}\Gamma_3^{-1} = \overline{T}(sI - \overline{A} + \overline{KC})^{-1}\overline{G}\Gamma_3^{-1}.$$

Further, we have

$$TG = \overline{T}\Gamma_1^{-1}\Gamma_1\overline{G}\Gamma_3^{-1} = \overline{TG}\Gamma_3^{-1},$$

and

$$\begin{aligned} FT - TA + LC &= \overline{FT}\Gamma_1^{-1} - \overline{T}\Gamma_1^{-1}\Gamma_1\overline{A}\Gamma_1^{-1} + \overline{L}\Gamma_2^{-1}\Gamma_2\overline{C}\Gamma_1^{-1} \\ &= (\overline{FT} - \overline{TA} + \overline{LC})\Gamma_1^{-1}. \end{aligned}$$

Since Γ_1 and Γ_3 are nonsingular, $TG = 0$ if and only if $\overline{TG} = 0$, $FT - TA + LC = 0$ if and only if $\overline{FT} - \overline{TA} + \overline{LC} = 0$. This completes the proof. ∎

Lemma 2.2.2 shows the equivalence of the design based on original system matrices and their corresponding SCB transform matrices.

Lemma 2.2.3 *For the given system (A, G, C), matrices K and T exist such that $A - KC$ is stable and $T(sI - A + KC)G = 0$, if and only if (A, C) is detectable and $(n_a^- + n_b) > 0$. Further, T can be any matrix satisfying*

$$\overline{T} = \Gamma_1 T \Gamma_3^{-1} = \begin{bmatrix} T_a^- & 0 & T_b & 0 & 0 \end{bmatrix} \tag{2.17}$$

where Γ_1, Γ_3 are SCB state and input transformations.

Lemma 2.2.3 is the main result for SCB based LTR theory [94]. If the requirement of $A - KC$ being stable is neglected, the following lemma can be proved.

Lemma 2.2.4 *For the given system (A, G, C), matrices K and T exist such that $T(sI - A + KC)G = 0$ if and only if $(n_a^- + n_a^+ + n_b) > 0$. Also, at least $n_a^- + n_b$ eigenvalues of $A - KC$ are negative, and at least n_a^+ eigenvalues of $A - KC$ are positive. Further, T must be matrix satisfying*

$$\overline{T} = \Gamma_1 T \Gamma_3^{-1} = \begin{bmatrix} T_a^- & T_a^+ & T_b & 0 & 0 \end{bmatrix} \tag{2.18}$$

In the other words, all rows of T must locate in the space spanned by the left eigenvectors corresponding to those $n_a^- + n_a^+ + n_b$ eigenvalues.

Lemma 2.2.4 is a simple extension of Remark 5.1 in [12].

2.3 Existence Conditions Of UIFO

Theorem 2.2 *For the system (2.1), the UIFO problem is solvable if and only if at least one of following two conditions hold,*

1. *The system (A, G, C) has stable transmission zeros (i. e. $n_a^- > 0$) or,*

2. *The system (A, G, C) is not right invertible (i.e. $n_b > 0$).*

Further the maximum row rank of T is $r = n_a^- + n_b$.

Proof. (*Sufficiency*) Without loss of generality, assume the system (A, G, C) is represented by the SCB form. If $r = n_a^- + n_b > 0$, according to Lemma 2.2.4, at least r eigenvalues of $A - KC$ can be made negative through design of K. For simplicity, assume those eigenvalues are distinct. Then we have

$$v_i(A - KC) = \lambda_i v_i, i = 1, ..., r$$

where v_i is the left eigenvector corresponding to a negative eigenvalue λ_i. Combining those r equations will result in

$$\Gamma A - \Gamma KC = \Lambda\Gamma \tag{2.19}$$

where $\Gamma = \begin{bmatrix} v_1^T & v_2^T & ... & v_r^T \end{bmatrix}^T$ and Λ is a diagonal matrix with elements λ_i. Multiplying (2.19) by any invertible matrix $\Omega_{r \times r}$ gives following general equation

$$TA - LC = \Omega\Lambda\Gamma = FT \tag{2.20}$$

where $T = \Omega\Gamma, F = \Omega\Lambda\Omega^{-1}, L = TK$ and F is stable. Due to Lemma 2.2.4, $T(sI - A + KC)^{-1}G = 0$, which implies $TG = 0$. In summary, the solution of UIFO can be derived if $(n_a^- + n_b) > 0$.

(*Necessity*) Assume F, L, T satisfying (2.4)- (2.6) have been obtained and that the dimension of F is r. Let $K = T^+ L$, where T^+ is the Moore Penrose inverse of T. Equation (2.5) implies that

$$T(A - KC) = FT. \tag{2.21}$$

Assume $\Omega\Lambda\Omega^{-1} = F$, so that (2.21) becomes

$$\Omega^{-1}T(A - KC) = \Lambda\Omega^{-1}T. \tag{2.22}$$

This means $\hat{T} = \Omega^{-1}T$ is composed of left eigenvector of $A - KC$. Because $TG = 0$, this implies that $\hat{T}G = 0$. Let $\bar{T}$ be the other $n - r$ left eigenvectors orthogonal to $\hat{T}$, $\tilde{T} = \begin{bmatrix} \hat{T} \\ \bar{T} \end{bmatrix}$. Note $\tilde{T}^{-1} = [\hat{T}^+ \quad \bar{T}^+]$, where $(.)^+$ means pseudo-inverse. Let $\bar{\Lambda}$ represents diagonal matrix whose elements are all other eigenvalues of $A - KC$ in additions to those eigenvalues in Λ. Then $\bar{T}(A - KC) = \bar{\Lambda}\bar{T}$, and

$$T(sI - A_c)^{-1}G = T\tilde{T}^{-1}\begin{pmatrix} sI - \Lambda & 0 \\ 0 & sI - \bar{\Lambda} \end{pmatrix}^{-1}\tilde{T}G$$

$$= \begin{bmatrix} T\hat{T}^{+} & 0 \end{bmatrix}\begin{pmatrix} sI - \Lambda & 0 \\ 0 & sI - \bar{\Lambda} \end{pmatrix}^{-1}\begin{bmatrix} 0 \\ \bar{T}G \end{bmatrix} = 0$$

where $A_c = A - KC$. In summary, if a UIFO exists, there exist T and K satisfying (2.16). Because T is located in the space spanned by some left eigenvectors whose corresponding eigenvalues are stable, it means $(n_a^- + n_b) > 0$. On the other hand, $rank(T) > (n_a^- + n_b)$ contradicts Lemma 2.2.4. Therefore, the maximum row rank of T is $r = n_a^- + n_b$. Further, the T with maximum rank can be represented as

$$\overline{T} = \Gamma_1 T \Gamma_3^{-1} = \Omega \begin{bmatrix} V_a^- & 0 & 0 & 0 & 0 \\ 0 & 0 & V_b & 0 & 0 \end{bmatrix}$$

where Ω is nonsingular linear transform matrix, V_a^- and V_b are nonsingular matrices of dimension n_a^- and n_b respectively. ∎

Remark 2.3.1 Theorem 1 of [110] states that if the plant (A, G, C) has at least one stable transmission zero or if $p > q$, a UIFO which estimates $Tx(t)(T \in \mathbb{R}^{r \times n})$ with order $n - p > r > 0$ can be constructed. Theorem 2.2 gives the exact upper limit on r. Because $p > q$ assures $n_b > 0$, $p > q$ is a sufficient condition for existence of UIFO. However, it is possible that $n_b > 0$ even if $p \leq q$, therefore, $p > q$ is not a necessary condition.

Theorem 2.3 *Solutions to the GUIO problem exist if and only if the system (A, G, C) satisfies at least one of the following conditions,*

1. *The system (A, G, C) has $n_a^-(> 0)$ stable transmission zeros.*

2. $(p - m_d) < n_b$.

Further, the maximum rank of T is $r = n_a^- + n_b + m_d - p$.

Proof: Without loss of generality, assume (A, G, C) is in SCB form. The proof of Theorem 2.2 shows that T of UIFO must be in the form of

$$T = \Omega \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \Omega \begin{bmatrix} V_a^- & 0 & 0 & 0 & 0 \\ 0 & 0 & V_b & 0 & 0 \end{bmatrix} \tag{2.23}$$

then

$$T_c = \begin{bmatrix} T \\ C \end{bmatrix} = \begin{bmatrix} \Omega & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} V_a^- & 0 & 0 & 0 & 0 \\ 0 & 0 & V_b & 0 & 0 \\ 0 & 0 & 0 & 0 & C_d \\ 0 & 0 & C_b & 0 & 0 \end{bmatrix}. \tag{2.24}$$

Note that dim $C_d = m_d \times n_d$, dim $C_b = (p - m_d) \times n_b$. C_b is always full row rank, $(p - m_d) \leq n_b$. Therefore, there are at the most $n_b - (p - m_d)$ vectors that can be linearly independent of C_b. Obviously, n_a^- rows in V_1 are linearly independent of rows in C, thus, the maximum row number of T, which makes T_c full rank, is $r = n_a^- + n_b + m_d - p$.

If the maximum $r = n_a^- + n_b + m_d - p = n - p$, then one ends up with the standard UIO, and all states can be estimated. Thus, in the context of the above, the following theorem on the existence of UIO can be stated.

Theorem 2.4 *Solutions to the standard UIO problem exist if and only if the system (A, G, C) satisfies the following conditions.*

1. *All of the system's invariant zeros are negative (i.e. of minimum phase).*
2. *It is left invertible (i.e. $n_c = 0$).*
3. *the matrix CG is of maximal rank (i.e. C_d is an identity matrix or the number of infinite zeros of each order, $q_i (i = 1, ..., m_d)$, is one).*

These conditions are equivalent to $n_a^- + n_b + m_d = n$.

Remark 2.3.2 Kudva *et al.* showed that a UIO exists if and only if the system (A, G, C) is minimum phase and $rank(CG) = rank(G)$ [63]. It is easy to show that the requirement $rank(CG) = rank(G)$ is equivalent to conditions (2) and (3) in Theorem 2.4 using the SCB form of the given system.

Remark 2.3.3 A constrained state-feedback control can be built upon a GUIO as

$$u = [H_z \;\; H_y] \begin{bmatrix} T \\ C \end{bmatrix} = \bar{H} T_c x.$$

If row rank of T_c is between n and p, it represents a control format between the full state feedback and the static output feedback. However, Table 1 of [110] stated that if $p > q$ or if there is at least one stable transmission zero, then there must exist a T to make $p < rank(T_c) < n$. This claim is wrong. For example, consider the following triple (A, G, C), where

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; G = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}; C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix};$$

For this system, $p > q, n_b = 1$. However the matrix T of UIFO can only be in the form of

$$T = \begin{bmatrix} 0 & t_1 & t_2 & 0 \end{bmatrix}$$

which is in output space and $rank(T_c) = p = 2$.

Remark 2.3.4 In summary, although the proof of the above theorems may appear complex, the ultimate results provide a simple and intuitive understanding of the UIFO design problem. Furthermore, the main conclusion is that only subsystems without the unknown inputs are useful for state function observation.

Remark 2.3.5 If the disturbance corrupts the output as well as the state equation, that is

$$y(t) = C(t) + Dd(t), \tag{2.25}$$

then the UIFO design can still be developed using the LTR theory for non-strictly proper systems [12].

2.4 Design Algorithm

Two explicit design algorithms for UIFO are presented in this section. They are suitable for implementation using a numerical software package such as MATLAB. Algorithm I is based on the SCB transform and implied in the proof of Theorem 2.2. It is summarized in the following steps.

UIFO Design Algorithm I

Step 1: The system (A, G, C) is transformed into its SCB form by non-singular state, output, and input transformations $\Gamma_1, \Gamma_2, \Gamma_3$.

(a) If $n_b + n_a^- = 0$, stop. No UIFO exists.

(b) Otherwise, continue.

Step 2: Since (A_b, C_b) forms an observable pair, choose a gain K_b such that eigenvalues $\lambda(A_b - K_b C_b) = \Lambda^b$ lie at desired locations in C^-. Calculate the corresponding left eigenvectors V_b and left eigenvectors V_{a^-} for A_{aa}^-.

Step 2: Define A_g and C_g as

$$A_g = \begin{bmatrix} A_{aa}^+ & 0 & L_{ad}^+ C_d \\ G_c E_{ca}^+ & A_{cc} & L_{cd} C_d \\ G_d E_a^+ & G_d E_c & A_d \end{bmatrix}, C_g = \begin{bmatrix} 0 & 0 & C_d \end{bmatrix}:$$

If (A, C) is detectable, (A_g, C_g) must be detectable. Choose a gain K_g such that eigenvalues $\lambda(A_g - K_g C_g) = \Lambda^g$ lie at desired locations in C^-. Otherwise, let K_g be any matrix with a suitable dimension. Partition K_g as

$$K_g = \begin{bmatrix} K_a^{+\prime} & K_c^{\prime} & K_d^{\prime} \end{bmatrix}^{\prime}.$$

Step 4: Construct a matrix $\overline{K}$ as

$$\overline{K} = \begin{bmatrix} L_{ad}^- & L_{ab}^- \\ K_a^+ & L_{ab}^+ \\ L_{bd} & K_b \\ K_c & L_{cb} \\ K_d & 0 \end{bmatrix} \tag{2.26}$$

and $\overline{T}$ as

$$\overline{T} = \begin{bmatrix} V_a^- & 0 & 0 & 0 & 0 \\ 0 & 0 & V_b & 0 & 0 \end{bmatrix}. \tag{2.27}$$

Calculate $\overline{L}$ via $\overline{L} = \overline{TK}$.

Step 5: Construct F, K, T and L as

$$F = \begin{bmatrix} \lambda(A_{aa}^-) & 0 \\ 0 & \lambda(A_b - K_b C_b) \end{bmatrix} \tag{2.28}$$

and

$$K = \Gamma_1 \overline{K} \Gamma_2^{-1}, T = \overline{T} \Gamma_1^{-1}, L = \overline{L} \Gamma_2^{-1}. \tag{2.29}$$

This concludes the design steps in the first approach. Its validity is proved below.

Theorem 2.5 *The matrices F, T, L given by equations (2.28)-(2.29) form the solution to UIFO design, namely they satisfy UIFO existence conditions (2.4)-(2.6).*

Proof. Lemma 2.2.2 shows that $FT - TA - LC = 0$ if and only if $F\overline{T} - \overline{TA} + \overline{LC} = 0$. Note $\overline{L} = \overline{TK}$, we need to show that

$$F\overline{T} = \overline{T}(\overline{A} - \overline{KC}) \tag{2.30}$$

which means each row of $\overline{T}$ is a left eigenvector of $\overline{A} - \overline{KC}$ and diagonal elements of F are corresponding eigenvalues. Based on equations of $\overline{A}, \overline{K}$ and $\overline{C}$, we have

$$\overline{A} - \overline{KC} = \begin{bmatrix} A_{aa}^- & 0 & 0 & 0 & 0 \\ 0 & A_{aa}^+ & 0 & 0 & (L_{ad}^+ - K_a^+)C_d \\ 0 & 0 & A_b - K_b C_b & 0 & 0 \\ G_c E_{ca}^- & G_c E_{ca}^+ & 0 & A_{cc} & (L_{cd} - K_c)C_d \\ G_d E_a^- & G_d E_a^+ & G_d E_b & G_d E_c & (A_d - K_d C_d) \end{bmatrix}. \tag{2.31}$$

Conditions (2.30) and (2.4)-(2.5) are proved easily by direct calculation.

Next, since $G = \Gamma_1 \overline{G} \Gamma_3^{-1}$,

$$TG = \overline{T}\Gamma_1^{-1}\Gamma_1\overline{G}\Gamma_3^{-1} = \overline{TG}\Gamma_3^{-1} = \begin{bmatrix} V_a^- & 0 & 0 & 0 & 0 \\ 0 & 0 & V_b & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & G_c \\ G_d & 0 \end{bmatrix} \Gamma_3^{-1} = 0.$$

∎

It may be desirable to design the UIFO in original system space. Thus, we provide an alternative design method using direct eigenstructure assignment theory, that is Algorithm II described below, which does not involve any transformation, and which may have better computational stability. It is a modification and extension of the results of [104] which provide a direct solution to ELTR for square systems.

UIFO Design Algorithm II

Step 1: Calculate the stable transmission zeros $Z_o = \{z_i, i = 1, ..., n_a^-\}$, and the structure invariant index n_b of the system (A, G, C) through MATLAB. For further details on the structure invariant index, see [77]. If $n_a^- = n_b = 0$, stop here.

Step 2: Choose z_i as the desired eigenvalue λ_i, calculate its corresponding left eigenvectors v_i and dummy vectors o_i by solving

$$\begin{bmatrix} v_i & o_i \end{bmatrix} \begin{bmatrix} \lambda_i I - A & G \\ -C & 0 \end{bmatrix} = 0. \tag{2.32}$$

where $i = 1, ..., n_a^-$. Note that v_i should be selected to be mutually linearly independent.

Step 3: Choose a distinct negative number set $\Lambda^b = \{\lambda_i, i = 1 + n_a^-, ..., n_b + n_a^-\}$ as desired eigenvalues, calculate their corresponding left eigenvectors v_i and dummy vectors o_i by solving (2.32). Note that v_i should be selected such that $v_i (i = 1 + n_a^-, ..., n_b + n_a^-)$ are mutually linearly independent and are linearly independent from those v_i obtained in Step 2.

Step 4: Let $r = n_a^- + n_b$, choose the remaining $n - r$ eigenvalues $\Lambda^g = \{\lambda_i, i = r + 1, ..., n\}$ and find their corresponding left eigenvectors v_i, and dummy vector o_i by solving the following equations,

$$v_i(\lambda_i I - A) - o_i C = 0, i = r + 1, ..., n.$$

Note that all unobservable fixed modes must be included in Λ^g and the resulting v_i should be linearly independent of v_i obtained in Steps 2 and 3.

Step 5: K is parameterized as

$$K = -\begin{bmatrix} v_1 \\ v_2 \\ \cdots \\ v_n \end{bmatrix}^{-1} \begin{bmatrix} o_1 \\ o_2 \\ \cdots \\ o_n \end{bmatrix} = -V^{-1}O \tag{2.33}$$

and T is constructed as

$$T = \Omega\Gamma = \Omega \begin{bmatrix} v_1 \\ v_2 \\ \cdots \\ v_r \end{bmatrix}. \tag{2.34}$$

where $\dim \Omega = r \times r$, Ω is any invertible matrix such that T has full rank.

Step 6: F and L are calculated as

$$F = \Omega\Lambda\Omega^{-1}, L = TK \tag{2.35}$$

where Λ is diagonal with elements composed by Z_o and Λ_b.

Theorem 2.6 *The matrices F, T, L given by equations (2.34)-(2.35) constitute the UIFO solution, namely, they satisfy UIFO conditions (2.4)-(2.6).*

Proof. Based on the well-known eigenstructure assignment theory [80], gain K given by (2.33) will assign $Z_o \cup \Lambda_b \cup \Lambda_g$ as the desired eigenvalues, and V is the left eigenvector set. Note Λ and Γ are composed of the eigenvalues and corresponding left eigenvectors, thus

$$\Lambda\Gamma = \Gamma(A - KC).$$

Multiplying both sides by Ω, we have

$$\Omega\Lambda\Omega^{-1}\Omega\Gamma = \Omega\Gamma(A - KC)$$

and

$$FT = TA - TKC = TA - LC.$$

On the other hand, equation (2.32) implies that

$$v_i G = 0, i = 1, ..., r.$$

Immediately, equation (2.34) tells us that $TG = 0$. This completes the proof. ∎

2.5 Illustrative Example

Let the system (2.1) have the coefficient matrices

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 0 & -2 & -4 \\ 2 & 0 & -1 & 1 \end{bmatrix}, G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}.$$

There are no transmission zeros in this system, and $rank(G) = 2 > rank(C) = 1$. According to the results in [110], no UIFO can be designed for this system. However, it is easy to show that this system has $n_b = 2$. Theorem 2.2 tells us that UIFO does exist for this system and there are at most two rows in T. It is noted also that there are two unobservable modes at -3 and 2. Let the two desired eigenvalues be at $\lambda_1 = -2$ and $\lambda_1 = -4$. The matrices K and T, which satisfy (2.16), are obtained through Algorithm II as:

$$K = \begin{bmatrix} 8.0 & 9.5 & -4.667 & 10.0833 \end{bmatrix}^T;$$

$$T = \begin{bmatrix} -0.6 & 0.4 & 0 & 0 \\ -0.2381 & 0.0952 & 0 & 0 \end{bmatrix}.$$

T will be used for UIFO directly. The coefficient matrices F, L for UIFO are

$$F = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix}, L = \begin{bmatrix} -1 \\ -1 \end{bmatrix}.$$

The structure of T shows that only the second state can be estimated. Further, let

$$N = \begin{bmatrix} 2.5 & 0 \end{bmatrix}; M = 1.5$$

The resulting w will be the estimation of x_2.

2.6 Conclusions

This chapter provides a complete solution of the unknown input functional observer problem for continuous time linear systems. It also shows that LTR theory has a close relationship with UIFO problem, and can be used effectively in solving UIFO problems. Despite the importance of regular UIO, its application is very limited because many plant systems do not satisfy its strict existence conditions. The results of UIFO are significantly more general than the existing UIO results, and have direct application in robust control and FDI research.

Chapter 3

Output Derivative Free Reduced-Order Unknown Input Estimator

Considerable attention has been focused on the problem of designing an observer to estimate the state of a system that is subject to unknown inputs. However, relatively little research has been carried out on estimating unknown inputs, which is the topic of this chapter. Two separate reduced-order input estimators, using the same design methodology, are proposed. The first is an extension of the state/input estimator reported in [17]. The second is based on the adaptive observer technique. The proposed estimators work for certain non-minimum phase systems and use only measured outputs.

3.1 Introduction

The problem of estimating unknown inputs is motivated mainly by the following three applications:

1. In certain situations where it is either too expensive or perhaps not possible to measure some of the system's input, the input estimator is a powerful and

valuable "soft" sensor. An example of this is in machine tool applications [81, 111], where the cutting force is often unavailable. Other measurements based on displacement sensors or accelerometers are usually more feasible. If one regards the cutting force as an unknown input into the machine tool system, one can estimate the cutting force using an input estimator based on the more feasible measurements.

2. When unknown inputs are certain signals such as incipient fault of actuator or plant components, the unknown input estimator is a powerful tool for fault detection and isolation (FDI) [97, 136].

3. With exact or approximate knowledge of unknown inputs, the effects of unknown inputs can be compensated and robust controller or fault-tolerant controller can be implemented [57, 108].

The research in unknown input estimation started in the mid of 1970s. A common approach models the unknown input as the output of a linear system and incorporates the input dynamics with the plant dynamics [41, 57]. This approach is limited to specific types of unknown inputs. In [98], the problem of unknown, constant or slowly varying input estimation, using a proportional integral observer (PIO), is discussed by Saif. Using an adaptive observer technique, Wang and Daley [119] presented an actuator fault diagnosis method, where actuator faults were estimated directly. Mathematically, the problem in [119] is equivalent to estimating the unknown inputs for linear time invariant systems. The adaptive input estimator, like the PIO in [98], can only be used to detect constant or slow unknown input signals.

Several researchers proposed the input estimator, which assumes no prior knowledge of the unknown input. Park and Stein [81] estimated unknown inputs by differentiating the output measurement. Their state and input observer is a combination of a reduced order unknown input observer and an algebraic equation relating the unknown input to the measured output and its derivatives. Hou and Patton [51] provided a complete analysis of input observability and input reconstruction problem. It is shown that, except for some trivial cases, derivatives of the measurements are unavoidable when exact input reconstruction is required. Obviously, differentiation of the output is undesirable in practice, because unexpected measurement noise can make the estimation unreliable. Recently, Corless and Tu [17] proposed a combined state/input estimator which does not require differentiation of the measured output. Although exact asymptotic estimation is not achieved, one can asymptotically estimate the unknown inputs to any degree of accuracy. The input estimator in [17] is based on a full-order UIO, which exists under strict conditions. The unknown inputs and their rate of change must satisfy some additional bound conditions.

It should be noted that in certain applications, such as synthesis of fault diagnosis schemes, an estimation of the entire state might not be required. Therefore, we propose a new reduced-order input estimator which is an extension of the state/input estimator in [17]. The proposed input estimator is based on unknown input functional observers researched in Chapter 2. It is shown that input estimation can be achieved for certain non-minimum phase systems using this new input estimator, while it is impossible using the method in [17]. Furthermore, we shall address the principle of the input estimator from a new viewpoint. As a result, the bound condition on inputs, which was assumed in [17], can be removed. It is interesting to note that the existence condition for the adaptive fault estimator in [119] are the same as that for the input estimator in [17]. The adaptive observer in [119] is in the form of the full-order Luenberger observer. Here we shall extend it to a reduced-order case. Finally, the two proposed input estimators are illustrated through a simulation study.

3.2 Background

In this section we shall highlight some of the input estimators proposed in the literature. Consider a linear dynamic system driven by both known and unknown inputs as in (2.1), which is repeated below for convenience,

$$\begin{aligned} \dot{x} = & \; Ax + Bu + Gd(t) \\ y = & \; Cx \end{aligned} \tag{3.1}$$

Here, as usual $x \in \mathcal{R}^n$ is the state, $u \in \mathcal{R}^m$ is the known control input, $y \in \mathcal{R}^p$ is the output. The matrices A, B, G and C are of appropriate dimensions, and G and C

are of full rank. The continuous function $d(t) \in \mathcal{R}^q$ models all uncertain, nonlinear, and time-varying terms. The full rank assumption for G is necessary for estimating d. Clearly, if this rank assumption on G is not satisfied, any input vector in its null space (which is not empty), cannot be distinguished from zero.

A general input estimator proposed in [51] is in the form of

$$\begin{aligned} \dot{z} = & \; Fz + Ly + Ju \\ \hat{d} = & \; G_1 z + \textstyle\sum_{i=0}^{k} G_{2,i} y^{(i)} \end{aligned} \tag{3.2}$$

where $y^{(i)}$ denotes the ith derivative of y. The design will determine the lowest k under which F, L, J, G_1 and $G_{2,i}(i = 1, ..., k)$ exist such that $\hat{d} \rightarrow d$ for any initial value $x(0), z(0)$. It is proved in [51] that $k > 0$ for most cases.

The work in [17] addresses the estimation of both state and input, and does not require differentiation of the measured output. That state/input estimator is described by

$$\begin{aligned} \dot{\hat{x}} = & \; A\hat{x} + Bu + G\hat{d} + K(y - C\hat{x}) \\ \hat{d} = & \; \gamma W(y - C\hat{x}) \end{aligned} \tag{3.3}$$

where γ is a positive scalar.

Theorem 3.1 *[17] If there exist matrices K and W, and two symmetric positive-definite (SPD) matrices P and Q such that*

$$P(A - KC) + (A - KC)^T P = -Q \text{ and } G^T P = WC. \tag{3.4}$$

then state and input can be asymptotically estimated to any desired degree of accuracy, provided the unknown input and its rate of change satisfy some additional bound conditions.

Considering a slight modification of the adaptive diagnostic observer proposed in [119], we have the following adaptive unknown input estimator,

$$\begin{aligned} \dot{\hat{x}} = & \; A\hat{x} + Bu + G\hat{d} + K(y - C\hat{x}) \\ \dot{\hat{d}} = & \; \rho W(y - C\hat{x}) \end{aligned} \tag{3.5}$$

where ρ is a positive value which defines the learning rate. It is easy to show that if conditions in Theorem 3.1 are satisfied, exact asymptotic estimation can be achieved for both states and unknown inputs satisfying $\dot{d} = 0$. It means the above adaptive algorithm can only follow constant or slowly varying inputs.

3.3 Problem Statement

In this chapter, we will extend the full-order state/input estimator (3.3) and (3.5) to their corresponding reduced-order case. In our extension, we wish to estimate the unknown inputs under less conservative conditions than those in [17]. The first proposed reduced-order state-function/input estimator is described by

$$\begin{aligned} \dot{z} &= Fz + Ly + TBu + TG\hat{d} \\ \hat{d} &= \gamma(My - Nz) \end{aligned} \tag{3.6}$$

where γ is a positive scalar, z is an estimate of linear combination of state, $z \to Tx$, the initial value of $z(0)$ is arbitrary, and $\hat{d}$ is the estimate of $d(t)$. Matrices F, T, L, N and M need to be designed to achieve the problem objective. The following theorem gives conditions that these matrices have to satisfy.

Theorem 3.2 *The system (3.6) is a functional observer for the system (3.1), if the following conditions are satisfied*

$$FT - TA + LC = 0; F \text{ is stable;} \tag{3.7}$$

$$N = (TG)^T P, P \text{ is Lyapunov matrix of } F; \tag{3.8}$$

$$NT = G^T T^T PT = MC; \tag{3.9}$$

$$rank(TG) = rank(G) = q; \tag{3.10}$$

and $\|d(t)\| \leq \beta_1, \|\dot{d}(t)\| \leq \beta_2$. *In this case the following hold for any* $\varepsilon_1, \varepsilon_2 > 0$.

1. *There exists a* $\gamma_1 \geq 0$ *such that for all* $\gamma \geq \gamma_1$ *and all* $x(0), z(0)$, *we have*

$$\lim sup_{t\to\infty} \|z(t) - Tx(t)\| \leq \varepsilon_1.$$

2. *There exists a* $\gamma_2 \geq 0$ *such that for all* $\gamma \geq \gamma_2$ *and all* $x(0), z(0)$, *we have*

$$\lim sup_{t\to\infty} \|\hat{d} - d\| \leq \varepsilon_2.$$

Proof. Defining the estimation error, $e = Tx - z$, and using (3.1) and (3.6) we get

$$\dot{e} = Fe + (TA - FT - LC)x + TGd - TG\hat{d}. \tag{3.11}$$

Using $NT = MC, N = (TG)^T P$,

$$My - Nz = MCx - Nz = N(Tx - z) = Ne = (TG)^T Pe.$$

If $FT - TA + LC = 0$, the estimation error equation becomes

$$\dot{e} = (F - \gamma(TG)(TG)^T P)e + TGd. \tag{3.12}$$

Consider the Lyapunov function candidate $V(e) = e^T Pe$, where P is a solution of $PF + F^T P = -Q$, and Q is positive definite. The derivative of $V(e)$ with respect to time evaluated on the trajectories of the error equation is

$$\dot{V}(e) = e^T(PF + F^T P)e - 2\gamma e^T PTG(TG)^T Pe + 2e^T PTGd(t)$$

if $\|d(t)\| \leq \beta_1$,

$$\dot{V}(e) \leq -e^T Qe - 2\gamma\|(TG)^T Pe\|^2 + 2\beta_1\|(TG)^T Pe\|.$$

For any positive scalar μ, we have the following inequality:

$$2\beta_1\|(TG)^T Pe\| \leq \mu^{-1}\beta_1^2\|(TG)^T Pe\|^2 + \mu.$$

hence,

$$\dot{V}(e) \leq -e^T Qe + \mu - [2\gamma - \mu^{-1}\beta_1^2]\|(TG)^T Pe\|^2.$$

Let $\alpha = \lambda_{min}[P^{-1}Q]/2 > 0$, and choose γ to satisfy $2\gamma \geq \mu^{-1}\beta_1^2$, then

$$\dot{V}(e) \leq -2\alpha V(e) + \mu.$$

From this, one may deduce that

$$\|e(t)\| \leq r + c\|e(0)\|e^{-\alpha t}$$

for all $t \geq 0$, where

$$c = (\lambda_{max}(P)/\lambda_{min}(P))^{1/2}, r = c(\mu/2\alpha)^{1/2}.$$

Considering any $\varepsilon_3 > 0$ and choosing any μ so that $c(\mu/2\alpha)^{1/2} \leq \varepsilon_3$ yields

$$\|e(t)\| \leq \varepsilon_3 + c\|e(0)\|e^{-\alpha t} \tag{3.13}$$

for all $t \geq 0$. Choosing $\varepsilon_3 \leq \varepsilon_1$, we prove part (1).

Using an analysis similar to that used in the analysis of the error dynamics, it is easy to demonstrate that for any $\varepsilon_4 > 0$, there exists a $\gamma_3 > 0$ such that for all $\gamma \geq \gamma_3$,

$$\|\dot{e}(t)\| \leq \varepsilon_4 + c\|\dot{e}(0)\|e^{-\alpha t} \tag{3.14}$$

where $\alpha > 0$. Consider now the estimation error (3.12) which can be rewritten as

$$\dot{e} = Fe + TG(d - \hat{d}).$$

If TG has a full column rank, that is $rank(TG) = rank(G) = q$, TG has a left inverse $(TG)^L$, and

$$d - \hat{d} = (TG)^L\dot{e} - (TG)^L Fe.$$

Due to (3.13) and (3.14), part (2) is proved. ∎

The proposed reduced-order adaptive state-function/input estimator is described by

$$\begin{aligned} \dot{z} &= Fz + Ly + TBu + TG\hat{d} \\ \dot{\hat{d}} &= \rho(My - Nz) \end{aligned} \tag{3.15}$$

where ρ is a positive scalar. The following theorem provides sufficient conditions for the convergence of the state-function and unknown input estimates.

Theorem 3.3 *Suppose system (3.1), with the initial condition $x(0)$, is subject to the estimator (3.15). If coefficient matrices of the estimator (3.15) satisfy (3.7)- (3.10), and d is constant, then*

$$z \to Tx \text{ and } \hat{d} \to d \text{ as } t \to \infty.$$

Proof. Let $e = Tx - z$ and $\tilde{d} = d - \hat{d}$. Similar to the proof of Theorem 3.2, the evolution of the estimation error can be described by

$$\dot{e} = Fe + TGd - TG\hat{d} \tag{3.16}$$

if (3.7) is satisfied.

$$\dot{V} = e^T(PF + F^TP)e + 2\tilde{d}^T(TG)^TPe + 2\tilde{d}^T\dot{\tilde{d}}/\rho$$

Because $NT = MC, N = (TG)^TP$, we have $\dot{\tilde{d}} = -\rho(My - Nz) = -\rho Ne$, and

$$2\tilde{d}^T(TG)^TPe + 2\tilde{d}^T\dot{\tilde{d}}/\rho = 0.$$

Then

$$\dot{V} \leq -e^TQe$$

and $e \to 0$. From (3.16), this implies $TG\tilde{d} \to 0$. Because TG is full column rank, it is easy to prove that $\tilde{d} \to 0$. ∎

The proposed adaptive input estimator (3.15) has the same design requirements as that of the input estimator (3.6). Theorem 3.2 and 3.3 provide basic guidelines for our input estimator design. However, these two theorems are not very useful for the observer's existence test and design because all the matrices in the above equations are unknown. Next, we propose a theorem which provides the equivalence conditions with requirements (3.7)-(3.10). Because these conditions are explained as properties of the original system, they can be checked easily to determine if the proposed input estimators exists or not for a given system. In addition, since it is very difficult to solve equations (3.7)-(3.10) directly, a constructive design algorithm will also be proposed, which can easily be implemented in MATLAB. A new proof for Theorem 3.2 is given based on that design algorithm. Futhermore, that algorithm and proof show that bound assumption on $\|d(t)\|, \|\dot{d}(t)\|$ is in fact unnecessary.

3.4 Preliminary Results

Before we present our main result, we present the following lemmas.

Lemma 3.4.1 *For given matrices P, C, T and G, a matrix M, which satisfies $MC = (TG)^TPT$, exists if and only if*

$$(TG)^TPT(I - C^\dagger C) = 0. \tag{3.17}$$

In this case, a general solution is given by

$$M = (TG)^T PTC^\dagger + Z - ZCC^\dagger \tag{3.18}$$

where Z is an arbitrary matrix of the same size as F, and $C^\dagger$ is the generalized inverse of C.

Lemma 3.4.1 is a straightforward application of the fundamental result of the linear matrix equation $WXQ = L$, where W, Q and L are given with adequate dimensions, and X is unknown.

Lemma 3.4.2 *[94] For any given system triple (A, G, C), the special coordinate basis(SCB) transformation $\Gamma_1, \Gamma_2, \Gamma_3$ transform G and C into the form of*

$$\overline{G} := \Gamma_1^{-1} G \Gamma_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & G_c \\ G_d & 0 \end{bmatrix}, \overline{C} := \Gamma_2^{-1} C \Gamma_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & C_d \\ 0 & 0 & C_b & 0 & 0 \end{bmatrix} \tag{3.19}$$

where G_d, C_d are

$$G_d = \begin{bmatrix} G_{q_1} & 0 & \dots & 0 \\ 0 & G_{q_2} & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & G_{qm_d} \end{bmatrix}; C_d = \begin{bmatrix} C_{q_1} & 0 & \dots & 0 \\ 0 & C_{q_2} & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & C_{qm_d} \end{bmatrix}. \tag{3.20}$$

The matrices G_{q_i} and C_{q_i} have the following forms:

$$G_{q_i} = \begin{bmatrix} 0_{(q_i-1)\times 1} \\ 1 \end{bmatrix}; C_{q_i} = \begin{bmatrix} 1 & 0_{1\times(q_i-1)} \end{bmatrix}; \tag{3.21}$$

q_i represents the number of infinite zeros of the order i ($i = 1, ..., m_d$). (Obviously for the case $q_i = 1$, we have $G_{qi} = 1, C_{qi} = 1$).

Lemma 3.4.2 states a property of SCB transformation, and is proved in the original paper dealing with SCB theory[94]. As will be seen, SCB also plays a pivotal role in our input estimator analysis and synthesis.

Lemma 3.4.3 *Let $(\overline{A},\overline{G},\overline{C})$ be the SCB transform matrices of (A,G,C). Matrices $\overline{F},\overline{T},\overline{L},\overline{N},\overline{M}$ satisfy conditions (3.7)-(3.10) for system $(\overline{A},\overline{G},\overline{C})$ if and only if $F = \overline{F}, T = \overline{T}\Gamma_1^{-1}, L = \overline{L}\Gamma_2^{-1}, M = \Gamma_3^{-T}\overline{M}\Gamma_2^{-1}, N = \Gamma_3^{-T}\overline{N}$ satisfy conditions (3.7)-(3.10) for system (A,G,C).*

Proof. As we show in the Lemma 2.2.2, $FT - TA - LC = 0$ if and only if $\overline{FT} - \overline{TA} + \overline{LC} = 0$.

Next, since $G = \Gamma_1\overline{G}\Gamma_3^{-1}$,

$$N = (TG)^T P = (\overline{T}\Gamma_1^{-1}\Gamma_1\overline{G}\Gamma_3^{-1})^T P = \Gamma_3^{-T}(\overline{TG})^T P = \Gamma_3^{-T}\overline{N}$$

Finally, $NT = \Gamma_3^{-T}\overline{NT}\Gamma_1^{-1} = MC = M\Gamma_2\overline{C}\Gamma_1^{-1}$, which means

$$\overline{NT} = \Gamma_3^T M\Gamma_2\overline{C}.$$

In other words, $\overline{M} = \Gamma_3^T M\Gamma_2$ and $M = \Gamma_3^{-T}\overline{M}\Gamma_2^{-1}$. This completes the proof. ∎

Lemma 3.4.4 *Suppose a symmetrical matrix P is partitioned as*

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}$$

where P_{11} and P_{22} are square matrices. Matrix P is positive definite if and only if P_{11} is positive definite and $P_{22} > P_{12}^T P_{11}^{-1} P_{12}$. Futhermore, this condition is equivalent to having $\rho(P_{12}^T P_{11}^{-1} P_{12} P_{22}^{-1}) < 1$, where $\rho(.)$ represents spectral radius.

Lemma 3.4.5 *Let $P \in R^k$ be positive definite. If $T \in R^{k\times n}, k \leq n$, then $T^T PT$ is positive definite if and only if T is full row rank.*

Lemmas 3.4.4 and 3.4.5 are well known results in matrix theory [48].

3.5 Input Estimator Analysis and Synthesis

Theorem 3.4 *Consider the system (3.1). Matrices F, T, L, N and M, which satisfy conditions (3.7)-(3.10), exist if and only if*

1. $rank(CG) = rank(G)$;
2. *all unstable transmission zeros of the system (A, G, C) are unobservable modes of (A, C).*

Proof. Necessity. Without loss of generality, and due to Lemma 3.4.3, we assume the system (A, G, C) is in its SCB form. If F, T, and L satisfy (3.7)-(3.10), then according to Lemma 3.4.1, T must satisfy (3.17).

Partitioning T according to the SCB index we get

$$T = \begin{bmatrix} T_a^- & T_a^+ & T_b & T_c & T_d \end{bmatrix}.$$

Note that

$$I - C^\dagger C = \begin{bmatrix} I_{n_a^-} & 0 & 0 & 0 & 0 \\ 0 & I_{n_a^+} & 0 & 0 & 0 \\ 0 & 0 & I_{n_b} - C_b^\dagger C_b & 0 & 0 \\ 0 & 0 & 0 & I_{n_c} & 0 \\ 0 & 0 & 0 & 0 & I_{n_d} - C_d^\dagger C_d \end{bmatrix}, \tag{3.22}$$

(3.17) can be described as

$$\begin{bmatrix} G_d^T T_d^T \\ G_c^T T_c^T \end{bmatrix} P \begin{bmatrix} T_a^- & T_a^+ & T_b(I - C_b^\dagger C_b) & T_c & T_d(I - C_d^\dagger C_d) \end{bmatrix} = 0. \tag{3.23}$$

(3.23) implies

$$G_c^T T_c^T P T_c = 0. \tag{3.24}$$

It is easy to show that (3.24) is true if and only if $T_c G_c = 0$. Therefore, n_c must be zero, in which case G_c will actually disappear. Otherwise, TG cannot be full rank.

Further, if some $q_i > 1$, namely the number of infinite zeros of the order i is greater than one, then $C_d \neq I_{n_d}$ according to Lemma 3.4.2. (3.23) implies

$$G_d^T T_d^T P T_d (I - C_d^\dagger C_d) = 0 \tag{3.25}$$

Equation (3.25) will be satisfied if and only if $T_d G_d = 0$, and this contradicts the fact that TG should be full rank. Therefore, $q_i = 1 (i = 1, ..., m_d)$. Because the conditions of $n_c = 0$ and $q_i = 1, i = 1, ..., m_d$ are equivalent to $rank(CG) = rank(G)$, the necessity of condition (1) is proved.

Condition $rank(CG) = rank(G)$ means $p \geq q, n_d = q$ and $C_d = G_d = I_q$. Under this condition, (3.23) implies

$$T_d^T P T_a^+ = 0. \tag{3.26}$$

Next we prove that (3.26) can be true if and only if $T_a^+ = 0$.

If $T_a^+ \neq 0$, (3.26) means that there exists transformation matrix S such that

$$ST_a^+ = \hat{T}_a^+ = \begin{bmatrix} T_{a1}^+ \\ 0 \end{bmatrix} \tag{3.27}$$

where T_{a1}^+ is full row rank. Due to $n_c = 0$ and $G_d = C_d = I$, equation $FT - TA + LC = 0$ can be rewritten as,

$$F \begin{bmatrix} T_a^+ & T_a^- & T_b & T_d \end{bmatrix} - \begin{bmatrix} T_a^+ & T_a^- & T_b & T_d \end{bmatrix} \begin{bmatrix} A_{aa}^- & 0 & L_{ab}^- C_b & L_{ad}^- C_d \\ 0 & A_{aa}^+ & L_{ab}^+ C_b & L_{ad}^+ C_d \\ 0 & 0 & A_b & L_{bd} C_d \\ E_a^- & E_a^+ & E_b & A_d \end{bmatrix}$$

$$+L \begin{bmatrix} 0 & 0 & 0 & I_{n_d} \\ 0 & 0 & C_b & 0 \end{bmatrix} = 0. \tag{3.28}$$

It's easy to derive the following equation from (3.28):

$$FT_a^+ - T_a^+ A_{aa}^+ - T_d E_a^+ = 0 \tag{3.29}$$

By replacing T_a^+ in the above equation with (3.27), we have

$$S^{-1} F S \begin{bmatrix} T_{a1}^+ \\ 0 \end{bmatrix} - \begin{bmatrix} T_{a1}^+ \\ 0 \end{bmatrix} A_{aa}^+ - S^{-1} T_d E_a^+ = 0. \tag{3.30}$$

Partition $\hat{F} = S^{-1}FS, \hat{P} = S^TPS$ and $\hat{T}_d = S^{-1}T_d$ as

$$\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}, \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \text{ and } \begin{bmatrix} T_{d1} \\ T_{d2} \end{bmatrix}.$$

Then $\hat{T}_d^T \hat{P} \hat{T}_a^+ = 0$ means

$$T_{d1}^T P_{11} + T_{d2}^T P_{12}^T = 0.$$

(3.30) can be reorganized as

$$\begin{bmatrix} F_{11} & T_{d1} \\ F_{21} & T_{d2} \end{bmatrix} \begin{bmatrix} T_{a1}^+ \\ E_a^+ \end{bmatrix} = \begin{bmatrix} T_{a1}^+ A_{aa}^+ \\ 0 \end{bmatrix}. \tag{3.31}$$

Multiplying both sides of (3.31) with $\hat{P}$, we can derive

$$(P_{11}F_{11} + P_{12}F_{12})T_{a1}^+ = P_{11}T_{a1}^+ A_{aa}^+. \tag{3.32}$$

Let $Q_{11} = (P_{11}F_{11} + P_{12}F_{12}), P_a = T_{a1}^{+^T} P_{11} T_{a1}^+$, so that (3.32) will lead to

$$T_{a1}^{+^T}(Q_{11} + Q_{11}^T)T_{a1}^+ = Q_a = P_a A_{aa}^+ + A_{aa}^{+^T} P_a.$$

Since P is the Lyapunov matrix of F. Q_a must be negative definite. Due to Lemma 3.4.5, P_a is positive definite. This is impossible because all eigenvalues of A_a^+ are positive. To that end. $T_a^+ = 0$.

Based on the above developments T must be in the form of

$$\begin{bmatrix} T_a^- & 0 & T_b & T_d \end{bmatrix}.$$

Since $TG = T_dG_d = T_d$. T_d should be full column rank q in order for $rank(TG) = rank(G)$ to be satisfied. On the other hand, as shown in the proof of Theorem 2.2 in Chapter 2, $FT - TA + LC = 0$ implies that rows of T must be the left eigenvectors of $A - KC$ for a suitable K, thus T satisfies

$$(\lambda[F]I_n - A^T)T^T \in Im(C^T). \tag{3.33}$$

Note that with the SCB form of A, C and T, it is easy to show that (3.33) implies

$$T_dE_a^+ = 0.$$

Obviously, T_d can be full column rank if and only if $E_a^+ = 0$. In the case of $n_c = 0$, the property 4 of SCB in Chapter 2 tell us that (A, C) is observable if and only if (A_a, E_a) is observable, where

$$A_a = \begin{bmatrix} A_{aa}^- & 0 \\ 0 & A_{aa}^+ \end{bmatrix}; E_a = \begin{bmatrix} E_a^- & E_a^+ \end{bmatrix}.$$

Since $E_a^+ = 0$ and the eigenvalues of A_{aa}^+ are unstable transmission zeros of the system, condition (2) is proved.

Sufficiency. Under conditions (1) and (2), the SCB form of the system is reduced to

$$\overline{A} = \begin{bmatrix} A_{aa}^- & 0 & L_{ab}^- C_b & L_{ad}^- \\ 0 & A_{aa}^+ & L_{ab}^+ C_b & L_{ad}^+ \\ 0 & 0 & A_b & L_{bd} \\ E_a^- & 0 & E_b & A_d \end{bmatrix}, \overline{G} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_q \end{bmatrix}, \overline{C} = \begin{bmatrix} 0 & 0 & 0 & I_q \\ 0 & 0 & C_b & 0 \end{bmatrix}.$$

SCB theory has shown that (A_b, C_b) and (A_{dd}, I_q) form observable pairs. Let K_b and K_d be gains such that $\hat{A}_b = A_b - K_b C_b$ and $\hat{A}_d = A_d - K_d$ are stable. Define the feedback gain $\overline{K}$ as

$$\overline{K} = \begin{bmatrix} L_{ad}^- & L_{ab}^- \\ L_{ad}^+ & L_{ab}^+ \\ L_{bd} & K_b \\ K_d & 0 \end{bmatrix}, \tag{3.34}$$

then

$$A_0 = \overline{A} - \overline{KC} = \begin{bmatrix} A_{aa}^- & 0 & 0 & 0 \\ 0 & A_{aa}^+ & 0 & 0 \\ 0 & 0 & \hat{A}_b & 0 \\ E_a^- & 0 & E_b & \hat{A}_d \end{bmatrix}.$$

Hence, it is easy to show by direct matrix computation that $\lambda(A_{aa}^-)$,$\lambda(\hat{A}_b)$ and $\lambda(\hat{A}_d)$ are amongst the eigenvalues of $\overline{A} - \overline{KC}$ and that

$$[V_{a-}, 0, 0, 0], [0, 0, V_b, 0], [V_{ad}, 0, V_{bd}, V_d] \tag{3.35}$$

are the associated left eigenvectors of $\overline{A} - \overline{KC}$, where V_a^-, V_b and V_d are the left eigenvector sets of A_{aa}^-, $\hat{A}_b$ and $\hat{A}_d$ respectively, and are nonsingular.

Let

$$\overline{F} = \begin{bmatrix} \lambda(A_{aa}^{-}) & 0 & 0 \\ 0 & \lambda(\hat{A}_b) & 0 \\ 0 & 0 & \lambda(\hat{A}_d) \end{bmatrix}, \overline{T} = \begin{bmatrix} V_a^{-} & 0 & 0 & 0 \\ 0 & 0 & V_b & 0 \\ V_{ad} & 0 & V_{bd} & V_d \end{bmatrix}, \tag{3.36}$$

we have

$$\overline{FT} = \overline{T}(\overline{A} - \overline{KC}).$$

Let $\overline{L} = \overline{TK}$, the above equation can be rewritten as

$$\overline{FT} - \overline{TA} + \overline{LC} = 0$$

Next we show there exists a Lyapunov matrix P of $\overline{F}$, which satisfies

$$(\overline{TG})^T P\overline{T}(I - \overline{C}^{\dagger}\overline{C}) = 0$$

such that $\overline{NT} = \overline{MC}$ is solvable when $\overline{N} = (\overline{TG})^T P$.

Given any positive definite matrix Q_d, compute the Lyapunov matrix P_d of $\hat{A}_d$. Let

$$V_{ab} = \begin{bmatrix} V_a^{-} & 0 \\ 0 & V_b \end{bmatrix}, V_{dd} = \begin{bmatrix} V_{ad} & V_{bd} \end{bmatrix}, F_{ab} = \begin{bmatrix} \lambda(A_{aa}^{-}) & 0 \\ 0 & \lambda(\hat{A}_b) \end{bmatrix}, F_d = \lambda(\hat{A}_d).$$

Obviously, V_{ab} is invertible. We calculate

$$P_{bd} = -P_d V_{dd} V_{ab}^{-1}, Q_{bd} = -(P_{bd} * F_{ab} + F_d^T * P_{bd}).$$

Let P_{ab} be the solution for $P_{ab}F_{ab} + F_{ab}^T P_{ab} = -\alpha I$, where α is a positive number which is large enough so that

$$\rho(\alpha^{-1} Q_{bd} Q_{bd}^T Q_d^{-1}) < 1 \text{ and } \rho(P_{bd} P_{ab}^{-1} P_{bd}^T P_d^{-1}) < 1 \tag{3.37}$$

Note that $\rho(P_{ab})$ increases as long as α increases, while all other matrices do not change with α, thus equation (3.37) always can be satisfied. Define P, Q as,

$$P = \begin{bmatrix} P_{ab} & P_{bd}^T \\ P_{bd} & P_d \end{bmatrix} \tag{3.38}$$

$$Q = \begin{bmatrix} \alpha I & (Q_{bd})^T \\ Q_{bd} & Q_d \end{bmatrix} \tag{3.39}$$

According to Lemma 3.4.4, P and Q are positive definite. Further,

$$PF + F^T P = \begin{bmatrix} P_{ab} & P_{bd}^T \\ P_{bd} & P_d \end{bmatrix} \begin{bmatrix} F_{ab} & 0 \\ 0 & F_d \end{bmatrix} + \begin{bmatrix} F_{ab}^T & 0 \\ 0 & F_d^T \end{bmatrix} \begin{bmatrix} P_{ab} & P_{bd}^T \\ P_{bd} & P_d \end{bmatrix}$$

$$= \begin{bmatrix} P_{ab}F_{ab} + F_{ab}^T P_{ab} & (P_{bd} * F_{ab} + F_d^T * P_{bd})^T \\ P_{bd} * F_{ab} + F_d^T * P_{bd} & P_d F_d + F_d^T P_d \end{bmatrix} = -\begin{bmatrix} \alpha I & (Q_{bd})^T \\ Q_{bd} & Q_d \end{bmatrix} = -Q$$

thus P and Q are the Lyapunov matrix pair of F. On the other hand, note

$$\overline{TG} = \begin{bmatrix} V_a^- & 0 & 0 & 0 \\ 0 & 0 & V_b & 0 \\ V_{ad} & 0 & V_{bd} & V_d \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_p \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_d \end{bmatrix}.$$

Decompose $P_{bd} = [P_a \quad P_b]$ where P_a, P_b have n_a^-, n_b columns respectively, we know $(\overline{TG})^T P\overline{T}(I - \overline{C}^\dagger \overline{C})$ is calculated as

$$V_d^T \begin{bmatrix} (P_a V_a^- + P_d V_{ad}) & 0 & (P_b V_b + P_d V_{bd}) & P_d V_d \end{bmatrix} \begin{bmatrix} I_{n_a^-} & 0 & 0 & 0 \\ 0 & I_{n_a^+} & 0 & 0 \\ 0 & 0 & I_{n_b} - C_b^\dagger C_b & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= V_d^T \begin{bmatrix} P_a V_a^- + P_d V_{ad} & 0 & (P_b V_b + P_d V_{bd})(I_{n_b} - C_b^\dagger C_b) & 0 \end{bmatrix}.$$

Because $P_{bd} = -P_d V_{dd} V_{ab}^{-1}$, namely

$$0 = P_{bd} V_{ab} + P_d V_{dd} = [P_a \quad P_b] \begin{bmatrix} V_a^- & 0 \\ 0 & v_b \end{bmatrix} + P_d [V_{ad} \quad V_{bd}]$$

$$= \begin{bmatrix} P_a V_a^- + P_d V_{ad} & P_b V_b + P_d V_{bd} \end{bmatrix}$$

therefore $(\overline{TG})^T P\overline{T}(I - \overline{C}^\dagger \overline{C}) = 0$. Due to Lemma 3.4.3, we conclude that there exist matrices F, T, L, N and M that satisfy (3.7)-(3.10), if conditions (1) and (2) are satisfied. ∎

The proof of Theorem 3.4 imbeds in it an algorithm for the construction of an input estimator (3.6) that is suitable for implementation. The algorithm is detailed below:

Unknown Input Estimator Design Algorithm

Step 1: Transform (A, G, C) into the SCB form $(\overline{A}, \overline{G}, \overline{C})$ by non-singular state, output, and input transformations $\Gamma_1, \Gamma_2, \Gamma_3$.

Step 2: Compute K_d and K_b to stabilize $\hat{A}_b = A_b - K_b C_b$ and $\hat{A}_d = A_d - K_d$ respectively. Construct $\overline{K}$ as (3.34) and calculate the left eigenvector set of $\overline{A} - \overline{KC}$ like (3.35). Formulate $\overline{F}, \overline{T}$ as (3.36) and $\overline{L} = \overline{TK}$.

Step 3: Use Lemma 3.4.3 to get the F, T, L. Compute the Lyapunov matrix P for F as (3.38), and calculate $N = (TG)^T P$ and $M = NTC^\dagger$.

At this stage, consider an alternative proof for Theorem 3.2.

Proof. As we showed before in Section 3.3, the error equation of estimator (3.6) is

$$\dot{e} = (F - \gamma(TG)N)e + TGd. \tag{3.40}$$

We know $F = \overline{F}, TG = \overline{TG}\Gamma_3^{-1}, N = \Gamma_3^{-T}\overline{N}$, and

$$\overline{N} = (\overline{TG})^T P = \begin{bmatrix} V_d^T P_{bd} & V_d^T P_d \end{bmatrix}.$$

It is easy to show that

$$\dot{e} = \begin{bmatrix} F_{ab} & 0 \\ \gamma V_d \Gamma V_d^T P_{bd} & F_d - \gamma V_d \Gamma V_d^T P_d \end{bmatrix} e + \begin{bmatrix} 0 \\ V_d \end{bmatrix} \overline{d}(t) \tag{3.41}$$

where $\Gamma = \Gamma_3^{-1}\Gamma_3^{-T}, \overline{d}(t) = \Gamma_3^{-1} d(t)$. Let $F_r = F_d - \gamma V_d \Gamma V_d^T P_d$, we have

$$E(s) = \begin{bmatrix} 0 \\ (sI - F_r)^{-1} \end{bmatrix} V_d \overline{D}(s)$$

Obviously, $V_d \Gamma V_d^T P_d$ is positive definite, and the eigenvalue of F_r can be pushed far into the left hand plane letting γ approach infinity such that H_∞ norm of $T_2(s) = (sI - F_r)^{-1} V_d$ would be approximately zero. Therefore the norm of e can be made as small as possible no matter what d is, thus we prove part (1) of Theorem 3.2. Further, $\hat{d} = \gamma(My - Nz) = \gamma Ne = \gamma\Gamma_3^{-T}\overline{N}e$, we have

$$\lim_{\gamma\to\infty}\hat{D}(s) = \lim_{\gamma\to\infty}\gamma\Gamma_3^{-T}V_d^T P_d(sI - F_d + \gamma V_d\Gamma V_d^T P_d)^{-1}V_d\Gamma_3^{-1}D(s) = D(s)$$

and part (2) of Theorem 3.2 is proved. ∎

Remark 3.5.1 Theorem 3.4 shows that we can estimate input for certain non-minimum phase systems, if we are not required to estimate all states using the same estimator. If system (A, G, C) is minimum phase, T can be full rank such that all states can be estimated. In this case, our proposed estimator is reduced to that in [17].

Remark 3.5.2 It is valuable to point out that matrix N may have a more general form as $N = W(TG)^T P$, where W is any positive definite matrix with a compatible dimension. The introduction of W does not change the estimation property at all. Due to this fact, in Step 4 of the design algorithm, we can let $N = \Gamma_3\overline{N}$, $M = \Gamma_3\overline{M}\Gamma_2^{-1}$. Under this case, $W = \Gamma_3\Gamma_3^T$, and $\overline{TGN} = TGN$. This may lead to more numerical robustness as we note in the simulation study.

Remark 3.5.3 In an adaptive input estimator, estimation of the input is based on the integral of state-function estimation error, thus measurement noise will not result in severe performance degradation.

3.6 Numerical Example

Example 3.6.1 In this example, the algorithm is applied to implement an unknown input estimator for a double-effect pilot plant evaporator [89]. As shown in Figure 3.1, this system, with a total of three inputs (F_1, F_2, T_s) and five outputs $(W_1, C_1, T_1, W_2, C_2)$, can be represented by a 5th order linear state-space model. In the operating process, the feed solution (flow F_0 and concentration C_0) is pumped into the first effect. Then the first effect solution (hold-up w_1) is heated by saturated steam (temperature T_s), and the boil-off travels into the second effect steam jacket.

The concentrated solution from the first effect (flow F_1 and concentration C_1) enters the second effect, which operates under vacuum; the hold-up in the second effect is W_2. Finally, the concentrated product (flow F_2 and concentration C_2) is produced. The state equations of the evaporator are the following:

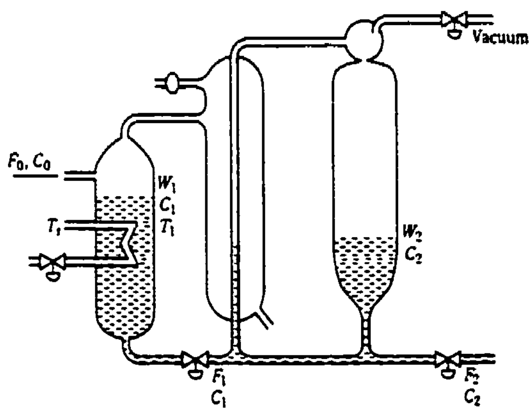

Figure 3.1: Schematic diagram of the double-effect pilot plant evaporator

$$\begin{aligned} \dot{x} &= Ax + Bu + Gd(t) \\ y &= Cx \end{aligned}$$

where $x = \begin{bmatrix} W_1 & C_1 & T_1 & W_2 & C_2 \end{bmatrix}^T, u = \begin{bmatrix} F_1 & F_2 & T_s \end{bmatrix}^T, d = \begin{bmatrix} C_0 & F_0 \end{bmatrix}^T$. For the purpose of illustration, we hypothesize that only state $x_1 = W_1$ and $x_2 = C_1$ are measured. The objective is to estimate unknown inputs $d(t)$ using measured value of W_1 and C_1. The matrices A, B, G and C are

$$A = \begin{bmatrix} 0 & 0 & -0.0034 & 0 & 0 \\ 0 & -0.041 & 0.0013 & 0 & 0 \\ 0 & 0 & -1.1471 & 0 & 0 \\ 0 & 0 & -0.0036 & 0 & 0 \\ 0 & 0.094 & 0.0057 & 0 & -0.051 \end{bmatrix}; B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ -0 & 0 & 0.948 \\ 0.916 & -1 & 0 \\ -0.598 & 0 & 0 \end{bmatrix};$$

$$G = \begin{bmatrix} 0 & 1 \\ 0.062 & -0.132 \\ 0 & -7.189 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

The system has two stable transmission zeros, $z_1 = -1.1715, z_2 = -0.051$, and an unstable transmission zero, $z_3 = 0$. It satisfies conditions for the existence of the input estimators (3.6) and (3.5). However, the input estimators proposed in [17] and [119] does not exist for the above system. Following the design algorithm step by step and choosing the set of desired eigenvalues to be $\{-1.1715, -0.0515, -1, -1\}$, we have

$$T = \begin{bmatrix} -7.1907 & 0 & -1.0002 & 0 & 0 \\ -0.0366 & 0 & -0.0051 & 0 & -1 \\ 0.0545 & 1 & 0.0076 & 0 & 0 \\ 0.8575 & 0 & -0.0198 & 0 & 0 \end{bmatrix}; F = \begin{bmatrix} -1.1715 & 0 & 0 & 0 \\ 0 & -0.051 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix};$$

$$L = \begin{bmatrix} -8.4243 & 0 \\ -0.0019 & -0.9400 \\ 0.0545 & 0.9590 \\ 0.8575 & -0 \end{bmatrix};$$

$$N = \begin{bmatrix} 0.0400 & 0 & 8.0645 & 1.0645 \\ -0.0099 & 0 & 0 & 0.5 \end{bmatrix}; M = \begin{bmatrix} 1.0645 & 8.0645 \\ 0.5 & 0 \end{bmatrix}$$

In the simulations that follow, a full state feedback controller is incorporated. The unknown input vector is assumed to be $d(t) = \begin{bmatrix} 5sin(5\pi t) \\ 10sin(0.5\pi t) \end{bmatrix}$, which is physically unrealistic but does demonstrate the capability of the input estimator very well. Figure 3.2 shows the result of the input estimator (3.6), where $\gamma = 100$, and a small random sequence (the maximum magnitude is 0.01) is added to the output to test its performance under measurement noise.

Figure 3.3 shows the result of the adaptive input estimator (3.15), where $\rho = 10$. Obviously, it cannot give the right estimation for a high frequency signal. If we let $d_1(t) = 5sin(\pi t)$, Figure 3.4 shows a satisfactory result, although the estimated

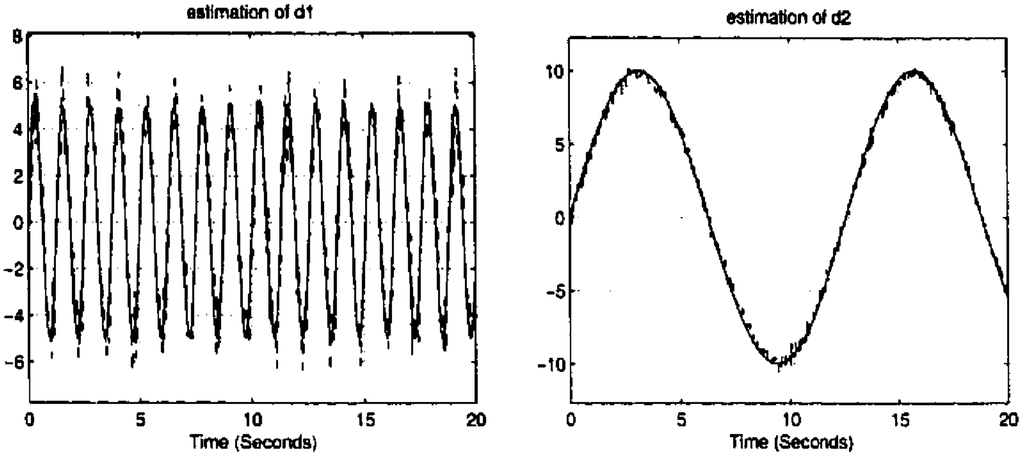

Figure 3.2: Simulation result for inputs estimator

magnitude is still not exact. However, compared with the result in Figure 3.2. we note its superior performance under measurement noise, as was discussed in Remark 3.5.3.

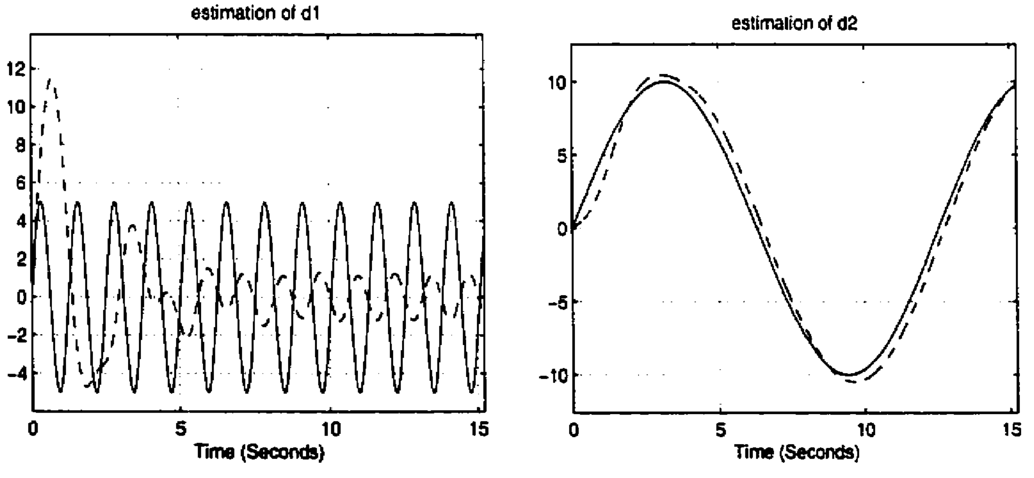

Figure 3.3: Simulation result for adaptive inputs estimator

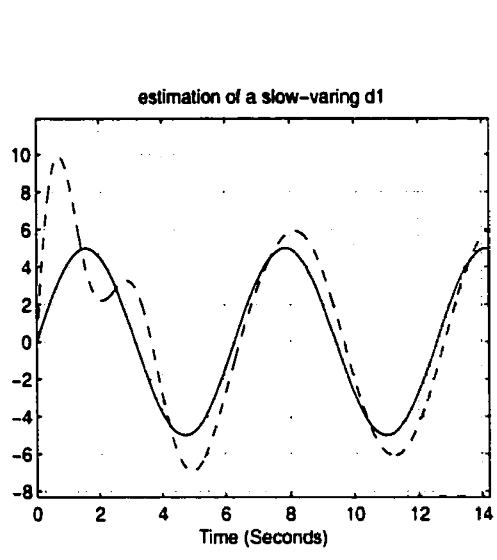

Figure 3.4: Adaptive inputs estimation simulation of slow time-varying signal

3.7 Conclusions

We considered the problem of estimating unknown inputs for linear systems using only the measurements. Our results extended the input estimators proposed by Corless [17] and Wang *et al.* [119] to a more general class of systems. Since faults can be considered as unknown inputs, the input estimator can be used for actuator and component fault diagnosis by estimating the fault signal directly. Generally, some other kinds of unknown inputs, such as external disturbance and model uncertainty, appear in the system together with faults. In these cases we can handle them in the same way as faults by estimating them. Clearly, the inclusion of the disturbance reduces the number of faults to be estimated, if the number of sensors was to remain fixed. Therefore it is unnecessary to estimate those disturbance signals in fault diagnosis problems. In Chapter 4, we will discuss how the unknown inputs decoupling and unknown inputs estimation can be combined to form an efficient robust fault diagnosis scheme, which would isolate the maximum number of faults.

Chapter 4

Robust Fault Diagnostic Observer Design For Linear Uncertain Systems

This chapter explores the design of a robust fault diagnostic observer for a class of linear uncertain systems, where all uncertainties acting on a system are modeled as unknown inputs. After briefly discussing fault detection observer design based on an unknown input decoupled residual generator, the relatively more difficult problem of mutual fault isolation is examined extensively. Two kinds of design approaches are proposed. The first method is carried out through transformation of the system into its Special Coordinate Basis (SCB) form, such that the unknown input decoupled residual generator can be easily combined with Beard-Jones Detection Filter (BJDF) theory, or input estimator, to produce a robust fault isolation observer. The second method uses a direct eigenstructure assignment scheme to make a diagonal transfer function between the faults and the residual, and achieves unknown input decoupling as well. We believe that these results shed additional light on some of the earlier work in the literature.

4.1 Introduction

Given that the analytical redundancy based approaches to FDI are essentially model-based, the issue of robustness in such FDI problems is of significant importance, because in practice, no accurate mathematical model is available. Over the past decade or so, a great deal of effort has gone in this direction [28]. Amongst various approaches to the design of a robust FDI scheme, one of the most powerful ways to achieve robustness in FDI is to build the unknown input (or disturbance) decoupled residuals. That is to say, the residual is made independent of all disturbances. Perfect decoupling designs can be achieved by using the *unknown input observer (UIO)* or *eigenstructure assignment* theory, where there are no assumptions made for knowledge of the unknown inputs. The unknown input insensitive residuals can also be realized by making use of the sliding mode approach if the upper bound of the unknown inputs is known and is finite. The sliding mode approach will be examined in Chapter 7.

The UIO based methods are of three kinds:

1. those which directly use the results of UIO to obtain the unknown inputs decoupled state estimation, then use a function of the observation error as a residual [89, 114];

2. those which estimate fault signals directly under the existence of unknown inputs by using the UIO theory [97];

3. those which generate the residual using the robust estimation of the outputs [11, 24, 35, 50]. This is called as unknown input fault detection observer (UIFDO).

Because of the disadvantage of the first and second methods, that is, strict existence conditions for UIO, the third kind is revisited and further developed in this chapter. In general, a bank of observers [24] is applied to deal with the problem of localizing the faults uniquely, after the alarm has been set by UIFDO. The basic idea of multiple observers scheme (MOS) is to make each observer sensitive to only one fault, and all other faults and unknown inputs are decoupled. However, the structure of multiple observer scheme is too large and is computationally intensive, especially for time-critical systems. To simplify the structure of MOS, the concept of the structured residual was introduced by Ben-Haim [3]. However, this method is not suitable for isolating multiple faults when different sources of faults occur simultaneously. An alternative approach, which performs multiple fault isolation, is the well-known Beard-Jones detection filter (BJDF). Unlike the structured residual, BJDF produces fixed-direction residual that confines each fault effect to a fault-specific direction through the design of observers with suitable dynamics. The fixed-direction residual can also be generated by parity relations [38] and it is proved to be equivalent to BJDF. The main advantage of BJDF is a considerable reduction of the on-line requirements in the implementation, because a single observer is used. The original BJDF depends on an accurate system model. Recently, several ways to enhance its robustness were proposed in [13, 16, 19]. [16, 19] were based on H_∞ and differential game theory and were very complex. The idea in [13] is more attractive, where the remaining freedom after the UIO design is used to provide the residual with directional properties.

The eigenstructure assignment was originally used to design an UIFDO by Patton *et al.* [85]. The fault isolation observers, which makes use of the remaining free parameters available in the parametric observer gain, were developed by Wang *et al.* for sensor faults [118], and by Shen *et al.* for actuator faults [101]. Those design algorithms are complicated, and the conditions for the existence of a solution are unclear. The work in [101, 118] is a special case of a robust BJDF, where the residual direction is fixed to a unit vector by a suitable eigenvector assignment.

In this chapter, the unknown input residual generator (UIRG) problem and fault detectability under UIRG are considered first. The necessary and sufficient conditions for the existence of UIRG are provided. The existence conditions for UIRG are not simply coincidental with those of UIO, which has been mentioned before in [24, 50, 72]. However, although the conditions for the existence of UIO were given two decades ago, no explicit existence conditions (in terms of original system matrices) for UIRG are ever provided. It is considered very difficult and perhaps impossible to provide general existence conditions for UIRG in [50]. The derivation of this chapter is based on the observation of the relationship between the unknown input functional observer (UIFO) and UIRG. Especially, our algorithm achieves the maximum number of independent residuals for a given system.

Multiple fault isolation is discussed carefully in the second part of this chapter. A new approach is proposed which combines UIRG with BJDF or input estimator. Explicit sufficient conditions are derived, and as well a design procedure to achieve a diagonal fault/residual map is given. On the other hand, some interesting property regarding multiple actuator/sensor fault isolation is revealed. In order to derive their results, [101] assumed that the number of outputs is greater than the number of unknown inputs and faults. The existence of an UIO is necessary in [13]. By exploiting the structure invariant property of the system itself, we achieve better robust FDI ability than previous schemes. Finally, two examples are given to illustrate the effectiveness of the proposed approach. The previous schemes are inapplicable to these two examples.

4.2 Unknown Inputs Residual Generator Problem

Consider the following formulation of a linear system, where uncertainty and fault models are included as follows:

$$\begin{aligned} \dot{x} =& \; Ax + Bu + Gd + F_a f_a \\ y =& \; Cx + F_s f_s \end{aligned} \tag{4.1}$$

where $x \in \mathcal{R}^n$ is the state, $u \in \mathcal{R}^m$ is the control input, $y \in \mathcal{R}^p$ is the output. A, B and C are nominal system matrices with compatible dimensions. The vector $d \in \mathcal{R}^q$ denotes the unknown inputs, and $G \in \mathcal{R}^{n \times q}$ represents the distribution matrix of the unknown input signal. The terms f_a, f_s are unknown vector time-varying functions that represent the evolution of actuator/component faults and sensor faults, respectively, while F_a and F_s denote their distribution. It is assumed that the matrices G, F_a and F_s are perfectly known and G is of full rank. However, no assumptions are made for knowledge of d, f_a, f_s. In Chapters 2 and 3, actuator and component faults are also considered as a kind of unknown input. For fault diagnosis, they are represented independently, so that the unknown input in this chapter only represents the model uncertainty and disturbance.

A full-order Luenberger observer for residual generation has the following form:

$$\begin{aligned} \dot{\hat{x}} = & \ A\hat{x} + Bu + K(\hat{y} - y) \\ \hat{y} = & \ C\hat{x} \end{aligned} \tag{4.2}$$

Defining the state estimation and output estimation error as $e = x - \hat{x}$ and $e_y = y - \hat{y}$ respectively, the estimation error equation is described by

$$\begin{aligned} \dot{e} = & \ (A - KC)e + Gd + F_a f_a - KF_s f_s \\ e_y = & \ Ce + F_s f_s \end{aligned} \tag{4.3}$$

A general expression for the residual vector can be written as

$$r = We_y = WCe + WF_s f_s. \tag{4.4}$$

Given the above, the transfer function between the residual and the unknown input signal can be written as

$$r(s) = G_{rd}(s)d(s) = WC(sI - A + KC)^{-1}Gd(s). \tag{4.5}$$

The full-order UIRG problem is to find the matrices K and $W.(WC \neq 0)$ such that $A - KC$ is stable and $G_{rd}(s) = 0$. The problem has been formulated earlier in [85], which proposed a trial and error eignestructure assignment algorithm to design the matrices K and W. Recall that the full-order UIFO problem in Chapter 2 is to find matrix T and K such that $A - KC$ is stable and $T(sI - A + KC)G = 0$. It is noted that full-order UIRG can be considered as a special full-order UIFO. Unlike general UIFO, the matrix T for UIRG is confined to make $NT = WC$ solvable.

A reduced-order UIRG has the following more general form:

$$\dot{z} = Fz + Ly + TBu \tag{4.6}$$

with the residual

$$r = My - Nz \tag{4.7}$$

where F, L, T, N and M are matrices to be designed with appropriate dimensions. It is desired that $z \to Tx$ with $t \to \infty$ under existence of unknown inputs. The corresponding equation for the estimation error of the above observer becomes

$$\dot{e} = T\dot{x} - \dot{z} = Fe - (FT - TA + LC)x - LF_s f_s + TGd + TF_a f_a \tag{4.8}$$

and the residual becomes

$$r = Ne + (MC - NT)x + MF_s f_s \tag{4.9}$$

To meet the unknown input decoupling requirements, it is straightforward to show that the following conditions must hold simultaneously:

$$F \text{ is stable;} \tag{4.10}$$

$$FT - TA + LC = 0; \tag{4.11}$$

$$TG = 0; \tag{4.12}$$

$$MC = NT. \tag{4.13}$$

In comparison with UIFO, the solution of reduced-order UIRG needs to satisfy only one more equation (4.13), which is equivalent to

$$T_c = \begin{bmatrix} T \\ C \end{bmatrix} \text{ is rank deficient.} \tag{4.14}$$

This condition is opposite to that for observer-based controller synthesis, which is $rank(T_c) > rank(C)$.

4.3 Solvability of Unknown Inputs Residual Generator Problem

The understanding of the similarities and differences between the observers for residual generation and state estimation is an important step in deriving solvable conditions for UIRG. A complete solution for the UIFO and special coordinate basis (SCB) theory described in Section 2.2.1 greatly simplifies the proof procedure.

Theorem 4.1 *The full-order UIRG problem is solvable if and only if* (A, C) *is detectable and the system* (A, G, C) *is not right invertible (i.e. SII* $n_b > 0$*), and the maximum dimension of the residual vector is* $rank(C) - m_d$*, where* m_d *is the highest order of an infinite zero.*

Proof. Without loss of generality, we assume (A, G, C) is already in the SCB form. According to Lemma 2.2.3, $T(sI - A + KC)G = 0$ can be attained by some K if and only if WC has the form of

$$WC := W_c = \begin{bmatrix} W_{ca}^- & 0 & W_{cb} & 0 & 0 \end{bmatrix} \tag{4.15}$$

where W_{cb} has n_b columns. Note the SCB form of matrix C, and divide $W = \begin{bmatrix} W_d & W_b \end{bmatrix}$ where W_d, W_b have compatible dimensions with C, then

$$WC = \begin{bmatrix} 0 & 0 & W_bC_b & 0 & W_dC_d \end{bmatrix}. \tag{4.16}$$

By comparing (4.15) and (4.16), we know that W_dC_d must be zero. The term W_bC_b would not be zero if and only if $n_b \neq 0$ (i.e. the system is not right invertible). Thus, if and only if $n_b \neq 0$, we can always find matrix W such that $WC \neq 0$, and WC has the form of (4.15) (in fact, in our case, W_{ca}^- must be zero).

Because $W_dC_d = 0$, (4.16) means that the maximum dimension of the residual vector is equal to $rank(C_b) = rank(C) - rank(C_d) = rank(C) - m_d$. ∎

Next we present the existence conditions for the reduced-order UIRG problem.

Theorem 4.2 *The reduced-order UIRG problem is solvable if and only if the system (A, G, C) is not right invertible (i.e. SII $n_b > 0$), and the maximum dimension of residual vector is $rank(C) - m_d$.*

Proof. The solution of the reduced-order UIRG must satisfy equations (4.10)-(4.13). Assume (A, G, C) is in SCB form. Theorem 2.2 shows that matrix T_c must be in the following form if F, T, and L satisfy conditions (4.10)-(4.12),

$$T_c = \begin{bmatrix} T \\ C \end{bmatrix} = \begin{bmatrix} \Omega & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} V_a^- & 0 & 0 & 0 & 0 \\ 0 & 0 & V_b & 0 & 0 \\ 0 & 0 & 0 & 0 & C_d \\ 0 & 0 & C_b & 0 & 0 \end{bmatrix} \tag{4.17}$$

Obviously, through suitable linear transformation, only rows in $\begin{bmatrix} 0 & 0 & V_b & 0 & 0 \end{bmatrix}$ can be made to be linearly dependent with rows in C. Therefore, UIRG exists if and only if $n_b > 0$. Since dim $C_b = (rank(C) - m_d) \times n_b$, there are at most $rank(C) - m_d$ rows which are linearly dependent with C_b. For that reason, $rank(C) - m_d$ is the maximum dimension of the residual vector. ∎

Remark 4.3.1 Theorem 4.2 implies that full-order UIRG is a special case of reduced-order UIRG. This is a reasonable result. Full-order UIRG is based on a full-order observer and requires $A - KC$ to be stable. From the viewpoint of system decomposition provided by SCB, the solution of the UIRG problem relies on the existence of the unknown input free observable subsystem.

Remark 4.3.2 The rank(C) >rank(G) is a simple and well-known sufficient condition for the reduced-order UIRG problem. Because if rank(C) >rank(G), the system (A, G, C) cannot be right invertible. However, the reverse is not true. It means that even if $rank(C) \leq rank(G)$, it is still possible to build a UIRG.

Remark 4.3.3 Generally, it is known that the design freedom is defined by the number of independent outputs and unknown inputs. However, it is the first time that the exact maximum dimension of the residual vector is found and achieved by a systematic algorithm. This has consequences in the number of simultaneous faults that can be isolated using a single observer, and is very important for a complete robust fault isolation solution.

4.4 Fault Detection Using Unknown Input Residual Generator

The emphasis up to this point has been on the unknown input decoupled residual generation (UIRG). However, this is not all that is required for fault diagnosis design. In order to accomplish the robust fault detection task, the residual has to be sensitive to sensor and actuator faults, and at the same time insensitive to the unknown inputs. In this section, we present the design algorithm for UIRG, and analyze the fault detectability using the UIRG design.

The design of UIRG can be accomplished easily by modifying the UIFO design algorithms in Section 2.4 due to the close relationship between the UIFO and UIRG problems. For completeness of presentation, two algorithms for UIRG are described below, although they are similar to the UIFO design algorithms. The design for both full-order and reduced-order UIRG are combined together.

UIRG Design Algorithm I

Step 1: System (A, G, C) is transformed into its SCB form by non-singular state, output, and input transformations $\Gamma_1, \Gamma_2, \Gamma_3$.

(a) If $n_b = 0$, stop. No UIRG exists.

(b) Otherwise, continue.

Step 2: Choose a gain K_b such that the eigenvalues $\lambda(A_b - K_b C_b) = \Lambda^b$ are negative, calculate out the corresponding left eigenvectors V_b.

Step 3: Define A_g and C_g as

$$A_g = \begin{bmatrix} A_{aa}^- & 0 & 0 & L_{ad}^- C_d \\ 0 & A_{aa}^+ & 0 & L_{ad}^+ C_d \\ G_c E_{ca}^- & G_c E_{ca}^+ & A_{cc} & L_{cd} C_d \\ G_d E_a^- & G_d E_a^+ & G_d E_c & A_d \end{bmatrix}, C_g = \begin{bmatrix} 0 & 0 & 0 & C_d \end{bmatrix}.$$

If (A, C) is detectable, (A_g, C_g) must be detectable. Choose a gain K_g such that the eigenvalues $\lambda(A_g - K_g C_g) = \Lambda^g$ lie at desired locations in C^-. Otherwise, let K_g be any matrix with a suitable dimension. Partition K_g as

$$K_g = \begin{bmatrix} K_a^- \\ K_a^+ \\ K_c \\ K_d \end{bmatrix}.$$

Step 4: Combine a matrix $\overline{K}$ as

$$\overline{K} = \begin{bmatrix} K_a^- & L_{ab}^- \\ K_a^+ & L_{ab}^+ \\ L_{bd} & K_b \\ K_c & L_{cb} \\ K_d & 0 \end{bmatrix} \tag{4.18}$$

and $\overline{T}$ as

$$\overline{T} = \begin{bmatrix} 0 & 0 & V_b & 0 & 0 \end{bmatrix}. \tag{4.19}$$

Calculate $\overline{L}$ via $\overline{L} = \overline{T}\overline{K}$, and select matrix $\overline{W} = [W_d \quad W_b]$ such that $W_b C_b \neq 0$ and $W_d C_d = 0$. Note that the row number of C_b is the maximum dimension of the residual vector, m_r. In order to achieve the m_r, W_b should be a nonsingular matrix.

Step 5: Compute K, W as

$$K = \Gamma_1 \overline{K} \Gamma_2^{-1}, W = \overline{W} \Gamma_2^{-1}. \tag{4.20}$$

It is a solution of full-order UIRG if (A, C) is detectable.

Step 6: Construct F as $F = \Lambda^b$ and $L = \overline{L}\Gamma_2^{-1}, T = \overline{T}\Gamma_1^{-1}, M = W, N = WCT^+$.

UIRG Design Algorithm II

Step 1: Find indices n_b of the system (A, G, C) from its system matrix. If n_b is zero, stop here. The way to calculate n_b without using the SCB transformation can be found in [76].

Step 2: Choose a distinct negative number set $\Lambda^b = \{\lambda_i, i = 1, ..., n_b\}$ as the desired eigenvalues, calculate its corresponding left eigenvectors v_i, and dummy vectors o_i by solving

$$\begin{bmatrix} v_i & o_i \end{bmatrix} \begin{bmatrix} \lambda_i - A & G \\ -C & 0 \end{bmatrix} = 0, \tag{4.21}$$

where $i = 1, ..., n_b$. Note that v_i should be selected to be mutually linearly independent.

Step 3: Choose the remaining $n - n_b$ eigenvalues $\Lambda^g = \{\lambda_i, i = n_b + 1, ..., n\}$ and find their corresponding left eigenvectors v_i, and dummy vector o_i, by solving the following equations,

$$v_i(\lambda_i I - A) - o_i C = 0, i = r + 1, ..., n.$$

Note that all unobservable fixed modes must be included in Λ^g and the resulting v_i should be linearly independent of those v_i obtained in Steps 2.

Step 4: K is parameterized as

$$K = -\begin{bmatrix} v_1 \\ v_2 \\ \cdots \\ v_n \end{bmatrix}^{-1} \begin{bmatrix} o_1 \\ o_2 \\ \cdots \\ o_n \end{bmatrix} = -V^{-1}O \tag{4.22}$$

and T is constructed as

$$T = \Omega\Gamma = \Omega \begin{bmatrix} v_1 \\ v_2 \\ \cdots \\ v_{n_b} \end{bmatrix}. \tag{4.23}$$

where Ω is any invertible matrix such that T has full rank.

Step 5: F and L are calculated as

$$F = \Omega\Lambda\Omega^{-1}, L = TK \tag{4.24}$$

where Λ is diagonal with the elements composed by Λ_b. Find W by solving $WCQ_g = 0$, where Q_g is the last $n - n_b$ columns of $Q = V^{-1}$. $M = W, N = WCT^+$. If (A, C) is detectable, (K, W) is the solution for the full-order UIRG problem.

Remark 4.4.1 Compared with algorithms for UIFO in Section 2.4, all rows in T, which are independent of rows in C, are removed. It is easy to prove that these two algorithms achieve unknown input decoupling following the lines of proof for Theorems 2.5 and 2.6.

Given the residual expression (4.4) which uses a full-order observer, the relationship between the residual and fault signal can be written in the transfer function form as

$$r(s) = G_{af}(s)f_a(s) + G_{sf}(s)f_s(s) \tag{4.25}$$

where $G_{af}(s) = WC(sI - A_0)^{-1}F_a; G_{sf}(s) = WF_s - WC(sI - A_0)^{-1}KF_s$ and $A_0 = A - KC$.

For the robust sensor fault detection, it is required that $G_{sf}(s) \neq 0$. One simple sufficient condition for $G_{af}(s) \neq 0$ is that $WCF_a \neq 0$. It is noted that $G_{rd}(s) = 0$ implies that $WCG = 0$. Consequently, CF_a should not belong to $Im(CG)$ in order that $WCF_a \neq 0$. The following lemma tell us the necessary and sufficient conditions for $G_{af}(s) \neq 0$.

Lemma 4.4.1 *Let*

$$\overline{F_a} := \Gamma_1^{-1}F_a = \begin{bmatrix} (F_{aa}^-)^T & (F_{aa}^+)^T & F_{ab}^T & F_{ac}^T & F_{ad}^T \end{bmatrix}^T, \tag{4.26}$$

where Γ_1 is the SCB state transformation matrix for the system (A, G, C). The gain matrices K and W given by (4.20) will result in

$$G_{af}(s) = W_bC_b(sI - A_b + K_bC_b)^{-1}F_{ab}. \tag{4.27}$$

Therefore, under the condition that $G_{rd}(s) = 0$, $G_{af}(s) \neq 0$ if and only if $F_{ab} \neq 0$.

Proof. By matrix computation, we know

$$G_{af}(s) = WC(sI - A + KC)^{-1}F_a = \overline{WC}(sI - \overline{A} + \overline{KC})^{-1}\overline{F_a}$$

where $\overline{X}$ means the SCB form of a matrix X. For simplicity of presentation, we assume $\overline{A} - \overline{KC}$ is non defective. Then

$$G_{af}(s) = \sum_{i=1}^{n} \frac{\overline{WC}q_iv_i\overline{F_a}}{s - \lambda_i} \tag{4.28}$$

where v_i, q_i are the left eigenrow and the right eigencolumn vector respectively corresponding to eigenvalue λ_i of $\overline{A} - \overline{KC}$. It is shown in [94] that n_b left eigenvectors corresponding to $\lambda(A_b - K_bC_b)$ are constrained to be the form $V_b = [0, 0, V_{bb}, 0, 0]$, where V_{bb} is the left eigenvector matrix of $A_b - K_bC_b$. The $n_g(n_g = n - n_b)$ right eigenvectors corresponding to $\lambda(A_g - K_gC_g)$ have the special matrix form $Q_g = [(Q_a^-)^T, (Q_a^+)^T, 0, (Q_c)^T, (Q_d)^T]^T$. On the other hand, $\overline{WC} = \begin{bmatrix} 0 & 0 & W_bC_b & 0 & 0 \end{bmatrix}$, therefore,

$$\overline{WC}Q_g = 0, V_b\overline{F_a} = V_{bb}F_{ab}. \tag{4.29}$$

Assume the n_b right eigenvectors corresponding to $\lambda(A_b - K_bC_b)$ are

$$Q_b = [(Q_a^-)^T, (Q_a^+)^T, (Q_{bb})^T, (Q_c)^T, (Q_d)^T]^T$$

Consequently,

$$G_{af}(s) = \sum_{i=1}^{n_b} \frac{W_bC_bq_{bb_i}v_{bb_i}F_{ab}}{s - \lambda_i}, \tag{4.30}$$

where q_{bb_i}, v_{bb_i} is the ith column of Q_{bb} and the ith row of V_{bb} respectively. $\lambda_i \in \lambda(A_b - K_bC_b)$.

According to the definition of the right eigenvector. we know

$$(\overline{A} - \overline{KC})Q_b = \lambda_bQ_b \tag{4.31}$$

It is noted that

$$\overline{A} - \overline{KC} = \begin{bmatrix} A_{aa}^- & 0 & 0 & 0 & (L_{ad}^- - K_a^-)C_d \\ 0 & A_{aa}^+ & 0 & 0 & (L_{ad}^+ - K_a^+)C_d \\ 0 & 0 & A_b - K_bC_b & 0 & 0 \\ G_cE_{ca}^- & G_cE_{ca}^+ & 0 & A_{cc} & (L_{cd} - K_c)C_d \\ G_dE_a^- & G_dE_a^+ & G_dE_b & G_dE_c & A_d - K_dC_d \end{bmatrix}. \tag{4.32}$$

(4.31) and (4.32) impliy that $(A_b - K_bC_b)Q_{bb} = \lambda_bQ_{bb}$, which means the Q_{bb} must be the right eigenvectors of $A_b - K_bC_b$. Combining this conclusion with (4.30), equation (4.27) is proved.

It is obvious that $F_{ab} \neq 0$ is necessary for $G_{af}(s) \neq 0$. Further, we prove it is also a sufficient condition.

Let $A_k = A_b - K_bC_b$, and

$$W_b \begin{bmatrix} C_b \\ C_bA_k \\ \cdots \\ CA_k^{n_b-1} \end{bmatrix} F_{ab} = W_bA_{ob}F_{ab}.$$

Because (A_b, C_b) is observable, then (A_k, C_b) is observable too, which means that A_{ob} is nonsingular. Let W_b is unitary matrix, thus $W_bA_{ob}F_{ab} \neq 0$ if $F_{ab} \neq 0$, or $X_i = W_bC_bA_k^iF_{ab}$ is nonzero for some i, $(i \leq (n_b - 1)$. It is well known that $G_{af}(s) \neq 0$ if and only if $X_i \neq 0$ for some i. This completes the proof. ∎

For robust sensor fault detection, $G_{sf}(s) \neq 0$ is required. Let

$$\overline{F_s} := \Gamma_2^{-1}F_s = \begin{bmatrix} F_{sd}^T & F_{sb}^T \end{bmatrix}^T. \tag{4.33}$$

It is easy to show that

$$G_{sf}(s) = F_{sb} - W_bC_b(sI - A_b + K_bC_b)^{-1}(L_{bd}F_{sd} + K_bF_{sb}). \tag{4.34}$$

Therefore, under the condition that $G_{rd}(s) = 0$, $G_{sf}(s) \neq 0$ if and only if $F_{sb} \neq 0$ or $L_{bd}F_{sd} + K_bF_{sb} \neq 0$.

Using the reduced-order observer, the residual (4.7) for actuator faults can be represented as

$$r(s) = N(sI - F)^{-1}TF_af_a(s). \tag{4.35}$$

The existence of W such that $WC(sI - A + KC)^{-1}G = 0$ implies that a matrix N must exist such that $WC = NT$, where T is a general solution of UIFO. To that end, the design of W in full-order UIRG is actually equivalent to solvability of $NT = MC$, the condition for reduced-order UIRG. It is simple to prove that $G_{af}(s) = F_{af}(s)$ using the above UIRG design algorithms, namely by choosing $M = W, N = WCT^+$. This concludes that fault detection using full-order and reduced-order UIRG have the same restrictive condition for fault distribution matrix. However, the computational load of the reduced-order UIRG is less than that of the full-order UIRG.

4.5 Fault Isolation Using Unknown Inputs Residual Generator

This section proposes three approaches to use the remaining design freedom to isolate faults if the maximum dimension of the residual vector, $m_r > 1$. The multiple actuator fault isolation is considered first. The sensor faults are expressed as the actuator faults of an augmented system, using the method in [83, 97], then processed by the same way as actuator faults. The special property of the multiple actuator/sensor FDI is given in Section 4.6 to show the difference between the multiple actuator/sensor FDI and multiple actuator FDI.

4.5.1 Unknown Inputs Decoupled Beard-Jones Detection Filter

Let us describe the basic principle of the Beard-Jones detection filter (BJDF) at the beginning. Consider a system without unknown inputs in the state-space format as

$$\begin{aligned} \dot{x}(t) = &\ Ax(t) + Bu(t) + f_i f_{a_i}(t) \\ y(t) = &\ Cx(t) \end{aligned} \tag{4.36}$$

The term $f_i f_{a_i}(t)(i = 1, ..., h)$ denotes that an actuator or component fault occurs, where f_i is a column vector (called the fault event direction) and $f_{a_i}(t)$ is an arbitrary function of time.

A BJDF is just a full-order observer, and its structure and the residual can be described as

$$\begin{aligned} \dot{\hat{x}}(t) = &\ A\hat{x}(t) + Bu(t) + K(\hat{y}(t) - y(t)) \\ r(t) = &\ y(t) - C\hat{x}(t) \end{aligned} \tag{4.37}$$

where r is the residual vector, and K is the observer gain matrix. If the state estimation error is defined as: $e(t) = x(t) - \hat{x}(t)$, the residual and $e(t)$ will be governed by the following error system:

$$\begin{aligned} \dot{e}(t) = &\ (A - KC)e(t) + f_i f_{a_i}(t) \\ r(t) = &\ Ce(t) \end{aligned} \tag{4.38}$$

The task of BJDF design is to make $Ce(t)$ have a fixed direction in the output space responding to $f_i f_{a_i}(t)$. The definition of the "isolability" of a fault with known direction f_i is given by Beard [2] as stated below.

Definition 4.5.1 Isolability of a fault with a given direction: the fault associated with f_i in the system described by (4.38) is isolable if a filter gain K exists such that

1. $r(t)$ maintains a fixed direction in the output space;

2. $A - KC$ can be stabilized.

Condition (1) guarantees that the residual has uni-directional characteristics. This condition is equivalent to $rank(CW_i) = 1$, where W_i is defined as

$$W_i = [f_i, (A - KC)f_i, ..., (A - KC)^{n-1}f_i].$$

Condition (2) ensures the convergence of the filter.

In order to isolate faults associated with the h isolable faults with direction f_i, $(i = 1, ..., h)$, the following output separability condition must be satisfied.

Definition 4.5.2 Output separability of faults: the faults associated with h fault event directions f_i, $(i = 1, ..., h)$ are separable in the residual space if $Cf_1, Cf_2, ..., Cf_h$ are linearly independent.

The output separability is necessary for a group of faults to be isolated in the residual space because the residual of BJDF will be actually fixed in the direction parallel to Cf_i. The direction Cf_i is called the fault signature direction in the residual space.

Definition 4.5.3 Mutual isolability: the faults associated with the fault event directions f_i, $(i = 1, ..., h)$ are mutually isolable if there exists a filter gain K that satisfies the isolability conditions of Definition 4.5.1 for all $f_i (i = 1, ..., h)$.

The condition for mutual isolability is given in Theorem 4.3, which is proved by White and Speyer in [124].

Theorem 4.3 *Given the system matrix A, output matrix C and fault direction matrix $F = [f_1, ..., f_h], h \leq rank(C)$, the f_i's in F are mutually isolable if and only if $rank(CF) = rank(F) = h$ and*

$$v = n - rank \begin{bmatrix} M \\ MN \\ \cdots \\ \cdots \\ MN^{n-1} \end{bmatrix} = \sum_{i=1}^{h}(n - rank \begin{bmatrix} M_i \\ M_iN_i \\ \cdots \\ \cdots \\ M_iN_i^{n-1} \end{bmatrix}) = \sum_{i=1}^{h} v_i$$

where

$$M = (I - (CF)(CF)^{+})C, M_i = (I - (Cf_i)(Cf_i)^{+})C,$$

and

$$N = A(I - F(CF)^{+}C), N_i = A(I - f_i(Cf_i)^{+}C).$$

If $h = rank(C)$, then $v = n$. v_i is defined as the detection order *of f_i.*

A group of mutually isolable faults can be isolated using the residual generated by a single BJDF by comparing the residual direction with the fault signature directions. The design techniques for choosing K which makes the residual uni-directional can be found in [2, 124].

It can be seen that uncertain factors associated with a dynamic system have not been considered in the design of BJDF. This is the main disadvantage of BJDF because uncertain factors are unavoidable in real systems. For system (4.1), with unknown input term $Gd(t)$ and possible actuator/component faults, we have built the residual so that unknown input effects are decoupled. Consequently, we have to make the residual with the correct directional properties as well in order to achieve robust fault isolation. The following lemma shows the necessary condition for a robust BJDF (in the unknown input decoupling sense).

Lemma 4.5.1 *The faults associated with fault event directions $F_a = [f_1, ..., f_h]$ are separable under the unknown inputs decoupling condition $G_{rd}(s) = 0$ only if $[G \;\; F_a]$ is full column rank.*

Proof. Assume $[G \quad F_a]$ is not full column rank. It is noted that G is full column rank, thus a nonzero matrix X exists such that $F_a = GX + F_g$ and F_g is not full column rank. Since $G_{rd}(s) = 0$, the residual becomes

$$\begin{aligned} r(s) &= WC(sI - A + KC)^{-1}F_a \\ &= WC(sI - A + KC)^{-1}(GX + F_g) \\ &= G_{rd}(s)X + WC(sI - A + KC)^{-1}F_g \\ &= WC(sI - A + KC)^{-1}F_g. \end{aligned}$$

Obviously, since F_g is not full rank, CF_g is not full rank, which is contradictory to the output separability condition. This completes the proof. ∎

Remark 4.5.1 Lemma 4.5.1 implies that the maximum number of faults that can be mutually isolated using UIRG could be less than the number that can be isolated by a standard BJDF. This is because some of the design freedom for gain K has been used to achieve robustness. However, the price is worth paying if the robustness is an essential property.

It is noted that Lemma 4.4.1 in Section 4.4 points out not only the fault detectability condition, but also shows exactly what is the remaining freedom for gain K design, after unknown input decoupling has been achieved. For convenience, let us repeat the residual equation (4.27) here,

$$G_{af}(s) = WC(sI - A + KC)^{-1}F_a = W_bC_b(sI - A_b + K_bC_b)^{-1}F_{ab} \tag{4.39}$$

Equation (4.39) means that the residual under unknown input decoupling design completely depends on the (A_b, C_b) subsystem. In fact, the residual in state-space format is

$$\begin{aligned} \dot{e}_b = &\ (A_b - K_bC_b)e_b + F_{ab}f_a \\ r_b = &\ C_be_b \end{aligned} \tag{4.40}$$

This means that the unknown input decoupled residual for an original linear uncertain system (4.1) is mathematically equivalent to the residual for an unknown input free subsystem

$$\begin{aligned} \dot{x}_b = &\ A_bx_b + F_{ab}f_a \\ y_b = &\ C_bx_b \end{aligned}$$

From the design of UIRG, we know that the matrix K_b can be designed arbitrarily as long as $A_b - K_b C_b$ is stable. This design freedom can be exploited to give the residual the uni-directional property. In conclusion, the unknown input decoupled BJDF can be designed by modifying the UIRG algorithm in Section 4.4 as follows:

1. Replace step 2 as follows:

   If the output separability condition $rank(F_{ab}) = h \leq m_r$ and $rank(C_b F_{ab}) = h$ are satisfied, compute gain K_b such that residual r_b, given by (4.40), has a uni-directional property.

2. In Step 4, set $W_b = I_{m_r}$.

Remark 4.5.2 The observer gain is the only design parameter in BJDF, thus W_b is simply set to be an identity matrix. For a linear time invariant system without unknown inputs, [68] proposes a fault isolation filter design method to find matrices K and W, such that the transfer function matrix from the actuator faults to the residual is diagonal. If we use the theory in [68] to design K_b, W_b for the system (A_b, C_b) with the fault matrix F_{ab}, another unknown input decoupled fault isolation observer can result.

4.5.2 Fault Estimation Based On Unknown Input Residual Generator

Compared with fault isolation using a uni-directional residual, estimating the fault evolution function directly is a more attractive fault isolation scheme. With fault estimation, not only the source of the faults can be identified, but the actual shape and magnitude of the faults can also be perceived. This information on faults is very useful for fault-tolerant control application.

If the faults can be represented as a change of system model parameters, fault detection and isolation using parameter estimation may be considered as a fault estimation approach [54]. For additive faults in the system model (4.1), Patton *et al.* [87] discussed the reconstruction of faults in discrete-time systems using a de-convolution approach. The faults can be uniquely reconstructed using the approach under a very restrictive condition, namely, that the system must have as many independent measurements as states. Saif and Guan [97] proposed an approach to compute the actuator fault using system inverse dynamics. At the heart of this approach is an unknown input observer. This approach has been extended to a certain class of nonlinear and time-delay system by Yang [136]. The fault signal function is not always known *a priori*, and can be considered as an unknown input in many cases. Therefore, the input estimator discussed in Chapter 3 can be used for fault estimation directly where no robust property is addressed.

For system (4.1) with both unknown inputs and faults, Lemma 4.4.1 implies that the unknown input decoupled residual can be described by the following equation:

$$\begin{aligned} \dot{e}_b = & \ (A_b - K_b C_b) e_b + F_{ab} f_a \\ r_b = & \ C_b e_b \end{aligned} \tag{4.41}$$

Instead of designing K_b so that r_b has a uni-directional property, we can build an unknown input estimator for the error system (4.41).

$$\begin{aligned} \dot{z} = & \ F_e z + L_e r_b + T_e F_{ab} \hat{f} \\ \hat{f} = & \ \gamma(M_e r_b - N_e z) \end{aligned} \tag{4.42}$$

which is actually a post-filter for the unknown input decoupled residual r_b. Because unknown input of the error system (4.41) is fault signal, 4.42 is called fault estimator.

The design is based on the system matrices $\Sigma = (A_b - K_b C_b, F_{ab}, C_b)$. As invariant properties under state feedback, the observability and transmission zeros of Σ are the same as those for system $\Xi = (A_b, F_{ab}, C_b)$. The property of SCB shows that (A_b, C_b) is always observable. On that ground, the existence conditions of the input estimator for Σ can be described by the following corollary.

Corollary 4.5.1 Consider the system (4.41). The fault estimator (4.42) exists to make $\hat{f} \to f_a$ if and only if

1. $rank(C_b F_{ab}) = rank(F_{ab})$;

2. all transmission zeros of system $\Xi = (A_b, F_{ab}, C_b)$ are stable.

The corollary 4.5.1 is an explicit expression of Theorem 3.4 for observable systems.

Remark 4.5.3 It is interesting to compare the mutually isolable conditions of Beard-Jones detection filter (BJDF) with the existence conditions for fault estimator. The following three examples demonstrate that neither one has a definite weaker existence condition in comparison to another one.

1. Let

$$A = \begin{bmatrix} 0 & 3 & 4 \\ 1 & 2 & 3 \\ 0 & 2 & 5 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, f_1 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, f_2 = \begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix}.$$

   Since $rank(CF) = rank(F) = 2$, then the fault direction of f_1 and f_2 are output separable. The test for mutual isolability using Theorem 4.3 produces a detection order of $f_i, i = 1,2$ are $v_1 = 2, v_2 = 1$. Since $n = v_1 + v_2$, then the system A, C, F is mutually isolable. However, no fault estimator (4.42) exists for this system because it has an unstable transmission zero, $z = 3$.

2. The system of example (1) is used again here, except that $f_1 = [0 \quad 0 \quad 1]^T$. From Theorem 4.3, $v_1 = 1$. Since $n > v_1 + v_2 = 2$, the system A, C, F is not mutually isolable. Because it has an unstable transmission zero, $z = 2$, fault estimator (4.42) does not exists either for this system.

3. Let

$$A = \begin{bmatrix} -2 & 3 & 4 \\ 1 & 2 & 3 \\ 0 & 2 & 5 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, f_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, f_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix},$$

   then $rank(CF) = rank(F) = 2$. Because the detection order of $f_i, i = 1,2$ are $v_1 = v_2 = 1$, this system A, C, F is not mutually isolable. However, the fault estimator (4.42) exists for this system because it has a stable transmission zero, $z = -2$.

For this reason, we should try another approach if any one of the BJDF and fault estimator methods fails.

Remark 4.5.4 Both BJDF and the fault estimator can be used directly for uncertain systems by considering unknown inputs in the same way as faults, similar to the idea in [50]. Following this simple logic,

$$rank(C) \geq rank([G \quad F]) = rank(G) + rank(F) \tag{4.43}$$

is a necessary condition for both BJDF and the fault/unknown input estimator. However, if we build the robust fault isolation scheme based on UIRG, even if

$$rank(C) < rank([G \quad F]) = rank(G) + rank(F)$$

it is still possible to succeed, as we will show in a numerical example. The reason for this is that unknown input decoupling always uses the same or less number of outputs, in comparison with BJDF and input estimation. Combining BJDF or input estimation technique with the unknown input decoupling technique for robust fault detection and isolation is not only beautiful in theory, but can also help us use a minimum number of sensors to achieve maximum FDI capability in application.

4.5.3 An Eigenstructure Assignment Algorithm for Robust Fault Isolation

This section presents an eigenstructure assignment algorithm to achieve unknown input decoupling and multiple fault isolation simultaneously. As we show before, the transfer function between unknown inputs and residual is

$$G_{rd}(s) = WC(sI - A + KC)^{-1}G,$$

and the transfer function between actuator faults and residual is

$$G_{af}(s) = WC(sI - A + KC)^{-1}F_a.$$

Mathematically, we consider how to design W and K such that $G_{rd}(s) = 0$ and $G_{af}(s)$ is in diagonal form, when the maximum residual dimension $m_r > 1$. Obviously, if $G_{af}(s)$ is a diagonal matrix, each element of the residual is affected only by a certain fault and thus, multiple faults can be isolated. Compared with the method which uses SCB transformation, this algorithm is easier to understand and apply because all calculations involve only original system matrices.

Lemma 4.5.2 *Assume* $F_a = [f_1, ..., f_{m_r}]$. *Let*

$$F_i = [f_1, ..., f_{i-1}, f_{i+1}, ..., f_{m_r}], G_F^i = [G \quad F_i], i = 1, ..., m_r. \tag{4.44}$$

Then system (A, G_F^i, C) *has SII* $n_b > 0$.

Proof. For system (A, G_F^i, C), assume the SCB form for G_F^i, C becomes

$$\hat{G}_F^i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & G_{cf} \\ G_{df} & 0 \end{bmatrix}, \hat{C} = \begin{bmatrix} 0 & 0 & 0 & C_{df} \\ 0 & C_{bf} & 0 & 0 \end{bmatrix}$$

Because $rank(\hat{C}) = rankC = p$, the term C_{bf} will disappear ($n_b = 0$) if and only if rank(C_{df})=p. On the other hand, it is known that

$$rank(C_{df}) = rank(G_{df}) \leq rank(G_d) + m_r - 1 = m_d + p - m_d - 1 = p - 1$$

Therefore, SII n_b for system (A, G_F^i, C) must be larger than zero. ∎

Next, we state the algorithm and prove its validity. Due to Lemma 4.5.1 and 4.5.2, it is necessary to assume that $G_F = [G \quad F_a]$ is full rank, where F_a has m_r independent columns.

Eigenstructure Assignment Algorithm For Robust Fault Isolation

Step 1: Calculate SII n_b and m_r. Then choose m_r negative eigenvalues $\Lambda^b = \{\lambda_i, i = 1, ..., m_r\}$. For each λ_i, calculate its corresponding left eigenvector v_i and dummy vector o_i by solving

$$\begin{bmatrix} v_i & o_i \end{bmatrix} \begin{bmatrix} \lambda_i - A & G_F^i \\ -C & 0 \end{bmatrix} = 0. \tag{4.45}$$

Note that Lemma 4.5.2 implies (4.45) is solvable. Note that v_i should be selected to be mutually linearly independent.

Step 2: If $n_h = n_b - m_r > 0$, and the system (A, G_F, C) has n_h stable zeros $Z = \{z_i, i = 1, .., n_h\}$, choose n_h eigenvalues as $\lambda_i = z_i, i = 1, ..., n_h$. For each λ_i, calculate its corresponding left eigenvector v_i and dummy vector o_i by solving

$$\begin{bmatrix} v_i & o_i \end{bmatrix} \begin{bmatrix} \lambda_i - A & G_F \\ -C & 0 \end{bmatrix} = 0. \tag{4.46}$$

Note that (4.46) has a solution if the system (A, G_F, C) has n_h zeros. The eigenvectors, v_i, should be selected such that $v_i (i = 1, ..., n_b)$ are linearly independent.

Step 3: Choose the remaining $n - n_b$ eigenvalues $\Lambda^g = \{\lambda_i, i = n_b + 1, ..., n\}$, and find their corresponding left eigenvectors v_i, and dummy vectors o_i by solving the following equations.

$$v_i(\lambda_i I - A) - o_i C = 0, i = n_b + 1, ..., n$$

Note that all unobservable fixed modes must be included in Λ^g and the resulting v_i should be linearly independent of those v_i obtained in Steps 1 and 2. K is parameterized as

$$K = - \begin{bmatrix} v_1 \\ v_2 \\ \cdots \\ v_n \end{bmatrix}^{-1} \begin{bmatrix} o_1 \\ o_2 \\ \cdots \\ o_n \end{bmatrix} = -V^{-1} O \tag{4.47}$$

Step 4: Let the right eigenvector matrix $Q = [q_1, ..., q_{n_b}, q_{n_b+1}, ..., q_n] = V^{-1}$. Assume $Q_g = [q_{n_b+1}, ..., q_n], Q_i = [q_1, ..., q_{i-1}, q_{i+1}, ..., q_{m_r}]$. There are m_r rows in W, The ith row of W is calculated by

$$w_i^T \in \Psi = Ker((C[Q_i \quad Q_g])^T). \tag{4.48}$$

Note, Ψ must be non empty since the dimension of $Ker((CQ_g)^T)$ is m_r.

Step 5: Construct the corresponding reduced-order observer.

$$T = \Omega\Gamma = \Omega \begin{bmatrix} v_1 \\ v_2 \\ \cdots \\ v_r \end{bmatrix}, \tag{4.49}$$

$F = \Omega\Lambda_b\Omega^{-1}, L = TK, M = W, N = WCT^+$, where dim $\Omega = r \times r$, Ω is any invertible matrix such that T has full rank.

Theorem 4.4 *A diagonal unknown input decoupled fault isolation observer can be designed if any one of the following two conditions is satisfied:*

(a) system (A, G, C) has $m_r = n_b$,

(b) system (A, G, C) has $n_h = n_b - m_r > 0$, but system (A, G_F, C) has n_h stable zeros.

Proof. Using the above constructive algorithm, $v_iG = 0, i = 1, ..., n_b$ and $WCq_i = 0, i = n_b + 1, ..., n$, thus $G_{rd}(s) = 0$. Further, the residual/fault transfer function becomes

$$G_{af}(s) = \sum_{i=1}^{n_b} \frac{WCq_iv_iF_a}{s - \lambda_i} \tag{4.50}$$

If $m_r = n_b$, step 2 will be omitted. Then step 1 and 4 will result in

$$v_if_j = \begin{cases} 0 & \text{if } i \neq j \\ a_{fi} & \text{if } i = j \end{cases} \text{ and } w_iCq_j = \begin{cases} 0 & \text{if } i \neq j \\ a_{wi} & \text{if } i = j \end{cases} \tag{4.51}$$

for $i, j = 1, 2, ..., m_r$, where $a_{fi} \neq 0$ and $a_{wi} \neq 0$. Assume $a_i = a_{fi} * a_{wi}$. According to (4.51), we have

$$G_{af}(s) = diag[a_1/(s - \lambda_1), ..., a_{m_r}/(s - \lambda_{m_r})]. \tag{4.52}$$

If $n_h = n_b - m_r > 0$, generally $G_{af}(s)$ becomes

$$G_{af}(s) = diag[a_1/(s - \lambda_1), ..., a_{m_r}/(s - \lambda_{m_r})] + \sum_{i=m_r+1}^{n_b} \frac{WCq_iv_iF_a}{s - \lambda_i} \tag{4.53}$$

If condition (b) is satisfied, step 2 will be executed, which will lead to

$$v_iF_a = 0 \text{ and } v_iG = 0; i = m_r + 1, ..., n_b \tag{4.54}$$

From (4.53), it is obvious that $G_{af}(s)$ becomes a diagonal matrix. ∎

4.6 Properties Of Multiple Actuator/Sensor Fault Isolation

It is well known that the sensor FDI has some special properties that is different from the actuator FDI. The response of sensor faults cannot be restricted to a line direction using BJDF, but only to a plane. [83, 97] presented a method for representing any sensor faults in the form of actuator faults of an augmented system, thereby permitting the use of actuator FDI methods to sensor FDI. In this section, we first recall the way to represent sensor faults as actuator faults of an augmented system from [83, 97], then we show that there is an inherent difference between the robust sensor and actuator FDI, although the design method can be almost the same.

Proposition 4.6.1 For any piecewise continuous vector function $f_s \in R^k$, and a stable $k \times k$ matrix A_f, an input $\xi \in R^k$ will always exist such that

$$\dot{f}_s = A_f f_s + \xi. \tag{4.55}$$

Augmenting the original system (4.1) with (4.55) results in the following $(n+k)$th order system:

$$\begin{pmatrix} \dot{x} \\ \dot{f}_s \end{pmatrix} = \begin{bmatrix} A & 0 \\ 0 & A_f \end{bmatrix} \begin{pmatrix} x \\ f_s \end{pmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} G \\ 0 \end{bmatrix} d + \begin{bmatrix} F_a \\ 0 \end{bmatrix} f_a + \begin{bmatrix} 0 \\ I_k \end{bmatrix} \xi \tag{4.56}$$

$$y = \begin{bmatrix} C & F_s \end{bmatrix} \begin{bmatrix} x \\ f_s \end{bmatrix} \tag{4.57}$$

The system (4.56)-(4.57) has actuator faults f_a, ξ.

Assume the maximum residual dimension of the system (A, G, C) is m_r and $rank(F_a) + rank(F_s) = m_r$. Now the augmented system matrices are

$$A_u = \begin{bmatrix} A & 0 \\ 0 & A_f \end{bmatrix}; G_u = \begin{bmatrix} G \\ 0 \end{bmatrix}; F_u = \begin{bmatrix} F_a & 0 \\ 0 & I_k \end{bmatrix}; C_u = \begin{bmatrix} C & F_s \end{bmatrix}. \tag{4.58}$$

It is proved that (A_u, C_u) is detectable if and only if (A, C) is detectable [83]. Here we further analyze how the structural properties of the augmented system relate to that of the original system, because the structural index defines the fault isolability. Let SII of systems $\Sigma_u = (A_u, G_u, C_u)$ to be $n_{au}, n_{bu}, n_{cu}, n_{du}$.

Lemma 4.6.1 *The structure invariant index of the augmented system is related to that of the original system according to:*

(a) $n_a \leq n_{au} \leq n_a + k$. All transmission zeros of the system (A, G, C) will be transmission zeros of the system (A_u, G_u, C_u). If $n_{au} > n_a$, those new transmission zeros must be eigenvalues of A_f;

(b) the infinite zero structure of augmented systems is the same as that of the original system, and $n_{du} = n_d$;

(c) $n_{cu} = n_c$;

(d) $n_{au} + n_{bu} = n_a + n_b + k$.

Proof.

(a) The set of transmission zeros of the system Σ_u is defined as those numbers λ for which the $(n+m+k) \times (n+p+k)$ matrix

$$P_u(\lambda) = \begin{bmatrix} A - \lambda I_n & 0 & G \\ 0 & A_f - \lambda I_k & 0 \\ C & F_s & 0 \end{bmatrix}$$

lost its rank. With a simple matrix operation, we can get

$$P_u(\lambda) = \begin{bmatrix} A - \lambda I_n & G & 0 \\ C & 0 & F_{sk} \\ 0 & 0 & A_f - \lambda I_k \end{bmatrix}$$

Obviously, if $\begin{bmatrix} A - \lambda I_n & G \\ C & 0 \end{bmatrix}$ lost its rank, $P_u(\lambda)$ lost its rank also, and all transmission zeros of the system Σ will be transmission zeros of the system Σ_u. Further, only if $rank(A_f - \lambda I_k) < k$(this holds if and only if λ is an eigenvalue of A_f), it is possible, but it is not certain, that $P_u(\lambda)$ lost its rank. Thus, (a) is proved.

(b) Let $H(s) = C(sI - A)^{-1}G, H_u(s) = C_u(sI - A_u)^{-1}G_u$. It is easy to show that $H_u(s) = H(s)$. Thus the systems Σ_u and Σ have the same infinite zero structure, and $n_{du} = n_d$.

(c) and (d) We need to prove that

$$n_{eu} = n_{du} + n_{cu} = n_d + n_c = n_e \tag{4.59}$$

If (4.59) is true, then (c) and (d) are proved by considering (b) and the fact that $n = n_a + n_b + n_c + n_d$.

Define $S(\Sigma)$ as the minimal $(A + LC)$-invariant subspace containing $Im(G)$ for some L, $S(\Sigma_u)$ as the minimal $(A_u + L_uC_u)$-invariant subspace containing $Im(G_u)$ for some L_u. Then

$$n_e = \text{the dimension of } S(\Sigma); n_{eu} = \text{the dimension of } S(\Sigma_u)$$

(see [94]). Note that $G_u = [G' \;\; 0]'$. It is easy to show that the basis of $S(\Sigma_u)$ can always have the form of $v_{ui} = [v_i' \;\; 0]', i = 1, ..., n_{eu}$, where $v_i \in Im(G)$. Let $L_u = [L_1' \;\; L_2']'$, then

$$A_u v_{ui} = \begin{bmatrix} A + L_1C & L_1F_{sk} \\ L_2C & A_f + L_2F_{sk} \end{bmatrix} * \begin{bmatrix} v_i \\ 0 \end{bmatrix} = \begin{bmatrix} (A + L_1C)v_i \\ L_2Cv_i \end{bmatrix} \in S(\Sigma_u). \quad (4.60)$$

Obviously, the minimal number of v_i satisfying (4.60) is n_e by making $L_1 = L$. In other words, $n_{eu} = n_e$. ∎

Theorem 4.5 *The maximum residual dimension of the augmented system, m_{ru}, is equal to that of the original system, or $m_{ru} = m_r$ and $n_b \leq n_{bu} \leq n_b + k$.*

Proof. The SCB form of C is

$$\overline{C} = \begin{bmatrix} 0 & 0 & 0 & C_d \\ 0 & C_b & 0 & 0 \end{bmatrix}$$

The column number of C_b, C_d equals n_b, n_d respectively. For the system Σ_u , express the SCB form of C_u as

$$C_{su} = \begin{bmatrix} 0 & 0 & 0 & C_{du} \\ 0 & C_{bu} & 0 & 0 \end{bmatrix}$$

Due to part (b) of Lemma 4.6.1, we have $C_d = C_{du}$. Since C_u and C have the same row number, thus

$$m_{ru} = \text{ row number of } C_{bu} = \text{ row number of } C_b = m_r.$$

$n_b \leq n_{bu} \leq n_b + k$ can be derived directly from part (a) and (d) of Lemma 4.6.1. ∎

Remark 4.6.1 Theorem 4.5 shows that augmentation does not improve the isolation ability of the original system. On the contrary, it implies that the isolation condition of the augmented system is often stricter than that of the original system. According to Theorem 4.4, $m_{ru} = n_{bu}$, which will ensure the diagonalization of the residual/fault transfer function. Since $n_b \leq n_{bu}, m_{ru} = m_r$, it is more likely that $m_{ru} < n_{bu}$ even if the original system has $m_r = n_b$. In this case, the system $\Sigma_{fu} = (A_u, [G_u \quad F_u], C_u)$ must have $n_{hu} = n_{bu} - m_{ru}$ stable zeros. Since $n_{hu} \geq n_h = n_b - m_r$, more stable zeros may be required if sensor faults are included in the augmented system.

Next, we propose a theorem about zeros of the system Σ_{fu}. For a simple expression, let $G_{AF} = [G \quad F_a]$. Assume $F_{sk} = [f_{si_1}, ... f_{si_k}]$. C_k is the result of $i_1, i_2, ..., i_k$ rows of C are taken out. For example, if there are four sensor outputs and $m_r = 2$, we want to detect and isolate both the first and the third sensor faults, then $i_1 = 1, i_2 = 3$, $F_{sk} = [f_{s1}, f_{s3}]$ and $C_k = \begin{bmatrix} C_2 \\ C_4 \end{bmatrix}$, where C_2 and C_4 are the second and fourth rows in C respectively. It is assumed that the ith sensor fault vector f_{si} is a standard unit vector e_i, the ith column of the $p \times p$ identity matrix. In practice, most sensor faults can be expressed in this form, although this is not always true.

Theorem 4.6 *The transmission zeros of the system $\Sigma_{fu} = (A_u, [G_u \quad F_u], C_u)$ are the same as the transmission zeros of the system (A, G_{AF}, C_k).*

Proof. The set of transmission zeros of the system Σ_{fu} is defined as those numbers λ for which the $(n+m+k) \times (n+p+m_r+k)$ matrix

$$\Gamma = \begin{bmatrix} A - \lambda I_n & 0 & -G_{AF} & 0 \\ 0 & A_f - \lambda I_k & 0 & -I_k \\ C & F_s & 0 & 0 \end{bmatrix}$$

has a rank less than its normal rank. With a simple matrix operation which does not alter the rank of Γ, we get

$$\Gamma = \begin{bmatrix} A - \lambda I_n & -G_{AF} & 0 & 0 \\ C & 0 & 0 & F_s \\ 0 & 0 & -I_k & 0 \end{bmatrix}$$

Due to the special structure of $F_s = [e_{i_1}, ..., e_{i_k}]$, the $i_i, ..., i_k$ rows in C will be changed to zero vectors by column operation. Rearranging the rows of $C_F = \begin{bmatrix} C & 0 & 0 & F_{sk} \end{bmatrix}$, C_F can yield

$$C_F = \begin{bmatrix} C_k & 0 & 0 & 0 \\ 0 & 0 & 0 & I_k \end{bmatrix}$$

Thus

$$\Gamma = \begin{bmatrix} A - \lambda I_n & -G_{AF} & 0 & 0 \\ C_k & 0 & 0 & 0 \\ 0 & 0 & 0 & I_k \\ 0 & 0 & -I_k & 0 \end{bmatrix}$$

which implies that

$$rank\Gamma = rank \begin{bmatrix} A - \lambda I_n & -G_{AF} \\ C_k & 0 \end{bmatrix} + rank \begin{bmatrix} 0 & I_k \\ -I_k & 0 \end{bmatrix}$$

Therefore, all the values of λ for which Γ lost rank are zeros of the system (A, G_{AF}, C_k).

∎

Remark 4.6.2 Theorem 4.6 means that transmission zeros of the system Σ_{fu} are actually independent of the dynamics of A_f, unlike the transmission zeros of the system Σ_u.

4.7 Numerical Example

Example 4.7.1 In this example, a fault detection and isolation design for the vertical takeoff and landing (VTOL) aircraft is considered. The linearized model of the VTOL aircraft in the vertical plane was obtained by Narendra and Tripathi [78], and given in the state space formulation as,

$$\dot{x} = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.3681 & -0.707 & 1.420 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix} x + \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.52 & 4.49 \\ 0.0 & 0.0 \end{bmatrix} u$$

with

$$x = \begin{pmatrix} \text{horizontal velocity}(kt) \\ \text{vertical velocity}(kt) \\ \text{pitch rate}(deg/s) \\ \text{pitch angle}(deg) \end{pmatrix} \text{ and } u = \begin{pmatrix} \text{collective pitch control } u_1 \\ \text{longitudal cyclic pitch control } u_2 \end{pmatrix}.$$

The above dynamics hold for a typical loading and flight condition of the VTOL at an air speed of 135 kt. As the airspeed changes, the dynamic equation of the model changes. The most significant of these changes occurs at the a_{32}, a_{34} and b_{21} elements of the A and B matrices.

The open loop poles of the nominal system are located at

$$\Lambda = \{-2.0727, 0.2758 \pm j0.2576, -0.2325\}$$

Clearly, the system is unstable and needs to be stabilized. Since the purpose of the example is to illustrate the robust fault diagnosis observer design, we simply stabilize the system using a state feedback type controller. The system dynamics after stabilization control is

$$\dot{x} = \begin{bmatrix} -9.9477 & -0.7476 & 0.2632 & 5.0337 \\ 52.1659 & 2.7452 & 5.5532 & -24.4221 \\ 26.0922 & 2.6361 & -4.1975 & -19.2774 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix} x +$$

$$\begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.52 & 4.49 \\ 0.0 & 0.0 \end{bmatrix} u + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} d;$$

where

$$d = \begin{bmatrix} \begin{bmatrix} 0 & \Delta a_{32} & 0 & \Delta a_{34} \end{bmatrix} x \\ \begin{bmatrix} \Delta b_{21} & 0 \end{bmatrix} u \end{bmatrix}$$

The unknown inputs d actually represent the uncertainties of parameters a_{32}, a_{34} and b_{21}. We consider two cases.

Case 1: Assume there are three sensor outputs, that is

$$y = Cx = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x.$$

In this case, it is noted that $rank(CG) < rank(G)$. Hence, no UIO exists, and all FDI methods based on UIO are inapplicable. However, it is easy to find that SII n_b of (A, G, C) is 1, therefore UIRG exists, and at least fault detection can be achieved.

Because (A, C) is observable, both full-order and reduced-order UIRG exist. As an example, one full-order solution is obtained as

$$K = \begin{bmatrix} -16.0877 & -0.7737 & 8.6810 \\ -31.5371 & 8.8302 & 1.9283 \\ 68.3185 & 3.7503 & -37.9043 \\ -30.9270 & -0.0992 & 15.8575 \end{bmatrix}; W = \begin{bmatrix} -9.671 & 0 & 2.545 \end{bmatrix}$$

where the assigned eigenvalues are $\Lambda = \{-2, -4, -6, -8\}$. The corresponding reduced-order UIRG solution is

$$F = -2; T = \begin{bmatrix} -0.1091 & 0 & 0 & 0.0287 \end{bmatrix};$$

$$L = \begin{bmatrix} 0.8670 & 0.0816 & -0.4917 \end{bmatrix}; N = 88.654; M = W.$$

The two actuator fault matrix is $F_a = B$, and the three sensor fault matrix is $F_s = I_3$. It is easy to show that all transfer functions between the residual and the faults are nonzero. Therefore, any actuator or sensor faults can be detected. Figure 4.1 demonstrates the residuals for the first actuator and the third sensor fault respectively, where the fault signal is assumed to be

$$f_a(t) = f_s(t) = \begin{cases} 0 & 0 \le t \le 4; \\ 0.5 & 4 < t; \end{cases}$$

It is clear from Figure 4.1 that the residual is nonzero at the beginning even if no fault exists. This is the transient period of the observer due to the initial estimation error and it is assumed that no fault occurs during this time. Obviously, once the transient

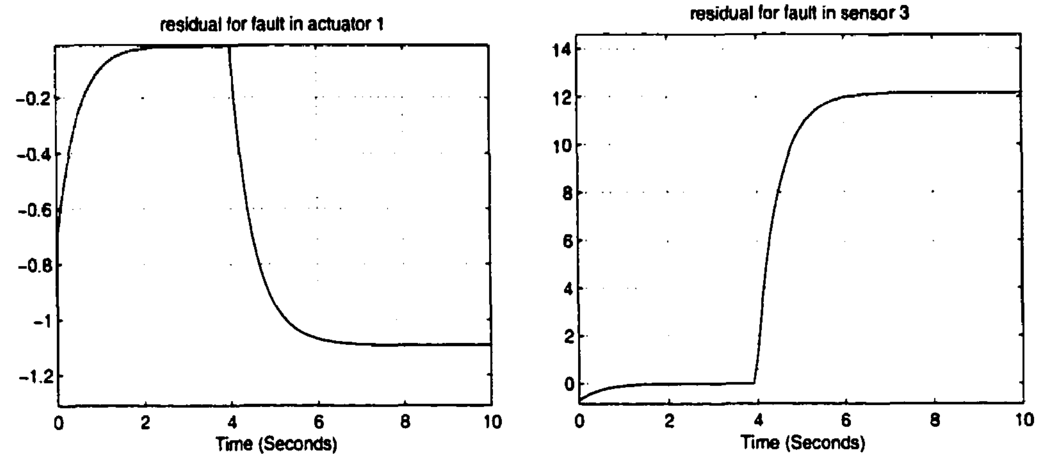

Figure 4.1: Robust detection of faults in actuator 1 and sensor 3 of a VTOL aircraft

period has washed out, any future nonzero value of the residual would indicate the existence of at least one fault. However, it is impossible to identify the source of the fault from the scalar residual information.

Case 2: In this case, assume the output was taken as

$$y = Cx = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} x$$

With a greater number of outputs available, the maximum residual dimension $m_r = 2$. Therefore, multiple sensor and actuator fault isolation is possible. Because the matrix $[G \;\; F_a]$ is not full rank, the two actuator faults are not separable. Assume we detect soft failures of the third sensor and the first actuator. Set the model of sensor fault signal as $\dot{f}_s = -4 * f_s + \xi$, then the augmented system becomes

$$\begin{bmatrix} \dot{x} \\ \dot{f}_s \end{bmatrix} = \begin{bmatrix} -9.9477 & -0.7476 & 0.2632 & 5.0337 & 0 \\ 52.1659 & 2.7452 & 5.5532 & -24.4221 & 0 \\ 26.0922 & 2.6361 & -4.1975 & -19.2774 & 0 \\ 0.0 & 0.0 & 1.0 & 0.0 & 0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -4.0 \end{bmatrix} \begin{bmatrix} x \\ f_s \end{bmatrix} +$$

$$\begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.52 & 4.49 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix} u + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} d + \begin{bmatrix} 0.4422 & 0 \\ 3.5446 & 0 \\ -5.52 & 0 \\ 0.0 & 0 \\ 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} f_a \\ \xi \end{bmatrix};$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ f_s \end{bmatrix} f_s$$

This augmented system has $n_b = 3, m_r = 2$. The zeros of the system Σ_{fu} is -1.0. The eigenstructure assignment algorithm is applied to obtain gain K and W by selecting the desired eigenvalues $\Lambda^b = \{-1, -2, -3\}$ and $\Lambda^g = \{-8, -6\}$, and dummy vectors o_i as

$$O = \begin{bmatrix} o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \end{bmatrix} = \begin{bmatrix} -0.8044 & 0.3805 & 0 & -0.4562 \\ -0.0000 & 0.6396 & 0.4264 & -0.6396 \\ 0 & 0.7071 & 0 & -0.7071 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

The matrices K and W are

$$K = \begin{bmatrix} -7.948 & -1.011 & 0 & 0.263 \\ -843.627 & -31.067 & 21.15 & 11.53 \\ -1006.34 & -38.96 & 22.05 & 10.03 \\ 0.0 & -1.0 & 0.0 & 1.0 \\ 0.0 & -1.0 & -1.0 & 1.0 \end{bmatrix}; W = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 5.774 & 5.774 & -5.774 \end{bmatrix}$$

The residual transfer function is

$$r(s) = \begin{bmatrix} 4.422/(s-2) & 0 \\ 0 & 5.774/(s-3) \end{bmatrix} \begin{pmatrix} f_a \\ \xi \end{pmatrix}.$$

Figure 4.2 is the simulation result. Obviously, $r_i (i = 1, 2)$ follows the variation of the ith fault only after the transient period, and multiple faults can be isolated promptly and correctly.

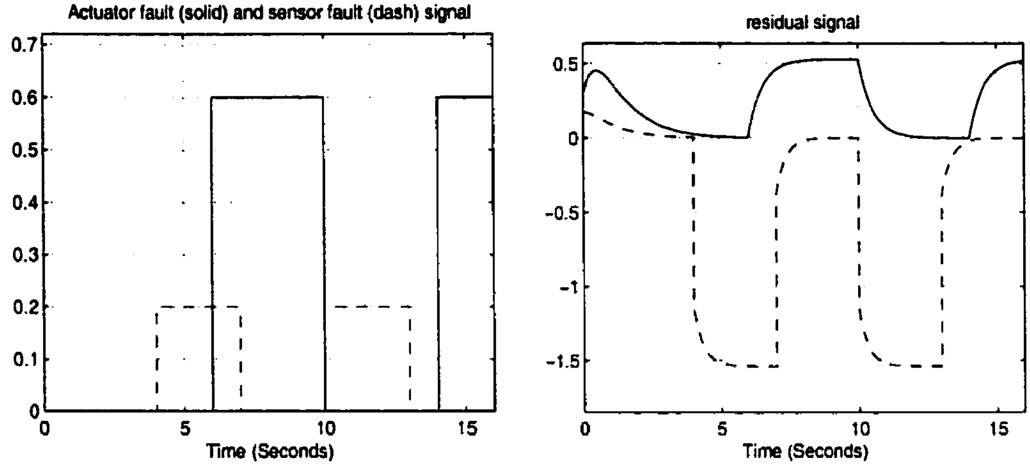

Figure 4.2: Isolation of faults in actuator 1 and sensor 3 of a VTOL aircraft

Example 4.7.2 This example deals with the implementation of robust fault isolation algorithms of Section 4.5.2. The system of example 1 in Chapter 3, a double-effect pilot plant evaporator(see Figure 3.1), is used again here, except that

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

The objective is to estimate actuator faults, but not unknown inputs. There are three inputs to this system. We want to identify the faults of the first and second inputs, which are two flow actuators. The third input represents the temperature of saturated steam, and no actuator is used directly for this input. The fault direction matrix is $F_a = [b_1 \quad b_2]$.

There are three output measurements, and four independent unknown inputs and faults. It is impossible to estimate both unknown inputs and faults. The UIO based method is also inapplicable to this system. The test for unknown input decoupling conditions using the SCB transformation produces A_b, C_b, F_{ab} as

$$A_b = \begin{bmatrix} -0.0222 & 0.0005 \\ 0 & 0 \end{bmatrix}; C_b = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; F_{ab} = \begin{bmatrix} -0.9574 & -0.0220 \\ 0.9379 & -0.9998 \end{bmatrix}.$$

The two faults can be estimated based on UIRG. The UIRG solution is designed as

$$K = \begin{bmatrix} 1.9625 & 0.1131 & -0.0221 \\ 0 & 0 & 0 \\ 0.2730 & 0.0157 & -0.0031 \\ -0.0221 & -0.0067 & 2.9995 \\ 0 & 0 & 0 \end{bmatrix}; W = \begin{bmatrix} 0.9776 & 0.1360 & 0.0220 \\ -0.0221 & -0.0031 & 0.9998 \end{bmatrix}$$

and the corresponding fault estimator matrices are given by

$$F = \begin{bmatrix} -3 & 0 \\ 0 & -4 \end{bmatrix}; T = L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; M = N = \begin{bmatrix} -0.1596 & 0.1172 \\ -0.0037 & -0.1250 \end{bmatrix}.$$

Figure 4.3 is the simulation result, where estimation of the fault is shown with the dotted line. It can be seen that the robust fault estimator provides us with the capability to detect the soft fault almost immediately, and in addition, it gives us the actual shape of the particular fault, even with the existence of disturbance. A slight estimation error exists for the reason we explained in Chapter 3 regarding the input estimator.

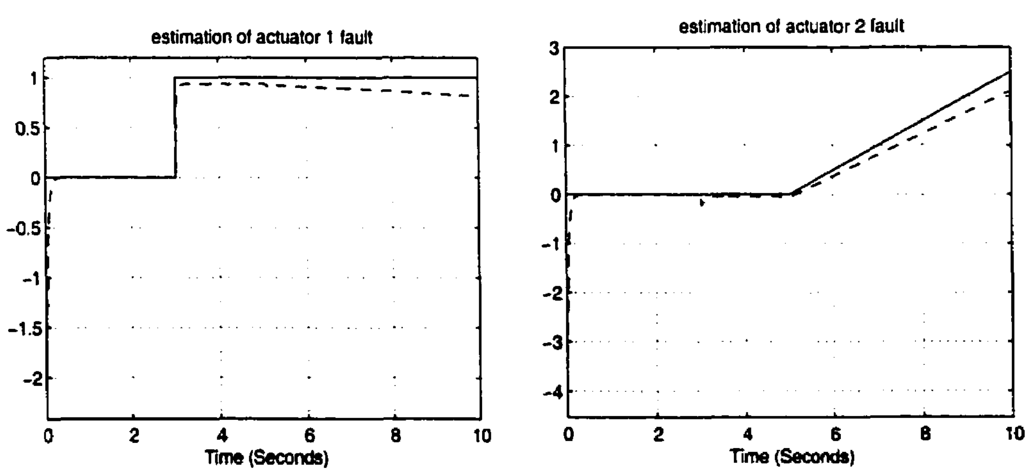

Figure 4.3: Robust actuator fault estimation for a double-effect pilot plant evaporator

4.8 Conclusions

In this chapter, we provide the necessary and sufficient conditions as well as design procedure for an unknown input residual generator (UIRG). The fault detectability under UIRG is discussed. The unique feature of our algorithm is to achieve the maximum residual dimension, which is defined by the system matrices. Based on the complete solution for UIRG, we present three new fault isolation observer design methods to achieve multiple and simultaneous robust fault isolation. They are unknown inputs decoupled BJDF, unknown inputs decoupled fault estimator, and direct eigenstructure assignment approach. Some special properties for the sensor FDI are revealed. Further research will focus on how to enhance the ability to handle unstructured uncertainties [102].

Chapter 5

Robust Diagnostic Observer for Bilinear Systems

This chapter explores three designs of bilinear fault detection observers with the special property that the unknown inputs are decoupled from the residuals. Although observability of bilinear system is input dependent, the first two can be input independent, however they rely on strong structural conditions. If the control inputs are bounded, it is possible to decouple the unknown inputs from the residuals under less conservative conditions. The main contribution of this chapter is to provide conditions as well as procedures for designing such observers.

5.1 Introduction

Many fault detection and isolation (FDI) methods, including those we propose in Chapter 4, are based on linear uncertain system models. These robust FDI schemes may be applied to a certain class of nonlinear systems, where the linearization error around an operation point can be represented by an unknown input vector. However, for highly nonlinear systems and a large operation region, the unknown input distribution matrix is difficult to construct, and linear robust FDI methods are often inapplicable. A trend in extending these methodologies to nonlinear systems is currently underway [28].

As a class of nonlinear systems, bilinear systems are used to represent a wide variety of industrial processes and systems, such as nuclear reactor systems, suspension systems, hydraulic drive systems, heat-exchange processes and gas-burning furnace systems [74]. In the bilinear model, the control appears in both additive and multiplicative terms. Although the control of bilinear systems is well developed [75], research progress for the FDI of bilinear systems has been limited.

It is not surprising that observer design plays a crucial role in the development of bilinear FDI schemes. As shown in [121], the observability as well as the existence of the observer, depends on the inputs. This is totally different with the linear observer and makes the observer design more difficult. Observers of bilinear systems are designed according to the required stability of observation error dynamics. Hara and Furuta [46] proposed a class of observers with linear error dynamics for systems which satisfy a number of strong structural constraints. The same problem has been considered by Funahashi [30] in that the Lyapunov method was used to assure the estimation error decay to zero exponentially, irrespective of the input. For bilinear systems with bounded inputs, Derese and Noldus [18] presented a design algorithm involving the computation of the maximal solution of the algebraic Riccati equation.

Like linear systems, bilinear ones are sometimes affected by time-varying unknown disturbances. Such unknown inputs can arise in the dynamic equation of the bilinear system due to actuator faults, and internal and/or external disturbances in the plant. Based on the results in Hara and Furuta [46], Hac [43] and Saif [99] considered the design of unknown input observers (UIO) for bilinear systems in which the estimation error dynamics are linear, and *a priori* knowledge about the unknown inputs is not required. Zasadzinski *et al.* [143] reformulated the observers proposed in [43] and [99] as an equivalent linear unknown input observer and simplified the design procedure. Yang and Saif [136] proposed an unknown input bilinear observer design using the knowledge of the control input bounds, which is applicable to a wider class of bilinear systems.

Recently, there have been several studies dealing with robust FDI of bilinear systems, where the residual generation is the most important part. There are two possible avenues towards robust observer based FDI in bilinear systems. The first is to design an UIO for bilinear systems, and use a function of observer error to represent the residual to be monitored [138], or estimate the fault signal directly based on complete state information [136]. The second approach is an extension of the unknown input residual generator (UIRG) for linear system [24], where what is observed is a linear function of the. This method is proposed in [140, 141, 143], where some existence conditions as well as design methods are provided. Although observability of a bilinear system is input dependent, as in any nonlinear systems, several proposed bilinear fault detection observers have the property whereby the residual decays to zero irrespective of the input [99, 140, 141]. It is generally true that for this class of observers, stronger existence conditions need to be satisfied than those that exist under a certain class of inputs [61, 136]. In [139], the parity space method for linear fault diagnosis is extended to bilinear systems. However, this method cannot be used for fast systems of high order due to the excessive computation involved.

In the first part of this chapter, we shall treat bilinear unknown input fault diagnosis observer (UIFDO) with the property of input independent. The bilinear UIFDO proposed in [140, 143] is revisited in Section 5.3, where linear error dynamics is achieved by decoupling both bilinear terms and unknown inputs. We provide some new results to simplify the design and solvability test. The derivation is based on linear UIFDO results in Chapter 4. Further, we propose a new bilinear UIFDO design approach, where the error dynamics is bilinear. Although the error itself depends on the input, the residual decays to zero for any control input if no fault exists.

As anticipated, it is shown that strict conditions on the structure of bilinear systems have to be satisfied for input independent robust residual generation. Therefore, we further discuss robust residual generation and FDI for bilinear systems with bounded input, where the constraints can be alleviated. The applicability and effectiveness of the proposed FDI scheme is demonstrated through simulation study of a vehicle semi-active suspension and an electromechanical actuator.

5.2 Problem Formulation

Consider the following formulation of a bilinear system where uncertainty and faults are included:

$$\begin{aligned} \dot{x} &= Ax + \textstyle\sum_{i=1}^{h} A_i u_i x + Bu + Gd + F_a(x) f_a \\ y &= Cx + F_s f_s \end{aligned} \tag{5.1}$$

where $x \in \mathcal{R}^n$, $d \in \mathcal{R}^q$ and $y \in \mathcal{R}^p$ are the state, unknown disturbance and output vectors, respectively. The control input is given by $u \in \mathcal{R}^m$. Vectors f_a and f_s represent vectors of actuator or component faults and sensor faults, respectively, and matrices $F_a(x)$ and F_s represent their distribution. A, A_i, B, C, and G are constant matrices with compatible dimensions. It is assumed C and G are of full rank, namely $rank(C) = p$ and $rank(G) = q$.

Remark 5.2.1 It should be noted that in many studies concerning FDI in bilinear systems, actuator faults are modeled as $F_a f_a$, where F_a is a constant matrix. A broader representation would be to represent the ith actuator fault distribution matrix as $F_a(x) = A_i x + B_i$.

A state-function observer for the bilinear system (5.1) is given by

$$\dot{z} = Fz + Ly + TBu + \sum_{i=1}^{h} L_i u_i y + \sum_{i=1}^{h} F_i u_i z \tag{5.2}$$

with the residual

$$r = My - Nz \tag{5.3}$$

where $F, L, T, N, M, L_i, F_i, (i = 1, ...h)$ are matrices to be designed with appropriate dimensions. It is desired that $z \to Tx$ as $t \to \infty$ under the presence of unknown input, d. Note T is not specified *a priori*.

Definition 5.2.1 An observer (5.2)-(5.3) is called a input-independent bilinear UIFDO (5.1) if for any $d, x(0), z(0)$ and any u, $r \to 0$ as $t \to \infty$ and $r(t) \neq 0, t \geq t_0$, when $f_a(t) \neq 0$ or $f_s(t) \neq 0$ for $t \geq t_0$. It is called a bounded-input bilinear UIFDO if $r \to 0$ for any $d, x(0), z(0)$ and any u that satisfy $\|u\| < \beta$, $r(t) \neq 0, t \geq t_0$, when $f_a(t) \neq 0$ or $f_s(t) \neq 0$ for $t \geq t_0$.

Note that the observer (5.2) does contain a natural term $\sum_{i=1}^{h} F_i u_i z$, which is not included in many of the past bilinear observer design studies. This term makes the observer structure more general, but the design more difficult. We shall first consider the structure restricted case, where $F_i = 0 (i = 1, ..., h)$. It is shown that structure restricted bilinear UIFDO is equivalent to the design of a UIFDO for a linear system. The general structure case, where $F_i \neq 0$ for some i, is the main topic of discussion of this chapter.

5.3 Structure Restricted Input Independent Bilinear UIFDO

If we let all $F_i = 0(i = 1, ..., h)$, the observer error dynamics will be linear and render the observer structure less general. The following lemma gives the necessary and sufficient conditions so that the residual is not effected by unknown inputs.

Lemma 5.3.1 *Residual (5.3) is unknown input decoupled if and only if there exists matrices F, T, L, N, M and $L_i(i = 1, ..., h)$ satisfying the following conditions simultaneously,*

$$FT - TA + LC = 0 : F \text{ is stable;} \tag{5.4}$$

$$TG = 0; \tag{5.5}$$

$$NT = MC; \tag{5.6}$$

$$TA_i - L_i C = 0 (i = 1, ..., h). \tag{5.7}$$

Proof. Let $e = Tx - z$. The corresponding equation for e is

$$\begin{aligned} \dot{e} \;=\; & Fe - (FT - TA + LC)x - (L + \textstyle\sum_{i=1}^{h} L_i u_i) F_s f_s + \\ & TGd + \textstyle\sum_{i=1}^{h} (TA_i - L_i C) u_i x + TF_a(x) f_a \end{aligned} \tag{5.8}$$

and the residual becomes

$$r = Ne + (MC - NT)x + MF_s f_s \tag{5.9}$$

It is easy to see that if and only if the constraints (5.4)-(5.7) are satisfied, equations (5.8) and (5.9) are simplified to

$$\begin{aligned} \dot{e} =& \ Fe + TF_a(x)f_a + (L + \textstyle\sum_{i=1}^{h} L_i u_i)F_s f_s \\ r =& \ Ne + MF_s f_s. \end{aligned} \tag{5.10}$$

Thus the residual will vanish exponentially if F is Hurwitz and $f_a = f_s = 0$ for any $x(0), z(0)$. This completes the proof. ∎

Nevertheless, the above conditions are very difficult to verify and use for design and analysis directly. Note that equations (5.4)-(5.6) correspond to the constraints to be satisfied for the design of a UIFDO for linear systems and the constraints (5.7) relate to the bilinear part. We propose an equivalent form of constraints (5.5) and (5.7) based on the matrix G_b defined as

$$G_b = [G \ \hat{A}(I - D_c^+ D_c]$$

where $\hat{A} = \begin{bmatrix} A_1 & \dots & A_h \end{bmatrix}$, and

$$D_c = \begin{bmatrix} C & 0 & \dots & 0 \\ 0 & C & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & C \end{bmatrix} \in R^{(p\times h)\times(n\times h)}$$

Now, the constraints given in Lemma 5.3.1 can be expressed equivalently in terms of matrix G_b.

Corollary 5.3.1 The residual (5.3) is unknown input decoupled if and only if there exist matrices F, T, L, N and M which satisfy the following conditions simultaneously,

$$FT - TA + LC = 0; F \text{ is stable;} \tag{5.11}$$

$$TG_b = 0; \tag{5.12}$$

$$NT = MC. \tag{5.13}$$

Proof. The residual (5.3) is unknown input decoupled if and only if Lemma 5.3.1 holds. The constraint (5.7) in Lemma 5.3.1 can be rewritten as

$$T\hat{A} = \hat{L}D_c \tag{5.14}$$

where $\hat{L} = \begin{bmatrix} L_1 & \dots & L_h \end{bmatrix}$. The matrix equation (5.14) admits a solution $\hat{L}$ for a certain T, if and only if T satisfies

$$T\hat{A}(I - D_c^+ D_c] = 0$$

Thus, the conditions (5.5) and (5.7) can be combined together as

$$TG_b = T[G \;\; \hat{A}(I - D_c^+ D_c] = 0 \tag{5.15}$$

Then Lemma 5.3.1 and Corollary 5.3.1 are equivalent. ∎

Since equations (5.7) and (5.5) have been expressed in a similar form as those of unknown input residual generator (UIRG) constraints for linear systems, the following theorem can be stated immediately.

Theorem 5.1 *For the bilinear system (5.1) an UIRG of the form (5.2)-(5.3) exists if and only if the system (A, G_b, C) is not right invertible, where $G_b = [G \;\; \hat{A}(I - D_c^+ D_c]$, D_c^+ is pseudo-inverse of D_c. In this case, $F_i (i = 1, \dots, h)$ can all be set equal to zero.*

Proof. Based on the previous Theorem 4.2, the constraints (5.11)-(5.13) in Corollary 5.3.1 are satisfied if and only if the system (A, G_b, C) is not right invertible. Combining the results of Corollary 5.3.1 and Theorem 4.2 completes the proof. ∎

Remark 5.3.1 The conditions in Theorem 5.1 are easy to verify using straightforward matrix calculus. Corollary 5.3.1 implies that a structure restricted bilinear observer realizes a UIRG by considering the terms $u_i x, (i = 1, \dots h)$ as unknown inputs. Corollary 5.3.1 actually transforms the design of a bilinear UIFDO to the design of a UIFDO for the following linear system

$$\begin{aligned} \dot{x} &= Ax + Bu + G_b d + F_a(x) f_a \\ y &= Cx + F_s f_s \end{aligned} \tag{5.16}$$

This method simplifies the design procedure. However, the observer's existence condition is stricter than those based on the general bilinear observer (5.2).

Remark 5.3.2 In order to accomplish a robust FDI task, the residual has to be insensitive to unknown inputs and sensitive to sensor and actuator faults. Under conditions (5.4)-(5.7), the residual vector then satisfies

$$\begin{aligned} \dot{e} &= Fe + TF_a(x)f_a + (L + \textstyle\sum_{i=1}^{h} L_i u_i)F_s f_s \\ r &= Ne + MF_s f_s \end{aligned} \tag{5.17}$$

Therefore, (5.2) is a bilinear UIFDO if $N(sI - F)^{-1}TF_a(x) \neq 0$ and $MF_s \neq 0$ or $(L+\sum_{i=1}^{h} L_i u_i)F_s \neq 0$. Because our proposed design procedure actually parameterized all solutions of (5.4)-(5.7), it is easy to know if it is useful for detecting specific faults by checking the above inequalities.

5.4 General Structure Input Independent Bilinear UIFDO

If the condition stated in Theorem 5.1 is not satisfied, we may be able to design an observer by allowing some F_i to be nonzero. It is straightforward to derive the equation of the estimation error $e = Tx - z$ in the absence of any faults:

$$\dot{e} = Fe + \sum_{i=1}^{h} F_i u_i e - (FT - TA + LC)x - \sum_{i=1}^{h}(F_i T - TA_i + L_i C)u_i x + TGd. \tag{5.18}$$

Therefore, if equation (5.19) below,

$$TA_i - F_i T = L_i C; (i = 1, ..., h) \tag{5.19}$$

and (5.4)-(5.5) are satisfied simultaneously, the error dynamics (5.18) reduces to

$$\dot{e} = Fe + \sum_{i=1}^{h} F_i u_i e. \tag{5.20}$$

Based on Theorem 4.2, a T satisfying (5.4)-(5.5) always exists if (A, G, C) is not right invertible, and all solution of T can be found using the algorithm described in Section 4.4. Thus, we may check the existence of the general structure bilinear observer by checking the solvability of (5.19) for a given T. We have the following proposition.

Proposition 5.4.1 *For a given T and A_i, F_i and L_i exist which satisfy (5.19) if and only if*

$$TA_iC_I(I-(TC_I)^+TC_I)=0(i=1,...,h) \tag{5.21}$$

where $C_I = I - C^+C$.

Proof. It is well known that there exists $L_i(i=1,..h)$ to satisfy (5.19) if and only if

$$(TA_i - F_iT)(I-C^+C)=0(i=1,..h \tag{5.22}$$

(5.22) can be written as $TA_iC_I = F_iTC_I(i=1,..h)$. Obviously, there exist F_i to satisfy (5.22) if and only if (5.21) is satisfied. ∎

Remark 5.4.1 It should be noted that Proposition 5.4.1 does not exclude the solution of $F_i = 0$. If $F_i = 0$ is possible, $TA_iC_I = 0$, and (5.21) is satisfied automatically. Proposition 5.4.1 provides a simple way to verify the existence of the solution of (5.19) such that the observer error dynamics can be bilinear.

However, the condition in Proposition 5.4.1 is not sufficient to make the bilinear error dynamics (5.20) stable. Next, we propose the conditions for making the error dynamics stable for any control input u_i. The conditions for the error dynamics being stable for the bounded control input u_i are provided in the next section.

Proposition 5.4.2 *The error dynamics (5.20) will be asymptotically stable for any input if there exist symmetric positive definite (SPD) matrices P and Q such that.*

$$PF+F^TP=-Q \tag{5.23}$$

$$PF_i+F_i^TP=0, i=1,...,h. \tag{5.24}$$

Proof. Let P be a SPD matrix, set the Lyapunov function as $V = e^TPe$. Then take the time derivative of V:

$$\frac{d}{dt}e^TPe = e^T\{P[F+\sum_{i=1}^{h}F_iu_i]+[F+\sum_{i=1}^{h}F_iu_i]^TP\}e. \tag{5.25}$$

If conditions (5.23)- (5.24) are satisfied, then (5.25) becomes

$$\frac{d}{dt}e^T Pe = -e^T Qe$$

Since P and Q are assumed to be SPD, $lim_{t\to\infty}e(t) = 0$ by the well-known Lyapunov stability theorem. ∎

Proposition 5.4.2 is very difficult to be applied since it requires the solution of a set of Lyapunov equations. At present, there is no technique to solve the set of Lyapunov equations (5.23)-(5.24). However, a linear matrix inequalities (LMI) approach can be used to verify Proposition 5.4.2 for the given matrices F, F_i by formulating (5.23)-(5.24) as an equivalent LMI problem which is stated as follows:

Solve for P, β which minimizes β subject to

$$PF + F^T P < 0 \tag{5.26}$$

$$\begin{bmatrix} \beta I & PF_i + F_i^T P \\ PF_i + F_i^T P & \beta I \end{bmatrix} \geq 0, i = 1, ...h \tag{5.27}$$

$$P > 0 \tag{5.28}$$

When the above optimization problem has a minimum of $\beta = 0$, Proposition 5.4.2 is satisfied. Recently, computationally feasible techniques have been developed for solving such problems (see [31]).

Remark 5.4.2 The solution of LMI (5.26)-(5.28) does not provide a direct basis for verification of the observer existence because F, F_i is unknown *a priori*. A basic guidance for the design of F and F_i comes from the following fact: equation (5.23) is equivalent to the condition of F being stable, and equation (5.24) means that all eigenvalues of $F_i, i = 1, ..., h$ must have zero real part. It is very useful to check if F, F_i satisfy these necessary conditions for the solvability of the Lyapunov equations (5.23)-(5.24) before solving the LMI problem (5.26)-(5.28). On the other hand, if $dim(F) = dim(F_i) = 1$, F_i must be zero. Therefore $dim(F) > 1$ is necessary for existence of general structure bilinear observer.

Next, we propose a design algorithm for general structure input independent bilinear UIFDO. It is summarized as following steps:

1. Design matrices F, T, and L which satisfy (5.4) and (5.5) using the linear UIRG algorithm in Section 4.4. If $dim(F) > 1$, continue.

2. If T satisfies (5.21), calculate F_i, L_i by solving the following equations,

$$F_i = T * A_i * C_I * (TC_I)^+ + X(I - (TC_I) * (TC_I)^+), i = 1, ...h; \tag{5.29}$$

$$L_i = (T * A_i - F_i * T) * C^+, i = 1, ...h \tag{5.30}$$

   where X is any matrix with the same dimension as F_i.

3. Adjust X such that all eigenvalues of $F_i, i = 1, ..., h$ have a zero real part.

4. Verify the feasibility of the F_i by solving LMI (5.26)-(5.28). If the error dynamics are stable, calculate N and M from (5.6).

Remark 5.4.3 In step 1, the resultant F can always be a diagonal matrix, such that $P = I$ may be a solution for (5.26). $P = I$ is a solution for (5.28) in the same time if $F_i + F_i^T = 0$. Therefore, the key part of the algorithm is reduced to find the X such that $F_i + F_i^T = 0$, which can be rewritten as a set of linear equations and solved easily. It is valuable to check the existence of this specific solution because we have no other systematic method to find the X to make all engenvalues of F_i have zero real part. The trial and error characteristic of step 3 limits the applicability of the above algorithm. However, it does provide another choice if the method described in Section 5.3 does not work.

Remark 5.4.4 Even if the structure restricted bilinear UIFDO exists, the general structure method is still worth to try because it will allow a higher residual vector dimension, which increases the capability to isolate faults.

5.5 UIFDO for Bilinear Systems With Bounded Control Input

In order to be able to design an input independent observer, either all F_i need to be zeros, or F_i must satisfy a strict constraint condition (5.24). If it is not possible to design an input independent bilinear observer, then one can explore the possibility of designing an observer whose error dynamics are input dependent.

Consider the general structure bilinear observer (5.2). The previous section has shown that if the conditions (5.4)-(5.5) and (5.21) are satisfied, then the estimation error dynamics becomes

$$\dot{e} = Fe + \sum_{i=1}^{h} F_i u_i e. \tag{5.31}$$

If the control input is bounded, the error dynamics can be considered as a linear time-invariant system with time-varying nonlinear but bounded perturbation.

Lemma 5.5.1 *[84] If F is Hurwitz, and*

$$\|\sum_{i=1}^{h} F_i u_i\| \leq \sum_{i=1}^{h} \|F_i\| \|\overline{u_i}\| \leq \frac{1}{2\lambda_{max}(P)} \tag{5.32}$$

where P is the solution of $PF + F^T P = -I, \overline{u_i} = max\{u_i\}$, then system (5.31) is asymptotically stable.

In Section 4.4, we have developed the UIFDO solutions for a linear system. They are matrices F, T, L, N and M satisfying equations (5.4)-(5.5). It is noted that eigenvalues of F can be assigned to arbitrary far locations in the half left-plane, and F may be diagonal. To that end, $\frac{1}{\lambda_{max}(P)}$ can be made as large as possible .

Proposition 5.4.1

$$F_i = TA_iC_I(TC_I)^+ \tag{5.33}$$

Recall that T is composed of left eigenvectors of $A - KC$, where K is the gain for eigenstructure assignment. T can always be normalized to have unit norm. Thus, norm of F_i can be kept under a certain bound, such that condition (5.32) is satisfied.

Combining the solvable condition for linear UIRG problem, we conclude that if (A, G, C) is not right invertible and condition (5.21) can be satisfied, a bounded-input bilinear UIRG always exists. Next we shall outline the design algorithm:

1. Design matrices F, T, and L which satisfy (5.4) and (5.5) using the linear UIRG algorithm in Section 4.4.

2. If condition (5.21) is satisfied, then calculate F_i as (5.33). If (5.32) is satisfied by F_i, go to step 3. Otherwise enlarge the desired eigenvalue for F and go back to step 1.

3. Design $L_i = (TA_i - F_iT)C^+$, calculate N, M from (5.6).

The fault detectability with the general structure bilinear observer is more complicated. The residual under the existence of fault signal will be

$$\begin{aligned} \dot{e} = &\ Fe + \textstyle\sum_{i=1}^{h} F_i u_i e + TF_a(x) f_a + (L + \textstyle\sum_{i=1}^{h} L_i u_i) F_s f_s \\ r = &\ Ne + MF_s f_s \end{aligned} \tag{5.34}$$

Unlike the structure restricted UIFDO, where the transfer function can be used easily to verify if it can be used for fault detection, the error dynamics are bilinear here. In general, we should use the complex bilinear analysis technique, such as the Volterra series method [75], to examine the condition of the nonzero output r. The conditions for undetectable faults are:

1. If $TF_a(x) = 0$, the actuator fault f_a will not be detectable;

2. If $MF_s = 0, (L + \sum_{i=1}^{h} L_i u_i)F_s = 0$, the sensor fault f_s will not be detectable.

In practice, simulation is a good way to check the shape of r for different fault signals.

Remark 5.5.1 For bounded input design, eigenvalues of F have to be large enough to compensate for the perturbation $F_i u_i e$. The residual with too large eigenvalues for F may not be sensitive to the actuator faults, since it arises in the error dynamics equation as the same form as the control input. However, the residual will still be sensitive to the sensor faults as the residual equation (5.34) shows that f_s effect residual directly.

5.6 Numerical Examples

In this section, two examples are given to illustrate the above developments. The first example is an electromechanical actuator [92] and the second is the quarter-car semi-active suspension [43, 99].

Example 5.6.1 This numerical example is an electromechanical actuator frequently used in robotics and consists of a direct-current motor with an elastic coupling and the load shaft as shown in Figure 5.1 [92].

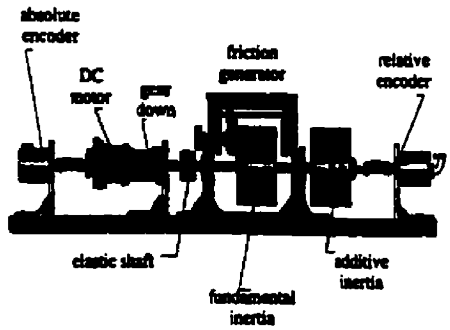

Figure 5.1: Structure of an electromechanical actuator

This plant can be described by the following bilinear state space model:

$$\dot{x} = Ax + A_1 u_1 x + B u_2 + G d, y = C x$$

where

$$A = \begin{bmatrix} -R_a/L_a & 0 & 0 & 0 & 0 \\ 0 & -F_m/J_m & 0 & -k_r/(NJ_m) & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1/N & 0 & 0 & -1 \\ 0 & 0 & 0 & k_r/J_c & -F_c/J_c \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1/L_a \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$x = \begin{bmatrix} i_a \\ \omega_m \\ \theta_m \\ \Delta\Gamma \\ \omega_c \end{bmatrix}, A_1 = \begin{bmatrix} 0 & -k_a/L_a & 0 & 0 & 0 \\ k_a/J_m & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, G = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1/J_c \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad \text{and} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} i_e \\ v_a \end{bmatrix}.$$

The state variables are the armature current $i_a(t)$, the motor shaft velocity $\omega_m(t)$, the motor shaft angular position $\theta_m(t)$, the angular rotation $\Delta\Gamma(t)$ between the motor shaft and the load shaft due to the elastic coupling and the load shaft angular velocity $\omega_c(t)$. The control inputs are the stator current $i_e(t)$ and the armature voltage $v_a(t)$. The disturbance $d(t)$ is the torque due to friction, and load reactions. In the state space description, J_m and J_c represent the motor and the load shaft inertia, F_m and F_c represent the motor and the load viscous friction coefficients, k_a represents the motor torque constant, k_r represents the coupling rigidity coefficient, and N represents the gear ratio. The numerical values of these parameters are $N = 20, R_a = 1[\Omega], L_a = 0.05[H], k_a = 0.156[m^2kgsec^{-2}A^{-2}], k_r = 37.7[m^2kgsec^{-2}], F_m = 0.0032[m^2kgsec^{-1}], F_c \cong 0[m^2kgsec^{-1}], J_m = 2.4e^{-4}[m^2kg], J_c = 0.0825[m^2kg]$. Because of $CG = 0$, the method in [136] is inapplicable for this system.

Using Corollary 5.3.1, the equivalent linear disturbance matrix G_b is

$$G_b = [G \quad A_1(I - C^+C)] = \begin{bmatrix} 0 & 0 & -k_a/L_a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1/J_c & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

This system satisfies conditions for the existence of the structure restricted bilinear input independent UIFDO, because SII index n_b for (A, G_b, C) is 2. The matrices of the observer (5.2) are,

$$F = \begin{bmatrix} -4 & 0 \\ 0 & -1 \end{bmatrix}, T = \begin{bmatrix} 0 & -0.0125 & 0.0483 & 0 & 0 \\ 0 & -0.0127 & 0.0110 & 0 & 0 \end{bmatrix}, L = \begin{bmatrix} 0 & 0.1932 & 0.9811 \\ 0 & 0.0110 & 0.9999 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} -0.0812 & 0 & 0 \\ -0.0828 & 0 & 0 \end{bmatrix}, N = \begin{bmatrix} -1.2731 & 1.2492 \end{bmatrix}, M = \begin{bmatrix} 0 & 0.0477 \end{bmatrix}.$$

In the simulation, we check the response of the residual to two actuator bias faults. It is easy to know that $F_{a1}(x) = A_1 x$ and $F_{a2}(x) = B$. Let $u_1(t) = 10sin(\pi t), u_2(t) = 130sin(2\pi t)$. Figure 5.2 displays the residual corresponding to the following actuator fault signals, respectively.

$$f_{a1}(t) = \begin{cases} 0 & t \leq 10 \\ 5 * sin(\pi t) & t > 10 \end{cases}; f_{a2}(t) = \begin{cases} 0 & t \leq 12 \\ 10 * sin(2\pi t) & t > 12 \end{cases}$$

It is assumed that $x_0 = \begin{bmatrix} 0.3 & 20 & 0.5 & 0.1 & 10 \end{bmatrix}, z_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}$.

Initially, during the observer's transient, the residual quickly decays to zero. It becomes nonzero as soon as faults occur. Unfortunately, fault isolation cannot be achieved by this simple design since the maximum dimension of residual vector is 1.

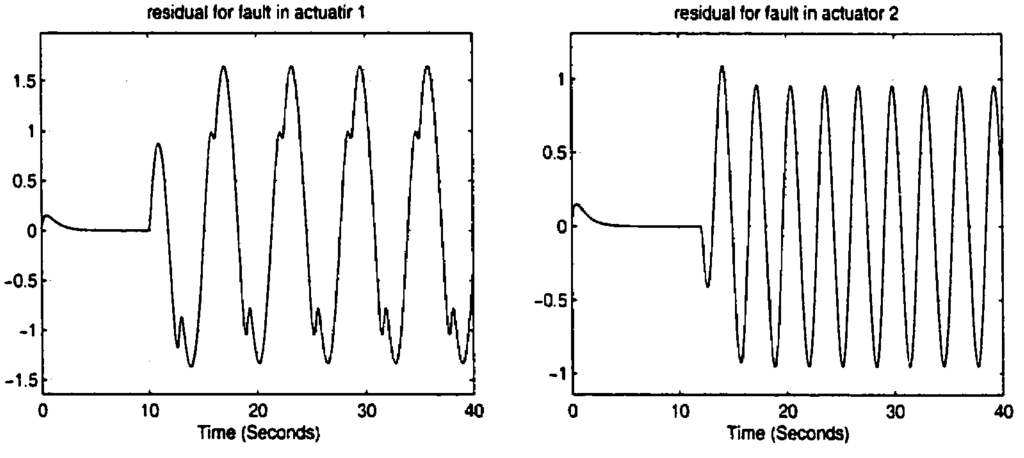

Figure 5.2: Residuals for faults in an electromechanical actuator

Further, we design an input independent bilinear UIFDO of general structure. It is easy to check that SII n_b for the system matrices (A, G, C) is 3, and the maximum residual dimension is 2. The matrices F, T and L for the observer (5.2) are designed as

$$F = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix}, L = \begin{bmatrix} 0 & -0.1932 & -0.9811 \\ 1 & 0 & 0 \\ 0 & -0.0110 & -0.9999 \end{bmatrix},$$

$$T = \begin{bmatrix} 0 & 0.0125 & -0.0483 & 0 & 0 \\ -0.0556 & 0 & 0 & 0 & 0 \\ 0 & 0.0127 & -0.0110 & 0 & 0 \end{bmatrix}.$$

All eigenvalues of F_1 must have a zero real part. F_1 is calculated according to the simplified procedure mentioned in Remark 5.4.3. Using equation (5.29),

$$F_1 = \begin{bmatrix} 0 & 0 & 0 \\ 6.8061 & 0 & 6.9364 \\ 0 & 0 & 0 \end{bmatrix} + X \begin{bmatrix} 0.5095 & 0 & -0.4999 \\ 0 & 1.0000 & 0 \\ -0.4999 & 0 & 0.4905 \end{bmatrix}$$

with

$$X = \begin{bmatrix} 0 & -6.8061 & 0 \\ 0 & 0 & 0 \\ 0 & -6.9364 & 0 \end{bmatrix}$$

The final solution is given by

$$F_1 = \begin{bmatrix} 0 & -6.8061 & 0 \\ 6.8061 & 0 & 6.9364 \\ 0 & -6.9364 & 0 \end{bmatrix}, L_1 = \begin{bmatrix} -0.2969 & 0 & 0 \\ 0 & 0.4053 & 0.0000 \\ -0.3026 & 0 & 0 \end{bmatrix}.$$

$$N = \begin{bmatrix} 0.5091 & 0 & -0.4996 \\ 0 & 0.9969 & 0 \end{bmatrix}, M = \begin{bmatrix} 0 & 0.0191 & 0 \\ 0.0554 & 0 & 0 \end{bmatrix}.$$

Because of the bilinear estimation error dynamics, the residual direction cannot be predicted for each fault. Figure 5.3 and 5.4 show the residuals for actuator faults 1 and 2, respectively. Obviously, the residual shape is totally different for these two actuator faults. Fault isolation can be achieved using bilinear UIFDO of general structure.

Example 5.6.2 This example deals with the implementation of the detection algorithm of Section 5.5. A model of a vehicle with a semi-active suspension is considered. It is introduced by Hac [43] and is a practically important example of a bilinear system driven by unknown disturbances. The model is shown in Figure 5.5, which represents a quarter of a vehicle. Here, m_2 denotes the body mass, m_1, the mass of the wheel and semi-axle, k_1, the tire stiffness and k_2, the suspension stiffness. The damping

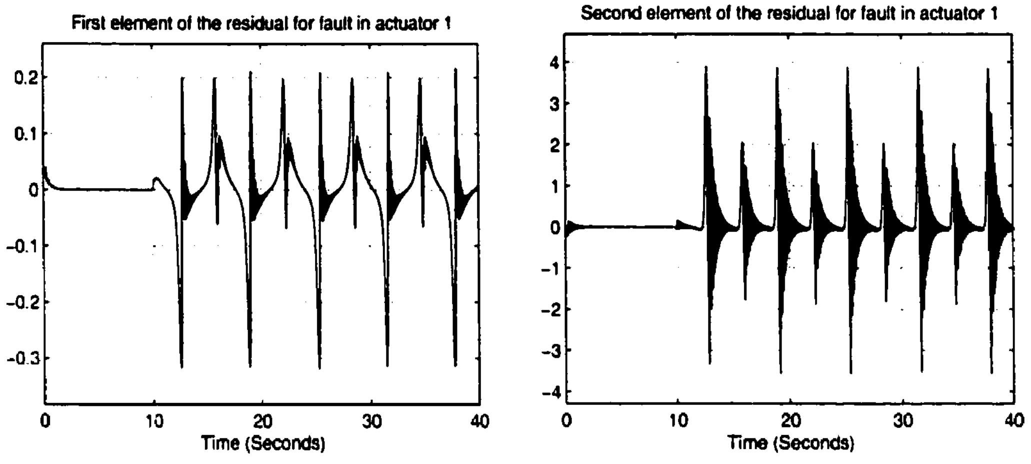

Figure 5.3: Residual vector for fault isolation of an electromechanical actuator (case 1)

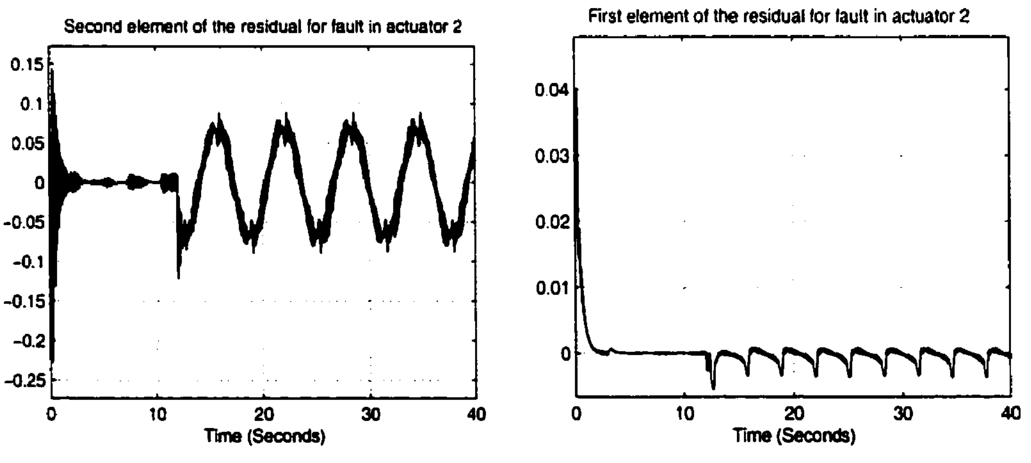

Figure 5.4: Residual vector for fault isolation of an electromechanical actuator (case 2)

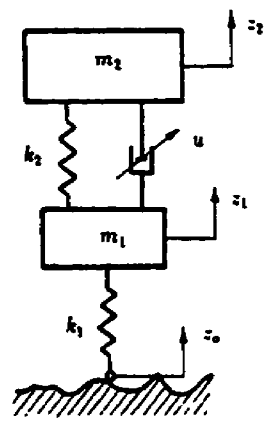

Figure 5.5: A 2-DOF vehicle model with semi-active suspension

ratio of an electronically controlled shock absorber, $u(t)$, can be varied within a given range according to a prescribed control law. This can be achieved by changing the size of the orifice in the shock absorber. Select the following state vector:

$$x = [z_1 - z_0, \dot{z}_1, z_2 - z_1, \dot{z}_2]^T$$

where z_1, z_2 denote the absolute displacement of the wheel and the body, respectively. z_0 is the road elevation at the point of contact with the tire. Then, the system represented in Figure 5.5 can be described by the state equation

$$\dot{x} = Ax + A_1 u_1 x + Gd$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1}{m_1} & 0 & \frac{k_2}{m_1} & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & \frac{-k_2}{m_1} & 0 \end{bmatrix}, A1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{m_1} & 0 & \frac{1}{m_1} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_2} & 0 & \frac{-1}{m_2} \end{bmatrix}, G = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and the disturbance $d(t) = \dot{z}_0$ is the rate of change of the road elevation. In the simulation, the following values for the vehicle parameters are used:

$$m_1 = 40[kg], m_2 = 250[kg], k_1 = 10^5[m/m], k_2 = 5000[m/m]$$

Assume the wheel displacement relative to the road and the suspension deflection are measured. Thus the matrix C is

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

It is easy to verify that no input independent bilinear UIFDO exists for this system using Theorem 5.1. However, because the input is bounded($\|u\| \leq 1000$), we can build the following fault detection observer:

$$F = \begin{bmatrix} -30 & 0 \\ 0 & -40 \end{bmatrix}, F_1 = \begin{bmatrix} -0.0211 & -0.0129 \\ -0.0129 & -0.0079 \end{bmatrix}, L = \begin{bmatrix} 2.3267 & -9.7256 \\ 1.4182 & -9.8989 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 0 & -0.0088 \\ 0 & -0.0054 \end{bmatrix}, T = \begin{bmatrix} 0 & -0.0093 & -0.2792 & 0.0093 \\ 0 & -0.0057 & -0.2269 & 0.0057 \end{bmatrix}.$$

$$N = \begin{bmatrix} 107.45 & -176.28 \end{bmatrix}, M = \begin{bmatrix} 0 & -10 \end{bmatrix}.$$

It is easy to check $\|F_1\| = 0.029$. Therefore thè observer is stable for all allowable u while we set the eigenvalues of F to be -30 and -40. Figure 5.6 shows the residual response due to two sensor bias faults, which are assumed to be

$$f_{s1}(t) = \begin{cases} 0 & t \leq 2 \\ 0.1 & t > 2 \end{cases} ; f_{s2}(t) = \begin{cases} 0 & t \leq 3 \\ 0.1 & t > 3 \end{cases}.$$

However, here the residual is insensitive to the bias actuator faults due to its large eigenvalues.

5.7 Conclusions

In this chapter we propose three approaches to design the input independent and input-bounded reduced-order robust fault diagnosis observers for bilinear systems. For the structure restricted bilinear fault detection observer, we provide the necessary and sufficient conditions for its existence, as well as a simple design procedure. The general structure bilinear fault diagnosis observer exists under less restrictive conditions. Future research is to needed to find necessary and sufficient existence conditions

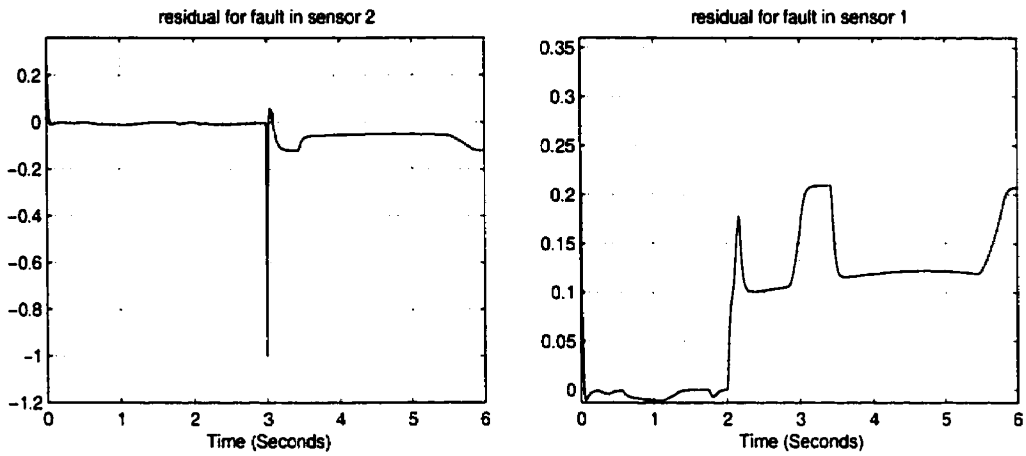

Figure 5.6: Residuals to the sensor faults in a semi-active suspension

as well as more systematic design methods for robust bilinear fault diagnosis observers with general structures.

Chapter 6

Sliding Mode Functional Observers For Linear and Nonlinear Uncertain Systems

The main thread in the last five chapters was the notion of decoupling the effects of unknown inputs from the estimation of a state function Tx, where x represents state variable and T is a constant matrix. The resulted unknown input decoupled estimation was then used for generating residuals which were only sensitive to fault signal and robust fault diagnosis was achieved. In all of those chapters, the system uncertainties, nonlinearity part and internal/external disturbance are lumped together to be expressed as an unknown input term $Gd(t)$, where G is a constant matrix and $d(t)$ is an arbitrary vector function of time. The heart of the decoupling technique is to make the state function matrix T satisfy $TG = 0$, such that Tx becomes an unknown input free variable and can be estimated using the classical Luenberger observer. Seliger and Frank [100] extended the unknown input observer (UIO) for linear systems to the following class of nonlinear systems:

$$\begin{aligned} \dot{x} = & \ A(x,u) + G(x)d \\ y = & \ H(x) \end{aligned} \tag{6.1}$$

Similarly, the unknown input decoupling is implemented by finding a nonlinear state transformation $T(x)$ such that $\frac{\partial T(x)}{\partial x}G(x) = 0$, then using different kinds of nonlinear observer approaches to estimate $T(x)$. The decoupling techniques based on the state transformation concept are applicable to a limited class of control systems where some strict system structural conditions have to be satisfied. Research into the designing of the robust observer and the observer-based robust fault diagnosis scheme, applicable to a wide class of uncertain systems, has been carried out for several years and will be the prevailing subject of research in the years to come. This chapter makes some progress in this direction. It is built upon the existing sliding mode observer techniques.

In recent years a considerable number of researchers have addressed the design of observer based on the variable structure systems theory, and sliding mode concept [22, 103, 112, 115, 116]. These existing methods can be classified into two categories: 1) the equivalent control based methods, and 2) the sliding mode observer (SMO) designs based on the method of Lyapunov. Since these techniques are important to the developments in this chapter, we shall briefly expand on these approaches in Section 6.1. The SMO design using the Lyapunov method was suggested by Walcott and Zak in [115, 116], and further developed by Misawa in [73]. It is called the Walcott-Zak observer in [23] and we shall also refer to it by the same name. A new and simple systematic design algorithm for the Walcott-Zak SMO is provided in Section 6.2. It is proved that the Walcott-Zak SMO is equivalent to UIO because of their same existence condition.

The SMO based on equivalent control method was originally proposed by Utkin [112]. This method, which we will term as the Utkin SMO, cannot provide the exact state estimation under the existence of unknown input. However, by using the equivalent control information carefully, we propose a novel SMO design method in Section 6.3, which works under much less conservative conditions than that of the Walcott-Zak SMO. We propose a sliding mode functional observer (SMFO) design so that estimating a function of the state is achieved when estimating all states is impossible. In addition, we address the issue of estimating unknown inputs under SMFO framework. Section 6.4 extend the SMFO design for linear uncertain systems to a general class of nonlinear uncertain systems. Numerical examples are used to illustrate the validity of the proposed observer design strategy in Section 6.5.

6.1 Introduction to Sliding Mode Observer

The notion of sliding mode has been investigated mostly as a mean of robust nonlinear control [112]. Sliding mode control is also called variable structure control in the literature due to its high-speed switching feedback control structure. The purpose of the switching control law is to drive the plant's state trajectory onto a pre-specified surface in the state space and to maintain the plant's state trajectory on this surface for all subsequent time. The plant's state trajectory then slides along this surface. Because of that, this surface is called a *sliding surface* (sliding manifold), which is defined as

$$S = \{(x,t) \in R^{n+1} : \sigma(x,t) = 0\}$$

where

$$\sigma(x,t) = [\sigma_1(x,t), ..., \sigma_m(x,t)]^T = 0.$$

These surfaces are designed so that the system state trajectory, restricted to $\sigma(x,t) = 0$, has a desired behavior such as stability or tracking. Although general nonlinear time-varying surfaces are possible, linear and time-invariant ones are more prevalent in design. The system is in a *sliding mode* when the state trajectory remains on the sliding surface. Figure 6.1 illustrates the existence of a sliding mode on the intersection of the two surfaces, where the tangent of the state trajectory must point towards the sliding surface in the vicinity of the sliding surface. The phenomenon of non-ideal but fast switching is called *chattering*, which stems from noise.

Sliding mode controllers exhibit excellent robust properties in the face of model uncertainty and disturbance. The main drawback is that they involve large control authority and control chattering. Despite fruitful research and development activity in the area of sliding mode control theory, relatively few authors have considered the dual problem of designing observers using the sliding mode principle. Utkin presents a SMO strategy for linear systems whereby the error between the estimated and measured outputs is forced to exhibit a sliding mode, and measurement noise effects are reduced [112]. The idea of the Utkin SMO is extended to a general class of nonlinear systems by Drakunov [22]. Walcott and Zak use a Lyapunov-based approach to formulate a SMO design which exhibits asymptotic state error decay

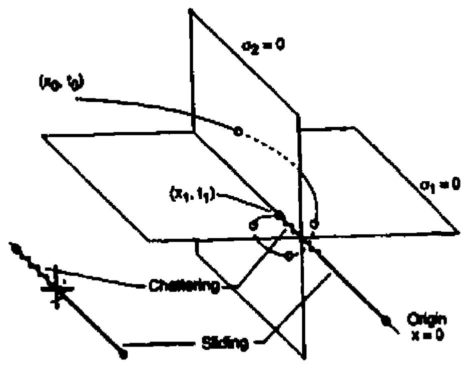

Figure 6.1: Illustration of Sliding Mode

in the presence of bounded nonlinearity and uncertainties that satisfy the so-called matching condition [115]. Misawa [73], Edwards and Spurgeon [23] further developed the numerical tractable algorithm for the Walcott-Zak SMO.

Consider a linear uncertain dynamic system described by

$$\begin{aligned} \dot{x} &= Ax + Bu + Gd(x, u, t) \\ y &= Cx \end{aligned} \tag{6.2}$$

where $x \in \mathcal{R}^n$ is the state, $u \in \mathcal{R}^m$ is the control input, $y \in \mathcal{R}^p$ is the output. The matrices A, B and C are of appropriate dimensions. It is assumed that $d(x, u, t)$ is unknown, but bounded, so that

$$\|d(x, u, t)\| \leq \rho \quad \forall x \in \mathcal{R}^n, u \in \mathcal{R}^m, t \geq 0$$

where $\|\cdot\|$ refers to the Euclidean norm. G is a full rank matrix in $\mathcal{R}^{n \times q}$. $Gd(x, u, t)$ represents the system uncertainties or nonlinearities, namely the unknown input. In addition, the matrices B and C are assumed to be of full rank. A detailed review of the SMO design approaches of Utkin [112] and Walcott and Zak [115, 116] is provided in the following.

6.1.1 Equivalent control concepts and Utkin sliding mode observer

Consider initially the system described above under the added assumptions that the pair (A, C) is observable and $Gd(x, u, t) \equiv 0$. Without loss of generality, it can be assumed that the output distribution matrix can be written as

$$C = [C_1 \quad C_2]$$

where $C_1 \in \mathcal{R}^{p\times(n-p)}, C_2 \in \mathcal{R}^{p\times p}$ and $det(C_2) \neq 0$. Consequently, the transformation

$$T = \begin{bmatrix} I_{n-p} & 0 \\ C_1 & C_2 \end{bmatrix} \tag{6.3}$$

is non-singular and with respect to this new coordinate system it can be seen that the new output distribution matrix $CT^{-1} = [0 \quad I_p]$. If the other system matrices are written as

$$TAT^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad \text{and } TB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

then the nominal system can be written as

$$\begin{aligned} \dot{x}_1 &= A_{11}x_1 + A_{12}y + B_1u \\ \dot{y} &= A_{21}x_1 + A_{22}y + B_2u \end{aligned} \tag{6.4}$$

The corresponding sliding mode observer for the y subsystem is

$$\dot{\hat{y}} = A_{22}\hat{y} + A_{21}\hat{x}_1 + B_2u + L_1 sign(y - \hat{y}) \tag{6.5}$$

where $(\hat{x}_1, \hat{y})$ are the state estimates for (x_1, y), L_1 is a constant nonsingular feedback gain matrix, and $sign(y - \hat{y})$ function is defined as

$$sign(y-\hat{y}) = \begin{cases} 1; & y - \hat{y} > 0; \\ 0; & y - \hat{y} = 0; \\ -1; & y - \hat{y} < 0. \end{cases}$$

If the error between the estimates and the true states are written as $e_y = y - \hat{y}$ and $e_1 = x_1 - \hat{x}_1$, then the following error system is obtained:

$$\dot{e}_y = A_{22}e_y + A_{21}e_1 - L_1 sign(e_y). \tag{6.6}$$

It can be shown using singular perturbation theory that for a large enough L_1, a sliding mode motion can be induced on the outputs' error state in (6.6) [112]. It follows that, after some finite time t_s, for all subsequent time, $e_y = 0$ and $\dot{e}_y = 0$.

For the second subsystem, the observer equation is

$$\dot{\hat{x}}_1 = A_{11}\hat{x}_1 + A_{12}y + B_1 u + L_2 L_1 sign(e_y) \tag{6.7}$$

which gives the following estimation error equation

$$\dot{e}_1 = A_{11}e_1 - L_2 L_1 sign(e_y). \tag{6.8}$$

According to the equivalent control method, the system in sliding mode behaves as if $L_1 sign(e_y)$ is replaced by its equivalent value $(L_1 sign(e_y))_{eq}$, which can be calculated from the subsystem (6.6) assuming $e_y = 0$ and $\dot{e}_y = 0$. Hence

$$(L_1 sign(e_y))_{eq} = A_{21}e_1. \tag{6.9}$$

Substituting (6.9) into (6.8) we obtain

$$\dot{e}_1 = (A_{11} - L_2 A_{21})e_1. \tag{6.10}$$

Since the pair (A_{11}, A_{21}) is observable if (A, C) is observable, $e_1 \to 0$ by appropriate choice of L_2.

6.1.2 Walcott-Zak Sliding Mode Observer

The problem considered by Walcott and Zak [115, 116] involves estimating states of a system such as that described in (6.2) so that the error tends to zero exponentially despite the presence of **matched** uncertainty. Because the model (6.2) can describe a certain class of nonlinear systems, Walcott-Zak SMO was considered as a state estimation method for nonlinear systems in some reference papers. Unlike several other techniques for the observer design of nonlinear systems, where the exact knowledge of the plant's nonlinearities must be known and incorporated (either directly or indirectly into the dynamics of the observer), only the bounds of the nonlinearities of the plant are used in the Walcott-Zak observer dynamics. However, we feel that it is more suitable to consider it as a robust observer technique for linear uncertain systems, which render the observer error system totally insensitive to uncertainty. This is because $d(x, u, t)$ can represent not only nonlinearity, but also time-varying term and internal/external disturbance.

Due to the assumption that the pair (A, C) is completely observable, a matrix $K \in \mathcal{R}^{n\times p}$ exists such that $A_0 = A - KC$ has stable eigenvalues. Therefore, for every real symmetrical positive definite (SPD) matrix $Q \in \mathcal{R}^{n\times n}$, there exists a real SPD matrix P as the unique solution to the following Lyapunov equation:

$$P(A - KC) + (A - KC)^T P = -Q \tag{6.11}$$

It is also assumed that a Lyapunov pair (P, Q) for A_0 exists such that the structural constraint

$$WC = G^T P \tag{6.12}$$

is satisfied for some $W \in \mathcal{R}^{q\times p}$.

The proposed observer in [115] has the form

$$\dot{\hat{x}} = A\hat{x} + Bu + K(y - C\hat{x}) + \nu \tag{6.13}$$

where

$$\nu = \begin{cases} \rho\frac{GWCe}{\|WCe\|} & \text{if } WCe \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{6.14}$$

or

$$\nu = \rho G sign(WCe) \tag{6.15}$$

where $e = x - \hat{x}$, $sign(\cdot)$ is the sign function. This sliding observer is basically the conventional Luenberger observer with the additional nonlinear, discontinuous "switching" term ν. By using $V(e) = e^T Pe$ as a candidate for a Lyapunov function it can be shown that $\dot{V}(e) < 0$ for $e \neq 0$ and thus $e \to 0$ exponentially, and the surface $S = \{e : WCe = 0\}$ is the sliding manifold.

The crucial problem is therefore to compute the matrix pair (P, W) such that (6.11) and (6.12) are satisfied. An algorithm given by Walcott and Zak [116] can be summarized as follows:

1 choose the spectrum of A_0, and compute K accordingly;

2 solve the structure constraint **symbolically** to obtain an expression for P_W in terms of entries of W, ensuring that P_W is symmetrical;

3 compute $Q(P_W)$ symbolically in terms of entries of P_W using the expression $-(P_W A_0 + A_0^T P_W)$;

4 choose elements of W and P_W to ensure $Q(P_W)$ is SPD.

The idea of imposing the structural constraint (6.12) to ensure, if possible, total insensitivity to the matched uncertainty, is intuitively appealing. Moreover, the observer dynamics may be easily implemented. However, from a computational point of view, this framework is impractical for high order systems because of the manipulation and solution of the associated constrained Lyapunov problem defined by equations (6.11) and (6.12). It is difficult to check if the Walcott-Zak SMO exists or not for systems (A, G, C) based on the original matrices. [107] shows that a sufficient condition for (6.12) being valid is that the modified transfer function defined as $G_F(s) = WC(sI - A_0)^{-1}G$ is strictly positive real. However, [107] did not consider the problem of finding a suitable W. The bottleneck in finding matrices P and W obstructs the application of the Walcott-Zak SMO. Edwards and Spurgeon provide a numerical tractable algorithm in [23] by transforming the system (A, G, C) into a canonical form. One may follow the design procedure outlined in [23] to find the matrices P and W, and if one fails to design a Walcott-Zak SMO using the algorithm, one knows that a Walcott-Zak SMO does not exist. The necessary and sufficient existence condition for the Walcott-Zak SMO has been a continuing problem for a long time.

In theory, the sliding-mode observer has good robustness to bounded modeling errors. However, there are some differences in terms of the robust properties of the above two sliding observer design methodologies. The analysis in [103, 120] has shown that bounded estimation error exists for bounded unknown inputs when an equivalent control based sliding mode observer is employed. In other words, the estimation will not be accurate when uncertainties are present. The Lyapunov based sliding-mode observer (6.13) design makes $\hat{x} \to x$. Because $d(x, u, t)$ may represent both nonlinearities and unknown inputs due to uncertainties (modeling error or disturbance), it means the Walcott-Zak SMO (6.13) results in an exact estimation for certain class of nonlinear uncertain systems. This is an important difference between the two design techniques.

The next section explores the reason for the above difference. The relationship between the UIO and Walcott-Zak SMO is also revealed. However, the main contribution of this chapter is to extend the design of the Walcott-Zak observer to a more general class of linear uncertain systems based on a new explanation about the principle of the Walcott-Zak observer. Further, the design is extended to a large class of nonlinear uncertain systems.

6.2 The Principle of Walcott-Zak SMO

It is difficult to find the Lyapunov pair $\{P, Q\}$ and gain matrices K, W to satisfy conditions (6.11) and (6.12). The difficulty has limited the use of the Walcott-Zak SMO. The two conditions, (6.11) and (6.12), in a roundabout way impose some structural constraints on the system under consideration. Recently, an explicit equivalent condition for (6.11) and (6.12), in terms of original system matrices, was derived by Corless and Tu [17], and Xiong and Saif [129]. That result is Theorem 3.1 in Chapter 3. Corless and Tu's work focuses on a robust state and input estimator and does not address the connection of their result to SMO design. Here we shall show how important it is for the analysis and design of the Walcott-Zak SMO.

Theorem 3.1 shows that equations (6.11) and (6.12) are solvable if and only if $rank(CG) = rank(G)$ and triplet $\{A, G, C\}$ is minimum phase. The minimum phase system with $rank(CG) = rank(G)$ has a canonical form, which is given by following Lemma 6.2.1.

Lemma 6.2.1 *For system (6.2), if and only if $rank(CG) = rank(G)$ and the triplet $\{A, G, C\}$ is minimum phase, there exist non-singular transformations Γ_1, and Γ_2 such that*

$$x = \Gamma_1[x_{ab}^T, x_d^T]^T, y = \Gamma_2[y_d^T, y_b^T]^T,$$

in which case the transformed system can be written as

$$\begin{aligned}\dot{x}_{ab} &= A_{ab}x_{ab} + L_d x_d + B_{ab}u \\ \dot{x}_d &= E_{ab}x_{ab} + A_d x_d + B_d u + G_d d(x,u,t) \\ y_b &= C_{ab}x_{ab} \\ y_d &= C_d x_d\end{aligned} \tag{6.16}$$

where G_d and C_d are invertible, and the pair (A_{ab}, C_{ab}) is detectable.

Lemma 6.2.1 is a direct result of the work on the special coordinate transformation (SCB) theory [94]. This lemma and Theorem 3.1 imply that the Walcott-Zak observer can actually be designed only for systems which can be transformed into the form given in (6.16). For the transformed system (6.16), it is noted that the nonlinear term $Gsign(WCe)$ in the Walcott-Zak observer actually appears only in the subsystem which is affected directly by unknown inputs. Based on this observation, we propose an algorithm to construct the Walcott-Zak SMO.

Walcott-Zak SMO Design Algorithm

Step 1: If $rank(CG) = rank(G)$ and system (A, G, C) is minimum phase, transform it into canonical form (6.16) by non-singular state and output transformations Γ_1, Γ_2.

Step 2: Compute K_{ab} and K_d to stabilize $\hat{A}_{ab} = A_{ab} - K_{ab}C_{ab}$ and $\hat{A}_d = A_d - K_d C_d$ respectively. Build the observer in the following form:

$$\begin{aligned}\dot{\hat{x}}_{ab} &= A_{ab}\hat{x}_{ab} + L_d\hat{x}_d + B_{ab}u + K_{ab}(y_b - C_{ab}\hat{x}_{ab}) \\ \dot{\hat{x}}_d &= E_{ab}\hat{x}_{ab} + A_d\hat{x}_d + B_d u + K_d(y_d - C_d\hat{x}_d) + \rho sign(y_d - C_d\hat{x}_d)\end{aligned} \tag{6.17}$$

Step 3: The original state is calculated as $x = \Gamma_1[x_{ab}', x_d']'$.

This algorithm considerably simplifies the Walcott-Zak SMO design because no constrained Lyapunov equation is involved in the design process. Of course, it is impossible to acknowledge that the complex Walcott-Zak SMO design can be simplied so much without the proof of Theorem 3.1.

Remark 6.2.1 Note that the existence conditions for the UIO [63] are exactly the same as those stated in Theorem 3.1. This is interesting in that it is generally perceived that the sliding mode observers can be designed under less restrictive conditions than say UIOs. Considering that the dynamics of UIO is much simpler than that of the Walcott-Zak observer, and the assumption of bounded unknown inputs is unnecessary, Theorem 3.1 puts the applicability and advantage of SMO in a new light. Another interesting point is that UIO and SMO seems to rely on different operating principles to achieve their robustness to matched uncertainties. The above analysis to some extent sheds light on the similarity between them. Both of them can be built only if an unknown input free subsystem exists. The Walcott-Zak SMO estimate those measurable and unknown input driving states through sliding mode method. The UIO simply neglects those measurable states.

Recalling the Utkin SMO (6.5)-(6.7), which uses the equivalent control method, we can see why, under existence of matched uncertainty, the estimation error would remain bounded. Using the transformation (6.3) for Utkin SMO, the linear uncertain system (6.2) can be transformed into following canonical form,

$$\begin{aligned} \dot{x}_1 = &\quad A_{11}x_1 + A_{12}y + B_1 u + G_1 d(x,u,t) \\ \dot{y} = &\quad A_{21}x_1 + A_{22}y + B_2 u + G_2 d(x,u,t) \end{aligned} \tag{6.18}$$

Under the sliding observer (6.5), the equivalent control signal will be

$$(L_1 sign(e_y))_{eq} = A_{21}e_1 + G_2 d(x,u,t). \tag{6.19}$$

The error dynamics of e_1 in this case will become,

$$\dot{e}_1 = (A_{11} - L_2 A_{21})e_1 + (G_1 + L_2 G_2)d(x,u,t). \tag{6.20}$$

Clearly, unless $G_1 + L_2G_2 = 0$, the error e_1 will not approach zero if $d(x,u,t)$ is nonzero. Even if the x_1 subsystem is unknown input free ($G_1 = 0$), the equivalent control signal may introduce an unknown input into it. The Utkin SMO never put $G_1 + L_2G_2 = 0$ as a constraint for designing of L_2. Generally, it is also impossible to find L_2 to achieve $G_1 + L_2G_2 = 0$. For example, if $G_1 = 0$ and G_2 is nonsingular, L_2

must be zero. However, the estimation error may be controlled within an acceptable level by a suitable choice of the gain L_2 under the assumption that $\|d(x,u,t)\|$ is small enough, as discussed in [103, 120]. In such a case however, the performance is not guaranteed because it is difficult to ensure that $\|d(x,u,t)\|$ is always small.

The Walcott-Zak observer requires each unknown input element to be compensated by each output directly, and uses the remaining outputs to design the observer for the unknown input free subsystem. If the number of unknown inputs are equal to outputs, it will require the unknown input free subsystem itself to be stable. In this case no equivalent control information is used. As such, the Walcott-Zak observer imposes a strong structural constraint on the system and limits its application. On the other hand, we show that the drawback of the current equivalent control method is to introduce an unknown input into error dynamics for those unmeasurable states. In the next section we propose a novel sliding mode observer design technique by exploiting the structural property of the subsystem upon an unknown input and using the equivalent control signal carefully. Our design significantly reduces the structural constraint.

6.3 A Novel Sliding Mode Functional Observer For Linear Uncertain systems

For simplicity of presentation, the component expanded form of the SCB theorem is given below.

Theorem 6.1 *[94] For system (6.2), there exist non-singular transformations Γ_1, Γ_2 and Γ_3, and integer $m_d \le q$, and interges $q_i (i = 1, ..., m_d)$, such that*

$$\overline{x} = \Gamma_1^{-1} x, \overline{y} = \Gamma_2^{-1} y, \overline{d} = \Gamma_3^{-1} d;$$

$$\overline{x} = [(x_a)', x_b', x_c', x_d']'$$

$$\overline{y} = [y_d', y_b']', \overline{d} = [d_d', d_c']'$$

$$y_d = [y_1, y_2, ..., y_{m_d}]', d_d = [d_1, d_2, ..., d_{m_d}]'$$

and

$$\begin{aligned}
\dot{x}_a &= A_a x_a + L_{ad} y_d + L_{ab} y_b + B_a u \\
\dot{x}_b &= A_b x_b + L_{bd} y_d + B_b u, \quad y_b = C_b x_b \\
\dot{x}_c &= A_c x_c + L_{cd} y_d + L_{cb} y_b + G_c E_{ca} x_a + G_c d_c + B_c u
\end{aligned} \tag{6.21}$$

and for each $i = 1, ..., m_d$,

$$\dot{x}_{id} = A_{q_i} x_{id} + L_{id} y_d + G_{q_i}[E_{ia} x_a + E_{ib} x_b + E_{ic} x_c + \sum_{j=1}^{m_d} E_{ij} x_{jd} + d_i] + B_{id} u \tag{6.22}$$

$$y_i = C_{q_i} x_{id}, \quad y_d = C_d x_d \tag{6.23}$$

Here the states x_a, x_b, x_c, x_d *are respectively of the dimensions* n_a, n_b, n_c *and* $n_d = \sum_{i=1}^{m_d} q_i$, *while* x_{id} *is of the dimension* q_i *for each* $i = 1, ..., m_d$. *The matrices* A_{q_i}, G_{q_i} *and* C_{q_i} *have the following forms:*

$$A_{q_i} = \begin{bmatrix} 0 & I_{q_i-1} \\ 0 & 0 \end{bmatrix}; G_{q_i} = \begin{bmatrix} 0_{(q_i-1)\times 1} \\ 1 \end{bmatrix}; C_{q_i} = \begin{bmatrix} 1 & 0_{1\times(q_i-1)} \end{bmatrix}; \tag{6.24}$$

and the last row of L_{id} *is identically zero.*

To this end, the x_d subsystem is further decomposed into m_d subsystems. Let

$$x_{id} = \begin{bmatrix} x_1^i & x_2^i & \dots & x_{q_i}^i \end{bmatrix}'.$$

The special form of A_{q_i}, G_{q_i} implies that the equations of the subsystem x_{id} in (6.22) can be rewritten as

$$\begin{aligned}
\dot{x}_1^i &= x_2^i + L_1^i y_d + B_1^i u \\
\dot{x}_2^i &= x_3^i + L_2^i y_d + B_2^i u \\
\dots & \quad \dots \\
\dot{x}_{q_i-1}^i &= x_{q_i}^i + L_{q_i-1}^i y_d + B_{q_i-1}^i u \\
\dot{x}_{q_i}^i &= E_{ia} x_a + E_{ib} x_b + E_{ic} x_c + \textstyle\sum_{j=1}^{m_d} E_{ij} x_{jd} + B_{q_i}^i u + d_i \\
y_i &= x_1^i
\end{aligned} \tag{6.25}$$

We propose the following sliding mode observer for each x_{id} subsystem based on the equivalent control method,

$$\begin{array}{ll} \dot{\hat{x}}_1^i & = \hat{x}_2^i + L_1^i y_d + B_1^i u + \lambda_1^i sign(y_i - \hat{x}_1^i) \\ \dot{\hat{x}}_2^i & = \hat{x}_3^i + L_2^i y_d + B_2^i u + \lambda_2^i sign(\overline{e_2^i}) \\ \dots & \dots \\ \dot{\hat{x}}_{q_i-1}^i & = \hat{x}_{q_i}^i + L_{q_i-1}^i y_d + B_{q_i-1}^i u + \lambda_{q_i-1}^i sign(\overline{e_{q_i-1}^i}) \\ \dot{\hat{x}}_{q_i}^i & = E_{ia}\hat{x}_a + E_{ib}\hat{x}_b + E_{ic}\hat{x}_c + \sum_{j=1}^{m_d} E_{ij}\hat{x}_{jd} + B_{q_i}^i u + \lambda_{q_i}^i sign(\overline{e_{q_i}^i}) \end{array} \tag{6.26}$$

where $\hat{x}_a, \hat{x}_b, \hat{x}_c$ are estimations for the states x_a, x_b, x_c respectively, coming from sub-observers which are given later. And

$$\overline{e_j^i} = (\lambda_{j-1}^i sign(\overline{e_{j-1}^i}))_{eq} \tag{6.27}$$

for $j = 2, ..., q_i$, and $e_1 = \hat{x}_1^i - y_i$ can be calculated directly. The equivalent control signal $(v)_{eq}$ for signal v is calculated by a low pass filtering signal v [22]. Further, we do not inject the observation error information before reaching the sliding manifold linked with this information. Moreover, we reach the manifold one by one. More precisely,

$$\overline{e_j^i} = (\lambda_{j-1}^i sign(\overline{e_{j-1}^i}))_{eq} = 0$$

before $\overline{e_{j-1}^i}$ reaches its sliding manifold.

Theorem 6.2 *For system (6.25) and observer (6.26) with any initial state and any bounded unknown input $d_i(x, u, t)$, there exists a choice of*

$$\lambda_j^i, i = 1, ..., m_d, j = 1, ..., q_i$$

such that the state estimation converges in finite time to its real value.

Proof. From (6.25) and (6.26), we obtain the following observation error dynamics of $e_j^i = x_j^i - \hat{x}_j^i, j = 1, ..., q_i$,

$$\begin{array}{ll} \dot{e}_1^i & = e_2^i - \lambda_1^i sign(e_1^i) \\ \dot{e}_2^i & = e_3^i - \lambda_2^i sign(\overline{e_2^i}) \\ \dots & \dots \\ \dot{e}_{q_i-1}^i & = e_{q_i}^i - \lambda_{q_i-1}^i sign(\overline{e_{q_i-1}^i}) \\ \dot{e}_{q_i}^i & = E_{ia}e_a + E_{ib}e_b + E_{ic}e_c + \sum_{j=1}^{m_d} E_{ij}e_{jd} + d_i - \lambda_{q_i}^i sign(\overline{e_{q_i}^i}) \end{array} \tag{6.28}$$

where $e_c = x_c - \hat{x}_c, e_a = x_a - \hat{x}_a, e_b = x_b - \hat{x}_b$. Thus, as the known and unknown inputs are bounded, the state does not go to infinity in finite time. Consequently, the observation error state is also bounded. By choosing $\lambda_1^i > |e_2^i|$, e_1^i goes to zero in finite time t_1. Moreover, after t_1, we have

$$\overline{e_2^i} = (\lambda_1^i sign(e_1))_{eq} = e_2^i.$$

Therefore, after t_1, the second error equation becomes

$$\dot{e}_2^i = e_3^i - \lambda_2^i sign(e_2^i).$$

If $\lambda_2^i > |e_3^i|$, e_2 goes to zero in finite time $t_2 > t_1$. Therefore after $t > t_2$,

$$\overline{e_3^i} = (\lambda_2^i sign(e_2))_{eq} = e_3^i,$$

$$\dot{e}_3^i = e_4^i - \lambda_3^i sign(e_3^i).$$

We run the procedure up to step q_i, thus after t_{q_i-1}, we have

$$\dot{e}_{q_i}^i = E_{ia}e_a + E_{ib}e_b + E_{ic}e_c + \sum_{j=1}^{m_d} E_{ij}e_{jd} + d_i - \lambda_{q_i}^i sign(e_{q_i}^i).$$

Let

$$\lambda_{q_i}^i > |E_{ia}e_a + E_{ib}e_b + E_{ic}e_c + \sum_{j=1}^{m_d} E_{ij}e_{jd} + d_i|, \tag{6.29}$$

e_{q_i} converges to zero in finite time $t_{q_i} > t_{q_i-1}$. ∎

Now, going back to provide the estimation of x_a, x_b and x_c in (6.26). A classical Luenberger observer is applied to subsystem x_b,

$$\dot{x}_b = A_b x_b + L_{bd} y_d + B_b u, \quad y_b = C_b x_b$$

because it is unknown input free, and (A_b, C_b) forms an observable pair. Of course, the Utkin SMO can be used also.

The subsystem x_a can be further decomposed into two subsystems,

$$\begin{aligned} \dot{x}_a^- &= A_a^- x_a^- + L_a^- \overline{y} + B_a^- u \\ \dot{x}_a^+ &= A_a^+ x_a^+ + L_a^+ \overline{y} + B_a^+ u \end{aligned}$$

A_a^- is stable, and the Luenberger observer exists for subsystem x_a^-. However, because A_a^+ is unstable, some other way has to be found in order to estimate x_a^+. Next, the possibility to estimate x_a^+ is discussed together with the estimation of x_c.

After $e_{q_i}^i$ reaches sliding mode, we have the following equivalent control signal:

$$(\lambda_{q_i}^i sign(e_{q_i}^i))_{eq} = E_{ia}^- e_a^- + E_{ia}^+ e_a^+ + E_{ib} e_b + E_{ic} e_c + \sum_{j=1}^{m_d} E_{ij} e_{jd} + d_i, i = 1, \dots, m_d,$$

and after all states of x_a^-, x_b and x_d have been estimated, it will equal to

$$(\Lambda sign(e_q))_{eq} = E_{dc} e_c + E_{da}^+ e_a^+ + d_d \tag{6.30}$$

where all m_d equivalent control signals are written together as a vector. Considering the above equivalent control signal as the output of the x_c, x_a^+ subsystem, this subsystem can be rewritten as

$$\begin{aligned} \dot{x}_{ac} &= A_{ac} x_{ac} + L_{ac} \overline{y} + B_{ac} u + G_{ac} d \\ y_{ac} &= E_{ac} x_{ac} - E_{ac} \hat{x}_{ac} + G_o d \end{aligned} \tag{6.31}$$

where

$$x_{ac} = \begin{bmatrix} x_a^+ \\ x_c \end{bmatrix}, A_{ac} = \begin{bmatrix} A_a^+ & 0 \\ G_c E_{ca}^+ & A_c \end{bmatrix}, G_{ac} = \begin{bmatrix} 0 & 0 \\ G_c & 0 \end{bmatrix},$$

$$E_{ac} = \begin{bmatrix} E_{da}^+ & E_{dc} \end{bmatrix}, G_o = \begin{bmatrix} 0 & I \end{bmatrix},$$

An interesting fact is that (A_{ac}, E_{ac}) is detectable if (A, C) is detectable. Unfortunately, the system (6.31) has unknown inputs, and there is no UIO for this system according to the following lemma.

Lemma 6.3.1 *[50] A UIO for system (6.31) exists only if*

$$rank \begin{bmatrix} G_o & E_{ac} G_{ac} \\ 0 & G_o \end{bmatrix} = rank G_o + rank \begin{bmatrix} G_{ac} \\ G_o \end{bmatrix} \tag{6.32}$$

It is easy to find that condition (6.32) will never be satisfied due to the particular form of G_s, G_o. In this case, we may use H_2 or H_∞ optimal observer design techniques proposed in [95] to make the estimation error as small as possible.

On the other hand, [103] has shown that exact or approximate estimation using sliding mode observer is impossible if all the measurements of a system are disturbed by the unknown inputs. Assuming the disturbance on the measurement is bounded by some constant v_0, the estimation error will not be bigger than v_0. Finally, although e_a^+ and e_c will not be zero, as long as it is bounded, its value does not affect the convergence of the sliding mode observer (6.26) with a gain satisfying (6.29).

At this point, based on the above analysis, we shall summarize our new sliding mode functional observer design procedure for a linear uncertain system as follows.

SMFO Design Algorithm

Step 1: Transform system (6.2) it into its SCB form (6.21)-(6.22) by non-singular transformations $\Gamma_1, \Gamma_2, \Gamma_3$.

Step 2: Estimate x_d using the sliding mode observer (6.26), and estimate x_a^-, x_b using a regular Luenberger observer. The measurement variable for the transformed system y_d, y_b is derived from the original output by $[y_d, y_b]' = \Gamma_2^{-1} y$, and the known input distribution matrix is transformed by $\Gamma_1^{-1} B$.

Step 3: If $\{A, G, C\}$ is left invertible (or equivalently $n_c = 0$) and is minimum phase (or $n_a^+ = 0$), the original state can be derived by

$$x = \Gamma_1 [x_a^- \quad x_b \quad x_d]^T.$$

Step 4: If $n_c \neq 0$ or $n_a^+ \neq 0$, any linear function of the states, $T_{k \times n} x$ can be estimated, where T must satisfy the following condition:

$$T\Gamma_1 = [T_a^- \quad 0 \quad T_b \quad 0 \quad T_d]_{k \times n} = \hat{T}_{k \times n}$$

where T_a^-, T_b, T_d are any matrices of dimension $k \times n_a^-, k \times n_b$ and $k \times n_d$ respectively. Obviously, the maximum rank of T is $n_a^- + n_b + n_d$.

Remark 6.3.1 The Walcott-Zak SMO requires $rank(CG) = rank(G)$, which immediately implies that $n_c = 0$ and $q_i = 1, i = 1, ..., m_d$, namely, the number of infinite zeros of order i is one. The restriction on the system $\{A, G, C\}$ infinite zero structure is removed in our algorithm. Unfortunately, the requirement of $n_c = 0$ is still necessary for estimating all states, and it implies $rank(C) \geq rank(G)$.

Remark 6.3.2 In Chapter 2, the linear state function Tx is estimated using the unknown input functional observer (UIFO), and the maximum rank of T is $n_a^- + n_b$. Using our new SMFO design, the maximum rank of T has been increased significantly to $n_a^- + n_b + n_d$.

In Chapter 3, we discussed the problem of estimating unknown inputs based on the UIFO. Checking the equivalent control signal (6.30), we have the following corollary of unknown input estimation.

Corollary 6.3.1 *If the system $\{A, G, C\}$ is left invertible (i.e. $n_c = 0$) and all unstable transmission zeros are unobservable modes (i.e. in SCB form, $E_{da}^+ = 0$), all unknown inputs can be estimated exactly using the proposed SMFO. If system $\{A, G, C\}$ is not left invertible (i.e. $n_c > 0$) but all eigenvalues of corresponding n_c subsystem are unobservable modes (i.e. $E_{dc} = 0$), and all unstable transmission zeros are unobservable modes (i.e. $E_{da}^+ = 0$), at least m_d unknown inputs can be estimated exactly using the proposed SMFO.*

Remark 6.3.3 Compared with the input estimator discussed in [17] and Chapter 3, our new SMFO has better capability at estimating the unknown inputs as well.

6.4 Sliding Mode Functional Observer For Nonlinear Uncertain systems

6.4.1 Introduction to Nonlinear Observer

Various methods for designing observers for nonlinear systems have been reported in the literature. A common approach to the problem of observer design for nonlinear systems has been to extend the linear Luenberger observer or Kalman Filter design approach to nonlinear systems. In this respect, one design approach deals with nonlinear systems for which observers with linearizable error dynamics can be designed (see e.g. [5], [66], [67], [93]). In several works (see eg. [7], [9],[33], [90]) systems which are composed of a linear unforced part and a nonlinear state dependent controlled part are considered. In such cases, the nonlinearity is usually accounted for via its Lipschitz constant in the observer design strategy.

In [65], under some uniform detectability conditions, the authors have proposed a constant gain observer for a class of nonlinear systems without inputs. This condition is usually not easy to verify and as a result, the gain of the observer can not be explicitly calculated. In [34], an observer is given for a class of nonlinear systems which are not necessarily control affine. However, the gain of the proposed observer is not easily computable. In recent works [10, 52], observers based on some ideas from the high gain approach, whose gain could easily be designed, were proposed for multivariable nonlinear systems.

A different class of observer design methodology for nonlinear systems is that based on sliding mode principle. [22] extended sliding mode observer design for linear systems [112] to nonlinear systems of the form

$$\begin{aligned} \dot{x} &= f(x) + B(x)u \\ y &= h(x) \end{aligned} \tag{6.33}$$

This extension was also applied to nonlinear systems in triangular input form in [1], output and output derivative injection form in [8]. In papers [103] and [120], a framework similar to a Luenberger observer were used by appending a switching function with constant or time-varying gains as part of feedback corrections, where nonzero estimation error were introduced inevitably.

On the other hand, observer design for nonlinear uncertain systems has been studied very rarely. Nonlinear unknown input observers (NUIO) were subjects of few studies with a main motivation of applying such an observer to robust model based fault diagnosis problems. In this realm, [100] extended the method of linear unknown input observer to a class of nonlinear systems, where if existence conditions are satisfied, a nonlinear transformation is used to produce a reduced-order model which is unaffected by unknown inputs. Then a nonlinear observer is constructed for this transformed model. Similarly, [135] considered the NUIO design for nonlinear systems which can be transformed into output injection form.

In this section, a sliding mode functional observer, which is insensitive to unknown inputs, is developed. The previous work on sliding mode nonlinear observer does not consider how to reduce the effect of the unknown input, because it is believed that the estimation error will be small if the magnitude of unknown input signal is small. This section also sheds extra light on the NUIO design problem.

6.4.2 A Nonlinear Coordinate Transformation

The multivariable nonlinear systems we consider are described in state space form by equations of the following form

$$\begin{aligned} \dot{x} &= f(x) + B(x,u) + \textstyle\sum_{i=1}^{m} g_i(x) d_i(x,u,t) \\ y_1 &= h_1(x) \\ \dots & \quad \dots \\ y_p &= h_p(x) \end{aligned} \tag{6.34}$$

in which $x \in M$, a C^∞ connected manifold of dimension n. $f(x), B(x,u), G(x) = [g_1(x), ..., g_m(x)]$ are smooth vector fields on M, and $h_i(x). i = 1, ..., p$ are smooth functions from M to R. The term $d(x,u,t)$ represents the uncertainty due to modeling error or component/actuator faults, namely the unknown input. In what follows, local coordinates are generally used. When global properties are considered, notions are simplified by assuming that M accepts a global coordinate system.

Assumption 1. We assume that $p \geq m$, and the first m outputs have relative degree $\{q_1, ..., q_m\}$ corresponding to $G(x)$ at each point $x_0 \in M$. This means

$$L_{g_j} L_f^k h_i(x) = 0$$

for all $j = 1, ..., m$, for all $k < q_i - 1$, for all $i = 1, ..., m$, and for all x in M. Further, the $m \times m$ matrix

$$A(x) = \begin{pmatrix} L_{g_1} L_f^{q_1-1} h_1(x) & \dots & L_{g_m} L_f^{q_1-1} h_1(x) \\ L_{g_1} L_f^{q_2-1} h_2(x) & \dots & L_{g_m} L_f^{q_2-1} h_2(x) \\ \dots & \dots & \dots \\ L_{g_1} L_f^{q_m-1} h_m(x) & \dots & L_{g_m} L_f^{q_m-1} h_m(x) \end{pmatrix}$$

is nonsingular at each point $x_0 \in M$.

Assumption 2. The distribution spanned by the vector fields $g_1(x), ..., g_m(x)$ is involutive.

Lemma 6.4.1 *Given the system (6.34), if assumptions 1-2 are valid, then*

$$q_1 + ... + q_m \leq n$$

Set, for $i = 1, ..., m$,

$$\begin{aligned} \phi_1^i =& \ h_i(x) \\ \phi_2^i =& \ L_f h_i(x) \\ ... & \ ... \\ \phi_{q_i}^i =& \ L_f^{q_i-1} h_i(x) \end{aligned} \tag{6.35}$$

if $q_1 + ... + q_m = n$, the mapping

$$\Phi(x) = col(\phi_1^1(x), ..., \phi_{q_1}^1(x), ..., \phi_1^m(x), ..., \phi_{q_m}^m(x)) \tag{6.36}$$

has a Jacobian matrix which is nonlinear at each $x_0 \in M$ and therefore qualifies as a local coordinate transformation in M.

Set

$$x_d^i = \begin{pmatrix} x_1^i \\ x_2^i \\ ... \\ x_{q_i}^i \end{pmatrix} = \begin{pmatrix} \phi_1^i(x) \\ \phi_2^i(x) \\ ... \\ \phi_{q_i}^i(x) \end{pmatrix} \tag{6.37}$$

for $i = 1, ..., m$, and $x_d = (x_d^1, ..., x_d^m)$, the equations under new coordinates can be written as

$$\begin{aligned} \dot{x}_1^i &= x_2^i + b_1^i(x_d, u) \\ ... & \quad ... \\ \dot{x}_{q_i-1}^i &= x_{q_i}^i + b_{q_i-1}^i(x_d, u) \\ \dot{x}_{q_i}^i &= a_i(x_d) + b_{q_i}^i(x_d, u) + \textstyle\sum_{j=1}^m c_{ij}(x_d) d_j \\ y_i &= x_1^i \end{aligned} \tag{6.38}$$

for $i = 1, ..., m$, where

$$a_i(x_d) = L_f^{q_i} h_i(\Phi^{-1}(x_d)), c_{ij} = L_{g_j} L_f^{q_i-1} h_i(\Phi^{-1}(x_d))$$

and

$$b_k^i(x_d, u) = \frac{\partial(L_f^{k-2} h_i)}{\partial x} B(\Phi^{-1}(x_d), u)$$

The above lemma is a special case of Proposition 5.1.2 in [56]. Under the new coordinates, the original system is decomposed into m subsystems with triangular form, where disturbance terms appear only in their last equations. Next, we shall discuss the observer design for the system of form (6.38).

6.4.3 Observability of Subsystems

If a linear system is observable, for any control input the initial state can be reconstructed. This property is in general not true for nonlinear systems, and the observability of nonlinear systems is associated with inputs. The observability of nonlinear subsystem (6.38) is great interest to us.

Generally, unknown inputs can make a nonlinear system to become unobservable, just the same as known inputs. The input to make a nonlinear system unobservable is called "bad input" or "not universal input". For known input signals, one can avoid applying those "bad inputs". However, the unknown inputs which may be the result of a fault or certain other disturbances are beyond our control. Therefore, observability for all unknown inputs is in general necessary in order to design a nonlinear unknown input observer, unless it can be guaranteed *a priori* that the unknown inputs do not belong to the class of bad inputs. Based on the work in [34], we have following lemma that shows the conditions such that the observability of x_d^i subsystem is independent of unknown inputs.

Lemma 6.4.2 *Let* $\overline{x_d^i} = \{x_d^1, ..., x_d^{i-1}, x_d^{i+1}, ..., x_d^m\}$. *Assume each* x_d^i *subsystem in (6.38) has its input term in the following form*

$$\begin{array}{ll} b_1^i(x_d, u) & = b_1^i(x_1^i, x_2^i, \overline{x_d^i}, u) \\ b_2^i(x_d, u) & = b_2^i(x_1^i, x_2^i, x_3^i, \overline{x_d^i}, u) \\ \cdots & \cdots \\ b_{q_i-1}^i(x_d, u) & = b_{q_i-1}^i(x_d, u) \\ b_{q_i}^i(x_d, u) & = b_{q_i}^i(x_d, u) \end{array} \tag{6.39}$$

and the functions

$$1 + \frac{\partial b_j^i}{\partial x_{j+1}^i} \neq 0, j = 1, ..., q_i - 1$$

and state $\overline{x_d^i}$ is considered as inputs for x_d^i subsystem. Then x_d^i subsystem is uniformly observable for all inputs $u, d, \overline{x_d^i}$.

6.4.4 Robust sliding mode observer design

Theorem 6.3 *If each x_d^i subsystem in (6.38) has its input term in the following form*

$$\begin{array}{ll} b_1^i(x_d, u) & = b_1^i(y, u) \\ b_2^i(x_d, u) & = b_2^i(x_2^i, y, u) \\ \dots & \dots \\ b_{q_i-1}^i(x_d, u) & = b_{q_i-1}^i(x_2^i, \dots, x_{q_i-1}^i, y, u) \\ b_{q_i}^i(x_d, u) & = b_{q_i}^i(x_d, u) \end{array} \tag{6.40}$$

there exists a choice of $\lambda_j^i, i = 1, \dots, m, j = 1, \dots, q_i$ such that following sliding mode observer can be built to estimate all states,

$$\begin{array}{ll} \dot{\hat{x}}_1^i & = \hat{x}_2^i + b_1^i(y, u) + \lambda_1^i sign(y_i - \hat{x}_1^i) \\ \dot{\hat{x}}_2^i & = \hat{x}_3^i + b_2^i(\hat{x}_2^i, y, u) + \lambda_2^i sign(\overline{e_2^i}) \\ \dots & \dots \\ \dot{\hat{x}}_{q_i-1}^i & = \hat{x}_{q_i}^i + b_{q_i-1}^i(\hat{x}_2^i, \dots, \hat{x}_{q_i-1}^i, y, u) + \lambda_{q_i-1}^i sign(\overline{e_{q_i-1}^i}) \\ \dot{\hat{x}}_{q_i}^i & = a_i(\hat{x}_d) + b_{q_i}^i(\hat{x}_d, u) + \lambda_{q_i}^i sign(\overline{e_{q_i}^i}) \end{array} \tag{6.41}$$

where $\overline{e_j^i}, j = 1, \dots, q_i$ are given by

$$\overline{e_j^i} = (\lambda_{j-1}^i sign(\overline{e_{j-1}^i}))_{eq} \tag{6.42}$$

for $j = 2, \dots, q_i$, and $\overline{e_1^i} = \hat{x}_1^i - y_i$ can be got directly.

Proof. From (6.38), (6.40) and (6.41), we obtain the following observation error dynamics of $e_j^i = x_j^i - \hat{x}_j^i, j = 1, \dots, q_i$,

$$\begin{array}{ll} \dot{e}_1^i = & e_2^i - \lambda_1^i sign(e_1^i) \\ \dot{e}_2^i = & e_3^i + b_2^i(x_2^i, y, u) - b_2^i(\hat{x}_2^i, y, u) - \lambda_2^i sign(\overline{e_2^i}) \\ \dots & \dots \\ \dot{e}_{q_i-1}^i = & e_{q_i}^i + b_{q_i-1}^i(x_2^i, \dots, x_{q_i-1}^i, y, u) - b_{q_i-1}^i(\hat{x}_2^i, \dots, \hat{x}_{q_i-1}^i, y, u) \\ & -\lambda_{q_i-1}^i sign(\overline{e_{q_i-1}^i}) \\ \dot{e}_{q_i}^i = & a_i(x_d) + b_{q_i}^i(x_d, u) + \sum_{j=1}^m c_{ij}(x_d) d_j - a_i(\hat{x}_d, \hat{x}_o) \\ & -b_{q_i}^i(\hat{x}_d, \hat{x}_o, u) - \lambda_{q_i}^i sign(\overline{e_{q_i}^i}) \end{array} \tag{6.43}$$

As the known and unknown inputs are bounded, the state does not go to infinity in finite time. Consequently, the observation error is also bounded. By choosing $\lambda_1^i > |e_2^i|$, e_1^i goes to zero in finite time t_1. Moreover, after t_1, we have

$$\overline{e_2^i} = (\lambda_1^i sign(e_1))_{eq} = e_2^i$$

Therefore, after t_1, the second error equation becomes

$$\dot{e}_2^i = e_3^i + b_2^i(x_2^i, y, u) - b_2^i(\hat{x}_2^i, y, u) - \lambda_2^i sign(e_2^i)$$

If $\lambda_2^i > |e_3^i + b_2^i(x_2^i, y, u) - b_2^i(\hat{x}_2^i, y, u)|$, e_2^i goes to zero in finite time $t_2 > t_1$. Therefore after $t > t_2$,

$$\overline{e_3^i} = (\lambda_2^i sign(e_2))_{eq} = e_3^i$$

$$\dot{e}_3^i = e_4^i + b_3^i(x_2^i, x_3^i, y, u) - b_3^i(\hat{x}_2^i, \hat{x}_3^i, y, u) - \lambda_3^i sign(e_3^i)$$

We run the procedure up to step q_i, thus after t_{q_i-1}, we have

$$\dot{e}_{q_i}^i = a_i(x_d) + b_{q_i}^i(x_d, u) + \sum_{j=1}^{m} c_{ij}(x_d)d_j - a_i(\hat{x}_d) - b_{q_i}^i(\hat{x}_d, u) - \lambda_{q_i}^i sign(e_{q_i}^i)$$

Let $\lambda_{q_i}^i > |a_i(x_d) + b_{q_i}^i(x_d, u) + \sum_{j=1}^{m} c_{ij}(x_d)d_j - a_i(\hat{x}_d) - b_{q_i}^i(\hat{x}_d, u)|$, e_{q_i} converges to zero in finite time $t_{q_i} > t_{q_i-1}$. ∎

Remark 6.4.1 The system form (6.40) is more general than the output injection or triangular input form, but more conservative than the form (6.39), which assures uniform observability.

Remark 6.4.2 It is not always necessary to design the proposed sliding mode observer based on the transformed system model as it is not always easy to find the suitable transformation. For the nonlinear systems with linear output equation, we suggest to check the possibility of designing based on original system equation. The basic rule is to make sure unknown inputs will not appear in the equivalent control signal. Our examples illustrate this intuitive design procedure.

Remark 6.4.3 It should be noted that if $q_i = 1, i = 1, ..., m$, above theorem does not put any special constraint on the input term. $q_i = 1, i = 1, ..., m$ means all states of subsystem x_d are measurable. An observer for such a system may be unnecessary for controller synthesis, but will be useful in fault detection and isolation (FDI) applications.

Remark 6.4.4 The m observers for $x_d^i (i = 1, ..., m)$ subsystems can run in parallel, although in each observer, e_j^i converges to zero if all the e_k^i with $k < j$ have already converged to zero. If we allow states x_d^i to be estimated after x_d^k with $k < i$ have been estimated correctly, then the input term can be following a more general form,

$$\begin{aligned}
b_1^i(x_d, u) &= b_1^i(y_d^i, x_d^1, ..., x_d^{i-1}, u) \\
b_2^i(x_d, u) &= b_2^i(x_2^i, y_d^i, x_d^1, ..., x_d^{i-1}, u) \\
\dots &\quad \dots \\
b_{q_i-1}^i(x_d, u) &= b_{q_i-1}^i(x_2^i, ..., x_{q_i-1}^i, y_d^i, x_d^1, ..., x_d^{i-1}, u) \\
b_{q_i}^i(x_d, u) &= b_{q_i}^i(x_d, u)
\end{aligned}$$

where $y_d^i = [y_i, y_{i+1}, ..., y_m]$.

6.4.5 Robust sliding mode functional observer design

In this section, we consider the nonlinear uncertain systems with more general structural properties.

Lemma 6.4.3 *Given the system (6.34), if assumptions 1-2 are valid and*

$$q = (q_1 + ... + q_m) < n$$

it is always possible to find $n - q$ functions $\phi_{q+1}(x), ..., \phi_n(x)$ such that

$$Lg_j \phi_i(x) = 0 \tag{6.44}$$

for all $i = q + 1, ..., n, j = 1, ..., m$, and all x in M. Further the mapping

$$\Phi_o(x) = col(\Phi(x), \phi_{q+1}(x), ..., \phi_n(x))$$

has a Jacobian matrix which is nonlinear at each $x_0 \in M$ and therefore qualifies as a local coordinate transformation in M, where $\Phi(x)$ is given by (6.36).

Set $x_d^i, i = 1, ..., m$ as (6.37) and

$$x_o = \begin{pmatrix} x_o^1 \\ x_o^2 \\ \cdots \\ x_o^{n-q} \end{pmatrix} = \begin{pmatrix} \phi_{q+1}(x) \\ \phi_{q+2}(x) \\ \cdots \\ \phi_n(x) \end{pmatrix}$$

the equations under new coordinates can be written as

$$\begin{array}{ll} \dot{x}_1^i & = x_2^i + b_1^i(x_d, x_o, u) \\ \cdots & \cdots \\ \dot{x}_{q_i-1}^i & = x_{q_i}^i + b_{q_i-1}^i(x_d, x_o, u) \\ \dot{x}_{q_i}^i & = a_i(x_d, x_o) + b_{q_i}^i(x_d, x_o, u) + \sum_{j=1}^m c_{ij}(x_d, x_o) d_j \\ y_i & = x_1^i \end{array} \tag{6.45}$$

for $i = 1, ..., m$, and

$$\begin{array}{ll} \dot{x}_o & = q(x_d, x_o) + p(x_d, x_o, u) \\ y_{m+1} & = h_{m+1}(x_d, x_o) \\ \cdots & \cdots \\ y_p & = h_p(x_d, x_o) \end{array} \tag{6.46}$$

where

$$a_i(x_d, x_o) = L_f^{q_i} h_i(\Phi^{-1}(x_d, x_o)), c_{ij} = L_{g_j} L_f^{q_i-1} h_i(\Phi^{-1}(x_d, x_o))$$

and

$$b_k^i(x_d, x_o, u) = \frac{\partial(L_f^{k-2} h_i)}{\partial x} B(\Phi^{-1}(x_d, x_o), u)$$

The above lemma is a trivial extension of Proposition 5.1.2 in [56]. The nonlinear transformation $\Phi_o(x)$ decomposes the system (6.34) into two parts. Of the two parts, x_d subsystem is affected by unknown inputs, while x_o subsystem is unknown input free. Of course, this is not a complete decomposition because it is relies on Assumption 1 and 2. The development of a complete transformation for general nonlinear system will be studied in the future.

The observability and sliding mode observer design for subsystems (6.45) are similar to subsystems (6.38) discussed in section 2. Considering x_o as input to subsystems (6.45), the x_d^i subsystem is uniformly observable for all inputs $u, d, \overline{x_d^i}$ and x_o if its input term has following form

$$\begin{aligned}
b_1^i(x_d, x_o, u) &= b_1^i(x_1^i, x_2^i, \overline{x_d^i}, x_o, u) \\
b_2^i(x_d, x_o, u) &= b_2^i(x_1^i, x_2^i, x_3^i, \overline{x_d^i}, x_o, u) \\
\dots &\quad \dots \\
b_{q_i-1}^i(x_d, x_o, u) &= b_{q_i-1}^i(x_d, x_o, u) \\
b_{q_i}^i(x_d, x_o, u) &= b_{q_i}^i(x_d, x_o, u)
\end{aligned} \tag{6.47}$$

and the functions

$$1 + \frac{\partial b_j^i}{\partial x_{j+1}^i} \neq 0, j = 1, \dots, q_i - 1$$

An open problem which does not effect the development here is whether x_o subsystem is observable if and only if the original system (6.34) is observable.

We can build the robust sliding mode observer for subsystems (6.38) if its input term has following form,

$$\begin{aligned}
b_1^i(x_d, x_o, u) &= b_1^i(y, u) \\
b_2^i(x_d, x_o, u) &= b_2^i(x_2^i, y, u) \\
\dots &\quad \dots \\
b_{q_i-1}^i(x_d, x_o, u) &= b_{q_i-1}^i(x_2^i, \dots, x_{q_i-1}^i, y, u) \\
b_{q_i}^i(x_d, x_o, u) &= b_{q_i}^i(x_d, x_o, u)
\end{aligned} \tag{6.48}$$

The observer design is the similar to (6.41), where the last equation is replaced by

$$\dot{\hat{x}}_{q_i}^i = a_i(\hat{x}_d, \hat{x}_o) + b_{q_i}^i(\hat{x}_d, \hat{x}_o, u) + \lambda_{q_i}^i sign(\overline{e_{q_i}^i}) \tag{6.49}$$

and $\hat{x}_o$ is the estimation of x_o. The proof for the validity of the observer design is similar to that for Theorem 6.3 and omitted here.

The x_o subsystem in (6.46) is free of unknown inputs. Therefore, an observer can be designed using any of existing techniques, including the standard sliding mode observer. Although, no uniform design methodology for general nonlinear systems exists.

In observer equation (6.49), $\hat{x}_o$ is the estimate of x_o. If x_o subsystem is unobservable, or too complex to build an observer for it, $\hat{x}_o$ will not approximate real value of x_o. However, as long as both $\hat{x}_o$ and x_o are bounded, observer (6.41) will always converge as long as gain $\lambda^i_{q_i}$ is big enough. For simplicity, we can let $\hat{x}_o = 0$. Therefore, a sliding mode functional observer for a general class of nonlinear uncertain systems can be designed. The reason for this desirable property is that x_o is only confined to the last equation.

The nonlinear unknown input observer (NUIO) design method reported in [100], transforms the system (6.34) by $z = T(x)$, where

$$\frac{\partial T(x)}{\partial x}G(x) = 0 \tag{6.50}$$

such that z is actually an unknown input free subsystem. Then an observer for the reduced order z subsystem is built. Equation (6.50) is the same as condition (6.44) of Lemma 1. According to Frobenius theorem [56], (6.50) is solvable if and only if $G(x)$ is involutive. Therefore, although involutivity of $G(x)$ is a very strong assumption, we cannot avoid it if totally unknown input decoupling is required.

In NUIO design, the original state is obtained through $x = \Phi(z, y^*)$. where y^* is obtained from a nonlinear transformation $y^* = S(y)$. It stated that the inverse function $x = \Phi(z, y^*)$ exists, if and only if

$$rank \begin{pmatrix} \frac{\partial T(x)}{\partial x} \\ \frac{\partial S(y)}{\partial x} \end{pmatrix} = n; lim_{\|x\| \to \infty} \|(T(x) \quad S(y))^T\| = \infty \tag{6.51}$$

However, the question as to under what conditions there exists $S(y)$ that satisfy conditions (6.51) is still unresolved. In [100] it is stated that $p > m$ is a necessary condition. Here we show that this is not true at least in theory by analyzing the existence of NUIO using the transformation given in Lemma 1. Note that the relative degree $q_i = 1 (i = 1, ..., m)$ means all states of x_d subsystem are measurable. Therefore, the following conditions are sufficient for the existence of a NUIO:

1. $p_i = 1, i = 1, ..., m$;

2. x_o subsystem are detectable,

Above conditions may be satisfied even if $p = m$. On the other hand, if any relative degree $q_k > 1 (1 \leq k \leq m)$, it is expected that no NUIO exists. Because under this case, the states $x_2^k, ..., x_{q_k}^k$ will not be able to derive through a nonlinear transform on y and z, the state of unknown input free subsystem. Therefore, the proposed SMO works under much less conservative conditions than NUIO.

Remark 6.4.5 If x_o subsystem is independent of x_d and can be estimated correctly by certain observer, then it is not a problem for x_o to appear in all input terms of x_d subsystem (6.45).

Remark 6.4.6 Although sliding mode concept provides a nice framework for observer design of a general class of nonlinear uncertain systems, due to the inherent property of sliding mode, there exist two basic drawbacks for practical applications. First, although the bound of unknown input is known, the estimation error bound is not known *a priori*. This makes the selection of the gain difficult. If the gain is too large, observer will exhibit excessive chattering before reaching sliding mode. If the gain is too small, observer may never be able to reach sliding mode and converge to real state value. Secondly, it is difficult to choose a suitable time to use the equivalent control signal. The equivalent control signal should be used only if its corresponding estimation error is near zero, or in sliding mode. However, except for those states which are measured, we do not know if a state estimation is in sliding mode or not. If the equivalent control is used too early, peaking phenomena is unavoidable. If the equivalent control is used too late, a correct estimation in time cannot be achieved which is unacceptable for high quality control.

6.5 Illustrative Examples

Example 6.5.1 The system under consideration is a one-link manipulator with revolute joints actuated by a DC motor, as shown in Figure 6.2. The elasticity of the joint can be well-modelled by a linear torsional spring [105]. The elastic coupling of the motor shaft to the link introduces an additional degree of freedom. The states of this system are the motor position and velocity, and the link position and velocity.

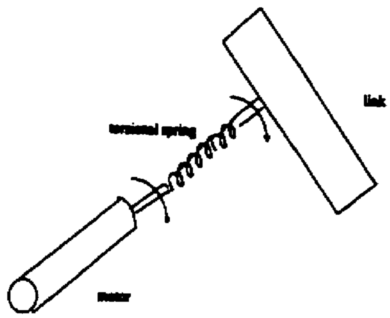

Figure 6.2: Framework of a Flexible Link Robot

The corresponding state-space model is

$$\begin{aligned}
\dot{\theta}_m &= \omega_m \\
\dot{\omega}_m &= \tfrac{k}{J_m}(\theta_l - \theta_m) - \tfrac{B}{J_m}\omega_m + \tfrac{K_r}{J_m}u \\
\dot{\theta}_l &= \omega_l \\
\dot{\omega}_l &= -\tfrac{k}{J_l}(\theta - \theta_m) - \tfrac{mgh}{J_l} sin(\theta_l)
\end{aligned}$$

with J_m being the inertia of the motor; J_l being the inertia of the link; θ_m being the angular rotation of the motor; θ_l begin the angular position of the link; ω_m being the angular velocity of the motor; and ω_l being the angular velocity of the link.

Thus, the system dynamics are nonlinear and are of the form

$$\dot{x} = Ax + \Phi(x) + Bu$$

with $x = [\theta_m, \omega_m, \theta_l, \omega_l]^T$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 0 & -19.5 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix}, \Phi(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -3.33 sin(x_3) \end{bmatrix}.$$

The above parameters for the system are typical and have been taken from [105].

A nonlinear I/O linearizing control law for this system is presented in [105]. The control law guarantees closed-loop stability and tracking of any desired trajectory by the robot link. However, this control law requires measurement of all the states, and the measurement of the angular position (x_3) and the velocity (x_4) of the link is difficult. [91] proposed an observer by considering this system as a Lipschitz nonlinear system, where motor position and velocity are measured. Obviously, this system can be considered as a linear system subject to a bounded unknown input, namely the nonlinear term $-3.33sin(x_3)$. Further, it is easy to verify that even if only the motor position is measured, or

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix},$$

a sliding mode observer can be designed using our algorithm in Section 6.3. Our observer design will save money because it does not require measurement of motor velocity.

It is noted that no Walcott-Zak SMO or linear UIO exists for the above robotic system, because $rank(CG) = 0 < rank(G) = 1$. The transformation

$$\Gamma_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0.0257 & 0.0206 & 0 \\ 0 & 1 & 0.0257 & 0.0206 \end{bmatrix}; \Gamma_2 = 1; \Gamma_3 = 0.0206$$

will transform the system to

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -24.375 & -68.1 & -1.25 \end{bmatrix} x + \begin{bmatrix} 0 \\ 21.6 \\ -27.0 \\ 1016.0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} d$$

and C is the same as before. The observer is

$$\dot{\hat{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -24.375 & -68.1 & -1.25 \end{bmatrix} \hat{x} + \begin{bmatrix} 0 \\ 21.6 \\ -27.0 \\ 1016.0 \end{bmatrix} u + \begin{bmatrix} \lambda_1 sign(y_1 - \hat{x}_1) \\ \lambda_2 sign(\overline{e_2}) \\ \lambda_3 sign(\overline{e_3}) \\ \lambda_4 sign(\overline{e_4}) \end{bmatrix}$$

The design based on SCB transform is systematic and works well no matter how complicate the system structure is and how large its dimension is. Actually, the design can be done easily without using SCB transformation for this simple system. First, build the observer for x_1 as

$$\dot{\hat{x}}_1 = \hat{x}_2 + \lambda_1 sign(y_1 - \hat{x}_1).$$

After e_1 approximates zero, we know $e_2 = (\lambda_1 sign(e_1))_{eq}$. Next, build the observer for x_2 as

$$\dot{\hat{x}}_2 = -48.6y_1 - 1.25\hat{x}_2 + 48.6\hat{x}_3 + 21.6u + \lambda_2 sign(e_2).$$

After e_2 approximates zero, we know that

$$e_3 = (\lambda_2 sign(e_2))_{eq}/48.6$$

and the observer for x_3 is built as

$$\dot{\hat{x}}_3 = \hat{x}_4 + \lambda_3 sign(e_3)$$

Finally, $e_4 = (\lambda_3 sign(e_3))_{eq}$ and the observer for x_4 is

$$\dot{\hat{x}}_4 = -19.5y_1 + 19.5\hat{x}_4 + \lambda_4 sign(e_4)$$

e_4 will go to zero even with the nonlinear term $-3.33sin(x_3)$. Actually, it does not matter if the nonlinear term is more complicated, Lipschitz or not.

In simulation for above design using the original system equations, we use the saturation function to replace the sign function, and the upper limit is set to be 0.02. The switching gains are $\lambda_1 = 800, \lambda_2 = 120, \lambda_2 = 120$ and $\lambda_4 = 1400$. The equivalent control signal is applied to the second, third and fourth state estimation at $t_1 = 0.2sec, t_2 = 0.8sec$ and $t_3 = 1.8sec$ respectively. The initial states are assumed to be

$$x_0 = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}, \hat{x}_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}.$$

Figure 6.3 shows that all estimated states converge rapidly to the correct values. The SMO is a practical solution to an important robotics application.

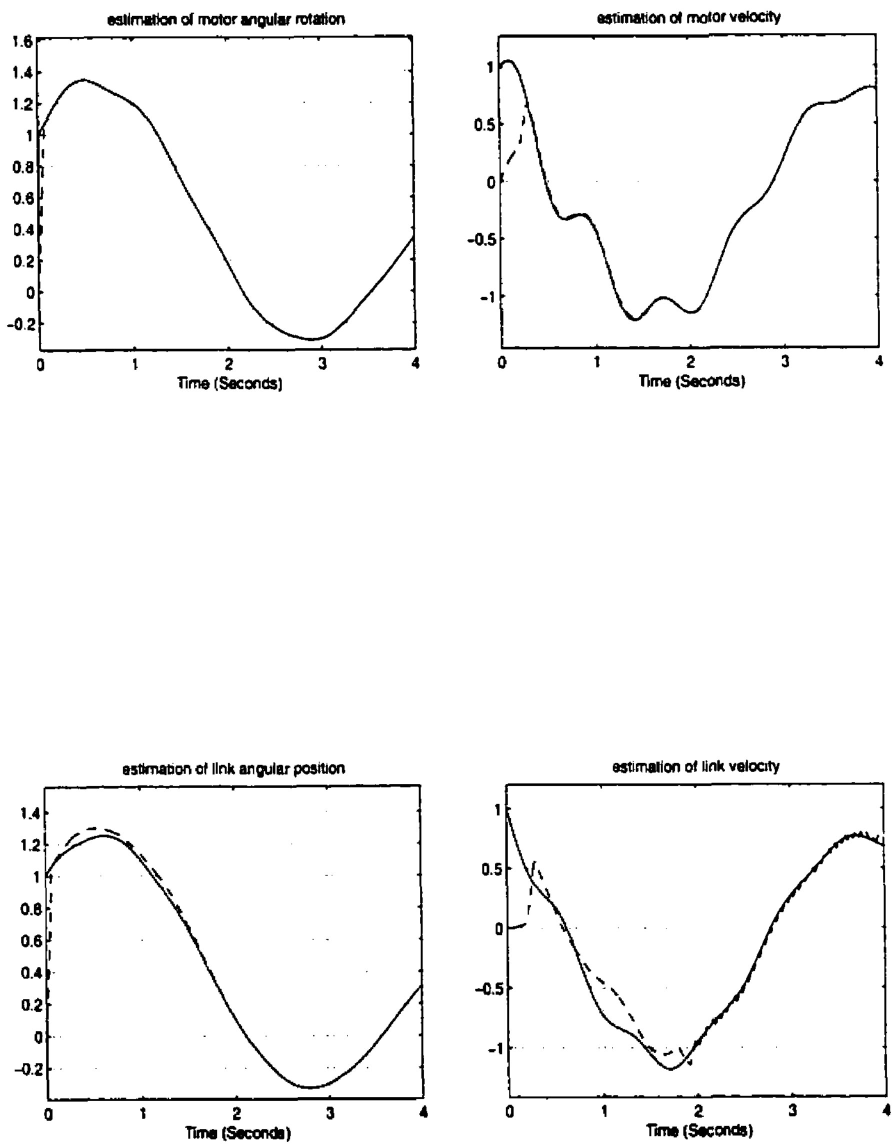

Figure 6.3: Results of state estimation for a flexible link robot using SMO

Example 6.5.2 A three-phase current motor model [6] is described by the following nonlinear equations:

$$\dot{x} = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} + B(x)u + g(x)d$$

$$= \begin{bmatrix} x_2 \\ -A_1x_2 - A_2x_3sinx_1 - A_3sin2x_1 \\ -D_1x_3 + D_2cosx_1 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

where $x = [x_1, x_2, x_3]^T$, x_1, x_2 and x_3 denote the model states rotor angle, speed deviation and field flux linkage, respectively. The known inputs are u_1 (nominal mechanical power input) and u_2 (field voltage), the and unknown inputs are

$$d_1 = \Delta A_1x_2 + \Delta A_2x_3sinx_1 + \Delta A_3sin2x_1$$

which represent uncertainties of parameters A_1, A_2 and A_3, and

$$d_2 = \Delta D_1x_3 + \Delta D_2cosx_1$$

which represents uncertainties of parameters D_1, D_2. All changes are induced by the operating temperature, or component incipient faults. d_1 or d_2 may be small enough to be neglected in different operational conditions. To illustrate the robust SMFO design, we consider several cases.

Case 1: Both d_1 and d_2 are nonzero

In this case, it is noted that x_1, x_2 is a sub-system in the triangular form of (6.38) if x_1 is measured, namely if $y_1 = x_1$, which will make x_2 observable. If we have $y_2 = x_3$, the all states will be estimated by following observer,

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + \lambda_1 sign(y_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 &= -A_1\hat{x}_2 - A_2y_2siny_1 - A_3sin2y_1 + \lambda_2 sign((\lambda_1 sign(y_1 - \hat{x}_1)_{eq}) \\ \dot{\hat{x}}_3 &= -D_1\hat{x}_3 + D_2cosy_1 + \lambda_1 sign(y_2 - \hat{x}_3) \end{aligned}$$

Case 2: d_1 is nonzero, d_2 is zero

In this case, x_3 is an unknown input free sub-system. Fortunately, it is detectable because $D_1 > 0$. Therefore, we can build following observer using only one measurement $y_1 = x_1$,

$$\begin{aligned}\dot{\hat{x}}_1 =& \ \hat{x}_2 + \lambda_1 sign(y_1 - \hat{x}_1)\\ \dot{\hat{x}}_2 =& -A_1\hat{x}_2 - A_2\hat{x}_3 sin y_1 - A_3 sin 2y_1 + \lambda_2 sign((\lambda_1 sign(y_1 - \hat{x}_1)_{eq})\\ \dot{\hat{x}}_3 =& -D_1\hat{x}_3 + D_2 cos y_1\end{aligned}$$

Case 3: d_1 is zero, d_2 is nonzero

In this case, with only one measurement for x_1, the system can be transformed into triangular form and all states can be estimated. Note that $g(x) = [0 \quad 0 \quad 1]^T$, and the relative degree of output $y = h(x) = x_1$ corresponding to $g(x)$ can be calculated as

$$\frac{\partial h_1}{\partial x} = (0 \quad 0 \quad 1), L_g h_1(x) = 0, L_f h_1(x) = f_1(x) = x_2$$

$$\frac{\partial(L_f h_1)}{\partial x} = \frac{\partial f_1}{\partial x} = (0 \quad 1 \quad 0), L_g L_f h_1(x) = 0; L_f^2 h_1(x) = f_2(x)$$

$$\frac{\partial(L_f^2 h_1)}{\partial x} = (-A_2 x_3 cos x_1 - A_3 cos 2x_1 \quad -A_1 \quad -A_2 sin x_1),$$

$$L_{g_1} L_f^2 h_1(x) = -A_2 sin x_1.$$

Note that $L_g L_f^2 h_1(x) \neq 0$ if $x_1 \neq k\pi$, thus the relative degree is $r_1 = 3$ at point $x_1 \neq k\pi$. This means that we shall be able to find a transformation only locally, away from any point such that $x_1 = k\pi$. The transformation is

$$\begin{aligned}\xi_1 = \phi_1(x) =& \ h_1(x) = x_1\\ \xi_2 = \phi_2(x) =& \ L_f h_1(x) = x_2\\ \xi_3 = \phi_3(x) =& \ L_f^2 h_1(x) = f_2(x).\end{aligned}$$

The Jacobian matrix of the transformation thus defined

$$\frac{\partial \Phi}{\partial x} = \begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ -A_2 x_3 cos x_1 - A_3 cos 2x_1 & -A_1 & -A_2 sin x_1 \end{bmatrix}$$

which is nonsingular for all $x_1 \neq k\pi$, and the inverse transformation is given by

$$\begin{aligned} x_1 =& \ \xi_1 \\ x_2 =& \ \xi_2 \\ x_3 =& \ z(\xi) = \tfrac{\xi_3 + A_1\xi_2 + A_3 sin2\xi_1}{-A_2 sin\xi_1}. \end{aligned}$$

In these new coordinates the system is described by

$$\begin{aligned} \dot{\xi}_1 =& \ \xi_2 \\ \dot{\xi}_2 =& \ \xi_3 \\ \dot{\xi}_3 =& \ -A_1\xi_3 - \xi_2(A_2 z(\xi) cos\xi_1 + 2A_3 cos2\xi_1) - A_2 sin\xi_1(-D_1 z(\xi) + D_2 cos\xi_1) \\ y_1 =& \ \xi_1 \end{aligned}$$

Actually, the SMO can be done without the above complicated transformation calculation. The observer is

$$\begin{aligned} \dot{\hat{x}}_1 =& \ \hat{x}_2 + \lambda_1 sign(y_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 =& \ -A_1\hat{x}_2 - A_2\hat{x}_3 siny_1 - A_3 sin2y_1 + \lambda_2 sign((\lambda_1 sign(y_1 - \hat{x}_1)_{eq}) \\ \dot{\hat{x}}_3 =& \ -D_1\hat{x}_3 + D_2 cosy_1 + \lambda_3 sign(\overline{e_3}). \end{aligned}$$

Note that the equivalent control signal, based on the second equation, is

$$(\lambda_2 sign(\overline{e_2}))_{eq} = -A_2 e_3 siny_1$$

thus

$$\overline{e_3} = \frac{(\lambda_2 sign(\overline{e_2}))_{eq}}{-A_2 siny_1} = e_3.$$

Obviously, it is true only if $siny_1 \neq 0$.

The parameters in the model have the value. $A_1 = 0.2703, A_2 = 12.01, A_3 = -48.04, D_1 = 0.3222, D_2 = 1.9$, and $\Delta D_1 = 0.1, \Delta D_2 = 0.6$. The control input $u_1 = 36.19, u_2 = 1.9333$. The gain $\lambda_1 = \lambda_2 = \lambda_3 = 200$. The initial state is assumed to be $x_0 = \{0.88, 0.0, 6.5\}$, and the initial value of the observer is $\hat{x} = \{0.8, 0.0, 5.0\}$. The simulation result for case 3 is shown in Figure 6.4.

6.6 Conclusions

This chapter first explored some underlying similarities and connections between two seemingly different methodologies for designing robust observers, namely the unknown input observer and sliding mode observer. Based the special coordinate basis (SCB) form for linear systems, this chapter extends the Walcott-Zak SMO to a general class of linear uncertain systems, and the sliding mode functional observer design is proposed. In second part of this chapter, we extended our proposed observer to a certain class of nonlinear uncertain systems. It is of future research interest to focus on the sliding mode observer for more general classes of nonlinear systems.

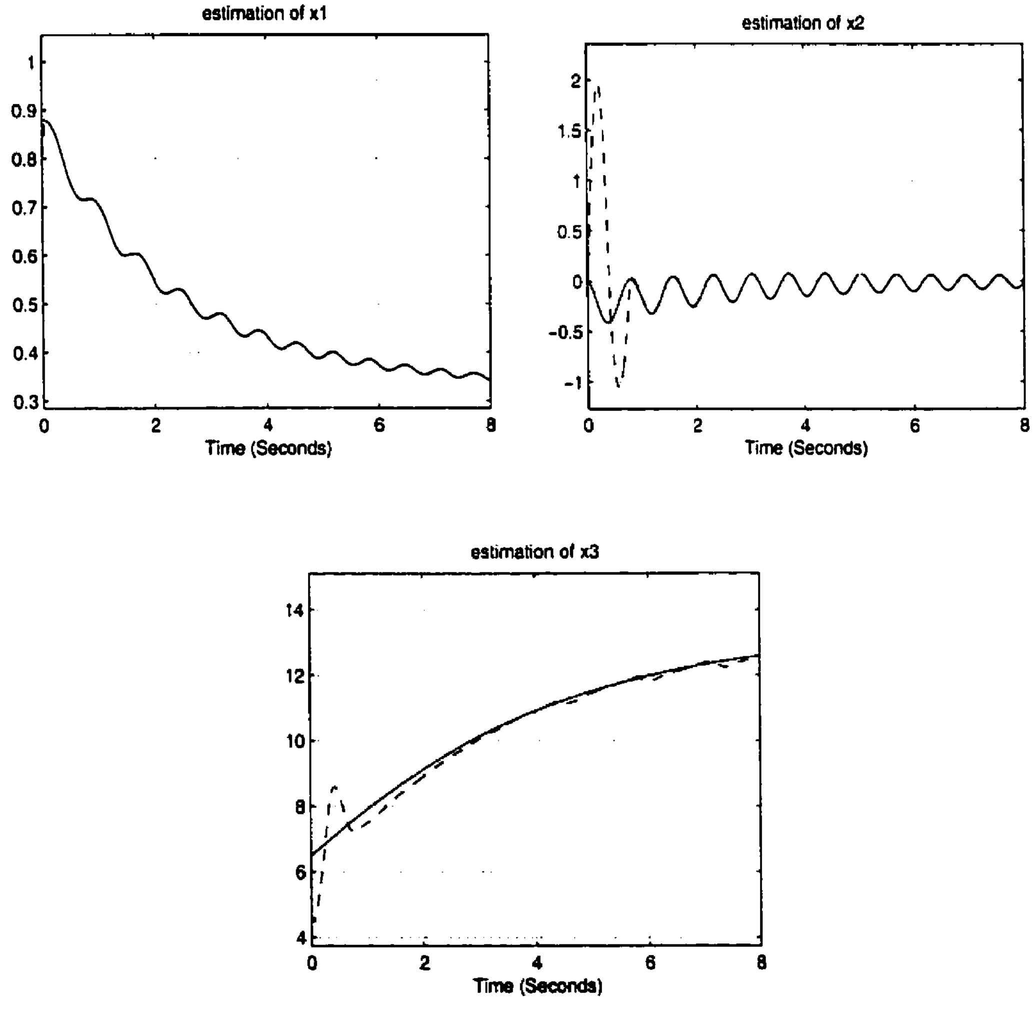

Figure 6.4: Results of state estimation for a three-phase current motor using SMO

Chapter 7

Robust and Nonlinear Fault Diagnosis Using Sliding Mode Observers

The aim of this chapter is to introduce and discuss the pros and cons of sliding mode functional observers (SMFO) for robust fault diagnosis of linear as well as nonlinear uncertain systems. We show that, under specific conditions, SMFO can be used to improve the capability of fault diagnosis schemes which use an unknown input functional observer (UIFO).

7.1 Introduction

The development of UIFO theory has had a great impact on the development of sophisticated on-line fault diagnosis techniques. Another approach to produce robust residuals is the use of a sliding mode observer (SMO). Despite SMO's excellent ability to generate unbiased estimates of the system states under modeling errors, relatively few researchers have investigated the area of fault diagnosis using SMO. Sreedhar and Fernadez presented a fault detection and identification scheme for a twin continuously-stirred tank reactor using SMO in 1993 [106]. Krishnaswami and Rizzoni explored health monitoring of vehicle steering systems based on SMO in 1995 [62], where an extended Utkin SMO [22] was used. Hermans and Zarrop discussed SMO for robust fault diagnosis within a general framework in 1996 [47]. Their work focused on the design requirements relevant to fault detection. Their experimental results for a nonlinear thermodynamic system under feedback control were promising. The observer used by [47] is the Walcott-Zak SMO [23, 115].

The entire state of the system is estimated in much of the previous research of SMO based fault diagnosis. This is unnecessary for fault diagnosis and puts unnecessary structural constraints on the system. In this chapter, we present an alternative to this trend by applying SMFO. In Section 7.2, a Sliding mode output observer (SMOO) (which is a specific type of SMFO) and its use for fault diagnosis are discussed for a general class of nonlinear uncertain systems. This observer works well only for abrupt sensor faults and large actuator faults, because the fault-free model uncertainties affect the residuals in SMOO. For incipient fault diagnosis, the SMFO proposed in Chapter 6 is used. The SMFO based fault diagnosis schemes for linear and nonlinear uncertain systems are discussed in Section 7.3 and Section 7.4 respectively. We compare our fault diagnosis scheme using SMFO with that based on UIFO, and will show how SMFO can be used to enhance the robust fault diagnosis capability. Simulation studies are presented in Section 7.5 to illustrate the validity of the proposed fault diagnosis schemes.

7.2 A Universal Sliding Mode Output Observer For Fault Diagnosis

Consider a multivariable nonlinear system described in state space form by equations of the following kind:

$$\begin{aligned} \dot{x} &= A(x, u, d, f_a(t)) \\ y &= H(x) + f_s(t) \end{aligned} \tag{7.1}$$

where $x \in M$, a C^∞ connected manifold of dimension n, $H(x) = [h_1(x), ..., h_p(x)]$ are smooth vector fields on M, u and d represent the control input and the signal representing the system uncertainties respectively. It is assumed that all x, u and d

are bounded. $f_a(t)$ represents actuator and component faults. $f_s(t) = [f_s^1(t), ..., f_s^p(t)]$ represent the sensor faults, which can be any function of time.

A general sliding mode output observer (SMOO) has the following form,

$$\dot{z} = L(y, u) + \Lambda sign(y - z) \tag{7.2}$$

where z is an estimation of $H(x)$, $L(y, u)$ is a function of y and u, Λ is a diagonal gain matrix with elements $\lambda_i, i = 1, ..., p$. The design of $L(y, u)$ is discussed later.

The dynamics of the estimation error $e = H(x) - z$ is

$$\dot{e} = \frac{\partial H(x)}{\partial x} A(x, u, d, f_a(t)) - L(H(x) + f_s(t), u) - \Lambda sign(H(x) + f_s(t) - z). \tag{7.3}$$

Note that e cannot be measured, and only $r = y - z = H(x) + f_s(t) - z$ is measured. r is the sliding surface and can be considered as a residual signal. Assume $e_i = h_i(x) - z_i$ and

$$\frac{\partial H(x)}{\partial x} A(x, u, d, f_a(t)) = \begin{bmatrix} m_1(x, u, d, f_a(t)) \\ m_2(x, u, d, f_a(t)) \\ \cdots \\ m_p(x, u, d, f_a(t)) \end{bmatrix} \quad \text{and} \quad L(y, u) = \begin{bmatrix} l_1(y, u) \\ l_2(y, u) \\ \cdots \\ l_p(y, u) \end{bmatrix}.$$

Then the equation of e_i is

$$\dot{e}_i = m_i(x, u, d, f_a(t)) - l_i(H(x) + f_s(t), u) - \lambda_i sign(e_i(x) + f_s^i(t)).$$

Under the condition of $f_a(t) = f_s(t) = 0$, if $|m_i(x, u, d, 0) - l_i(H(x), u)| \leq \lambda_i$, we have

$$\frac{d}{dt} e_i^2 = 2e_i\dot{e}_i = 2e_i(m_i(x, u, d, 0) - l_i(H(x), u) - \lambda_i sign(e_i)).$$

Therefore,

$$\begin{aligned} &\text{if } e_i > 0, \ \ \tfrac{d}{dt}e_i^2 = 2e_i(m_i(x, u, d, 0) - l_i(H(x), u) - \lambda_i) < 0; \\ &\text{if } e_i < 0, \ \ \tfrac{d}{dt}e_i^2 = 2e_i(m_i(x, u, d, 0) - l_i(H(x), u) + \lambda_i) < 0. \end{aligned}$$

Thus $r_i = e_i$ exponentially decreases to zero according to the Lyapunov principle. Next we investigate how a fault interacts with the sliding surfaces and how the sliding performance of the observer is affected.

Case 1: Sensor Faults

If sensor i becomes faulty at time t_i, namely $f_s^i(t) \neq 0$ after $t > t_i$, the residual $r_i = e_i + f_s^i(t)$, then

$$\dot{r}_i = m_i(x, u, d, 0) - l_i(H(x) + f_s(t), u) + \dot{f}_s^i(t) - \lambda_i sign(r_i). \tag{7.4}$$

The fault may produce two results:

1. If $|m_i(x, u, d, 0) - l_i(H(x) + f_s(t), u) + \dot{f}_s^i(t)| \leq \lambda_i$, the sliding behavior, if any, can then only occur on the surface

$$r_i = h_i(x) + f_s^i(t) - z_i = 0. \tag{7.5}$$

   In other words, the estimation error $h_i(x) - z_i$ is nonzero already. However, the estimation error cannot be measured, and the measured residual signal will still be zero. Fortunately, if the fault is an abrupt change, or $f_s^i(t)$ jumps from zero to a significantly large value at time t_i (see Fig. 7.1(a)), a corresponding abrupt change will be observed in r_i. The reason for such a phenomena is that the residual will change from zero to $f_s^i(t_i)$, and then decrease to zero quickly. Unfortunately, not all abrupt changes of $f_s^i(t)$ represent real faults. Consider the shape of f_s^i shown in Fig. 7.1(b), which represents a sensor jitter. This sensor jitter will produce almost the same residual shape as an abrupt sensor fault. In such a case, it is difficult to distinguish between the real abrupt sensor fault and sensor jitter.

2. If

$$|m_i(x, u, d, 0) - l_i(H(x) + f_s(t), u) + \dot{f}_s^i(t)| > \lambda_i \tag{7.6}$$

   persistently holds, the observer will be disturbed from its surface and sliding will cease. In this case, the ith residual element will become nonzero persistently, and will alarm the occurrence of the sensor fault.

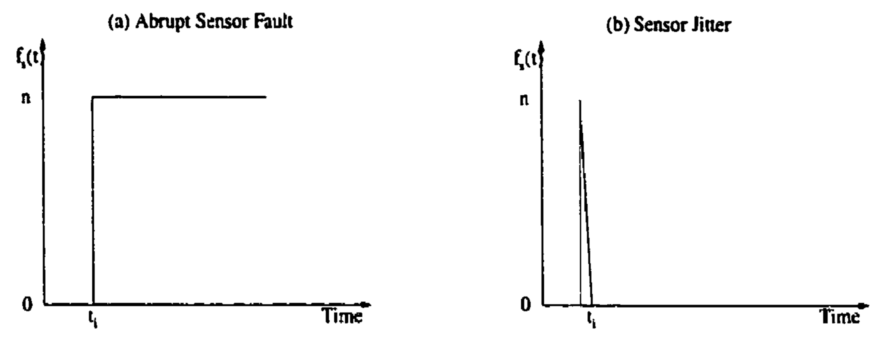

Figure 7.1: Two kinds of Sensor Fault Signal

For multiple sensor fault identification, we can design $l_i(y,u) = l_i(y_i,u)$. Thus r_i is sensitive only to $f_s^i(t)$.

Case 2: Actuator Faults

If actuators fail, namely if $f_a(t) \neq 0$ when $t > t_a$, we may also observe two effects on the residual:

1. If $|m_i(x,u,d,f_a(t)) - l_i(H(x),u)| \leq \lambda_i$ for all $i = 1,...,k$, the observer. will keep on the sliding surface,
$$r_i = \epsilon_i = 0, i = 1,...,k.$$

2. If
$$|m_i(x,u,d,f_a(t)) - l_i(H(x),u)| > \lambda_i \tag{7.7}$$
is true persistently for some $i \in \{1,2,...k\}$, the residual element, r_i, will become nonzero.

In practice, because of an unavoidable measurement noise, we always use the saturation function to replace the sign function. When there is no fault present, the residual is not exactly zero, but less than a small threshold. Once its value passes the threshold, an alarm is registered.

Amongst the past studies, [106] only considered the abrupt sensor fault detection, and [47] simply assumed that the faults were persistent and had moved outside the interval of robustness. While these studies are based on the linearized system model, we have conceptually shown in the above discussion that fault diagnosis using the sliding mode concept can actually be used for general nonlinear uncertain systems, as long as

1. The upper bound of model mismatch $|m_i(x,u,d,0) - l_i(H(x),u)|$ is known *a priori*, such that a suitable gain λ_i can be selected;

2. The faults result in a system model mismatch out of the boundary, namely condition (7.6) or (7.7) is satisfied;

Therefore, in order to improve the capability of SMOO based fault diagnosis scheme, we must minimize the model mismatch through the design of $L(y,u)$. Recall the error dynamics of SMOO given by (7.3), we know that $L(y,u)$ should be designed such that

$$\|\frac{\partial H(x)}{\partial x}A(x,u,d,0) - L(H(x),u)\|$$

is minimized for all possible value of state x and input u. The solution of $L(y,u)$ will be strongly dependent on the specific form of the nonlinear equation (7.1). Consider a simple case of linear uncertain systems described by the following equations:

$$\begin{aligned} \dot{x} &= Ax + Bu + Gd + F_a f_a \\ y &= Cx + F_s f_s. \end{aligned} \tag{7.8}$$

With the general form of SMOO (7.2), the estimation error equation (7.3) is simplified as

$$\dot{e} = CAx + CBu + CGd + CF_a f_a - L(Cx + F_s f_s(t), u) - \Lambda sign(Cx + F_s f_s - z).$$

For the linear system (7.8), $L(y,u)$ is designed as $L(y,u) = Ky + CBu$. The problem of minimizing the model mismatch is formulated as finding the gain K, such that $\|CA - KC\|$ is minimized. Obviously, if and only if $CG = 0$ and there exists K to satisfy $CA - KC = 0$, the residual is totally independent of the unknown input d and the unknown states x, namely

$$\dot{e} = CF_a f_a - KF_s f_s(t) - \Lambda sign(e + F_s f_s).$$

However, this is impossible in most cases.

Most observer-based fault diagnosis schemes build the residual by comparing the measurement and its corresponding estimate provided by observers. Because most observers will produce wrong estimation when faults appear, a nonzero residual will indicate the occurrence of faults. This section shows that SMOO can produce the estimation of outputs for general nonlinear systems, which is useful for detecting faults of large magnitude. However, it should be pointed out that the framework of SMOO is not useful for the detection of incipient faults in many practical applications, where system model mismatch is unavoidable and is significant. Under this case, when SMOO removes the effect of the system model mismatch and the unknown internal state variables, the effects of incipient faults will be removed at the same time. An incipient fault slowly develops from zero to a certain significant value, it will neither produce a sudden peak in the residual, nor will it push the system out of sliding mode, at least when its value is small in its early stage. Many previous studies on SMO based fault diagnosis neglected this important limitation.

7.3 Incipient Fault Diagnosis Using SMFO for Linear Uncertain Systems

In order to detect incipient fault, we have to exploit the structure property of the system. Therefore, we use special coordinate basis (SCB) to transform the linear uncertain system (7.8). The SCB transformation is applied to matrices (A, G, C) of the system (7.8). It is easy to show that the fault distribution matrices F_a and F_s will be transformed as

$$\overline{F_a} = \Gamma_1^{-1} F_a = \begin{bmatrix} F_{aa}^T & F_{ab}^T & F_{ac}^T & F_{ad}^T \end{bmatrix}^T, \tag{7.9}$$

and

$$\overline{F_s} = \Gamma_2^{-1} F_s = \begin{bmatrix} F_{sd} \\ F_{sb} \end{bmatrix}. \tag{7.10}$$

Therefore, the SCB form of the system (7.8) will be

$$\begin{aligned} \dot{x}_a &= A_a x_a + L_{ad}C_d x_d + L_{ab}C_b x_b + B_a u + F_{aa}f_a, \\ \dot{x}_b &= A_b x_b + L_{bd}C_d x_d + B_b u + F_{ab}f_a, \quad y_b = C_b x_b + F_{sb}f_s, \\ \dot{x}_c &= A_c x_c + L_{cd}C_d x_d + L_{cb}C_b x_b + G_c E_{ca} x_a + G_c d_c + B_c u + F_{ac}f_a \end{aligned} \tag{7.11}$$

and $x_d = \begin{bmatrix} x_{1d} & x_{2d} & \dots & x_{m_d d} \end{bmatrix}^T$. Each $x_{id}, i = 1, \dots, m_d$ is further composed of q_i states, or

$$x_{id} = \begin{bmatrix} x_1^i & x_2^i & \dots & x_{q_i}^i \end{bmatrix}';$$

so $F_{ad} = [F_{d1}^1, \dots, F_{dq_1}^1, \dots, F_{d1}^{m_d}, \dots, F_{dq_{m_d}}^{m_d}]^T$, and the equations of x_{id} are

$$\begin{aligned} \dot{x}_1^i &= x_2^i + L_1^i C_d x_d + B_1^i u + F_{d1}^i f_a \\ \dot{x}_2^i &= x_3^i + L_2^i C_d x_d + B_2^i u + F_{d2}^i f_a \\ \dots & \quad \dots \\ \dot{x}_{q_i-1}^i &= x_{q_i}^i + L_{q_i-1}^i C_d x_d + B_{q_i-1}^i u + F_{d(q_i-1)}^i f_a \\ \dot{x}_{q_i}^i &= E_{ia}x_a + E_{ib}x_b + E_{ic}x_c + \textstyle\sum_{j=1}^{m_d} E_{ij}x_{jd} + B_{q_i}^i u + F_{dq_i}^i f_a + d_i \\ y_{id} &= x_1^i + f_{sd}^i \end{aligned} \tag{7.12}$$

where f_{sd}^i is the ith element of $F_{sd}f_s$. Note that $y_d = C_d x_d$ if no sensor faults occur.

7.3.1 Residual Generation Using Utkin SMO for Unknown Input Free Subsystem

If observable and unknown input free subsystem, x_b, does exist, we can use FDI schemes discussed in Chapter 4. As we point out in Chapter 4, fault diagnosis using UIFO actually relies on existence of the x_b subsystem. Here, we discuss if a SMO can be used to generate the residual. It is well known that an Utkin SMO can be built for a linear system without unknown input. In order to design an Utkin SMO, we further transform the x_b subsystem into following form:

$$\begin{aligned} \dot{x}_{b1} &= A_{11}x_{b1} + A_{12}x_{b2} + L_{bd1}C_d x_d + B_{b1}u + F_{ab1}f_a \\ \dot{x}_{b2} &= A_{21}x_{b1} + A_{22}x_{b2} + L_{bd2}C_d x_d + B_{b2}u + F_{ab2}f_a \\ y_b &= x_{b2} + F_{sb}f_s. \end{aligned} \tag{7.13}$$

The corresponding Utkin SMO is

$$\begin{aligned} \dot{\hat{x}}_{b1} &= A_{12}y_b + A_{11}\hat{x}_{b1} + L_{bd1}y_d + B_{b1}u + \Lambda_2(\Lambda_1 sign(y_b - \hat{x}_{b2}))_{eq} \\ \dot{\hat{x}}_{b2} &= A_{22}\hat{x}_{b2} + A_{21}\hat{x}_{b1} + L_{bd2}y_d + B_{b2}u + \Lambda_1 sign(y_b - \hat{x}_{b2}) \end{aligned} \tag{7.14}$$

where Λ_1 is a nonsingular diagonal matrix and its diagonal elements must be large enough to compensate initial estimation errors. $(\Lambda_1 sign(y_b - \hat{x}_{b2}))_{eq}$ is the equivalent control of $\Lambda_1 sign(y_b - \hat{x}_{b2})$, Λ_2 is a matrix selected to make $A_{11} - \Lambda_2 A_{21}$ stable.

The estimation error equations for the observer (7.14) are

$$\begin{aligned} \dot{e}_{b1} &= -A_{12}F_{sb}f_s + A_{11}e_{b1} - L_{bd1}F_{sd}f_s + F_{ab1}f_a - \Lambda_2(\Lambda_1 sign(e_{b2} + F_{sb}f_s))_{eq} \\ \dot{e}_{b2} &= A_{22}e_{b2} + A_{21}e_{b1} - L_{bd2}F_{sd}f_s + F_{ab2}f_a - \Lambda_1 sign(e_{b2} + F_{sb}f_s) \end{aligned} \tag{7.15}$$

where $e_{b2} = x_{b2} - \hat{x}_{b2}, e_{b1} = x_{b1} - \hat{x}_{b1}$.

Now, we set the residual as $r = y_b - \hat{x}_{b2} = e_{b2} + F_{sb}f_s$. Its dynamic equation is

$$\dot{r} = A_{22}r - A_{22}F_{sb}f_s + A_{21}e_{b1} - L_{bd2}F_{sd}f_s - F_{sb}\dot{f}_s + F_{ab2}f_a - \Lambda_1 sign(r).$$

If faults are incipient, it is expected that r will keep on the sliding plane, thus

$$\begin{aligned} r_{eq} = (\Lambda_1 sign(r))_{eq} &= -A_{22}F_{sb}f_s + A_{21}e_{b1} - L_{bd2}F_{sd}f_s - F_{sb}\dot{f}_s + F_{ab2}f_a \\ &= A_{21}e_{b1} + f_{w2}, \end{aligned} \tag{7.16}$$

then

$$\begin{aligned} \dot{e}_{b1} &= (A_{11} - \Lambda_2 A_{21})e_{b1} + A_{12}F_{sb}f_s - L_{bd1}F_{sd}f_s + F_{ab1}f_a - \Lambda_2 f_{w2} \\ &= (A_{11} - \Lambda_2 A_{21})e_{b1} + f_{w1} - \Lambda_2 f_{w2}. \end{aligned}$$

Finally, we have the following transfer function of r_{eq},

$$r_{eq}(s) = A_{21}(sI - A_{11} + \Lambda_2 A_{21})^{-1}(f_{w1}(s) - \Lambda_2 f_{w2}(s)) + f_{w2}(s). \tag{7.17}$$

r_{eq} will be the real residual for fault detection. r_{eq} is the function of fault signal and independent of unknown inputs or unknown state variable. In practice, we can easily check if r_{eq} will be nonzero for certain faults using equation (7.17).

Remark 7.3.1 If C_b happens to be full rank, the above Utkin SMO reduces into the simple SMOO as discussed in Section 7.1. Because e_{b1} term actually disappears under this condition, r_{eq} in equation (7.16) is simplified as

$$r_{eq} = -A_b F_{sb} f_s - L_{bd} F_{sd} f_s - F_{sb} \dot{f}_s + F_{ab} f_a.$$

If there are no sensor faults and F_{ab} is full column rank, F_{ab} has the left inverse $= (F_a)^L$, and the actuator faults can be identified directly as $f_a = (F_a)^L (r)_{eq}$.

Remark 7.3.2 In the future, we need to compare the fault detectability conditions using UIFO and the above Utkin SMO. We should study how to build a post filter for $r_{eq}(s)$ to produce a directional residual, so that fault isolation is achievable.

7.3.2 Residual Generation Using SMFO for Subsystem with Unknown Input

Next, we examine how fault information in the x_d subsystem can be extracted, although the x_d subsystem is affected by unknown input. In Chapter 6, the following SMO is built for x_{id} subsystems

$$\begin{array}{ll}
\dot{\hat{x}}_1^i & = \hat{x}_2^i + L_1^i y_d + B_1^i u + \lambda_1^i sign(y_{id} - \hat{x}_1^i) \\
\dot{\hat{x}}_2^i & = \hat{x}_3^i + L_2^i y_d + B_2^i u + \lambda_2^i sign(r_2^i) \\
\dots & \dots \\
\dot{\hat{x}}_{q_i-1}^i & = \hat{x}_{q_i}^i + L_{q_i-1}^i y_d + B_{q_i-1}^i u + \lambda_{q_i-1}^i sign(r_{q_i-1}^i) \\
\dot{\hat{x}}_{q_i}^i & = E_{ia}\hat{x}_a + E_{ib}\hat{x}_b + E_{ic}\hat{x}_c + \sum_{j=1}^{m_d} E_{ij}\hat{x}_{jd} + B_{q_i}^i u + \lambda_{q_i}^i sign(r_{q_i}^i)
\end{array} \tag{7.18}$$

where $\hat{x}_a, \hat{x}_b, \hat{x}_c$ are estimations for states x_a, x_b, x_c respectively, and

$$r_k^i = (\lambda_{k-1}^i sign(r_{k-1}^i))_{eq} \tag{7.19}$$

for $k = 2, ..., q_i$, and $r_1^i = y_{1d} - \hat{x}_1^i$ and can be obtained directly. The $\hat{x}_a, \hat{x}_b, \hat{x}_c$ are given by

$$\begin{array}{rl}
\dot{\hat{x}}_a = & A_a\hat{x}_a + L_{ad}y_d + L_{ab}y_b + B_a u, \\
\dot{\hat{x}}_b = & A_b\hat{x}_b + L_{bd}y_d + B_b u + K_b y_b, \\
\dot{\hat{x}}_c = & A_c\hat{x}_c + L_{cd}y_d + L_{cb}y_b + G_c E_{ca}\hat{x}_a + B_c u.
\end{array} \tag{7.20}$$

We can make $\hat{x}_b \to x_b$ by designing K_b such that $A_b - K_b C_b$ is a stable matrix. However, $e_a = x_a - \hat{x}_a$ will not be zero if A_a is an unstable matrix and the initial estimation error is nonzero. Also, $e_c = x_c - \hat{x}_c$ will not be zero because of the unknown input. However, if no sensor faults appear and

$$\lambda_k^i > \|x_{k+1}^i - \hat{x}_{k+1}^i\|$$

$k = 1, 2, ..., q_i - 1$, as well as

$$\lambda_{q_i}^i > \|E_{ia} e_a + E_{ic} e_c + d_i\|$$

the observer (7.18) will give the right estimation of x_{id}.

To analyze the effect of the faults, let us consider the dynamic equation of the estimation error $e_1^i = x_1^i - \hat{x}_1^i$, it is

$$\dot{e}_1^i = e_2^i - L_1^i f_{sd} + F_{d1}^i f_a - \lambda_1^i sign(e_1^i + f_{sd}^i)$$

and the residual is

$$r_1^i = y_{id} - \hat{x}_1^i = e_1^i + f_{sd}^i \tag{7.21}$$

The static equation of r_1^i implies that r_1^i will be nonzero at the moment f_{sd}^i becomes nonzero, because $e_1^i = 0$ when $f_{sd}^i = 0$. On the other hand, we derive the dynamic equation of r_1^i as below

$$\dot{r}_1^i = e_2^i - L_1^i f_{sd} + \dot{f}_{sd}^i + F_{d1}^i f_a - \lambda_1^i sign(r_1^i). \tag{7.22}$$

Because we consider detection of incipient faults, it is expected that magnitude of $-L_1^i f_{sd} + \dot{f}_{sd}^i + F_{d1}^i f_a$ will be small. Note that we already chose $\lambda_1^i > \|x_2^i - \hat{x}_2^i\| = e_2^i$, and it is expected that

$$\|e_2^i - L_1^i f_{sd} + \dot{f}_{sd}^i + F_{d1}^i f_a\| < \lambda_1^i.$$

Under above condition, r_1^i will approximate zero. With $r_1^i = 0$, its equivalent control signal will be

$$r_2^i = (\lambda_1^i sign(r_1^i))_{eq} = e_2^i - L_1^i f_{sd} + \dot{f}_{sd}^i + F_{d1}^i f_a. \tag{7.23}$$

r_2^i is used for the estimation of x_2^i, therefore, we have

$$\dot{e}_2^i = e_3^i - L_2^i f_{sd} + F_{d2}^i f_a - \lambda_2^i sign(r_2^i).$$

Therefore, the dynamic equation of r_2^i becomes

$$\dot{r}_2^i = e_3^i - L_2^i f_{sd} - L_1^i \dot{f}_{sd} + \dot{f}_{sd}^i + F_{d2}^i f_a + F_{d1}^i \dot{f}_a - \lambda_2^i sign(r_2^i).$$

As long as λ_2^i is large enough, r_2^i will approximate zero. However, if we compare the static equations of r_1^i and r_2^i, namely equations (7.21) and (7.23), we note the derivate of sensor fault f_{sd}^i appears in r_2^i, which is much more significant than f_{sd}^i itself in r_1^i.

The equivalent control signal of r_2^i is used further for the estimation of x_3^i, and so on, until $r_{q_i}^i$ is calculated. Following above derivation process, we have following static equation for $r_k^i (k = 1, ..., q_i)$ if no fault is large enough to make the system stop in sliding mode,

$$r_k^i = e_k^i - \sum_{j=1}^{k-1} L_j^i f_{sd}^{(k-j-1)} + \sum_{j=1}^{k-1} F_{dj}^i f_a^{(k-j-1)} + (f_{sd}^i)^{(k-1)} = e_k^i + f_k^i.$$

where $(\cdot)^j$ means the jth derivative. On the other hand, all $r_k^i (1 \leq k \leq q_i, 1 \leq i \leq m_d)$ will dynamically approximate zero. However, we may observer peak phenomenon in $r_k^i (2 \leq k)$ if the derivative of the fault signal changes abruptly from zero to a nonzero value, which is likely for incipient faults. When k is bigger, a higher order of the derivate appears in r_k^i. Fault isolation is possible if the fault distribution matrices $L_k^i, F_{dk}^i (k = 1, ..., q_i, i = 1, ..., m_d)$ have some special structure such that r_k^i is only sensitive to certain faults.

The drawback of fault detection based on residual r_k^i is that we cannot distinguish sensor or actuator jitter from real sensor or actuator fault, because both of them will produce peak phenomena. The bottom line is that no any other system uncertainties or unknown inputs will produce such a peak. Therefore, it is worth to check the system further once peaks appear in r_k^i.

Remark 7.3.3 We can derive the equivalent control signal of $r_{q_i}^i$ as

$$r_{q_i+1}^i = (\lambda_{q_i}^i sign(r_{q_i}^i))_{eq} = E_{ia} e_a + E_{ib} e_b + E_{ic} e_c + \sum_{j=1}^{m_d} E_{ij} e_{jd} + \dot{f}_{q_i} + F_{dq_i}^i f_a + d_i.$$

Even under healthy conditions, $r^i_{q_i+1}$ will be nonzero because of the estimation errors of e_a, e_c and the unknown input d_i. We may also observer peak phenomenon in $r^i_{q_i+1}$ because it contains the derivative of faults. However, since we never know how d_i changes, an occurrence of a peak phenomenon in $r^i_{q_i+1}$ is unreliable for fault detection.

Remark 7.3.4 Note that if $q_i = 1$ for all $i = 1, ..., m_d$, the above SMO is reduced to SMOO for x_d subsystem. There is no way to distinguish between the fault and the unknown input under this condition.

In summary, when we design a robust fault diagnosis scheme for a linear uncertain system, we may decomposition it into SCB subsystem form. Because no output for x_a and x_c subsystem, no residual can be generated based on observer for these two subsystems. If x_b subsystem exists, the residual generation method discussed in Chapter 4 or section 7.3.1 can be applied. If x_d subsystem exist and not all $q_i = 1, (i = 1, ..., m_d)$, we can build SMFO for x_d subsystem and generate the residual (7.19). The form of SCB transform implies that either x_b or x_d subsystem will exist within a linear system subject to unknown input. In practical applications, if the number of independent unknown input is larger than the number of independent output, it is very often that there is no unknown input free subsystem. Therefore, the SMFO for x_d subsystem significantly improves the robust fault detection ability for the linear uncertain systems.

7.4 SMFO Based Incipient Fault Diagnosis for Nonlinear Uncertain Systems

We consider the nonlinear uncertain systems as described by the following equations:

$$\begin{aligned} \dot{x} = & \ A(x) + B(x,u) + F_a(x,u)f_a + \textstyle\sum_{i=1}^m g_i(x)d_i(x,u,t) \\ y = & \ H(x) + f_s \end{aligned} \tag{7.24}$$

where $H(x) = [h_1(x), ..., h_p(x)]^T$. In Chapter 6, Lemma 6.4.3 shows that if the nonlinear system (7.24) satisfies the following conditions:

1. $p \geq m$, where p, m are dimensions of the output and the unknown inputs respectively,

2. First m outputs have the relative degree $\{q_1, ..., q_m\}$ corresponding to the unknown input matrix $G(x) = [g_1(x), ..., g_m(x)]$, and the distribution spanned by the vector fields $g_1(x), ..., g_m(x)$ is involutive,

then there exists a nonlinear transformation $\Phi_o(x)$ such that (7.24) can be decomposed into one subsystem x_d with unknown inputs, and one subsystem without unknown inputs. The x_d subsystem can be further decomposed into m subsystems with triangular structure, or $x_d = [x_{1d}, ..., x_{md}]^T$, and each x_{id} subsystem has q_i states, or $x_{id} = [x_1^i, ..., x_{q_i}^i]^T, i = 1, ..., m$. The equations are

$$\begin{aligned}
\dot{x}_1^i &= x_2^i + b_1^i(x_d, x_o, u) + F_{d1}^i(x, u) f_a \\
\cdots & \quad \cdots \\
\dot{x}_{q_i-1}^i &= x_{q_i}^i + b_{q_i-1}^i(x_d, x_o, u) + F_{d(q_i-1)}^i(x, u) f_a \\
\dot{x}_{q_i}^i &= a_i(x_d, x_o) + b_{q_i}^i(x_d, x_o, u) + \textstyle\sum_{j=1}^m c_{ij}(x_d, x_o) d_j + F_{dq_i}^i(x, u) f_a \\
y_{id} &= x_1^i + f_{sd}^i
\end{aligned} \tag{7.25}$$

The unknown input free subsystem can be written as

$$\begin{aligned}
\dot{x}_o &= q(x_d, x_o) + p(x_d, x_o, u) + s(x_d, x_o, u, f_a) \\
y_{m+1} &= h_{m+1}(x_d, x_o) + F_{m+1}(x) f_s \\
\cdots & \quad \cdots \\
y_p &= h_p(x_d, x_o) + F_p(x) f_s
\end{aligned} \tag{7.26}$$

As we showed in Chapter 6, if the input term of subsystem (7.25) is in the following form

$$\begin{aligned}
b_1^i(x_d, x_o, u) &= b_1^i(y_r, u) \\
b_2^i(x_d, x_o, u) &= b_2^i(x_2^i, y_r, u) \\
\cdots & \quad \cdots \\
b_{q_i-1}^i(x_d, x_o, u) &= b_{q_i-1}^i(x_2^i, ..., x_{q_i-1}^i, y_r, u)
\end{aligned} \tag{7.27}$$

where y_r represents the sensor fault-free output, a nonlinear SMO can be built because of its triangular structure,

$$\begin{array}{ll} \dot{\hat{x}}_1^i & = \hat{x}_2^i + b_1^i(y,u) + \lambda_1^i sign(y_{id} - \hat{x}_1^i) \\ \dot{\hat{x}}_2^i & = \hat{x}_3^i + b_2^i(\hat{x}_2^i, y, u) + \lambda_2^i sign(r_2^i) \\ \dots & \dots \\ \dot{\hat{x}}_{q_i-1}^i & = \hat{x}_{q_i}^i + b_{q_i-1}^i(\hat{x}_2^i, ..., \hat{x}_{q_i-1}^i, y, u) + \lambda_{q_i-1}^i sign(r_{q_i-1}^i) \\ \dot{\hat{x}}_{q_i}^i & = a_i(\hat{x}_d, \hat{x}_o) + b_{q_i}^i(\hat{x}_d, \hat{x}_o, u) + \lambda_{q_i}^i sign(r_{q_i}^i) \end{array} \tag{7.28}$$

where $r_k^i(k = 2, ..., q_i)$ are given by (7.19). We can derive the formula of r_k^i as

$$r_k^i = e_k^i + \sum_{j=1}^{k-1}(\Delta b_j^i)^{(k-j-1)} + \sum_{j=1}^{k-1}(F_{dj}^i(x,u)f_a)^{(k-j-1)} + (f_{sd}^i)^{(k-1)} = e_k^i + f_k^i$$

where

$$\Delta b_j^i = b_j^i(x_2^i, ..., x_j^i, y_r, u) - b_j^i(\hat{x}_2^i, ..., \hat{x}_j^i, y_r + f_s, u).$$

Finally, the $r_{q_i+1}^i = (\lambda_{q_i}^i sign(r_{q_i}^i))_{eq}$ is equal to

$$\begin{array}{rcl} r_{q_i+1}^i & = & a_i(x_d, x_o) - a_i(\hat{x}_d, \hat{x}_o) + b_{q_i}^i(x_d, x_o, u) - b_{q_i}^i(\hat{x}_d, \hat{x}_o, u) \\ & & + \sum_{j=1}^m c_{ij}(x_d, x_o)d_j + F_{dq_i}^i(x,u)f_a + \dot{f}_{q_i}. \end{array} \tag{7.29}$$

Similar to the linear case, the usage of $r_k^i(2 \leq k \leq q_i)$ is based on a peak phenomenon due to derivative of fault signal.

To exploit the usage of an unknown input free subsystem, we should try to build an observer for the x_o subsystem. If x_o is an observable subsystem and we can build an observer for it, then the following residuals can be formulated:

$$r_j = h_j(\hat{x}_d, \hat{x}_o) - y_j, j = m+1, ..., p.$$

Unfortunately, the complete solution for nonlinear systems in the form of (7.26) is an open problem and needs to be studied in the future.

Here we propose a sliding mode output functional observer (SMOFO) and discuss its usability for incipient fault diagnosis. Assume $y_{r,j} = h_j(x_d, x_o)(j = m+1, ..., p)$, $Y(x) = [y_{r,m+1}, ..., y_{r,p}]^T$, $f_{ts} = [F_{m+1}(x)f_s, ..., F_p(x)f_s]$ and $Y_o =$

$[y_{m+1}, \ldots, y_p]^T$. Obviously $Y_o = Y(x) + f_{ts}$ and $Y_o = Y(x)$ when $f_s = 0$. The equation for SMOFO is

$$\dot{z} = L(Y_o, \hat{x}_d, u) + \Lambda sign(W(Y_o) - z). \tag{7.30}$$

It is expected that $z \to W(Y(x))$. Let $e = W(Y(x)) - z$, we have

$$\begin{aligned} \dot{e} &= \tfrac{\partial W(Y_o)}{\partial Y_o}\tfrac{\partial Y_o}{\partial x_o}(q(x_d, x_o) + p(x_d, x_o, u) + s(x_d, x_o, u, f_a)) \\ &\quad -L(Y_o, \hat{x}_d, u) - \Lambda sign(W(Y_o) - z) \\ &= M(x_d, x_o, u, f_a) - L(Y_o, \hat{x}_d, u) - \Lambda sign(W(Y_o) - z). \end{aligned}$$

Set $r = W(Y_o) - z = W(Y(x) + f_{ts}) - z$, then $W(Y(x) + f_{ts})$ can be represented in a Taylor series expansion as

$$W(Y(x) + f_{ts}) = W(Y(x)) + \sum_{k=1}^{\infty} \frac{W(Y(x))^{(k)} f_{ts}^k}{k!} = W(Y(x)) + Q(x, f_{ts})$$

where $W(Y(x))^{(k)}$ refers to the kth derivative. Thus, $r = e + Q(x, f_{ts})$. Represent the derivate of $Q(x, f_{ts})$ as

$$\frac{\partial Q}{\partial x}\dot{x} + \frac{\partial Q}{\partial f_{ts}}\dot{f}_{ts} = D(x, f_{ts}, \dot{f}_{ts}).$$

The equation of r becomes

$$\dot{r} = M(x_d, x_o, u, f_a) - L(Y_o, \hat{x}_d, u) - D(x, f_{ts}, \dot{f}_{ts}) - \Lambda sign(r).$$

Similar to the principle of SMOO, r will approximate zero due to the high gain design of Λ and will stay at zero for incipient faults. However, we can set residual as the equivalent control signal of r, it is

$$r_{eq} = (\Lambda sign(r))_{eq} = M(x_d, x_o, u, f_a) - L(Y_o, \hat{x}_d, u) - D(x, f_{ts}, \dot{f}_{ts}). \tag{7.31}$$

For the objective of fault diagnosis, It is expected that

1. $r_{eq} = 0$ when there is no fault, namely $f_s = f_a = 0$. It is easy to derive that $Q(x, f_{ts}) = 0$ and $D(x, f_{ts}, \dot{f}_{ts}) = 0$ if no fault appears. Therefore, r_{eq} is reduced to be

$$r_{eq} = M(x_d, x_o, u, 0) - L(Y(x), \hat{x}_d, u).$$

Note that $\hat{x}_d \to x_d$ when no fault appears, thus, x_d can be considered as a known input for the x_o subsystem. Therefore, the objective of $r_{eq} = 0$ under the fault free condition can be achieved if and only if

$$M(x_d, x_o, u, 0) = L(Y(x), x_d, u). \tag{7.32}$$

2. r_{eq} is nonzero when $f_a \neq 0$ or $f_s \neq 0$. In practical application, this property need to be checked using the equation (7.31).

Remark 7.4.1 Compared with SMOO, SMOFO introduces one more design parameter, function of output $W(Y(x))$, so that the effect of unknown states variable x_o can be removed through nonlinear state transformation. This design avoids the complex problem of estimating states for nonlinear systems. Unfortunately, we still need to find the general way to design $W(Y(x))$ and $L(Y(x), x_d, u)$ such that condition (7.32) is satisfied. As we show in example 2 of the next section, solutions can be found through try-and-error design for nonlinear systems with special format.

7.5 Illustrative Example

Example 7.5.1 The system considered is an inverted pendulum. This system has four states: x_1, the position of base; $x_3 = \dot{x}_1$, the velocity of the base; x_2, the angular position of the pendulum; $x_4 = \dot{x}_2$, the angular velocity of the pendulum. It is assumed that x_1, x_2 and x_3 are measurable. The input variable is the input voltage, u, to the power amplifier which drives the motor. This is a nonlinear unstable system which is stabilized by an observer-based feedback controller [28]. The linearized closed-loop system can be described as follows:

$$\begin{aligned} \dot{x} = & \ (A + \Delta A)x + Bu + Br_{ref} + B\xi \\ y = & \ Cx + f_s \end{aligned} \tag{7.33}$$

where $u = -K\hat{x}$, and $\hat{x}$ is the estimation of the state x. It also incorporates the system input r_{ref}, the desired position of the moving base, into the amplifier input. ξ represents the effects of the nonlinear friction in the drive train on the pendulum motion. The numerical values of the system matrix A, B, C and the controller gain K are,

$$A = \begin{bmatrix} 0 & 0 & -1.399 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.1389 & -0.546 & 0.001905 \\ 0 & 21.7 & 6.236 & -0.2902 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ -4.192 \\ 47.82 \end{bmatrix}, K = \begin{bmatrix} -3.7219 \\ 3.615 \\ 4.7994 \\ 0.7849 \end{bmatrix}^T,$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

The matrix ΔA is unknown, which represents model uncertainty due to the system's nonlinearity. This can be expressed by

$$\Delta A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \Delta a_{32} & \Delta a_{33} & \Delta a_{34} \\ 0 & \Delta a_{42} & \Delta a_{43} & \Delta a_{44} \end{bmatrix}.$$

All system uncertainty and disturbance can be grouped together as Gd, where

$$Gd = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta a_{32}x_2 + \Delta a_{33}x_3 + \Delta a_{34}x_4 - 4.192\xi_1 \\ \Delta a_{42}x_2 + \Delta a_{43}x_3 + \Delta a_{44}x_4 - 47.82\xi_2 \end{bmatrix}.$$

The states and disturbance ξ are bounded, thus the unknown input d is bounded. For details of controller design we refer the reader to [28]. Because actuator fault vector $F_a = B$ belongs to $Im(G)$, it is not separable from the unknown inputs. The detection and isolation of sensor faults is considered here.

We first note that state x_1 and output y_1 form an unknown input free and observable subsystem,

$$\dot{x}_1 = -1.399x_3; y_1 = x_1,$$

where x_3 is the state for a subsystem subject to the unknown input,

$$\dot{x}_3 = -0.1389x_2 - 0.546x_3 + 0.001905x_4 - 4.192u + d_1; y_3 = x_3.$$

The x_1 is equivalent to the x_b subsystem in SCB form, and all states are measurable. Thus, the Utkin SMO is simplified as SMOO, which is constructed as

$$\dot{z} = -1.399y_3 + \Lambda sign(y_1 - z).$$

If $r = y_1 - z$, we know that

$$\dot{r} = 1.399f_s^3 + \dot{f}_s^1 - \lambda sign(r).$$

and

$$r_{eq} = 1.399f_s^3 + \dot{f}_s^1.$$

Therefore, the residual will be sensitive to faults of sensor 1 and 3. Because the dimension of the residual vector is 1, it is impossible to isolate these two faults.

We note that x_2, x_4 is in the form of a two dimensional x_d subsystem. The following SMFO can be constructed:

$$\begin{aligned} \dot{\hat{x}}_2 &= \hat{x}_4 + \lambda_1 sign(y_2 - \hat{x}_2) \\ \dot{\hat{x}}_4 &= 21.7\hat{x}_2 - 0.2902\hat{x}_4 + 6.236y_3 + 47.82u + \lambda_2 sign((\lambda_1 sign(y_2 - \hat{x}_2))_{eq}) \end{aligned}$$

Thus, we can detect f_s^2 by the following fact:

$$r_2 = (\lambda_1 sign(y_2 - \hat{x}_2))_{eq} = e_4 + \dot{f}_s^2$$

Figure 7.2 shows the simulation result of SMFO where sensor noise with maximum magnitude 0.01 is introduced, and $\lambda_1 = 2000, \lambda_2 = 500$. We use the saturation function to replace the sign function, and the upper limit is set to be 0.02. The subplot a) is the shape of incipient sensor fault signal f_s^2, and subplot b) is the residual r_2. It is noted that a peak appears in the residual when the fault occurs.

Example 7.5.2 To illustrate the fault diagnosis for a nonlinear system using SMFO, the three-phase current motor studied in Chapter 6 is considered. Its model is repeated

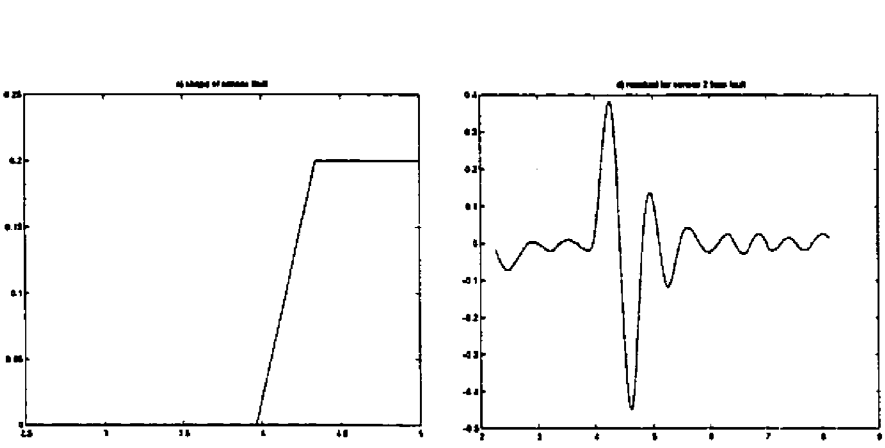

Figure 7.2: Fault diagnosis simulation using SMFO for an inverted pendulum

as below:

$$\dot{x} = \begin{bmatrix} x_2 \\ -A_1x_2 - A_2x_3sinx_1 - A_3sin2x_1 \\ -D_1x_3 + D_2cosx_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} f_{a1} \\ f_{a2} \end{pmatrix}$$

$$y = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}.$$

We consider the isolation of faults f_{a1} and f_{a2} through the multiple observer scheme. f_{a1} represents the fault of actuator 1 and the component faults which lead to change of parameters A_1, A_2 and A_3. f_{a2} represents the fault of actuator 2 and the component faults which lead to change of parameters D_1 and D_2.

First, we regard f_{a1} as an unknown input, and design SMFO as insensitive to f_{a1}. In this case, it is noted that x_3 is an unknown input free subsystem with one output. Because it is one-dimensional, the SMOFO design for x_3 is simplified as a simple SMOO. It is easy to show that x_1 and x_2 can be estimated using the SMFO technique proposed in Chapter 6, even if there is an unknown input in the x_d subsystem formed by x_1 and x_2. Generally, as we discussed in Section 7.4, the estimation of x_d is required in order to form a SMOFO for x_o subsystem without unknown input. However, it is unnecessary here because x_3 is only affected by x_1, and x_1 is an output. Finally, the SMOFO is

$$\dot{z} = -D_1y_2 + D_2cos(y_1) + u_2 + \lambda sign(y_2 - z). \tag{7.34}$$

It is easy to show that this SMOFO is only sensitive to f_{a2}. The corresponding residual is

$$r_1 = (\lambda sign(y_2 - z)_{eq} = f_{a2}$$

In the second observer design, f_{a2} is considered as an unknown input. In this case, there is no unknown input free subsystem. However, the following SMO can be designed using the techniques in Chapter 6:

$$\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x}_2 + \lambda_1 sign(y_1 - \hat{x}_1) \\
\dot{\hat{x}}_2 &= -A_1\hat{x}_2 - A_2\hat{x}_3 siny_1 - A_3 sin2y_1 + u_1 + \lambda_2 sign((\lambda_1 sign(y_1 - \hat{x}_1)_{eq}) \\
\dot{\hat{x}}_3 &= -D_1\hat{x}_3 + D_2 cosy_1 + u_2 + \lambda_3 sign(\overline{e_3})
\end{aligned}$$

where

$$\overline{e_3} = \frac{(\lambda_2 sign(\overline{e_2}))_{eq}}{-A_2 siny_1} = e_3 + \frac{f_{a1}}{-A_2 siny_1}. \tag{7.35}$$

Assuming $\hat{f}_{a1} = \frac{f_{a1}}{-A_2 siny_1}$, we know that

$$r_2 = (\lambda_3 sign(\overline{e_3}))_{eq} = D_1\hat{f}_{a1} + f_{a2} + \dot{\hat{f}}_{a1}.$$

Therefore, we have the following fault diagnosis logic:

1. If $r_1 = r_2 \neq 0$, only f_{a2} happens.

2. If $r_1 = 0, r_2 \neq 0$, only f_{a1} exists.

3. If $r_1 \neq 0$, $r_2 \neq 0$ and $r_1 \neq r_2$, both faults f_{a1} and f_{a2} exist.

If $y_2 = x_3$ is not available, the SMOFO for fault diagnosis of actuator 2 cannot be designed, or r_1 will not be available. In this case, fault detection based on r_2 is possible, and fault isolation will be difficult. However, an abrupt fault of f_{a1} can still be isolated because only the abrupt change of f_{a1} will make the signal $\overline{e_3}$ (given by equation (7.35)) becomes nonzero for a short while. Therefore, we can record the third residual signal as $r_3 = \overline{e_3}$.

Figure 7.3 is the simulation result. For simplicity, it is assumed that the original estimation error is zero. The solid line and dashed line in subplot a) are the shapes of f_{a1} and f_{a2} respectively, and subplot b) is the residual signal r_1, c) is the residual signal r_2, d) is the residual signal r_3. Obviously, r_3 stays near zero for slow-varying f_{a1}, and only produces a nonzero value peak for abrupt f_{a1} at time $t = 18s$. The residuals r_1 and r_2 validate our fault isolation logic very well.

7.6 Conclusions

In this chapter, we have extensively discussed robust fault diagnosis using SMFO. We have shown that the detection of a large and abrupt fault is feasible for a general class of nonlinear uncertain systems. Special attention has been paid to incipient fault diagnosis using SMFO.

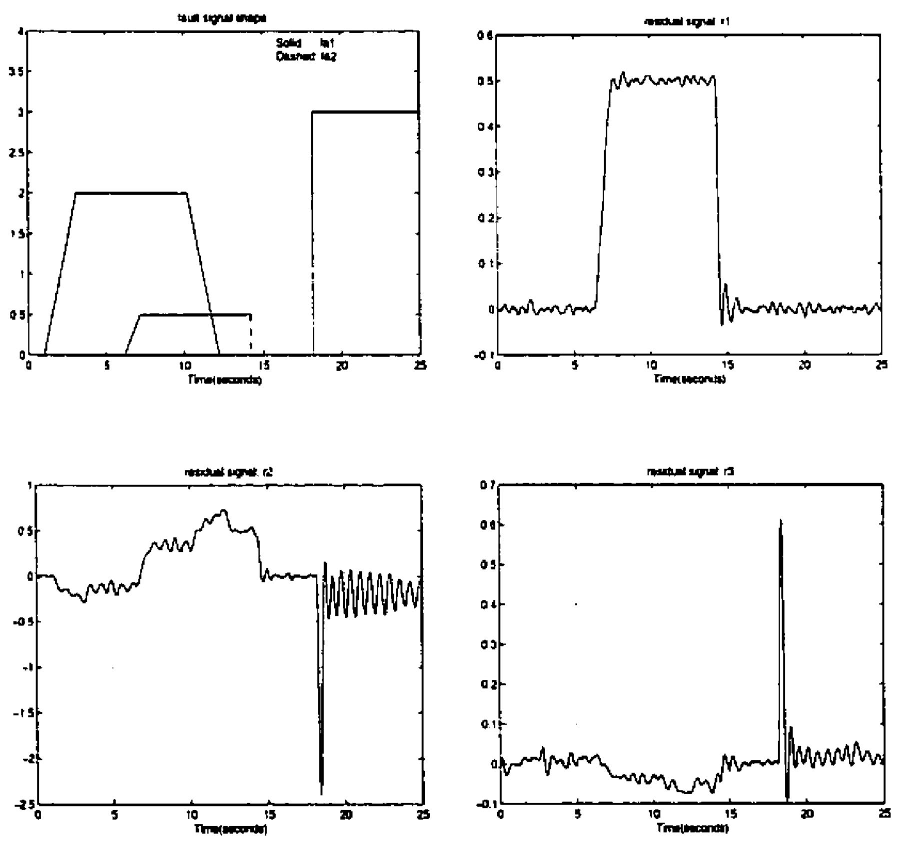

Figure 7.3: Fault diagnosis simulation using SMFO for a nonlinear three phase motor model

Chapter 8

Conclusions

Two kinds of robust observer, the unknown input functional observer (UIFO) and the sliding mode functional observer (SMFO), as well as their application for robust fault detection and isolation (FDI) of uncertain linear and nonlinear dynamic systems, have been considered in this thesis work. The contribution of the thesis have been the subjects of several publications [125, 127, 128, 129, 130, 131, 132], and are summarized as follows.

1. The necessary and sufficient conditions for the existence of UIFO are obtained. The solvable conditions of unknown input estimator, which does not use derivative of output, are obtained also. The systematic design procedures for UIFO and the input estimator are provided.

2. The relationship between an unknown input observer (UIO) and unknown input fault detection observer (UIFDO) is explained clearly within the UIFO framework. In addition, several new design approaches of UIFDO are proposed based on the complete solution of UIFO. The solution centers on finding the remaining design freedom after unknown input decoupling, such that robust FDI is achieved by combining unknown input decoupling theory and the Beard-Jones detection filter, or input estimator. The structural properties for multiple actuator and sensor faults FDI are provided, which are proved to be different from multiple actuator faults FDI.

3. The robust fault diagnostic observer for bilinear systems is studied and several new results are given. Two input independent bilinear fault diagnostic observers are proposed. The first one has linear estimation error dynamics, and owns the advantage of simplicity. The second one has bilinear estimation error dynamics, and can be designed under less conservative conditions. For the bilinear systems with bounded control input, the structural constraints for robust FDI are further alleviated.

4. The inherent and hidden principle of the Walcott-Zak sliding mode observer (SMO), as well as its connection with UIO, is revealed. SMFO using equivalent control method is proposed for both linear uncertain systems, and for a general class of nonlinear uncertain systems. Compared with UIFO, the proposed SMFO works for system with more general uncertainty structure.

5. The robust FDI schemes based on SMFO are developed, which achieve better FDI ability under certain conditions than do UIFO based FDI schemes.

The robust observer and observer-based fault diagnosis have been studied extensively over the last two decades. Several problems considered in this thesis were formulated more than ten years ago and are well known in the fault diagnosis community. However, only sufficient conditions for their solvability are found. The significance of our work is that we provide not only sufficient, but also necessary conditions for the existence of their solutions. Further, we propose several new robust fault diagnosis approaches, which outperform certain schemes proposed by earlier researchers. The results developed in this thesis suggest several immediate directions for extensions, among them the following:

1. We have focused on the complete decoupling of the disturbance or unknown inputs, which will lead to the optimal and prompt detection of incipient faults. Of course, the robust FDI schemes based on this idea require that the systems to satisfy strong structural conditions. If the disturbance to the system is not very significant, compared to the effects of incipient faults, it is valuable to use the H_∞ technique or linear matrix inequality (LMI) technique to find a suboptimal

residual solution. Several papers in this direction have been noted (see [27, 71, 79]), and many efforts have to be made before its practical application.

2. Unknown inputs can only represent structured uncertainties in the system. The way to enhance the ability to handle unstructured uncertainties is an important issue. It is desirable to make FDI schemes robust for both structure and unstructured uncertainty such that the false alarm ratio will be reduced further.

3. Time delay in states or outputs is a common phenomenon in many industrial systems. However, FDI for these systems has not been researched extensively. Yang [136] extended UIO theory to time-delay systems, and proposed the FDI scheme for time-delay systems based on their corresponding UIO theory. It is valuable to further extend the UIFO theory to time-delay systems to derive a better FDI scheme.

4. The FDI scheme using the proposed SMFO makes some progress for robust FDI of nonlinear uncertain systems. However, because of the difficulties in estimating the state or measurement vector of a nonlinear system, there is long way to go to get practical solutions of the FDI for general nonlinear model. Even if the nonlinearities are known and no disturbance is present, the nonlinear observer theory is far from being complete and is not ready for industrial application.

5. The results derived in this thesis assume continuous-time system models. Extending these results to discrete-time systems is necessary and is not trivial. There is very little work available in the literature on the subject of nonlinear discrete-time observers. The FDI for discrete-time nonlinear systems will be very tough. Lots of effort should be expected in order to develop a robust SMFO for a discrete-time nonlinear system.

The observer-based robust FDI methods are constrained to the faults of actuators, sensors and some special component faults. They are also limited to handle a certain class of disturbance, and model uncertainties. If the unknown input vectors are linearly dependent with the actuators faults vectors, the method in this thesis has no way to decouple the effects of the unknown inputs from the effects of the faults. On the other hand, due to fault propagation phenomena and complexity of the system, observer-based methods are unable to identify the real source of failure. Recently, people have begun to research the way to integrate different kinds of quantitative FDI methods. For example, the parameter estimation and robust observer techniques have been combined together to achieve more powerful FDI schemes [142].

While the quantitative method relies heavily on the mathematical model of the systems, it is valuable to consider the integration of quantitative and qualitative FDI methods for a wide variety of practical situations. It is very difficult or even impossible to obtain a full model of a complex object of diagnosis (e.g. a chemical plant or sugar factory). but models of specified parts are often obtainable, and operators know various functional relationships of those parts. There are many topics for future research in this direction, such as interface with the operator, scheduling, planning, and conflict coordination of different methods. It is believed that the integrated FDI scheme will be a popular research topic in the many years to come.

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