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Stochastic Analysis of Robot-Safety Systems

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By
© Ali R. M. Fashandi

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**THIS WORK IS DEDICATED TO MY MOTHER
WHOM I LOVE MORE THAN LIFE ITSELF**

ABSTRACT

Robot population is increasing at an incredible pace. Over the last fifteen years, robot population grew from 30,000 in 1983, to the forecasted 820,000 by the end of 1998. Their infancy period has come to an end and they are not just being used in the automotive industry or required to perform simple tasks. They are now being employed in various sectors of industry and handle much more complex operations.

Increased robot system complexity and their critical applications utilization have led to various reliability and safety problems. In 1982, the Machine Tool Trade Associations guidelines stated that a working robot can be a potential hazard to personnel under certain circumstances. The need for robot system safety was highlighted by a 10-million dollar lawsuit awarded to the family of a worker killed by an industrial robot in 1983.

This study presents a detailed introductory aspect of robot safety, an identification of the most appropriate robot systems reliability and safety assessment techniques, and probabilistic modelling of robot-safety systems. The domain of the probabilistic models include: a stochastic analysis of a system containing one robot with n-redundant safety units, a stochastic analysis of a system composed of n-redundant robots with one safety unit, and an availability analysis of robot systems susceptible to common-cause failure. The

primal intent of the analyses is to develop generalized and numerical expressions relating to the performance indices for robot systems operating with or without the safety unit. Generalized models are introduced and generalized expressions including reliability, time-dependent availability, steady-state availability, and mean time to failure (MTTF) are developed. In order to assess performance indices, some special cases of the generalized models are presented resulting in the formation of numerical values.

Robot system performance indices are determined by means of the Markovian and non-Markovian methods. The method of supplementary variables and the device of stages are used to deal with the non-Markovian models. Various failed system repair time distributions (i.e., exponential, gamma, Weibull, Rayleigh, and log-normal distributions) have been considered to obtain generalized steady state availability expressions. Markov method is utilized in models where failure and repair rates are assumed constant. With the aid of Laplace transforms, a system of first-order differential equations are solved and generalized reliability and MTTF expressions are developed.

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INTRODUCTION
AND OVERVIEW

1.1 Introduction

Human confidence in robots has been steadily increasing over the years. Robot utilization is no longer confined to simple arc or spot welding, but to more complicated applications such as underwater exploration, outer space exploration, fire fighting, and medicine. Exploring the earth's ocean floors and the recent mission to the plant Mars are indeed prime examples of the fact that robots have gone where no man has ever gone before.

Robots are now significantly complex machines handling critical responsibilities and are expected to operate flawlessly. As the robots make use of electrical, mechanical, pneumatic, and hydraulic components, the many possible sources of failures render the complete system's reliability quite challenging. A great deal of progress has been made to make robots safe and reliable, there is however still much room for improvement. According to the published literature, recorded robot mean time between failure is around 500 to 2500 hours and at least ten fatal accidents involving robots have occurred with corresponding millions

1

of dollars in loses [109]. Two examples of such accidents are as follows [318,429]:

- A maintenance person climbed over a safety fence without turning off power to the robot and performed necessary tasks in its area while it was temporarily halted. When the robot resumed movement, it pushed the maintenance person into a grinding machine and, consequently, the man died.

- A worker turned on a welding robot, meanwhile another person was still in its work zone, consequently, the person in its work zone was pushed into the positioning fixture by the robot and died later.

Just as for other engineering products, a robot must not only be reliable but also safe. An unreliable robot may become an unsafe robot and cause unsafe conditions, high maintenance costs, inconvenience, and so on.

This study is concerned with the techniques applicable to robot reliability and safety, the robot reliability and safety relationship, and considers the effect of safety mechanism failures on robot system overall performance indices, mainly, availability, reliability, and mean time to failure.

1.2 Robot Development: An Overview

The word "Robot" first entered the English language in 1923 when Karel Capeck's play Rossum's Universal Robots (R.U.R) was translated and introduced to the English speaking world, the Czech word for "Worker" [204]. Robots mean different things to different people. Many definitions have been suggested and discarded. Webster's dictionary defines

a robot as a mechanism guided by automatic controls. The Robotics Institute of America

(RIA) defines a robot as [336];

> "A reprogramable multi-functional manipulator designed
>
> to move material, parts, tools or specialized devices,
>
> through variable programmed motions for the performance
>
> of a variety of tasks."

The Japanese Industrial Robot Association (JIRA) defines a robot as [394];

> "An all purpose machine equipped with a memory device
>
> and terminal, capable of rotation and of replacing human
>
> labor by automatic performance of movements."

Automation is not a recent innovation, in fact it dates back 5000 years when Egyptians

built water-powered clocks and the Chinese built water and steam-powered toys [109]. The

idea of man made artificial intelligence or functional robot however, is first created in the

thoughts of the Greek philosopher Aristotle (4th century B.C.) when he wrote:

> "If every instrument could accomplish its own work,
>
> obeying or accomplish the will of others..." [90]

Aristotle's idea becomes reality when in 1801, the Frenchman Joseph Marie Jacquard

invented the first intelligent loom machine to weave patterns according to information on

punched paper cards [30].

The eighteenth century witnessed the birth of the industrial revolution, but the true

beginning of machine age occurred by the end of the nineteenth century when machines

were substituting human physical capabilities as large steam engines, gasoline engines, and

electric motors were introduced. Hard automation[a] vastly developed and took hold in factories through World War I and II. With the growth in population and rapid increase in demand, supplies were matched by automated production lines.

Although automated machines and artificial intelligence tend to look to us like more of the same, it would be a mistake to think that they are. Powered machines provide the physical amplification of human work, though incapable of human mental capabilities. Emergence of modern computers made it possible for the automation industry to enter a new era. It provided the missing ingredient, "the brain". This occured in 1948 when the world witnessed two major technological advancements. The first computer with program storage capability was built, and also the discovery of the transistor which revolutionized the computer industry [407]. In 1954, George Devol [489] put all the elements together and designed a programmable device that is generally considered to be the first industrial robot. In 1959, the first commercially available robot was manufactured and sold by the Planet Corporation and in 1967, Japan imported its first robot [109,489]. In 1970, the first symposium on industrial robots was held in Chicago, USA and in 1975, the Robot Institute of America (RIA) was founded.

For the first decade after its birth, robot population increased at 20% annually and by 1983, world robot population was estimated at 30,000 [406, 478]. According to the international federation of robotics the world wide robot population was 350,000 in 1987 610,000 in 1994, and it forecasted 820,000 robots by the end of 1998 [381,421]. Robot's

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[a]Hard automation is defined [415] as machines which are designed to perform specific functions. In these systems, every change in standard operation demands a change in machine hardware and setup.

population growth and a breakdown of robot population for major user countries in 1998

are shown in Figures 1.1(a) and 1.1(b), respectively.
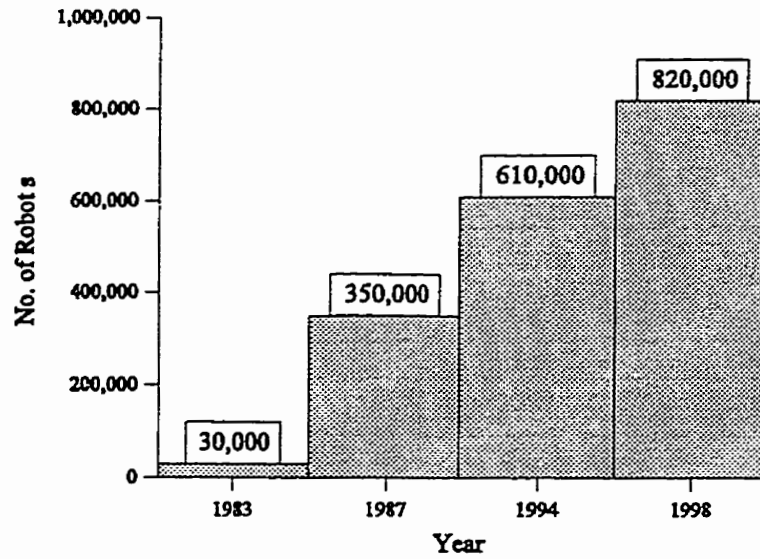
(a)
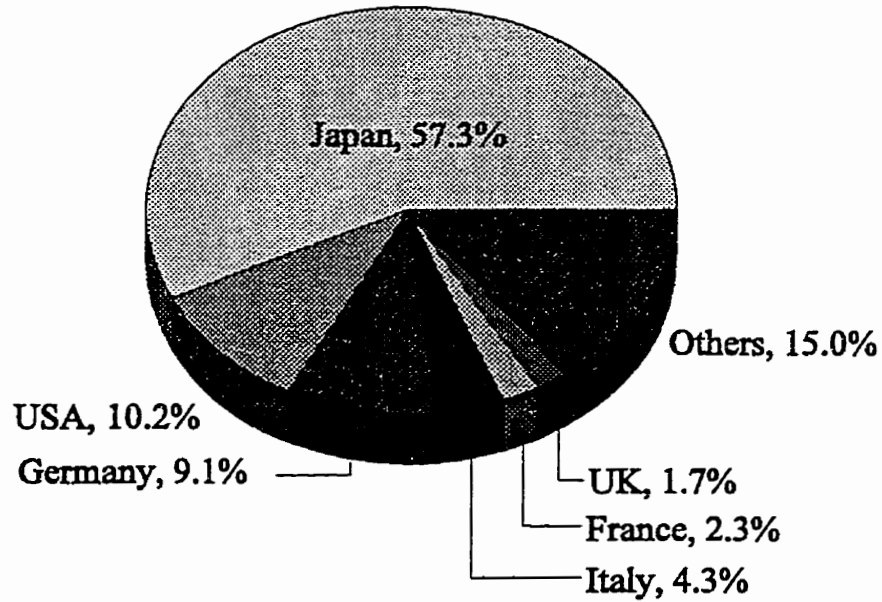
(b)

Figure 1.1.   Robots: (a) population growth, (b) population breakdowns as forecasted for
major user countries in 1998.

Although in the early years of robotics, applications were concentrated in the automotive industry, recently however, technology has diversified and is vastly utilized in other sectors of industry as well. A future potential market will undoubtedly be the general consumer where robots may well become another household item. By the year 2015, the predicted figure for the robots to be used for performing household tasks is over 5,000,000 [228]. This makes the reliability and safety factors even more crucial since personal robots have to work among human beings. Therefore, robots have to be much more reliable and safe, so as not to injure humans should a malfunction occur. In any case, the exponential increase in robot utilization underlines the fact that robots are here to stay. Not surprisingly, Polakoff expressed his passion for robots and wrote [354]:

Man's marriage to robotics: A " for better or worse"

1.3 Literature Review: Robot Reliability and Safety

This review is conducted by categorizing published literature on robot reliability and safety into different classifications. The collected publications listed include conference proceedings, technical reports, journals, and books from 1973 to 1997 [115a]. Table 1-1 illustrates the classification of references, and Table 1-2 presents sources of journal and conference proceeding papers. The publications on the subject are grouped into three main classifications: robot reliability; robot safety; and miscellaneous.

The robot reliability classification includes publications that discuss methods, evaluation, and modeling techniques to assess overall robot system reliability. The robot safety category contains publications that emphasize various aspects of robot safety. In turn, these

publications are classified under many categories: general safety, safeguarding techniques and methods, robot accidents, robot safety standards, safety systems/technologies, and human factors. The miscellaneous category covers published literature that discusses relevant topics dealing with both robot reliability and robot safety directly or indirectly.

Both robot reliability and robot safety classifications are reviewed separately in the following sections and deal with the most recent and the most important publications.

1.3.1   Robot Reliability

Enormous amounts of studies have been performed to perfect the robot's precision (i.e., positioning, repeatability, and accuracy), recently however, a careful attention has been given to robot system reliability [109]. Increased attention toward robot system reliability in the recent years is illustrated by Figure 1.2.
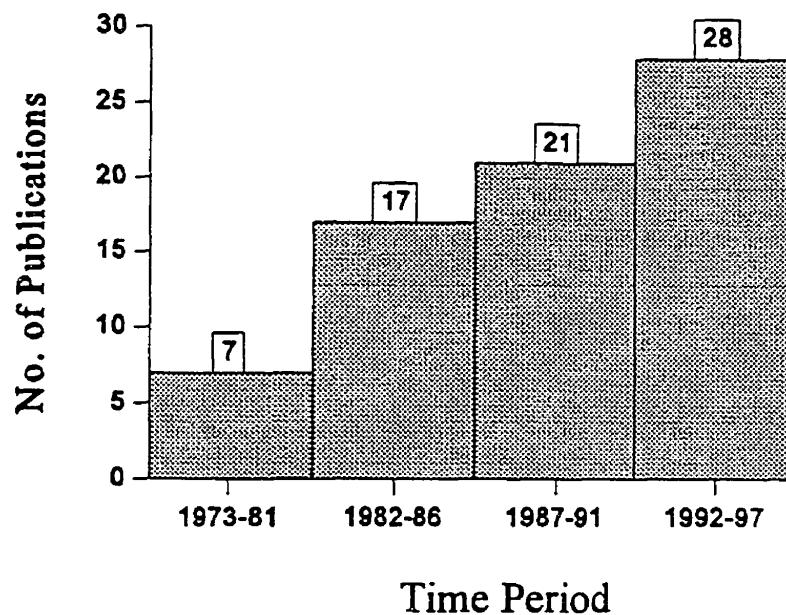
Figure 1.2.   Profile of publications on robot reliability.

Table 1-1:   Classification of Publications on Robot Reliability and Safety

<u>Categories</u>	<u>References</u>
(i) Robot Reliability	[3, 9, 28, 33, 36, 49, 55, 63, 76, 77, 78, 79, 84, 86, 93, 98, 100, 109, 111, 112, 113, 119, 122, 123, 124, 125, 126, 146, 147, 163, 171, 189, 212, 214, 215, 216, 231, 232, 233, 234, 239, 244, 245, 249, 272, 273, 277, 286, 290, 299, 304, 333, 355, 358, 389, 402, 403, 407, 418, 423, 446, 448, 462, 469, 472, 473, 481, 487, 490, 491, 492, 494]
(ii) Robot Safety	
● **General:**	[11, 12, 18, 26, 34, 40, 41, 44, 52, 59, 64, 94, 103, 120, 129, 135, 141, 154, 158, 166, 169, 172, 174, 183, 184, 213, 230, 278, 279, 282, 289, 298, 300, 303, 308, 318, 319, 326, 331, 349, 350, 359, 364, 367, 370, 373, 377, 378, 379, 380, 391, 406, 413, 425, 426, 427, 433, 438, 449, 458, 460, 463, 466, 486, 496, 497, 501]
● **Human Factors:**	[16, 30, 46, 47, 48, 54, 60, 95, 99, 104, 105, 106, 128, 131, 144, 156, 173, 177, 178, 179, 181, 182, 187, 190, 191, 198, 200, 217, 224, 225, 226, 257, 260, 280, 281, 309, 310, 311, 324, 325, 328, 329, 330, 332, 337, 338, 339, 341, 342, 343, 344, 365, 372, 390, 409, 411, 417, 441, 475, 482, 483, 493]
● **Accidents:**	[5, 58, 62, 67, 69, 70, 71, 72, 87, 88, 89, 90, 132, 136, 137, 162, 205, 206, 209, 221, 222, 223, 243, 251, 288, 292, 312, 316, 340, 347, 354, 383, 392, 397, 398, 412, 419, 420, 429, 430, 431, 434, 435, 437, 455, 456, 461]
● **Safety Systems:**	[1, 7, 13, 19, 20, 21, 22, 23, 24, 25, 35, 56, 57, 73, 81, 92, 97, 118, 127, 149, 150, 151, 152, 161, 164, 188, 193, 218, 235, 237, 238, 241, 247, 269, 270, 271, 274, 283, 284, 293, 294, 301, 305, 306, 307, 313, 314, 315, 352, 353, 360, 361, 369, 388, 401, 404, 422, 440, 447, 452, 454, 464, 474, 479, 485, 488, 498]
● **Safety Method:**	[2, 4, 6, 8, 17, 27, 32, 37, 38, 43, 53, 65, 74, 75, 80, 83, 96, 130, 142, 145, 153, 155, 157, 160, 180, 192, 210, 211, 219, 227, 248, 250, 252, 255, 259, 263, 265, 266, 291, 297, 317, 334, 335, 362, 363, 366, 375, 384, 385, 393, 394, 395, 396, 399, 400, 405, 410, 414, 428, 436, 439, 443, 444, 445, 467, 470, 471, 476, 499, 500, 502]
● **Safety Standards:**	[14, 39, 42, 50, 51, 68, 82, 101, 114, 116, 133, 138, 140, 143, 148, 159, 170, 185, 194, 199, 201, 207, 208, 246, 253, 256, 261, 262, 264, 275, 276, 285, 296, 302, 336, 345, 346, 348, 351, 356, 357, 374, 376, 382, 386, 387, 408, 432, 457, 480, 495]
(iii) Miscellaneous	[10, 15, 29, 31, 45, 61, 66, 85, 91, 102, 107, 108, 110, 115, 117, 121, 134, 139, 165, 167, 168, 175, 176, 186, 195, 196, 197, 202, 203, 204, 228, 229, 236, 240, 242, 258, 267, 268, 287, 295, 320, 321, 322, 323, 327, 368, 371, 381, 415, 416, 421, 424, 442, 450, 451, 453, 459, 465, 468, 477, 478, 484, 489]

Table 1-2:   Sources of the most journal and conference proceeding papers listed in the references

Journal

- American Machinist
- Chemical and Petroleum Engineering
- Computer World
- Computing and Control Engineering Journal
- Human Factors
- IEEE Transactions on Automatic Control
- IEEE Transactions on Software Engineering
- IEEE Transactions on Industrial Applications
- IEEE Transactions on Reliability
- Institute of Industrial Engineers (IIE) Transactions
- Industrierobotor
- International Journal of System Science
- Journal of Occupational Accidents
- JSME International Journal
- JSME Bulletin
- Machine Tool Research
- Microelectronics and Reliability
- National Safety News
- Nuclear Engineering International
- Plant Maintenance
- Plant Engineering
- Professional Safety
- Robot New International
- Robotics
- Robotics World
- Robotics and Autonomous Systems
- Robotics and Computer-Integrated Manufacturing
- Robotics Today
- Robotics Engineering
- Robotics Age
- Safety and Health
- Soviet Engineering Research

Conference Proceedings

- ASCE Speciality Conference Proceedings
- Proceedings of Robot Safety Conference
- Proceedings of the Conference on Remote System Technology
- Proceedings of the Robotic Industries Association's Robot Safety Seminar
- Proceedings of the 4th, 6th, and 7th British Robot Association Annual Conferences
- Proceedings of the IEEE Southeast Annual Conference
- Proceedings of the 4th National Reliability Conference
- Proceedings of the 1987 ASME Design Automation Conference
- Proceedings of the 1990 International Industrial Engineering Conference
- Proceedings of the International Seminar on Safety in Advanced Manufacturing
- Proceedings of the RI/SME AUTOMACH Conference
- Proceedings of the Robot 8 Conference
- Proceedings of the Robot 9 Conference
- Proceedings of the 1st Robotic Europe Conference
- Proceedings of the 8th International System Safety Conference
- Proceedings of the Annual Reliability and Maintainability Symposia
- Proceedings of the 3rd Canadian CAD/CAM and Robotics Conference
- Proceedings of an International Conference on Robotics and Factories of the future
- Proceedings of the IEEE International Conference on Robotics and Automation
- Proceedings of the 17th Annual Electronics and Aerospace Conference
- Proceedings of the 2nd Conference on Industrial Robot Technology
- Proceedings of the 31st Annual Meeting of the Human Factors Society
- Proceedings of the 37th Annual Meeting of the Human Factors Society
- Proceedings of the Annual International Industrial Ergonomics and Safety Conference
- Proceedings of the Robot VI Conference of the Society of Manufacturing Engineers
- Proceedings of Spie—the International Society for Optical Engineering
- Proceedings of the Workshop on Object Oriented Real Time Dependable Systems (WORDS)

Engelberger [123] in 1974, compared the reliability of an numerically controlled (NC) machine and of an industrial robot and concluded that since NC machines carry more than 90% up-time, an up-time of 97% should be expected from robots to satisfy its users. In the same year, Haugan [171] performed a study on the reliability in industrial robots for spray gun applications.

In 1975, Thomopoulos [446] presented a probabilistic method related to the design analysis of certain classes of industrial robots and, in general, of machines subjected to numerous combinations of stress, including cyclic loadings.

A year later in 1976, Engelberger [124] assessed the reliability of the unimate-2000 robot. The robot was examined in modules which consisted of components for which failure rate data was available from a data base. The Mean Time Between Failures (MTBF) was estimated to be around 500 hours.

In 1983, Jones and Dawson [215] on the basis of collected data, emphasized on the fact that robot installation do give some concern about variable reliability and potential for injury and harm.

In 1984, Khodabandehloo et al. [233] presented an assessment of robot reliability by applying Failure Mode and Effect Analysis (FMEA), Event-Tree Analysis (ETA), and Fault-Tree Analysis (FTA). Same researchers utilized similar methodologies to deal with other aspects of robot reliability with particular reference to safety [232]. Alayan et al. [9] discussed reliability of robot networks and presented techniques for its evaluation.

Critchlow [98] in 1985, discussed robots expected useful life and other parameters. He suggested that well-designed robots are expected to have useful life of at least 40,000

working hours, Mean Time Between Failures (MTBF) of at least 400 hours, and a Mean

Time To Repair (MTTR) of no more than 8 hours. Comprehensive bibliography on robot

reliability and safety was presented by Dhillon [108] in 1987.

Chamin et al. [79] in 1989, recognized the bulk of downtime related to breakdown of

equipment in robot based sections (RBS) and flexible manufacturing systems was due to

failure of secondary technological equipment. To improve the reliability, they developed a

standard loading device for piece-wise loading of cylindrical blanks for RBS used in

machining operations. Jin et al. [214] measured probabilistic behavior and reliability analysis

for a multi-robot system by applying Petri net and Markov renewal process theory.

Barabanov and Chirkov [36] in 1990, investigated the reliability of devices and systems

which tend to cause failure of robot-assisted manufacturing cells (RAMCs) in their initial

period. They developed a method for determining the probability of devices and systems

having a running in period and optimum test-run for the elimination of faults. Gopinath et

al. [146] designed and presented an overview of a special model of distributed computation

to handle the issues of robot planning. Dashui and Wells [100] presented a reliability model

and proposed a design approach for robotics assembly operation which could estimate the

assembly reliability over a robot work-space.

In 1991, Wewerink [472] considered a modeling approach to describe complex manned

robotics systems. The objective of the model was to answer questions related to the

reliability and efficiency, design alternatives, function allocation, automation, etc. In a book

titled "Robot Reliability and Safety", Dhillon [109] covered many important aspects

concerning robotics. The main objective of the book was to be wide in scope, in particular

with respect to reliability.

Zheng [490] in 1992, presented a novel approach which utilized explanation based learning as a framework for acquisition of failure recovery knowledge, and thus reducing the dependency of the automation system upon a human operator in recovering from failure states. Khodabandehloo [231] stated that the safe and reliable performance of robot systems depend on many factors, including the integrity of the robot's hardware and software, the way it communicates with sensory and other production equipment, the reliable function of the safety features and the way the robot interacts with its environment.

Celestine and Park [78] in 1993, conducted a reliability experiment with a laboratory scale, teach-pendant robot. They identified robot system failure modes and characterized the robot system reliability using a structure function model from system theory. Dhillon and Anude [110] extended earlier lists of references on robot reliability and safety.

In 1994, Schneider et al. [403] performed a case study and showed that the framework for the Reliability Performance Index is a useful empirical tool during the selection and design of a configuration of robotics modules of varying reliability and precision.

Maier et al. [277], in 1995, described the derivation of a reliability and safety analysis methodology, and its application in a case study of a handling device (i.e., robot) for the blankets that constitute the fusion reactor Torus.

In 1997, Dhillon and Fashandi [113] presented availability analysis of a system composed of a robot and its associated safety system. Supplementary variable and Markov techniques were employed to obtain expressions for state probability, Laplace transform of the state probabilities, and steady-state availability. The robot system's success based on

maintenance was outlined by Pollard [355] in 1980; Ottinger [334], Cop [93] in 1983;

Munson [304] in 1985; Anon [28] in 1987; Wells [469] in 1992; Schneider [402] in 1993.

1.3.2 Robot Safety

The earliest statement regarding robotics safety may be credited to the famous science

fiction writer Isaac Asimov [31]. At age 21, in 1942, he wrote the three laws of robotics as

presented in Figure 1.3.

Law I: A robot must not harm a human nor, through inaction permit a human to come to harm.

Robotic Laws

Law II: A robot must obey humans unless their orders conflict with law I.

Law III: A robot must safeguard its own existence unless it conflicts with laws I and II.
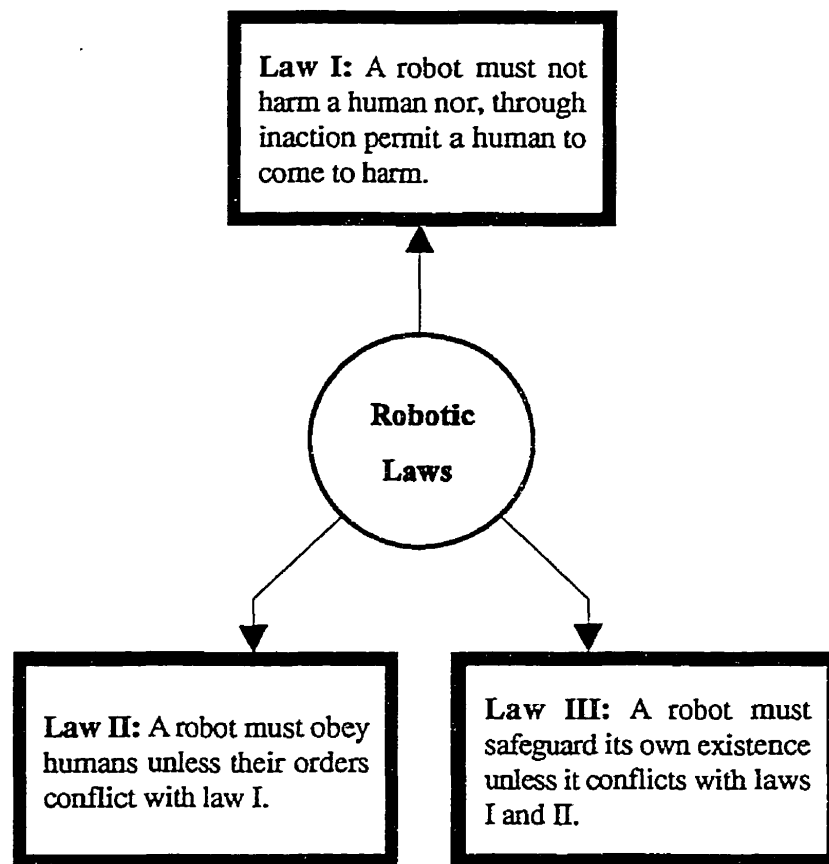
Figure 1.3.   Three laws of robotics.

Historically, one of the fundamental reasons for using robots in industrial applications was to remove human operators from a potentially hazardous work environment. The hazards in the workplace include heat, noise, fumes, radiation, toxic atmosphere, physical dangers and other health hazards. Since the Occupational Safety and Health Act (OSHA) was enacted [162] in 1971, worker safety has become a significant factor in promoting the substitution of robots for human labor in these kinds of dangerous jobs. Although robots may liberate humans from hazardous working conditions, they also bring about safety concerns. The Machine Tool Trade Association guidelines [276] in 1982 stated that a working robot can be a potential hazard to personnel under certain circumstances.

Robot safety was in the front line of robotics technology during the 1980's. In fact, a bulk of literature had been published (i.e., 286 articles) between 1982-88. The need for system safety was highlighted by a 10-million dollar lawsuit awarded to the family of a worker killed by an industrial robot in 1983 [90]. The probability of serious robot accidents and the legal actions have forced manufacturers and users to a quick adoption of safety devices and regulations. A breakdown of publications on robot safety are given in Figures 1.4 and 1.5. Each of the category is described below.
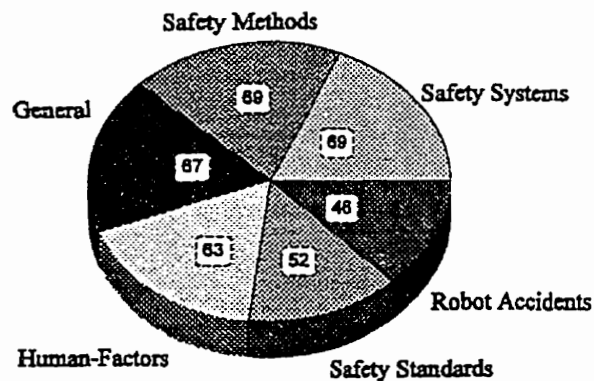
Figure 1.4.   Profile of publications on robot safety.
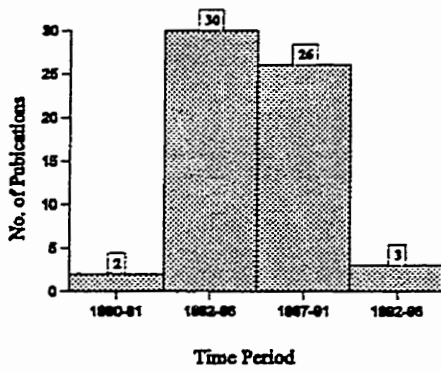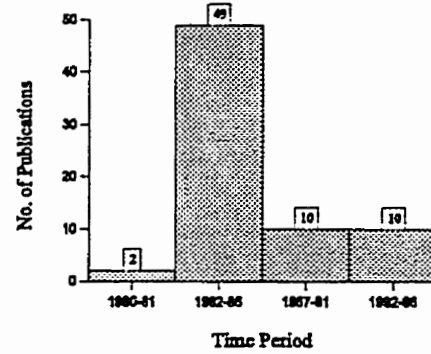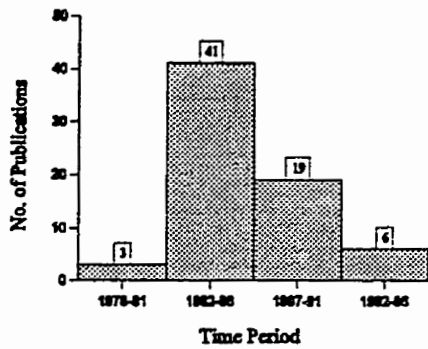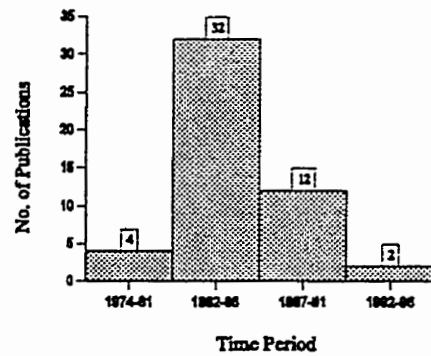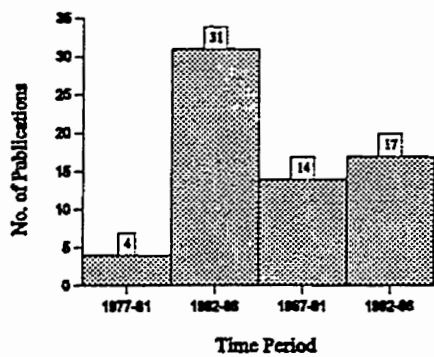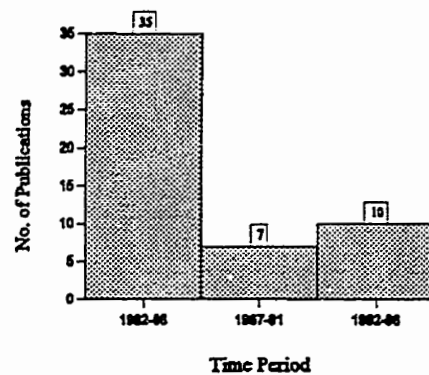
Figure 1.5.   Categories: (a) general, (b) human factors, (c) robot accidents, (d) safety systems, (e) safety methods, (f) safety standards.

1.3.2.1 General Safety

Safety engineering is generally defined [438] as "a system of knowledge of science and technology necessary to determine the cause, clarify the process of and prevent accidents".

In 1978, Park [335] to achieve overall safety made various suggestions: redundancy, fail-safe design of hazard detectors, protection against software failures, protection against hardware failures, intrusion monitoring, use of deadman switches and panic buttons, workplace design considerations, restricting arm motion, and operator training.

In 1980, Engelberger [125] reviewed ambient factors influencing decisions to use robots or presenting environmental requirements. Factors included ambient temperature, shock and vibration, electrical noise and interference, liquid sprays, gases, and harmful particles.

Hasegawa and Sugimoto [169] in 1982, stated that the safety problem of industrial robot is mainly composed of two categories: the one is promoting industrial safety by utilizing industrial robots and the other is avoiding unhappy accidents caused by robots themselves.

Ghosh et al. [142] in 1984, reviewed relevant data on accidents involving robots and discussed the actions that could be taken to increase the intrinsic safety of robots. In the same year, Noro [325] connected education and safety in relationship between man and robot. Similar studies on training coupled to safety were presented by Trouteaud [453], Finnerty [133], and Balanico [34].

In 1985, Ziskovsky [500] suggested that robot safety should start before the robot is introduced into the work environment. He pointed out three steps in establishing any robot system—application planning, installation and start up, and continuing operation. In the same year, Linger [270] proposed the concept of the 'production adapted' safety system, i.e.,

a safety system based on high knowledge about the process. A system safety for automated production to maintain the highest level of safety with the lowest loss of production was proposed by Kilmer [237] while Lee [259] described safety precautions for robot users including: risk analysis, safety consideration, and proper methods of safeguarding.

Jones and Dawson [219] in 1986, established strategies for ensuring safety with industrial robot systems while Deivanayagam [106] addressed the possible safety measures for controlling hazards unique to robotics work systems. DeGregoria [103] reviewed the fundamental of robotics and presented some of the critical aspects of robotics safety. He also suggested guidelines to reduce safety risks to an acceptable level.

In 1987, Ramachandran and Vajpayee [367] investigated robot safety and presented an analysis of the sources of accidents, and the accident-prone operational phases of robotic installations. Similar studies on safety consideration for robotics installation were presented in [38],[50],[143],[262], [460], and [461].

In 1988, Etherton [128,129,130,131] proposed safe maintenance guidelines for robotics workstation.

In 1990, Jiang and Cheng [211] reviewed several robot safety techniques, and presented a procedure analysis for the planning, installation, and operation stages of adding a robot to the workplace. The analysis covered safety measures that could be taken, risk of not taking them, causes of the risks, and corrective or preventive measures.

1.3.2.2 *Human-Factors*

Human-Factors [225] in robotics is the study of principles concerning human behavior and

characteristics for efficient design, evaluation, operation and maintenance of robots.

In 1985, Ghosh and Lemay [144] discussed man/machine interactions during maintenance and methods to improve safety.

Ryan [383] in 1986, presented a review of research efforts in determining the quantification of accident causation based on human behavior model.

Parsons [344] in 1987, explained how the applied science known as "human factors" can help prevent accidents in which robots may harm workers, damage equipment, or themselves. Nagamachi [310] found that approachable distance which people considered safe is when robot arm movement was at slower speed. The closest approach distance at 14 cm/s was 1.5 cm, whereas the median approach at 22 cm/s was 22.5 cm.

In 1988, Karwowski et al. [223] discovered the average approach distance of 20.9 cm for those who had seen the accident, compared to 15.3 cm for those who didn't.

Rahimi and Karwowski [365] in 1990, reviewed critical issues in human-robot interaction area, and proposed a research framework to study human aspects of robotics system design while Beauchamp et al. [48] briefly described and synthesized the empirical results generated in the human factors studies and suggested future research avenues in that area. Same researchers [47] evaluated factors that could effect human performance in the event of an unexpected robot motion.

In 1992, Sun and Sneckenberger [441] introduced conceptual safety guidelines for the design of a human-robot symbiotic system to achieve reasonable allocation of function and optimum match between the human and the robot, to reduce the possibility of human error, and to enhance the system safety and reliability.

1.3.2.3 *Accidents*

Almost all reports involving industrial robot accidents specify an actual or potential risk to those involved in programming, and teaching [72], [206], [210], [226], [292], [316], [437]. Jiang and gainer [213] defined the robot accident as "contact between the person and a robot either directly or indirectly, leading to a record of the accident".

In 1977, Sugimoto [430] in one of the earliest publications on robot accidents showed that the greatest risk of accident occurs during programming, and maintenance. He also pointed out that only 10% of the accidents occurred during normal operation.

On the basis of 1980 investigation by Carlsson [71] carried out in Swedish industry, out of 13 accidents examined, the prime hazard was either operator error or the entrance of worker into the envelope of the robot during its operation in the automatic mode.

The result of another survey conducted by the Japanese Ministry of labor [430] in 1982, reported 11 cases of accidents and 37 of near-accidents. Furthermore, 8 of the 11 accidents (73%) were due to unexpected start-moving. It was reported that more than a third of the accidents occurred because of operator error and nearly two-thirds were due to the robot problems.

Ziskovsky [496] in 1983, identified the common cause of fatal accidents involving industrial robots, i.e., the lack of proper respect for what the robot is and the accompanying appreciation for its capabilities and limitations. In the same year, Percical [347] studied accident reports from various countries and provided a summary of sources of hazards that may lead to an accident. A similar study was conducted by Parsons [340] in 1985.

In 1986, Collins [88, 89] pointed out the importance of space requirements for designing

a robotics work station to avoid pinch points, while Ryan [383] presented a review of research efforts in determining the quantification of accident causation based on human behavior model. Motosko [302] reviewed the literature on robot-related hazards and discussed, when and how they occur, as well as precautions and safeguards that may prevent industrial incidents and accidents involving robots. An analysis of the robot safety strategies [219] in six British Companies covering 84 robots showed that in 50% of the cases the hazard control devices were systematically circumvented by workers. Also, in the same year, to express hazard in the field of safety, Sugimoto and Houshi [437] formulated the following relationship:

Hazard = [(Error Frequency)]   ×   [(Hazard Rate) × (Severity of Injury)]

   = [(Reliability)]   ×   [(Safety)]

Jiang and Gainer [199], in 1987, reviewed and analyzed the reported accidents in various countries and their findings revealed that if recommended safety guidelines and standards were in effect, in all phases of robot implementation, the accidents could have been avoided. They concluded that more effective worker safety training and workplace design would help to prevent the accident.

Nagamachi [312] in 1988, investigated ten fatal accidents which occurred in Japan Between 1978 and 1987. The accidents occurred during the phases of normal operation of robot, repair, maintenance or installation work, or during manual adjustment of work-piece. A cause-and-effect analysis of reported accidents involving robots and humans was conducted by Japan [205] in 1983, United States [292] in 1984, Sweden [72] in 1985 and Germany [317] in 1986.

1.3.2.4 *Safety Systems*

Over the years, to counter safety problems various safeguard methods have been developed including intelligent systems, electronic devices, infrared rays, and flashing lights [181,226]. In 1983, Sugimoto and Kawaguchi [434] performed an extensive robot study and concluded that only when robots themselves are able to detect the approach of humans and perform appropriate action to avoid accident, will safety in the human robot work place be assured. Same researchers [435] analyzed available data on accidents to point out dangerous operations involving industrial robots and developed a document on fundamental technologies and appropriate safety measures.

Rahimi [363] in 1984, on the basis of earlier robot accident reports classified major factors contributing to robot accidents and outlined system safety as an appropriate approach to analyze safety of semi-automated and automated robot systems. In the same year, in two separate articles, an automatic robot safety shutdown system was proposed by Anon [24] and Ziskovesky [498]. A number of researchers have already explored various aspects of both of these sensors categories [232,235,238].

Harless and Donath [164] in 1985, suggested that conventional safety systems such as barriers, light fences, and pressure mats are incapable of providing protection during periods of unstructured interaction between people and robots. In their article, they presented an overview of monitoring system, controller architecture, and an example of a safety algorithm. Kilmer et al. [237] considered a protective safety computer that could be used to monitor robot movements in the workstation to detect operations and to stop the robot before collision or damage. Graham [150] discussed safety mat design and construction,

applications, features, and costs. In 1986, Millard [293] discussed results of the research at the center for manufacturing productivity (CMP) at Rensselaer Polytechnic Institute (RPI) to develop an intelligent sensory system that will monitor the working envelop of a robot.

Ward [464] in 1988, reviewed the robot safeguarding problem and presented an overview of programmable electronic based control systems.

In 1993, Motamed and Schmitt [301] discussed the development of 'intelligent' safety systems using computer vision while Hischfled et al. [188], studied a survey responses from 19 out of 55 industrial robot users of over 580 robots. Findings indicated that only 20% of robots were found to be completely enclosed, while 60% had a limited barrier or no barrier. Furthermore, safety measures such as light curtains and floor mats were found to be the most widely used safety devices at 67% and 59%, respectively. In addition, industrial robot users demonstrated poor adherence to the Occupational Safety and Health Administration (OSHA) requirements and to the American National Standards Institute/Robot Industries Association (ANSI/RIA) Standards. In the same year, Aghazadeh et al. [2] based upon these survey results, performed a hazard analysis to assist in the evaluation of robot workstation safety. The hazard analysis indicated that safety sensors should be integrated in a layered protection system with an external perimeter, an internal work zone area, and a software path monitoring system.

In 1994, Abdallah et al. [1] proposed a real time system that was able to detect an intruder in a dangerous area, even when there were disturbing illumination changes in the considered shopfloor. A year later in 1995, Stentz and Hebert [422], developed a complete system that integrates local and global navigation. The local system uses a scanning laser

range-finder to detect obstacles and recommends steering commands to ensure robot safety. These obstacles are passed to the global system which stores them in a map of the environment.

In 1996, Crane et al. [97] successfully implemented teleproprioception techniques for determining robot position and orientation in known environments with a single camera.

1.3.2.5 *Safety Methods*

In 1985, Lee [259] described safety precautions for robot users including risk analysis, safety consideration, and proper methods of safeguarding. He also presented safety measuring methods and devices. Bellino [51] pointed out that four groups of people who need to be safeguarded are; operators, programmers/teachers, maintenance personnel and bystanders or unauthorized personnel.

Weck and Schoenbohm [467] in 1987, presented a mechanism that allows improvement of the existing safety precautions in all fields of robot technology.

In 1991, Rahimi and Xiadong [366] presented a generic software safety verification and encoding for safety-critical actions of robots. Jiang and Cheng [213] presented the six severity level design concept for safely designing a manufacturing cell. The design philosophy integrated guarding techniques with control actions, considering both production needs and safety concerns and interfaces machine functions with process requirements.

Buckingham [65] in 1993, considered examples of robotic devices applied to different surgical tasks that illustrate the issue of intrinsically safe design and passive and active systems. A year later, Akeel et al. [8] addressed the factors that influence robotic

safeguarding with respect to the identification of potential hazards and the various controlling mechanisms instituted to prevent them.

Suita et al. [439] in 1995, proposed a concept and a design method of covering a robot with a viscoelastic material to achieve both impact force attenuation and contact sensitivity and keeping it within the human pain tolerance limit.

In 1996, Montgomery and Lauderbaugh [297] developed a space robot hazard identification checklist through the identification of sources of possible hazards, assessment of associated risks, and determination of necessary safeguards. Also in 1996, Atcitty and Robinson [32] demonstrated the use of failure modes and effects analysis (FMEA) approach for the safety assessment of a robotic system developed at Sandia National Laboratories.

1.3.2.6 *Safety Standards*

Ordinance Amending Parts of the Industrial Safety and Health Regulations' [246] were officially announced in 1983. Thus, the action to be taken by employers for the prevention of industrial accidents caused by industrial robots was established. In 1985, the Japanese Industrial Safety and Health Association (JISHA) developed a document entitled "An Interpretation of the Technical Guidance on Safety Standards, etc., of Industrial Robots" [207].

A year later, in 1986, a safety standard entitled "Industrial Robots and Robot Systems Safety Requirements" was published jointly by the American National Standards Institute and the Robotic Industries Association [14].

In 1992, Doming et al. [114] identified safety issues as well as the general safety

requirements necessary for the safe operation of the automated test bed (ATB) to handle

the processing of special nuclear materials (SNM). A year later, in 1993, the Robotic

Industries Association (RIA), in association with the American National Standards Institute

(ANSI), established stringent safety guidelines for robot manufacturers and users [408].

Pegman [346] in 1994, outlined the activities of the National Advanced Robotics

Research Center (NARRC) in the promotion of safety and standards for advanced robots.

Standards and user guidelines for robot safety were also recommended and discussed

extensively by Park [335] in 1978, Potter [356] in 1983, Ziskovsky [495] in 1985, and Clem

[82] in 1986.

1.4 Motivation and Objectives of the Thesis

Motivation:

- Most of the published works have addressed either robot reliability or robot safety, but
  not concurrently.

- The criticality of robot reliability and safety relationship have been emphasized in some
  studies, yet, no specific studies have been performed to investigate this correlation.

- Most robot reliability and safety problems were dealt with basic methods such as the
  failure mode and effect analysis (FMEA) or the fault tree analysis (FTA). This study
  attempts to identify other methods which can be equally applicable to robot reliability
  and safety problems.

- Usually, as robot systems' complexity increase their reliability decrease which in turn
  contribute to their safety problems. Although stochastic processes present a valuable

tool for investigating complex reliability problems, but, they have never been used to study robot systems reliability and safety relationship. This work is the first bold attempting at synthesizing robot system reliability and safety relationship by employing stochastic processes. In this study, the models considered to show this relationship are entirely new, original and relevant.

- The assumption of an appropriate distribution for the failure and repair times is the integral part of any model. Traditionally, the exponential distribution has enjoyed great popularity due to its analytical simplicity. This study attempts to incorporate non-exponential distributions and present a generalization of the robot failed system repair process which may be of more practical significance.

Objectives:

The main objectives of this study are:

- To collect literature and publications related to robot reliability and safety,

- Review the most suitable system safety methods and identify those which are equally applicable to robot safety,

- Review the most appropriate systems reliability evaluation techniques and identify those which are applicable for robot reliability assessment,

- Stochastic analysis of a system comprised of a robot with n–redundant safety units,

- Stochastic analysis of a system comprised of n–redundant robots with one safety unit,

- Availability analysis of robot systems with non-constant failure and repair rates,

- Availability analysis of robot systems susceptible to non-constant common cause failure.

1.5 Thesis Structure

This study is divided into six chapters, a reference section, and four appendices:

- **Chapter 1** presents an introduction to robot reliability and safety including a brief overview of robot evolvement and a literature survey.

- **Chapter 2** discusses existing robot safeguarding techniques as well as safety methodologies used to evaluate robot system safety. It also reviews the techniques that are available to assess robot system reliability.

- **Chapter 3** presents a stochastic analysis of a system containing one robot with n–redundant safety units. Robot failure rates are assumed constant, whereas the repair rates could be constant or non-constant. Markov method is used to perform analysis for robot systems with constant failure and repair rates. For the non-Markovian models where the robot repair rate is assumed non-constant, supplementary variables technique is used to obtain the steady-state and time-dependent availabilities. Various failed robot system repair time distributions such as exponential, gamma, Weibull, Rayleigh, and log-normal are considered to obtain generalized steady-state availability expressions. Gamma distribution is utilized to obtain robot system time-dependent availability.

- **Chapter 4** presents analysis for a system comprised of n–redundant robots with one safety unit. As for Chapter 3, Makrov and supplementary variables techniques are used to investigate robot system performance indices.

- **Chapter 5** extends the analysis presented in Chapters 3 and 4 by introducing robot systems with non-constant common-cause failure rates. The method of device of stages combined with the supplementary variables technique are employed to obtain steady

state availability expressions.

- **Chapter 6** presents conclusions and recommendations for future studies.

A comprehensive list of references collected from conference proceedings, technical reports, journals, and books from 1973 to 1997 are listed in the reference section. For the sake of completeness of this study, introductory material is provided in Appendix A – C. An overview of probability distributions and their properties such as probability density function (*p.d.f*), moment-generating function, hazard function, and expected mean value are given in Appendix A. The relevant reliability and availability characteristics are provided in Appendix B. Markov, supplementary variables, and the device of stages methods are described in detail in Appendix C. Symbols associated with the model in Chapter 3 are defined in Appendix D and miscellaneous plots are given in Appendix E.

2

SAFETY AND RELIABILITY ASSESSMENT TECHNIQUES IN ROBOTICS

2.1 Introduction

A primary concern in the design of any system is the determination of an acceptable level of risk of failure on the basis of economic and/or social consequences associated with such risks. This is normally accomplished by meticulous analysis of the reliability and safety of the system.

Reliability may be defined as the probability that an item will perform its function adequately for the desired period of time when operated according to specified conditions [107]. Safety is freedom from those conditions that can cause damage to or loss of equipment or injury or death to human beings [109]. These definitions clarify the fact that an industrial robot which fails to perform properly, due to either partial or total functional failure over extended periods of time, will not satisfy the required economics for implementation in an industrial application. Failures not only are uneconomical, but also can

have an unsafe outcome. Engelberger [123] stated that for most applications, up-time must

exceed 97% to satisfy most users of industrial robots. In 1989 Klafter [242] wrote;

> "a robot having the most innovative controller or
>
> programming language which if not mechanically
>
> reliable becomes nothing more than an expensive
>
> laboratory toy."

There are many methods and techniques which may be used to make a robot more

reliable and safer. This chapter presents the most suitable robot safety and reliability

assessment methods selected from published literature [111,112].

2.2 Safety Methods in Robotics

The major motive for investing in industrial robots is to enhance productivity and to relieve

human operators from adverse environment and difficult or hazardous tasks. Robots

however, can not function without human interference and if left unattended, they will

gradually be unable to continue their assigned tasks because breakdowns may occur which

have not been allowed for by the human designers. Industrial robots like traditional machines

can bring hazards for people who work with them. Human errors and component failures

make such compulsory interaction (man-robot) dangerous and costly at times. Errors and/or

failures which affect man-robot interface may be classified in various ways: What causes the

error? What are the consequences ? How can they be prevented? etc. Also, in order to

understand how failures or errors may lead to accidents, to estimate their probabilities, and

more importantly to reduce the likelihood of their happening, a number of analytical

methods have been developed. Thus, the following presents the most suitable robot related

safeguarding techniques and safety methodologies taken from published articles as well as

detailed introductory aspects of robot safety (i.e., the W5 of robot safety).

2.2.1   W5 (What, Why, Who, When, Where) of Robot Safety

Safety requirements differ for applications with or without human interfacing. In controlling

hazards in a system without human interfacing, the entire application environment affecting

the machine must be understood. In contrast, evaluating hazard potential with human

interfacing not only requires knowledge of the overall operation of the system, but also an

understanding of how a human operator relates to the robot. This may be achieved by

determining the universal questions (What?, Why?, Who?, When?, and Where?). The W5

of safety is illustrated in Figure 2.1.
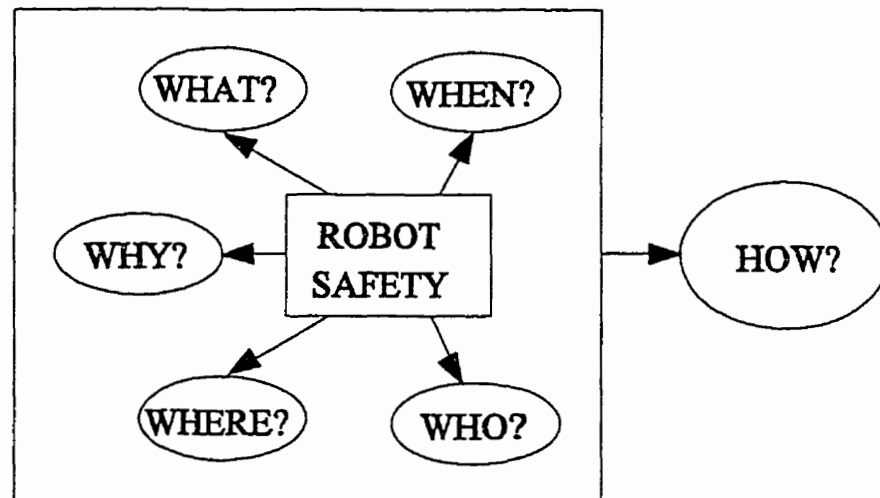
Figure 2.1.   The W5 of robot safety.

2.2.1.1 Why Robot Safety?

Like any other power-driven machine, there have been many reports of minor cuts, bumps,

pinches, and shocks to people working with robots. However, there have also been a few

fatalities. In Japan for instance, a robot pushed a maintenance worker into a grinding

machine and he subsequently died [11]. Later investigation revealed that the man did not

take proper safety precautions before entering the robot work envelope. Reports from other

fatal accidents conclude that in most cases, human negligence is the cause of these sad

accidents. Nevertheless, if robots could have been a bit more forgiving, these people did not

have to pay such a high price for their mistake. But, what makes robots unforgiving at

times? Attempting to answer this question is best done by identifying some of the

characteristices of a robot.

Generally speaking, a robot is blind, deaf, mute, dumb, and unconscious. Not many

robots are equipped with visionary systems, so a robot is incapable of locating a human who

is in its way or approaching it. It won't hear the human approaching, nor will it hear the

human say "ouch" if it hits him/her. Few robots have voices to warn humans away or to say

what its next move is. It will follow an operational sequence of instructions and will do only

what it is programmed to do. A robot is capable of moving suddenly or gradually in any

direction within its work envelope (or anywhere in the case of mobile robots) and is also

generally strong, fast, or both. The sum of these elements render robots dangerous and

unforgiving. Once the necessity of considering robot safety is realized, the next step is to

identify the sources of hazards.

2.2.1.2 What are the Sources of Hazards?

The sources of robot accidents can be grouped into three categorie as shown in Figure 2.2:

i) Those due to human error,

ii) Those caused by robots, and

iii) The environment in which man-robot interact.

The hazards due to man may arise as a result of the psychological behavior of the worker or the software errors of the programmer. Hazards due to the robot may occur from loss of structural integrity of the robot such as joint failure, material fatigue, erosion, etc. It can also originate from mechanical or electrical faults due to failures which occur randomly during the useful life of a component. There are also hazards from the environment such as accumulation of dust in the joints and motors which may cause malfunction of the robot.
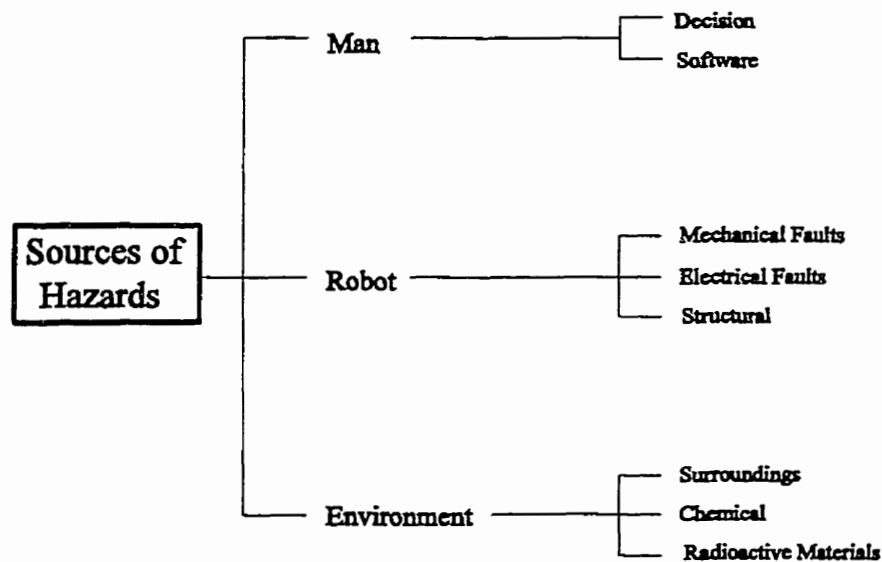
Figure 2.2.   Possible sources of hazards in robotic installations.

2.2.1.3 Who is Responsible, Who is at Risk, and Who Should be Protected?

Figure 2.3 shows the major causes of robot accidents. The highest percentage of robot-related accidents occur during programming, teaching, and manual operation.
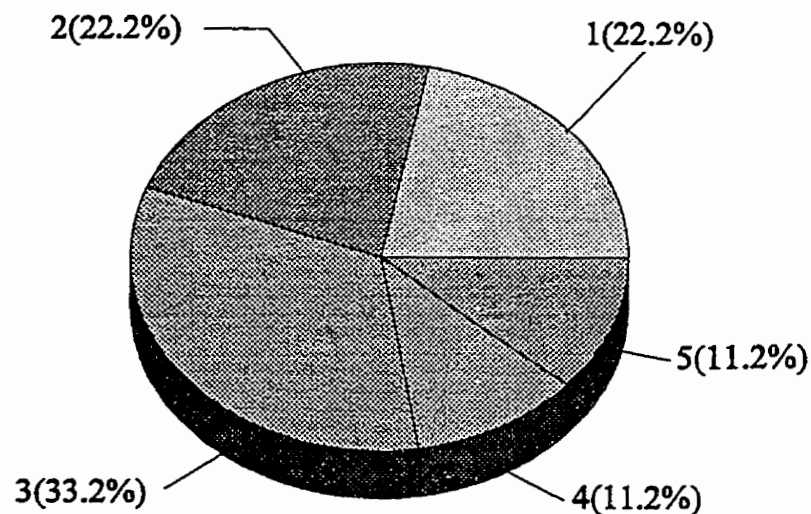
Figure 2.3.   Causes of robot accidents.

1. Erratic movement of peripheral equipment during teaching, testing, or normal operation.
2. Erratic movement of the robot during teaching, testing, or normal operation.
3. Erratic movement of the robot during repair or manual operation.
4. Unauthorized entry of the human to the robot work envelop.
5. Others.

A Swedish survey revealed that human error accounts for over 90% of accidents, whereas in another survey in Japan 2/3 are robot caused accidents [434]. With such discrepant data, it is not clear how much blame can be put on man and how much on the robot. Nevertheless, the primary objective of safeguarding is to protect humans from robots and the prevention of damage to the robots by the humans, particularly:

- Programmer/Teacher

- Maintenance personnel

- Operators

- Observers

- Equipment

- Work piece

2.2.1.4 *When to Consider Safety and When is the Critical Time?*

Historically new technologies are always implemented first and the safety factors incorporated afterwards because of lessons learned from unfortunate accidents resulting in injuries and property losses. The legal, social, and humanitarian considerations of our present world however, require that safety issues be addressed during the early stages of technology implementation. Sugimoto [438] stated that the principal of safety starts with the notion: "Safety is not the correction of accident that has already occurred and if a machine with no accident record has a potential hazard, safety measures should be instituted beforehand". This implies that:

- Safety measures should be incorporated before accidents occur,

- Safety is a planned and continuous process that is paramount to any successful robot application,

- The cost of safety is always acceptable compared to the cost of accident.

Robots typically have reliability, 98% or better. The 2% downtime (critical time) include factors like planned maintenance and programming or is due to failure of some sort. During

this period the robot is the most dangerous, because people directly interface with it. According to many studies, only 5 to 15 percent of the accidents occur in automatic mode [210,437]. This means 85 to 95 percent occur when the robot system is under manual operation control, such as during programming or maintenance of the robot.

2.2.1.5 Where to Consider Safety?

Types of injuries caused by robots are more diverse than those caused by other machines. Robots can strike, crush, or thrust to any location inside the point of operation. The likelihood of accidents taking place outside the work envelope must also be considered. One such situation can arise if a part being handled by the end effector slips and is thrown at varied trajectories well outside the point of operation. This can become more dangerous in the case of mobile robots which are to assume prominence in industry.

2.2.2   How Safe is Safe, and How to Achieve Safety?

Barret et al [41] stated that the degree of guarding depends upon the risks involved. The risks are determined by the frequency of access to the danger area, and severity of injury. Certainly, absolute safety is practically impossible, however, realistic goals can be achieved by blending safety with efficiency of operation. To be practical, the safety system itself must be able (to a reasonable degree) to satisfy the following criteria for any given application:

- Inexpensive
- Fail Safely

- Reliable

- Highly immune to false triggers

- Durable

- Capable of working in an industrial environment

- Easy to install

- Hard to bypass

- Fairly simple to program

- Easy to maintain

At present, the safety of robotics systems depends on a combination of the following items:

- Robot area protection

- Electronic sensing devices

- Training programs for personnel

The following is a brief discussion of the safety procedure and devices utilized in the robotics system safety.

2.2.2.1 *Robot Area Protection*

The two most critical areas to be protected are the work envelope and the working arm. The envelope can be protected in a number of ways, including visual barriers, isolation, intrusion detection devices, and common practice and procedures. Visual barriers provide the first step in protection of the envelope and if obeyed by the operators, can be quite effective. Warning signs and placarding, describing the hazards and related precautions, are a minimum requirement. Isolation of the envelope may offer the safest method by which

intrusion into the robot envelope is prevented. Some of the methods which are physical barriers include fences, chains, and curtains or without physical barriers such as pressure pads and light beams. Fences are not only the least expensive and most reliable but also have the capability to stop most parts if they slip from the robot gripper during very high speed motions. They may also prove to be hazardous, however, if there is any necessary human exposure, such as employees who may become trapped inside the fenced area if the robot malfunctions. All types of enclosures require interlocking to stop the robot whenever something enters the robot movement envelope. This can be inefficient in the case of isolation without physical barriers for nuisance stoppage. An overview of the most common robot area protection is given in Ref. [181,226].

2.2.2.2 *Electronic Sensing Devices*

Robot area protection may offer high safety standards, but, since people do not always obey warning signs or respect fences, additional precautions must be taken. Altamuro [11] in 1983 suggested that workers should be equipped "with protective gear or special devices to reduce the incidence or severity of accidents".

Programmers / teachers and maintenance personnel who must interact with robots might wear a device that returns a signal to the robot's safety sensor indicating a human presence. The robot itself should be aware of its immediate surrounding and differentiate between human and a piece-work. This is accomplished by duplicating human sensing capabilities. Of man's five senses, sight, touch, hearing, smell, and taste, those whose artificial parallels have been most extensively developed are sight and touch. These sensors will monitor both

the entire working envelope of the robot and a volume around the robot's arm and end

effector. A computer system which monitors the sensors can cause corrective action to be

taken if an unsafe condition occurs. Some significant examples of sensors are described in

Ref. [181,226]. These sensors are selected on the basis of many variables including sensing

characteristics, durability, cost and reliability.

2.2.2.3 *Training Programs for Personnel*

Accidents cannot be prevented by safety devices alone. Human errors during operation and

maintenance are highly possible. Therefore, good working practices and training of all

personnel in the vicinity of the robot is essential and at least in theory reduce sources of

human error. Furthermore, common sense is as important as any other safety measure. No

matter how many safety devices are planted in or around the robot, it still boils down to

human responsibility and thus, good common sense in all aspects of the application should

be used.

2.2.2.4 *Maintenance*

For successful operation of any robotics system, the maintenance of the robots, especially

of the preventive type, is essential. Periodic system checkup minimizes disruption due to

breakdown and exposure of workers to robot envelope.

2.2.3 Safety Analysis Methodologies

As a branch of system analysis, system safety analysis has come of age in many areas of design and manufacture. The system safety concept requires timely identification and evaluation of system hazards before losses occur. In other words, the ultimate aim is to produce a better understanding of the potential safety problems for a given system (e.g. robot) and to suggest actions which may improve system safety. To achieve this aim, the process of safety analysis often calls for the use of different methodologies which may include qualitative and quantitative analysis or both. These techniques are used to identify and evaluate system hazards and assure that safety is properly designed into each subsystem component of a major system.

Clemens [83] outlined 25 methods for hazard identification and evaluation while Rahimi [363] provided an overview of the system safety techniques applied to robotics. Among the qualitative techniques are preliminary hazard analysis, subsystem analysis, failure mode and effects analysis (FMEA), energy barrier analysis (EBA), critical incident technique, task analysis approach, system simulation, deviation analysis, and near accident analysis.

An example of a more quantitatively inclined technique is fault tree analysis (FTA) which usually involves application of probability theory to quantify the hazard probability of each event or component of a system. Some of these techniques have a broad application base (eg., Preliminary hazard analysis, Task analysis, system simulation) and need to be further detailed out for an application such as robotics safety. Deviation analysis is a new method which is used principally in other methods such as FMEA.

Among the techniques listed above, FTA and FMEA appear to be the most appropriate

techniques for robot safety analysis. Almost all of the potential dangers in the robot-man work environment are the result of combinations of unsafe conditions and unsafe actions. A deductive analysis of the conditions for combination of these factors and their cause-and-effect logical construction can be made by fault-tree analysis. FTA concentrates on accidents arising from characteristic function or structural features of robots. Both FTA and FMEA are described below.

2.2.3.1 *Fault-Tree Analysis (FTA)*

The fault tree method is a systematic, descriptive form of analysis that has been widely used for quantitative analysis of the safety and reliability of nuclear power generation systems. Sugimoto [434,438], Nagamachi [309,310], and Devianayagam [106] proposed the use of fault tree analysis in robot safety. As used in system safety analysis, the tree leads to an undesired event (i.e., top event) and determines the sets of fault events that cause the outcome in question. For example, sudden transfer of kinetic energy of the robot to the human body due to unexpected robot motion is responsible for the majority of accidents. The top event in Figure 2.4 is a classical example of such a case. The fault-tree method is described in detail in Ref. [358].

The symbols associated with Figure 2.4 are described below:
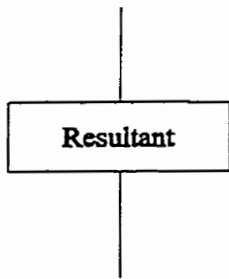
| Resultant |

The resultant of combination of fault events through a logic gate is represented by a rectangle.

Input   AND   Output

This symbol indicates that the output fault event can occur if and only if all the input fault events occur.

Input   OR   Output
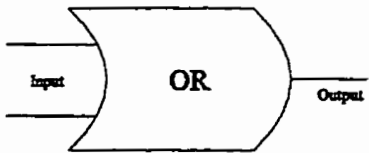
This symbol indicates that the output fault event can occur if any one or more of the input fault events occur.
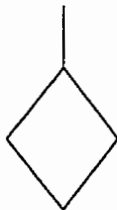
A fault event whose causes have not been fully identified and can be further investigated is denoted by a diamond.
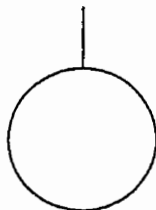
A circle indicates a basic fault event.
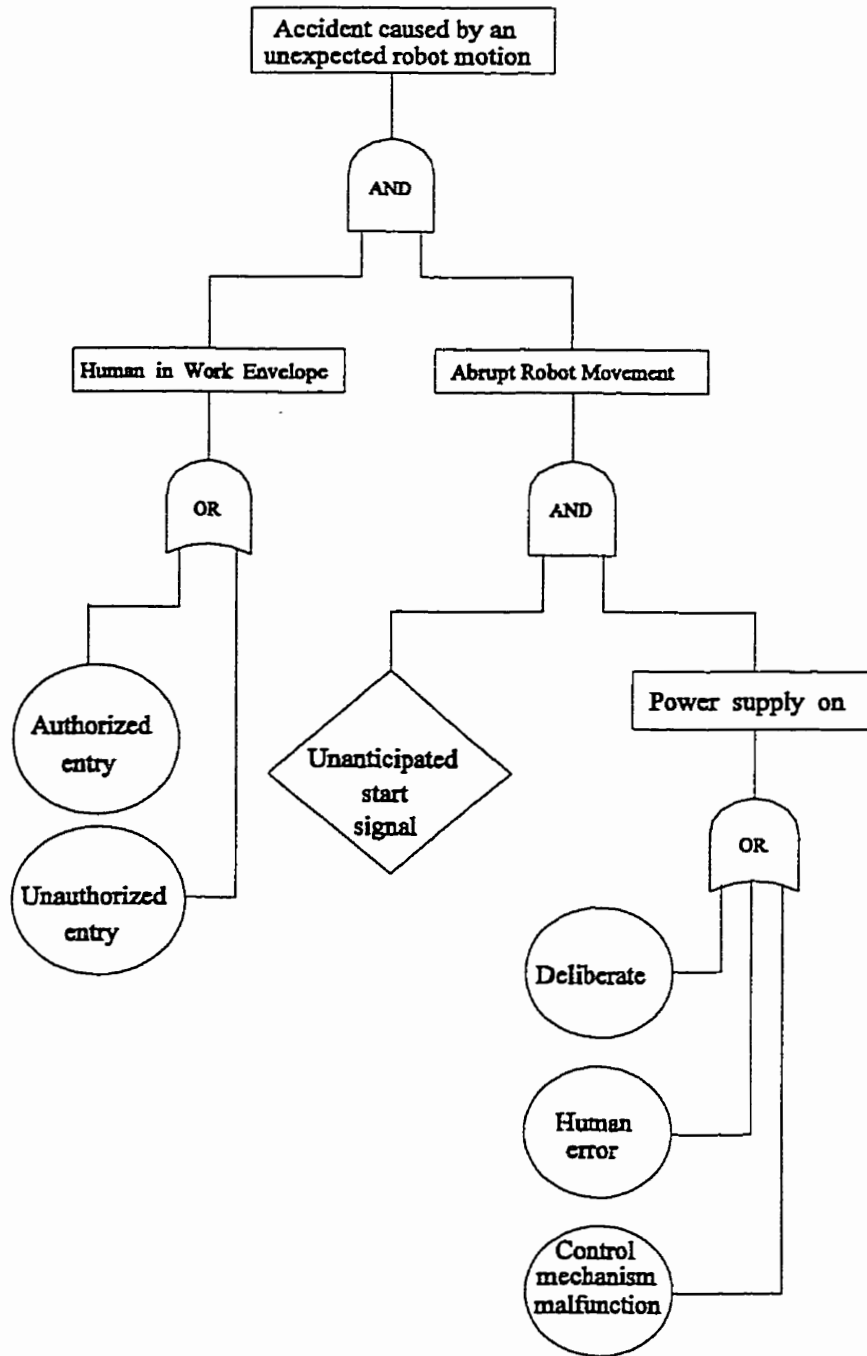
Figure 2.4.   Fault-tree analysis for the top event: accident caused by an unexpected robot movement [109].

2.2.3.2 *Failure Mode and Effect Analysis (FMEA)*

Failure mode and effect analysis (FMEA) is one of the most widely used methods that uses a tabular form to identify the modes of failure for the components of a system along with their occurrence probability, severity, and effects of failure. In general terms, FMEA is used to do the following [442]:

- Ensure that all conceivable failure modes and their effects are understood.

- Assist in the identification of system weaknesses.

- Provide a basis for selecting alternatives during each stage of operation.

- Provide a basis for recommending test programs and improvements.

- Provide a basis for corrective action priorities.

The analysis traces the problem back into its root by answering the following questions;

- How can the component fail (cause)?

- What are the consequences (effects) of the failure?

Once these questions are properly answered, a list of symptoms or methods of detection of each failure mode is formulated for safe operation. Thus, the next step would be to answer the following;

- How is failure detected? and,

- What are the safeguards against the failure?

One example of studies concerning FMEA in robotics safety is performed by Jiang and Gainer [210]. The study is motivated as a result of 32 robot accidents which occurs between 1984 to 1986 in the U.S., West Germany, Sweden and Japan. The analysis included the accident source, the accident cause, the accident effect in terms of human injury, recommended guidelines to be implemented, and applicable safety standards in each case.

The accidents' causes were grouped into four categories (human error, workplace design, robot design and others). The accidents' effects were grouped according to the person injured (line worker, maintenance worker or programmer), the type of injury (pinch-point, impact or other), and the degree of injury (fatal or non-fatal). By using this approach (i.e., FMEA), it was concluded that one of the most important tasks in eliminating or reducing the probability of robot accidents is to identify the cause by which they may take place, and to pay particular attention to safety throughout all phases of robot life cycle including workplace design, robot installation, robot testing, and robot operation.

2.3 Reliability Techniques in Robotics

There are two main categories of reliability evaluation techniques: analytical and simulation (real time). Analytical techniques represent the system by a mathematical model and evaluate the reliability indices from this model using mathematical solutions. Simulation (e.g., Monte Carlo) methods, however, estimate the reliability indices by simulating the actual process and random behavior of the system. There are merits and demerits in both methods.

In the application of a stochastic model, one has to make a basic decision: whether to use a simulation technique or an analytical method. Simulation technique is usually chosen because it is readily understood, its feasibility is known beforehand, and it imposes, in principle, no restrictions on the complexity of the model. The fact that the simulation technique will permit one to use very complex models with apparent ease does not enhance the significance of its answers. Analytical methods, yield explicit functional relations instead of specific numerical values, therefore, shed more light on general properties and are capable

of a broader range of applications, such as parametric studies. Thus, it is in this spirit that

the analysis in this study should be viewed, being not a method for generating numbers, but

rather an approach which gives rise to deeper perception and increased judgement. This

does not imply that quantitative statements are to be disregarded. It merely means that we

will readily accept approximate solutions whose emphasis is on the right trend rather than

on numerical accuracy.

2.3.1   Analytical Methods

In performing the reliability analysis of a robot system, an important task is to identify the

most suitable available analytical methods. In the field of reliability engineering, there are

many techniques and methods available that are equally effective to perform robot reliability

analysis and prediction studies. Some of these techniques are network reduction method,

minimal cut set method, fault tree analysis (FTA), failure mode and effect analysis (FMEA),

Markov method, delta-star method, and parts count method, etc [107]. After considering

such factors as simplicity and effectiveness, the most appropriate of the analytical methods

for the stated purpose are as follows:

- Failure Mode and Effects Analysis (FMEA)

- Fault Tree Analysis (FTA)

- Block diagram

- Combinational models (i.e., combined Fault Trees and block diagram)

- Markov and Non-Markovian Models

Each of the above methods is described below briefly.

2.3.1.1 Failure Mode and Effect Analysis (FMEA)

As in robot safety analysis, FMEA can also be used in reliability evaluation of a robot system. It is used to systematically analyze the failure modes of components of a robot and determine the effects of these failures on robot performance. In robot reliability assessment, FMEA is performed to identify potential design weaknesses through systematic documented consideration of the following items:

- All possible ways in which robot can fail.

- Causes for each mode of failure.

- Effects of each failure mode on robot system reliability.

- Probability of occurrence of each failure mode.

One main advantage of FMEA is hypothesizing the source of failure, thereby reducing the probability of failure or reducing the severity of failure by redesign to produce a fail-safe, or system redundancy, etc. For instance, the robot joint in Figure 2.5 can fail due to various faults. Each component and its associated failure modes are considered individually and their effect on other components as well as on the whole system, (i.e., the joint) is identified. One major draw back of FMEA is its singularity-failure analysis [268]. This means that FMEA is not well suited for assessing the combined effects of two or more failures.

2.3.1.2 Fault Tree Analysis (FTA)

In many complex systems where a single failure may not adversely affect the system, but two or more failures together may adversely affect the system, deductive methods such as fault-tree analysis are more suitable. This method is equally applicable to robot

safety/reliability analyses. FTA is a valuable tool that can be applied by which the subsystem or component failure events leading to system failure are related using simple logical relationships. These relationships are a structural-model of cause and effect that represent the system failure modes. FTA can provide information on how a system may fail, what the probability of the occurrence of the top event (i.e., undesired event) is, and what remedies may be used to overcome the causes of failure. Figure 2.5 presents possible sources of failure of the robot joint and Figure 2.6 shows the hierarchy of the combination of events that contribute to the top event of joint failure. The joint can fail either because of actuator failure, structural failure, or failure of one of the hinges. Hinge failure is a basic fault which can be assigned an independent probability. Actuator failure however, can either be caused by pipe failure or by valve malfunction. Pipe failure may be treated as a basic fault whereas valve malfunction can be traced back further. FTA is described in detail in Ref. [358].
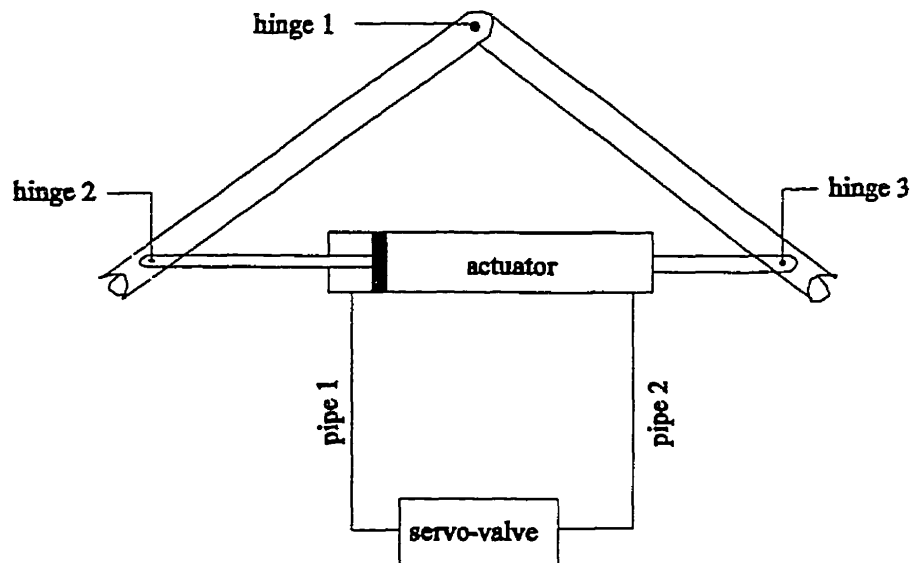
Figure 2.5.   Possible sources of a robot joint failure [407].
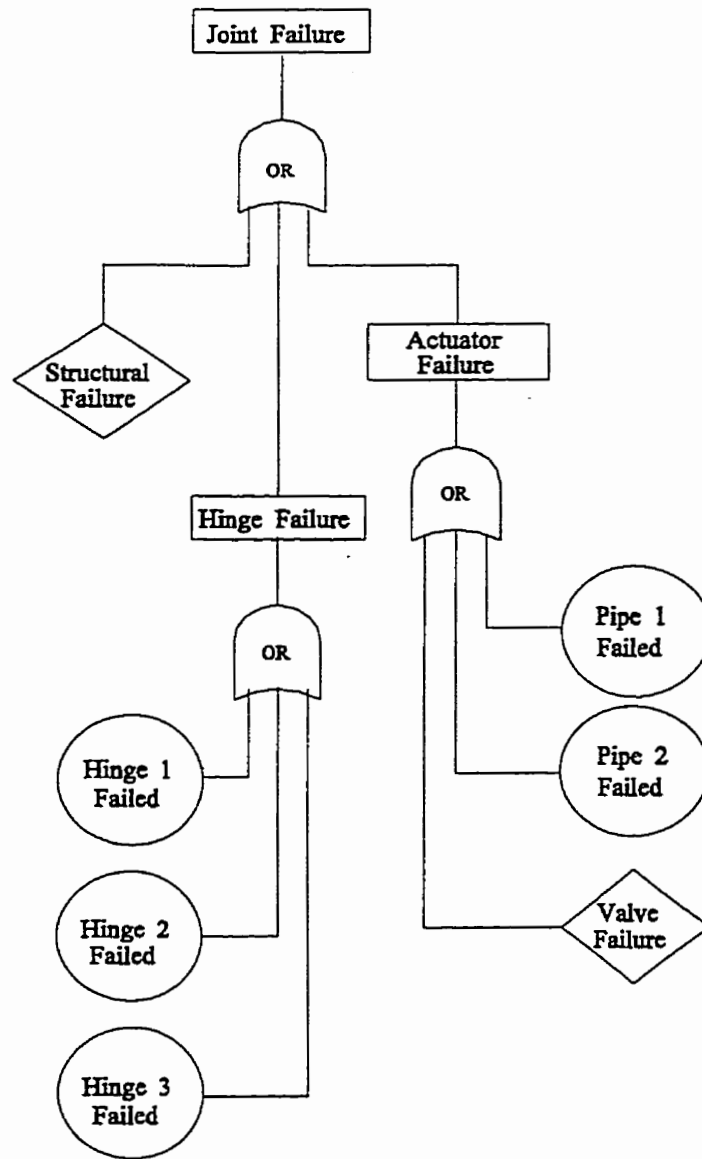
Figure 2.6.   Fault-tree diagram for a robot joint failure.

2.3.1.3 *Reliability Block Diagram*

In general, the main objective of a reliability study is to predict the performance of a complete system. Block diagram [107] (also called reliability network) is one of the simple and effective methods which enables the system failure probability to be evaluated in terms of the component performance characteristics. The first step in the reliability prediction is to identify failure modes of the system and collect reliability information on all of its components. A block with assigned probability of success or failure rate represents each component. Blocks are then connected together so as to form a reliability network which represents the reliability dependencies between components of the system. Figure 2.7 represents a system whose components are placed in series (also known as non-redundant system). In this fashion component failure cannot be tolerated and any component failure will break the single path, thus cause system failure.
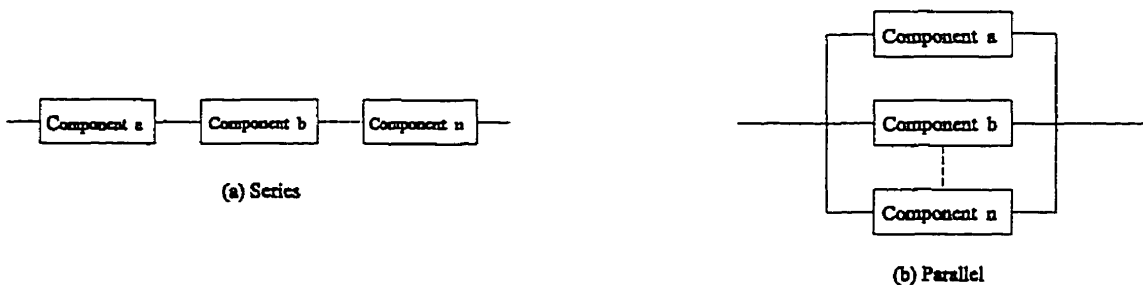
Figure 2.7. Reliability block diagrams: (a) series configuration, (b) parallel configuration.

For a general series network containing n components, the system failure and success expressions are

$$Q_{ss} = 1 - \prod_{i=1}^{n}(1 - Q_i) \tag{2.1}$$

and

$$R_{SS} = \prod_{i=1}^{n} R_i \tag{2.2}$$

where Q_i = Probability of failure of the ith component.

Q_{ss} = Probability of failure of the series system.

R_i = Probability of success of the ith component.

R_{ss} = Probability of success of the series system.

n = Number of components.

Systems with a higher degree of reliability usually require redundancy in part or all of

the system. The simplest form of a redundant system is the parallel configuration shown in

Figure 2.7(b). Here, all paths must fail to cause system failure. The system reliability for a

parallel configuration is given by

$$Q_{PS} = \prod_{i=1}^{n} Q_i \tag{2.3}$$

and

$$R_{PS} = 1 - \prod_{i=1}^{n}(1 - R_i) \tag{2.4}$$

where Q_{PS} = Probability of failure of the parallel system.

R_{PS} = Probability of success of the parallel system.

n = Number of components.

The primary advantage of this method is that it is easy to understand and apply.

However, generally it is not suitable for components and sub-systems with degraded failure

modes.

2.3.1.4 Combinational Models

Combinational models have been widely used and have become a standard method for reliability prediction because they are conceptually simple and easy to understand. These models are basically represented by combination of fault trees and block diagrams. There are however, some limitations to this approach [45]:

- It is difficult, if not impossible to allow for various types of dependencies such as repair, near coincident faults, transient and intermittent faults and standby systems with spares.

- The nature of the Combinational approach requires that all combinations of events for the entire time period must be included. For complex systems, this results in complicated models.

- A fault tree is constructed to predict the probability of a single failure condition (i.e., top event). If a robot has many failure conditions, separate fault trees must be constructed for each one of them.

2.3.1.5 Markov Models

A stochastic process is a family of random variables observed at different times and defined on a specified probability space. When all the random variables are exponentially distributed, the associated stochastic process is a time homogenous Markov[b] process [368]. Any Markov model is defined by a set of probabilities p_{ij}, which define the probability of

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[b]*Russian mathematician A. A. Markov (1856 - 1922).*

transition from any state i to any state j. One of the most important features of any Markov model is that the transition probability p_{ij} depends only on states i and j and is completely independent of all past states except the last one, state i. Markov models can be formulated as long as the failure and repair rates are exponentially distributed.

Aside from theoretical considerations the exponential distribution has found wide acceptance since Davis [102] presented goodness of fit tests on failure data which favored exponential distribution. Drenick [117], also proved that the time between failures of complex equipment tends to an exponential distribution as the number of components become large, irrespective of the component failure laws. The major assumption underlying his proof is that every failed component is immediately replaced by a new one and that the failure of any component causes the system to fail.

Markov method is applied only when certain restrictions are fulfilled. In a capsule form, the assumptions such as the following are made in developing reliability analysis of a system [371]:

1. The state of the system changes as time progresses.

2. The transition rates are constant.

3. All failure occurrences are independent.

4. The probability of two or more failure occurrences in a finite time interval is negligible.

An example of Markov method and its applicability to robot systems is discussed in Appendix C.

2.3.1.6 Non-Markovian Models

When some of the random variables have non-exponential distributions, the interstate transition rates become function of the time spent in the states and the process becomes non-Markovian. The solution of non-Markovian models is in general difficult and the analytical techniques are usually of limited application in practical problems.

Traditionally the exponential distribution has enjoyed great popularity. Reasons for this are simple analytical from of the exponential distribution and its giving rise to Markov processes. In fact, in most cases exponentially distributed failure times are assumed in reliability analysis of components during their useful life period. This is the middle portion of so called "bathtub" curve. Exponentiality, as is well known, that the component does not deteriorate with age and is referred as memory–less distribution. But it is this very property which makes the exponential distribution rather unsuited for the repair time of the component.

An exponential repair time implies that the time which has been spent on a repair in the past has no influence on the probability of completing that repair during some time interval in the future. This is contrary to the concept of an organized repair. On intuitive grounds one expects to have zero probability of completing a repair immediately after it is begun, and to have most of the probability mass concentrated in the neighborhood of the mean repair time. This situation is represented by a two parameter distribution such as the gamma, lognormal, or Weibull distribution.

Thus, if the distribution cannot be represented by a single exponential form then the process becomes non-Markovian and different techniques are required for system solution.

There are several techniques available for solving this problem [139,416]. Two frequently proposed methods to deal with those cases are supplementary variables and the device of stages. Supplementary variable technique and the device of stages method are discussed in detail in Appendix C.

2.3.2 Simulation Techniques

Simulation (e.g., Monte-Carlo) techniques can be used to calculate system reliability by simulating the failure of the components at times distributed according to their failure rates. Random samples from item failure and repair distributions are taken which require the generation of very large quantities of random numbers. Since every event (failure, repair, movement, etc.) must be sampled for every unit of time, a simulation of a moderately large system over a reasonable period of time can require hours of computer time. This technique is recommended only when other methods cannot handle the problem or when the simplifying assumptions to be made to make the problem solvable by other methods are not acceptable. For example, failure of components may not be independent and prevent the use of a general fault tree analysis. In addition, component failure or repair distributions may not be exponentially distributed. This means that the transition from one state to another is not governed by the negative exponential distribution which therefore prevents the use of the Markov method. As indicated earlier, the major drawback with this method is its extensive use of computer time. Also, if a minor change occurs the simulation must be rerun at considerable cost.

2.4 Disscussion and Conclusions

In addition to better productivity, robots can make the work place safer for workers by relieving humans from hazardous or repetitive jobs where monotony can lead to operator error and possible injury. However, some appropriate safety precautions must be observed. These precautions include:

- Use of safety guards and other devices,

- Periodic scheduled maintenance to ensure the robot is always in ideal operating condition which minimizes breakdown and exposure of workers to robot envelope.

- Constant training of personnel to reduce sources of human errors.

Accidents can still happen even with the above precautions. The ultimate goal is, however, to minimize the risk of an accident. This can be done by performing risk analysis.

Robot reliability is a very complex issue. There are many interlocking variables in predicting and achieving various levels of reliability. Components with varying degrees of sophistication are used in many different combinations to make today's industrial robots. As mechanical, hydraulic, pneumatic, and electronic parts are used in their design, this brings with them many sources of failures. In general, failures of a robot system may be classified into the following categories [460, 461]:

- Failures due to structural malfunctions,

- Failures due to technological malfunctions, and

- Failures due to behavioral malfunctions.

In robot systems, structural failures could be related to the materials that are susceptible

to external and internal changes in temperature, pressure, etc. Technological failures can be due to random component failures, systematic hardware faults, and software error. Behavioral failures can be the result of decisional (human) errors during operation or maintenance. Sources of failure must be considered right from the design and development phase to implementation and continuous operational phases. It is therefore necessary to assess the reliability of the robot in depth so that modifications of the design as well as component selection can be allowed for achieving a higher degree of reliability.

This chapter reviewed available techniques that can be applied to the safety and reliability evaluation of robot systems.

There are many approaches available for reliability analysis of general systems which can also be applicable to robotics. The most suitable proposed methods (analytical and simulation) are:

- Failure Mode and Effect Analysis (FMEA),

- Fault-Tree Analysis (FTA),

- Block diagram,

- Combinational models (e.g., Fault Trees and reliability block diagram),

- Markov Models,

- Non-Markovian Models,

- Simulation Technique (Monte-Carlo).

Generally, Simulation technique requires a large amount of computing time and is not used extensively if alternative analytical methods are available.

3

STOCHASTIC ANALYSIS OF A SYSTEM CONTAINING ONE ROBOT WITH N-REDUNDANT SAFETY UNITS

3.1 Introduction

The success of a robot in a certain operation is dependent on many important parameters.

Robot accuracy to carry out a task and being able to repeat that task over and over again

indeed contributes to the robot's success. Another factor which contributes to the overall

success of a robot is its safe operation. Just as for other engineering products, a robot must

not only be reliable but also safe. An unreliable robot may become an unsafe robot and cause

unsafe conditions, high maintenance costs, inconvenience, and so on. Thus, in performing

robot reliability analysis, the coupling between reliability and safety must also be considered,

otherwise, the end result may not be realistic. In other words, robot reliability parameters

must be estimated by paying particular attention to the safety and reliability relationship.

To improve reliability of a system, one concept often practiced is known as *Redundancy*. The term *Redundant* implies the existence of more than one means, identical or non-identical. For example, redundant pumps are crucial for the reliable and therefore safe operation of a nuclear reactor. This concept is equally applicable to robot systems and robot reliability and safety can be enhanced by increasing the number of safety units associated with them. The number of safety units that should be incorporated within the robot system is the matter of desired level of safety.

This chapter presents a stochastic analysis of a system containing a robot with redundant safety units. A generalized model of the robot-safety system is introduced and generalized expressions are developed. Generalized expressions include steady state availability, time-dependent availability, reliability, and mean time to failure (MTTF). For the sake of comparison, robot system performance indices are examined when the robot is working with one ($n = 1$) and/or two ($n = 2$) safety units. Robot system performance indices are determined by means of the Markovian and non-Markovian methods. The method of supplementary variables is used to deal with the non-Markovian models where robot repair rates are non-constant. Various robot failed system repair time distributions (i.e., exponential, gamma, Weibull, Rayleigh, and log-normal distributions) have been considered to obtain generalized steady state availability expressions. To obtain generalized time-dependent availability expressions, gamma distribution is utilized to fit the robot failed system repair time distribution. The gamma distribution offers some advantages: it is of simple analytical form and it has a rational Laplace transform.

Markov method is utilized in models where failure and repair rates are assumed

constant. A system of first-order differential equations are obtained and with the aid of

Laplace transforms, generalized reliability and MTTF expressions are developed.

3.2 Robot-Safety System Description

The block diagram of a system containing one robot with n-redundant safety units is shown

in Figure 3.1. The corresponding state space transition diagram of this robot-safety system

is shown in Figure 3.2 [115b].

A: The robot
B: n–identical safety units
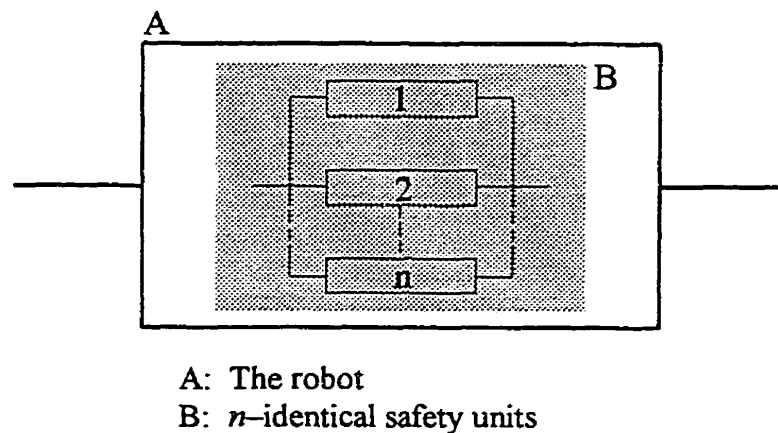
Figure 3.1.   The block diagram of a system containing one robot with n–identical safety
units.

At time $t = 0$, the robot and n–safety units begin operating. The basic system can fail due

to the failure of the robot or it may degrade due to the failure of the first safety unit. From

its degradation state, the basic system may degrade further after the failure of the second,

third, fourth, ... n^{th} working safety units or it can fail totally due to the failure of the robot.

Once all safety units have failed, the robot may continue operating without any safety

mechanism. The operating robot can either fail safely or with an incident. The degraded or

fully failed system may be repaired.
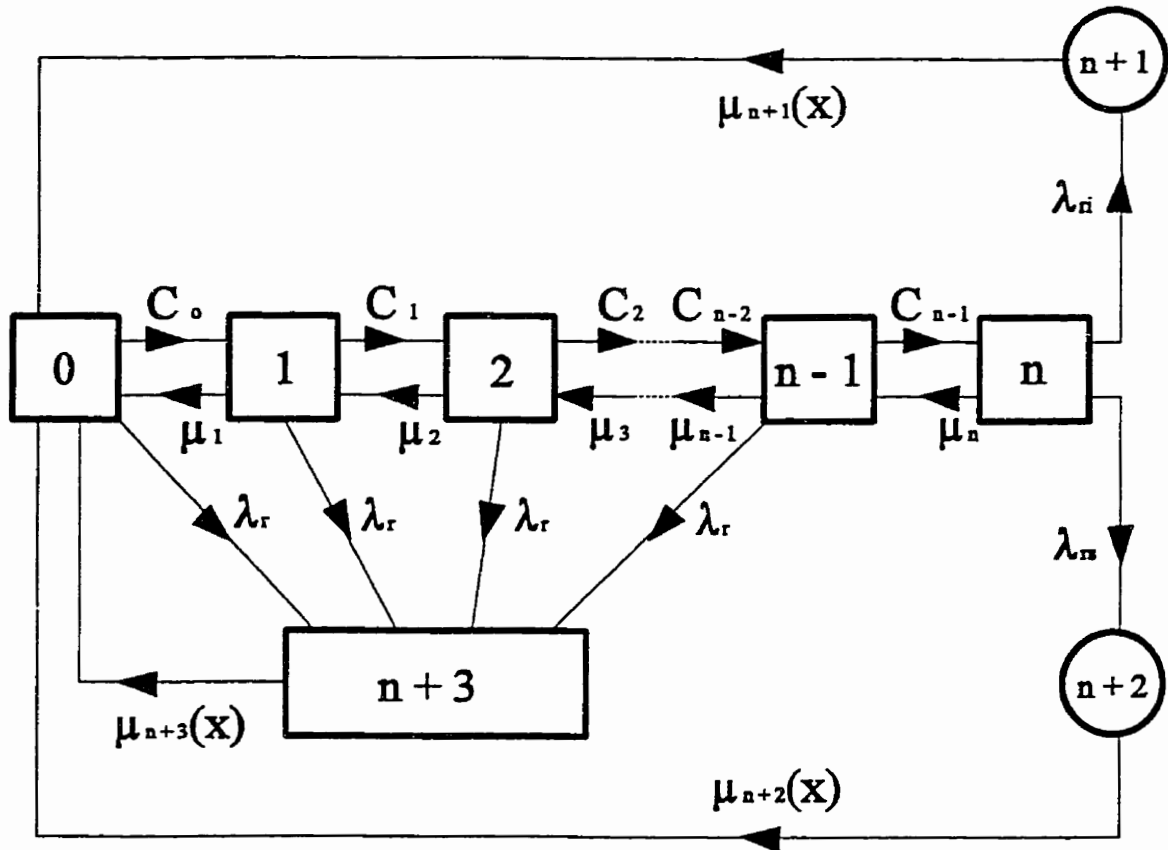
Figure 3.2.   The state space transition diagram of a system comprising a single robot and
n-identical safety units. The numeral in squares, rectangle, and circles
denotes the system state.

The numerals or letters (as applicable) in each box in Figure 3.2 represent corresponding

robot system states. For $i = 0$, robot and n safety units are working normally. For $i = 1$,

robot and n - 1 safety units operating normally while one safety unit has failed. For $i = 2$,

robot and n - 2 safety units operating normally while two safety units have failed.   For

$i = n$ - 1, robot and one safety unit operating normally while n - 1 safety units have failed.

For $i = n$, robot operating normally while n safety units have failed. For $k = n + 1$, robot

failed with an incident, for $k = n + 2$, robot failed safely, and for $k = n + 3$, robot failed while

at least one safety unit is functioning.

3.3 Generalized Robot-Safety System Analysis

The following symbols are associated with the model in this chapter.

i	i^{th} state of the overall robot system: $i = 0, 1, \dots , n$.
k	k^{th} state of the overall robot system: $k = n + 1, n + 2, n + 3$.
t	time
λ_s	Constant failure rate of the safety unit.
λ_{ri}	Constant failure rate of the robot failing with an incident.
λ_{rs}	Constant failure rate of the robot failing safely.
λ_r	Constant failure rate of the robot.
μ_i	Constant repair rate of the safety mechanism in state i; $i = 1, 2, \dots , n$.
Δx	Finite repair time interval.
$\mu_k(x)$	Time-dependent repair rate when the failed robot system is in state k and has an elapsed repair time of x; for $k = n + 1, n + 2, n + 3$.
$p_k(x,t)\Delta x$	The probability that at time t, the failed robot system is in state k and the elapsed repair time lies in the interval $[x, x + \Delta x]$; for $k = n + 1, n + 2, n + 3$.
pdf	Probability density function.

$q_k(x)$ *pdf* of repair time when the failed system is in state k and has an elapsed time

of x; for $k = n + 1, n + 2, n + 3$.

$P_i(t)$ Probability that the robot system is in state i at time t; for $i = 0, 1, \dots, n$.

$P_k(t)$ Probability that the robot system is in state k at time t; for $k = n + 1, n + 2, n+3$.

P_i Steady-state probability that the robot system is in state i; for $i = 0, 1, \dots, n$.

P_k Steady-state probability that the robot system is in state k; for $k = n + 1, n + 2,$

$n + 3$.

s Laplace transform variable.

$P_i(s)$ Laplace transform of the probability that the robot system is in state i; for $i =$

$0, 1, \dots, n$.

$P_k(s)$ Laplace transform of the probability that the robot system is in state k; for $k =$

$k = n + 1, n + 2, n + 3$.

$SSAV_{rs}$ Robot system steady state availability when the robot working with at least one

safety unit.

$SSAV_r$ Robot system steady state availability when the robot working with or without

the safety unit(s).

$SSAV_{rs}(s)$ Laplace transform of the robot system availability when the robot working

with at least one safety unit.

$SSAV_r(s)$ Laplace transform of the robot system availability when the robot working

with or without the safety unit(s).

$SSAV_{rs}(t)$ Robot system time-dependent availability when the robot working with at least

one safety unit.

SSAV$_r$(t)   Robot system time-dependent availability when the robot working with or without the safety unit(s).

R$_{rs}$(s)   Laplace transform of the robot system reliability when the robot working with at least one safety unit.

R$_r$(s)   Laplace transform of the robot system reliability when the robot working with or without the safety unit(s).

MTTF$_{rs}$ Robot system mean time to failure when the robot working with at least one safety unit.

MTTF$_r$ Robot system mean time to failure when the robot working with or without the safety unit(s).

The analysis presented in this chapter are subject to assumptions such as follows:

- The system is composed of one robot and n identical safety units,

- The redundant safety units are active or operating simultaneously,

- Statistically independent robot and safety unit failures,

- Robot and its associated safety units' failure rates are constant,

- Robot and safety units' repair rates are constant,

- Failed robot system (i.e., total system) repair rates can be constant or non-constant,

- A repaired robot-safety unit is as good as new,

- The overall system fails only when the active robot fails.

Using the supplementary variables technique, the corresponding system of integro-differential equations for the model given in Figure 3.2 is

$$P_0'(t) + a_0 P_0(t) = \mu_1 P_1(t) + \sum_{k=n+1}^{n+3} \int_0^\infty p_k(x,t)\mu_k(x)dx \tag{3.1}$$

$$P_1'(t) + a_1 P_1(t) = C_0 P_0(t) + \mu_2 P_2(t) \tag{3.2}$$

$$P_i'(t) + a_i P_i(t) = C_{i-1}P_{i-1}(t) + \mu_{i+1}P_{i+1}(t) \tag{3.3}$$

$$(for \quad i = 1, 2, 3, \dots, n-1)$$

$$P_n'(t) + a_n P_n(t) = C_{n-1}P_{n-1}(t) \tag{3.4}$$

where

$$a_i = C_i + \mu_i + \lambda_r$$
$$C_i = (n - i)\lambda_s \qquad (for \qquad i = 0,1,2,\dots,n-1)$$
$$a_n = \lambda_{rl} + \lambda_{rs} + \mu_n$$
$$\mu_0 = 0$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_k(x)\right]p_k(x,t) = 0 \qquad (for \quad k = n+1, n+2, n+3) \tag{3.5}$$

The associated boundary conditions are as follows:

$$p_{n+1}(0,t) = \lambda_{rl}P_n(t) \tag{3.6}$$

$$p_{n+2}(0,t) = \lambda_{rs}P_n(t) \tag{3.7}$$

$$p_{n+3}(0,t) = \lambda_r\left(\sum_{i=0}^{n-1} P_i(t)\right) \tag{3.8}$$

At time $t = 0$, $P_0(0) = 1$, and all other initial state probabilities are equal to zero. The prime denotes differentiation with respect to time t.

3.3.1   Steady State Availability Analysis

As time approaches infinity, Equations $(3.1) - (3.5)$ reduce to Equations $(3.9) - (3.13)$, respectively.

$$a_0 P_0 = \mu_1 P_1 + \sum_{k=n+1}^{n+3} \int_0^\infty P_k(x) \mu_k(x) dx \tag{3.9}$$

$$a_1 P_1 = C_0 P_0 + \mu_2 P_2 \tag{3.10}$$

$$a_i P_i = C_{i-1} P_{i-1} + \mu_{i+1} P_{i+1} \qquad (for \quad i = 1,2,3,...,n-1) \tag{3.11}$$

$$a_n P_n = C_{n-1} P_{n-1} \tag{3.12}$$

$$\frac{d}{dx} p_k(x) = -\mu_k(x) p_k(x) \qquad (for \quad k = n+1, n+2, n+3) \tag{3.13}$$

The associated boundary conditions become

$$p_{n+1}(0) = \lambda_{rt} P_n \tag{3.14}$$

$$p_{n+2}(0) = \lambda_{rs} P_n \tag{3.15}$$

$$p_{n+3}(0) = \lambda_r \left(\sum_{i=0}^{n-1} P_i \right) \tag{3.16}$$

Solving Equation (3.13), the resulting expression is

$$p_k(x) = p_k(0) e^{-\int_0^x \mu_k(\theta) \, d\theta} \qquad (for \quad k = n+1, n+2, n+3) \tag{3.17}$$

The steady state condition of the probability, P_k, that due to a failure the robot system is

under repair, is

$$P_k = \int_0^\infty p_k(x)dx \qquad (for \quad k = n+1, n+2, n+3) \qquad (3.18)$$

Substituting Equation (3.17) into Equation (3.18), yields

$$P_k = \int_0^\infty p_k(0)e^{-\int_0^x \mu_k(\theta)\,d\theta}dx \qquad (for \quad k = n+1, n+2, n+3) \qquad (3.19)$$

Substituting Equations (3.14) – (3.16) into Equation (3.19), we get:

$$P_{n+1} = \int_0^\infty \lambda_{rl} P_n e^{-\int_0^x \mu_{n+1}(\theta)\,d\theta}dx = \lambda_{rl} P_n E_{n+1}[x] \qquad (3.20)$$

$$P_{n+2} = \int_0^\infty \lambda_{rs} P_n e^{-\int_0^x \mu_{n+2}(\theta)\,d\theta}dx = \lambda_{rs} P_n E_{n+2}[x] \qquad (3.21)$$

$$P_{n+3} = \int_0^\infty \lambda_r \sum_{l=0}^{n-1} P_l e^{-\int_0^x \mu_{n+3}(\theta)\,d\theta}dx = \lambda_r \sum_{l=0}^{n-1} P_l E_{n+3}[x] \qquad (3.22)$$

where

$$E_k[x] = \int_0^\infty e^{-\int_0^x \mu_k(\theta)\,d\theta}dx \qquad (for \quad k = n+1, n+2, n+3)$$

which is the mean time to robot system repair when the failed robot system is in state k and

has an elapsed repair time of x, or the expected value of x. The failed robot system mean

time to repair, $E_k[x]$, can be expressed for various repair time distributions and is given by

$$E_k[x] = \int_0^\infty x q_k(x)\,dx \qquad (for \quad k = n+1, n+2, n+3) \qquad (3.23)$$

where $q(x)$ is the repair time probability density function (*pdf*).

For various repair time probability density functions, we can get different steady state

probability, P_k, solutions which are the probabilities that the robot system is in the failed

state k, for $k = n + 1, n + 2, n + 3$. The failed robot system mean time to repair represented

by various repair time distributions are explained in details in Appendix A1. Solving

Equations (3.10) – (3.12), and (3.20) – (3.22), together with

$$\sum_{i=0}^{n} P_i + \sum_{k=n+1}^{n+3} P_k = 1 \tag{3.24}$$

leads to the following general form of the steady state probability solutions:

$$P_i = \frac{\omega_i}{D} \qquad (for \quad i = 0, 1, 2, \dots, n) \tag{3.25}$$

$$P_k = \frac{\omega_k}{D} E_k[x] \qquad (for \quad k = n + 1, n + 2, n + 3) \tag{3.26}$$

$$D = \sum_{i=0}^{n} \omega_i + \sum_{k=n+1}^{n+3} \omega_k E_k[x] \tag{3.27}$$

where $\omega_0, \omega_1, \omega_2, \dots, \omega_{n+3}$ are the constants associated with the state probabilities P_0, P_1,

$P_2, \dots, P_{n+3}$. Consequently, the generalized robot system steady state availability when the

robot operating with at least one working safety unit is

$$SSAV_{rs} = \sum_{i=0}^{n-1} P_i = \frac{\displaystyle\sum_{i=0}^{n-1} \omega_i}{\displaystyle\sum_{i=0}^{n} \omega_i + \sum_{k=n+1}^{n+3} \omega_k E_k[x]} \tag{3.28}$$

Similarly, the general form of the robot-safety system steady state availability with or

without the working safety unit(s) is given by

$$SSAV_r = \sum_{i=0}^{n} P_i = \frac{\displaystyle\sum_{i=0}^{n} \omega_i}{\displaystyle\sum_{i=0}^{n} \omega_i + \sum_{k=n+1}^{n+3} \omega_k E_k[x]} \tag{3.29}$$

For various failed robot system repair time distributions, the values of D [i.e., Equation (3.27)] are obtained as follows:

3.3.1.1 *Gamma Distribution*

When the failed robot system repair time x is gamma distributed, the probability density function (*pdf*) of the repair time is expressed by

$$q_k(x) = \frac{\mu_k (\mu_k x)^{\beta-1} e^{-\mu_k x}}{\Gamma(\beta)} \quad ; \ x \geq 0, \ \beta > 0 \tag{3.30}$$

where x is the repair time, $\Gamma(\beta)$ is the gamma function, β and μ are the shape and scale parameters, respectively. Thus, the mean time to robot system repair, $E_k[x]$, for the gamma distribution is

$$E_k[x] = \int_0^\infty x q_k(x) \ dx = \beta / \mu_k \qquad (for \quad k = n+1, n+2, n+3) \tag{3.31}$$

Substituting Equation (3.31) into Equation (3.27), we get

$$D = D_G = \sum_{i=0}^{n} \omega_i + \sum_{k=n+1}^{n+3} \omega_k \beta / \mu_k \tag{3.32}$$

Exponential distribution is a special case of gamma distribution for $\beta = 1$. If β is an integer, where $\Gamma(\beta) = (\beta - 1)!$, then, Equation (3.32) becomes **Erlangian distribution**.

3.3.1.2 *Weibull Distribution*

When the failed robot system repair time x is Weibull distributed, the *pdf* of the repair time

is expressed by

$$q_k(x) = \mu_k \, \beta \, x^{\beta-1} \, e^{-(\mu_k x)^{\beta}} \quad ; \ x > 0, \ \beta > 0 \tag{3.33}$$

where x is the repair time, β and μ are shape and scale parameters of the Weibull distribution, respectively. Thus, the mean time to robot system repair, $E_k[x]$, for the Weibull distribution is given by

$$E_k[x] = \int_0^{\infty} x q_k(x) \, dx = \left(\frac{1}{\mu_k}\right)^{1/\beta} \frac{1}{\beta} \, \Gamma\left(\frac{1}{\beta}\right) \quad (for \quad k = n+1, n+2, n+3) \tag{3.34}$$

Substituting Equation (3.34) into Equation (3.27), we get

$$D = D_W = \sum_{l=0}^{n} \omega_l + \sum_{k=n+1}^{n+3} \omega_k \left(\frac{1}{\mu_k}\right)^{1/\beta} \frac{1}{\beta} \, \Gamma\left(\frac{1}{\beta}\right) \tag{3.35}$$

Exponential distribution is also a special case of the Weibull distribution for $\beta = 1$.

3.3.1.3 *Rayleigh Distribution*

Rayleigh distribution is another special case of Weibull distribution for $\beta = 2$. The probability distribution function (*pdf*) of the Rayleigh distribution is expressed by

$$q_k(x) = \mu_k x \, e^{-\mu_k x^2/2} \quad ; \ x > 0, \ \mu_k > 0 \tag{3.36}$$

where x is the repair time and μ the scale parameter. Therefore, the failed robot system mean time to repair, $E_k[x]$, for the Rayleigh distribution is

$$E_k[x] = \int_0^\infty x q_k(x) \, dx = \sqrt{\frac{\pi}{4\mu_k}} \qquad (for \quad k = n+1, n+2, n+3) \tag{3.37}$$

Substituting Equation (3.37) into Equation (3.27), we get

$$D = D_R = \sum_{i=0}^n \omega_i + \sum_{k=n+1}^{n+3} \omega_k \sqrt{\frac{\pi}{4\mu_k}} \tag{3.38}$$

3.3.1.4 *Lognormal Distribution*

When the robot system repair time x is lognormally distributed, the *pdf* of the repair time is

$$q_k(x) = \frac{1}{x\sigma_{y_k}\sqrt{2\pi}} \, e^{\left[\frac{-(lnx - \mu_{y_k})^2}{2\sigma_{y_k}^2}\right]} \qquad (for \quad k = n+1, n+2, n+3) \tag{3.39}$$

where x is the repair time, lnx is the natural logarithm of x with a mean and variance μ and σ, respectively. The conditions on parameters are as follows:

$$\sigma_{y_k} = \ln\sqrt{1 + \left(\frac{\sigma_{x_k}}{\mu_{x_k}}\right)^2} \ , \qquad \mu_{y_k} = \ln\sqrt{\frac{\mu_{x_k}^4}{\mu_{x_k}^2 + \sigma_{x_k}^2}}$$

Hence, the failed robot system mean time to repair, $E_k[x]$, for the log-normal distribution is

$$E_k[x] = e^{(\mu_{y_k} + \frac{\sigma_{y_k}^2}{2})} \qquad (for \quad k = n+1, n+2, n+3) \tag{3.40}$$

Substituting Equation (3.40) into Equation (3.27), we get

$$D = D_L = \sum_{i=0}^n \omega_i + \sum_{k=n+1}^{n+3} \omega_k e^{(\mu_{y_k} + \frac{\sigma_{y_k}^2}{2})} \tag{3.41}$$

3.3.2   Time Dependent Availability Analysis

Taking the Laplace transforms of Equations (3.1) – (3.8), we get

$$(s + a_0)P_0(s) = 1 + \mu_1 P_1(s) + \sum_{k=n+1}^{n+3} \int_0^\infty p_k(x,s)\mu_k(x)dx \tag{3.42}$$

$$(s + a_1)P_1(s) = C_0 P_0(s) + \mu_2 P_2(s) \tag{3.43}$$

$$(s + a_i)P_i(s) = C_{i-1}P_{i-1}(s) + \mu_{i+1}P_{i+1}(s) \qquad (for \quad i = 1, 2, 3, \ldots, n-1) \tag{3.44}$$

$$(s + a_n)P_n(s) = C_{n-1}P_{n-1}(s) \tag{3.45}$$

$$\left[\frac{\partial}{\partial x} + s + \mu_k(x)\right] p_k(x,s) = 0 \qquad (for \quad k = n+1, n+2, n+3) \tag{3.46}$$

$$p_{n+1}(0,s) = \lambda_{rf} P_n(s) \tag{3.47}$$

$$p_{n+2}(0,s) = \lambda_{rs} P_n(s) \tag{3.48}$$

$$p_{n+3}(0,s) = \lambda_r \left(\sum_{i=0}^{n-1} P_i(s)\right) \tag{3.49}$$

Solving differential Equation (3.46), we get the following resulting expression:

$$p_k(x,s) = p_k(0,s)e^{-sx}e^{-\int_0^x \mu_k(\theta)\,d\theta} \qquad (for \quad k = n+1, n+2, n+3) \tag{3.50}$$

The Laplace transform of the probability, $P_k(t)$, that due to a failure the robot system is under repair, is

$$P_k(s) = \int_0^\infty p_k(x,s)dx \qquad (for \quad k = n+1, n+2, n+3) \tag{3.51}$$

Substituting Equation (3.50) into Equation (3.51), we get

$$P_k(s) = \int_0^\infty p_k(0,s) e^{-sx} e^{-\int_0^x \mu_k(\theta)\, d\theta} dx \qquad (for \quad k = n+1, n+2, n+3) \qquad (3.52)$$

Substituting Equations (3.47) – (3.49) into Equation (3.52) and we obtain the following

generalized probability expressions when the robot is in the failed state.

$$P_{n+1}(s) = \int_0^\infty \lambda_{n} P_n(s) e^{-sx} e^{-\int_0^x \mu_{n+1}(\theta)\, d\theta} dx$$
$$= \lambda_{n} P_n(s) \frac{1 - W_{n+1}(s)}{s} \qquad (3.53)$$

$$P_{n+2}(s) = \int_0^\infty \lambda_{rs} P_n(s) e^{-sx} e^{-\int_0^x \mu_{n+2}(\theta)\, d\theta} dx$$
$$= \lambda_{rs} P_n(s) \frac{1 - W_{n+2}(s)}{s} \qquad (3.54)$$

$$P_{n+3}(s) = \int_0^\infty \lambda_r \sum_{i=0}^{n-1} P_i(s) e^{-sx} e^{-\int_0^x \mu_{n+3}(\theta)\, d\theta} dx$$
$$= \lambda_r \sum_{i=0}^{n-1} P_i(s) \frac{1 - W_{n+3}(s)}{s} \qquad (3.55)$$

where

$$\frac{1 - W_k(s)}{s} = \int_0^\infty e^{-sx} e^{-\int_0^x \mu_k(\theta)\, d\theta} dx \qquad (for \quad k = n+1, n+2, n+3) \qquad (3.56)$$

or

$$W_k(s) = \int_0^\infty e^{-sx} q_k(x) dx \qquad (for \quad k = n+1, n+2, n+3) \qquad (3.57)$$

$$q_k(x) = \mu_k(x) e^{-\int_0^x \mu_k(\theta)\, d\theta}$$

where $q_k(x)$ is the robot failed system repair time probability density function.

Solving Equations (3.43)–(3.45) and (3.53)– (3.55), together with

$$\sum_{i=0}^{n} P_i(s) + \sum_{k=n+1}^{n+3} P_k(s) = 1/s \tag{3.58}$$

we obtain the following general forms of Laplace transforms of state probabilities:

$$P_i(s) = \frac{\alpha_i(s)}{s \cdot B(s)} \qquad (for \quad i = 0, 1, 2, ..., n) \tag{3.59}$$

$$P_k(s) = \frac{\alpha_k(s)}{s \cdot B(s)} [1 - W_k(s)] \qquad (for \quad k = n+1, n+2, n+3) \tag{3.60}$$

$$B(s) = \sum_{i=0}^{n} \alpha_i(s) + \sum_{k=n+1}^{n+3} \alpha_k(s)[1 - W_k(s)] \tag{3.61}$$

where $\alpha_1(s)$, $\alpha_2(s)$, ... , $\alpha_{n+3}(s)$ are the coefficients associated with $P_0(s)$, $P_1(s)$, ... , $P_{n+3}(s)$, respectively. For a known robot failed system repair time distribution, one can invert Equations (3.59) – (3.61) and obtain the corresponding time-dependent probability expressions $P_0(t)$, $P_1(t)$, ... , $P_{n+3}(t)$. The repair time distribution for a failed system is often assumed to fit the gamma distribution. One important property of the gamma distribution which is not shared by the Weibull and log-normal distributions is that its Laplace transform is an elementary function. In other words, gamma distribution possesses rational Laplace transform and therefore, it is best suited to fit the robot failed system repair time distribution.

When the robot system repair time x is gamma distributed, the probability density function of the repair time is expressed by Equation (3.30). For $\beta = 1$ in Equation (3.30), the failed robot system repair rate is constant and its repair time is exponentially distributed.

Thus, Equation (3.30) yields:

$$q_k(x) = \mu_k \, e^{-\mu_k x} \qquad (for \quad k = n+1, n+2, n+3)$$

The equivalent Laplace transform expression is

$$W_k(s) = \frac{\mu_k}{s + \mu_k} \qquad (for \quad k = n+1, n+2, n+3) \tag{3.62}$$

Substituting the above result into Equations (3.59) – (3.61), we obtain the following set of Laplace transforms of the state probability expressions:

$$P_i(s) = \frac{\alpha_i(s)}{s \cdot B(s)} \qquad (for \quad i = 0, 1, 2, \dots n) \tag{3.63}$$

$$P_k(s) = \frac{\alpha_k(s)}{(s + \mu_k)B(s)} \qquad (for \quad k = n+1, n+2, n+3) \tag{3.64}$$

$$B(s) = \sum_{i=0}^{n} \alpha_i(s) + \sum_{k=n+1}^{n+3} \frac{s \cdot \alpha_k(s)}{(s + \mu_k)} \tag{3.65}$$

Using Equation (3.63), the generalized Laplace transform of the robot system availability when the robot is operating with at least one working safety unit, is

$$AV_{rs}(s) = \sum_{i=0}^{n-1} P_i(s) = \frac{\displaystyle\sum_{i=0}^{n-1} \alpha_i(s)}{s\left(\displaystyle\sum_{i=0}^{n} \alpha_i(s) + \sum_{k=n+1}^{n+3} \frac{s \cdot \alpha_k(s)}{(s + \mu_k)}\right)} \tag{3.66}$$

Similarly, using Equations (3.63), the generalized Laplace transform of the robot system availability with or without a working safety unit(s) is expressed by

$$AV_r(s) = \sum_{i=0}^{n} P_i(s) = \frac{\displaystyle\sum_{i=0}^{n} \alpha_i(s)}{s\left(\displaystyle\sum_{i=0}^{n} \alpha_i(s) + \sum_{k=n+1}^{n+3} \frac{s \cdot \alpha_k(s)}{(s + \mu_k)}\right)} \tag{3.67}$$

Inverting Equations (3.63) – (3.67), we can obtain the robot system time-dependent probability and availability expressions.

For $\beta = 2$, the failed robot system repair rate becomes non-constant and its repair time is represented by an Erlangian distribution. Thus, Equation (3.30) yields:

$$q_k(x) = \mu_k^2 \, x \, e^{-\mu_k x} \qquad (for \quad k = n+1, n+2, n+3)$$

The equivalent Laplace transform expression is

$$W_k(s) = \frac{\mu_k^2}{(s + \mu_k)^2} \qquad (for \quad k = n+1, n+2, n+3) \tag{3.69}$$

Substituting the above result into Equations (3.59) – (3.61), we obtain the following set of Laplace transforms of the state probability expressions

$$P_i(s) = \frac{\alpha_i(s)}{s \cdot B(s)} \qquad (for \quad i = 0, 1, 2, \ldots n) \tag{3.70}$$

$$P_k(s) = \frac{s \cdot \alpha_k(s)}{(s + \mu_k)^2 B(s)} \qquad (for \quad k = n+1, n+2, n+3) \tag{3.71}$$

$$B(s) = \sum_{i=0}^{n} \alpha_i(s) + \sum_{k=n+1}^{n+3} \frac{s^2 \cdot \alpha_k(s)}{(s + \mu_k)^2} \tag{3.72}$$

Using Equation (3.70), the generalized Laplace transform of the robot system availability when the robot is operating with at least one working safety unit, is

$$AV_{rs}(s) = \sum_{l=0}^{n-1} P_l(s) = \frac{\displaystyle\sum_{l=0}^{n-1} \alpha_l(s)}{s\left(\displaystyle\sum_{l=0}^{n} \alpha_l(s) + \sum_{k=n+1}^{n+3} \frac{s^2 \cdot \alpha_k(s)}{(s + \mu_k)^2}\right)} \tag{3.73}$$

Similarly, using Equation (3.70), the generalized Laplace transform of the robot system availability with or without a working safety unit(s) is expressed by

$$AV_r(s) = \sum_{l=0}^{n} P_l(s) = \frac{\displaystyle\sum_{l=0}^{n} \alpha_l(s)}{s\left(\displaystyle\sum_{l=0}^{n} \alpha_l(s) + \sum_{k=n+1}^{n+3} \frac{s^2 \cdot \alpha_k(s)}{(s + \mu_k)^2}\right)} \tag{3.74}$$

Inverting Equations (3.70) – (3.74), we can obtain the robot system time-dependent probability and availability expressions.

3.3.3   Robot System Reliability and MTTF

Setting $\mu_{n+1}(x) = \mu_{n+2}(x) = \mu_{n+3}(x) = 0$ in Equations (3.1) – (3.8), we can investigate system reliability of an irreparable robot when its associated safety units can be repaired. Utilizing the Markov method, the system of differential equations becomes

$$P_0'(t) + a_0 P_0(t) = \mu_1 P_1(t) \tag{3.75}$$

$$P_1'(t) + a_1 P_1(t) = C_0 P_0(t) + \mu_2 P_2(t) \tag{3.76}$$

$$P_i'(t) + a_i P_i(t) = C_{i-1} P_{i-1}(t) + \mu_{i+1} P_{i+1}(t) \quad (for \ i = 1, 2, 3, \dots, n-1) \quad (3.77)$$

$$P_n'(t) + a_n P_n(t) = C_{n-1} P_{n-1}(t) \quad\quad\quad\quad\quad\quad\quad\quad\quad (3.78)$$

where

$$a_i = C_i + \mu_i + \lambda_r$$
$$C_i = (n - i)\lambda_s \quad\quad (for \quad i = 0,1,2,\dots,n - 1)$$
$$C_n = \lambda_{ri} + \lambda_{rs} + \mu_n$$
$$\mu_0 = 0$$

$$P_{n+1}'(t) = \lambda_{ri} P_n(t) \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (3.79)$$

$$P_{n+2}'(t) = \lambda_{rs} P_n(t) \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (3.80)$$

$$P_{n+3}'(t) = \lambda_r \left(\sum_{i=0}^{n-1} P_i(t) \right) \quad\quad\quad\quad\quad\quad\quad\quad\quad (3.81)$$

At time $t = 0$, $P_0(0) = 1$, and all other initial state probabilities are equal to zero. The prime denotes differentiation with respect to time t. Taking the Laplace transforms of Equations (3.75) – (3.81) and solving the resulting set of equations, we obtain the following generalized Laplace transforms of state probabilities:

$$P_i(s) = \frac{\gamma_i(s)}{s \cdot Z(s)} \quad\quad (for \quad i = 0, 1, 2,\dots, n) \quad\quad\quad\quad (3.82)$$

$$P_k(s) = \frac{\gamma_k(s)}{s \cdot Z(s)} \quad\quad (for \quad k = n + 1, n + 2, n + 3) \quad\quad (3.83)$$

$$Z(s) = \sum_{i=0}^{n} \gamma_i(s) + \sum_{k=n+1}^{n+3} \gamma_k(s) \quad\quad\quad\quad\quad\quad (3.84)$$

where $\gamma_1(s), \gamma_2(s), \dots, \gamma_{n+3}(s)$ are the coefficients associated with $P_0(s)$, $P_1(s)$, ... , $P_{n+3}(s)$,

respectively. Using Equation (3.82), the generalized Laplace transform of the robot system

reliability with at least one working safety unit is

$$R_{rs}(s) = \sum_{i=0}^{n-1} P_i(s) = \frac{\sum_{i=0}^{n-1} \gamma_i(s)}{s \cdot Z(s)} \tag{3.85}$$

Utilizing Equation (3.85), the robot system mean time to failure can be obtained as follows:

$$MTTF_{rs} = \lim_{s \to 0} R_{rs}(s) = \frac{\sum_{i=0}^{n-1} \gamma_i}{\sum_{k=n+1}^{n+3} \gamma_k} \tag{3.86}$$

Similarly, the generalized Laplace transform of the robot system reliability with or without

the safety units(s) is given by

$$R_r(s) = \sum_{i=0}^{n} P_i(s) = \frac{\sum_{i=0}^{n} \gamma_i(s)}{s \cdot Z(s)} \tag{3.87}$$

The mean time to failure under this condition is

$$MTTF_r = \lim_{s \to 0} R_r(s) = \frac{\sum_{i=0}^{n} \gamma_i}{\sum_{k=n+1}^{n+3} \gamma_k} \tag{3.88}$$

3.4 Special Case Model I: (n = 1)

Figure 3.3 represents the transition diagram for a system containing one robot and one

safety unit [113]. It can be obtained from the generalized model in Figure 3.2 for $n = 1$.
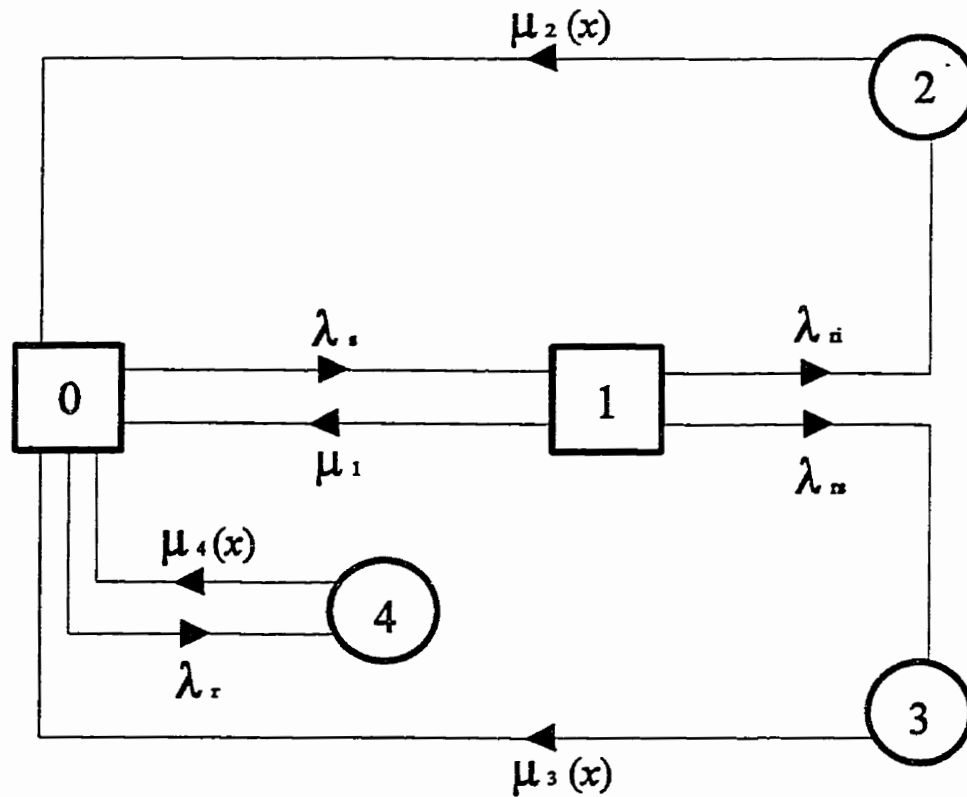
Figure 3.3.   State space transition diagram for a system containing one robot and one safety unit $(n = 1)$.

The corresponding system of integro-differential equations for the model in Figure 3.3 can

be extracted from the generalized Equations $(3.1) - (3.8)$ by setting $n = 1$. Thus, the set of

differential equations becomes

$$P_0'(t) + a_0 P_0(t) = \mu_1 P_1(t) + \sum_{k=2}^{4} \int_0^\infty P_k(x,t)\mu_k(x)dx \tag{3.89}$$

$$P_1'(t) + a_1 P_1(t) = C_0 P_0(t) \tag{3.90}$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_k(x)\right] P_k(x,t) = 0 \qquad (for \quad k = 2,3,4) \tag{3.91}$$

where

$$a_0 = \lambda_s + \lambda_r$$
$$C_0 = \lambda_s$$
$$a_1 = \lambda_{rl} + \lambda_{rs} + \mu_1$$

The associated boundary conditions are as follows:

$$p_2(0,t) = \lambda_{rl} P_1(t) \tag{3.92}$$

$$p_3(0,t) = \lambda_{rs} P_1(t) \tag{3.93}$$

$$p_4(0,t) = \lambda_r P_0(t) \tag{3.94}$$

At time $t = 0$, $P_0(0) = 1$, $P_1(0) = 0$, and $p_k(x,0) = 0$ for $k = 2, 3, 4$. The prime denotes differentiation with respect to time t.

3.4.1   Steady State Availability Analysis

As time approaches infinity, state probabilities reach the steady state. Inserting $n = 1$ into the generalized Equations (3.25) – (3.27) developed in Section 3.3.1, we get the following steady state probabilities:

$$P_i = \frac{\omega_i}{D} \qquad (for \quad i = 0, 1) \tag{3.95}$$

$$P_k = \frac{\omega_k}{D} E_k[x] \qquad (for \quad k = 2, 3, 4) \tag{3.96}$$

$$D = \sum_{i=0}^{1} \omega_i + \sum_{k=2}^{4} \omega_k E_k[x] \tag{3.97}$$

where

$$\omega_0 = a_1$$
$$\omega_1 = C_0$$
$$\omega_2 = \lambda_{ri} C_0$$
$$\omega_3 = \lambda_{ri} C_0$$
$$\omega_4 = a_1 \lambda_r$$

The robot system steady state availability with an operating safety unit is:

$$SSAV_{ri} = P_0 = \frac{\omega_0}{\displaystyle\sum_{i=0}^{1} \omega_i + \sum_{k=2}^{4} \omega_k E_k[x]} \tag{3.98}$$

Similarly, the robot system steady state availability with or without the working safety units

is given by

$$SSAV_r = \sum_{i=0}^{1} P_i = \frac{\displaystyle\sum_{i=0}^{1} \omega_i}{\displaystyle\sum_{i=0}^{1} \omega_i + \sum_{k=2}^{4} \omega_k E_k[x]} \tag{3.99}$$

For various robot system repair time distributions, the values of D [i.e., Equation (3.97)] are

as follows:

For the robot system repair time x, represented by a **gamma** distribution, we get

$$D = D_G = \sum_{i=0}^{1} \omega_i + \sum_{k=2}^{4} \omega_k \beta / \mu_k \qquad (3.100)$$

When $\beta = 1$ in Equation (3.100), the robot system repair time x is represented by a

exponential distribution, therefore

$$D = D_E = \sum_{i=0}^{1} \omega_i + \sum_{k=2}^{4} \omega_k / \mu_k \qquad (3.101)$$

For the robot system repair time x, represented by a **Weibull** distribution, we get

$$D = D_W = \sum_{i=0}^{1} \omega_i + \sum_{k=2}^{4} \omega_k (\frac{1}{\mu_k})^{1/\beta} \frac{1}{\beta} \Gamma(\frac{1}{\beta}) \qquad (3.102)$$

When $\beta = 2$ in Equation (3.102), the robot system repair time x is represented by a

Rayleigh distribution, thus

$$D = D_R = \sum_{i=0}^{1} \omega_i + \sum_{k=2}^{4} \omega_k \sqrt{\frac{\pi}{4\mu_k}} \qquad (3.103)$$

For the robot system repair time x, represented by a **log-normal** distribution, we get

$$D = D_L = \sum_{i=0}^{1} \omega_i + \sum_{k=2}^{4} \omega_k e^{(\mu_{y_k} + \frac{\sigma_{y_k}^2}{2})} \qquad (3.104)$$

where

$$\sigma_{y_k} = \ln \sqrt{1 + (\frac{\sigma_{x_k}}{\mu_{x_k}})^2} \quad , \qquad \mu_{y_k} = \ln \sqrt{\frac{\mu_{x_k}^4}{\mu_{x_k}^2 + \sigma_{x_k}^2}} \qquad (for \quad k = 2, 3, 4)$$

3.4.1.1 *Steady State Availability Numerical Examples*

Setting:

$\lambda_s = 0.0005$, $\lambda_{si} = 0.0003$, $\lambda_{rs} = 0.0004$, $\lambda_r = 0.0005$
$\mu_1 = 0.0006$, $\mu_2 = 0.0007$, $\mu_3 = 0.0008$, $\mu_4 = 0.0009$

into Equations (3.98) and (3.99) and performing numerical analysis, we can obtain robot-safety system steady state availability numerical values.

Figures 3.4 – 3.6 show plots of the robot-safety system steady state availability for gamma, Weibull, and log-normal distributions, respectively. These plots indicate the steady state availability as a function of safety unit (mechanism) failure rate, λ_s. The objective is to examine the robot system long term availability for different distributions' properties (e.g., different shape parameters of gamma distribution).

To inspect robot system repairability, Tables 3-1 to 3-3 present steady state availability for gamma, Weibull, and log-normal distributions, respectively. These tables indicate the steady state values as a function of safety unit (mechanism) repair rate, μ_1. Again, the objective is to examine the robot long term availability for different distributions' properties.

For the sake of comparison, all distributions are presented on the same figure. For all distributions, Figures 3.7 and 3.8 show plots of the robot-safety system steady state availability as a function of safety unit (mechanism) failure (λ_s) and repair (μ_1) rates, respectively. More detailized inspection of Figures 3.7 and 3.8 can be made by referring to their associated tabular values which are given in Tables 3-4 and 3-5, respectively.

The trends shown by figures and tables are discussed in the concluding section of this chapter.
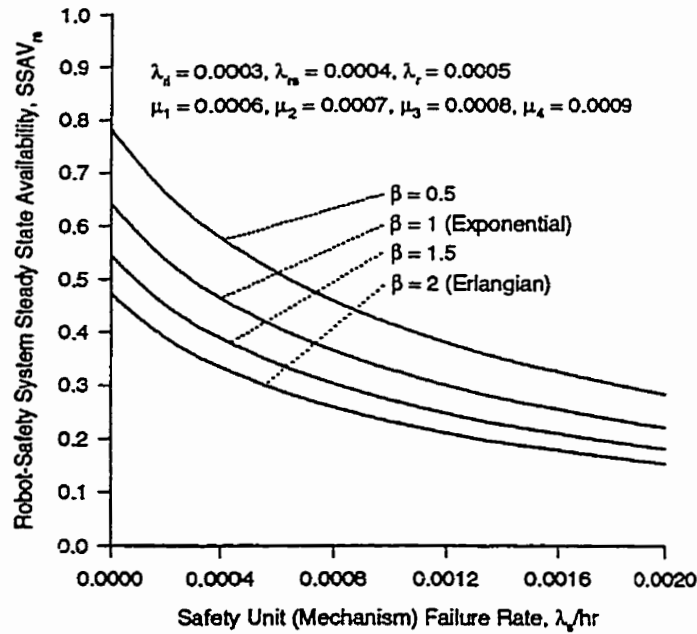
(a)
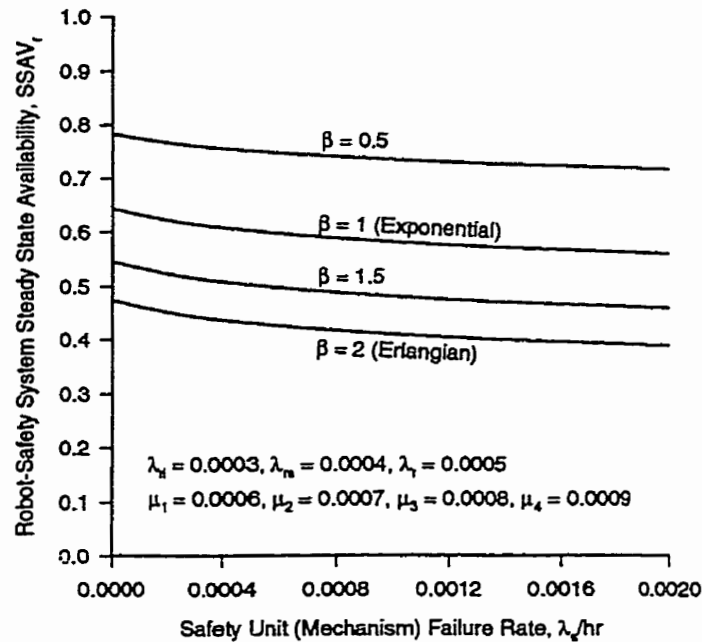
(b)

Figure 3.4.   Steady state availability ($n = 1$) vs λ_s plots for a robot with constant failure rate and for **gamma** distributed failed system repair times; (a) robot working with an operating safety mechanism, (b) robot working with or without the safety mechanism. More specifically, these plots were obtained using Equations (3.98) − (3.100).
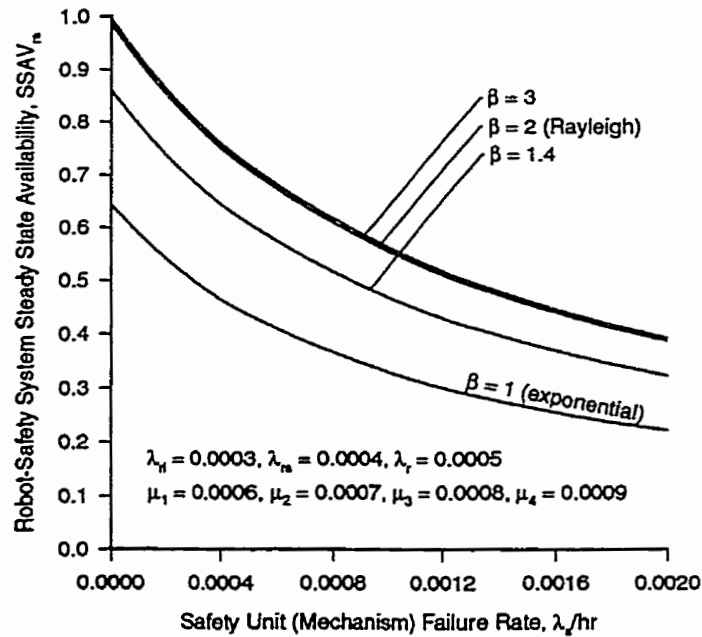
(a)
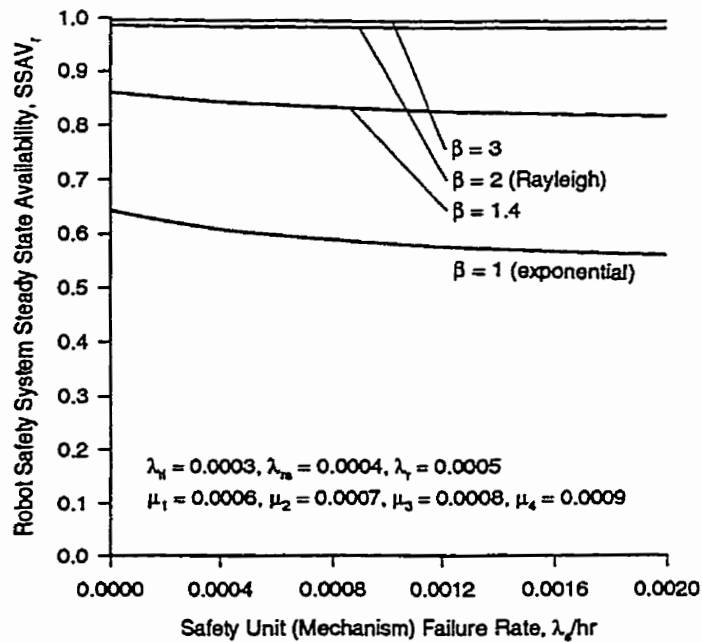
(b)

Figure 3.5.   Steady state availability ($n = 1$) vs λ_s plots for a robot with constant failure rate and for **Weibull** distributed failed system repair times; (a) robot working with an operating safety mechanism, (b) robot working with or without the safety mechanism. More specifically, these plots were obtained using Equations (3.98), (3.99), (3.102), and (3.103).
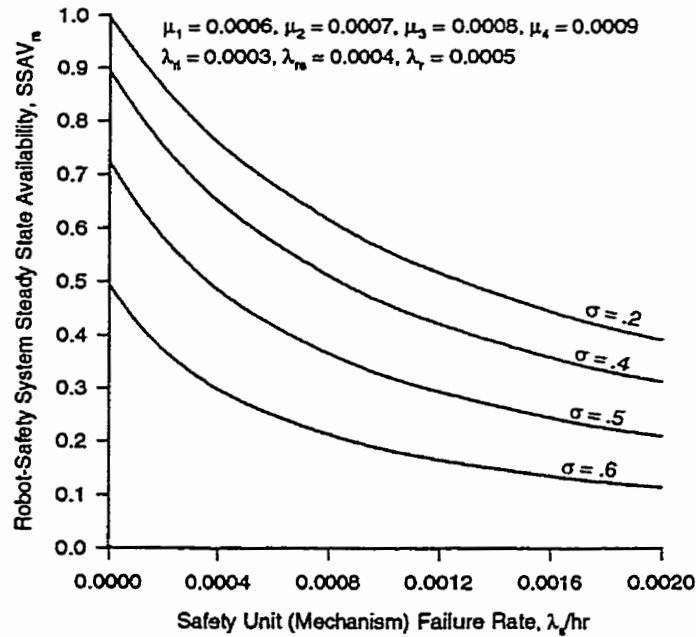
(a)
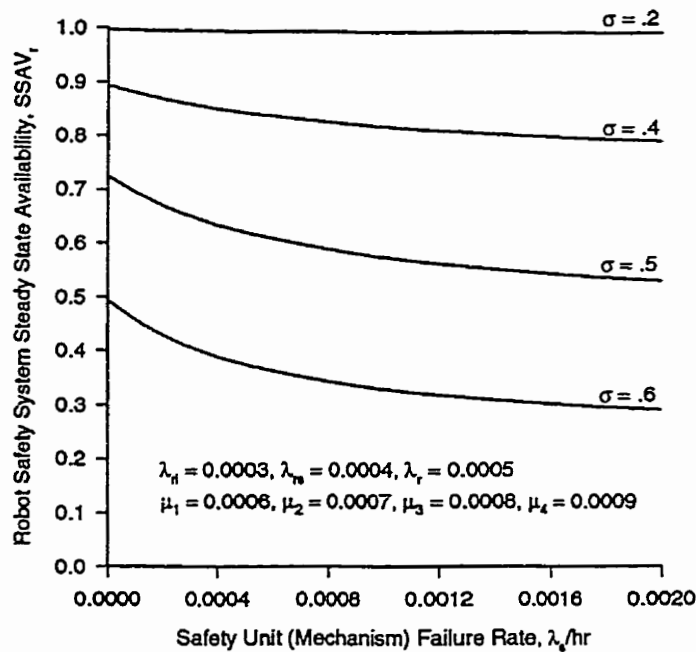
(b)

Figure 3.6.   Steady state availability ($n = 1$) vs λ_s plots for a robot with constant failure rate and for **log-normal** distributed failed system repair times; (a) robot working with an operating safety mechanism, (b) robot working with or without the safety mechanism. More specifically, these plots were obtained using Equations (3.98), (3.99), and (3.104).

Table 3-1: SSAV ($n = 1$) vs μ_1 values for a robot-safety system with constant failure rate and **gamma** distributed failed system repair times.

μ_1	$\beta = 0.5$		$\beta = 1$ (Exponential)		$\beta = 1.5$		$\beta = 2$ (Erlangian)	
	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$
0.0000	0.4303	0.7377	0.3409	0.5845	0.2823	0.4839	0.2409	0.4129
0.0005	0.5297	0.7504	0.4239	0.6005	0.3533	0.5005	0.3029	0.4291
0.0010	0.5853	0.7575	0.4711	0.6096	0.3941	0.5101	0.3388	0.4385
0.0015	0.6209	0.7620	0.5015	0.6155	0.4207	0.6163	0.3623	0.4446
0.0020	0.6456	0.7652	0.5228	0.6196	0.4393	0.5206	0.3788	0.4489

Table 3-2: SSAV ($n = 1$) vs μ_1 values for a robot-safety ystem with constant failure rate and **Weibull** distributed failed system repair times.

μ_1	$\beta = 1$ (Exponential)		$\beta = 1.4$		$\beta = 2$ (Rayleigh)		$\beta = 3$	
	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$
0.0000	0.3409	0.5845	0.5376	0.9217	0.5730	0.9823	0.5801	0.9945
0.0005	0.4239	0.6005	0.6535	0.9259	0.6940	0.9832	0.7022	0.9947
0.0010	0.4711	0.6096	0.7172	0.9282	0.7601	0.9837	0.7688	0.9949
0.0015	0.5015	0.6155	0.7575	0.9296	0.8018	0.9840	0.8107	0.9950
0.0020	0.5228	0.6196	0.7852	0.9306	0.8305	0.9843	0.8396	0.9950

Table 3-3: SSAV ($n = 1$) vs μ_1 values for a robot-safety system with constant failure rate and **log-normal** distributed failed system repair times.

μ_1	$\sigma = 0.2$		$\sigma = 0.4$		$\sigma = 0.5$		$\sigma = 0.6$	
	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$
0.0000	0.5790	0.9926	0.4778	0.8191	0.3377	0.5789	0.1944	0.3333
0.0005	0.7013	0.9934	0.5927	0.8396	0.4342	0.6151	0.2602	0.3686
0.0010	0.7680	0.9939	0.6578	0.8512	0.4921	0.6368	0.3023	0.3912
0.0015	0.8101	0.9942	0.6997	0.8587	0.5307	0.6513	0.3315	0.4069
0.0020	0.8391	0.9944	0.7290	0.8640	0.5583	0.6616	0.3530	0.4184

Table 3-4:   SSAV ($n = 1$) vs λ_s values for a robot with constant failure rate and various failed system repair time distributions.

λ_s	Erlangian (gamma, $\beta =2$)		exponential (gamma, $\beta =1$)		Lognormal ($\sigma = 0.4$)		Weibull ($\beta=1.2$)		Rayleigh (Weibull, $\beta=2$)	
	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$
.0000	.4737	.4737	.6429	.6429	.8933	.8933	.8603	.8603	.9854	.9854
.0004	.3344	.4373	.4653	.6085	.6499	.8498	.6447	.8431	.7522	.9837
.0008	.2584	.4175	.3646	.5890	.5107	.8250	.5155	.8327	.6083	.9826
.0012	.2106	.4050	.2998	.5765	.4206	.8089	.4294	.8258	.5105	.9818
.0016	.1777	.3964	.2545	.5677	.3576	.7976	.3680	.8209	.4399	.9813
0.002	.1537	.3901	.2211	.5613	.3109	.7893	.3219	.8172	.3864	.9809

System parameter values: $\lambda_{ri} = 0.0003$, $\lambda_{rs} = 0.0004$, $\lambda_r = 0.0005$

$\mu_1 = 0.0006$, $\mu_2 = 0.0007$, $\mu_3 = 0.0008$, $\mu_4 = 0.0009$

Table 3-5: SSAV ($n = 1$) vs μ_1 values for a robot with constant failure rate and various failed system repair time distributions.

μ_1	Erlangian (gamma, $\beta=2$)		exponential (gamma, $\beta =1$)		Lognormal ($\sigma = 0.4$)		Weibull ($\beta=1.2$)		Rayleigh (Weibull, $\beta=2$)	
	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$
.0000	.2409	.4129	.3409	.5845	.4778	.8191	.4843	.8302	.5730	.9823
.0004	.2933	.4266	.4112	.5980	.5751	.8365	.5758	.8375	.6759	.9831
.0008	.3264	.4352	.4549	.6065	.6354	.8473	.6315	.8420	.7377	.9836
.0012	.3493	.4412	.4847	.6123	.6766	.8546	.6690	.8450	.7789	.9839
.0016	.3660	.4456	.5064	.6165	.7064	.8599	.6959	.8472	.8084	.9841
.0020	.3788	.4489	.5228	.6196	.7290	.8640	.7162	.8488	.8305	.9843

System parameter values: $\lambda_s = 0.0005$, $\lambda_{ri} = 0.0003$, $\lambda_{rs} = 0.0004$, $\lambda_r = 0.0005$

$\mu_2 = 0.0007$, $\mu_3 = 0.0008$, $\mu_4 = 0.0009$
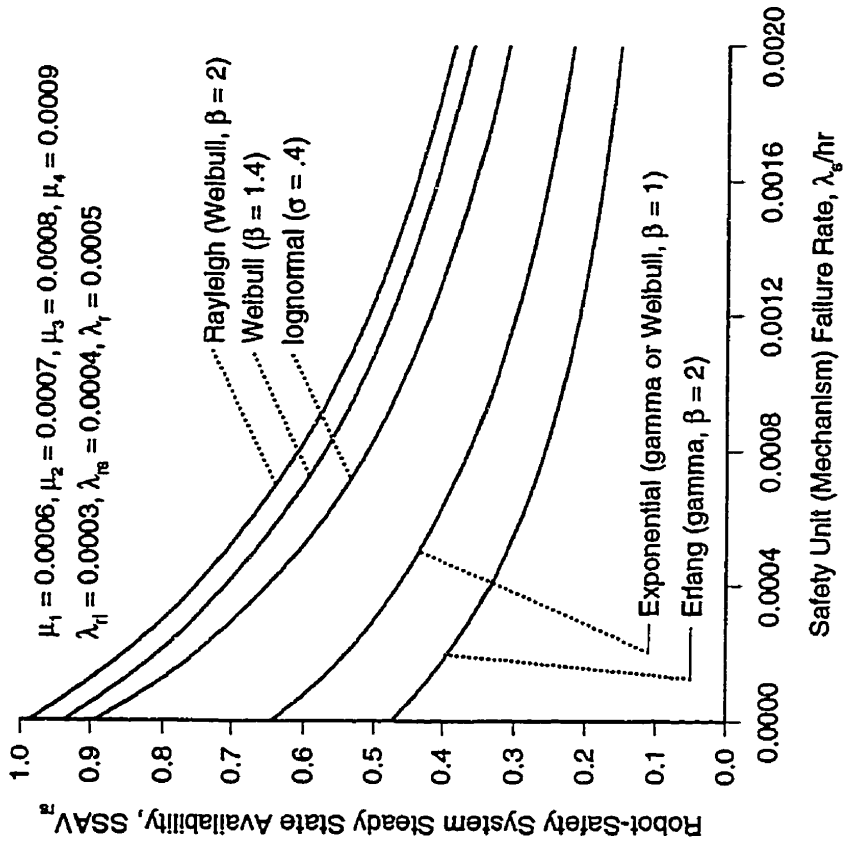
(a)
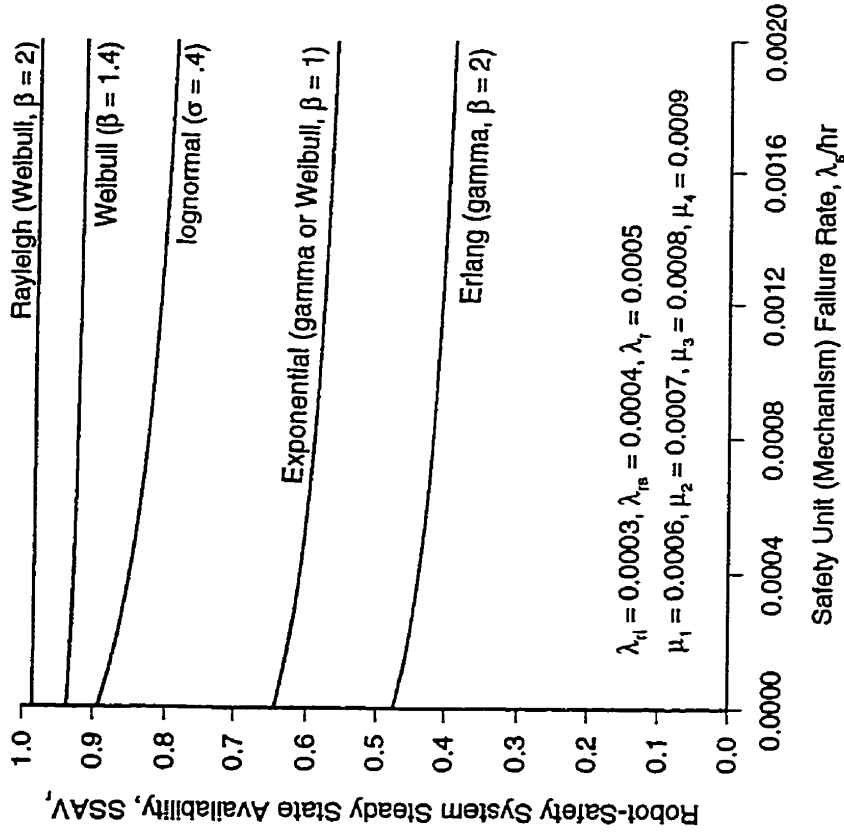
(b)

Figure 3.7. Steady state availability ($n = 1$) vs λ_s plots for a robot with constant failure rate and for various failed system repair time distributions when (a) robot working with an operating safety mechanism, (b) robot working with or without the safety mechanism. More specifically, these plots were obtained using Equations (3.98) – (3.104).
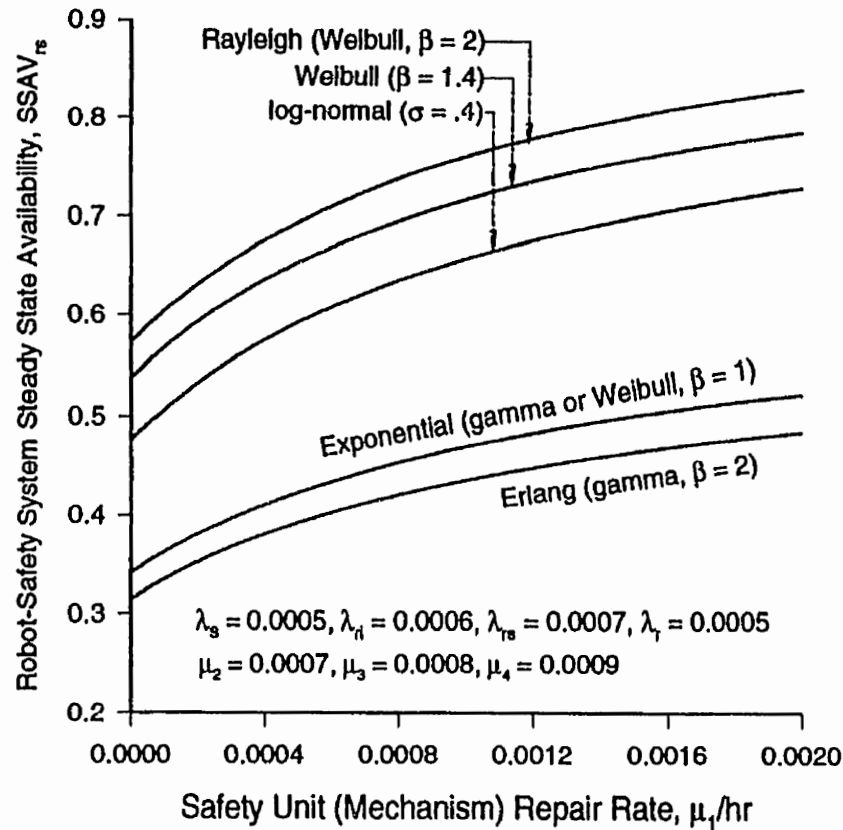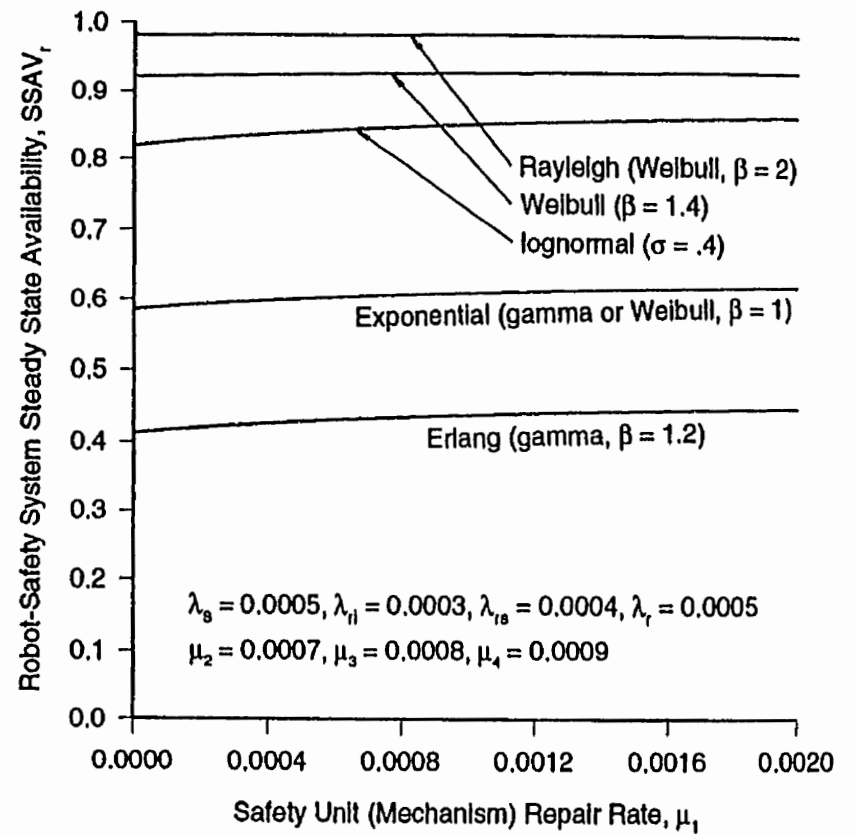
Figure 3.8.   Steady state availability ($n = 1$) vs μ_1 plots for a robot with constant failure rate and for various failed system repair time distributions when (a) robot working with an operating safety mechanism, (b) robot working with or without the safety mechanism. More specifically, these plots were obtained using Equations (3.98) – (3.104).

3.4.2   Time Dependent Availability Analysis

Inserting $n = 1$ into the generalized Equations (3.63) – (3.65) developed in Section 3.3.2, we obtain the following Laplace transforms of the robot system probability expressions with constant repair rate:

$$P_i(s) = \frac{\alpha_i(s)}{s \cdot B(s)} \qquad (for \quad i = 0, 1) \tag{3.105}$$

$$P_k(s) = \frac{\alpha_k(s)}{(s + \mu_k)B(s)} \qquad (for \quad k = 2, 3, 4) \tag{3.106}$$

$$B(s) = \sum_{i=0}^{1} \alpha_i(s) + \sum_{k=2}^{4} \frac{s \cdot \alpha_k(s)}{(s + \mu_k)} \tag{3.107}$$

where

$$\alpha_0(s) = s(s + a_1)$$
$$\alpha_1(s) = C_0 s$$
$$\alpha_2(s) = \lambda_{rs} C_0$$
$$\alpha_3(s) = \lambda_{rs} C_0$$
$$\alpha_4(s) = \lambda_r(s + a_1)$$

Using Equation (3.105), the Laplace transform of the robot system availability with an operating safety mechanism is given by

$$AV_{rs}(s) = P_0(s) = \frac{\alpha_0(s)}{s\left(\sum_{i=0}^{1} \alpha_i(s) + \sum_{k=2}^{4} \frac{s \cdot \alpha_k(s)}{(s + \mu_k)}\right)} \tag{3.108}$$

Similarly, using Equation (3.105), the Laplace transform of the robot system availability with or without a working safety mechanism is expressed by

$$AV_r(s) = P_0(s) + P_1(s) = \cfrac{\displaystyle\sum_{i=0}^{1} \alpha_i(s)}{s\left(\displaystyle\sum_{i=0}^{1} \alpha_i(s) + \sum_{k=2}^{4} \cfrac{s \cdot \alpha_k(s)}{(s + \mu_k)}\right)} \tag{3.109}$$

Inserting $n = 1$ into the generalized Equations (3.70) – (3.72) developed in Section 3.3.2, we obtain the following Laplace transforms of the robot system probability expressions with non-constant repair rate:

$$P_i(s) = \frac{\alpha_i(s)}{s \cdot B(s)} \qquad (for \quad i = 0, 1) \tag{3.110}$$

$$P_k(s) = \frac{s \cdot \alpha_k(s)}{(s + \mu_k)^2 B(s)} \qquad (for \quad k = 2, 3, 4) \tag{3.111}$$

$$B(s) = \sum_{i=0}^{1} \alpha_i(s) + \sum_{k=2}^{4} \frac{s^2 \cdot \alpha_k(s)}{(s + \mu_k)^2} \tag{3.112}$$

Using Equation (3.110), the Laplace transform of the robot system availability with an operating safety mechanism is

$$AV_{rs}(s) = P_0(s) = \cfrac{\alpha_0(s)}{s\left(\displaystyle\sum_{i=0}^{1} \alpha_i(s) + \sum_{k=2}^{4} \cfrac{s^2 \cdot \alpha_k(s)}{(s + \mu_k)^2}\right)} \tag{3.113}$$

Similarly, using Equation (3.110), the Laplace transform of the robot system availability with or without a working safety mechanism is expressed by

$$AV_r(s) = P_0(s) + P_1(s) = \cfrac{\displaystyle\sum_{i=0}^{1} \alpha_i(s)}{s\left(\displaystyle\sum_{i=0}^{1} \alpha_i(s) + \sum_{k=2}^{4} \cfrac{s^2 \cdot \alpha_k(s)}{(s + \mu_k)^2}\right)} \tag{3.114}$$

3.4.2.1 Time-Dependent Availability Numerical Examples

Generalized time dependent probability and availability expressions can be obtained by inverting Equations (3.105) – (3.114). For example, using Equation (3.105), the robot system up-time probability expressions are given by

$$P_0(t) = F_1 e^{k_1 t} + F_2 e^{k_2 t} + F_3 e^{k_3 t} + F_4 e^{k_4 t} + F_5 e^{k_5 t} \qquad (3.115)$$

$$P_1(t) = F_6 e^{k_1 t} + F_7 e^{k_2 t} + F_8 e^{k_3 t} + F_9 e^{k_4 t} + F_{10} e^{k_5 t} \qquad (3.116)$$

where $F_1, \ldots, F_{10}$ are constants and $k_1, \ldots, k_5$ are real and unique roots of the polynomial, $B(s)$ [i.e,. Equation (3.107)]. The symbols F_1 to F_{10} are defined in Appendix D.

Using Equations (3.115) and (3.116), the robot system generalized availability expressions, $AV_{rs}(t)$ (with an operating safety unit) and $AV_r(t)$ (with or without an operating safety unit) are given by

$$AV_{rs}(t) = P_0(t) = F_1 e^{k_1 t} + F_2 e^{k_2 t} + F_3 e^{k_3 t} + F_4 e^{k_4 t} + F_5 e^{k_5 t} \qquad (3.117)$$

and

$$\begin{aligned} AV_r(t) &= P_0(t) + P_1(t) \\ &= (F_1 + F_6) e^{k_1 t} + (F_2 + F_7) e^{k_2 t} + (F_3 + F_8) e^{k_3 t} + \\ &\quad (F_4 + F_9) e^{k_4 t} + (F_5 + F_{10}) e^{k_5 t} \end{aligned} \qquad (3.118)$$

The algebraic generalized derivation can become tediousm, specially, as the number of robots increases. Alternatively, numerical solutions and approximation are always feasible. They play an important role in applications, for which a digital computer becomes an indispensable tool. Numerical probability expressions can be obtained for the robot system

with constant repair rate (i.e., $\beta = 1$) and non-constant repair rate (i.e., $\beta = 2$) by setting:

$\lambda_r = 0.0005,$ $\lambda_{ri} = 0.0003,$ $\lambda_{rs} = 0.0004,$ $\lambda_r = 0.0005,$
$\mu_1 = 0.0006,$ $\mu_2 = 0.0007,$ $\mu_3 = 0.0008,$ $\mu_4 = 0.0009$

in Equations (3.105) − (3.107) and (3.110) − (3.112), respectively, and taking inverse

Laplace transform of the resulting equations. For $\beta = 1$, the up-time probability expressions

are

$$P_0(t) = 0.4353\, e^{k_1 t} + 0.0032\, e^{k_2 t} + 0.0032\, e^{k_3 t}$$
$$+ e^{k_4 t}\big(0.5583\, Cos\,(k_5 t) - 5.8817\, Sin\,(k_5 t)\big) \tag{3.119}$$

$$P_1(t) = 0.1674\, e^{k_1 t} + 0.0035\, e^{k_2 t} + 0.0028\, e^{k_3 t}$$
$$- e^{k_4 t}\big(0.1738\, Cos\,(k_5 t) - 11.2094\, Sin\,(k_5 t)\big) \tag{3.120}$$

where

$$k_1 = 0 \qquad\qquad k_2 = -0.0008$$
$$k_3 = -0.0007 \qquad k_4 = -0.0016$$
$$k_5 = 0.00002$$

Utilizing Equations (3.119) and (3.120), the robot system availability numerical expressions

are

$$AV_{rn}(t) = P_0(t) = 0.4353 + 0.0032\, e^{k_2 t} + 0.0032\, e^{k_3 t}$$
$$+ e^{k_4 t}\big(0.5583\, Cos\,(k_5 t) - 5.8817\, Sin\,(k_5 t)\big) \tag{3.121}$$

and

$$AV_r(t) = P_0(t) + P_1(t)$$
$$= 0.6027 + 0.0067\, e^{k_2 t} + 0.006\, e^{k_3 t} + \tag{3.122}$$
$$e^{k_4 t}\big(0.3845\, Cos\,(k_5 t) + 5.3277\, Sin\,(k_5 t)\big)$$

For $\beta = 2$, the up-time probability expressions become

$$P_0(t) = 0.3115 + 0.2496 \, e^{k_1 t} +$$

$$e^{k_2 t}\left(0.0012 \cos(k_3 t) - 0.0007 \sin(k_3 t)\right) +$$

$$e^{k_4 t}\left(0.0003 \cos(k_5 t) - 0.001 \sin(k_5 t)\right) +$$

$$e^{k_6 t}\left(0.4374 \cos(k_7 t) - 0.1304 \sin(k_7 t)\right)$$

$$(3.123)$$

$$P_1(t) = 0.1198 - 0.3128 \, e^{k_1 t} +$$

$$e^{k_2 t}\left(0.0011 \cos(k_3 t) - 0.0005 \sin(k_3 t)\right) +$$

$$e^{k_4 t}\left(0.0004 \cos(k_5 t) - 0.001 \sin(k_5 t)\right) +$$

$$e^{k_6 t}\left(0.1914 \cos(k_7 t) + 0.3064 \sin(k_7 t)\right)$$

$$(3.124)$$

where

$$k_1 = -0.0017$$
$$k_2 = -0.0007$$
$$k_3 = 0.00005$$
$$k_4 = -0.0008$$
$$k_5 = 0.00005$$
$$k_6 = -0.0011$$
$$k_7 = 0.0006$$

Using Equations (3.123) and (3.124), the robot system availability numerical expressions are

$$AV_{rs}(t) = P_0(t) = 0.3115 + 0.2496 \, e^{k_1 t} +$$

$$e^{k_2 t}\left(0.0012 \cos(k_3 t) - 0.0007 \sin(k_3 t)\right) +$$

$$e^{k_4 t}\left(0.0003 \cos(k_5 t) - 0.001 \sin(k_5 t)\right) +$$

$$e^{k_6 t}\left(0.4374 \cos(k_7 t) - 0.1304 \sin(k_7 t)\right)$$

$$(3.125)$$

and

$$AV_r(t) = P_0(t) + P_1(t) = 0.4313 - 0.0632\, e^{k_1 t} +$$

$$e^{k_2 t}\Big(0.0023 \cos(k_3 t) - 0.0012 \sin(k_3 t)\Big) +$$

$$e^{k_4 t}\Big(0.0007 \cos(k_5 t) - 0.002 \sin(k_5 t)\Big) +$$

$$e^{k_6 t}\Big(0.6288 \cos(k_7 t) + 0.176 \sin(k_7 t)\Big) \qquad (3.126)$$

State probability plots of the robot system with constant repair rate ($\beta = 1$) and the specified system parameter values are shown in Figure 3.9. Using Equations (3.121) and (3.122), time-dependent availability plots of the robot system with an operating or a failed safety mechanism are shown in Figure 3.10. More detailed inspection of the robot system state probability and availability can be made by referring to Tables 3-6 and 3-7, respectively.

State probability plots for the robot system with non-constant repair rate ($\beta = 2$) and the specified system parameter values are shown in Figure 3.11. Using Equations (3.125) and (3.126), time-dependent availability plots of the robot system with an operating or a failed safety mechanism are shown in Figure 3.12. More detailed inspection of the robot system state probability and availability can be made by referring to Tables 3-8 and 3-9, respectively.

Table 3-6: Time-dependent probability ($n = 1$) values for a robot with constant failure and repair ($\beta = 1$) rates.

Time (Hrs)	Time Dependent Probability ($\beta = 1$)				
	$P_0(t)$	$P_1(t)$	$P_2(t)$	$P_3(t)$	$P_4(t)$
0	1	0	0	0	0
500	0.6671	0.1456	0.0116	0.0152	0.1605
1000	0.5295	0.1828	0.0297	0.0382	0.2198
1500	0.4732	0.1863	0.0445	0.0563	0.2398
2000	0.4504	0.1816	0.0546	0.0681	0.2454
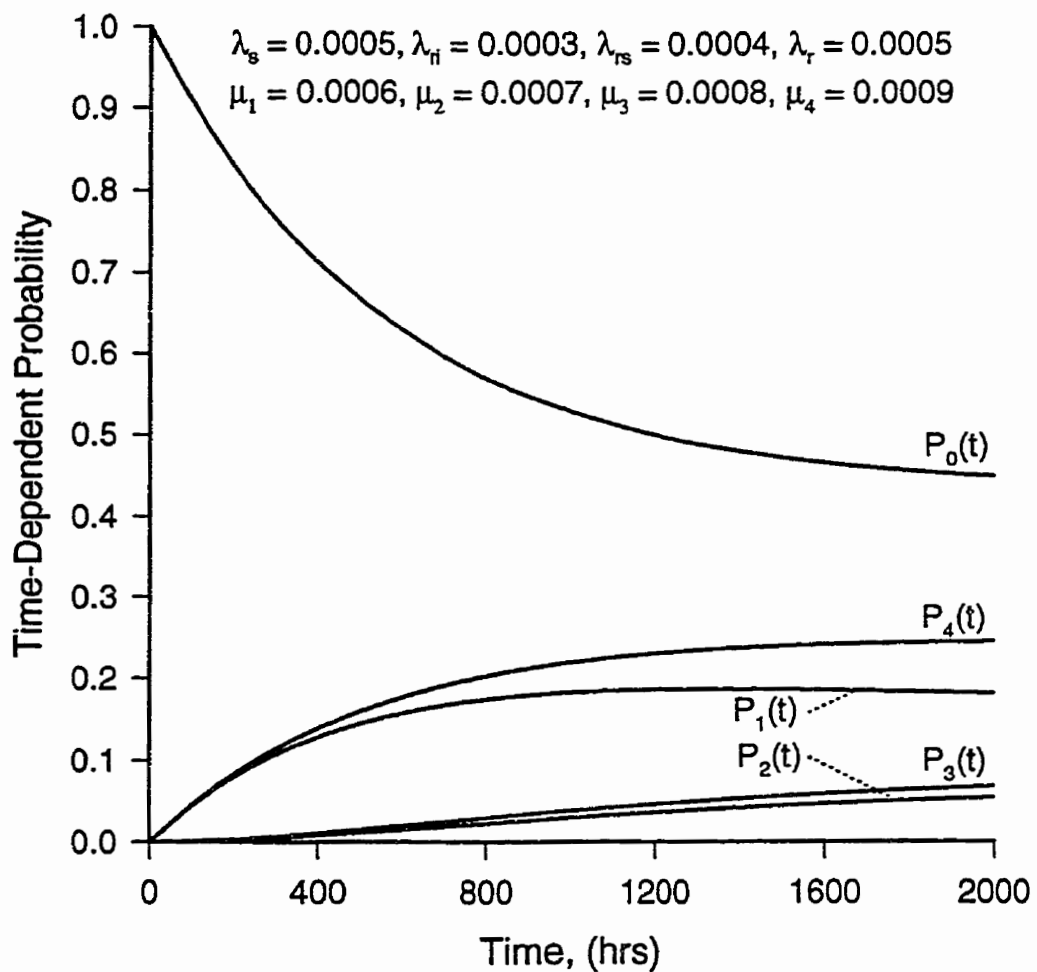
Figure 3.9.   Time-dependent probability ($n = 1$) plots for a robot with constant failure and repair ($\beta = 1$) rates.

Table 3-7: Time-dependent availability ($n = 1$) values for a robot with constant failure and repair ($\beta = 1$) rates.

Time (Hrs)	Time-Dependent Availability ($\beta = 1$)	
	$AV_{rs}(t)$	$AV_r(t)$
0	1	1
500	0.6671	0.8127
1000	0.5295	0.7123
1500	0.4732	0.6595
2000	0.4504	0.6320

$\lambda_s = 0.0005$, $\lambda_{ri} = 0.0003$, $\lambda_{rs} = 0.0004$, $\lambda_r = 0.0005$

$\mu_1 = 0.0006$, $\mu_2 = 0.0007$, $\mu_3 = 0.0008$, $\mu_4 = 0.0009$
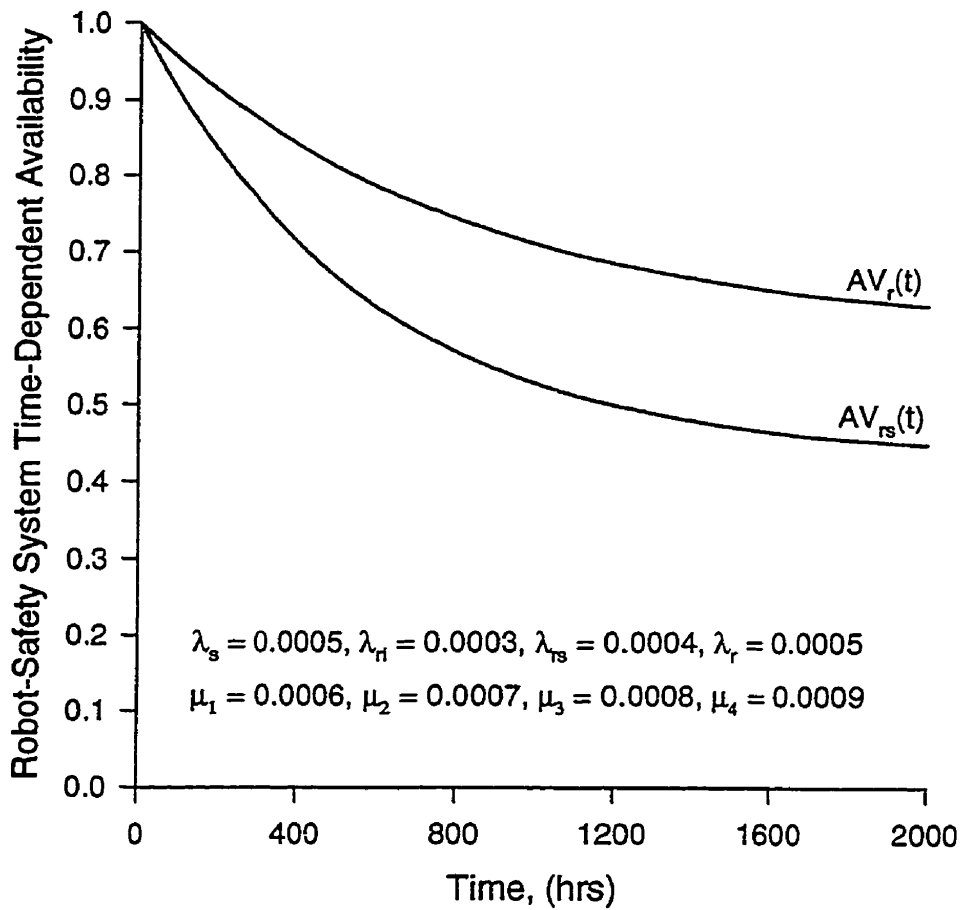
Figure 3.10.   Availability ($n = 1$) plots for a robot with constant failure and repair rates. More specifically, the plots were obtained using Equations (3.121) and (3.122).

Table 3-8: Time-dependent probability ($n = 1$) values for a robot with constant failure rate and **gamma** distributed ($\beta = 2$) failed system repair time distribution.

Time (Hrs)	Time-Dependent Probability ($\beta = 2$)				
	$P_0(t)$	$P_1(t)$	$P_2(t)$	$P_3(t)$	$P_4(t)$
0	1	0	0	0	0
500	0.6339	0.1429	0.0129	0.0171	0.1932
1000	0.4494	0.1704	0.0358	0.0472	0.2973
1500	0.3615	0.1616	0.0570	0.0742	0.3457
2000	0.3232	0.1464	0.0731	0.0939	0.3636
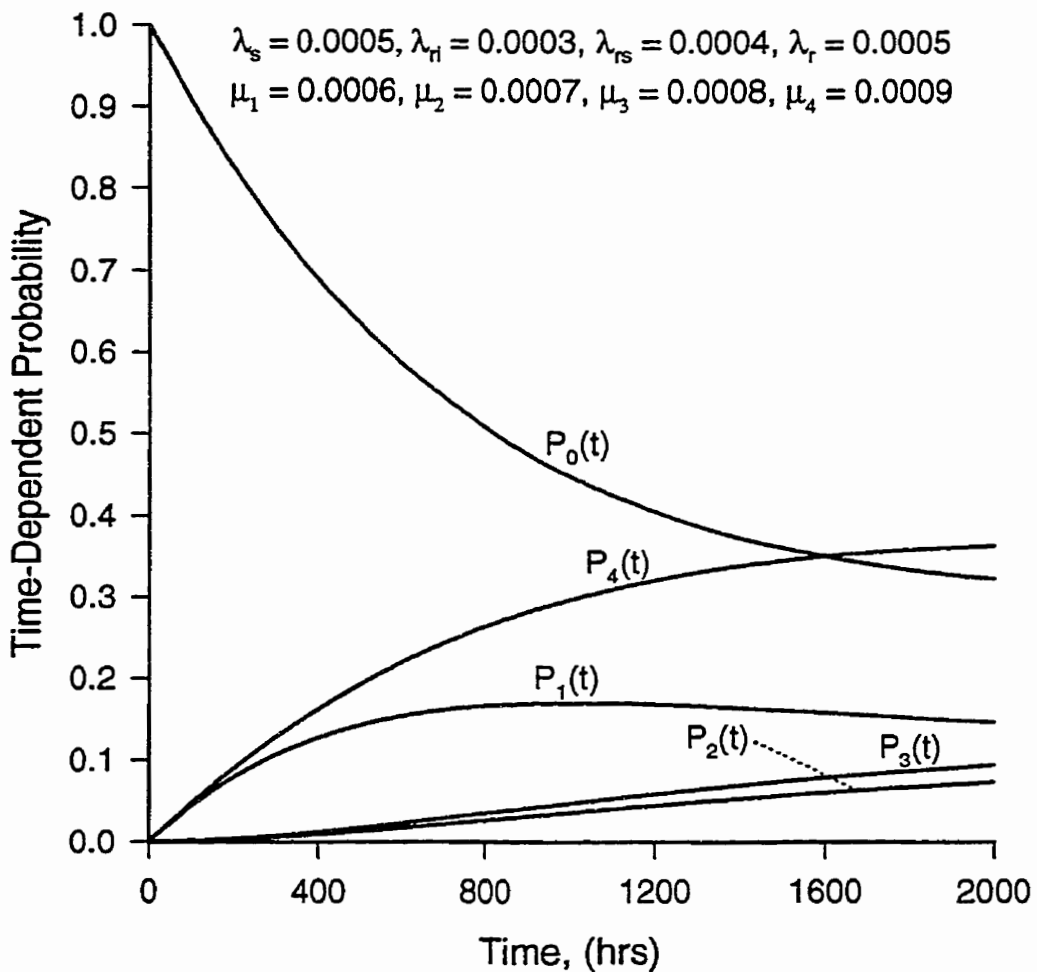
Figure 3.11.   Time-dependent probability ($n = 1$) plots for a robot with constant failure rate and **gamma** distributed ($\beta = 2$) failed system repair time distribution.

Table 3-9: Time-dependent availability ($n = 1$) values for a robot with constant failure rate and **gamma** distributed ($\beta = 2$) failed system repair time distribution.

Time (Hrs)	Time-Dependent Availability ($\beta = 2$)	
	$AV_{rs}(t)$	$AV_r(t)$
0	1	1
500	0.6339	0.7765
1000	0.4494	0.6198
1500	0.3615	0.5231
2000	0.3232	0.4696
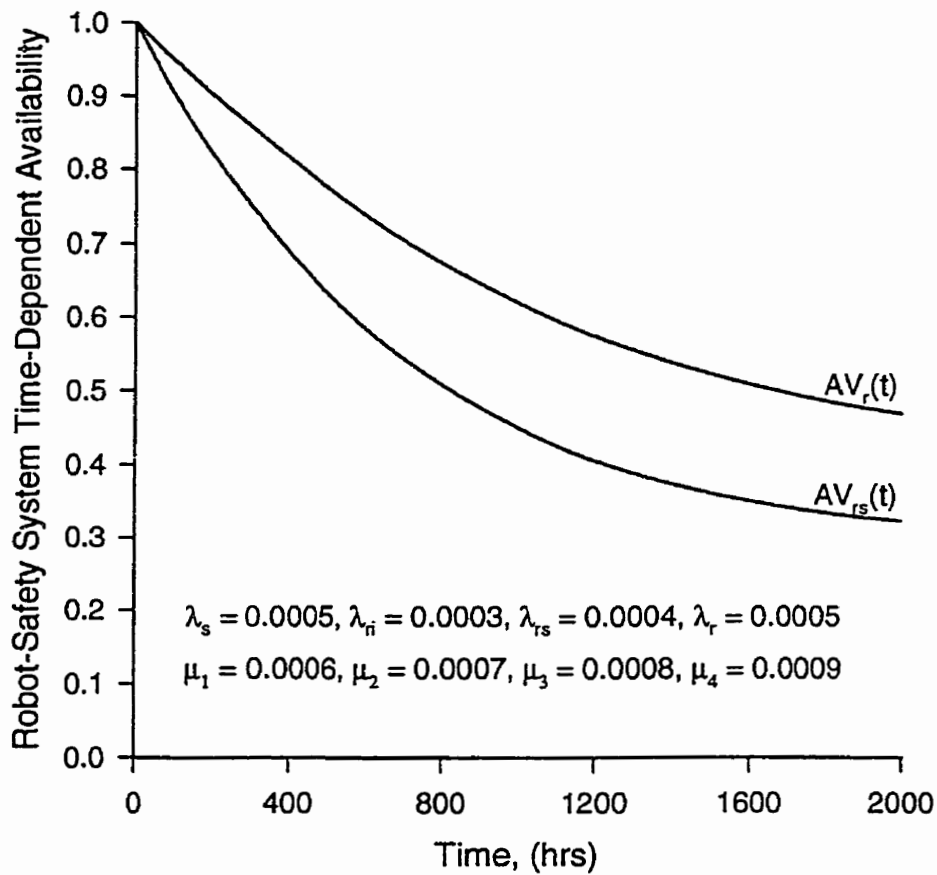
Figure 3.12.   Availability ($n = 1$) plots for a robot with constant failure rate and **gamma** distributed ($\beta = 2$) failed system repair time distribution. More specifically, the plots were obtained using Equations (3.125) and (3.126).

3.4.3   Robot System Reliability and MTTF

Inserting $n = 1$ into the generalized Equations (3.82) – (3.84) developed in Section 3.3.3,

we obtain the following Laplace transforms of the state probabilities:

$$P_i(s) = \frac{\gamma_i(s)}{s \cdot Z(s)} \qquad (\textit{for} \quad i = 0, 1) \tag{3.127}$$

$$P_k(s) = \frac{\gamma_k(s)}{s \cdot Z(s)} \qquad (\textit{for} \quad k = 2, 3, 4) \tag{3.128}$$

$$Z(s) = \sum_{i=0}^{1} \gamma_i(s) + \sum_{k=2}^{4} \gamma_k(s) \tag{3.129}$$

where

$$\gamma_0(s) = (s + a_1)s$$
$$\gamma_1(s) = C_0 s$$
$$\gamma_2(s) = \lambda_{ri} C_0$$
$$\gamma_3(s) = \lambda_{rs} C_0$$
$$\gamma_4(s) = \lambda_r(s + a_1)$$

Inverting Equations (3.127) and (3.128), yield the following general time-dependent

probability expressions:

$$P_0(t) = \frac{(a_1 - x_1)e^{-x_1 t} - (a_1 - x_2)e^{-x_2 t}}{(x_2 - x_1)} \tag{3.130}$$

$$x_1, x_2 = \frac{g_1 \pm \sqrt{g_1^2 - 4g_2}}{2}$$

where

$$g_1 = \lambda_r + a_1 + C_0$$
$$g_2 = a_1 \lambda_r + \lambda_{ri} C_0 + \lambda_{rs} C_0$$

Similarly,

$$P_1(t) = \frac{C_0(e^{-x_1 t} - e^{-x_2 t})}{(x_2 - x_1)} \tag{3.131}$$

$$P_2(t) = \lambda_{rt} C_0 \left[\frac{(x_2 - x_1) + x_1 e^{-x_2 t} - x_2 e^{-x_1 t}}{x_1 x_2 (x_2 - x_1)} \right] \tag{3.132}$$

$$P_3(t) = \lambda_{rs} C_0 \left[\frac{(x_2 - x_1) + x_1 e^{-x_2 t} - x_2 e^{-x_1 t}}{x_1 x_2 (x_2 - x_1)} \right] \tag{3.133}$$

$$P_4(t) = \frac{\lambda_r [a_1(x_2 - x_1) + x_1(a_1 - x_2)e^{-x_2 t} - x_2(a_1 - x_1)e^{-x_1 t}]}{x_1 x_2 (x_2 - x_1)} \tag{3.134}$$

Using Equation (3.130), the reliability of an irreparable robot system with an operating safety unit is

$$R_{rs}(t) = P_0(t) = \frac{(a_1 - x_1)e^{-x_1 t} - (a_1 - x_2)e^{-x_2 t}}{(x_2 - x_1)} \tag{3.135}$$

Similarly, the reliability of an irreparable robot system with or without a safety unit is given by

$$\begin{aligned} R_r(t) &= P_0(t) + P_1(t) \\ &= \frac{C_0(e^{-x_1 t} - e^{-x_2 t}) + (a_1 - x_1)e^{-x_1 t} - (a_1 - x_2)e^{-x_2 t}}{(x_2 - x_1)} \end{aligned} \tag{3.136}$$

Using Equation (3.135), the mean time to failure is

$$MTTF_{rs} = \int_0^\infty R_{rs}(t)\,dt = \frac{a_1}{x_1 x_2} = \frac{a_1}{g_2} \tag{3.137}$$

Similarly, integrating Equation (3.136), the mean time to failure is

$$MTTF_r = \int_0^\infty R_r(t)dt = \frac{C_0 + a_1}{x_1 x_2} = \frac{C_0 + a_1}{g_2}$$
(3.138)

3.4.3.1 Reliability and MTTF Numerical Examples

Setting:

$\lambda_s = 0.0005$, $\lambda_{ri} = 0.0003$, $\lambda_{rs} = 0.0015$, $\lambda_r = 0.0004$, $\mu_1 = 0.0005$

into Equations (3.130) – (3.134), time-dependent probability plots are shown in Figure 3.13.

Equation (3.132) represents the probability of the robot system failing with an incident. Inserting system parameter values into this equation, Figure 3.14 shows the robot incident probability for various safety unit repair rates.

Substituting the above parameter values into Equations (3.135) and (3.136), the reliability plots for various safety unit repair rates are shown in Figure 3.15. Utilizing Equations (3.137) and (3.138), MTTF plots of the robot as a function of safety system failure and repair rates are shown in Figure 3.16.

More detailed inspection of the robot system state probability, incident probability, reliability, and MTTF can be made by referring to Tables 3-10 to 3-13, respectively.

Table 3-10:   Time-dependent probability ($n = 1$) values for an irreparable robot
system.

Time (Hrs)	Time Dependent Probability				
	$P_0(t)$	$P_1(t)$	$P_2(t)$	$P_3(t)$	$P_4(t)$
0	1	0	0	0	0
500	0.6283	0.1426	0.0130	0.0173	0.1988
1000	0.4192	0.1670	0.0370	0.0494	0.3274
1500	0.2919	0.1504	0.0611	0.0815	0.4151
2000	0.2092	0.1232	0.0817	0.1089	0.4771
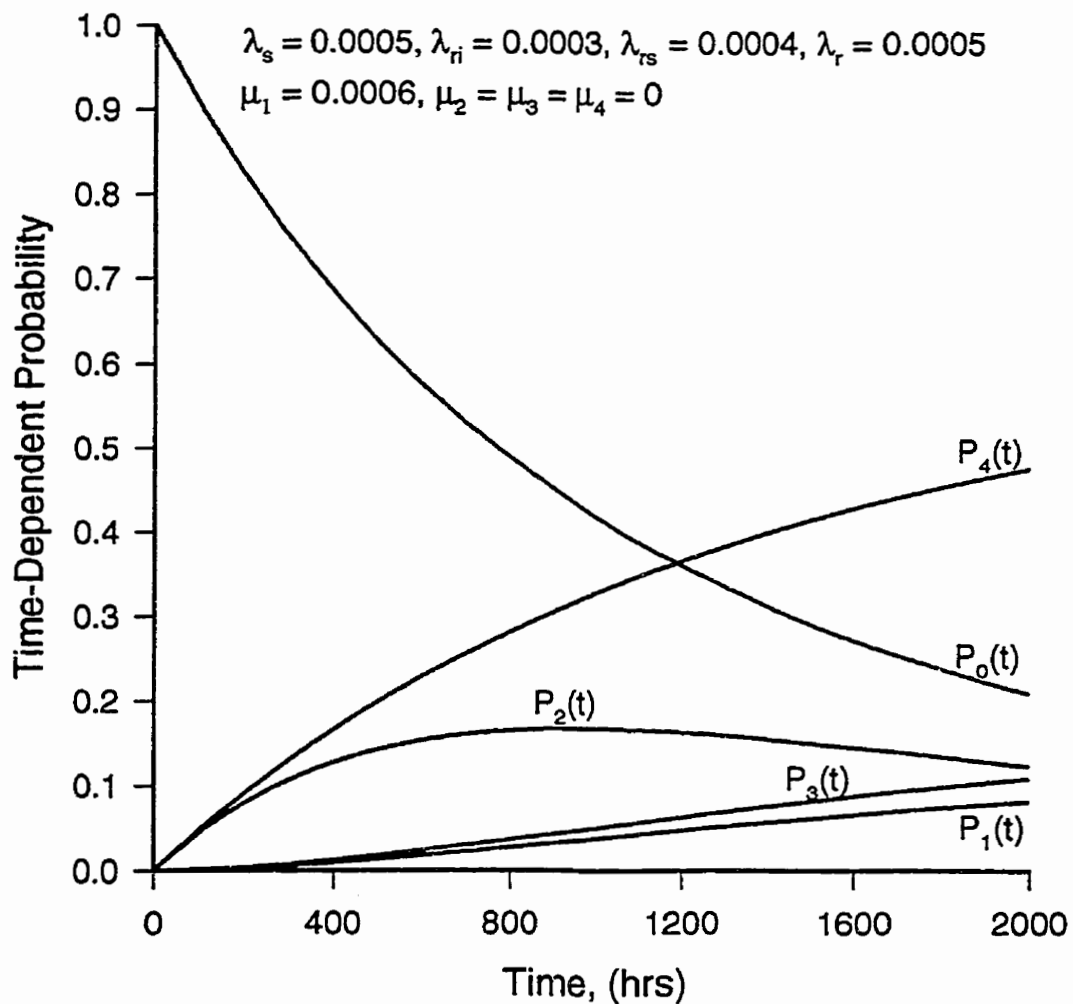
Figure 3.13.   Time-dependent probability ($n = 1$) plots for an irreparable robot system.

Table 3-11:   Robot failing with an incident probability values for various given values of the safety unit repair rates.

Time (Hrs)	Robot Failing With an Incident Probability, $P_2(t)$			
	$\mu_1 = 0$	$\mu_1 = .0006$	$\mu_1 = .0012$	$\mu_1 = .0018$
0	0	0	0	0
2000	0.1058	0.0817	0.0658	0.0549
4000	0.1800	0.1280	0.0989	0.0806
6000	0.2048	0.1431	0.1100	0.0893
8000	0.2118	0.1478	0.1136	0.0922
10,000	0.2137	0.1493	0.1148	0.0932
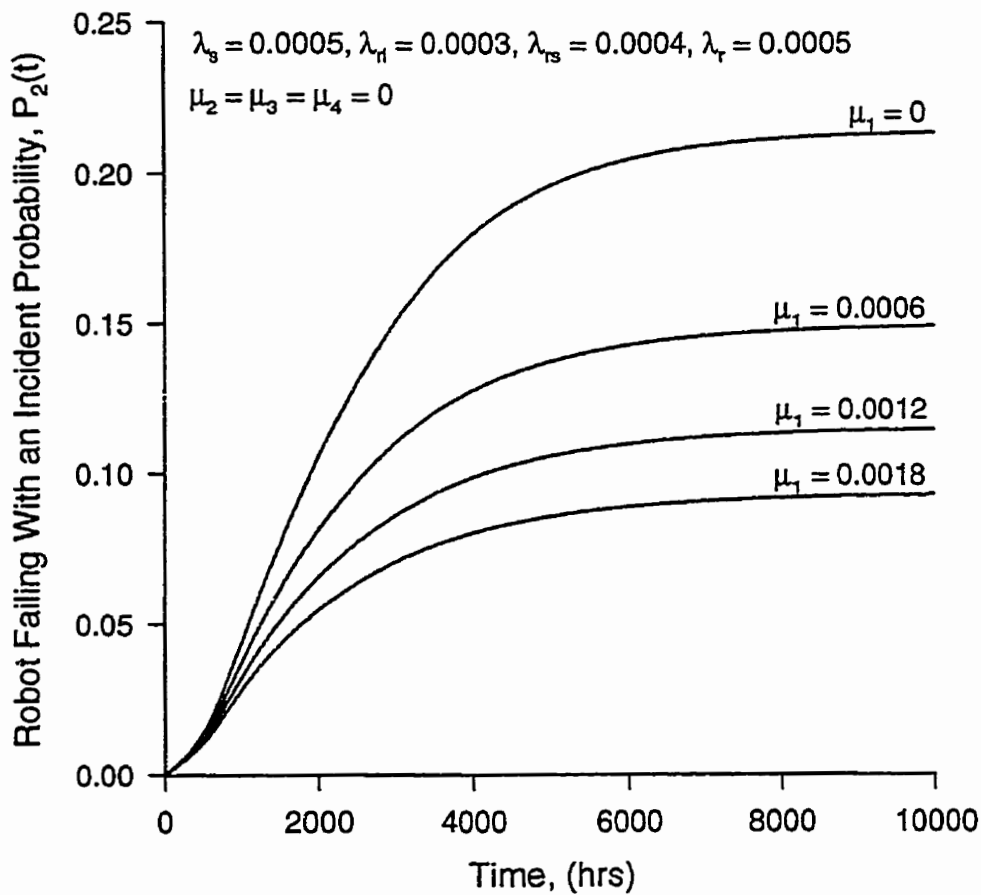
Figure 3.14.   Plots of Equation (3.132), robot system failing with an incident for various safety unit repair rates, μ_1.

Table 3-12:   Reliability $(n = 1)$ values of an irreparable robot system with various given values of safety mechanism repair rates.

Time (hrs)	$\mu_1 = 0$		$\mu_1 = 0.0006$		$\mu_1 = 0.006$		$\mu_1 = 1$	
	$R_{rs}(t)$	$R_r(t)$	$R_{rs}(t)$	$R_r(t)$	$R_{rs}(t)$	$R_r(t)$	$R_{rs}(t)$	$R_r(t)$
0	1	1	1	1	1	1	1	1
500	.6065	.7701	.6283	.7709	.7188	.7747	.7784	.7788
1000	.3679	.5824	.4192	.5861	.5541	.5989	.6062	.6065
1500	.2231	.4345	.2919	.4423	.4283	.4629	.4721	.4723
2000	.1353	.3208	.2092	.3324	.3311	.3578	.3676	.3678

Table 3-13:   MTTF $(n = 1)$ values of an irreparable robot system as a function of safety unit failure and repair rates, respectively.

Failure rate, λ_s	Mean Time To Failure		Repair rate, μ_1	Mean Time To Failure	
	$MTTF_{rs}$	$MTTF_r$		$MTTF_{rs}$	$MTTF_r$
0.0000	2000	2000	0.0000	1000	1714.3
0.0004	1397.8	1827.9	0.0004	1222.2	1777.8
0.0008	1074.4	1735.5	0.0008	1363.6	1818.2
0.0012	872.5	1677.8	0.0012	1461.5	1846.1
0.0016	734.5	1638.4	0.0016	1533.3	1866.7
0.0020	634.1	1609.7	0.0020	1588.2	1882.3

(a)

(b)
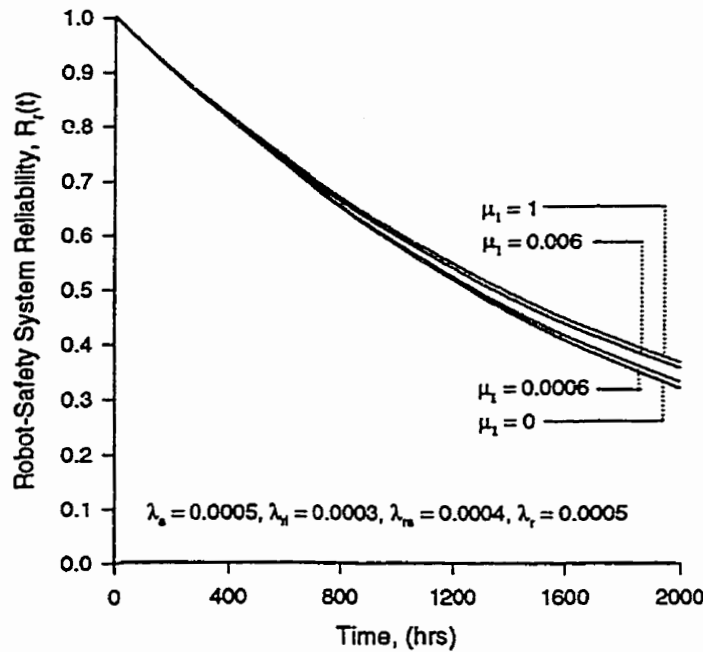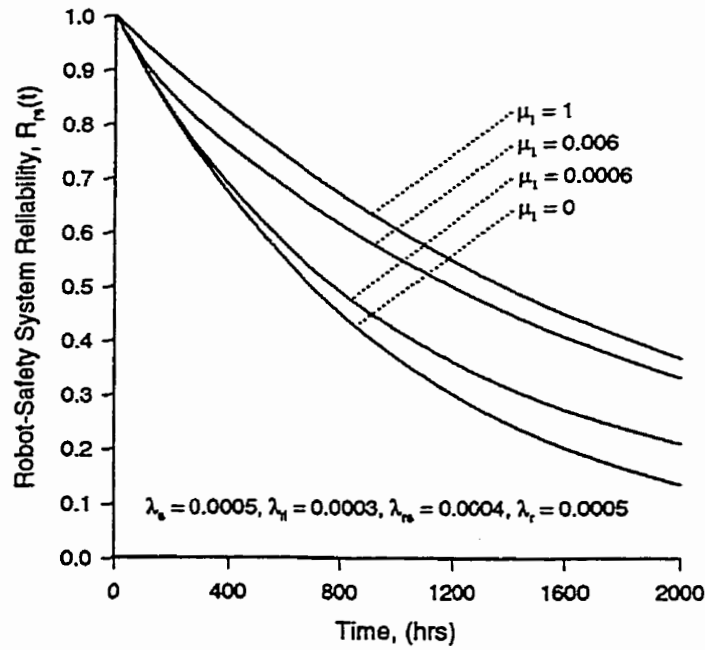
Figure 3.15.   Reliability $(n = 1)$ plots of an irreparable robot system with various specified values of safety mechanism repair rate; (a) robot working with an operating safety mechanism; (b) robot working with or without a safety mechanism. More specifically, the plots were obtained using Equations (3.135) and (3.136).
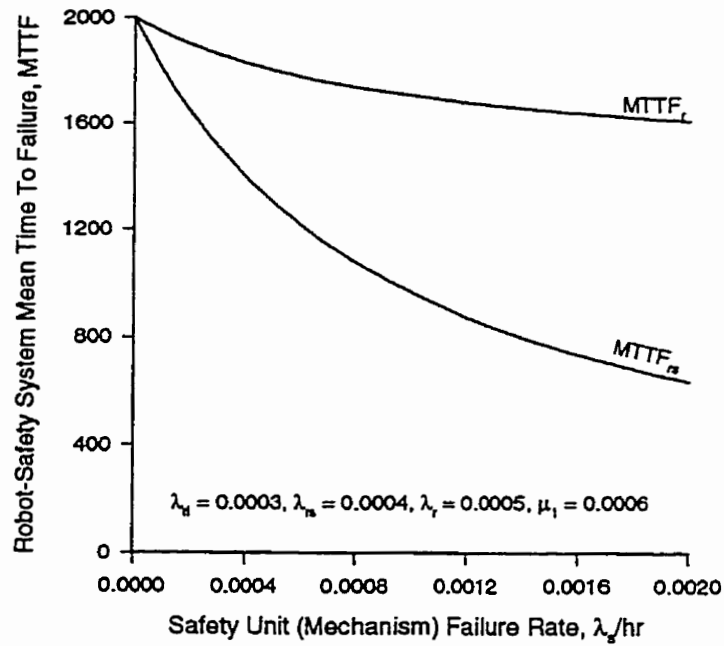
(a)
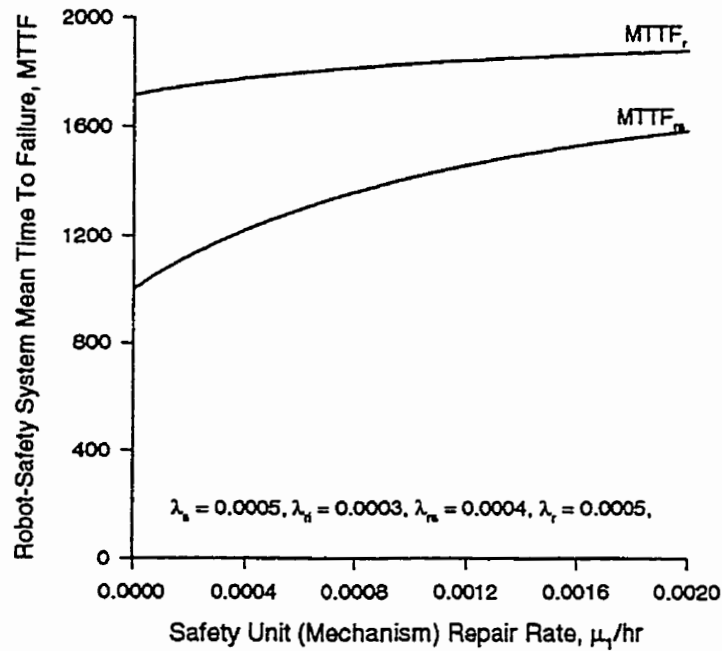
(b)

Figure 3.16.   MTTF ($n = 1$) plots of an irreparable robot system as a function of safety mechanism failure and repair rates. More specifically, the plots were obtained using Equations (3.137) and (3.138).

3.5 Special Case Model II: (n = 2)

Figure 3.17 represents the transition diagram for a system containing one robot and two

safety units. It can be obtained from the generalized model in Figure 3.2 for $n = 2$ [115].
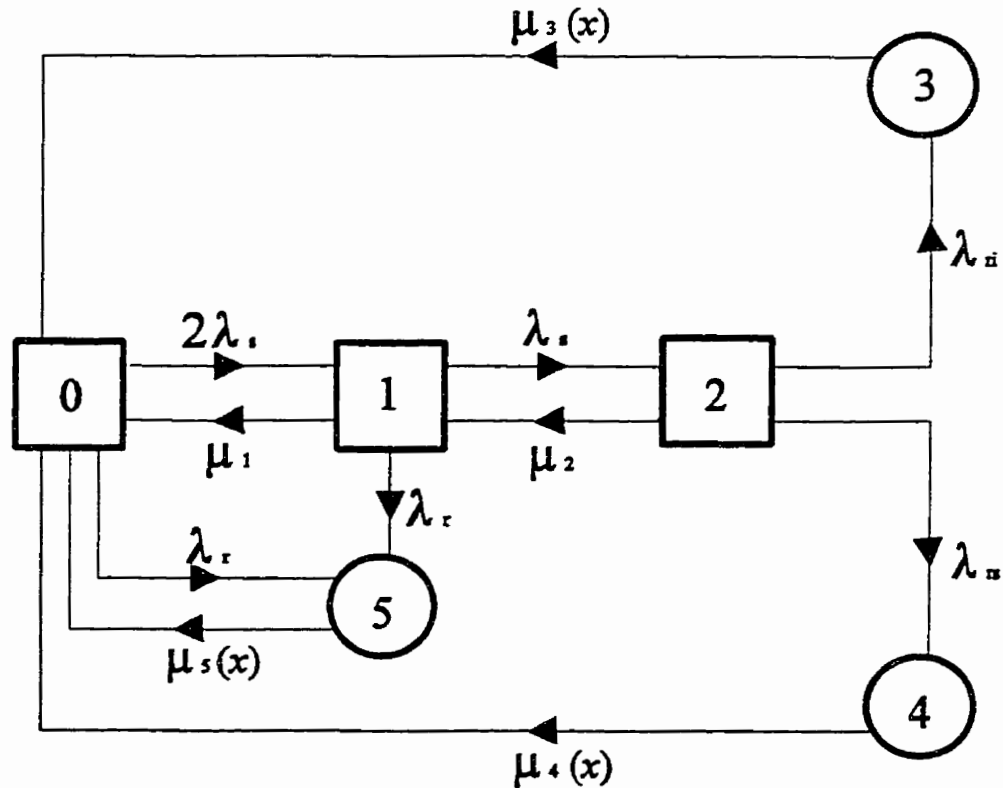
Figure 3.17.   State space transition diagram for a system containing one robot and two safety units ($n = 2$).

The corresponding system of integro-differential equations for the model in Figure 3.20 can

be extracted from the generalized Equations $(3.1) - (3.8)$ by setting $n = 2$. Thus, the set of

differential equations becomes

$$P_0'(t) + a_0 P_0(t) = \mu_1 P_1(t) + \sum_{k=3}^{5} \int_0^\infty p_k(x,t)\mu_k(x)dx \qquad (3.139)$$

$$P_1'(t) + a_1 P_1(t) = C_0 P_0(t) + \mu_2 P_2(t) \tag{3.140}$$

$$P_2'(t) + a_2 P_2(t) = C_1 P_1(t) \tag{3.141}$$

where

$$a_0 = 2\lambda_s + \lambda_r$$
$$a_1 = \lambda_s + \lambda_r + \mu_1$$
$$a_2 = \lambda_{rl} + \lambda_{rs} + \mu_2$$
$$C_0 = 2\lambda_s$$
$$C_1 = \lambda_s$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_k(x) \right] p_k(x,t) = 0 \qquad (for \quad k = 3,4,5) \tag{3.142}$$

The associated boundary conditions are as follows:

$$p_3(0,t) = \lambda_{rl} P_2(t) \tag{3.143}$$

$$p_4(0,t) = \lambda_{rs} P_2(t) \tag{3.144}$$

$$p_5(0,t) = \lambda_r [P_0(t) + P_1(t)] \tag{3.145}$$

At time $t = 0$, $P_0(0) = 1$, $P_1(0) = 0$, $p_k(x,0) = 0$, for $k = 3$, 4, 5. The prime denotes differentiation with respect to time t.

3.5.1   Steady State Availability Analysis

As time approaches infinity, state probabilities reach the steady state. Inserting $n = 2$ into the generalized Equations (3.25) – (3.27) developed in Section 3.3.1, we get the following steady state probabilities:

$$P_i = \frac{\omega_i}{D} \qquad (for \quad i = 0, 1, 2) \tag{3.146}$$

$$P_k = \frac{\omega_k}{D} E_k[x] \qquad (for \quad k = 3, 4, 5) \tag{3.147}$$

$$D = \sum_{i=0}^{2} \omega_i + \sum_{k=3}^{5} \omega_k E_k[x] \tag{3.148}$$

where

$$\omega_0 = a_1 a_2 - \mu_2 C_1$$
$$\omega_1 = a_2 C_0$$
$$\omega_2 = C_0 C_1$$
$$\omega_3 = \lambda_n C_0 C_1$$
$$\omega_4 = \lambda_n C_0 C_1$$
$$\omega_5 = \lambda_r (a_1 a_2 - \mu_2 C_1 + a_2 C_0)$$

The robot system steady state availability with at least one working safety unit is:

$$SSAV_{rs} = P_0 + P_1 = \frac{\omega_0 + \omega_1}{\sum_{i=0}^{2} \omega_i + \sum_{k=3}^{5} \omega_k E_k[x]} \tag{3.149}$$

Similarly, the robot system steady state availability with or without the working safety units

is given by

$$SSAV_r = \sum_{i=0}^{2} P_i = \frac{\sum_{i=0}^{2} \omega_i}{\sum_{i=0}^{2} \omega_i + \sum_{k=3}^{5} \omega_k E_k[x]} \tag{3.150}$$

For various robot system repair time distributions, the values of D [i.e., Equation (3.148)]

are obtained as follows:

For the robot system repair time x, represented by a **gamma** distribution, we get

$$D = D_G = \sum_{i=0}^{2} \omega_i + \sum_{k=3}^{5} \omega_k \beta / \mu_k \tag{3.151}$$

For $\beta = 1$ in Equation (3.151), the robot system repair time x is represented by an **exponential** distribution, therefore

$$D = D_E = \sum_{i=0}^{2} \omega_i + \sum_{k=3}^{5} \omega_k / \mu_k \tag{3.152}$$

For the robot system repair time x, represented by a **Weibull** distribution, we get

$$D = D_W = \sum_{i=0}^{2} \omega_i + \sum_{k=3}^{5} \omega_k \left(\frac{1}{\mu_k}\right)^{1/\beta} \frac{1}{\beta} \Gamma\left(\frac{1}{\beta}\right) \tag{3.153}$$

For $\beta = 2$ in Equation (3.153), the robot system repair time x is represented by a **Rayleigh** distribution, thus

$$D = D_R = \sum_{i=0}^{2} \omega_i + \sum_{k=3}^{5} \omega_k \sqrt{\frac{\pi}{4\mu_k}} \tag{3.154}$$

For the robot system repair time x, represented by a **log-normal** distribution, we get

$$D = D_L = \sum_{i=0}^{2} \omega_i + \sum_{k=3}^{5} \omega_k e^{\left(\mu_{y_k} + \frac{\sigma_{y_k}^2}{2}\right)} \tag{3.155}$$

where

$$\sigma_{y_k} = \ln \sqrt{1 + \left(\frac{\sigma_{x_k}}{\mu_{x_k}}\right)^2} \quad , \quad \mu_{y_k} = \ln \sqrt{\frac{\mu_{x_k}^4}{\mu_{x_k}^2 + \sigma_{x_k}^2}} \quad (for \quad k = 3, 4, 5)$$

3.5.1.1 *Steady State Availability Numerical Examples*

Setting:

$\lambda_s = 0.0005$, $\lambda_d = 0.0003$, $\lambda_{rs} = 0.0004$, $\lambda_r = 0.0005$
$\mu_1 = \mu_2 = 0.0006$, $\mu_3 = 0.0007$, $\mu_4 = 0.0008$, $\mu_5 = 0.0009$

into Equations (3.149) and (3.150) and performing numerical analysis, we can obtain robot-safety system steady state availability numerical values.

Figures 3.18 – 3.20 show plots of the robot-safety system steady state availability for gamma, Weibull, and log-normal distributions, respectively. These plots indicate the steady state availability as a function of safety unit (mechanism) failure rate, λ_s. The objective is to examine the robot long term availability for different distributions' properties and compare them with those obtained in Section 3.4.1.

To inspect robot system repairability, Tables 3-14 to 3-16 present steady state availability for gamma, Weibull, and log-normal distributions, respectively. These tables indicate the steady state values as a function of safety unit (mechanism) repair rate, μ_1. Again, the objective is to examine the robot long term availability for different distributions' properties and compare them with those obtained in Section 3.4.1.

For the sake of comparison, all distributions are presented on the same figure. For all distributions, Figures 3.21 and 3.22 show plots of the robot-safety system steady state availability as a function of safety unit (mechanism) failure (λ_s) and repair (μ_1) rates, respectively. More detalized inspection of Figures 3.21 and 3.22 can be made by referring to their associated tabular values which are given in Tables 3-17 and 3-18, respectively.
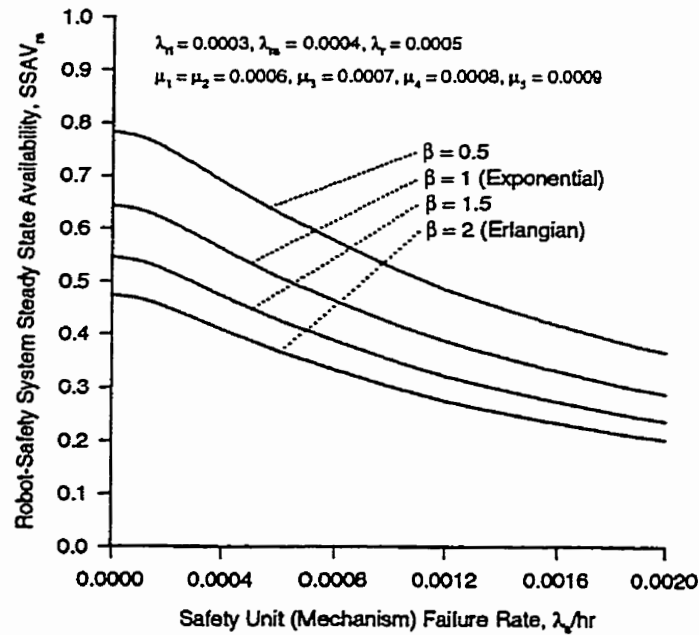
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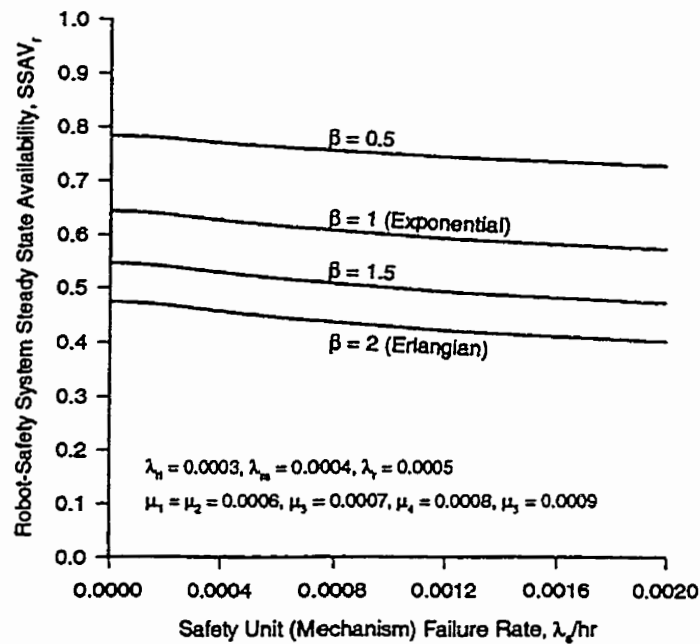
(b)

Figure 3.18.   Steady state availability ($n = 2$) vs λ_s plots for a robot with constant failure rate and for **gamma** distributed failed system repair times; (a) robot working with at least one safety unit, (b) robot working with or without a safety unit. More specifically, these plots were obtained using Equations (3.149) – (3.152).
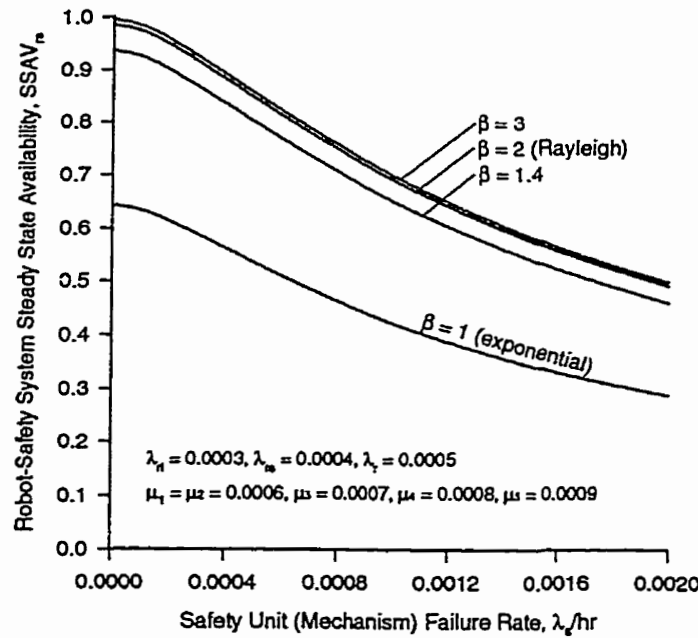
(a)
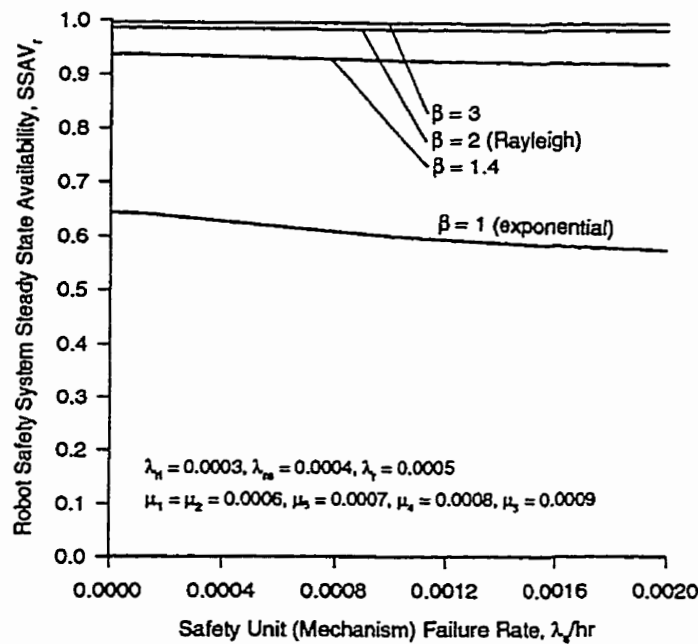
(b)

Figure 3.19.   Steady state availability ($n = 2$) vs λ_s plots for a robot with constant failure rate and for **Weibull** distributed failed system repair times; (a) robot working with at least one safety unit, (b) robot working with or without a safety unit. More specifically, these plots were obtained using Equations (3.149), (3.150), (3.153), and (3.154).
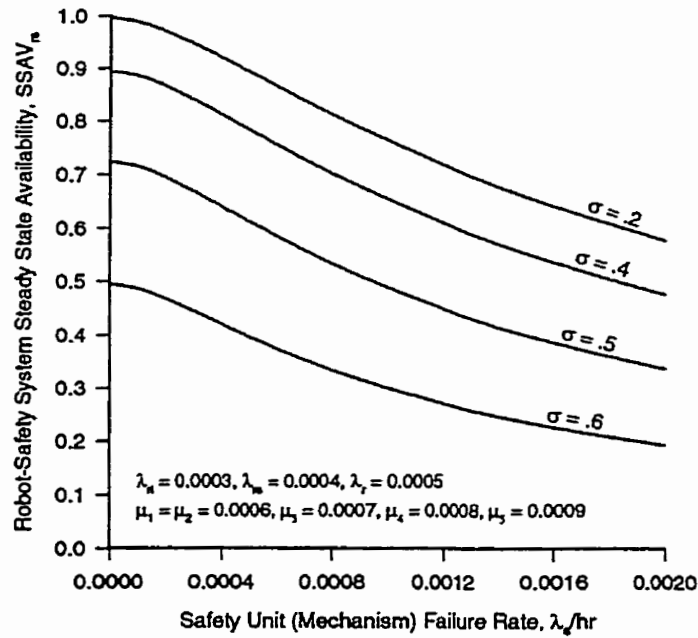
(a)
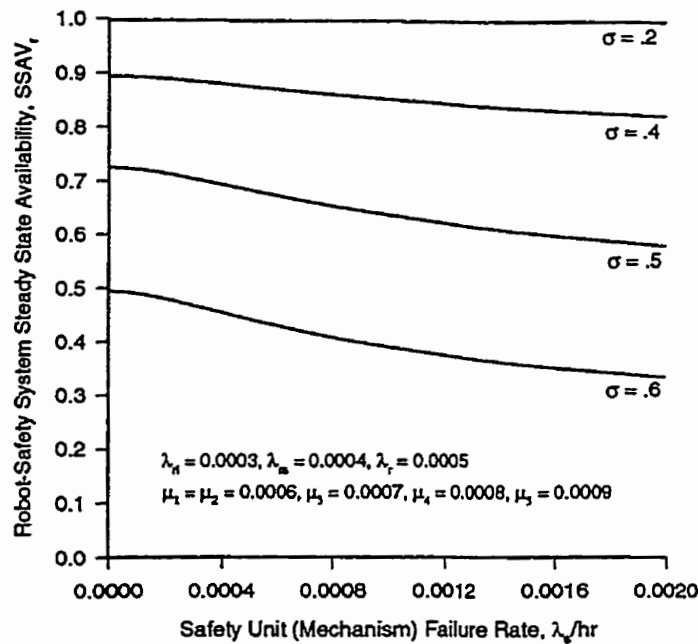
(b)

Figure 3.20.   Steady state availability ($n = 2$) vs λ_s plots for a robot with constant failure rate and for **log-normal** distributed failed system repair times; (a) robot working with at least one safety unit, (b) robot working with or without a safety unit. More specifically, these plots were obtained using Equations (3.149), (3.150), and (3.155).

Table 3-14:   SSAV $(n = 2)$ vs μ_1 values for a robot with constant failure rate and **gamma** distributed failed system repair times.

μ_1	$\beta = 0.5$		$\beta = 1$(Exponential)		$\beta = 1.5$		$\beta = 2$ (Erlangian)	
	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$
0.0000	0.6265	0.7627	0.5064	0.6165	0.4249	0.5173	0.3660	0.4456
0.0005	0.6553	0.7664	0.5312	0.6213	0.4466	0.5224	0.3853	0.4506
0.0010	0.6752	0.7689	0.5484	0.6246	0.4617	0.5259	0.3987	0.4541
0.0015	0.6896	0.7708	0.5610	0.6270	0.4728	0.5285	0.4086	0.4567
0.0020	0.7007	0.7722	0.5707	0.6289	0.4813	0.5305	0.4162	0.4587

Table 3-15:   SSAV $(n = 2)$ vs μ_1 values for a robot with constant failure rate and **Weibull** distributed failed system repair times.

μ_1	$\beta = 1$ (Exponential)		$\beta = 1.4$		$\beta = 2$ (Rayleigh)		$\beta = 3$	
	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$
0.0000	0.5064	0.6165	0.7638	0.9299	0.8084	0.9841	0.8173	0.9950
0.0005	0.5312	0.6213	0.7961	0.9310	0.8417	0.9843	0.8509	0.9951
0.0010	0.5484	0.6246	0.8182	0.9318	0.8645	0.9845	0.8738	0.9951
0.0015	0.5610	0.6270	0.8343	0.9324	0.8810	0.9846	0.8904	0.9952
0.0020	0.5707	0.6289	0.8465	0.9328	0.8936	0.9847	0.9030	0.9952

Table 3-16:   SSAV $(n = 2)$ vs μ_1 values for a robot with constant failure rate and **log normal** distributed failed system repair times.

μ_1	$\sigma = 0.2$		$\sigma = 0.4$		$\sigma = 0.5$		$\sigma = 0.6$	
	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$
0.0000	0.8167	0.9943	0.7064	0.8599	0.5369	0.6536	0.3363	0.4094
0.0005	0.8504	0.9945	0.7405	0.8660	0.5693	0.6658	0.3618	0.4231
0.0010	0.8734	0.9947	0.7641	0.8702	0.5921	0.6743	0.3802	0.4330
0.0015	0.8901	0.9948	0.7814	0.8733	0.6090	0.6807	0.3941	0.4404
0.0020	0.9028	0.9949	0.7946	0.8757	0.6221	0.6856	0.4049	0.4463

Table 3-17:   SSAV ($n = 2$) vs λ_s values for a robot with constant failure rate and various failed system repair time distributions.

λ_s	Erlangian (gamma,β =2)		exponential (gamma,β =1)		Lognormal ($\sigma = 0.4$)		Weibull (β=1.4)		Rayleigh (Weibull,β=2)	
	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$
.0000	.4737	.4737	.6429	.6429	.8933	.8933	.9361	.9361	.9854	.9854
.0004	.4092	.4569	.7825	.6272	.5618	.8735	.8352	.9324	.8820	.9847
.0008	.3323	.4368	.4625	.6080	.6460	.8491	.7058	.9277	.7483	.9836
.0012	.2749	.4218	.3867	.5933	.5412	.8304	.6022	.9240	.6405	.9828
.0016	.2330	.4108	.3303	.5824	.4631	.8165	.5225	.9211	.5571	.9822
0.002	.2017	.4027	.2876	.5741	.4037	.8059	.4603	.9189	.4918	.9817

System parameter values: $\lambda_{ri} = 0.0003$, $\lambda_{rs} = 0.0004$, $\lambda_r = 0.0005$
$\mu_1 = \mu_2 = 0.0006$, $\mu_3 = 0.0007$, $\mu_4 = 0.0008$, $\mu_5 = 0.0009$

Table 3-18:   SSAV ($n = 2$) vs μ_1 values for a robot with constant failure rate and various failed system repair time distributions.

μ_1	Erlangian (gamma,β=2)		exponential (gamma,β =1)		Lognormal ($\sigma = 0.4$)		Weibull (β=1.4)		Rayleigh (Weibull,β=2)	
	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$
.0000	.3660	.4456	.5064	.6165	.7064	.8599	.7638	.9299	.8084	.9841
.0004	.3820	.4497	.5270	.6205	.7347	.8650	.7906	.9308	.8361	.9843
.0008	.3939	.4528	.5422	.6234	.7556	.8687	.8105	.9315	.8563	.9845
.0012	.4030	.4552	.5539	.6257	.7716	.8716	.8252	.9321	.8717	.9846
.0016	.4103	.4571	.5632	.6274	.7843	.8739	.8770	.9325	.8838	.9847
.0020	.4162	.4587	.5707	.6289	.7946	.8757	.8465	.9328	.8936	.9847

System parameter values: $\lambda_s = 0.0005$, $\lambda_{ri} = 0.0003$, $\lambda_{rs} = 0.0004$, $\lambda_r = 0.0005$
$\mu_2 = 0.0006$, $\mu_3 = 0.0007$, $\mu_4 = 0.0008$, $\mu_5 = 0.0009$

(a)

(b)
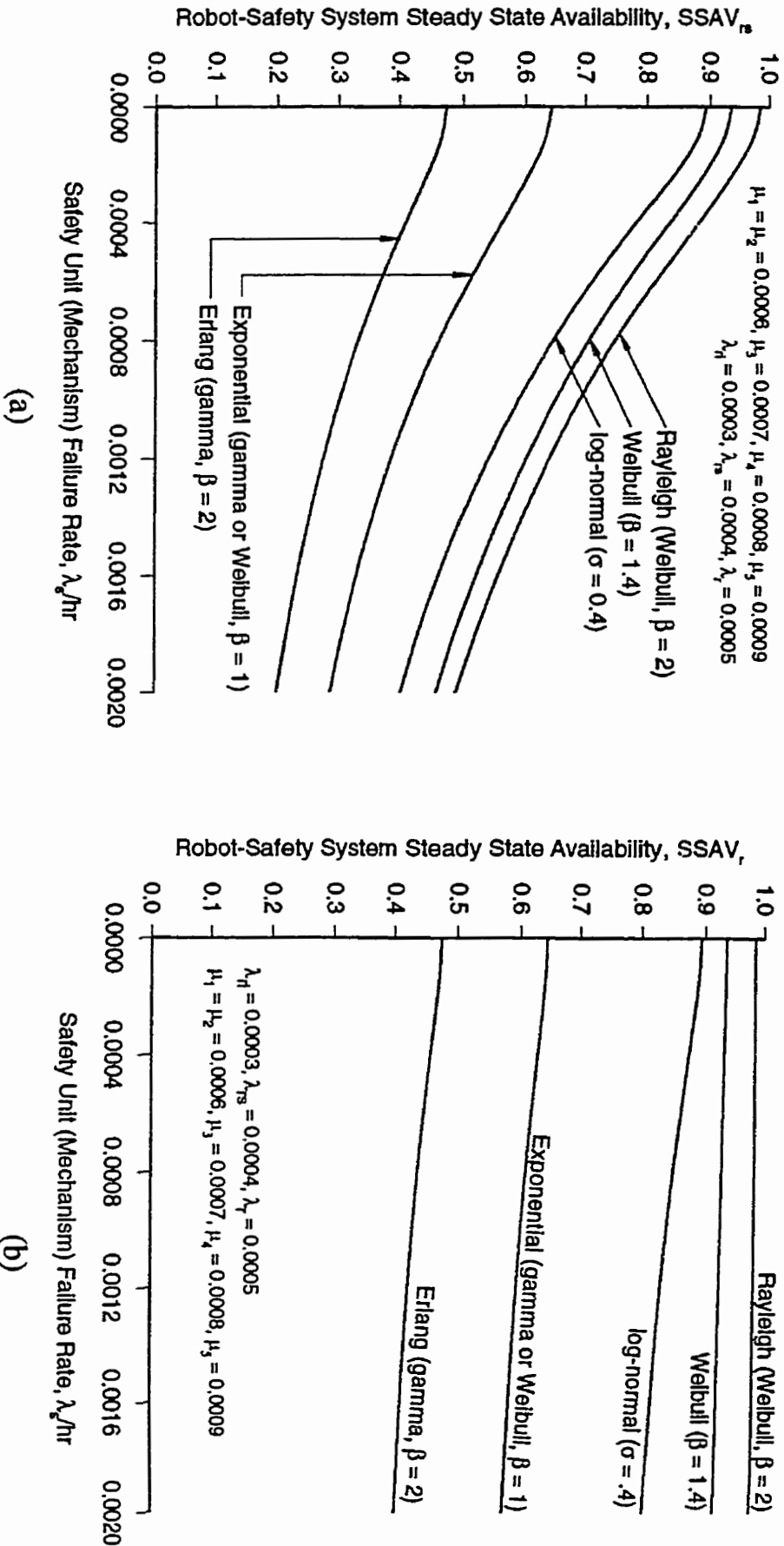
Figure 3.21.   Steady state availability ($n = 2$) vs λ_s plots for a robot with constant failure rate and for various failed system repair time distributions when (a) robot working with at least one safety unit, (b) robot working with or without a safety unit. More specifically, these plots were obtained using Equations (3.149) – (3.155).
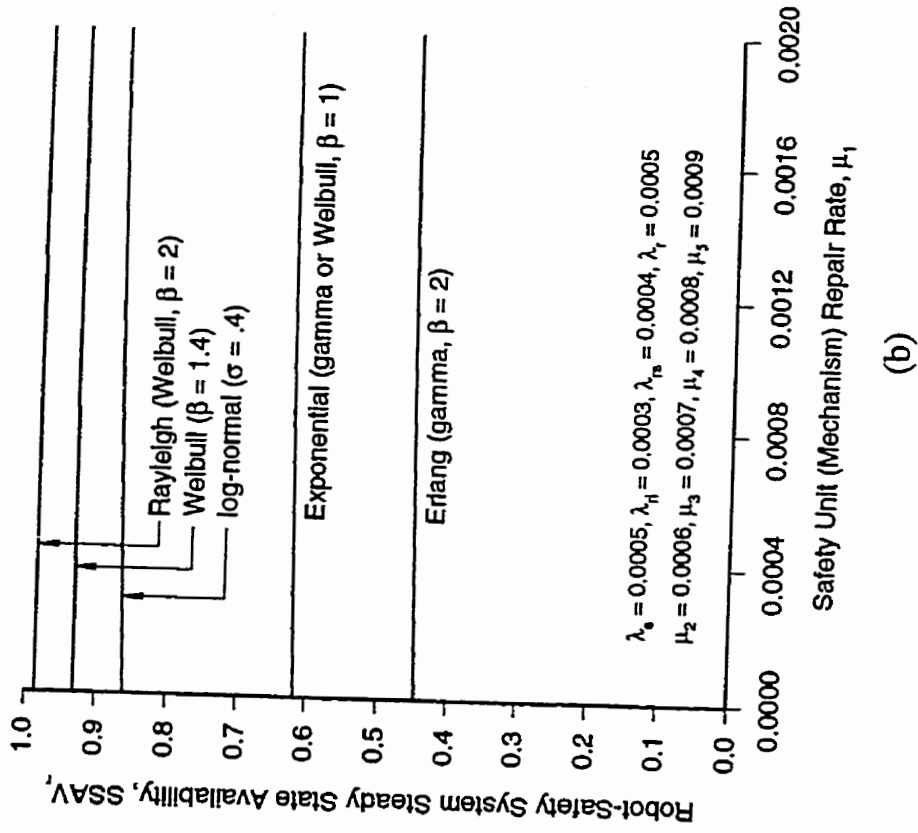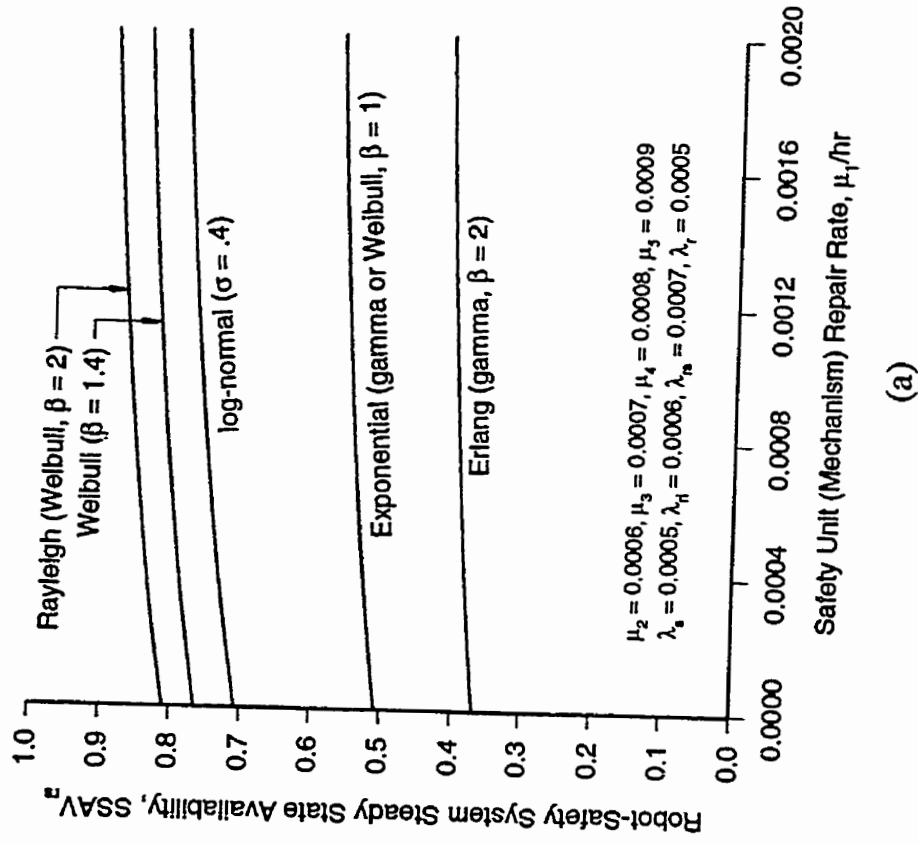
Figure 3.22.   Steady state availability (n = 2) vs μ_1 plots for a robot with constant failure rate and for various failed system repair time distributions when (a) robot working with at least one safety unit, (b) robot working with or without a safety unit. More specifically, these plots were obtained using Equations (3.149) – (3.155).

3.5.2   Time Dependent Availability Analysis

Substituting $n = 2$ into the generalized Equations (3.63) – (3.65) developed in Section 3.3.2,

we obtain the following Laplace transforms of the robot system probability expressions with

constant repair rate:

$$P_i(s) = \frac{\alpha_i(s)}{s \cdot B(s)} \qquad (for \quad i = 0, 1, 2) \tag{3.156}$$

$$P_k(s) = \frac{\alpha_k(s)}{(s + \mu_k)B(s)} \qquad (for \quad k = 3, 4, 5) \tag{3.157}$$

$$B(s) = \sum_{i=0}^{2} \alpha_i(s) + \sum_{k=3}^{5} \frac{s \cdot \alpha_k(s)}{(s + \mu_k)} \tag{3.158}$$

where

$$\alpha_0(s) = [(s + a_1)(s + a_2 - \mu_2 C_1]s$$
$$\alpha_1(s) = C_0(s + a_2)s$$
$$\alpha_2(s) = C_0 C_1 s$$
$$\alpha_3(s) = \lambda_{rt} C_0 C_1$$
$$\alpha_4(s) = \lambda_{rs} C_0 C_1$$
$$\alpha_5(s) = \lambda_r \left(\frac{\alpha_0(s) + \alpha_1(s)}{s} \right)$$

Using Equation (3.156), the Laplace transform of the robot system availability with at least

one working safety unit is given by

$$AV_{rs}(s) = P_0(s) + P_1(s) = \frac{\alpha_0(s) + \alpha_1(s)}{s \left(\sum_{i=0}^{2} \alpha_i(s) + \sum_{k=3}^{5} \frac{s \cdot \alpha_k(s)}{(s + \mu_k)} \right)} \tag{3.159}$$

Similarly, using Equation (3.156), the Laplace transform of the robot system availability

with or without a working safety mechanism is expressed by

$$AV_r(s) = \sum_{i=0}^{2} P_i(s) = \frac{\sum_{i=0}^{2} \alpha_i(s)}{s \left(\sum_{i=0}^{2} \alpha_i(s) + \sum_{k=3}^{5} \frac{s \cdot \alpha_k(s)}{(s + \mu_k)} \right)} \tag{3.160}$$

Substituting $n = 2$ into the generalized Equations (3.70) – (3.72) developed in Section 3.2.2, we obtain the following Laplace transforms of the robot system state probability expressions with non-constant repair rate:

$$P_i(s) = \frac{\alpha_i(s)}{s \cdot B(s)} \qquad (for \quad i = 0, 1, 2) \tag{3.161}$$

$$P_k(s) = \frac{s \cdot \alpha_k(s)}{(s + \mu_k)^2 B(s)} \qquad (for \quad k = 3, 4, 5) \tag{3.162}$$

$$B(s) = \sum_{i=0}^{2} \alpha_i(s) + \sum_{k=3}^{5} \frac{s^2 \cdot \alpha_k(s)}{(s + \mu_k)^2} \tag{3.163}$$

Using Equation (3.161), the Laplace transform of the robot system availability with at least one working safety unit is

$$AV_{rs}(s) = P_0(s) + P_1(s) = \frac{\alpha_0(s) + \alpha_1(s)}{s \left(\sum_{i=0}^{2} \alpha_i(s) + \sum_{k=3}^{5} \frac{s^2 \cdot \alpha_k(s)}{(s + \mu_k)^2} \right)} \tag{3.164}$$

Similarly, using Equation (3.161), the Laplace transform of the robot system availability with or without the working safety mechanism is expressed by

$$AV_r(s) = \sum_{i=0}^{2} P_i(s) = \frac{\displaystyle\sum_{i=0}^{2} \alpha_i(s)}{s\left(\displaystyle\sum_{i=0}^{2} \alpha_i(s) + \sum_{k=3}^{5} \frac{s^2 \cdot \alpha_k(s)}{(s + \mu_k)^2}\right)} \qquad (3.165)$$

3.5.2.1 *Time Dependent Availability Numerical Examples*

Numerical probability and availability expressions can be obtained for the robot system with

constant (i.e., $\beta = 1$) and non-constant (i.e., $\beta = 2$) repair rates by setting:

$\lambda_s = 0.0005,\quad \lambda_{ri} = 0.0003,\quad \lambda_{rs} = 0.0004,\quad \lambda_r = 0.0005$
$\mu_1 = \mu_2 = 0.0006,\ \mu_3 = 0.0007,\ \mu_4 = 0.0008,\quad \mu_5 = 0.0009$

into Equations (3.154) – (3.158) and (3.159) – (3.163), respectively, and taking inverse

Laplace transform of the resulting equations. For $\beta = 1$, the up-time probability expressions

are

$$P_0(t) = 0.3093 + 0.3948\,e^{k_1 t} + 0.0003\,e^{k_2 t} + 0.001\,e^{k_3 t}$$
$$+ e^{k_4 t}\big(0.2946\,Cos\,(k_5 t) - 0.7329\,Sin\,(k_5 t)\big) \qquad (3.166)$$

$$P_1(t) = 0.2259 - 0.6227\,e^{k_1 t} + 0.0032\,e^{k_2 t} + 0.0027\,e^{k_3 t}$$
$$+ e^{k_4 t}\big(0.3909\,Cos\,(k_5 t) - 0.4938\,Sin\,(k_5 t)\big) \qquad (3.167)$$

$$P_2(t) = 0.0868 + 0.2624\,e^{k_1 t} + 0.0035\,e^{k_2 t} + 0.0024\,e^{k_3 t}$$
$$- e^{k_4 t}\big(0.3552\,Cos\,(k_5 t) - 1.1023\,Sin\,(k_5 t)\big) \qquad (3.168)$$

where

$$k_1 = -0.0025 \qquad k_2 = -0.0008$$
$$k_3 = -0.0007 \qquad k_4 = -0.0014$$
$$k_5 = 0.0002$$

Using Equation (3.166), the robot system availability with two working safety units is

$$AV_{rs2}(t) = P_0(t) = 0.3093 + 0.3948 \, e^{k_1 t} + 0.0003 \, e^{k_2 t} + 0.001 \, e^{k_3 t}$$
$$+ \, e^{k_4 t} \big(0.2946 \, Cos \, (k_5 t) - 0.7329 \, Sin \, (k_5 t) \big) \tag{3.169}$$

Similarly, utilizing Equations (3.166) and (3.167), the robot system availability with one working safety unit is

$$AV_{rs1}(t) = P_0(t) + P_1(t)$$
$$= 0.5352 - 0.2279 \, e^{k_1 t} + 0.0035 \, e^{k_2 t} + 0.0037 \, e^{k_3 t} + \tag{3.170}$$
$$e^{k_4 t} \big(1.6855 \, Cos \, (k_5 t) - 1.2267 \, Sin \, (k_5 t) \big)$$

Also, robot system availability with or without a working safety unit can be obtained by summing Equations (3.166) – (3.168).

$$AV_r(t) = P_0(t) + P_1(t) + P_2(t)$$
$$= 0.6221 + 0.0345 \, e^{k_1 t} + 0.007 \, e^{k_2 t} + 0.0061 \, e^{k_3 t} + \tag{3.171}$$
$$e^{k_4 t} \big(1.3303 \, Cos \, (k_5 t) - 0.1244 \, Sin \, (k_5 t) \big)$$

For $\beta = 2$, the up-time probability expressions become

$$P_0(t) = 0.2244 + 0.3151 \, e^{k_1 t} + 0.1029 \, e^{k_2 t}$$
$$e^{k_3 t} \big(0.0004 \cos(k_4 t) - 0.0003 \sin(k_4 t) \big) +$$
$$e^{k_5 t} \big(-0.0001 \cos(k_6 t) - 0.0002 \sin(k_6 t) \big) +$$
$$e^{k_7 t} \big(0.3571 \cos(k_8 t) - 0.2898 \sin(k_8 t) \big)$$
$$\tag{3.172}$$

$$P_1(t) = 0.1639 - 0.509 \, e^{k_1 t} + 0.0019 \, e^{k_2 t}$$
$$e^{k_3 t} \big(0.0014 \cos(k_4 t) - 0.0004 \sin(k_4 t) \big) +$$
$$e^{k_5 t} \big(0.0006 \cos(k_6 t) - 0.0009 \sin(k_6 t) \big) +$$
$$e^{k_7 t} \big(0.3411 \cos(k_8 t) + 0.2271 \sin(k_8 t) \big)$$
$$\tag{3.173}$$

$$P_2(t) = 0.063 + 0.2166\, e^{k_1 t} - 0.1706\, e^{k_2 t}$$

$$e^{k_3 t}\Big(0.0012 \cos(k_4 t) - 0.0003 \sin(k_4 t)\Big) +$$

$$e^{k_5 t}\Big(0.0008 \cos(k_6 t) - 0.0009 \sin(k_6 t)\Big) +$$

$$e^{k_7 t}\Big(-0.1111 \cos(k_8 t) + 0.3105 \sin(k_8 t)\Big)$$

(3.174)

where

$$k_1 = -0.0025 \qquad k_2 = -0.0013$$
$$k_3 = -0.0007 \qquad k_4 = 0.00005$$
$$k_5 = -0.0008 \qquad k_6 = 0.00006$$
$$k_7 = -0.0012 \qquad k_8 = 0.0006$$

Using Equations (3.172) − (3.174), the numerical time-dependent availability of the robot

with safety units in various operating conditions are

$$AV_{rs2}(t) = P_0(t) = 0.2244 + 0.3151\, e^{k_1 t} + 0.1029\, e^{k_2 t}$$

$$e^{k_3 t}\Big(0.0004 \cos(k_4 t) - 0.0003 \sin(k_4 t)\Big) +$$

$$e^{k_5 t}\Big(-0.0001 \cos(k_6 t) - 0.0002 \sin(k_6 t)\Big) +$$

$$e^{k_7 t}\Big(0.3571 \cos(k_8 t) - 0.2898 \sin(k_8 t)\Big)$$

(3.175)

$$AV_{rs1}(t) = P_0(t) + P_1(t) = 0.3636 - 0.3575\, e^{k_1 t} + 0.1048\, e^{k_2 t}$$

$$e^{k_3 t}\Big(0.0018 \cos(k_4 t) - 0.0007 \sin(k_4 t)\Big) +$$

$$e^{k_5 t}\Big(0.0005 \cos(k_6 t) - 0.0011 \sin(k_6 t)\Big) +$$

$$e^{k_7 t}\Big(0.6982 \cos(k_8 t) - 0.0627 \sin(k_8 t)\Big)$$

(3.176)

and

$$AV_r(t) = P_0(t) + P_1(t) + P_1(t)$$

$$= 0.4266 - 0.1409\, e^{k_1 t} - 0.0658\, e^{k_2 t}$$

$$e^{k_3 t}\Big(0.003 \cos(k_4 t) - 0.001 \sin(k_4 t)\Big) +$$

$$e^{k_5 t}\Big(0.0013 \cos(k_6 t) - 0.002 \sin(k_6 t)\Big) +$$

$$e^{k_7 t}\Big(0.5871 \cos(k_8 t) + 0.2477 \sin(k_8 t)\Big)$$

(3.177)

where

$AV_{rs2}(t)$ =   Robot-safety system availability at time t when the robot is working with both safety units.

$AV_{rs1}(t)$ =   Robot-safety system availability at time t when the robot is working with only one safety unit.

$AV_r(t)$ =   Robot-safety system availability at time t when the robot is working without a safety unit (both safety units have failed).

State probability plots of the robot system with constant repair rates ($\beta = 1$) and the specified system parameter values are shown in Figure 3.23. Using Equations (3.169) – (3.171), time-dependent availability plots of the robot working with safety units in various operating conditions are shown in Figure 3.24. More detailed inspection of the robot system state probability and availability can be made by referring to Tables 3-19 and 3-20, respectively.

State probability plots for the robot system with non-constant repair rates ($\beta = 2$) and the specified system parameter values are shown in Figure 3.25. Using Equations (3.175)– (3.177), time-dependent availability plots of the robot working with safety units in various operating conditions are shown in Figure 3.26. More detailed inspection of the robot system state probability and availability can be made by referring to Tables 3-21 and 3-22, respectively.

Table 3-19:   Time-dependent state probability ($n = 2$) values for a robot with constant failure and repair ($\beta = 1$) rates.

Time (Hrs)	Time Dependent Probability ($\beta = 1$)					
	$P_0(t)$	$P_1(t)$	$P_2(t)$	$P_3(t)$	$P_4(t)$	$P_5(t)$
0	1	0	0	0	0	0
500	0.5435	0.2447	0.0309	0.0017	0.0022	0.1770
1000	0.3880	0.2728	0.0653	0.0076	0.0098	0.2565
1500	0.3343	0.2599	0.0831	0.0149	0.0191	0.2886
2000	0.3160	0.2458	0.0896	0.0216	0.0272	0.2998
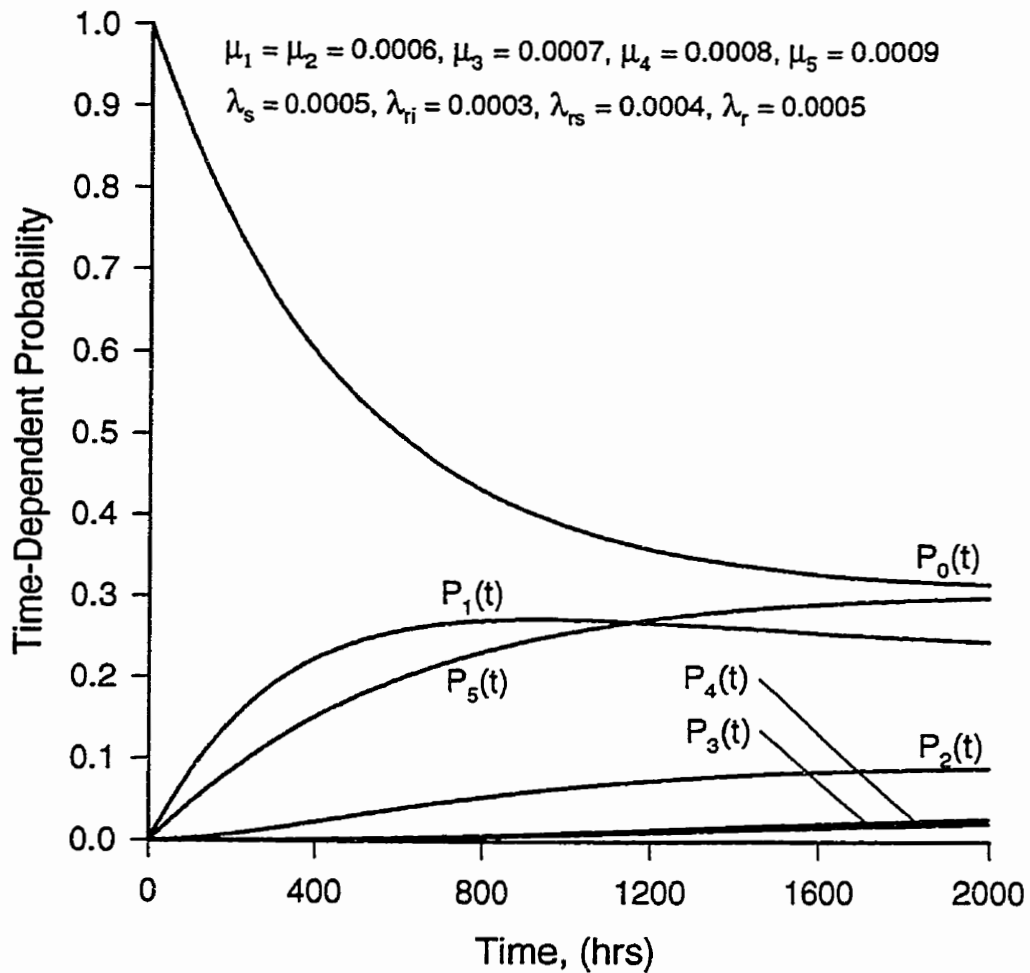
Figure 3.23.   Time-dependent state probability ($n = 2$) plots for a robot with constant failure and repair ($\beta = 1$) rates.

Table 3-20:   Time-dependent availability ($n = 2$) values for a robot with constant failure and repair ($\beta = 1$) rates.

Time (Hrs)	Time-Dependent Availability ($\beta = 1$)		
	$AV_{rs2}(t)$	$AV_{rs1}(t)$	$AV_r(t)$
0	1	1	1
500	0.5435	0.7882	0.8191
1000	0.3880	0.6608	0.7261
1500	0.3343	0.5943	0.6773
2000	0.3160	0.5618	0.6515
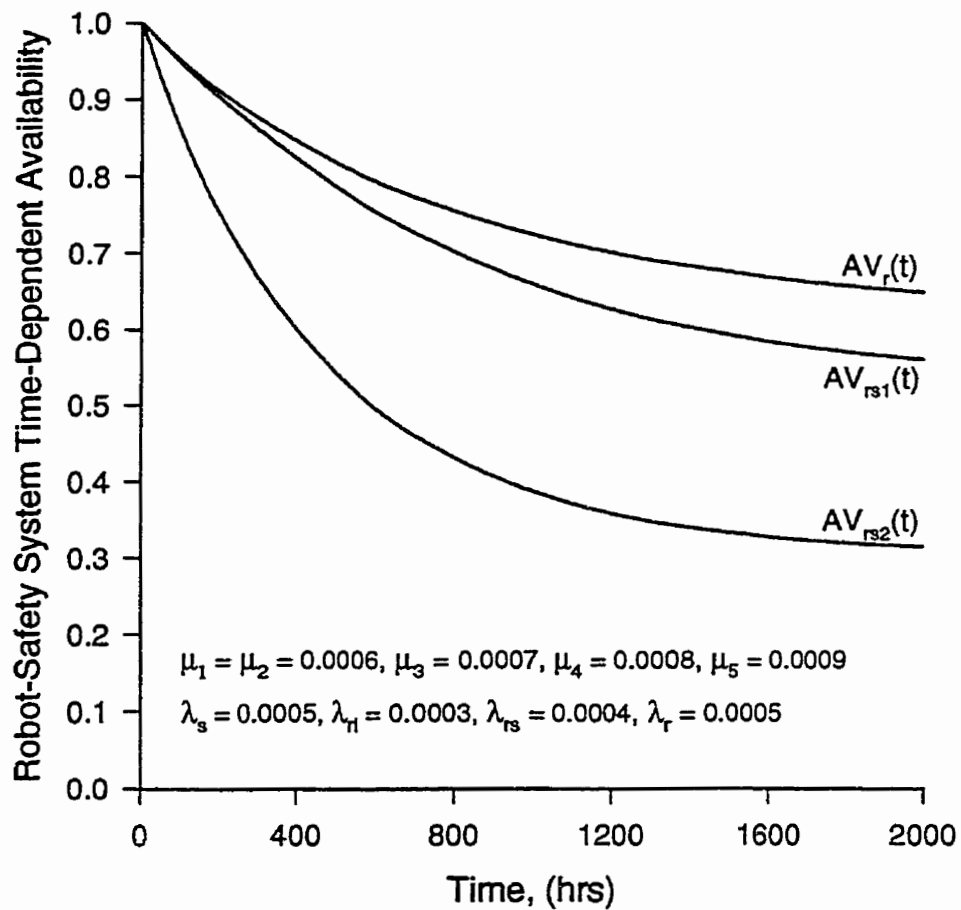
Figure 3.24.   Availability ($n = 2$) plots for a robot with constant failure and repair rates. More specifically, the plots were obtained using Equations (169) – (3.171).

Table 3-21:   Time-dependent state probability ($n = 2$) values for a robot with constant failure rate and **gamma** distributed ($\beta = 2$) failed system repair times.

Time (Hrs)	Time Dependent Probability ($\beta = 2$)					
	$P_0(t)$	$P_1(t)$	$P_2(t)$	$P_3(t)$	$P_4(t)$	$P_5(t)$
0	1	0	0	0	0	0
500	0.5132	0.2398	0.0306	0.0016	0.0024	0.2121
1000	0.3201	0.2522	0.0627	0.0080	0.0117	0.3444
1500	0.2455	0.2217	0.0761	0.0170	0.0243	0.4139
2000	0.2199	0.1943	0.0776	0.0261	0.0363	0.4439
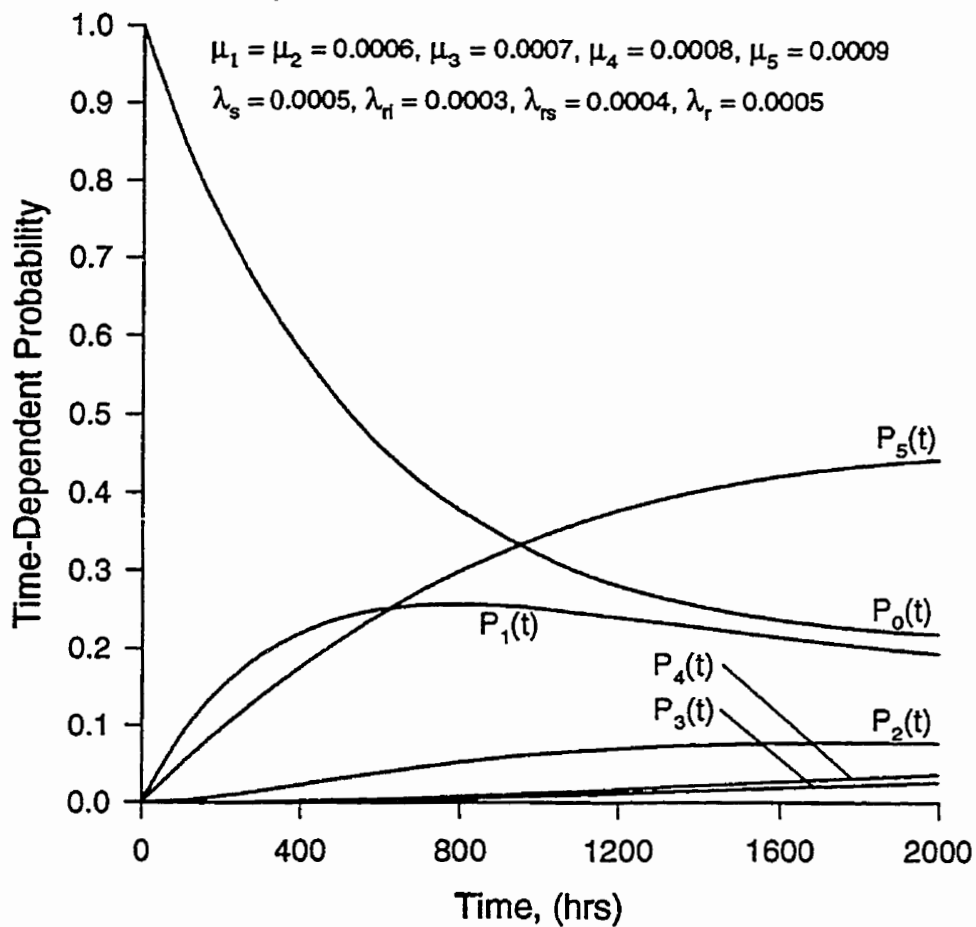
Figure 3.25.   Time-dependent state probability ($n = 2$) plots for a robot with constant failure rate and **gamma** distributed ($\beta = 2$) failed system repair times.

Table 3-22:   Time-dependent availability ($n = 2$) values for a robot with constant failure rate and **gamma** distributed ($\beta = 2$) failed system repair times.

Time (Hrs)	Time-Dependent Availability ($\beta = 2$)		
	$AV_{rs2}(t)$	$AV_{rs1}(t)$	$AV_r(t)$
0	1	1	1
500	0.5132	0.7530	0.7836
1000	0.3201	0.5723	0.6351
1500	0.2455	0.4672	0.5434
2000	0.2199	0.4143	0.4918
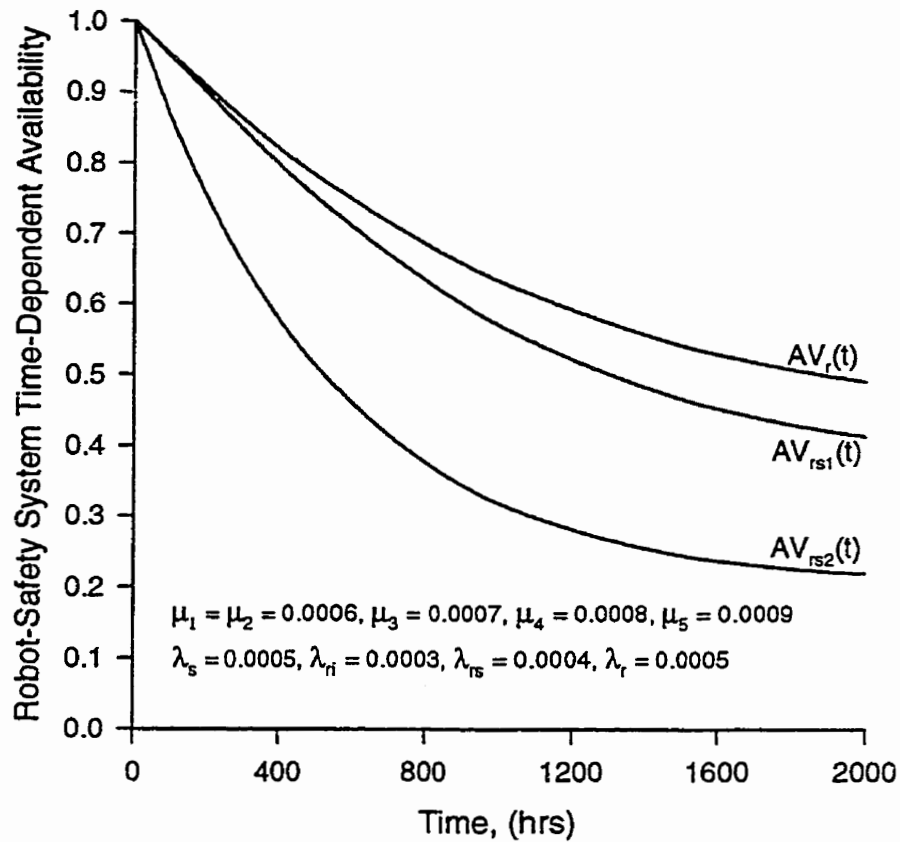
Figure 3.26.   Availability ($n = 2$) plots for a robot with constant failure rate and **gamma** distributed ($\beta = 2$) failed system repair times. More specifically, the plots were obtained using Equations (175) – (3.177).

3.5.3   Robot System Reliability and MTTF

Substituting $n = 2$ into the generalized Equations (3.82) – (3.84) developed in Section 3.3.3,

we obtain the following Laplace transforms of the state probabilities:

$$P_i(s) = \frac{\gamma_i(s)}{s \cdot Z(s)} \qquad (for \quad i = 0, 1, 2) \tag{3.178}$$

$$P_k(s) = \frac{\gamma_k(s)}{s \cdot Z(s)} \qquad (for \quad k = 3, 4, 5) \tag{3.179}$$

$$Z(s) = \sum_{i=0}^{2} \gamma_i(s) + \sum_{k=3}^{5} \gamma_k(s) \tag{3.180}$$

where

$$\gamma_0(s) = [(s + a_1)(s + a_2) - \mu_2 C_1]s$$
$$\gamma_1(s) = C_0(s + a_2)s$$
$$\gamma_2(s) = C_0 C_1 s$$
$$\gamma_3(s) = \lambda_{ri} C_0 C_1$$
$$\gamma_4(s) = \lambda_{rs} C_0 C_1$$
$$\gamma_5(s) = \lambda_r[(s + a_1)(s + a_2) - \mu_2 C_1 + C_0(s + a_2)]$$

Expanding equations (3.178) and (3.179), result in the following Laplace transforms of the

state probabilities.

$$P_0(s) = \frac{(a_1 a_2 - \mu_2 C_1) + (a_1 + a_2)s + s^2}{Z(s)} \tag{3.181}$$

$$P_1(s) = \frac{a_2 C_0 + C_0 s}{Z(s)} \tag{3.182}$$

$$P_2(s) = \frac{C_0 C_1}{Z(s)} \tag{3.183}$$

$$P_3(s) = \frac{\lambda_{ri} C_0 C_1}{Z(s)} \tag{3.184}$$

$$P_4(s) = \frac{\lambda_{rs} C_0 C_1}{Z(s)} \tag{3.185}$$

$$P_5(s) = \frac{\lambda_r[(a_1 a_2 + C_0 a_2 - \mu_2 C_1) + (a_1 + a_2 + C_0)s + s^2]}{Z(s)} \tag{3.186}$$

$$Z(s) = z_0 + z_1 s + z_2 s^2 + z_3 s^3 \tag{3.187}$$

where

$$z_0 = a_0 a_1 a_2 - \mu_1 a_2 C_0 - \mu_2 C_1 a_0$$
$$z_1 = a_0 a_1 + a_0 a_2 + a_1 a_2 - \mu_1 C_0 - \mu2 C_1$$
$$z_2 = a_0 + a_1 + a_2$$
$$z_3 = 1$$
$$a_0 = 2\lambda_s + \lambda_r$$
$$a_1 = \lambda_s + \lambda_r + \mu_1$$
$$a_2 = \lambda_{rl} + \lambda_{rs} + \mu_2$$
$$C_0 = 2\lambda_s$$
$$C_1 = \lambda_s$$

Using Equations (3.181) – (3.183), the Laplace transform reliability of an irreparable robot

system with safety units in various operating conditions are

$$R_{rs2}(s) = P_0(s) = \frac{(a_1 a_2 - \mu_2 C_1) + (a_1 + a_2)s + s^2}{Z(s)} \tag{3.188}$$

$$
\begin{aligned}
R_{rs1}(s) &= P_0(s) + P_1(s) \\
&= \frac{(a_1 a_2 + C_0 a_2 - \mu_2 C_1) + (a_1 + a_2 + C_0)s + s^2}{Z(s)}
\end{aligned} \tag{3.189}
$$

and

$$
\begin{aligned}
R_r(s) &= P_0(s) + P_1(s) + P_2(s) \\
&= \frac{(a_1 a_2 + C_0 a_2 + C_0 C_1 - \mu_2 C_1) + (a_1 + a_2 + C_0)s + s^2}{Z(s)}
\end{aligned} \tag{3.190}
$$

where

$R_{rs2}(s)$ = Laplace transform reliability of an irreparable robot system when the robot is working with two safety units.

$R_{rs1}(s)$ = Laplace transform reliability of an irreparable robot system when the robot is working with one safety unit.

$R_r(s)$ = Laplace transform reliability of an irreparable robot system when the robot is working without safety unit.

Inverting Equations (3.188) – (3.190), yield the general robot system reliability expressions

in time domain. For example, Equation (3.188) becomes

$$R_{rs2}(t) = \frac{e^{r_1 t}(r_a r_b - r_a r_1 - r_b r_1 - r_1^2)}{(r_1 - r_2)(r_1 - r_3)} + \frac{e^{r_2 t}(r_a r_b - r_a r_2 - r_b r_2 - r_2^2)}{(r_2 - r_1)(r_2 - r_3)} + \frac{e^{r_3 t}(r_a r_b - r_a r_3 - r_b r_3 - r_3^2)}{(r_3 - r_1)(r_3 - r_2)}$$

(3.191)

where r_a and r_b are the two roots of the numerator in Equation (3.188), and r_1, r_2, and r_3 are

the three real and unique roots of its polynomial function, $Z(s)$.

Using Equations (188) – (190), the robot system mean time to failure are

$$MTTF_{rs2} = \lim_{s \to 0} R_{rs2}(s) = \frac{a_1 a_2 - \mu_2 C_1}{a_0 a_1 a_2 - \mu_1 a_2 C_0 - \mu_2 C_1 C_0}$$

(3.192)

$$MTTF_{rs1} = \lim_{s \to 0} R_{rs1}(s) = \frac{a_1 a_2 + C_0 a_2 - \mu_2 C_1}{a_0 a_1 a_2 - \mu_1 a_2 C_0 - \mu_2 C_1 C_0}$$

(3.193)

$$MTTF_r = \lim_{s \to 0} R_r(s) = \frac{a_1 a_2 + C_0 a_2 + C_0 C_1 - \mu_2 C_1}{a_0 a_1 a_2 - \mu_1 a_2 C_0 - \mu_2 C_1 C_0}$$

(3.194)

where

$MTTF_{rs2}$ = Robot system mean time to failure when the robot is working with two safety units.

$MTTF_{rs1}$ = Robot system mean time to failure when the robot is working with one safety unit.

$MTTF_r$ = Robot system mean time to failure when the robot is working without a safety unit.

3.5.3.1 *Reliability and MTTF Numerical Examples*

Setting:

$\lambda_s = 0.0005$, $\lambda_{ri} = 0.0003$, $\lambda_{rs} = 0.0004$, $\lambda_r = 0.0005$, $\mu_1 = \mu_2 = 0.0006$

into Equations (3.181) – (3.187) and inverting the resulting equations, we can obtain state numerical probability expressions for the robot and plot them as a function of time. Time-dependent probability plots are shown in Figure 3.27.

Equation (3.184) represents the Laplace transform of the robot system failing with an incident. Inserting system parameter values into this equation and inverting the results, Figure 3.28 shows the robot incident probability for various safety unit repair rates.

Substituting the above parameter values into Equations (3.188) – (3.190) and inverting the results, the reliability plots for various safety unit repair rates are shown in Figure 3.29. Utilizing Equations (3.192) – (3.194), MTTF plots of the robot as a function of safety system failure and repair rates are shown in Figure 3.29.

More detailed inspection of the robot system state probability, Incident probability, reliability, and MTTF can be made by referring to their associated Tables 3-23 to 3-26, respectively.

Table 3-23:   Time-dependent probability $(n = 2)$ values for an irreparable robot system.

Time (Hrs)	Time Dependent Probability					
	$P_0(t)$	$P_1(t)$	$P_2(t)$	$P_3(t)$	$P_4(t)$	$P_5(t)$
0	1	0	0	0	0	0
500	0.5079	0.2392	0.0306	0.0018	0.0025	0.2180
1000	0.2929	0.2462	0.0621	0.0090	0.0121	0.3776
1500	0.1855	0.2031	0.0735	0.0195	0.0259	0.4925
2000	0.1250	0.1575	0.710	0.0304	0.0405	0.5756
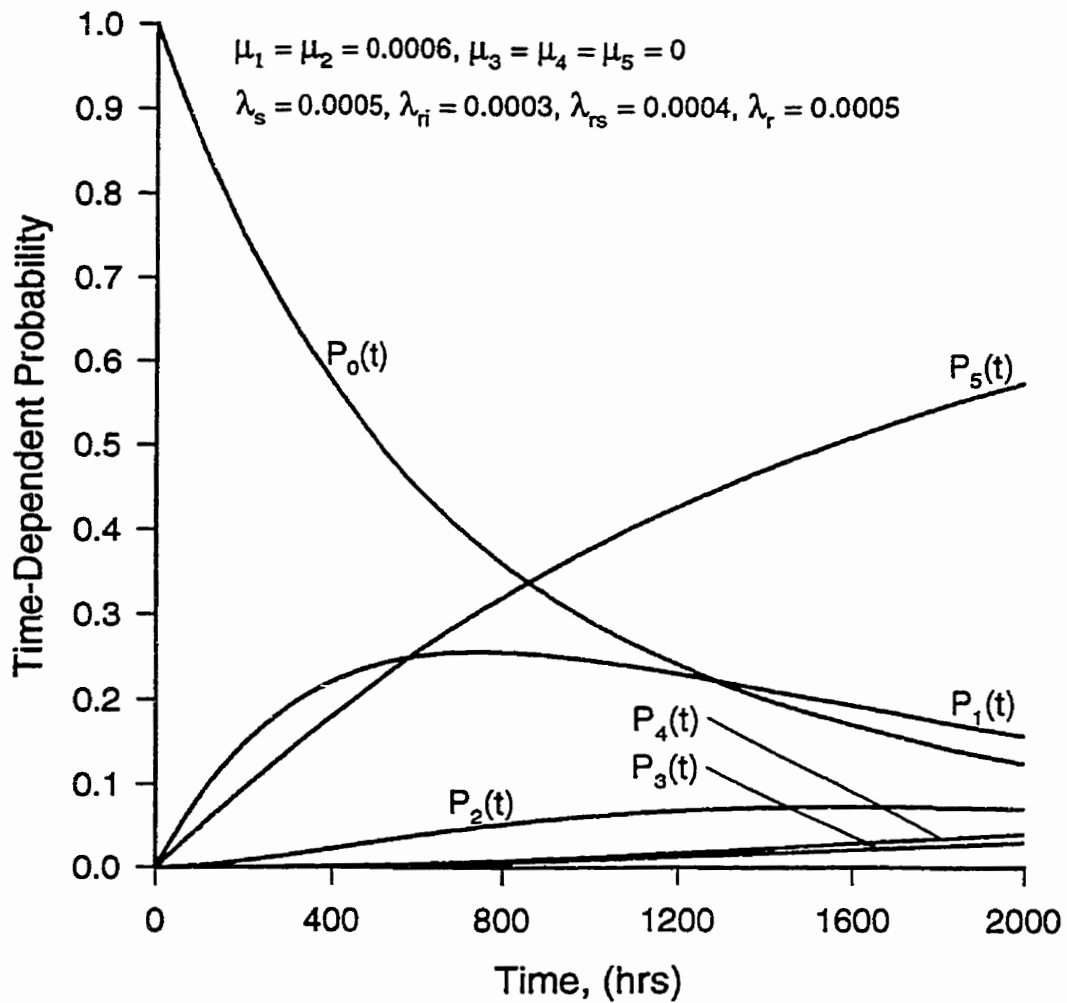
Figure 3.27.   Time-dependent probability $(n = 2)$ plots for an irreparable robot system.

Table 3-24:   Robot failing with an incident probability values for various given values
of the safety unit repair rates.

Time (Hrs)	Robot Failing With an Incident Probability, $P_3(t)$			
	$\mu_1 = \mu_2 = 0$	$\mu_1 = \mu_2 = .0006$	$\mu_1 = \mu_2 = .0012$	$\mu_1 = \mu_2 = .0018$
0	0	0	0	0
2000	0.0456	0.0304	0.0212	0.0154
4000	0.1063	0.0613	0.0386	0.0262
6000	0.1319	0.0733	0.0451	0.0301
8000	0.1399	0.0774	0.0473	0.0315
10,000	0.1421	0.0787	0.0481	0.0320
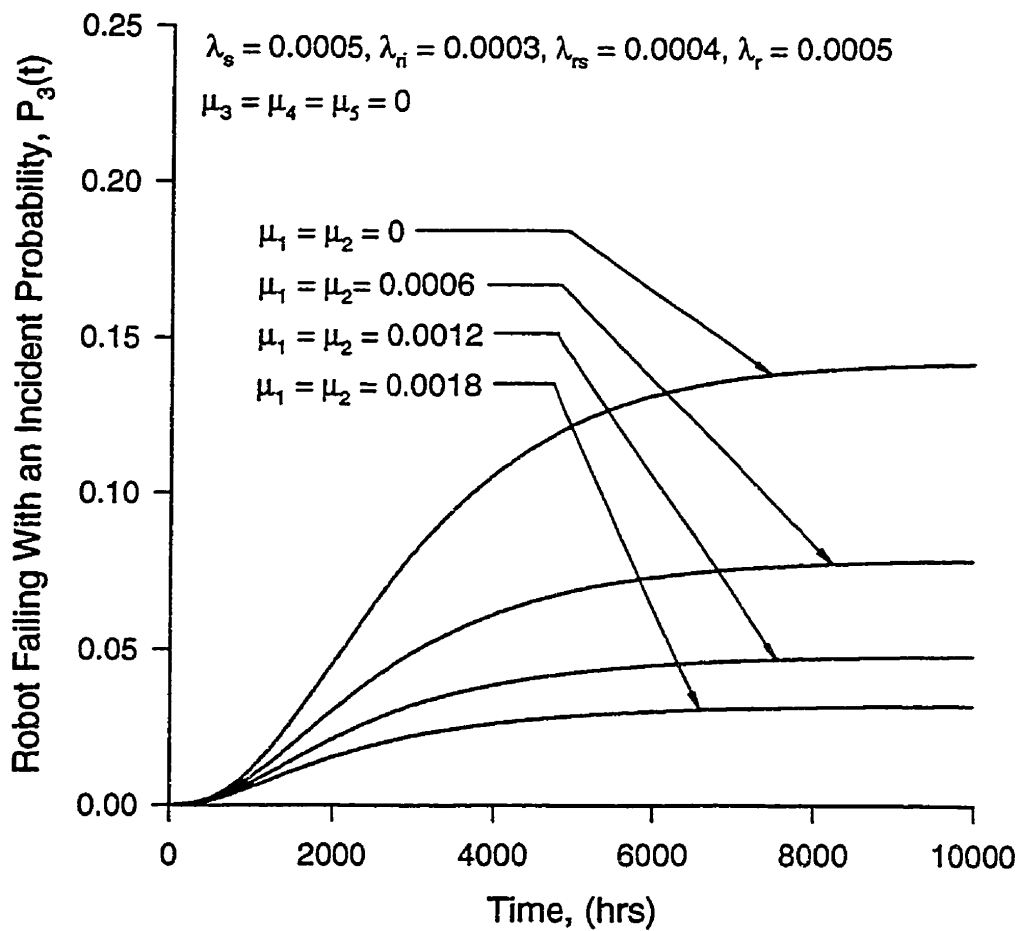
Figure 3.28.   Plots of Equation (3.184), robot system failing with an incident for various
safety unit repair rates, μ_1 and μ_2.

Table 3-25:   Reliability (n = 2) values of an irreparable robot under various conditions.

Time (hrs)	$\mu_1 = 0$			$\mu_1 = 0.0006$		
	$R_{rs2}(t)$	$R_{rs1}(t)$	$R_r(t)$	$R_{rs2}(t)$	$R_{rs1}(t)$	$R_r(t)$
0	1	1	1	1	1	1
500	0.4724	0.7407	0.7775	0.5079	0.7471	0.7777
1000	0.2231	0.5126	0.5998	0.2929	0.5392	0.6013
1500	0.1054	0.3408	0.4579	0.1855	0.3886	0.4622
2000	0.0498	0.2209	0.3457	0.1250	0.2825	0.3535

Table 3-26:   MTTF (n = 2) values of an irreparable robot under various conditions.

Failure rate, λ_s	Mean Time To Failure			Repair rate, μ_1	Mean Time To Failure		
	$MTTF_{rs2}$	$MTTF_{rs1}$	$MTTF_r$		$MTTF_{rs2}$	$MTTF_{rs1}$	$MTTF_r$
0.0000	2000.0	2000.0	2000.0	0.0000	606.1	1393.9	1696.9
0.0004	994.8	1600.0	1785.9	0.0004	795.8	1476.4	1738.2
0.0008	627.6	1283.5	1687.2	0.0008	940.1	1539.2	1769.6
0.0012	447.6	1062.9	1630.8	0.0012	1053.5	1588.5	1794.2
0.0016	343.7	904.4	1595.0	0.0016	1144.9	1628.3	1814.1
0.0020	277.0	786.1	1569.3	0.0020	1220.3	1661.0	1830.5
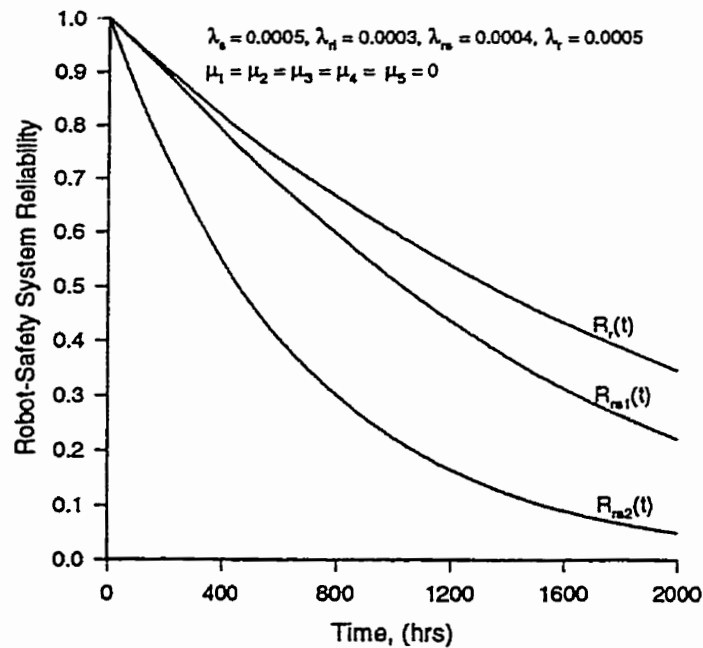
(a)
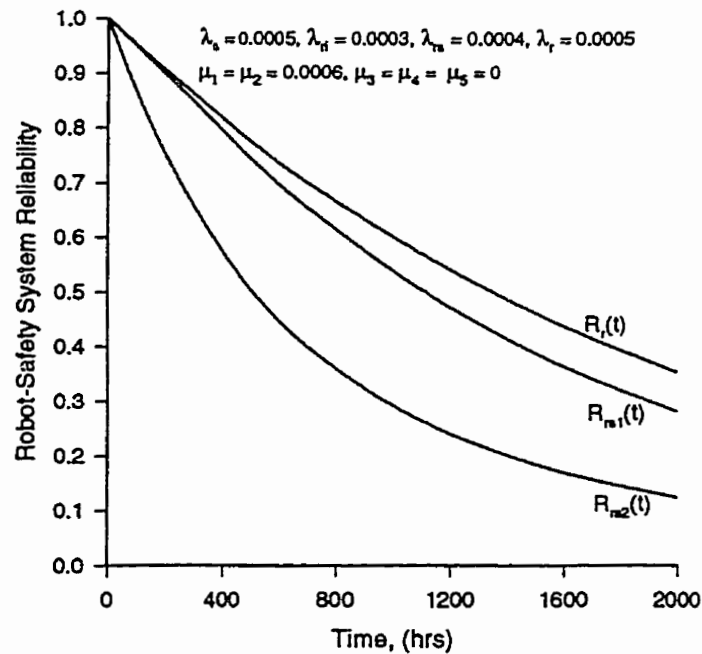
(b)

Figure 3.29.   Reliability ($n = 2$) plots of an irreparable robot system with various specified values of safety mechanism repair rates; (a) $\mu_1 = \mu_2 = 0$, (b) $\mu_1 = \mu_2 = 0.0006$. More specifically, the plots were obtained using Equations (3.188) -- (3.190).
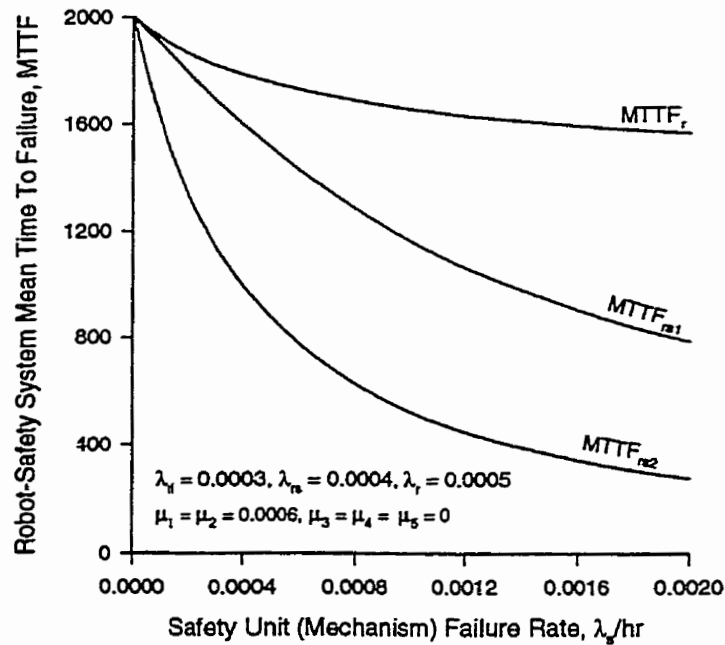
(a)
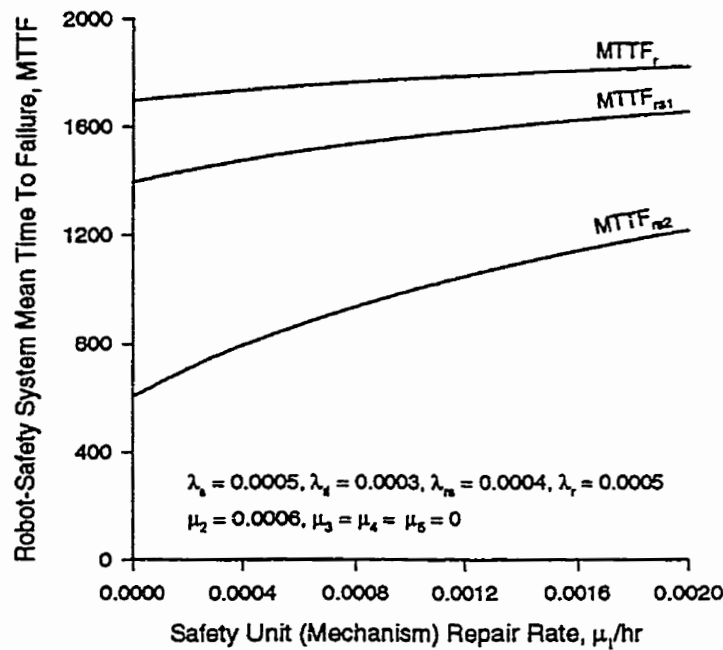
(b)

Figure 3.30.   MTTF $(n = 2)$ plots of an irreparable robot system as a function of safety unit failure and repair rates. More specifically, the plots were obtained using Equations (3.192) − (3.194).

3.6 Discussion and Conclusions

This chapter presented various mathematical models to investigate performance indices for a system comprised of a robot operating with redundant safety units. A block diagram of a system containing one robot with n-redundant safety units is shown in Figure 3.1 and the corresponding state space transition diagram is given in Figure 3.2. Generalized expressions were developed for the robot operating with a working safety unit and for the robot operating with/without the working safety unit(s). Developed expressions included, steady state availability, time-dependent availability, reliability, and mean time to failure.

Supplementary variables technique was used to produce a system of integro-differential equations. Time was set to infinity (or $P'(t) = 0$) which resulted in developing generalized steady state availability expressions for a robot with constant or non-constant repair rates. Various repair time distributions were considered for the robot in the failed state. With the exception of exponential (one parameter distribution), gamma, Erlang, Weibull, Rayleigh, and log normal were the two parameter distributions selected to fit the robot's failed system repair times. These distributions are popular lifetime model distributions with many engineering applicabilities. Each distribution is explained separately in Appendix A1.

Time-dependent probability and availability expressions were developed with the aid of the Laplace transforms. Gamma distribution was selected to represent the robot's failed system repair time distribution since it possesses a rational Laplace transform while other distributions do not. Although the Laplace transform of the robot system availability may be simple to find, inverting them is not an easy task. The analytical inversion of a rational function may not be attractive because of the inherent difficulties connected with the

calculation of the roots of a polynomial, especially, as the number of robots increases. For this, numerical method was considered simply because they do not require complex algebraic Laplace transform inversion. The resulting availability expressions were for the robot system with constant or non-constant repair rates.

Generalized reliability and mean time to failure (MTTF) expressions were developed by setting robot system repair rate equal to zero. A set of first-order differential equations were obtained and with the aid of Laplace transforms, generalized reliability and MTTF expressions were developed.

To examine robot system performance indices, mathematical analysis were performed for two special case models. Availability, reliability, and MTTF expressions were obtained for the robot system working with one safety unit (i.e., $n = 1$) and for the robot system working with two safety units (i.e., $n = 2$). To validate these expressions, numerical analysis were performed and the results were demonstrated by means of plots and tables. Plots were obtained by inserting failure and repair parameter values into the generalized expressions.

For most engineering applications, the value of the robot failure rate (λ_r) usually lies between the range $0.002 - 0.0004$ [124] and the safety system failure rate (λ_s) was assumed to be 0.0008 [30]. Furthermore, the repair parameter values were assumed to be greater than the failure rates. Nonetheless, there are many documents available that contain failure rates for various engineering parts [109].

The selected repair parameter values may yield unrealisticly high mean time to system repair (i.e., over 1000 hours). Although this fact may affect the degree of confidence in the numerical results, it will not affect any of the analysis or any expressions developed in the study. The role of the parameter values were to validate the generalized expressions only. In fact, any other repair parameter values may be chosen but the resulting plots will be

similar. Appendix E presents plots for some other repair parameter values obtained from an actual field data.

The following conclusions are associated with the models in this chapter.

1. When the failed robot system repair time x was represented by a gamma distribution, the steady state availability decreased as the shape parameter β, increased. This trend was evident in both special case models.

2. When failed robot system repair time x was represented by a Weibull distribution, the steady state availability increased as the shape parameter, β, increased. This trend was evident in both special case models.

3. When failed robot system repair time x was represented by a log normal distribution, the steady state availability dropped as the standard deviation, σ, increased. This trend was evident in both special case models.

4. For all failed system repair distributions, robot safety steady state availability decreased as the safety unit failure rate increased. This trend was evident in both special case models.

5. For all failed system repair distributions, robot steady-safety state availability increased as the safety unit repair rate increased. This trend was evident in both special case models.

6. The Weibull distributed failed system repair time displayed the highest values of system steady state availability while the Erlangian distributed failed system repair time produced the least values for the system steady state availability. This trend was evident in both special case models.

7. If the robot system performance indices is correlated with a working safety unit, then its steady state availability, time dependent availability, reliability, and mean time to

failure increased as the number of safety units increased. For example, system availability increased by about 18% when two safety units were employed instead of one.

8. If the robot system performance indices is independent of its safety unit operating condition, then its steady state availability, time dependent availability, reliability, and mean time to failure value marginally increased as the number of safety units increased. For example, system availability increased only by about 3% when two safety units were employed instead of one.

9. Trends for time-dependent availability results were similar with those found for the steady state availability.

10. Robot failing probability with an incident decreased as the safety mechanism repair rate increased.

11. Robot failing probability with an incident increased as the incident failure rate, λ_{ri}, increased.

12. Robot failing probability with an incident decreased as the number of safety units increased. For example, incident probability reduced by almost 50% when two safety units were employed instead of one.

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4

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STOCHASTIC ANALYSIS
OF A SYSTEM CONTAINING
N-REDUNDANT ROBOTS
WITH
ONE SAFETY UNIT

4.1 Introduction

Chapter 3 dealt with a system containing one robot with n-redundant safety units. This chapter is concerned with the reversed scenario of Chapter 3 in which analyses are conducted for a system containing n-redundant robots with one safety unit [114,115c]. An application of this chapter may be exemplified as the situations where identical robots are to be used to carry out similar activities. Situations like these arise where the degree of sensitivity of a particular task is high. For example, the success of the recent mission to the planet Mars is completely reliant on the inherent reliability of the robot "Galileo". If something was to go wrong with the Galileo, the whole mission will halt and billions of

dollars will be lost. Thus, as the degree of an operation's sensitivity increases, so does the importance of the redundancy. Construction of permanent space stations have already been proposed and robots will be the main workers to build them. Identical robots will be used to carry out similar activities and they will be working alongside with humans. This human–robot interaction must be safe to ensure success.

Similar to Chapter 3, a generalized model of the robot-safety system is introduced and generalized expressions are developed. For the sake of comparison, two special cases of the general model are also presented resulting in the formation of numerical expressions and various plots demonstrating the end result. In special case model I, two robots ($n = 2$) are working with one safety unit, whereas, in special case model II, three robots ($n = 3$) are working with one safety unit.

4.2 Robot-Safety System Description

The block diagram of n-redundant robots and a safety unit is shown separately in Figure 4.1. The corresponding state space transition diagram of this robot-safety system is shown in Figure 4.2.

(a)   (b)
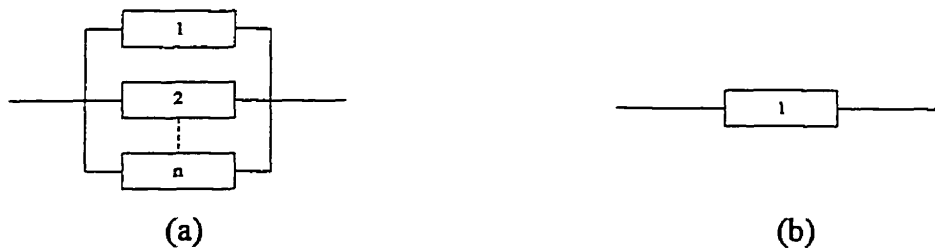
Figure 4.1.   The block diagram of; (a) n–identical robots, (b) safety unit.
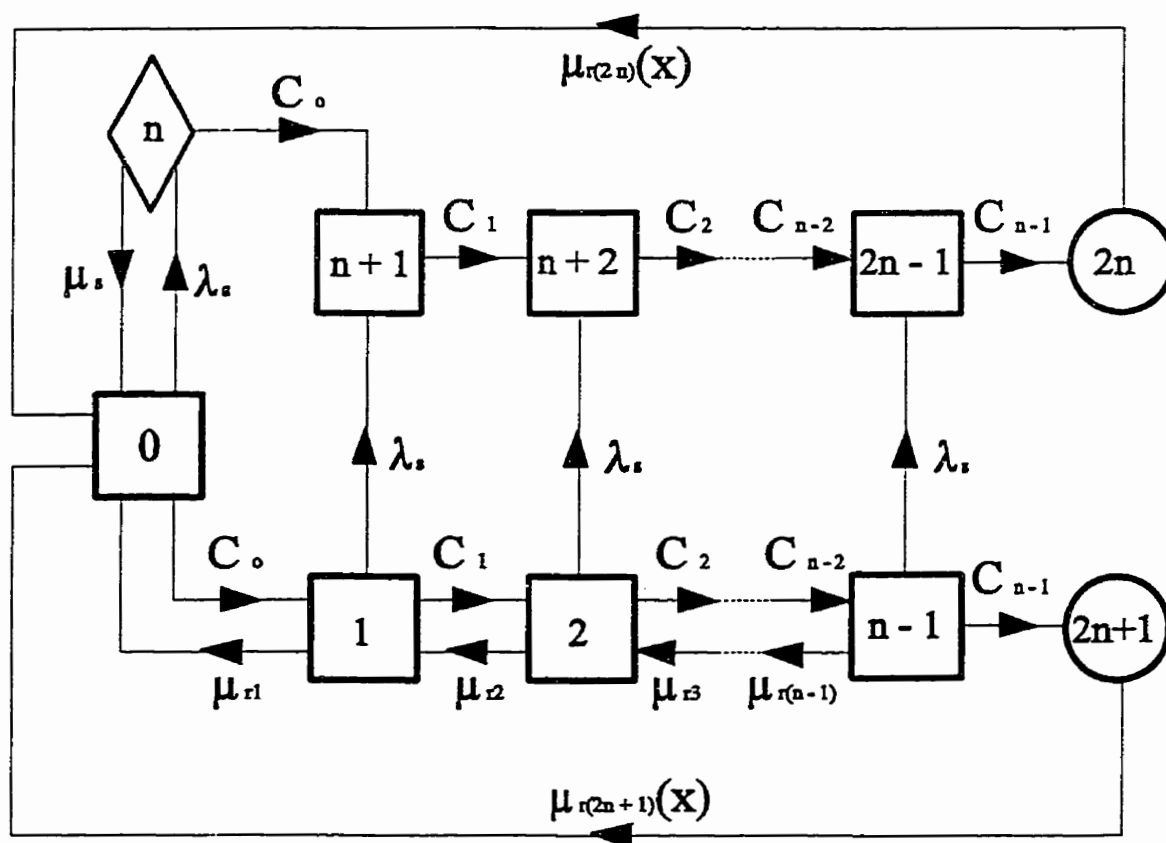
Figure 4.2.   The state space transition diagram of robot-safety system comprised of *n*-identical robots with one safety unit. The numeral in circles, diamond, and squares denotes the system state.

At time $t = 0$, the safety unit and *n*-robots begin operating. The basic system may be degraded either due to the failure of a robot or the safety unit. From degradation state due to the failure of a robot, the system may degrade further after the failure of the second, third, fourth, ... $(n-1)^{th}$ working robot or the failure of the safety unit. Both, the robot and the safety mechanism are subject to repair after each failure. From degradation state due to the

failure of the safety unit, the system may also degrade further after the failure of the second, third, fourth, ... $(n-1)^{th}$ working robot. Consequently, the overall robot system is degraded to states having only one robot working with a failed or an operating safety unit. The system fails whenever the single operating robot fails, and fully failed system may be repaired.

The numerals or letters (as applicable) in each box in Figure 4.2 represent corresponding robot system states.   For $i = 0$, safety unit and n-robots are working normally. For $i = 1$, safety unit and $n - 1$ robots operating normally while one robot has failed.   For $i = 2$, safety unit and $n - 2$ robots operating normally while two robots have failed and so on.   For $i = n - 1$, one robot working with an operating safety unit.

For $j = n$, safety unit has failed while n robots operating normally. For $j = n + 1$, one robot has failed while $n - 1$ robots are working with a failed safety unit.   For $j = n + 2$, two robots have failed while $n - 2$ robots working with a failed safety unit and so on. For $j = 2n - 1$, one robots working with a failed safety unit. For $k = 2n$ and $k = 2n + 1$ all robots have failed.

4.3 Generalized Robot-Safety System Analysis

The following symbols are associated with the models in Chapter 4.

i i^{th} state of the overall robot system: $i = 1, 2, ... , n - 1$.

j j^{th} state of the overall robot system: $j = n, n + 1, ... , 2n - 1$.

k k^{th} state of the overall robot system: $k = 2n, 2n + 1$.

λ_s Constant failure rate of the safety unit.

λ_r Constant failure rate of the robot(s).

μ_s Constant repair rate of the safety unit.

μ_{ri} Constant repair rate of the robot in state i; $i = 1, 2, \ldots , n - 1$.

$\mu_k(x)$ Time-dependent repair rate when the robot system is in state k and has an

elapsed repair time of x; for $k = 2n, 2n + 1$.

$p_k(x,t)\Delta x$ The probability that at time t, the failed robot system is in state k and the elapsed

repair time lies in the interval $[x, x + \Delta x]$; for $k = 2n, 2n + 1$.

pdf   Probability density function.

$q_k(x)$ *pdf* of repair time when the robot system is in state k and has an elapsed time of

x; for $k = 2n, 2n + 1$.

$P_i(t)$ Probability that the robot system is in state i at time t; for $i = 0, 1, \ldots , n - 1$.

$P_j(t)$ Probability that the robot system is in state j at time t; for $j = n, n + 1, \ldots , 2n\text{-}1$.

$P_k(t)$ Probability that the robot system is in state k at time t; for $k = 2n, 2n + 1$.

s Laplace transform variable.

$P_i(s)$ The Laplace transform of $P_i(t)$.

$P_j(s)$ The Laplace transform of $P_j(t)$.

$P_k(s)$ The Laplace transform of $P_k(t)$.

$SSAV_{rs}$ Robot system steady state availability when the robot system is working with an

operating safety unit.

$SSAV_r$ Robot system steady state availability when the robot system is working with or

without the safety unit.

$SSAV_{rs}(s)$ Laplace transform of the robot system availability when the robot system is

working with an operating safety unit.

$SSAV_r(s)$ Laplace transform of the robot system availability when the robot system is working with or without the safety unit.

$R_{rs}(s)$ Laplace transform of the robot system reliability when the robot system is working with an operating safety unit.

$R_r(s)$ Laplace transform of the robot system reliability when the robot system is working with or without the safety unit.

$MTTF_{rs}$ Robot system mean time to failure when the robot system is working with an operating safety unit.

$MTTF_r$ Robot system mean time to failure when the robot system is working with or without the safety unit.

The analyses presented in this chapter are subject to assumptions such as follows:

- The system is composed of one safety unit and *n* identical robots,
- The redundant robots are active or operating simultaneously,
- Statistically independent robots and safety unit failures,
- Robots and their associated safety unit's failure rates are constant,
- Safety unit and robot(s) repair rates are constant,
- Failed robot system (i.e., total system) repair rates can be constant or non-constant,
- A repaired robot(s) unit is as good as new,
- A repaired safety unit is as good as new,
- The overall system fails only when the all active robots fail,
- No repair is performed whenever robots and safety unit are in the same failure mode with the exception of all robot failures.

Using the supplementary variables technique, the corresponding system of integro-differential equations for the model given in Figure 4.2 is

$$P_0'(t) + a_0 P_0(t) = \mu_{r1} P_1(t) + \mu_s P_n(t) + \sum_{k=2n}^{2n+1} \int_0^\infty P_k(x,t) \mu_{r(k)}(x) dx \tag{4.1}$$

$$P_1'(t) + a_1 P_1(t) = C_0 P_0(t) + \mu_{r2} P_2(t) \tag{4.2}$$

$$P_i'(t) + a_i P_i(t) = C_{i-1} P_{i-1}(t) + \mu_{r(i+1)} P_{i+1}(t) \quad (for\ \ i = 1,2,\ ...\ ,\ n - 1) \tag{4.3}$$

$$P_n'(t) + a_n P_n(t) = \lambda_s P_0(t) \tag{4.4}$$

$$P_{n+1}'(t) + C_1 P_{n+1}(t) = C_0 P_n(t) + \lambda_s P_1(t) \tag{4.5}$$

$$P_j'(t) + C_j P_j(t) = C_{i-1} P_{j-1}(t) + \lambda_s P_i(t) \tag{4.6}$$

$$for \quad \begin{aligned} i &= 0,1,2,\ ...\ ,\ n - 1 \\ j &= n + 1,\ n + 2,\ ...\ ,\ 2n - 1 \end{aligned}$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{r(k)}(x) \right] P_k(x,t) = 0 \quad (for \quad k = 2n, 2n + 1) \tag{4.7}$$

where

$$\begin{aligned} a_i &= C_i + \lambda_s + \mu_{ri} \\ C_i &= (n - i)\lambda_r \\ \mu_{r0} &= 0 \\ a_n &= C_0 + \mu_s \\ i &= 0,1,2,\ ...\ ,\ n - 1 \\ n &= No.\ of\ Robots \\ No.\ &of\ states\ = 2n + 1 \end{aligned}$$

The associated boundary conditions are as follows:

$$P_{2n}(0,t) = C_{n-1} P_{2n-1}(t) \tag{4.8}$$

$$P_{2n+1}(0,t) = C_{n-1} P_{n-1}(t) \tag{4.9}$$

At time $t = 0$, $P_0(0) = 1$, and all other initial state probabilities are equal to zero. The prime denotes differentiation with respect to time t.

4.3.1   Steady State Availability Analysis

As time approaches infinity, Equations (4.1) – (4.7) reduce to Equations (4.10) – (4.16), respectively.

$$a_0 P_0 = \mu_{r1} P_1 + \mu_s P_n + \sum_{k=2n}^{2n+1} \int_0^\infty P_k(x) \mu_{r(k)}(x) dx \tag{4.10}$$

$$a_1 P_1 = C_0 P_0 + \mu_{r2} P_2 \tag{4.11}$$

$$a_i P_i = C_{i-1} P_{i-1} + \mu_{r(i+1)} P_{i+1} \quad (\text{for } i = 1,2, \dots , n - 1) \tag{4.12}$$

$$a_n P_n = \lambda_s P_0 \tag{4.13}$$

$$C_1 P_{n+1} = C_0 P_n + \lambda_s P_1 \tag{4.14}$$

$$C_j P_j = C_{i-1} P_{j-1} + \lambda_s P_i \tag{4.15}$$

$$\text{for} \quad \begin{matrix} i = 0,1,2, \dots , n - 1 \\ j = n + 1, n + 2, \dots , 2n - 1 \end{matrix}$$

$$\frac{d}{dx} P_k(x) = - \mu_{r(k)}(x) p_k(x) \quad (\text{for} \quad k = 2n, 2n + 1) \tag{4.16}$$

where

$$a_i = C_i + \lambda_s + \mu_{ri}$$
$$C_i = (n - i)\lambda_r$$
$$\mu_{r0} = 0$$
$$a_n = C_0 + \mu_s$$
$$i = 0,1,2, \ldots , n - 1$$
$$n = No. \ of \ Robots$$
$$No. \ of \ states = 2n + 1$$

The associated boundary conditions become:

$$P_{2n}(0) = C_{n-1}P_{2n-1} \tag{4.17}$$

$$P_{2n+1}(0) = C_{n-1}P_{n-1} \tag{4.18}$$

Solving Equation (4.16), the resulting expression is:

$$p_k(x) = p_k(0)e^{-\int_0^x \mu_{r(k)}(\theta) \, d\theta} \qquad (for \quad k = 2n, 2n+1) \tag{4.19}$$

The steady state condition of the probability, $P_k(t)$, that due to a failure the robot system is under repair, is

$$P_k = \int_0^\infty p_k(x)dx \qquad (for \quad k = 2n, 2n+1) \tag{4.20}$$

Substituting Equation (4.19) into Equation (4.20), we have

$$P_k = \int_0^\infty p_k(0)e^{-\int_0^x \mu_{r(k)}(\theta) \, d\theta}dx \qquad (for \quad k = 2n, 2n+1) \tag{4.21}$$

Inserting Equations (4.17) and (4.18) into Equation (4.21), we get:

$$P_{2n} = \int_0^\infty C_{n-1}P_{2n-1} e^{-\int_0^x \mu_{2n}(\theta) \, d\theta}dx = C_{n-1}P_{2n-1}E_{2n}[x] \tag{4.22}$$

$$P_{2n+1} = \int_0^\infty C_{n-1} P_{n-1} e^{-\int_0^x \mu_{2n+1}(\theta)\, d\theta}\, dx = C_{n-1} P_{n-1} E_{2n+1}[x] \qquad (4.23)$$

where

$$E_k[x] = \int_0^\infty e^{-\int_0^x \mu_{r(k)}(\theta)\, d\theta}\, dx \qquad (for \quad k = 2n, 2n+1)$$

which is the mean time to robot system repair when the failed robot system is in state k and has an elapsed repair time of x, or the expected value of x. The failed robot system mean time to repair, $E_k[x]$, represented by various repair time distributions are defined in Sections 3.2.1.1 – 3.2.1.4, and are explained in details in Appendix A1.

Solving Equations (4.11)– (4.15), (4.22), and (4.23), together with

$$\sum_{i=0}^{n-1} P_i + \sum_{j=n}^{2n-1} P_j + \sum_{k=2n}^{2n+1} P_i = 1 \qquad (4.24)$$

leads to the following general form of the steady state probabilities:

$$P_i = \frac{\omega_i}{D} \qquad (for \quad i = 0, 1, 2, \dots, n-1) \qquad (4.25)$$

$$P_j = \frac{\omega_j}{D} \qquad (for \quad j = n, n+1, n+2, \dots, 2n-1) \qquad (4.26)$$

$$P_k = \frac{\omega_k}{D} E_k[x] \qquad (for \quad k = 2n, 2n+1) \qquad (4.27)$$

$$D = \sum_{i=0}^{n-1} \omega_i + \sum_{j=n}^{2n-1} \omega_j + \sum_{k=2n}^{2n+1} \omega_k E_k[x] \qquad (4.28)$$

where $\omega_0, \omega_1, \omega_2, \dots, \omega_{2n+1}$ are the constants associated with the state probabilities P_0, P_1, $P_2, \dots, P_{2n+1}$. Consequently, the robot system steady state availability with an operating

safety unit is:

$$SSAV_{rs} = \sum_{i=0}^{n-1} P_i = \frac{\displaystyle\sum_{i=0}^{n-1} \omega_i}{\displaystyle\sum_{i=0}^{n-1} \omega_i + \sum_{j=n}^{2n-1} \omega_j + \sum_{k=2n}^{2n+1} \omega_k E_k[x]} \tag{4.29}$$

Similarly, the robot system steady state availability with or without a working safety unit is given by

$$SSAV_r = \sum_{i=0}^{n-1} P_i + \sum_{i=n}^{2n-1} P_j = \frac{\displaystyle\sum_{i=0}^{n-1} \omega_i + \sum_{j=n}^{2n-1} \omega_j}{\displaystyle\sum_{i=0}^{n-1} \omega_i + \sum_{j=n}^{2n-1} \omega_j + \sum_{k=2n}^{2n+1} \omega_k E_k[x]} \tag{4.30}$$

For various robot system repair time distributions, the values of D [i.e,. Equation (4.28)] are obtained as follows:

When the failed robot system repair time, x, is **gamma** distributed, the mean time to repair is given by Equation (3.31). Thus, substituting Equation (3.31) into Equation (4.28), we get

$$D = D_G = \sum_{i=0}^{n-1} \omega_i + \sum_{j=n}^{2n-1} \omega_j + \sum_{k=2n}^{2n+1} \omega_k \beta / \mu_k \tag{4.31}$$

Substituting $\beta = 1$ in Equation (4.31), we get the robot failed system repair time, x, represented by an **exponential** distribution. Thus, Equation (4.31) becomes

$$D = D_E = \sum_{i=0}^{n-1} \omega_i + \sum_{j=n}^{2n-1} \omega_j + \sum_{k=2n}^{2n+1} \omega_k / \mu_k \tag{4.32}$$

When the failed robot system repair time, x, is **Weibull** distributed, the mean time to repair is given by Equation (3.34). Thus, substituting Equation (3.34) into Equation (4.28), yields

$$D = D_W = \sum_{i=0}^{n-1} \omega_i + \sum_{j=n}^{2n-1} \omega_j + \sum_{k=2n}^{2n+1} \omega_k \left(\frac{1}{\mu_k}\right)^{1/\beta} \frac{1}{\beta} \Gamma\left(\frac{1}{\beta}\right) \tag{4.33}$$

Substituting $\beta = 2$ in Equation (4.33), we get the robot failed system repair time, x, represented by a **Rayleigh** distribution. Thus, Equation (4.33) becomes

$$D = D_R = \sum_{i=0}^{n-1} \omega_i + \sum_{j=n}^{2n-1} \omega_j + \sum_{k=2n}^{2n+1} \omega_k \sqrt{\frac{\pi}{4\mu_k}} \tag{4.34}$$

When the failed robot system repair time, x, is **log-normally** distributed, the mean time to repair is given by Equation (3.40). Thus, substituting Equation (3.40) into Equation (4.28), we get

$$D = D_L = \sum_{i=0}^{n-1} \omega_i + \sum_{j=n}^{2n-1} \omega_j + \sum_{k=2n}^{2n+1} \omega_k e^{\left(\mu_{y_k} + \frac{\sigma_{y_k}^2}{2}\right)} \tag{4.35}$$

4.3.2   Time Dependent Availability Analysis

Using Laplace Transform and the initial conditions in Equations (4.1) – (4.9), we get the following expressions:

$$(s + a_0)P_0(s) = 1 + \mu_{r1}P_1(s) + \mu_s P_n(s) + \sum_{k=2n}^{2n+1} \int_0^\infty P_k(x,s)\mu_{r(k)}(x)dx \tag{4.36}$$

$$(s + a_1)P_1(s) = C_0 P_0(s) + \mu_{r2}P_2(s) \tag{4.37}$$

$$(s + a_i)P_i(s) = C_{i-1}P_{i-1}(s) + \mu_{r(i+1)}P_{i+1}(s) \quad (for \ i = 1,2, \ldots , n - 1) \quad (4.38)$$

$$(s + a_n)P_n(s) = \lambda_s P_0(s) \tag{4.39}$$

$$(s + C_1)P_{n+1}(s) = C_0 P_n(s) + \lambda_s P_1(s) \tag{4.40}$$

$$(s + C_j)P_j(s) = C_{j-1}P_{j-1}(s) + \lambda_s P_i(s) \tag{4.41}$$

$$for \quad \begin{array}{l} i = 0,1,2, \ldots , n - 1 \\ j = n + 1, \ n + 2, \ldots , 2n - 1 \end{array}$$

$$\left[\frac{\partial}{\partial x} + s + \mu_{r(k)}(x) \right] p_k(x,s) = 0 \quad (for \quad k = 2n, 2n + 1) \tag{4.42}$$

where

$$\begin{aligned} a_i &= C_i + \lambda_s + \mu_{ri} \\ C_i &= (n - i)\lambda_r \\ \mu_{r0} &= 0 \\ a_n &= C_0 + \mu_s \\ i &= 0,1,2, \ldots , n - 1 \\ n &= No. \ of \ Robots \\ & \quad No. \ of \ states = 2n + 1 \end{aligned}$$

$$P_{2n}(0,s) = C_{n-1}P_{2n-1}(s) \tag{4.43}$$

$$P_{2n+1}(0,s) = C_{n-1}P_{n-1}(s) \tag{4.44}$$

Solving Equation (4.42), the resulting expression is:

$$p_k(x,s) = p_k(0,s)e^{-sx}e^{-\int_0^x \mu_{r(k)}(\theta) \, d\theta} \quad (for \quad k = 2n, 2n+1) \tag{4.45}$$

The Laplace transform of the probability, $P_k(t)$, that due to a failure the robot system is under repair, is

$$P_k(s) = \int_0^\infty p_k(x,s)dx \qquad (for \quad k = 2n, 2n+1) \tag{4.46}$$

Substituting Equation (4.45) into Equation (4.46), we get

$$P_k(s) = \int_0^\infty p_k(0,s)e^{-sx} e^{-\int_0^x \mu_{r(k)}(\theta)\, d\theta} dx \qquad (for \quad k = 2n, 2n+1) \tag{4.47}$$

Substituting Equations (4.43) and (4.44) into Equation (4.47), and we obtain the following generalized probability expressions when the robot system is in the failed state.

$$
\begin{aligned}
P_{2n}(s) &= \int_0^\infty C_{n-1} P_{2n-1}(s)\, e^{-sx} e^{-\int_0^x \mu_{2n}(\theta)\, d\theta} dx \\
&= C_{n-1} P_{2n-1}(s) \frac{1 - W_{2n}(s)}{s}
\end{aligned} \tag{4.48}
$$

$$
\begin{aligned}
P_{2n+1}(s) &= \int_0^\infty C_{n-1} P_{n-1}(s)\, e^{-sx} e^{-\int_0^x \mu_{2n+1}(\theta)\, d\theta} dx \\
&= C_{n-1} P_{n-1}(s) \frac{1 - W_{2n+1}(s)}{s}
\end{aligned} \tag{4.49}
$$

where

$$\frac{1 - W_k(s)}{s} = \int_0^\infty e^{-sx} e^{-\int_0^x \mu_{r(k)}(\theta)\, d\theta} dx \qquad (for \quad k = 2n, 2n+1) \tag{4.50}$$

or

$$W_k(s) = \int_0^\infty e^{-sx} q_k(x)dx \qquad (for \quad k = 2n, 2n+1) \tag{4.51}$$

$$q_k(x) = \mu_{r(k)}(x)e^{-\int_0^x \mu_{r(k)}(\theta)\, d\theta}$$

where $q_k(x)$ is the robot failed system repair time probability density function.

Solving Equations (4.37)– (4.41), (4.48), and (4.49), together with

$$\sum_{i=0}^{n-1} P_i(s) + \sum_{j=n}^{2n-1} P_j(s) + \sum_{k=2n}^{2n+1} P_k(s) = 1/s \qquad (4.52)$$

we obtain the following Laplace transforms of the state probabilities:

$$P_i(s) = \frac{\alpha_i(s)}{s \cdot B(s)} \qquad (for \quad i = 0, 1, 2, ..., n - 1) \qquad (4.53)$$

$$P_j(s) = \frac{\alpha_j(s)}{s \cdot B(s)} \qquad (for \quad j = n, n + 1, n + 2, ..., 2n - 1) \qquad (4.54)$$

$$P_k(s) = \frac{\alpha_k(s)}{s \cdot B(s)} [1 - W_k(s)] \qquad (for \quad k = 2n, 2n + 1) \qquad (4.55)$$

$$B(s) = \sum_{i=0}^{n-1} \alpha_i(s) + \sum_{j=n}^{2n-1} \alpha_j(s) + \sum_{k=2n}^{2n+1} \alpha_k(s)[1 - W_k(s)] \qquad (4.56)$$

where $\alpha_1(s)$, $\alpha_2(s)$, ... , $\alpha_{2n+1}(s)$ are the coefficients associated with $P_0(s)$, $P_1(s)$, ... , $P_{2n+1}(s)$, respectively. For a known robot failed system repair time distribution, one can invert Equations (4.53) – (4.56) and obtain the corresponding time-dependent probability expressions $P_0(t)$, $P_1(t)$, ... , $P_{n+3}(t)$.

For the robot system repair time x represented by a gamma distribution, the probability density function (*pdf*) of the repair time is given by Equation (3.30). For $\beta = 1$ in Equation (3.30), the failed robot system repair rate is constant and its repair time is exponentially distributed. Therefore, Equation (3.30) yields:

$$q_k(x) = \mu_k e^{-\mu_k x} \qquad (for \quad k = 2n, 2n + 1)$$

The equivalent Laplace transform expression is

$$W_k(s) = \frac{\mu_k}{s + \mu_k} \qquad (for \quad k = 2n, 2n + 1) \qquad (4.57)$$

Substituting the above result in Equations (4.53) – (4.56), we obtain the following set of state probability expressions:

$$P_i(s) = \frac{\alpha_i(s)}{s \cdot B(s)} \qquad (for \quad i = 0, 1, 2, ..., n - 1) \qquad (4.58)$$

$$P_j(s) = \frac{\alpha_j(s)}{s \cdot B(s)} \qquad (for \quad j = n, n + 1, n + 2, ..., 2n - 1) \qquad (4.59)$$

$$P_k(s) = \frac{\alpha_k(s)}{(s + \mu_k)B(s)} \qquad (for \quad k = 2n, 2n + 1) \qquad (4.60)$$

$$B(s) = \sum_{i=0}^{n-1} \alpha_i(s) + \sum_{j=n}^{2n-1} \alpha_j(s) + \sum_{k=2n}^{2n+1} \frac{s \cdot \alpha_k(s)}{(s + \mu_k)} \qquad (4.61)$$

Using Equation (4.58), the generalized Laplace transform of the robot system availability with an operating safety unit is given by

$$AV_n(s) = \sum_{i=0}^{n-1} P_i(s) = \frac{\displaystyle\sum_{i=0}^{n-1} \alpha_i(s)}{s\left(\displaystyle\sum_{i=0}^{n-1} \alpha_i(s) + \sum_{j=n}^{2n-1} \alpha_j(s) + \sum_{k=2n}^{2n+1} \frac{s \cdot \alpha_k(s)}{(s + \mu_k)}\right)} \qquad (4.62)$$

Using Equations (4.58) and (4.59), the generalized Laplace transform of the robot system availability with or without a working safety unit is

$$AV_r(s) = \sum_{i=0}^{n-1} P_i(s) + \sum_{j=n}^{2n-1} P_j(s)$$

$$= \frac{\displaystyle\sum_{i=0}^{n} \alpha_i(s) + \sum_{j=n}^{2n-1} \alpha_j(s)}{s\left(\displaystyle\sum_{i=0}^{n-1} \alpha_i(s) + \sum_{j=n}^{2n-1} \alpha_j(s) + \sum_{k=2n}^{2n+1} \frac{s\cdot\alpha_k(s)}{(s+\mu_k)}\right)}$$

(4.63)

Inverting Equations (4.58) – (4.63), we can obtain the robot-safety system time-dependent probability and availability expressions.

For $\beta = 2$, the failed robot system repair rate becomes non-constant and its repair time is represented by an Erlangian distribution. Thus, Equation (3.30) yields:

$$q_k(x) = \mu_k^2 \, x \, e^{-\mu_k x} \qquad (for \qquad k = 2n, 2n+1)$$

The equivalent Laplace transform expression is

$$W_k(s) = \frac{\mu_k^2}{(s+\mu_k)^2} \qquad (for \qquad k = 2n, 2n+1)$$

(3.64)

Substituting the above result in Equations (4.53) – (4.56), we obtain the following set of state probability expressions:

$$P_i(s) = \frac{\alpha_i(s)}{s\cdot B(s)} \qquad (for \qquad i = 0, 1, 2, ..., n-1)$$

(4.65)

$$P_j(s) = \frac{\alpha_j(s)}{s\cdot B(s)} \qquad (for \qquad j = n, n+1, n+2, ..., 2n-1)$$

(4.66)

$$P_k(s) = \frac{s\cdot\alpha_k(s)}{(s+\mu_k)^2 B(s)} \qquad (for \qquad k = 2n, 2n+1)$$

(4.67)

$$B(s) = \sum_{i=0}^{n-1} \alpha_i(s) + \sum_{j=n}^{2n-1} \alpha_j(s) + \sum_{k=2n}^{2n+1} \frac{s^2 \cdot \alpha_k(s)}{(s + \mu_k)^2} \tag{4.68}$$

Using Equation (4.65), the generalized Laplace transform of the robot system availability with an operating safety unit is given by

$$AV_{rs}(s) = \sum_{i=0}^{n-1} P_i(s) = \frac{\sum_{i=0}^{n-1} \alpha_i(s)}{s\left(\sum_{i=0}^{n-1} \alpha_i(s) + \sum_{j=n}^{2n-1} \alpha_j(s) + \sum_{k=2n}^{2n+1} \frac{s^2 \cdot \alpha_k(s)}{(s + \mu_k)^2}\right)} \tag{4.69}$$

Using Equations (4.65) and (4.66), the generalized Laplace transform of the robot system availability with or without a working safety unit is

$$AV_r(s) = \sum_{i=0}^{n-1} P_i(s) + \sum_{j=n}^{2n-1} P_j(s)$$

$$= \frac{\sum_{i=0}^{n-1} \alpha_i(s) + \sum_{j=n}^{2n-1} \alpha_j(s)}{s\left(\sum_{i=0}^{n-1} \alpha_i(s) + \sum_{j=n}^{2n-1} \alpha_j(s) + \sum_{k=2n}^{2n+1} \frac{s^2 \cdot \alpha_k(s)}{(s + \mu_k)^2}\right)} \tag{4.70}$$

Inverting Equations (4.65) – (4.70), we can obtain the robot-safety system time-dependent state probability and availability expressions.

4.3.3   Robot System Reliability and MTTF

Setting $\mu_{r(2n)}(x) = \mu_{r(2n+1)}(x) = 0$ in Equations (4.1) – (4.9), we can investigate system

reliability of an irreparable robot system with a repairable safety unit. Utilizing the Markov method, the system of differential equations become

$$P_0'(t) + a_0 P_0(t) = \mu_{rI} P_1(t) + \mu_s P_n(t) \tag{4.71}$$

$$P_1'(t) + a_1 P_1(t) = C_0 P_0(t) + \mu_{r2} P_2(t) \tag{4.72}$$

$$P_i'(t) + a_i P_i(t) = C_{i-1} P_{i-1}(t) + \mu_{r(i+1)} P_{i+1}(t) \quad (\textit{for } i = 1, 2, \dots, n-1) \tag{4.73}$$

$$P_n'(t) + a_n P_n(t) = \lambda_s P_0(t) \tag{4.74}$$

$$P_{n+1}'(t) + C_1 P_{n+1}(t) = C_0 P_n(t) + \lambda_s P_1(t) \tag{4.75}$$

$$P_j'(t) + C_i P_j(t) = C_{i-1} P_{j-1}(t) + \lambda_s P_i(t) \tag{4.76}$$

$$\textit{for} \quad \begin{matrix} i = 0,1,2, \dots, n-1 \\ j = n+1, n+2, \dots, 2n-1 \end{matrix}$$

$$P_{2n}(t) = C_{n-1} P_{2n-1}(t) \tag{4.77}$$

$$P_{2n+1}(t) = C_{n-1} P_{n-1}(t) \tag{4.78}$$

where

$$\begin{aligned}
a_i &= C_i + \lambda_s + \mu_{ri} \\
C_i &= (n-i)\lambda_r \\
\mu_{r0} &= 0 \\
a_n &= C_0 + \mu_s \\
i &= 0,1,2, \dots, n-1 \\
n &= \textit{No. of Robots} \\
\textit{No. of states} &= 2n+1
\end{aligned}$$

At time $t = 0$, $P_0(0) = 1$, and all other initial state probabilities are equal to zero. The

prime denotes differentiation with respect to time t. Taking Laplace transform in Equations (4.62) – (4.69) and solving the resulting set of equations, we obtain the following generalized Laplace transforms of state probabilities:

$$P_i(s) = \frac{\gamma_i(s)}{s \cdot Z(s)} \qquad (for \quad i = 0, 1, 2, ..., n - 1) \tag{4.79}$$

$$P_j(s) = \frac{\gamma_j(s)}{s \cdot Z(s)} \qquad (for \quad j = n, n + 1, ..., 2n - 1) \tag{4.80}$$

$$P_k(s) = \frac{\gamma_k(s)}{s \cdot Z(s)} \qquad (for \quad k = 2n, 2n + 1) \tag{4.81}$$

$$Z(s) = \sum_{i=0}^{n-1} \gamma_i(s) + \sum_{j=n}^{2n-1} \gamma_j(s) + \sum_{k=2n}^{2n+1} \gamma_k(s) \tag{4.82}$$

where $\gamma_1(s)$, $\gamma_2(s)$, ... , $\gamma_{2n+1}(s)$ are the coefficients associated with $P_0(s)$, $P_1(s)$, ... , $P_{2n+1}(s)$, respectively. Using Equation (4.79), the Laplace transform of the robot system reliability with an operating safety unit is

$$R_{rs}(s) = \sum_{i=0}^{n-1} P_i(s) = \frac{\displaystyle\sum_{i=0}^{n-1} \gamma_i(s)}{s \left(\displaystyle\sum_{i=0}^{n-1} \gamma_i(s) + \sum_{j=n}^{2n-1} \gamma_j(s) + \sum_{k=2n}^{2n+1} \gamma_k(s) \right)} \tag{4.83}$$

Utilizing Equation (4.83), the robot system mean time to failure is

$$MTTF_{rs} = \lim_{s \to 0} R_{rs}(s) = \frac{\displaystyle\sum_{i=0}^{n-1} \gamma_i}{\displaystyle\sum_{k=2n}^{2n+1} \gamma_k} \tag{4.84}$$

Using Equations (4.79) and (4.80), the generalized Laplace transform of the robot system

reliability with or without an operating safety unit is given by

$$R_r(s) = \sum_{i=0}^{n-1} P_i + \sum_{j=n}^{2n-1} P_j(s) = \frac{\displaystyle\sum_{i=0}^{n-1} \gamma_i(s) + \sum_{i=n}^{2n-1} \gamma_i(s)}{s\left(\displaystyle\sum_{i=0}^{n-1} \gamma_i(s) + \sum_{j=n}^{2n-1} \gamma_j(s) + \sum_{k=2n}^{2n+1} \gamma_k(s)\right)} \qquad (4.85)$$

The mean time to failure under this condition is

$$MTTF_r = \lim_{s \to 0} R_r(s) = \frac{\displaystyle\sum_{i=0}^{n-1} \gamma_i + \sum_{j=n}^{2n-1} \gamma_j}{\displaystyle\sum_{k=2n}^{2n+1} \gamma_k} \qquad (4.86)$$

4.4 Special Case Model I: (n = 2)

Figure 4.3 represents the state space transition diagram for a system containing two robots

and one safety unit. It can be obtained from the generalized model in Figure 4.2 for $n = 2$.
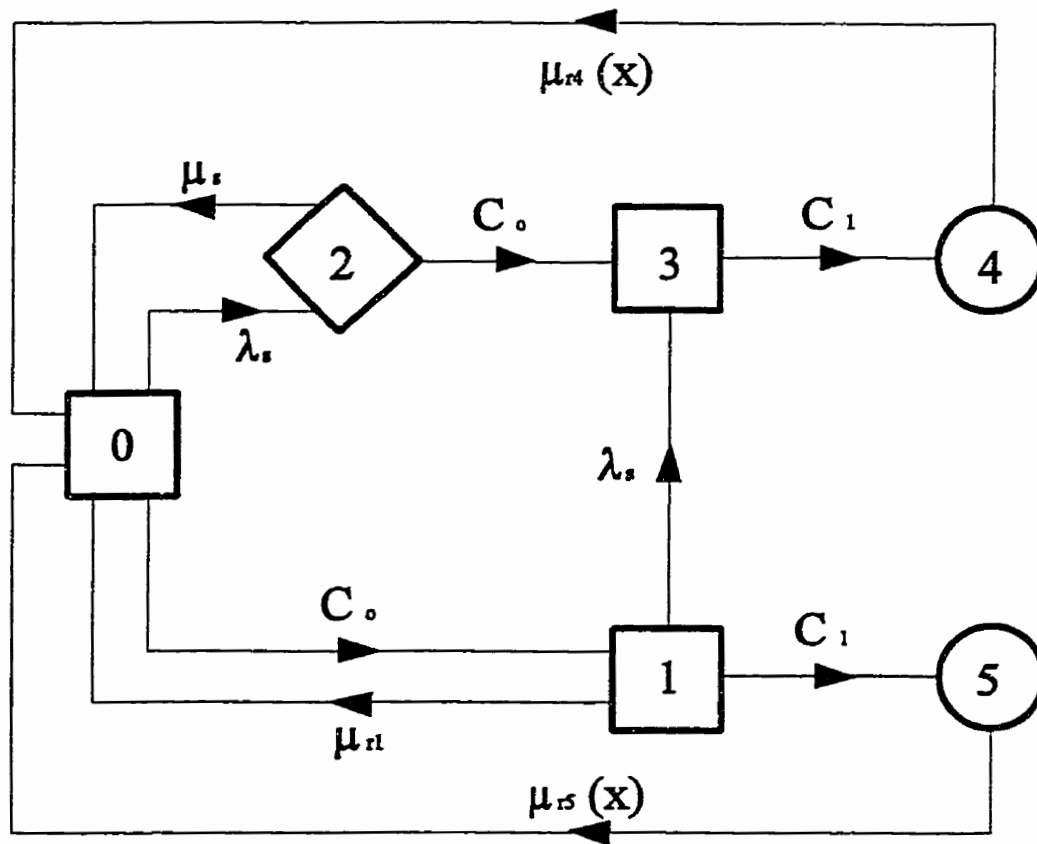
Figure 4.3.   State space transition diagram of a system containing two robots ($n = 2$) and
one safety unit.

The corresponding system of integro-differential equations for the model given in Figure 4.3

can be extracted from the generalized Equations $(4.1) - (4.9)$ by setting $n = 2$. Thus, the set

of differential equations becomes

$$P_0'(t) + a_0 P_0(t) = \mu_{rl} P_1(t) + \mu_s P_2(t) + \sum_{k=4}^{5} \int_0^\infty p_k(x,t) \mu_{rk}(x) dx \tag{4.87}$$

$$P_1'(t) + a_1 P_1(t) = C_0 P_0(t) \tag{4.88}$$

$$P_2'(t) + a_2 P_2(t) = \lambda_s P_0(t) \tag{4.89}$$

$$P_3'(t) + a_3 P_3(t) = C_0 P_2(t) + \lambda_s P_1(t) \tag{4.90}$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{rk}(x) \right] p_k(x,t) = 0 \qquad (for \quad k = 4, 5) \tag{4.91}$$

where

$$a_0 = C_0 + \lambda_s$$
$$a_1 = C_1 + \lambda_s + \mu_{rl}$$
$$a_2 = C_0 + \mu_s$$
$$a_3 = C_1$$
$$C_0 = 2\lambda_r$$
$$C_1 = \lambda_r$$

The associated boundary conditions are as follows:

$$p_4(0,t) = C_1 P_3(t) \tag{4.92}$$

$$p_5(0,t) = C_1 P_1(t) \tag{4.93}$$

At time $t = 0$, $P_0(0) = 1$, and all other initial state probabilities are equal to zero. The prime

denotes differentiation with respect to time t.

4.4.1   Steady State Availability Analysis

As time approaches infinity, state probabilities reach the steady state. Inserting $n = 2$ into

the generalized Equations (4.25) – (4.28) developed in Section 4.3.1, we get the following

steady state probabilities:

$$P_i = \frac{\omega_i}{D} \qquad (for \quad i = 0, 1) \qquad\qquad (4.94)$$

$$P_j = \frac{\omega_j}{D} \qquad (for \quad j = 2, ,3) \qquad\qquad (4.95)$$

$$P_k = \frac{\omega_k}{D} E_k[x] \qquad (for \quad k = 4, 5) \qquad\qquad (4.96)$$

$$D = \sum_{i=0}^{1} \omega_i + \sum_{j=2}^{3} \omega_j + \sum_{k=4}^{5} \omega_k E_k[x] \qquad\qquad (4.97)$$

where

$$\omega_0 = a_1 a_2 a_3$$
$$\omega_1 = a_2 a_3 C_0$$
$$\omega_2 = a_1 a_3 \lambda_s$$
$$\omega_3 = \lambda_s C_0 (a_1 + a_2)$$
$$\omega_4 = \lambda_s C_0 C_1 (a_1 + a_2)$$
$$\omega_5 = a_2 a_3 C_0 C_1$$

The robot system steady state availability with an operating safety unit is

$$SSAV_{rs} = \sum_{i=0}^{1} P_i = \sum_{i=0}^{1} \omega_i / D \qquad\qquad (4.98)$$

Similarly, the steady state availability of the robot system with or without an operating

safety unit is expressed by

$$SSAV_r = \sum_{i-0}^{1} P_i + \sum_{i-2}^{3} P_j = \frac{\sum_{i-0}^{1} \omega_i + \sum_{j-2}^{3} \omega_j}{D} \tag{4.99}$$

For various robot system repair time distributions, the values of D [i.e., Equation (4.97)] are obtained as follows:

For the robot system repair time x, represented by a **gamma** distribution, we get

$$D = D_G = \sum_{i-0}^{1} \omega_i + \sum_{j-2}^{3} \omega_j + \sum_{k-4}^{5} \omega_k \beta/\mu_k \tag{4.100}$$

When $\beta = 1$ in Equation (4.100), the robot system repair time x is represented by a **exponential** distribution, therefore

$$D = D_E = \sum_{i-0}^{1} \omega_i + \sum_{j-2}^{3} \omega_j + \sum_{k-4}^{5} \omega_k/\mu_k \tag{4.101}$$

For the robot system repair time x, represented by a **Weibull** distribution, we get

$$D = D_W = \sum_{i-0}^{1} \omega_i + \sum_{j-2}^{3} \omega_j + \sum_{k-4}^{5} \omega_k (\frac{1}{\mu_k})^{1/\beta} \frac{1}{\beta} \Gamma(\frac{1}{\beta}) \tag{4.102}$$

When $\beta = 2$ in Equation (4.102), the robot system repair time x is represented by a **Rayleigh** distribution, thus

$$D = D_R = \sum_{i-0}^{1} \omega_i + \sum_{j-2}^{3} \omega_j + \sum_{k-4}^{5} \omega_k \sqrt{\frac{\pi}{4\mu_k}} \tag{4.103}$$

For the robot system repair time x, represented by a **log-normal** distribution, we get

$$D = D_L = \sum_{i-0}^{1} \omega_i + \sum_{j-2}^{3} \omega_j + \sum_{k-4}^{5} \omega_k e^{(\mu_{y_k} + \frac{\sigma_{y_k}^2}{2})} \tag{4.104}$$

where

$$\sigma_{y_k} = \ln \sqrt{1 + \left(\frac{\sigma_{x_k}}{\mu_{x_k}}\right)^2} \; , \qquad \mu_{y_k} = \ln \sqrt{\frac{\mu_{x_k}^4}{\mu_{x_k}^2 + \sigma_{x_k}^2}} \qquad (for \quad k = 4, 5)$$

4.4.1.1 *Steady State Availability Numerical Examples*

Setting: $\lambda_s = \lambda_r = 0.0005$,

$\mu_s = 0.0006$, $\mu_{r1} = 0.0007$, $\mu_{r4} = 0.0008$, $\mu_{r5} = 0.0009$

into Equations (4.98) and (4.99) and performing numerical analysis, we can obtain robot-safety system steady state availability numerical values.

Figures 4.4 – 4.6 show plots of the robot-safety system steady state availability for gamma, Weibull, and log-normal distributions, respectively. These plots indicate the steady state availability as a function of safety unit (mechanism) failure rate, λ_s. The objective is to examine the robot system long term availability for different distributions' properties (e.g., different shape parameters of gamma distribution).

To inspect robot system repairability, Tables 4-1 to 4-3 present steady state availability for gamma, Weibull, and log-normal distributions, respectively. These tables indicate the steady state values as a function of safety unit (mechanism) repair rate, μ_s. Again, the objective is to examine the robot long term availability for different distributions' properties.

For the sake of comparison, all distributions are presented on the same figure. For all distributions, Figures 4.7 and 4.8 show plots of the robot-safety system steady state availability as a function of safety unit (mechanism) failure (λ_s) and repair (μ_s) rates, respectively. More detailized inspection of Figures 4.7 and 4.8 can be made by referring to their associated tabular values which are given in Tables 4-4 and 4-5, respectively.

The trends shown by these figures and tables are discussed in the concluding section of this chapter.
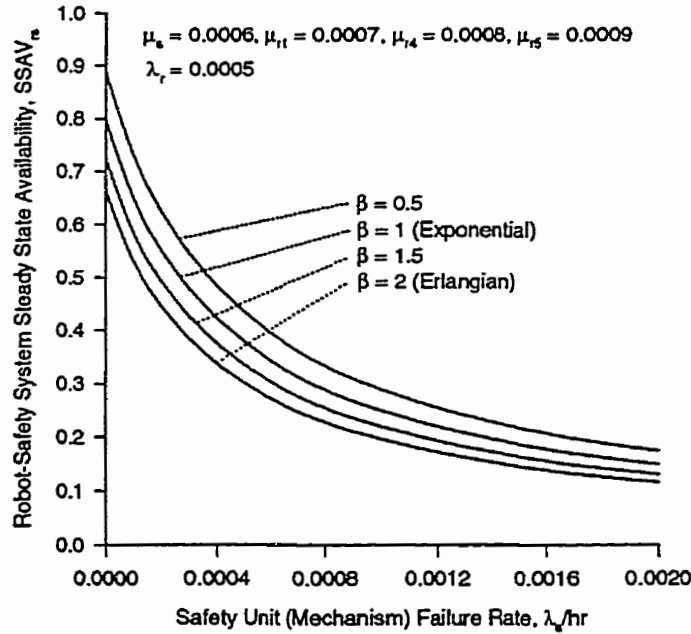
(a)
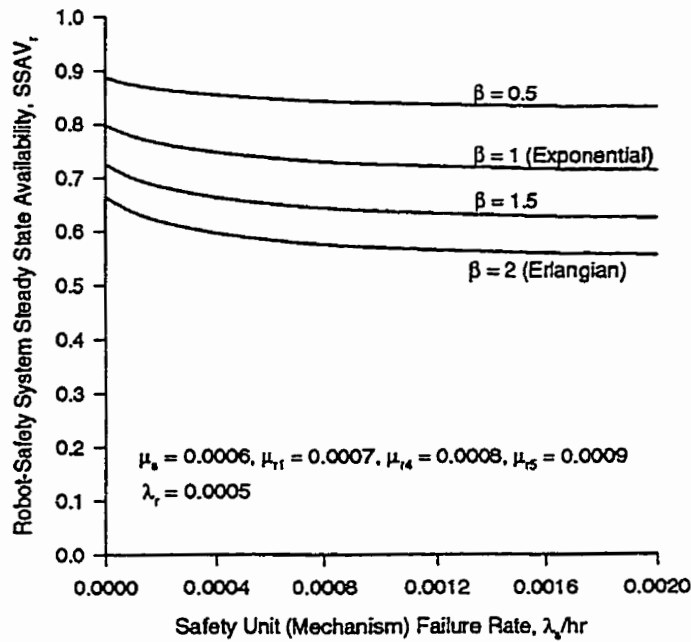
(b)

Figure 4.4.   Steady state availability ($n = 2$) vs λ_s plots for a robot system with constant failure rate and for **gamma** distributed failed system repair times; (a) robot system working with an operating safety unit, (b) robot system working with or without the safety unit. More specifically, the plots were obtained using Equations (4.98) – (4.101).
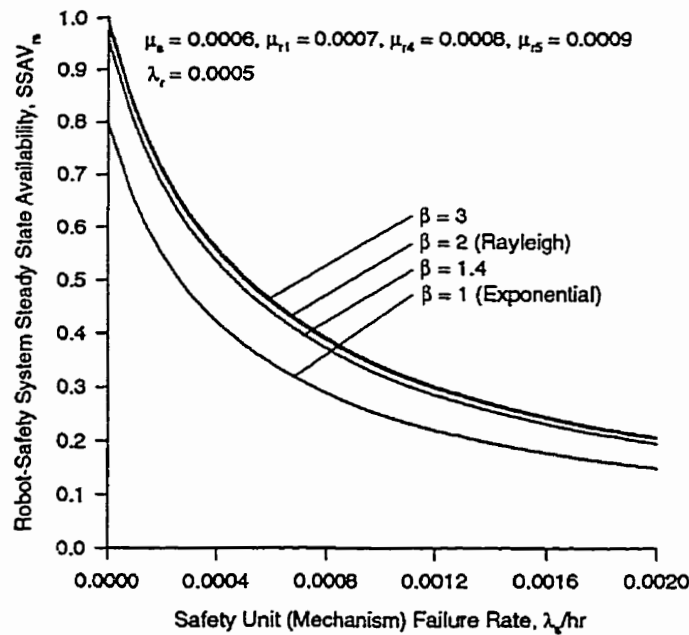
(a)
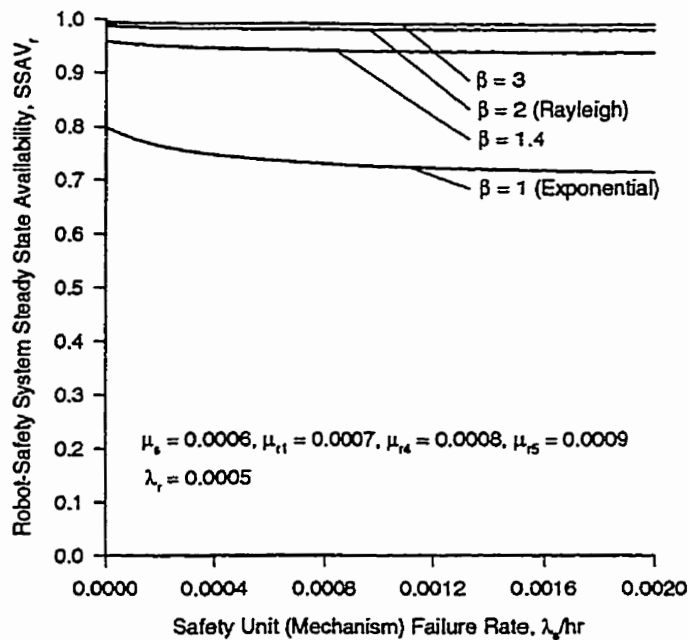
(b)

Figure 4.5.   Steady state availability ($n = 2$) vs λ_s plots for a robot system with constant failure rate and for **Weibull** distributed failed system repair times; (a) robot system working with an operating safety unit, (b) robot system working with or without the safety unit. More specifically, the plots were obtained using Equations (4.98), (4.99), (4.102), and (4.103).
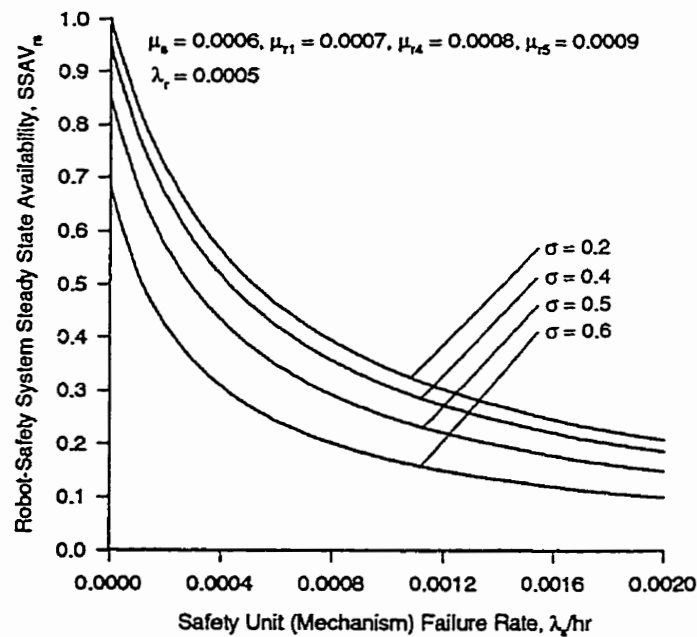
(a)
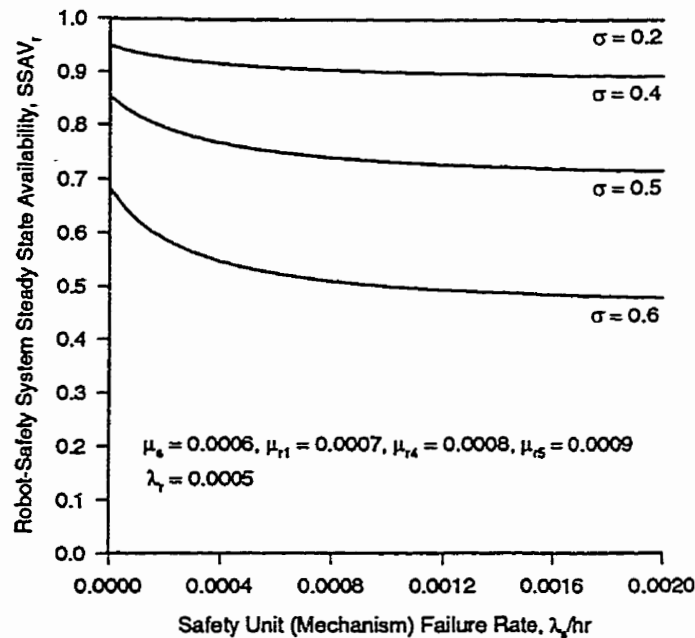
(b)

Figure 4.6.   Steady state availability ($n = 2$) vs λ_s plots for a robot system with constant failure rate and for **log-normal** distributed failed system repair times; (a) robot system working with an operating safety unit, (b) robot system working with or without the safety unit. More specifically, the plots were obtained using Equations (4.98), (4.99), and (4.104).

Table 4-1: SSAV (n = 2) vs μ_s values for a robot system with constant failure rate and **gamma** distributed failed system repair times.

μ_s	$\beta = 0.5$		$\beta = 1$ (Exponential)		$\beta = 1.5$		$\beta = 2$ (Erlangian)	
	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$
0.0000	0.3663	0.8479	0.3179	0.7359	0.2808	0.6501	0.2515	0.5821
0.0005	0.4159	0.8506	0.3619	0.7400	0.3203	0.6549	0.2872	0.5874
0.0010	0.4498	0.8525	0.3920	0.7429	0.3474	0.6583	0.3118	0.5910
0.0015	0.4744	0.8538	0.4139	0.7450	0.3671	0.6607	0.3298	0.5936
0.0020	0.5078	0.8557	0.4437	0.7478	0.3940	0.6640	0.3544	0.5972

Table 4-2: SSAV (n = 2) vs μ_s values for a robot system with constant failure rate and **Weibull** distributed failed system repair times.

μ_s	$\beta = 1$ (Exponential)		$\beta = 1.4$		$\beta = 2$ (Rayleigh)		$\beta = 3$	
	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$
0.0000	0.3179	0.7359	0.4142	0.9588	0.4281	0.9910	0.4308	0.9972
0.0004	0.3619	0.7400	0.4692	0.9596	0.4847	0.9911	0.4877	0.9972
0.0008	0.3920	0.7429	0.5066	0.9601	0.5231	0.9912	0.5262	0.9973
0.0012	0.4139	0.7450	0.5337	0.9605	0.5508	0.9913	0.5542	0.9973
0.0020	0.4437	0.7478	0.5703	0.9610	0.5883	0.9914	0.5918	0.9973

Table 4-3: SSAV (n = 2) vs μ_s values for a robot system with constant failure rate and **lognormal** distributed failed system repair times.

μ_s	$\sigma = 0.2$		$\sigma = 0.4$		$\sigma = 0.5$		$\sigma = 0.6$	
	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$
0.0000	0.4305	0.9964	0.3916	0.9064	0.3230	0.7478	0.2249	0.5206
0.0004	0.4873	0.9965	0.4446	0.9093	0.3689	0.7544	0.2590	0.5296
0.0008	0.5259	0.9966	0.4809	0.9113	0.4005	0.7590	0.2828	0.5359
0.0012	0.5538	0.9967	0.5072	0.9127	0.4236	0.7623	0.3004	0.5406
0.0020	0.5915	0.9967	0.5428	0.9146	0.4550	0.7668	0.3246	0.5470

Table 4-4: SSAV (*n* = 2) vs λ_s values for a robot system with constant failure rate and for various failed system repair time distributions.

λ_s	Erlangian (gamma,β =2)		exponential (gamma,β =1)		Lognormal (σ = 0.4)		Weibull (β=1.2)		Rayleigh (Weibull,β=2)	
	$SSAV_n$	$SSAV_r$	$SSAV_n$	$SSAV_r$	$SSAV_n$	$SSAV_r$	$SSAV_n$	$SSAV_r$	$SSAV_n$	$SSAV_r$
.0000	.6644	.6644	.7984	.7984	.9485	.9485	.9699	.9699	.9933	.9933
.0004	.3372	.5965	.4224	.7473	.5168	.9143	.5431	.9609	.5604	.9914
.0008	.2271	.5753	.2883	.7304	.3563	.9025	.3780	.9577	.3911	.9907
.0012	.1716	.5653	.2193	.7223	.2723	.8968	.2903	.9561	.3007	.9904
.0016	.1381	.5598	.1771	.7178	.2205	.8936	.2357	.9552	.2443	.9902
0.002	.1157	.5563	.1486	.7149	.1854	.8916	.1985	.9547	.2059	.9901

System parameters: λ_r = 0.0005,

μ_s = 0.0006, μ_{r1} = 0.0007, μ_{r4} = 0.0008, μ_{r5} = 0.0009

Table 4-5: SSAV (*n* = 2) vs μ_s values for a robot system with constant failure rate and for various failed system repair time distributions.

μ_s	Erlangian (gamma,β=2)		exponential (gamma,β =1)		Lognormal (σ = 0.4)		Weibull (β=1.2)		Rayleigh (Weibull,β=2)	
	$SSAV_n$	$SSAV_r$	$SSAV_n$	$SSAV_r$	$SSAV_n$	$SSAV_r$	$SSAV_n$	$SSAV_r$	$SSAV_n$	$SSAV_r$
.0000	.2515	.5821	.3179	.7359	.3916	.9064	.4142	.9588	.4281	.9910
.0004	.2872	.5874	.3619	.7400	.4446	.9093	.4692	.9596	.4847	.9911
.0008	.3118	.5910	.3920	.7429	.4809	.9113	.5066	.9601	.5231	.9912
.0012	.3298	.5936	.4139	.7450	.5072	.9127	.5337	.9605	.5508	.9913
.0016	.3435	.5956	.4306	.7465	.5271	.9138	.5542	.9608	.5719	.9914
.0020	.3544	.5972	.4437	.7478	.5428	.9146	.5703	.9610	.5883	.9914

System parameters: λ_s = λ_r = 0.0005,

μ_{r1} = 0.0007, μ_{r4} = 0.0008, μ_{r5} = 0.0009
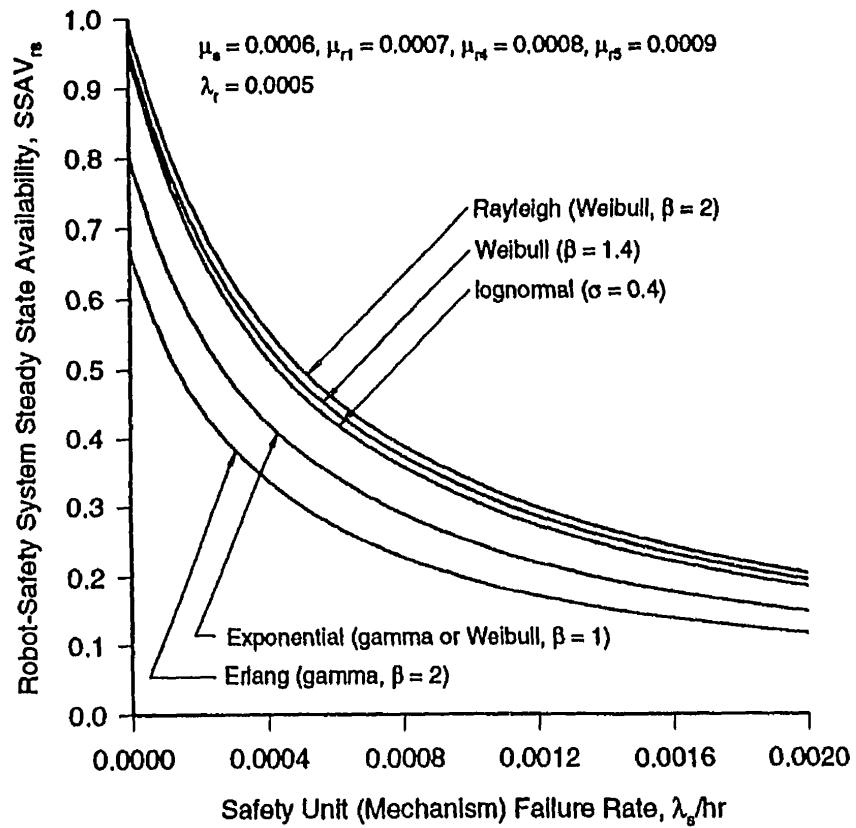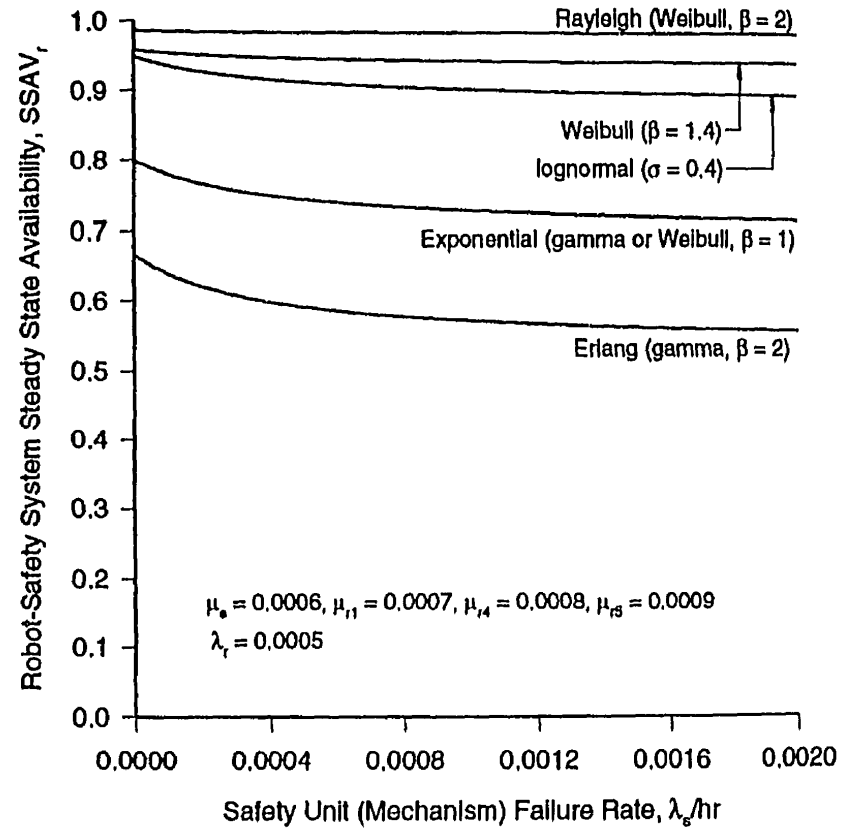
Figure 4.7.   Steady state availability ($n = 2$) vs λ_s plots for a robot system with constant failure rate and for various failed system repair time distributions when (a) robot system working with an operating safety unit, (b) robot system working with or without the safety unit. More specifically, these plots were obtained using Equations (4.98) – (4.104).
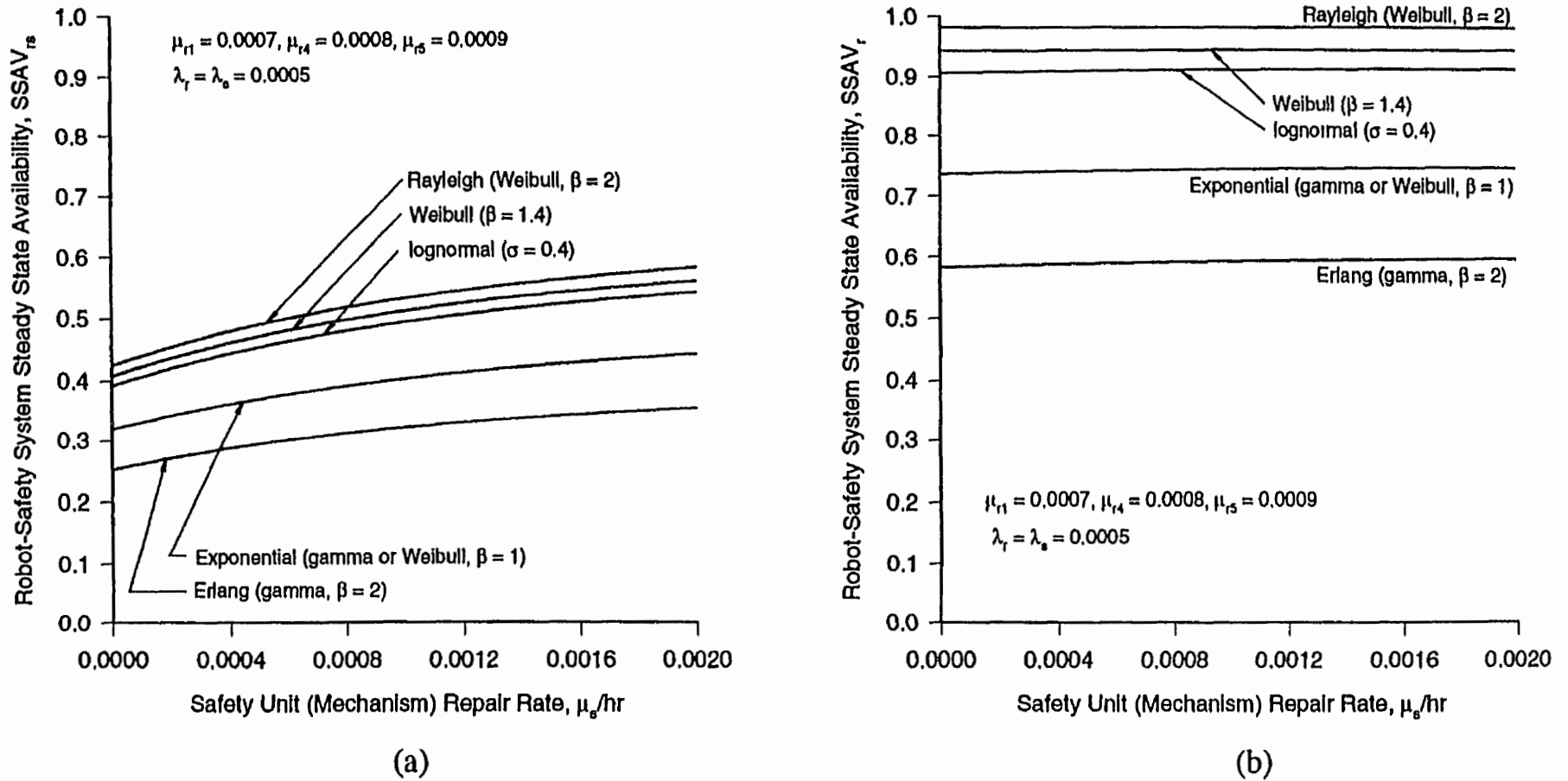
Figure 4.8.   Steady state availability ($n = 2$) vs μ_s plots for a robot system with constant failure rate and for various failed system repair time distributions when (a) robot system working with an operating safety unit, (b) robot system working with or without the safety unit. More specifically, these plots were obtained using Equations (4.98) – (4.104).

4.4.2   Time Dependent Availability Analysis

Substituting $n = 2$ into the generalized Equations (4.58) – (4.61) developed in Section 4.3.2, we obtain the following Laplace transforms of the robot system probability expressions with constant repair rate:

$$P_i(s) = \frac{\alpha_i(s)}{s \cdot B(s)} \qquad (for \quad i = 0, 1) \tag{4.105}$$

$$P_j(s) = \frac{\alpha_j(s)}{s \cdot B(s)} \qquad (for \quad j = 2, 3) \tag{4.106}$$

$$P_k(s) = \frac{\alpha_k(s)}{(s + \mu_k)B(s)} \qquad (for \quad k = 4, 5) \tag{4.107}$$

$$B(s) = \sum_{i=0}^{1} \alpha_i(s) + \sum_{j=2}^{3} \alpha_j(s) + \sum_{k=4}^{5} \frac{s \cdot \alpha_k(s)}{(s + \mu_k)} \tag{4.108}$$

where

$$\alpha_0(s) = s(s + a_1)(s + a_2)(s + a_3)$$
$$\alpha_1(s) = s(s + a_2)(s + a_3)C_0$$
$$\alpha_2(s) = s(s + a_1)(s + a_3)\lambda_s$$
$$\alpha_3(s) = s(2s + a_1 + a_2)\lambda_s C_0$$
$$\alpha_4(s) = (2s + a_1 + a_2)\lambda_r C_0 C_1$$
$$\alpha_5(s) = (s + a_2)(s + a_3)C_0 C_1$$

Using Equation (4.105), the Laplace transform of the robot system availability with an operating safety unit is given by

$$AV_{rs}(s) = \sum_{i=0}^{1} P_i(s) = \frac{\displaystyle\sum_{i=0}^{1} \alpha_i(s)}{s\left(\displaystyle\sum_{i=0}^{1} \alpha_i(s) + \sum_{j=2}^{3} \alpha_j(s) + \sum_{k=4}^{5} \frac{s \cdot \alpha_k(s)}{(s + \mu_k)} \right)} \tag{4.109}$$

Similarly, using Equations (4.105), and (4.106), the Laplace transform of the robot system availability with or without a working an operating safety unit is expressed by

$$
\begin{aligned}
AV_r(s) &= \sum_{i=0}^{1} P_i(s) + \sum_{j=2}^{3} P_j(s) \\
&= \frac{\displaystyle\sum_{i=0}^{1} \alpha_i(s) + \sum_{j=2}^{3} \alpha_j(s)}{s\left(\displaystyle\sum_{i=0}^{1} \alpha_i(s) + \sum_{j=2}^{3} \alpha_j(s) + \sum_{k=4}^{5} \frac{s \cdot \alpha_k(s)}{(s + \mu_k)} \right)}
\end{aligned}
\tag{4.110}
$$

Inserting $n = 2$ into the generalized Equations (4.65) – (4.68) developed in Section 3.3.2, we obtain the following Laplace transforms of the robot system probability expressions with non-constant repair rate:

$$
P_i(s) = \frac{\alpha_i(s)}{s \cdot B(s)} \qquad (\text{for} \quad i = 0, 1)
\tag{4.111}
$$

$$
P_j(s) = \frac{\alpha_j(s)}{s \cdot B(s)} \qquad (\text{for} \quad j = 2, 3)
\tag{4.112}
$$

$$
P_k(s) = \frac{s \cdot \alpha_k(s)}{(s + \mu_k)^2 B(s)} \qquad (\text{for} \quad k = 4, 5)
\tag{4.113}
$$

$$
B(s) = \sum_{i=0}^{1} \alpha_i(s) + \sum_{j=2}^{3} \alpha_j(s) + \sum_{k=4}^{5} \frac{s^2 \cdot \alpha_k(s)}{(s + \mu_k)^2}
\tag{4.114}
$$

Using Equations (4.111), the Laplace transform of the robot system availability with an operating safety unit is

$$AV_{rs}(s) = \sum_{i=0}^{1} P_i(s) = \frac{\sum_{i=0}^{1} \alpha_i(s)}{s\left(\sum_{i=0}^{1} \alpha_i(s) + \sum_{j=2}^{3} \alpha_j(s) + \sum_{k=4}^{5} \frac{s^2 \cdot \alpha_k(s)}{(s + \mu_k)^2}\right)} \tag{4.115}$$

Similarly, using Equations (4.111) and (4.112), the Laplace transform of the robot system availability with or without an operating safety unit is expressed by

$$AV_r(s) = \sum_{i=0}^{1} P_i(s) + \sum_{j=2}^{3} P_j(s)$$

$$= \frac{\sum_{i=0}^{1} \alpha_i(s) + \sum_{j=2}^{3} \alpha_j(s)}{s\left(\sum_{i=0}^{1} \alpha_i(s) + \sum_{j=2}^{3} \alpha_j(s) + \sum_{k=4}^{5} \frac{s^2 \cdot \alpha_k(s)}{(s + \mu_k)^2}\right)} \tag{4.116}$$

4.4.2.1 *Time-Dependent Availability Numerical Examples*

Numerical probability and availability expressions for the robot system with constant (i.e., $\beta = 1$) and non-constant (i.e., $\beta = 2$) repair rates can be obtained by setting:

$\lambda_s = \lambda_r = 0.0005$,
$\mu_s = 0.0006$, $\mu_{r1} = 0.0007$, $\mu_{r4} = 0.0008$, $\mu_{r5} = 0.0009$

into Equations (4.105) – (4.110) and (4.111) – (4.116), respectively, and taking inverse Laplace transform of the resulting equations. For $\beta = 1$, the up-time probability expressions are

$$P_0(t) = 0.2382 + 0.4704\, e^{k_1 t} + 0.0028\, e^{k_2 t} + 0.02 e^{k_3 t} +$$
$$e^{k_4 t}\left(0.2686\, Cos\,(k_5 t) - 0.1327\, Sin\,(k_5 t)\right) \tag{4.117}$$

$$P_1(t) = 0.1401 - 0.5895\, e^{k_1 t} + 0.0725\, e^{k_2 t} + 0.0278\, e^{k_3 t} +$$
$$e^{k_4 t}\big(0.3491\, Cos\,(k_5 t) + 0.0006\, Sin\,(k_5 t)\big) \tag{4.118}$$

$$P_2(t) = 0.0744 - 0.2619\, e^{k_1 t} - 0.0228\, e^{k_2 t} + 0.0161\, e^{k_3 t} +$$
$$e^{k_4 t}\big(0.1942\, Cos\,(k_5 t) + 0.0116\, Sin\,(k_5 t)\big) \tag{4.119}$$

$$P_3(t) = 0.2889 + 0.2786\, e^{k_1 t} - 0.0115\, e^{k_2 t} - 0.0627\, e^{k_3 t} +$$
$$e^{k_4 t}\big(-0.4933\, Cos\,(k_5 t) + 0.4091\, Sin\,(k_5 t)\big) \tag{4.120}$$

where

$$k_1 = -0.0025, \quad k_2 = -0.0017, \quad k_3 = -0.001,$$
$$k_4 = -0.0009, \quad k_5 = 0.0004$$

Using Equations (4.117) and (4.118), the numerical availability expression for the robot system operating with a working safety unit is

$$AV_{rs}(t) = P_0(t) + P_1(t)$$
$$= 0.3782 - 0.1191\, e^{k_1 t} + 0.0753\, e^{k_2 t} + 0.0478\, e^{k_3 t} + \tag{4.121}$$
$$e^{k_4 t}\big(0.6177\, Cos\,(k_5 t) - 0.1322\, Sin\,(k_5 t)\big)$$

Using Equations (4.117) – (4.120), the numerical availability expression for the robot system operating with or without a working safety unit is given by

$$AV_r(t) = P_0(t) + P_1(t) + P_2(t) + P_3(t)$$
$$0.7416 - 0.1024\, e^{k_1 t} + 0.0409\, e^{k_2 t} + 0.0012\, e^{k_3 t} + \tag{4.122}$$
$$e^{k_4 t}\big(0.3187\, Cos\,(k_5 t) + 0.2885\, Sin\,(k_5 t)\big)$$

Similarly, for $\beta = 2$, the up-time probability expressions are

$$P_0(t) = 0.1919 + 0.4508\, e^{k_1 t} + 0.0007\, e^{k_2 t} - 1.48 \cdot 10^{-9} e^{k_3 t} +$$
$$0.0001\, e^{k_4 t} + e^{k_5 t}\big(0.0833\, Cos\,(k_6 t) - 0.02983\, Sin\,(k_6 t)\big) +$$
$$e^{k_7 t}\big(0.2733\, Cos\,(k_8 t) - 0.071\, Sin\,(k_8 t)\big) \qquad (4.123)$$

$$P_1(t) = 0.1129 - 0.505\, e^{k_1 t} + 0.0076\, e^{k_2 t} + 5.09 \cdot 10^{-10} e^{k_3 t} +$$
$$0.0001\, e^{k_4 t} + e^{k_5 t}\big(0.1448\, Cos\,(k_6 t) - 0.0162\, Sin\,(k_6 t)\big) +$$
$$e^{k_7 t}\big(0.2396\, Cos\,(k_8 t) + 0.064\, Sin\,(k_8 t)\big) \qquad (4.124)$$

$$P_2(t) = 0.06 - 0.227\, e^{k_1 t} - 0.0473\, e^{k_2 t} - 1.75 \cdot 10^{-10} e^{k_3 t} +$$
$$0.0001\, e^{k_4 t} + e^{k_5 t}\big(0.0865\, Cos\,(k_6 t) - 0.0057\, Sin\,(k_6 t)\big) +$$
$$e^{k_7 t}\big(0.1278\, Cos\,(k_8 t) + 0.0406\, Sin\,(k_8 t)\big) \qquad (4.125)$$

$$P_3(t) = 0.2328 + 0.2291\, e^{k_1 t} + 0.0393\, e^{k_2 t} - 1.22 \cdot 10^{-9} e^{k_3 t} -$$
$$0.0001\, e^{k_4 t} + e^{k_5 t}\big(-0.2428\, Cos\,(k_6 t) + 0.0784\, Sin\,(k_6 t)\big) +$$
$$e^{k_7 t}\big(-0.2583\, Cos\,(k_8 t) + 0.3454\, Sin\,(k_8 t)\big) \qquad (4.126)$$

where

$$k_1 = -0.0026$$
$$k_2 = 0.0016$$
$$k_3 = -0.0016$$
$$k_4 = -0.0009$$
$$k_5 = -0.0011$$
$$k_6 = 0.0001$$
$$k_7 = -0.0007$$
$$k_8 = 0.0006$$

Using Equations (4.123) and (4.124), the numerical availability expression for the robot

system operating with a working safety unit is

$$
\begin{aligned}
AV_{rs}(t) = {} & 0.3047 - 0.0542\, e^{k_1 t} + 0.0083\, e^{k_2 t} - 9.75 \ast 10^{-10} e^{k_3 t} + \\
& 0.0001\, e^{k_4 t} + e^{k_5 t}\big(0.2281\, Cos\,(k_6 t) - 0.046\, Sin\,(k_6 t)\big) + \\
& e^{k_7 t}\big(0.5129\, Cos\,(k_8 t) - 0.007\, Sin\,(k_8 t)\big)
\end{aligned}
\tag{4.127}
$$

Using Equations (4.123) − (4.126), the numerical availability expression for the robot

system operating with or without a working safety unit is given by

$$
\begin{aligned}
AV_r(t) = {} & 0.5975 - 0.0521\, e^{k_1 t} + 0.0003\, e^{k_2 t} - 2.37 \ast 10^{-9} e^{k_3 t} - \\
& 0.00003\, e^{k_4 t} + e^{k_5 t}\big(0.0718\, Cos\,(k_6 t) + 0.0267\, Sin\,(k_6 t)\big) + \\
& e^{k_7 t}\big(0.3825\, Cos\,(k_8 t) + 0.3791\, Sin\,(k_8 t)\big)
\end{aligned}
\tag{4.128}
$$

State probability plots for the robot system with constant repair rates (i.e., $\beta = 1$) and

the specified system parameter values are shown in Figure 4.9. Using Equations (4.121) and

(4.122), time dependent availability plots of the robot system with an operating or a failed

safety unit are shown in Figure 4.10. More detailed inspection of the robot system state

probability and availability can be made by referring to Tables 4-6 and 4-7, respectively.

State probability plots for the robot system with non-constant repair rate (i.e., $\beta = 2$)

and the specified system parameters are shown in Figure 4.11. Using Equations (4.127) and

(4.128), time-dependent availability plots of the robot system with an operating or a failed

safety unit are shown in Figure 4.12. More detailed inspection of the robot system state

probability and availability can be made by referring to Tables 4-8 and 4-9, respectively.

Table 4-6: Time-dependent probability ($n = 2$) values for a robot system with constant failure and repair ($\beta = 1$) rates.

Time (Hrs)	Time Dependent Probability ($\beta = 1$)					
	$P_0(t)$	$P_1(t)$	$P_2(t)$	$P_3(t)$	$P_4(t)$	$P_5(t)$
0	1	0	0	0	0	0
500	0.5363	0.2349	0.1203	0.0700	0.0061	0.0323
1000	0.3637	0.2436	0.1273	0.1657	0.0293	0.0703
1500	0.2922	0.2113	0.1120	0.2323	0.0617	0.0905
2000	0.2602	0.1819	0.0971	0.2701	0.0938	0.0969
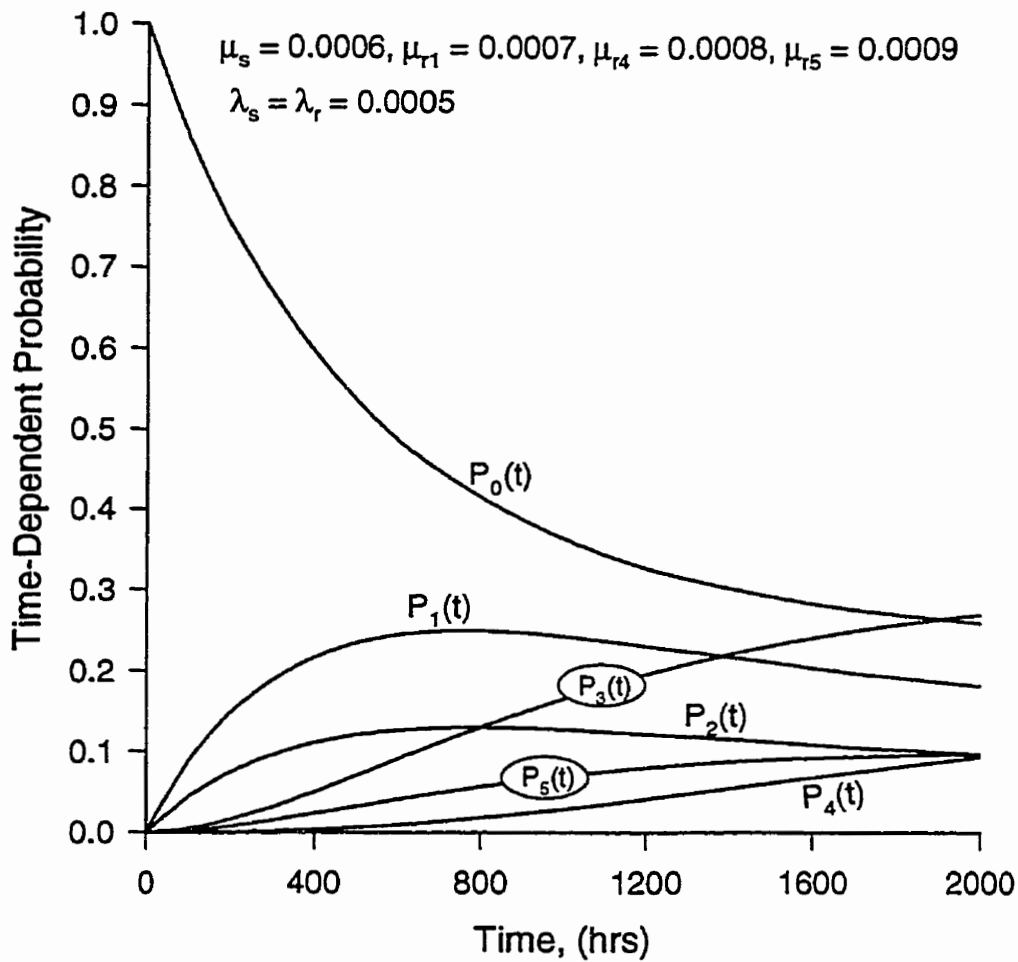
Figure 4.9.   Time-dependent probability ($n = 2$) plots for a robot system with constant failure and repair ($\beta = 1$) rates.

Table 4-7: Time-dependent availability ($n = 2$) values for a robot system with constant failure and repair ($\beta = 1$) rates.

Time (Hrs)	Time-Dependent Availability ($\beta = 1$)	
	$AV_{rs}(t)$	$AV_r(t)$
0	1	1
500	0.7713	0.9616
1000	0.6073	0.9003
1500	0.5035	0.8478
2000	0.4421	0.8093
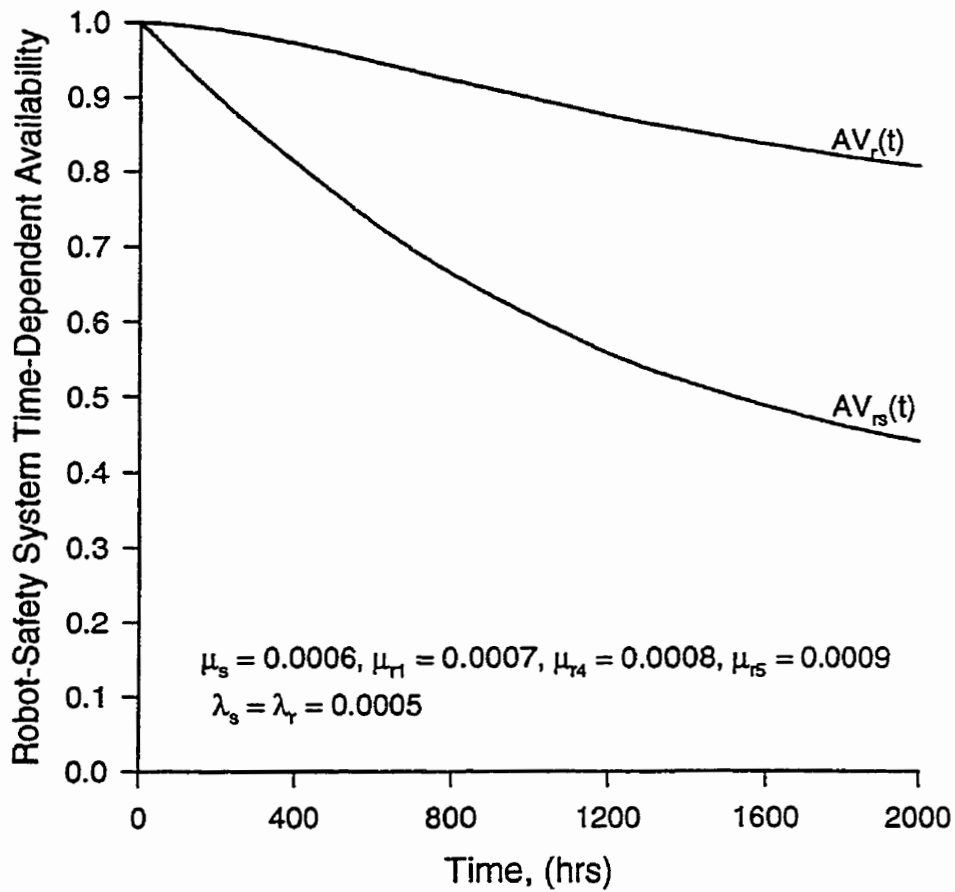
Figure 4.10.   Availability ($n = 2$) plots for a robot system with constant failure and repair rates. More specifically, the plots were obtained using Equations (4.121) and (4.122).

Table 4-8: Time-dependent probability ($n = 2$) values for a robot system with constant failure rate and **gamma** distributed ($\beta = 2$) failed system repair times.

Time (Hrs)	Time Dependent Probability ($\beta = 1$)					
	$P_0(t)$	$P_1(t)$	$P_2(t)$	$P_3(t)$	$P_4(t)$	$P_5(t)$
0	1	0	0	0	0	0
500	0.5325	0.2344	0.1201	0.0699	0.0067	0.0363
1000	0.3469	0.2398	0.1254	0.1648	0.0354	0.0877
1500	0.2601	0.2010	0.1068	0.2285	0.0805	0.1231
2000	0.2155	0.1642	0.0880	0.2608	0.1308	0.1408
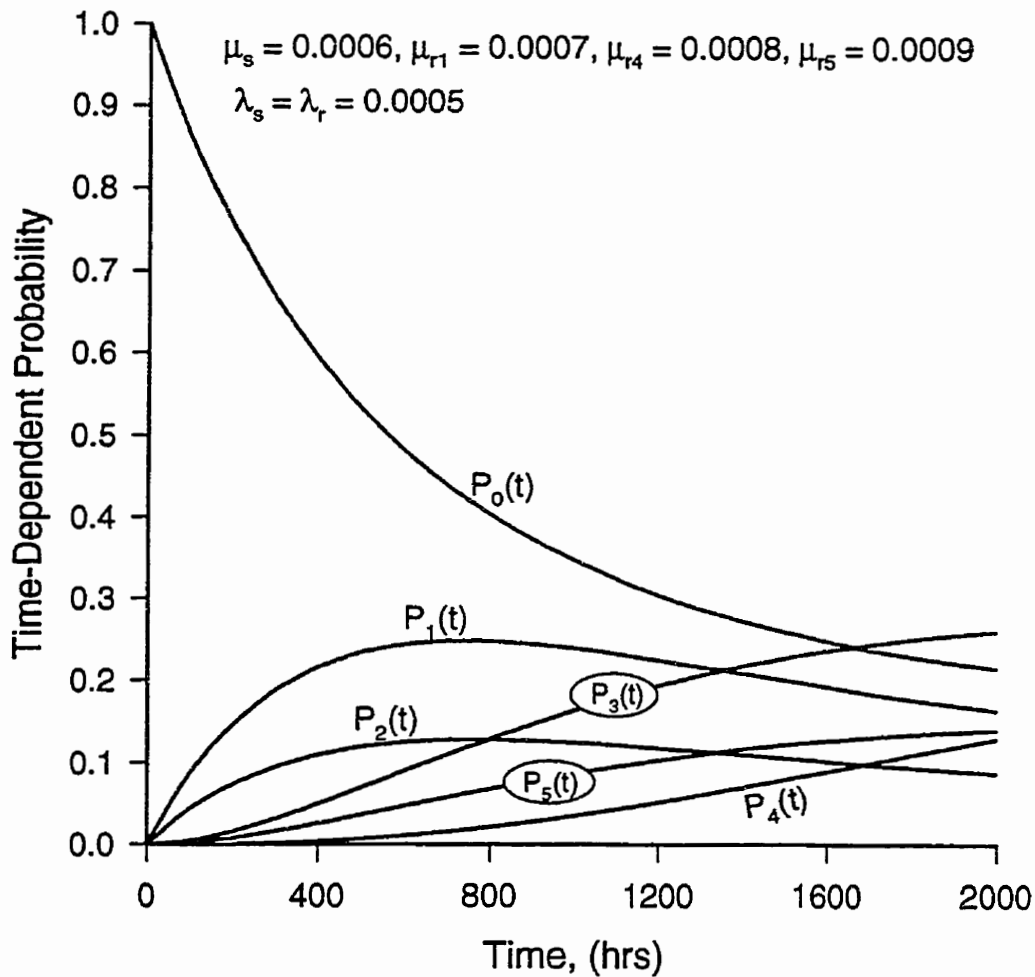
Figure 4.11.   Time-dependent probability ($n = 2$) plots for a robot system with constant failure rate and **gamma** distributed ($\beta = 2$) failed system repair times.

Table 4-9: Time-dependent availability ($n = 2$) values for a robot system with constant failure rate and **gamma** distributed ($\beta = 2$) failed system repair times.

Time (Hrs)	Time-Dependent Availability ($\beta = 2$)	
	$AV_{rs}(t)$	$AV_r(t)$
0	1	1
500	0.7670	0.9570
1000	0.5867	0.8770
1500	0.4611	0.7964
2000	0.3797	0.7285
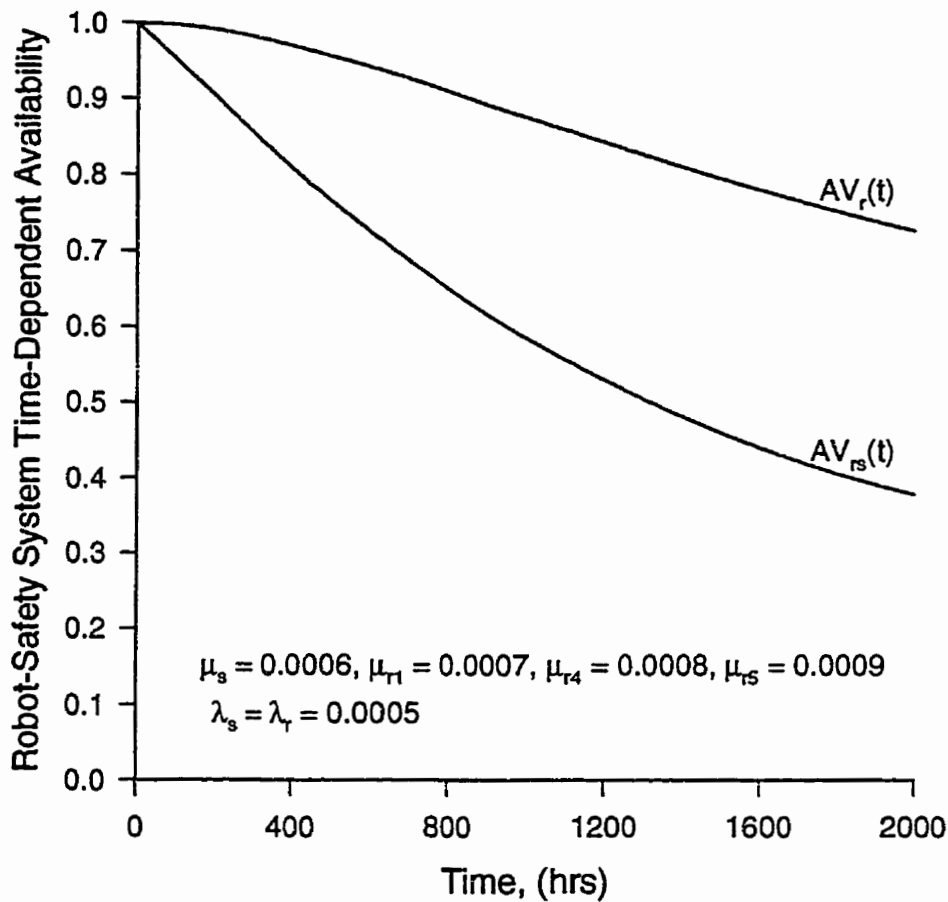
Figure 4.12.   Availability ($n = 2$) plots for a robot system with constant failure rate and **gamma** distributed ($\beta = 2$) failed system repair times. More specifically, the plots were obtained using Equations (4.127) and (4.128).

4.4.3   Robot System Reliability and MTTF

Inserting $n = 2$ into the generalized Equations (4.79) – (4.82) developed in Section 4.3.3, we obtain the following Laplace transforms of the state probabilities.

$$P_i(s) = \frac{\gamma_i(s)}{s \cdot Z(s)} \qquad (for \quad i = 0, 1) \tag{4.129}$$

$$P_j(s) = \frac{\gamma_j(s)}{s \cdot Z(s)} \qquad (for \quad j = 2, 3) \tag{4.130}$$

$$P_k(s) = \frac{\gamma_k(s)}{s \cdot Z(s)} \qquad (for \quad k = 4, 5) \tag{4.131}$$

$$Z(s) = \sum_{i=0}^{1} \gamma_i(s) + \sum_{j=2}^{3} \gamma_j(s) + \sum_{k=3}^{5} \gamma_k(s) \tag{4.132}$$

where

$$\gamma_0(s) = (s + a_1)(s + a_2)(s + a_3)s$$
$$\gamma_1(s) = C_0(s + a_2)(s + a_3)s$$
$$\gamma_2(s) = \lambda_s(s + a_1)(s + a_3)s$$
$$\gamma_3(s) = [C_0\lambda_s(s + a_1) + C_0\lambda_s(s + a_2)]s$$
$$\gamma_4(s) = C_0C_1\lambda_s(s + a_1) + C_0C_1\lambda_s(s + a_2)$$
$$\gamma_5(s) = C_0C_1(s + a_2)(s + a_3)$$

Rearranging Equation (4.132), we get

$$Z(s) = z_0 + z_1s + z_2s^2 + z_3s^3 + z_4s^4 \tag{4.133}$$

where

$$z_0 = C_0C_1(a_2a_3 + a_1\lambda_s + a_2\lambda_s)$$
$$z_1 = a_1a_2a_3 + C_0C_1(a_2 + a_3 + 2\lambda_s) + C_0\lambda_s(a_1 + a_2) + a_2a_3C_0 + a_1a_3\lambda_s$$
$$z_2 = a_1a_2 + a_1a_3 + a_2a_3 + C_0(a_2 + a_3) + \lambda_s(a_1 + a_3) + C_0(C_1 + 2\lambda_s)$$
$$z_3 = a_1 + a_2 + a_3 + C_0 + \lambda_s$$
$$z_4 = 1$$

Using Equation (4.129), the Laplace transform of the robot system reliability, $R_{rs}(s)$, with

an operating safety unit and with partial repair action is

$$R_{rs}(s) = \sum_{i-0}^{1} P_i(s) = \frac{\displaystyle\sum_{i-0}^{1} \gamma_i(s)}{s\left(\displaystyle\sum_{i-0}^{1} \gamma_i(s) + \sum_{j-2}^{5} \gamma_j(s)\right)} \tag{4.134}$$

Utilizing Equations (4.133) and (4.134), the robot system mean time to failure is

$$MTTF_{rs} = \lim_{s \to 0} R_{rs}(s) = \frac{a_2 a_3 (a_1 + C_0)}{z_0} \tag{4.135}$$

where

$$z_0 = C_0 C_1 (a_2 a_3 + a_1 \lambda_s + a_2 \lambda_r)$$

Similarly, Laplace transform of the robot system reliability, $R_r(s)$, with or without the safety

unit operating and with partial repair action is expressed by

$$R_r(s) = \sum_{i-0}^{1} P_i(s) + \sum_{j-2}^{3} P_j(s) = \frac{\displaystyle\sum_{i-0}^{1} \gamma_i(s) + \sum_{j-2}^{3} \gamma_j(s)}{s\left(\displaystyle\sum_{i-0}^{1} \gamma_i(s) + \sum_{j-2}^{5} \gamma_j(s)\right)} \tag{4.136}$$

The robot system mean time to failure $(MTTF_r)$ under this condition is

$$MTTF_r = \lim_{s \to 0} R_r(s) = \frac{a_1 a_2 a_3 + a_2 a_3 C_0 + \lambda_s a_1 a_3 + \lambda_s (a_1 + a_2) C_0}{z_0} \tag{4.137}$$

4.4.3.1   Reliability and MTTF Numerical Examples

Setting:

$\lambda_s = \lambda_r = 0.0005$,
$\mu_s = 0.0006, \quad \mu_{rl} = 0.0007$

in Equations (4.129) – (4.131) and inverting the resulting equations, we can obtain state

probability numerical expressions for the robot system and plot them as a function of time

as shown in Figure 4.13.

Inserting the above parameter values into Equations (4.134) and (4.136), and inverting

the results, Figure 4.14 shows robot system reliability plots for various values of safety unit

repair rates.

Also, using Equations (4.135) and (4.137), robot system mean time to failure plots are

shown in Figure 4.15.

More detailed inspection of the robot system state probability, reliability, and MTTF can

be made by referring to their associated values tabulated in Tables 4-10 to 4-12,

respectively.

Table 4-10:   Time-dependent probability ($n = 2$) values for an irreparable robot system.

Time (Hrs)	Time Dependent Probability					
	$P_0(t)$	$P_1(t)$	$P_2(t)$	$P_3(t)$	$P_4(t)$	$P_5(t)$
0	1	0	0	0	0	0
500	0.5310	0.2343	0.1200	0.0699	0.0068	0.0381
1000	0.3376	0.2379	0.1244	0.1644	0.0365	0.0991
1500	0.2362	0.1945	0.1034	0.2264	0.0861	0.1534
2000	0.1722	0.1497	0.0805	0.2544	0.1469	0.1963
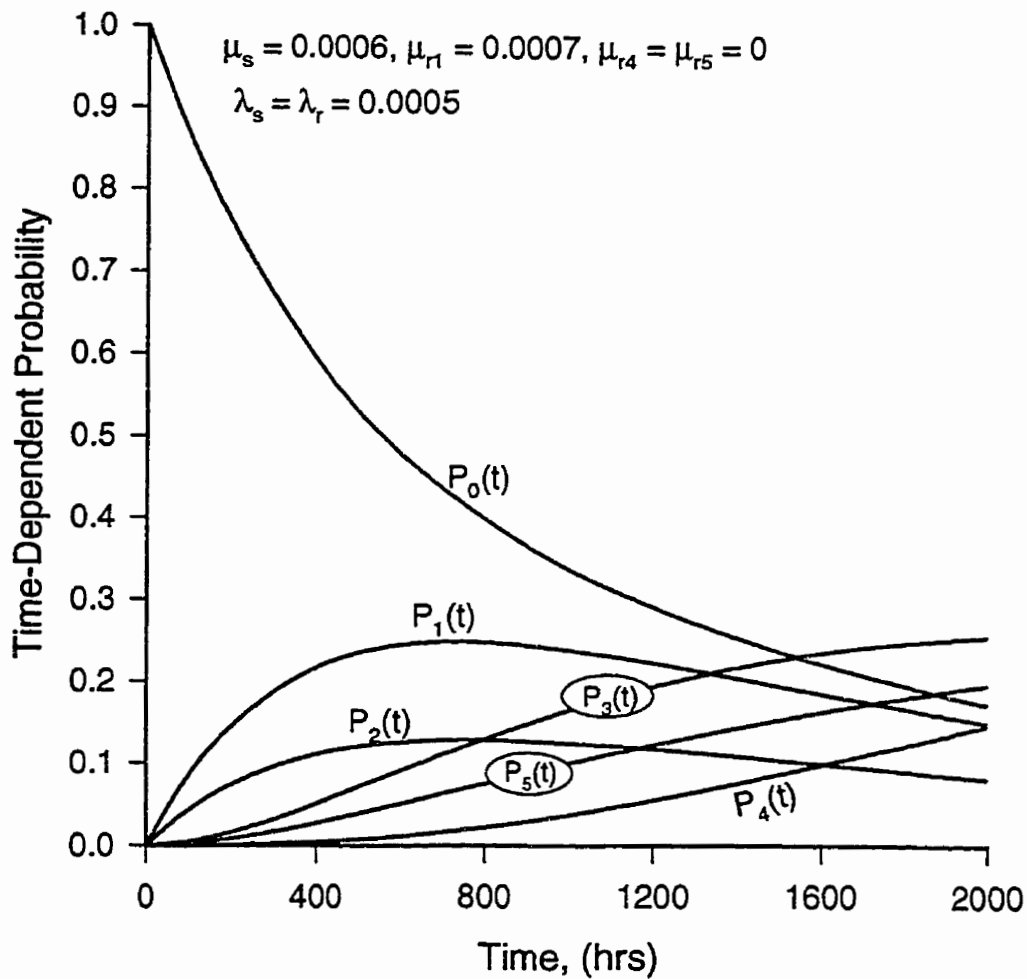
Figure 4.13.   Time-dependent probability ($n = 2$) plots for an irreparable robot system.

Table 4-11:   Reliability ($n = 2$) values of an irreparable robot system with various safety unit repair rates.

Time (hrs)	$\mu_s = 0.0006, \mu_{rl} = 0.0007$		$\mu_s = \mu_{rl} = 0$	
	$R_{rs}(t)$	$R_r(t)$	$R_{rs}(t)$	$R_r(t)$
0	1	1	1	1
500	0.7653	0.9552	0.7653	0.9511
1000	0.5755	0.8644	0.5126	0.8644
1500	0.4307	0.7605	0.3409	0.7216
2000	0.3219	0.6569	0.2209	0.6004

Table 4-12:   MTTF ($n = 2$) values of an irreparable robot system as a function of safety unit failure and repair rates.

Failure rate, λ_s	Mean Time To Failure		Repair rate, μ_s	Mean Time To Failure	
	$MTTF_{rs}$	$MTTF_r$		$MTTF_{rs}$	$MTTF_r$
0.0000	4400.0	4400.0	0.0000	1459.5	3378.4
0.0004	2000.0	3538.5	0.0004	1680.0	3435.6
0.0008	1304.3	3304.3	0.0008	1833.9	3475.5
0.0012	971.4	3200.0	0.0012	1947.5	3504.9
0.0016	775.5	3142.8	0.0016	2034.8	3527.5
0.0020	646.2	3107.7	0.0020	2103.9	3545.5

(a)

(b)
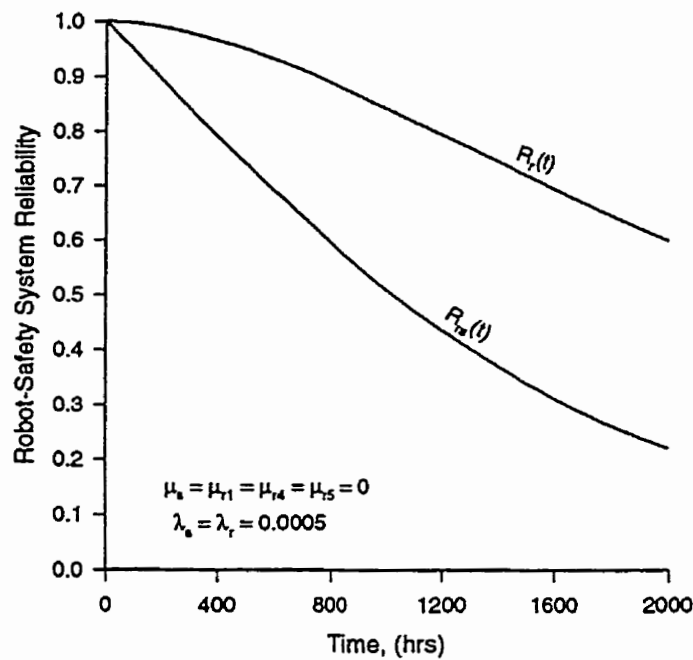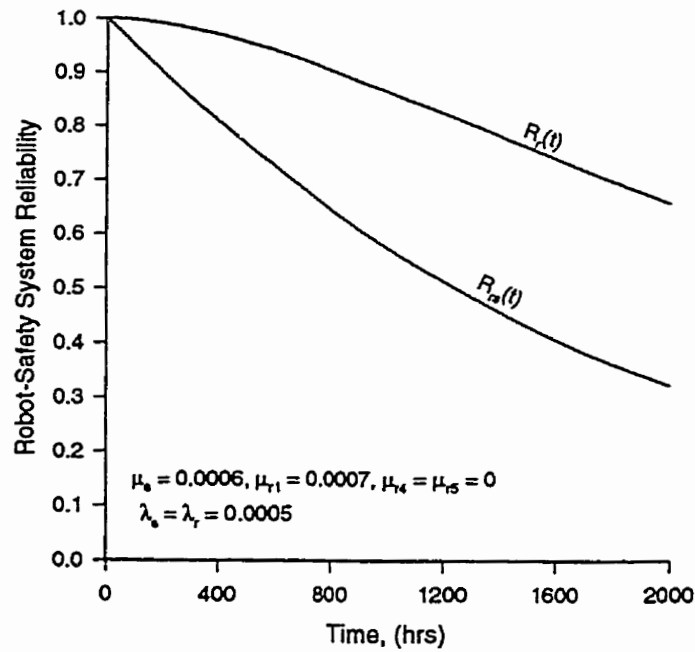
Figure 4.14.   Reliability ($n = 2$) plots of an irreparable robot system with various repair conditions: (a) $\mu_s = 0.0006$, $\mu_{rl} = 0.0007$, (b) $\mu_z = \mu_{rl} = 0$. More specifically, the plots were obtained using Equations (4.134) and (4.136).
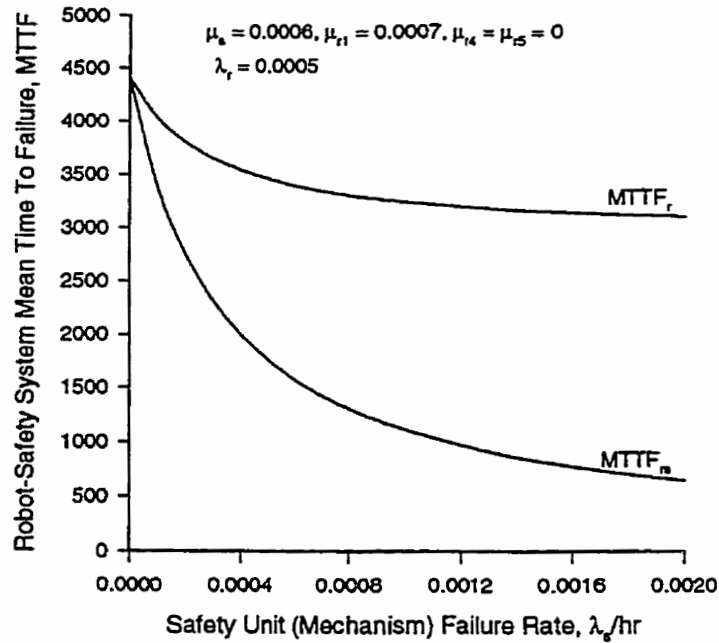
(a)
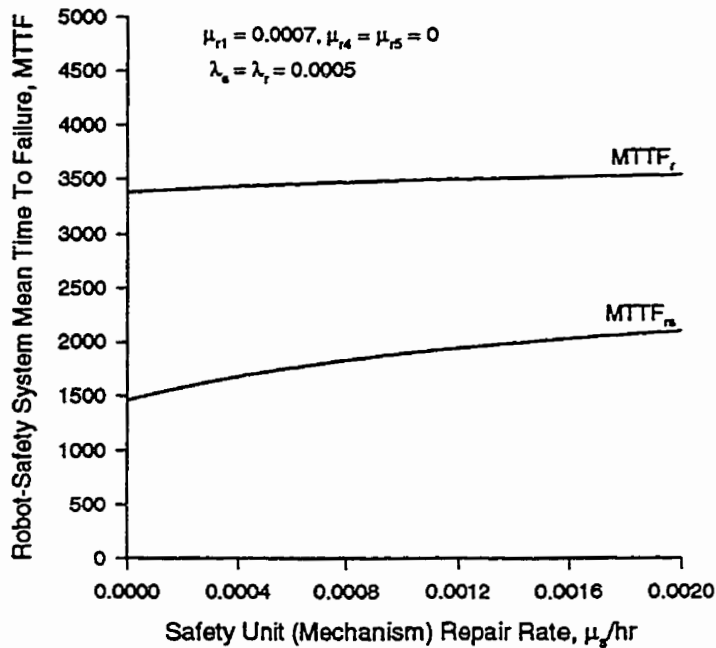
(b)

Figure 4.15.   MTTF (n = 2) plots of an irreparable robot system as a function of safety unit failure and repair rates. More specifically, the plots were obtained using Equations (4.135) and (4.137).

4.5 Special Case Model II: (n = 3)

Figure 4.16 represents the transition diagram for a system containing three robots and one

safety unit. It is obtained from the generalized model in Figure 4.2 for $n = 3$.
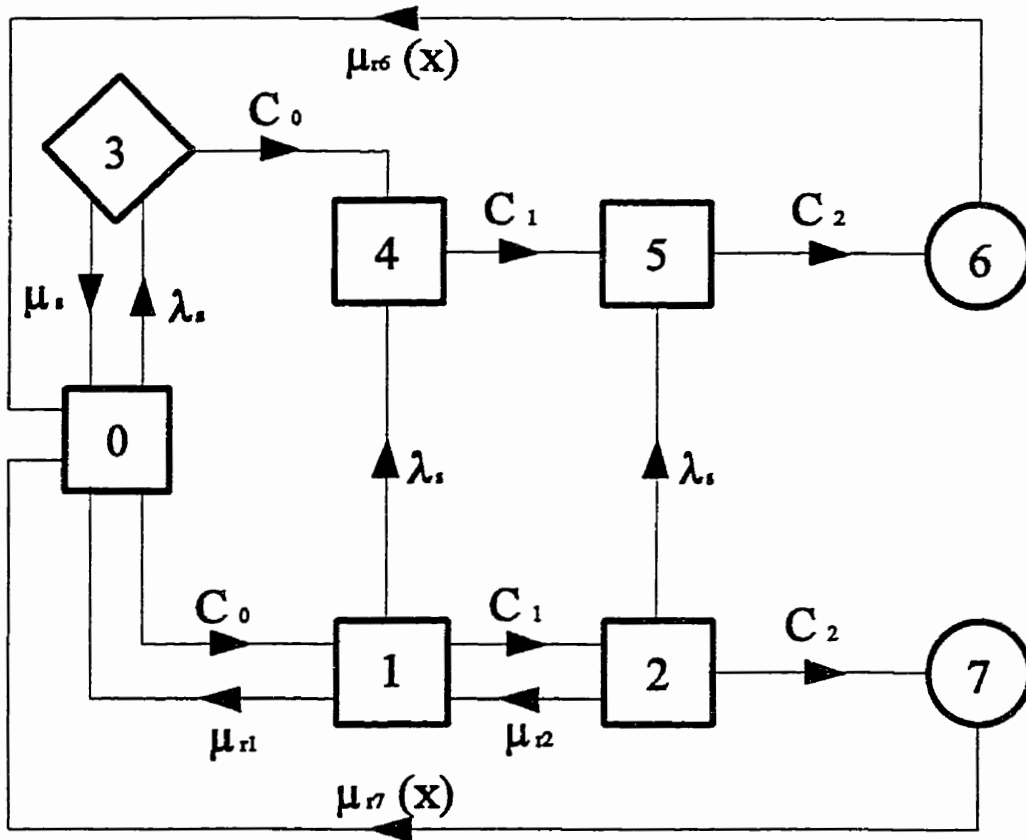
Figure 4.16.   State space transition diagram of a system containing three robots ($n = 3$)
and one safety unit.

The corresponding system of integro-differential equations for the model given in Figure

4.18 can be extracted from the generalized Equations $(4.1) - (4.9)$ by setting $n = 3$. Thus,

the set of differential equations becomes

$$P_0'(t) + a_0 P_0(t) = \mu_{rl} P_1(t) + \mu_s P_2(t) + \sum_{k=6}^{7} \int_0^\infty p_k(x,t) \mu_{rk}(x) dx \qquad (4.138)$$

$$P_1'(t) + a_1 P_1(t) = \mu_{r2} P_2(t) + C_0 P_0(t) \qquad (4.139)$$

$$P_2'(t) + a_2 P_2(t) = C_1 P_1(t) \qquad (4.140)$$

$$P_3'(t) + a_3 P_3(t) = \lambda_s P_0(t) \qquad (4.141)$$

$$P_4'(t) + a_4 P_4(t) = C_0 P_3(t) + \lambda_s P_1(t) \qquad (4.142)$$

$$P_5'(t) + a_5 P_5(t) = C_1 P_4(t) + \lambda_s P_2(t) \qquad (4.143)$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_{rk}(x)\right] p_k(x,t) = 0 \qquad (for \quad k = 6, 7) \qquad (4.144)$$

where

$$a_0 = C_0 + \lambda_s$$
$$a_1 = C_1 + \lambda_s + \mu_{rl}$$
$$a_2 = C_2 + \lambda_s + \mu_{r2}$$
$$a_3 = C_0 + \mu_s$$
$$a_4 = C_1$$
$$a_5 = C_2$$
$$C_0 = 3\lambda_r, \quad C_1 = 2\lambda_r, \quad C_2 = \lambda_r$$

The associated boundary conditions are as follows:

$$p_6(0,t) = C_2 P_5(t) \qquad (4.145)$$

$$p_7(0,t) = C_2 P_2(t) \qquad (4.146)$$

At time $t = 0$, $P_0(0) = 1$, and all other initial state probabilities are equal to zero. The prime denotes differentiation with respect to time t.

4.5.1   Steady State Availability Analysis

As time approaches infinity, state probabilities reach the steady state. Inserting $n = 3$ into

the generalized Equations (4.25) – (4.28) developed in Section 4.3.1, we get the following

steady state probabilities:

$$P_i = \frac{\omega_i}{D} \quad (for \quad i = 0, 1, 2) \tag{4.147}$$

$$P_j = \frac{\omega_j}{D} \quad (for \quad j = 3, 4, 5) \tag{4.148}$$

$$P_k = \frac{\omega_k}{D} E_i[k] \quad (for \quad k = 6, 7) \tag{4.149}$$

$$D = \sum_{i=0}^{2} \omega_i + \sum_{j=3}^{5} \omega_j + \sum_{k=6}^{7} \omega_k E_k[x] \tag{4.150}$$

where

$$\omega_0 = a_3 a_4 a_5 (a_1 a_2 - \mu_{r2} C_1)$$
$$\omega_1 = a_2 a_3 a_4 a_5 C_0$$
$$\omega_2 = a_3 a_4 a_5 C_0 C_1$$
$$\omega_3 = a_4 a_5 \lambda_s (a_1 a_2 - \mu_{r2} C_1)$$
$$\omega_4 = a_5 \lambda_s C_0 (a_1 a_2 + a_2 a_3 - \mu_{r2} C_1)$$
$$\omega_5 = \lambda_s C_0 C_1 (a_1 a_2 + a_2 a_3 + a_3 a_4 - \mu_{r2} C_1)$$
$$\omega_6 = \lambda_s C_0 C_1 C_2 (a_1 a_2 + a_2 a_3 + a_3 a_4 - \mu_{r2} C_1)$$
$$\omega_7 = a_3 a_4 a_5 C_0 C_1 C_2$$

The robot system steady state availability with an operating safety unit is

$$SSAV_{rs} = \sum_{i=0}^{2} P_i = \sum_{i=0}^{2} \omega_i / D \tag{4.151}$$

Similarly, the steady state availability of the robot system with or without an operating

safety unit is expressed by

$$SSAV_r = \sum_{i=0}^{2} P_i + \sum_{j=3}^{5} P_j = \frac{\sum_{i=0}^{2} \omega_i + \sum_{j=3}^{5} \omega_j}{D} \tag{4.152}$$

For various robot system repair time distributions, the values of D [i.e., Equation (4.150)] are obtained as follows:

For the robot system repair time x, represented by a **gamma** distribution, we get

$$D = D_G = \sum_{i=0}^{2} \omega_i + \sum_{j=3}^{5} \omega_j + \sum_{k=6}^{7} \omega_k \beta / \mu_k \tag{4.153}$$

For $\beta = 1$ in Equation (4.153), the robot system repair time x is represented by an **exponential** distribution, therefore

$$D = D_E = \sum_{i=0}^{2} \omega_i + \sum_{j=3}^{5} \omega_j + \sum_{k=6}^{7} \omega_k / \mu_k \tag{4.154}$$

For the robot system repair time x, represented by a **Weibull** distribution, we get

$$D = D_W = \sum_{i=0}^{2} \omega_i + \sum_{j=3}^{5} \omega_j + \sum_{k=6}^{7} \omega_k \left(\frac{1}{\mu_k}\right)^{1/\beta} \frac{1}{\beta} \Gamma\left(\frac{1}{\beta}\right) \tag{4.155}$$

For $\beta = 2$ in Equation (4.155), the robot system repair time x is represented by a **Rayleigh** distribution, thus

$$D = D_R = \sum_{i=0}^{2} \omega_i + \sum_{j=3}^{5} \omega_j + \sum_{k=6}^{7} \omega_k \sqrt{\frac{\pi}{4\mu_k}} \tag{4.156}$$

For the robot system repair time x, represented by a **log-normal** distribution, we get

$$D = D_L = \sum_{i=0}^{2} \omega_i + \sum_{j=3}^{5} \omega_j + \sum_{k=6}^{7} \omega_k e^{\left(\mu_{y_k} + \frac{\sigma_{y_k}^2}{2}\right)} \tag{4.157}$$

where

$$\sigma_{y_k} = \ln \sqrt{1 + \left(\frac{\sigma_{x_k}}{\mu_{x_k}}\right)^2} \, , \qquad \mu_{y_k} = \ln \sqrt{\frac{\mu_{x_k}^4}{\mu_{x_k}^2 + \sigma_{x_k}^2}} \qquad (for \quad k = 6, 7)$$

4.5.1.1 *Steady State Availability Numerical Examples*

Setting:

$\lambda_s = \lambda_r = 0.0005,$
$\mu_s = 0.0006, \quad \mu_{r1} = \mu_{r2} = 0.0007, \quad \mu_{r6} = 0.0008, \quad \mu_{r7} = 0.0009$

into Equations (4.151) and (4.152) and performing numerical analysis, we can obtain robot-safety system steady state availability numerical values.

Figures 4.17 – 4.19 show plots of the robot-safety system steady state availability for gamma, Weibull, and log-normal distributions, respectively. These plots indicate the steady state availability as a function of safety unit (mechanism) failure rate, λ_s. The objective is to examine the robot system long term availability for different distributions' properties and compare them with those obtained in Section 4.4.1.

To inspect robot system repairability, Tables 4-13 to 4-15 present steady state availability for gamma, Weibull, and log-normal distributions, respectively. These tables indicate the steady state values as a function of safety unit (mechanism) repair rate, μ_s. Again, the objective is to examine the robot long term availability for different distributions' properties and compare them with those obtained in Section 4.4.1.

For the sake of comparison, all distributions are presented on the same figure. For all distributions, Figures 4.20 and 4.21 show plots of the robot-safety system steady state availability as a function of safety unit (mechanism) failure (λ_s) and repair (μ_s) rates, respectively. More detalized inspection of Figures 4.20 and 4.21 can be made by referring to their associated tabular values which are given in Tables 4-16 and 4-17, respectively.

(a)

(b)
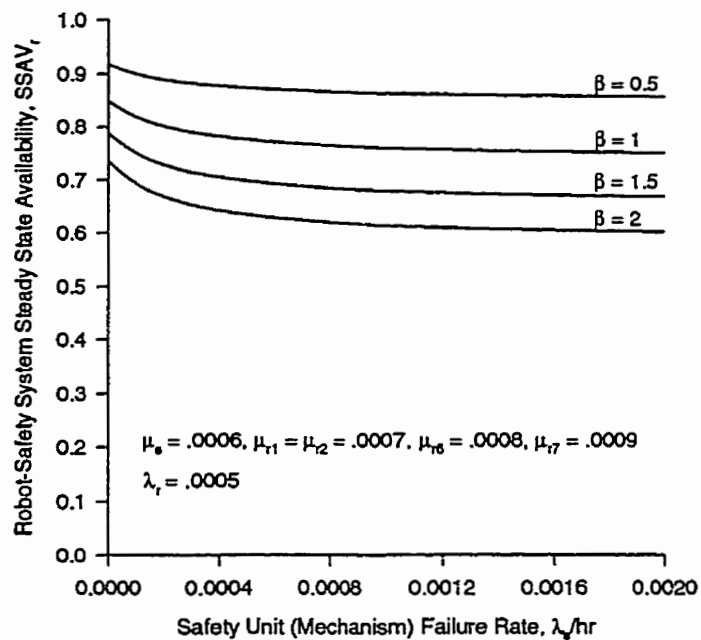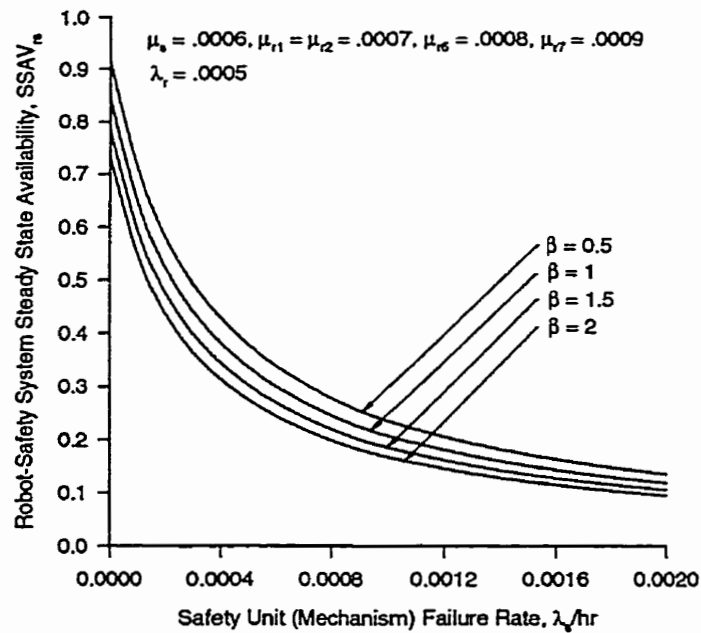
Figure 4.17.   Steady state availability ($n = 3$) vs λ_s plots for a robot system with constant failure rate and for **gamma** distributed failed system repair times; (a) robot system working with an operating safety unit (b) robot system working with or without the safety unit. More specifically, the plots were obtained using Equations (4.151) – (4.154).

(a)

(b)
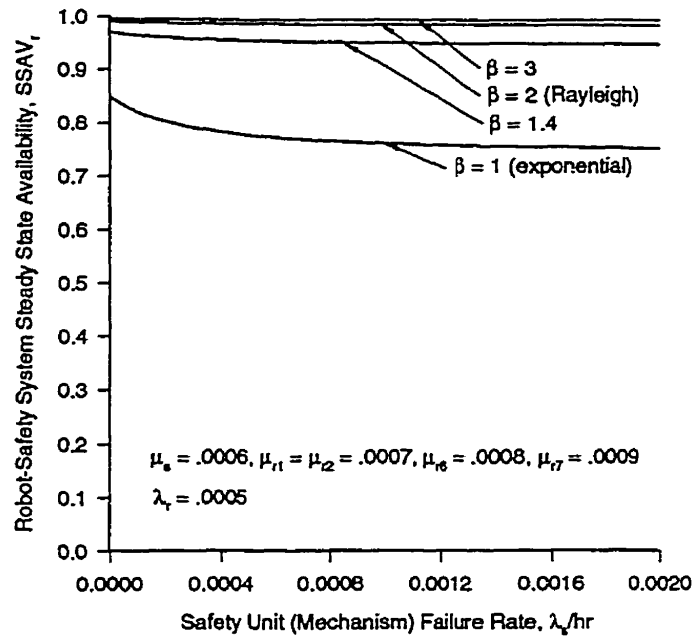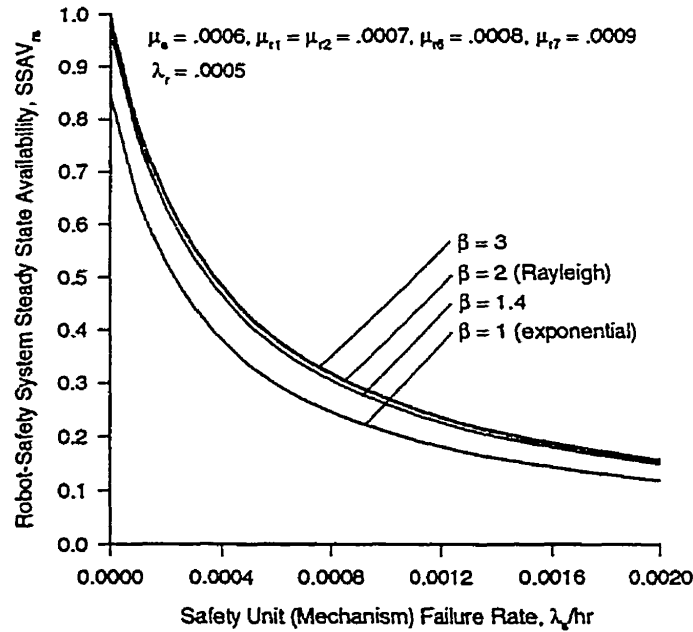
Figure 4.18.   Steady state availability ($n = 2$) vs λ_s plots for a robot system with constant failure rate and for **Weibull** distributed failed system repair times; (a) robot system working with an operating safety unit, (b) robot system working with or without the safety unit. More specifically, the plots were obtained using Equations (4.151), (4.152), (4.155), and (4.156).
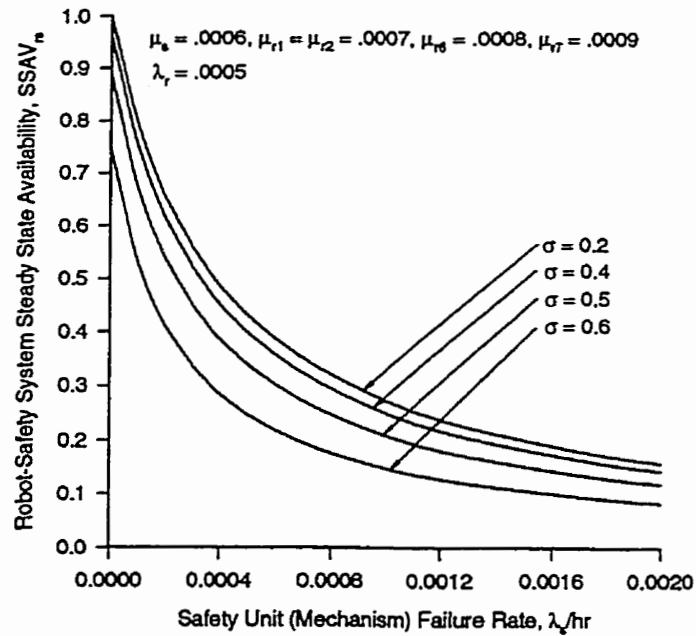
(a)
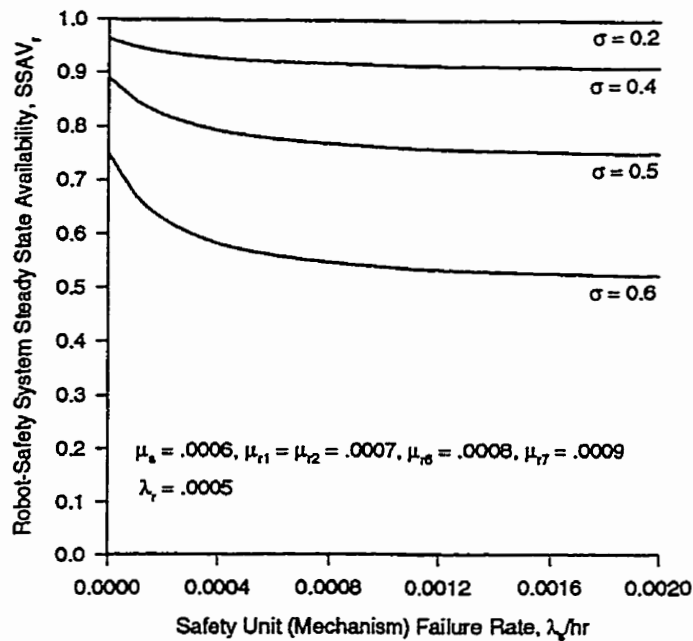
(b)

Figure 4.19.   Steady state availability ($n = 3$) vs λ_s plots for a robot system with constant failure rate and for **log-normal** distributed failed system repair times; (a) robot system working with an operating safety unit, (b) robot system working with or without the safety unit. More specifically, the plots were obtained using Equations (4.151), (4.152), and (4.157).

Table 4-13: SSAV ($n = 3$) vs μ_s values for a robot system with constant failure rate and for **gamma** distributed failed system repair times.

μ_s	$\beta = 0.5$		$\beta = 1$ (Exponential)		$\beta = 1.5$		$\beta = 2$	
	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$
0.0000	0.3438	0.8723	0.3049	0.7736	0.2739	0.6949	0.2486	0.6308
0.0005	0.3684	0.8736	0.3270	0.7756	0.2940	0.6974	0.2671	0.6335
0.0010	0.3863	0.8746	0.3433	0.7771	0.3089	0.6992	0.2807	0.6355
0.0015	0.4001	0.8753	0.3557	0.7782	0.3202	0.7006	0.2912	0.6370
0.0020	0.4197	0.8763	0.3735	0.7799	0.3365	0.7025	0.3061	0.6392

Table 4-14: SSAV ($n = 3$) vs μ_s values for a robot system with constant failure rate and for **Weibull** distributed failed system repair times.

μ_s	$\beta = 1$ (Exponential)		$\beta = 1.4$		$\beta = 2$ (Rayleigh)		$\beta = 3$	
	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$
0.0000	0.3049	0.7736	0.3758	0.9533	0.3884	0.9854	0.3915	0.9932
0.0005	0.3270	0.7756	0.4022	0.9538	0.4156	0.9856	0.4188	0.9932
0.0010	0.3433	0.7771	0.4215	0.9542	0.4354	0.9857	0.4388	0.9933
0.0015	0.3557	0.7782	0.4363	0.9545	0.4506	0.9858	0.4541	0.9934
0.0020	0.3735	0.7799	0.4573	0.9549	0.4722	0.9859	0.4758	0.9934

Table 4-15: SSAV ($n = 3$) vs μ_s values for a robot system with constant failure rate and for **log-normal** distributed failed system repair times.

μ_s	$\sigma = 0.2$		$\sigma = 0.4$		$\sigma = 0.5$		$\sigma = 0.6$	
	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$
0.0000	0.3930	0.9970	0.3627	0.9202	0.3070	0.7789	0.2219	0.5631
0.0005	0.4204	0.9971	0.3885	0.9213	0.3296	0.7817	0.2391	0.5671
0.0010	0.4405	0.9971	0.4074	0.9222	0.3462	0.7837	0.2518	0.5700
0.0015	0.4558	0.9971	0.4218	0.9228	0.3589	0.7852	0.2616	0.5723
0.0020	0.4776	0.9971	0.4424	0.9238	0.3772	0.7875	0.2757	0.5756

Table 4-16:   SSAV (n = 3) vs λ_s values for a robot system system with constant failure rate and for various failed system repair time distributions.

λ_s	Erlangian (gamma, β =2)		exponential (gamma, β =1)		Lognormal (σ = 0.4)		Weibull (β=1.2)		Rayleigh (Weibull, β=2)	
	$SSAV_n$	$SSAV_r$	$SSAV_n$	$SSAV_r$	$SSAV_n$	$SSAV_r$	$SSAV_n$	$SSAV_r$	$SSAV_n$	$SSAV_r$
.0000	.7357	.7357	.8477	.8477	.9628	.9628	.9784	.9784	.9952	.9952
.0004	.3140	.6426	.3823	.7824	.4521	.9254	.4728	.9678	.4852	.9930
.0008	.1990	.6199	.2457	.7653	.2938	.9152	.3097	.9647	.3186	.9923
.0012	.1457	.6104	.1809	.7581	.2174	.9109	.2299	.9634	.2368	.9920
.0016	.1150	.6054	.1432	.7542	.1726	.9087	.1828	.9627	.1884	.9919
0.002	.0950	.6025	.1186	.7520	.1431	.9074	.1517	.9623	.1564	.9918

System parameters: λ_r = 0.0005,

μ_s = 0.0006, μ_{r1} = μ_{r2} = 0.0007, μ_{r6} = 0.0008, μ_{r7} = 0.0009

Table 4-17:   SSAV (n = 3) vs μ_s values for a robot system with constant failure rate and for various failed system repair time distributions.

μ_s	Erlangian (gamma, β=2)		exponential (gamma, β =1)		Lognormal (σ = 0.4)		Weibull (β=1.2)		Rayleigh (Weibull, β=2)	
	$SSAV_n$	$SSAV_r$	$SSAV_n$	$SSAV_r$	$SSAV_n$	$SSAV_r$	$SSAV_n$	$SSAV_r$	$SSAV_n$	$SSAV_r$
.0000	.2486	.6308	.3049	.7736	.3627	.9202	.3808	.9662	.3912	.9926
.0004	.2671	.6335	.3270	.7756	.3885	.9213	.4076	.9666	.4186	.9927
.0008	.2807	.6355	.3433	.7771	.4074	.9222	.4271	.9668	.4386	.9928
.0012	.2912	.6370	.3557	.7782	.4218	.9228	.4420	.9670	.4538	.9928
.0016	.2994	.6382	.3655	.7791	.4332	.9233	.4538	.9672	.4658	.9929
.0020	.3061	.6392	.3735	.7799	.4424	.9238	.4633	.9673	.4755	.9929

System parameters: λ_s = λ_r = 0.0005,

μ_{r1} = μ_{r2} = 0.0007, μ_{r6} = 0.0008, μ_{r7} = 0.0009
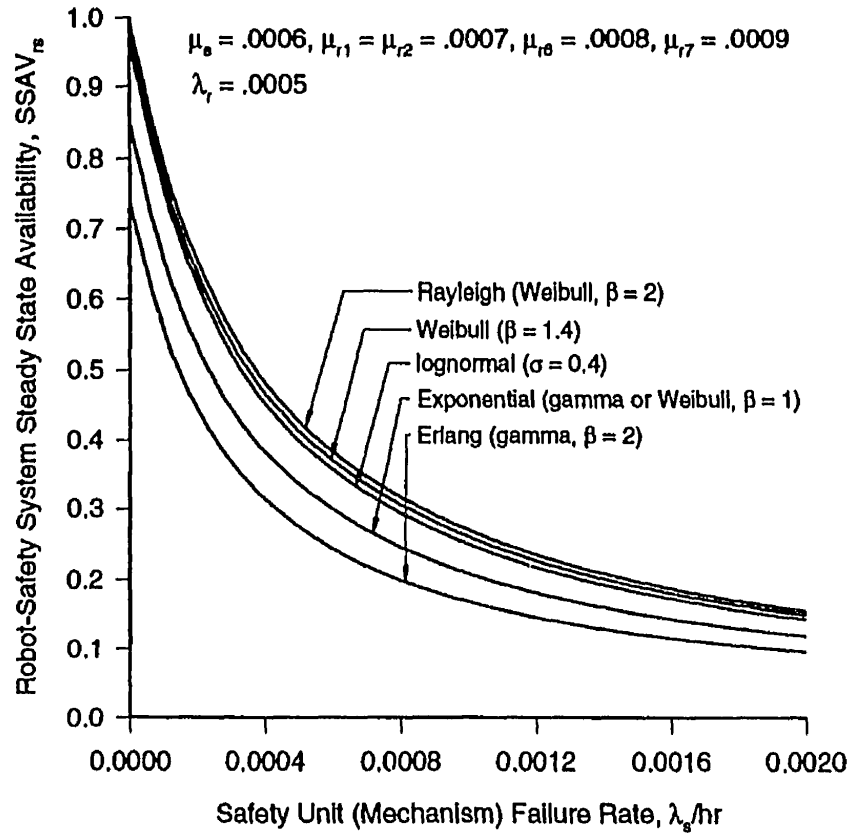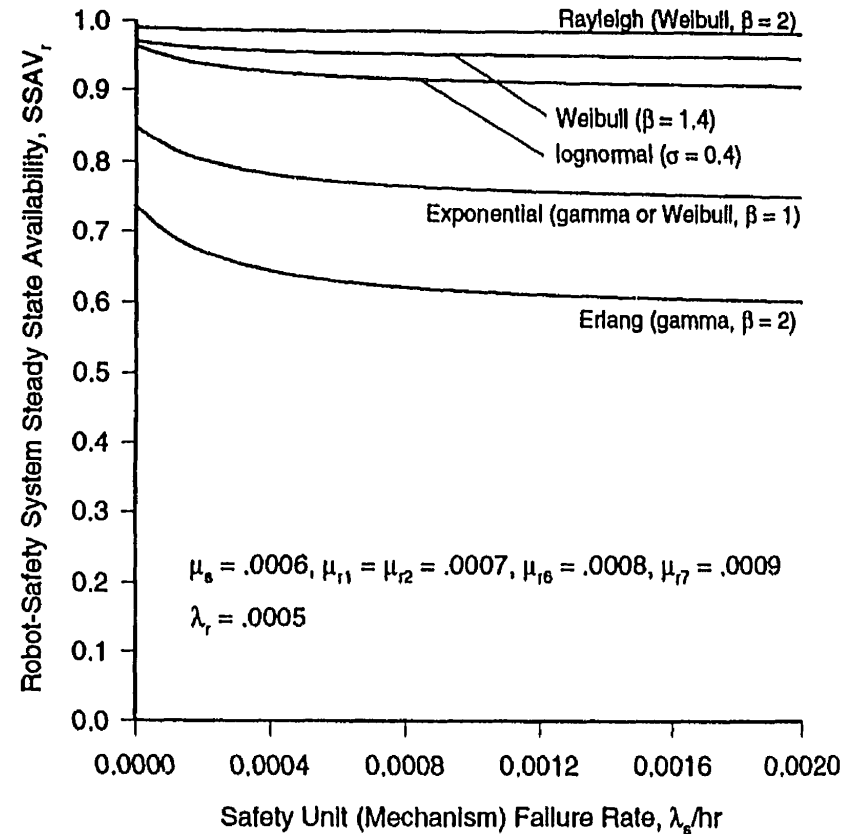
(a)   (b)

Figure 4.20.   Steady state availability ($n = 3$) vs λ_s plots for a robot system with constant failure rate and for various failed system repair time distributions when (a) robot system working with an operating safety unit, (b) robot system working with or without the safety unit. More specifically, the plots were obtained using Equations (4.151) – (4.157).
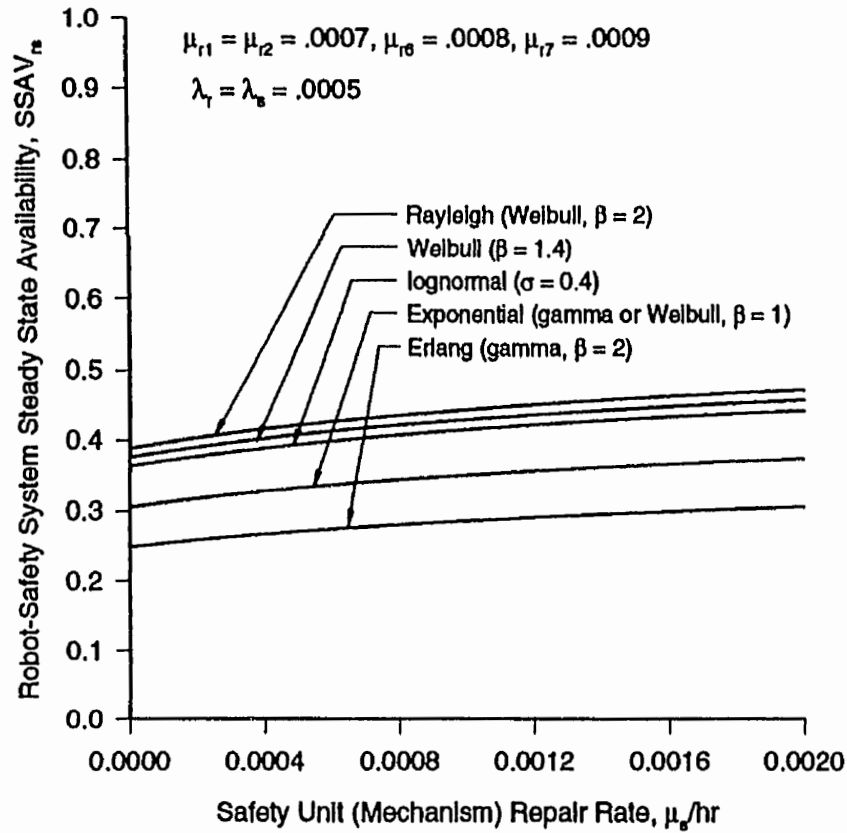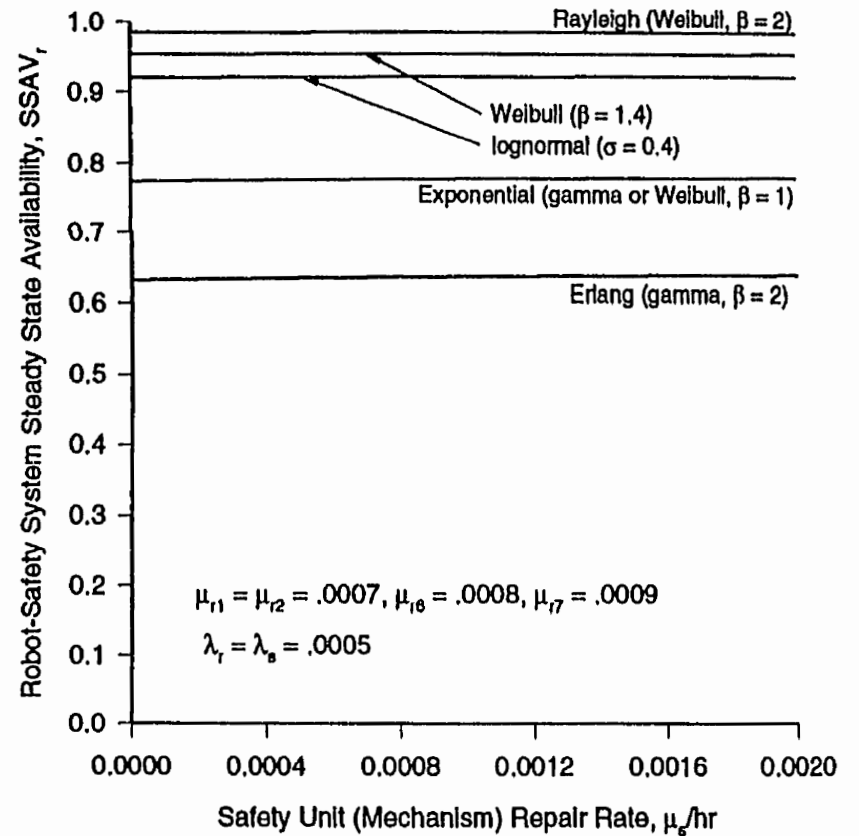
Figure 4.21.   Steady state availability ($n = 3$) vs μ_s plots for a robot system with constant failure rate and for various failed system repair time distributions when (a) robot system working with an operating safety unit, (b) robot system working with or without the safety unit. More specifically, the plots were obtained using Equations (4.151) – (4.157).

4.5.2   Time Dependent Availability Analysis

Substituting $n = 3$ into the generalized Equations (4.58) – (4.61) developed in Section 4.3.2,

we obtain the following Laplace transforms of the robot system state probability expressions

with constant repair rate:

$$P_i(s) = \frac{\alpha_i(s)}{s \cdot B(s)} \qquad (for \quad i = 0, 1, 2) \tag{4.158}$$

$$P_j(s) = \frac{\alpha_j(s)}{s \cdot B(s)} \qquad (for \quad j = 3, 4, 5) \tag{4.159}$$

$$P_k(s) = \frac{\alpha_k(s)}{(s + \mu_k)B(s)} \qquad (for \quad k = 6, 7) \tag{4.160}$$

$$B(s) = \sum_{i=0}^{2} \alpha_i(s) + \sum_{j=3}^{5} \alpha_j(s) + \sum_{k=6}^{7} \frac{s \cdot \alpha_k(s)}{(s + \mu_k)} \tag{4.161}$$

where

$$\alpha_0(s) = s(s + a_1)(s + a_2)(s + a_3)$$
$$\alpha_1(s) = s(s + a_2)(s + a_3)C_0$$
$$\alpha_2(s) = s(s + a_1)(s + a_3)\lambda_r$$
$$\alpha_3(s) = s(2s + a_1 + a_2)\lambda_r C_0$$
$$\alpha_4(s) = (2s + a_1 + a_2)\lambda_r C_0 C_1$$
$$\alpha_5(s) = (s + a_2)(s + a_3)C_0 C_1$$

Using Equation (4.158), the Laplace transform of the robot system availability with an

operating safety unit is given by

$$AV_{rs}(s) = \sum_{i=0}^{2} P_i(s) = \frac{\sum\limits_{i=0}^{2} \alpha_i(s)}{s\left(\sum\limits_{i=0}^{2} \alpha_i(s) + \sum\limits_{j=3}^{5} \alpha_j(s) + \sum\limits_{k=6}^{7} \frac{s \cdot \alpha_k(s)}{(s + \mu_k)} \right)} \tag{4.162}$$

Similarly, using Equations (4.158) and (4.159), the Laplace transform of the robot system

availability with or without a working safety unit is expressed by

$$
\begin{aligned}
AV_r(s) &= \sum_{i=0}^{2} P_i(s) + \sum_{j=3}^{5} P_j(s) \\
&= \frac{\displaystyle\sum_{i=0}^{2} \alpha_i(s) + \sum_{j=3}^{5} \alpha_j(s)}{s\left(\displaystyle\sum_{i=0}^{2} \alpha_i(s) + \sum_{j=3}^{5} \alpha_j(s) + \sum_{k=6}^{7} \frac{s\cdot\alpha_k(s)}{(s+\mu_k)}\right)}
\end{aligned}
\tag{4.163}
$$

Inserting $n = 3$ into the generalized Equations (4.65) – (4.68) developed in Section 4.3.2,

we obtain the following Laplace transforms of the robot system probability expressions with

non-constant repair rate:

$$
P_i(s) = \frac{\alpha_i(s)}{s\cdot B(s)} \qquad (for \quad i = 0, 1, 2)
\tag{4.164}
$$

$$
P_j(s) = \frac{\alpha_j(s)}{s\cdot B(s)} \qquad (for \quad j = 3, 4, 5)
\tag{4.165}
$$

$$
P_k(s) = \frac{s\cdot\alpha_k(s)}{(s+\mu_k)^2 B(s)} \qquad (for \quad k = 6, 7)
\tag{4.166}
$$

$$
B(s) = \sum_{i=0}^{2} \alpha_i(s) + \sum_{j=3}^{5} \alpha_j(s) + \sum_{k=6}^{7} \frac{s^2\cdot\alpha_k(s)}{(s+\mu_k)^2}
\tag{4.165}
$$

Using Equation (4.164), the Laplace transform of the robot system availability with an

operating safety unit is

$$
AV_{rs}(s) = \sum_{i=0}^{2} P_i(s) = \frac{\displaystyle\sum_{i=0}^{2} \alpha_i(s)}{s\left(\displaystyle\sum_{i=0}^{2} \alpha_i(s) + \sum_{j=3}^{5} \alpha_j(s) + \sum_{k=6}^{7} \frac{s^2\cdot\alpha_k(s)}{(s+\mu_k)^2}\right)}
\tag{4.168}
$$

Similarly, using Equations (4.164) and (4.165), the Laplace transform of the robot system

availability with or without a working safety unit is expressed by

$$
AV_r(s) = \sum_{i=0}^{2} P_i(s) + \sum_{j=3}^{5} P_j(s)
$$

$$
= \frac{\displaystyle\sum_{i=0}^{2} \alpha_i(s) + \sum_{j=3}^{5} \alpha_j(s)}{s\left(\displaystyle\sum_{i=0}^{2} \alpha_i(s) + \sum_{j=3}^{5} \alpha_j(s) + \sum_{k=6}^{7} \frac{s^2 \cdot \alpha_k(s)}{(s + \mu_k)^2}\right)}
$$

(4.169)

4.5.2.1 *Time-Dependent Availability Numerical Examples*

Numerical probability and availability expressions can be obtained for the robot system with

constant (i.e., $\beta = 1$) and non-constant (i.e., $\beta = 2$) repair rates by setting:

$\lambda_s = \lambda_r = 0.0005,$
$\mu_s = 0.0006, \quad \mu_{r1} = \mu_{r2} = 0.0007, \quad \mu_{r6} = 0.0008, \quad \mu_{r7} = 0.0009$

in Equations (4.158) − (4.163) and (4.164) − (4.167), respectively, and taking inverse

Laplace transform of the resulting equations. For $\beta = 1$ time-dependent availability, $AV_{rs}(t)$

(with the safety unit working) and $AV_r(t)$ (with or without an operating the safety unit) are

expressed by

$$
AV_{rs}(t) = 0.4365 - 0.1583\, e^{k_1 t} + 0.2584\, e^{k_2 t} - 0.0581\, e^{k_3 t} + 0.1745\, e^{k_4 t} +
$$
$$
0.0004\, e^{k_5 t} + e^{k_6 t}\left(0.3466\, Cos\,(k_7 t) + 0.1872\, Sin\,(k_7 t)\right)
$$

(4.170)

and

$$
AV_r(t) = 0.8247 + 0.4615\, e^{k_1 t} - 0.6165\, e^{k_2 t} + 0.0418\, e^{k_3 t} + 0.2369\, e^{k_4 t} -
$$
$$
0.0009\, e^{k_5 t} + e^{k_6 t}\left(0.0524\, Cos\,(k_7 t) + 0.3275\, Sin\,(k_7 t)\right)
$$

(4.171)

where

$$k_1 = -0.0029$$
$$k_2 = -0.0027$$
$$k_3 = -0.0019$$
$$k_4 = -0.0013$$
$$k_5 = -0.0009$$
$$k_6 = -0.0007$$
$$k_7 = 0.0004$$

Similarly, if the failed robot system repair rate is non-constant (i.e., $\beta = 2$), then, numerical availability expressions become

$$
\begin{aligned}
AV_{rs}(t) &= 0.3714 - 0.0306\, e^{k_1 t} - 0.0001\, e^{k_2 t} - 3.91 * 10^{-10} e^{k_3 t} + 0.0001\, e^{k_4 t} + \\
&\quad e^{k_5 t}\big(0.1449\, Cos\,(k_6 t) + 0.2323\, Sin\,(k_6 t)\big) + \\
&\quad e^{k_7 t}\big(0.1596\, Cos\,(k_8 t) + 0.1585\, Sin\,(k_8 t)\big) + \\
&\quad e^{k_9 t}\big(0.3547\, Cos\,(k_{10} t) + 0.1696\, Sin\,(k_{10} t)\big)
\end{aligned}
\tag{4.172}
$$

and

$$
\begin{aligned}
AV_r(t) &= 0.0.7017 + 0.0144\, e^{k_1 t} + 0.0005\, e^{k_2 t} - 1.3 * 10^{-9} e^{k_3 t} - 0.0001\, e^{k_4 t} - \\
&\quad e^{k_5 t}\big(0.0331\, Cos\,(k_6 t) + 0.1695\, Sin\,(k_6 t)\big) + \\
&\quad e^{k_7 t}\big(0.1505\, Cos\,(k_8 t) + 0.1139\, Sin\,(k_8 t)\big) + \\
&\quad e^{k_9 t}\big((0.1656\, Cos\,(k_{10} t) + 0.4256\, Sin\,(k_{10} t)\big)
\end{aligned}
\tag{4.173}
$$

where

$$k_1 = -0.0019, \quad k_2 = -0.0009, \quad k_3 = -0.0016$$
$$k_4 = -0.0009, \quad k_5 = -0.0029, \quad k_6 = 0.0004$$
$$k_7 = -0.0011, \quad k_8 = 0.0004, \quad k_9 = 0.0005$$
$$k_{10} = 0.0005$$

Using Equations (4.170) – (4.173), time-dependent availability plots for $\beta = 1$ and $\beta = 2$

are shown in Figures 4.22 and 4.23, respectively. More detailed inspection of the robot

system availability can be made by referring to Tables 4-18 and 4-19, respectively.

Table 4-18:   Time-dependent availability ($n = 3$) values for a robot system with constant failure and repair ($\beta = 1$) rates.

Time (Hrs)	Time-Dependent Availability ($\beta = 1$)	
	$AV_{rs}(t)$	$AV_r(t)$
0	1	1
500	0.8052	0.9925
1000	0.6826	0.9688
1500	0.5996	0.9399
2000	0.5419	0.9130
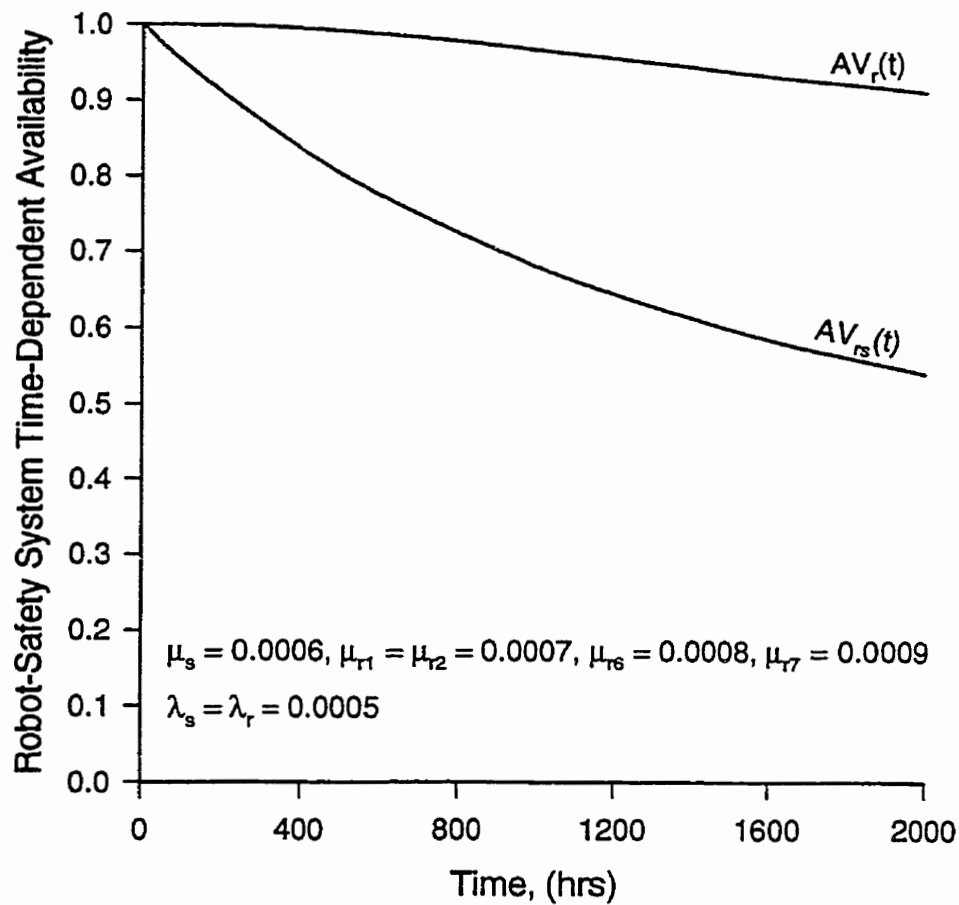
Figure 4.22.   Availability ($n = 3$) plots for a robot system with constant failure and repair rates. More specifically, the plots were obtained using Equations (4.170) and (4.171).

Table 4-19:   Time-dependent availability ($n = 3$) values for a robot system with constant failure rate and **gamma** distributed ($\beta = 2$) failed system repair time.

Time (Hrs)	Time-Dependent Availability ($\beta = 2$)	
	$AV_{rs}(t)$	$AV_r(t)$
0	1	1
500	0.8044	0.9917
1000	0.6758	0.9613
1500	0.5816	0.9188
2000	0.5101	0.8738
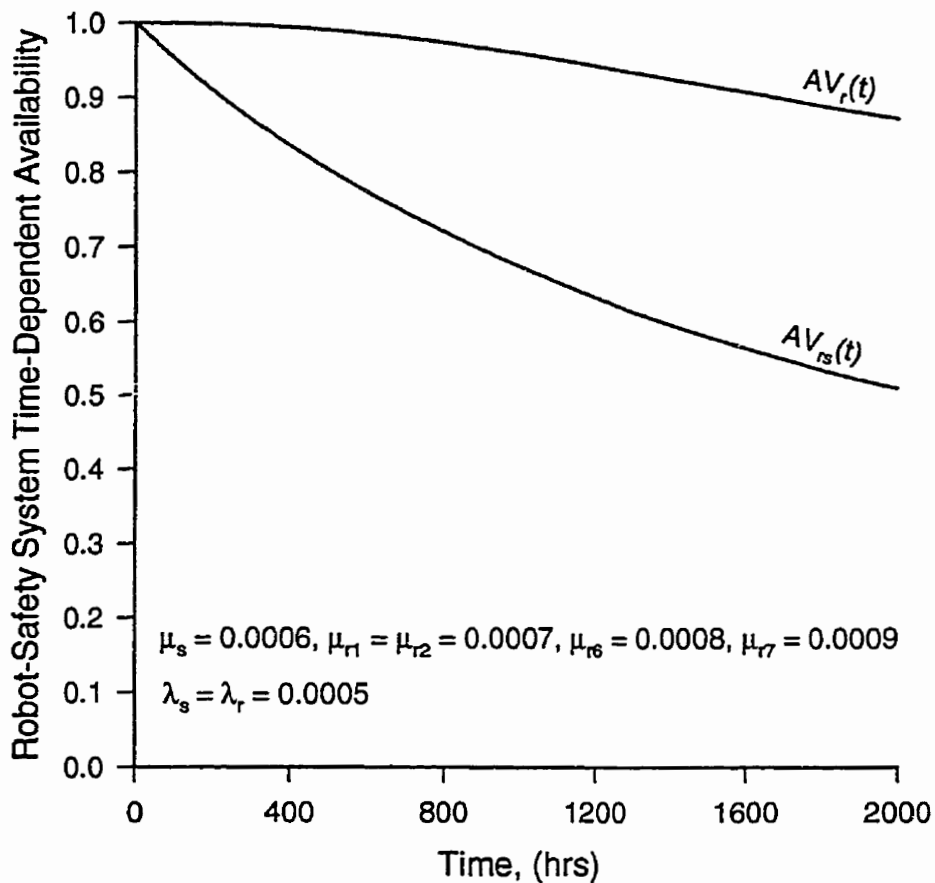
Figure 4.23.   Availability ($n = 3$) plots for a robot system with constant failure rate and for **gamma** distributed ($\beta = 2$) failed system repair time. More specifically, the plots were obtained using Equations (4.172) and (4.173).

4.5.3   Robot System Reliability and MTTF

Inserting $n = 3$ into the generalized Equations (4.79) – (4.82) developed in Section 4.3.3,

we obtain the following Laplace transforms of the state probabilities.

$$P_i(s) = \frac{\gamma_i(s)}{s \cdot Z(s)} \qquad (for \quad i = 0, 1, 2) \tag{4.174}$$

$$P_j(s) = \frac{\gamma_j(s)}{s \cdot Z(s)} \qquad (for \quad j = 3, 4, 5) \tag{4.175}$$

$$P_k(s) = \frac{\gamma_k(s)}{s \cdot Z(s)} \qquad (for \quad k = 6, 7) \tag{4.176}$$

$$Z(s) = \sum_{i=0}^{2} \gamma_i(s) + \sum_{j=3}^{7} \gamma_j(s) \tag{4.177}$$

where

$$\gamma_0(s) = [M(s)\Pi_{i=3}^{5}(s + a_i)]s$$

$$\gamma_1(s) = [C_0\Pi_{i=2}^{5}(s + a_i)]s$$

$$\gamma_2(s) = [C_0 C_1 \Pi_{i=3}^{5}(s + a_i)]s$$

$$\gamma_3(s) = [\lambda_j M(s)\Pi_{i=4}^{5}(s + a_i)]s$$

$$\gamma_4(s) = [\lambda_j M(s)C_0(s + a_5) + \lambda_s C_0(s + a_2)(s + a_3)(s + a_5)]s$$

$$\gamma_5(s) = [\lambda_j M(s)C_0 C_1 + \lambda_s C_0 C_1 \Pi_{i=2}^{3}(s + a_i) + \lambda_s C_0 C_1 \Pi_{i=3}^{4}(s + a_i)]s$$

$$\gamma_6(s) = C_2 \gamma_5(s)/s$$

$$\gamma_7(s) = C_2 \gamma_2(s)/s$$

$$M(s) = \Pi_{i=1}^{2}(s + a_i) - \mu_{r2}C_1$$

Using Equation (4.174), the Laplace transform reliability of an irreparable robot system with

an operating safety unit is

$$R_{rs}(s) = \sum_{i=0}^{2} P_i(s) = \frac{\displaystyle\sum_{i=0}^{2} \gamma_i(s)}{s\left(\displaystyle\sum_{i=0}^{2} \gamma_i(s) + \sum_{j=3}^{5} \gamma_j(s) + \sum_{k=6}^{7} \gamma_k(s)\right)} \tag{4.178}$$

Similarly, the Laplace transform reliability of an irreparable robot system with or without an operating safety unit is expressed by

$$R_r(s) = \sum_{i-0}^{2} P_i(s) + \sum_{j-3}^{5} P_j(s) = \frac{\sum\limits_{i-0}^{2} \gamma_i(s) + \sum\limits_{j-3}^{5} \gamma_j(s)}{s\left(\sum\limits_{i-0}^{2} \gamma_i(s) + \sum\limits_{j-3}^{7} \gamma_j(s)\right)} \tag{4.179}$$

Using Equations (4.178) and (4.179), the robot system mean time to failures are

$$MTTF_{rs} = \lim_{s \to 0} R_{rs}(s) = \frac{\sum\limits_{i-0}^{2} \gamma_i}{\sum\limits_{k-6}^{7} \gamma_k} \tag{4.180}$$

and

$$MTTF_r = \lim_{s \to 0} R_r(s) = \frac{\sum\limits_{i-0}^{2} \gamma_i + \sum\limits_{j-3}^{5} \gamma_j}{\sum\limits_{k-6}^{7} \gamma_k} \tag{4.181}$$

where

$$\gamma_0 = a_3 a_4 a_5 M$$
$$\gamma_1 = a_2 a_3 a_4 a_5 C_0$$
$$\gamma_2 = a_3 a_4 a_5 C_0 C_1$$
$$\gamma_3 = a_4 a_5 \lambda_s M$$
$$\gamma_4 = a_5 \lambda_s C_0 M + a_2 a_3 a_5 \lambda_s C_0$$
$$\gamma_5 = \lambda_s C_0 C_1 M + a_2 a_3 \lambda_s C_0 C_1 + a_3 a_4 \lambda_s C_0 C_1$$
$$\gamma_6 = C_2 \gamma_5$$
$$\gamma_7 = C_2 \gamma_2$$
$$M = a_1 a_2 - \mu_{r2} C_1$$

4.5.3.1 *Reliability and MTTF Numerical Examples*

Setting:

$\lambda_s = \lambda_r = 0.0005$,
$\mu_s = 0.0006$, $\mu_{rl} = \mu_{rl} = 0.0007$

in Equations (4.174) – (4.177) and inverting the resulting equations, numerical time-dependent probability expressions can be obtained for the robot system.

Inserting the above parameter values into Equations (4.178) and (4.179), and inverting the results, Figure 4.24 shows robot system reliability plots for various values of safety unit repair rates.

Also, using Equations (4.180) and (4.181), robot system mean time to failure plots are shown in Figure 4.25.

More detailed inspection of the robot system reliability and MTTF can be made by referring to their associated values tabulated in Tables 4-20 to 4-21, respectively.

Table 4-20:   Reliability ($n = 3$) values of an irreparable robot system with various safety unit repair rates' specified parameter values.

Time (hrs)	$\mu_s = 0.0006, \mu_{r1} = \mu_{r2} = 0.0007$		$\mu_s = \mu_{r1} = \mu_{r2} = 0$	
	$R_{rs}(t)$	$R_r(t)$	$R_{rs}(t)$	$R_r(t)$
0	1	1	1	1
500	0.7951	0.9916	0.7761	0.9904
1000	0.6702	0.9596	0.6415	0.9506
1500	0.5735	0.9111	0.5457	0.8887
2000	0.4921	0.8529	0.4698	0.8170

Table 4-21:   MTTF ($n = 3$) values of an irreparable robot system as a function of safety unit failure and repair rates.

Failure rate, λ_s	Mean Time To Failure		Repair rate, μ_s	Mean Time To Failure	
	$MTTF_{rs}$	$MTTF_r$		$MTTF_{rs}$	$MTTF_r$
0.0000	6186.7	6186.7	0.0000	1650.7	4188.2
0.0004	2136.8	4373.5	0.0004	1783.6	4230.2
0.0008	1292.7	4026.6	0.0008	1882.5	4261.4
0.0012	928.4	3889.8	0.0012	1958.9	4285.5
0.0016	725.4	3820.0	0.0016	2019.7	4304.7
0.0020	595.9	3779.1	0.0020	2069.2	4320.4

(a)

(b)
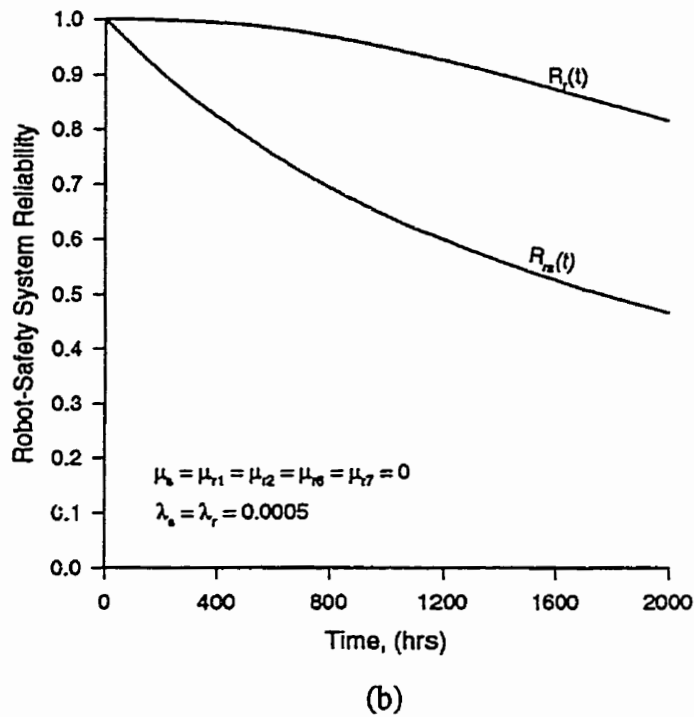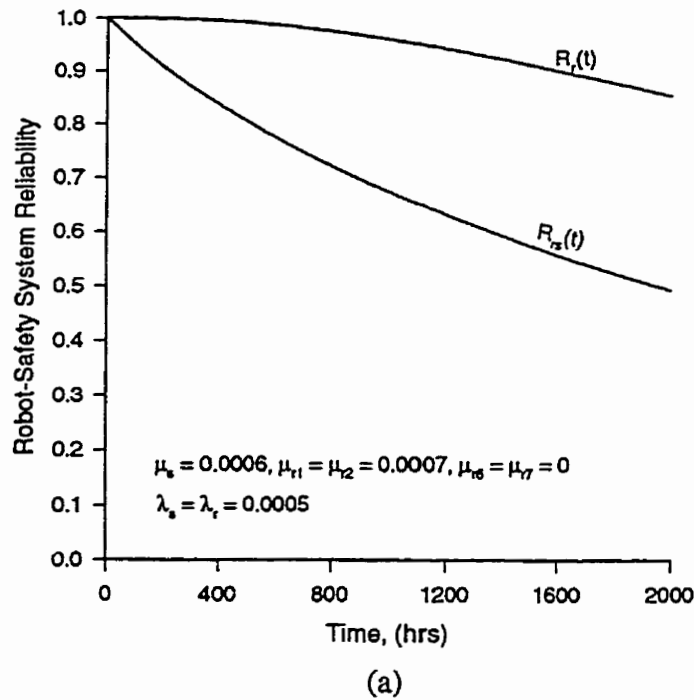
Figure 4.24.   Reliability ($n = 3$) plots of an irreparable robot system with various repair conditions: (a) $\mu_s = 0.0006$, $\mu_{r1} = \mu_{r2} = 0.0007$, (b) $\mu_s = \mu_{r1} = \mu_{r2} = \mu_{r6} = \mu_{r7} = 0$. More specifically, the plots were obtained using Equations (4.178) and (4.179).
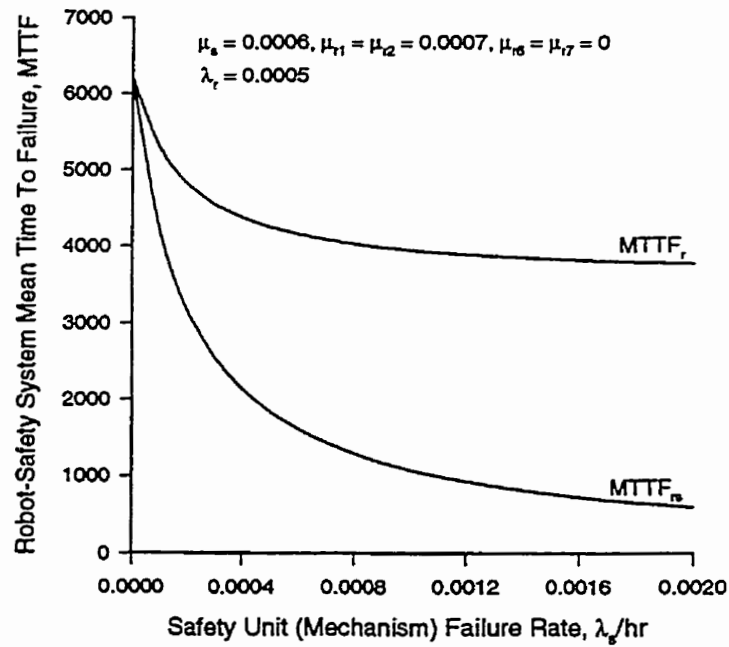
(a)
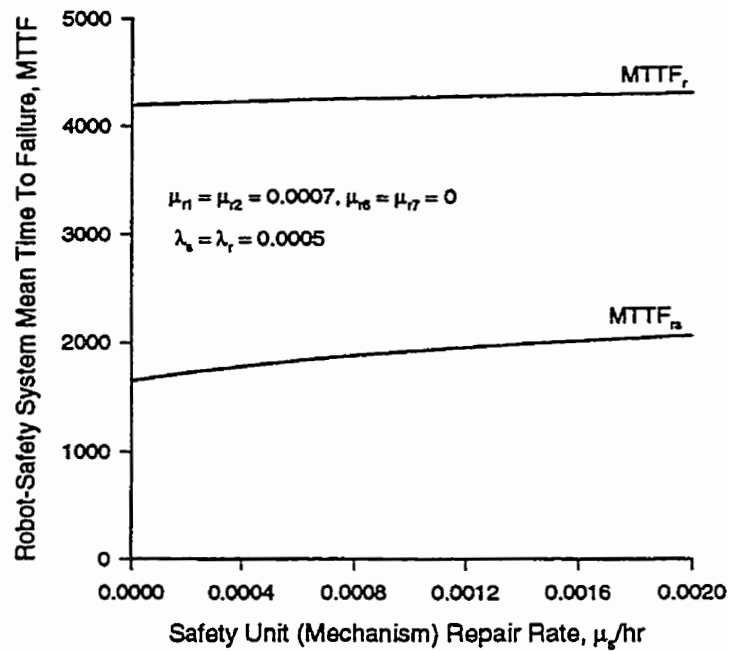
(b)

Figure 4.25.   MTTF ($n = 3$) plots of an irreparable robot system as a function of safety unit failure and repair rates. More specifically, the plots were obtained using Equations (4.180) and (4.181).

4.6 Discussion and Conclusions

The relation between safety and reliability can be very complex. The interests of reliability and safety coincide in preventing equipment failures, for when there are no failures no production time is lost and no public risk is caused. Reliability and safety however, may sometimes come into conflict. From a safety point of view design and procedures may be tilted toward shutting down the robot(s) at the first sign of trouble so that risk is minimized. From a reliability standpoint, however, management may be inclined to keep the robot(s) at pace and wait until actual failure before shutting down, or to make repairs whenever possible while the robot(s) are on line. Criteria must be carefully worked out for risk management so that the public is protected, and so that the robot(s) can be operated without frequent and undesired shutdowns.

In the model presented in this chapter, the robots were assumed to operate independently which meant they may continue working with or without the safety unit operating (i.e., working or failed). A failed robot however, may be repaired only when the safety unit is functioning. Certainly, other repair scenarios for the robots and its safety unit can be rationalized, this entirely depends on the objectives of the operation. For the purposes of this study, the objective of this model was restricted to maximize robots' productivity and to the certain extent ensure safety.

Thus, this chapter presented various mathematical models to investigate performance indices for a system comprised of redundant robots operating with one safety unit. A generalized model (Figure 4.2 on page 147) was developed illustrating the state space transition diagram of n-identical robots operating with one safety unit. With the aid of the

supplementary variables technique, generalized steady state and time-dependent availability expressions were developed. Using Laplace transforms, generalized reliability and mean time to failure (MTTF) were also developed for an irreparable robot system.

To examine robot system performance indices, mathematical analysis were performed for two special case models. For $n = 2$, the system consisted of two robots with one safety unit, and for $n = 3$, the system consisted of three robots with one safety unit. Availability, reliability, and MTTF expressions were obtained for these special case and were validated by means of numerical analysis.

For the purpose of comparison, failure and repair parameter values used in this chapter were kept similar to those used in Chapter 3. Again, the selected repair parameter values may yield unrealisticly high mean time to system repair (i.e., over 1000 hours). But, the main purpose here was to demonstrate the validity of the resulting expressions rather than to give real life values to these parameters. However, Appendix E presents steady state and time dependent availability plots for more realistic practical repair parameter values (i.e., mean time to repair of 8 hours).

The following conclusions are associated with the models in this chapter.

1. When the failed robot system repair time x was represented by a gamma distribution, the steady state availability decreased as the shape parameter β, increased. This trend was evident in both special case models.

2. When the failed robot system repair time x was represented by a Weibull distribution, the steady state availability increased as the shape parameter, β, increased. This trend was evident in both special case models.

3. When the failed robot system repair time x was represented by a log normal distribution, the steady state availability dropped as the standard deviation, σ, increased. This trend was evident in both special case models.

4. For all failed system repair distributions, robot system steady state availability decreased as the safety unit failure rate increased. This trend was evident in both special case models.

5. For all failed system repair distributions, robot system steady state availability increased as the safety unit repair rate increased. This trend was evident in both special case models.

6. The Weibull distributed failed system repair time displayed the highest values of system steady state availability while the Erlangian distributed failed system repair time produced the least values for the system steady state availability.

7. If robot system performance indices is correlated with a working safety unit, then its steady state availability, time dependent availability, reliability, and mean time to failure decreased as the number of robots increased. For example, system availability decreased by about 15% when three robots were employed instead of two.

8. If the robot system performance indices is independent of its safety unit operating conditions, then its steady state availability, time dependent availability, reliability, and mean time to failure value marginally increased as the number of robots increased. For example, system availability increased by about 5% when three robots were employed instead of two.

5

STOCHASTIC ANALYSIS OF ROBOT SYSTEMS WITH NON-CONSTANT FAILURE RATES

5.1 Introduction

In the models presented in Chapters 3 and 4, safety unit failure and repair rates were assumed to be constant. The exponential distribution is usually employed to represent the life time of electronic parts. Since the majority of the items that make up a safety unit are electronic parts, this assumption may well be suited. Although for many systems, assuming a constant failure rate is legitimate and simplifies computations, we unfortunately cannot verify how valid a constant-hazard model is unless the actual failure data is available. For example, robots are multi-disciplined systems which make use of mechanical, electronic, hydraulic, and pneumatic parts. Many factors could affect their failure rates including components durability, wear & tear, software or hardware failures, correct use of the robot,

environment where the robot is operating in, etc... Thus, all of the potential sources of failures make constant failure rate assumption impractical, and one should consider a non-constant failure rate. Furthermore, robots multiplicity and their utilization in sometimes hostile environments contribute to a phenomena commonly known as common-cause failures. Broadly stated, a common-cause failure may be defined as [107] multiple items malfunction due to a single cause. Examples of causes of common-cause failure include loss of power, design errors, irregular operating environment such as high (or low) temperatures and humidity, catastrophic external events such as floods, fires, and earthquakes.

This chapter presents a study of robot systems having non-constant failure rates. A method generally known as the *device of stages* or simply the *stages* method is used to deal with systems having non-constant failure rates. This method is described in Appendix *C* [416].

Basically, the chapter is divided into three sections. A robot system with *m* failure modes is modeled in Section 5.2. Using the stages method and the supplementary variables technique, generalized steady state availability expressions are obtained for a robot system with non-constant failure and repair rates. To validate the applicability of the stages method to this model, Markov method is also used to obtain the generalized steady state availability expressions and the results are compared with those obtained with the stages method.

Once the applicability of the stages method is validated, the method is used in the special cases of the generalized models presented in Chapters 3 and 4. Sections 5.3 and 5.5 present analyses for a system containing one robot with two safety units and a system containing two robots with one safety unit, respectively. These models are similar to those presented

in Sections 3.5 and 4.4, except that non-constant common-cause failure is added to the overall robot systems failed states. This adds an important dimension to the overall failure definition of the robot systems. Using the supplementary variables and the stages method, steady state availability expressions are developed and the results are demonstrated by means of plots. The applicability of the stages method to the models in Sections 5.3 and 5.4 are also verified by using the Markov method.

5.2 Robot Systems With Non-Constant Failure Rates

When the time to failure of a robot is assumed exponentially distributed, then the Markov method is quite useful to obtain performance indices. However, when the failure rate of a robot becomes non-constant, Markov method is no longer valid and the process becomes non-Markovian. There are not many methods available which deal with systems with non-constant failure rates. In Chapters 3 and 4, the method of supplementary variables was used to deal with models having arbitrary repair rates. Unfortunately, this method is not applicable in the robot's operating states. Block diagrams and the joint density function method may be suitable techniques but their practicality diminish as the systems' complexity increase.

Another approach to work with the non-constant failure or repair rates is called the device of stages. The basic principle surrounding the stages method is based on the assumption that the time till transition from the system's operating state to the system's failed state is the sum of infinite number of exponential distributions. The sum of exponentially distributed random variables arise naturally in the theory of Markov chains, as the time is

required to make several transitions. This mechanism, through insertion of dummy stages, can be used to generate non-exponential transition times.

The state space transition diagram of a robot system with m failure modes is shown in Figure 5.1. At time $t = 0$, the robot system begins operating and may fail due to m sources of failures. The failure times are the summation of n independent exponentially distributed random variables represented by dummy or sub-states (stages). The purpose of the dummy states is to extend the application of ordinary Markov processes so as to include non-exponential distributions of time to failure. The distributions which result from the dummy state approach are distribution of sums of independent exponential random variables. They belong in the family of general Erlangian distribution.

The fully failed robot system may be repaired and the failed robot system repair rates are also assumed non-constant. The analysis presented in this section are subject to assumptions such as follows:

- The system is composed of one robot,

- The robot may fail in m failures modes,

- Robot's failure rates may be constant or non-constant,

- Robot's repair rates may be constant or non-constant,

- Repaired robot is as good as new.
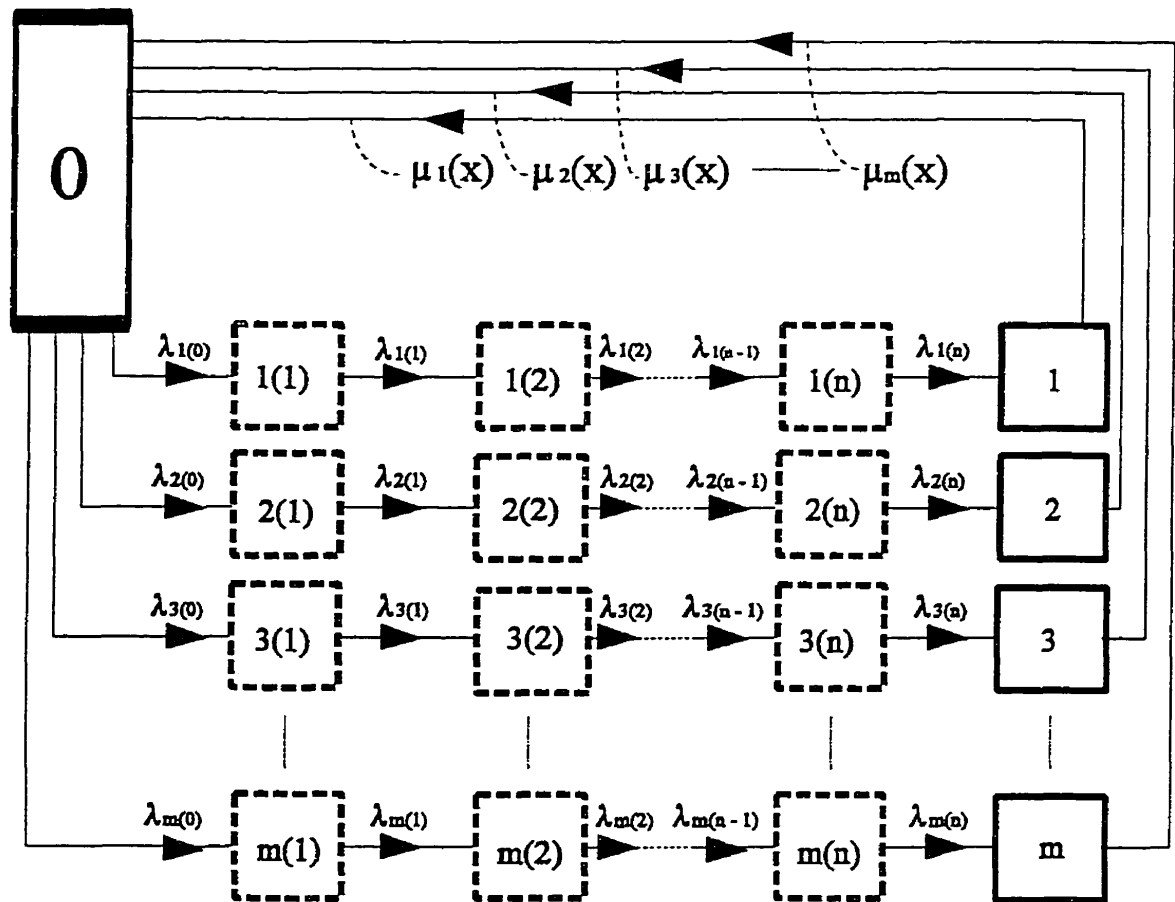
Figure 5.1.   The state space transition diagram of a robot system with m non-constant failure and repair rates.

The following symbols are associated with the model:

j j^{th} state of the robot: $j = 0, 1, 2, \dots , m$.

i i^{th} sub-state: $i = 0, 1, 2, \dots , n$.

$\lambda_{j(i)}$ Constant failure rate of the sub-states.

P_j Steady state probability of the robot in state j: for $j = 0, 1, 2, \dots , m$.

$\mu_j(x)$ Time-dependent repair rate when the robot is in state j and has an elapsed repair

time of x; for $j = 1, 2, \dots , m$.

$p_j(x, t)\Delta x$ The probability that at time t, the failed robot system is in state j and the elapsed

repair time lies in the interval $[x, x + \Delta x]$; for $j = 1, 2, \dots , m$.

pdf   Probability density function.

$q_j(x)$ *pdf* of repair time when the robot system is in state j and has an elapsed time of

x; for $j = 1, 2, \dots , m$.

SSAV   Robot system steady state availability.

Using the method of *device of stages* combined with the *supplementary variables* technique,

the differential equations associated with the model in Figure 5.1 are as follows:

$$P_0'(t) + \left[\sum_{j-1}^{m} \lambda_{j(0)}\right]P_0(t) = \sum_{j-1}^{m} \int_0^{\infty} p_j(x,t)\mu_j(x)dx \tag{5.1}$$

$$P_{1(1)}'(t) + \lambda_{1(1)}P_{1(1)}(t) = \lambda_{1(0)}P_0(t) \tag{5.2}$$

$$P_{1(2)}'(t) + \lambda_{1(2)}P_{1(2)}(t) = \lambda_{1(1)}P_{1(1)}(t) \tag{5.3}$$

$$P_{1(i)}'(t) + \lambda_{1(i)}P_{1(i)}(t) = \lambda_{1(i-1)}P_{1(i-1)}(t) \qquad (for \ \ i = 1, 2, 3, \dots , n) \tag{5.4}$$

$$P_{2(1)}'(t) + \lambda_{2(1)}P_{2(1)}(t) = \lambda_{2(0)}P_0(t) \tag{5.5}$$

$$P_{2(2)}'(t) + \lambda_{2(2)}P_{2(2)}(t) = \lambda_{2(1)}P_{2(1)}(t) \tag{5.6}$$

$$P_{2(i)}'(t) + \lambda_{2(i)}P_{2(i)}(t) = \lambda_{2(i-1)}P_{2(i-1)}(t) \qquad (for \ \ i = 1, 2, 3, \dots , n) \tag{5.7}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$P'_{j(i)}(t) + \lambda_{j(i)} P_{j(i)}(t) = \lambda_{j(i-1)} P_{j(i-1)}(t) \tag{5.8}$$

$$i = 1, 2, 3, \dots, n$$
$$j = 1, 2, 3, \dots, m$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_j(x)\right] p_j(x,t) = 0 \qquad (\text{for } j = 1, 2, 3, \dots m) \tag{5.9}$$

The associated boundary conditions are as follows:

$$p_1(0,t) = \lambda_{1(n)} P_{1(n)}(t) \tag{5.10}$$

$$p_2(0,t) = \lambda_{2(n)} P_{2(n)}(t) \tag{5.11}$$

$$p_j(0,t) = \lambda_{j(n)} P_{j(n)}(t) \qquad (\text{for } j = 1, 2, 3, \dots, m) \tag{5.12}$$

At time $t = 0$, $P_0(0) = 1$, and all other initial state probabilities are equal to zero. The prime denotes differentiation with respect to time t.

5.2.1   Steady State Availability Analysis

As time approaches infinity, Equations $(5.1) - (5.12)$ reduce to Equations $(5.13) - (5.24)$, respectively.

$$\left[\sum_{j-1}^{m} \lambda_{j(0)}\right] P_0 = \sum_{j-1}^{m} \int_0^\infty p_j(x)\mu_j(x)dx \tag{5.13}$$

$$\lambda_{1(1)} P_{1(1)} = \lambda_{1(0)} P_0 \tag{5.14}$$

$$\lambda_{1(2)} P_{1(2)} = \lambda_{1(1)} P_{1(1)} \tag{5.15}$$

$$\lambda_{1(i)}P_{1(i)} = \lambda_{1(i-1)}P_{1(i-1)} \qquad (for \ i = 1, 2, 3, \dots, n) \tag{5.16}$$

$$\lambda_{2(1)}P_{2(1)} = \lambda_{2(0)}P_0 \tag{5.17}$$

$$\lambda_{2(2)}P_{2(2)} = \lambda_{2(1)}P_{2(1)} \tag{5.18}$$

$$\lambda_{2(i)}P_{2(i)} = \lambda_{2(i-1)}P_{2(i-1)} \qquad (for \ i = 1, 2, 3, \dots, n) \tag{5.19}$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$\lambda_{j(i)}P_{j(i)} = \lambda_{j(i-1)}P_{j(i-1)} \tag{5.20}$$

$$i = 1, 2, 3, \dots, n$$
$$j = 1, 2, 3, \dots, m$$

$$\frac{d}{dx}p_j(x) = -\mu_j(x)p_j(x) \qquad (for \ j = 1, 2, 3, \dots m) \tag{5.21}$$

$$p_1(0) = \lambda_{1(n)}P_{1(n)} \tag{5.22}$$

$$p_2(0) = \lambda_{2(n)}P_{2(n)} \tag{5.23}$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$p_j(0) = \lambda_{j(n)}P_{j(n)} \qquad (for \quad j = 1, 2, 3, \dots, m) \tag{5.24}$$

Solving Equation (5.21), the resulting expression is:

$$p_j(x) = p_j(0)e^{-\int_0^z \mu_j(\theta) \, d\theta} \qquad (for \ j = 1, 2, 3, \dots, m) \tag{5.25}$$

The steady state condition of the probability, p_j, that due to a failure the robot system is under repair, is

$$P_j = \int_0^\infty p_j(x)dx \qquad (for \ j = 1, 2, 3, \dots, m) \tag{5.26}$$

Substituting Equation (5.25) into Equation (5.26), we have

$$P_j = \int_0^\infty p_j(0)e^{-\int_0^x \mu_j(\theta) \, d\theta}dx \qquad (for \ j = 1, 2, 3, \dots, m) \tag{5.27}$$

Substituting Equation (5.24) into Equation (5.27), we get:

$$\begin{aligned} P_j &= \int_0^\infty \lambda_{j(n)} P_{j(n)} e^{-\int_0^x \mu_j(\theta) \, d\theta}dx \\ &= \lambda_{j(n)} P_{j(n)} E_j[x] \qquad (for \quad j = 1, 2, 3, \dots, m) \end{aligned} \tag{5.28}$$

where

$$E_j[x] = \int_0^\infty e^{-\int_0^x \mu_j(\theta) \, d\theta}dx \qquad (for \ j = 1, 2, 3, \dots, m)$$

which is the mean time to robot system repair when the failed robot system is in state j and has an elapsed repair time of x, or the expected value of x. Solving Equations (5.14) – (5.20), and (5.28), together with

$$P_0 + \sum_{i-1}^n P_{1(i)} + \sum_{i-1}^n P_{2(i)} + \dots + \sum_{i-1}^n P_{m(i)} + \sum_{j-1}^m P_j = 1 \tag{5.29}$$

yield the following generalized steady state probabilities:

$$P_0 = \frac{1}{D} \tag{5.30}$$

where

$$D = 1 + \sum_{i-1}^n \frac{\lambda_{1(0)}}{\lambda_{1(i)}} + \sum_{i-1}^n \frac{\lambda_{2(0)}}{\lambda_{2(i)}} + \dots + \sum_{i-1}^n \frac{\lambda_{m(0)}}{\lambda_{m(i)}} + \sum_{j-1}^m \lambda_{j(0)} E_j[x] \tag{5.31}$$

$$P_{j(i)} = \frac{\lambda_{j(0)}}{\lambda_{j(i)}} \cdot \frac{1}{D} \tag{5.32}$$

$$i = 1, 2, 3, \dots, n$$
$$j = 1, 2, 3, \dots, m$$

$$P_j = \lambda_{j(0)} E_j[x] \cdot \frac{1}{D} \qquad (for \ j = 1, 2, 3, \dots, m) \tag{5.33}$$

Using Equations (5.30) – (5.32), the steady state availability of the robot system is given by

$$SSAV = P_0 + \sum_{i-1}^{n} P_{1(i)} + \sum_{i-1}^{n} P_{2(i)} + \dots + \sum_{i-1}^{n} P_{m(i)} \tag{5.34}$$

Thus,

$$SSAV = \frac{1}{D} \left[1 + \sum_{i-1}^{n} \frac{\lambda_{1(0)}}{\lambda_{1(i)}} + \sum_{i-1}^{n} \frac{\lambda_{2(0)}}{\lambda_{2(i)}} + \dots + \sum_{i-1}^{n} \frac{\lambda_{m(0)}}{\lambda_{m(i)}} \right] \tag{5.35}$$

If $\lambda_{1(0)} = \lambda_{1(1)} = \dots = \lambda_{1(n)} = \lambda_1$
$\lambda_{2(0)} = \lambda_{2(1)} = \dots = \lambda_{2(n)} = \lambda_2$
$\lambda_{m(0)} = \lambda_{m(1)} = \dots = \lambda_{m(n)} = \lambda_m$

Then,

$$SSAV = \frac{1 + m \cdot n}{1 + m \cdot n + \sum_{j-1}^{m} \lambda_j E_j[x]} \tag{5.36}$$

where

m = Number of robot failure modes; $m = 0, 1, 2, \dots$

n = Number of sub-states or stages before failure; $n = 0, 1, 2, \dots$

$E_j[x]$ = Robot mean time to repair

Robot mean time to repair, $E_j[x]$, can be expressed for various repair time distributions. The mean time to robot system repair, $E_j[x]$, represented by a **gamma** distribution is given by Equation (3.31), or

$$E_j[x] = \frac{\beta}{\mu_j} \qquad (for \ j = 1, 2, 3, ..., m) \tag{5.37}$$

Therefore, Equation (5.36) becomes

$$SSAV_{(m,n,\beta)} = \frac{1 + m \cdot n}{1 + m \cdot n + \sum_{j-1}^{m} \lambda_j \frac{\beta}{\mu_j}} \tag{5.38}$$

5.2.2   Markovian Representation of The Generalized Model

When there are no stages before system failure, one must obtain the conventional Markov processes. For a robot system with one failure mode ($m = 1$), no sub-states ($n = 0$), constant repair rate (i.e., $\beta = 1$), and the following parameter assumptions:

$$\lambda_0 = \lambda_1 = ... = \lambda_m = \lambda$$
$$\mu_1 = \mu_2 = ... = \mu_m = \mu$$

Equation (5.38) becomes

$$SSAV_{(1,0,1)} = \frac{1}{1 + \frac{\lambda}{\mu}} = \frac{\mu}{\mu + \lambda} \tag{5.39}$$

This result agrees with those obtained in Appendix C2 [Equation (C7)], Appendix C3 [Equation (C27)], and Appendix C4 [Equation (C43)].

5.2.3   Special Case Model Numerical Example

Setting :

$m = 1$ (or robot systems with one failure mode),
$\beta = 1$ (or robot systems with constant repair rate),
$\mu_1 = \mu_2 = ... = \mu_m = \mu = 0.0006$, and
$\lambda_0 = \lambda_1 = ... = \lambda_m = \lambda$

into Equation (5.38), Figure 5.2 shows the steady state availability plots of a robot system

with increasing number of sub-states before failure (i.e., $n \geq 0$). More detailized inspection

of Figures 5.2 can be made by referring to its associated tabular values given in Tables 5-1.

The trends shown by these values are discussed in the concluding section of this chapter.

Table 5-1: SSAV vs λ values for a robot with increasing number of stages before failure.

λ	Steady State Availability			
	$n = 0^*$	$n = 1$	$n = 2$	$n = 3$
0.0000	1	1	1	1
0.0004	0.6000	0.7500	0.8182	0.8571
0.0008	0.4286	0.6000	0.6923	0.7500
0.0012	0.3333	0.5000	0.6000	0.6667
0.0016	0.2727	0.4286	0.5294	0.6000
0.0020	0.2308	0.3750	0.4737	0.5455

*Exponential distribution (constant failure rate)
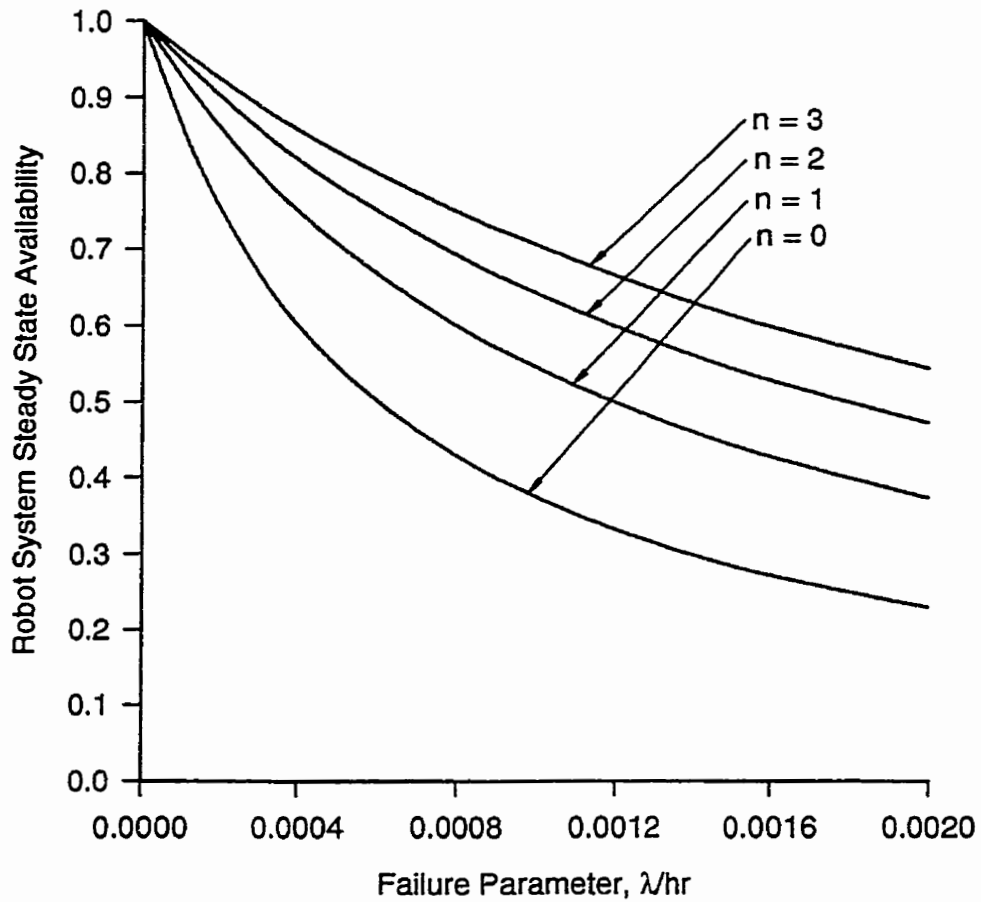
Figure 5.2.   Steady state availability plots of a robot system with one failure mode and increasing number of stages before failure ($n \geq 0$). More specifically, the plots were obtained using Equation (5.38).

5.3 Robot Systems With Redundant Safety Units

In the models presented in Chapters 3 and 4, failure rates were assumed constant and purely due to the system (mechanical or electrical) faults. But, what if failure rates were non-constant? Or, failures could not be blamed on neither mechanical nor electrical faults?

Furthermore, in the previous two chapters, two events were assumed to take place from the operating state (i.e., state 0) of the robot-safety system: either the robot fails or the safety unit fails. But, what if both the robot and the safety unit failed at the same time? An event such as this is generally known as a common-cause failure which was described in the introductory section of this chapter. At present, it is widely believed that the reliability and safety evaluation of engineering systems would be incomplete and would lead to overly optimistic results if common-cause failure possibilities and probabilities were discounted. An extensive study of redundant reliability systems with common-cause failures was presented by Anude[c], in 1994.

It is therefore in the spirit of these legitimate questions and the likelihood of the common-cause failures that the following analysis is aimed at obtaining the generalized steady state availability of robot-safety systems susceptible to common-cause failures, particularly non-constant.

The state space transition diagram of a system containing a robot with two safety units subjected to non-constant common-cause failure rate is shown in Figure 5.3. The robot and

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[c]Anude, O.C., The Analysis of Redundant Reliability Systems With Common-Cause Failures, Ph.D. Dissertation, Department of Mechanical Engineering, University of Ottawa, Ottawa, Canada, 1994.

its associated safety units start operating at $t = 0$ (i.e., state 0). From its operating state, the robot-safety system can move to three mutually exclusive states from their normal operating state:

- The robot operating with one safety unit while the other safety unit has failed (state 1),

- Safety units functioning normally but the robot has failed (state 5),

- Robot system fails due to a common-cause failure (state 6).

From the degradation state 1, the robot-safety system can move to two mutually exclusive states:

- The robot operating and both safety units have failed (state 2),

- The robot failed while one safety unit is still working (state 5),

From the degradation state 2, the robot-safety system can move further to two mutually exclusive states

- The robot fails with an incident (state 3), or,

- The robot fails safely (state 4).

The following assumptions are associated with the analysis presented in this section:

- Times to failure other than of common-cause failure rates are assumed constant.

- Times to common-cause failure are non-constant.

- To simplify the model, common-cause failure event is assumed to take place only from state 0, only.

- System fails when the robot fails.

- The failed system repair times other than of safety units repair rates are non-

constant.

• Once repaired, the robot or its safety units are as good as new.

Figure 5.3.   State space transition diagram of a system containing one robot and two
safety units. The robot is susceptible to common-cause failures which may
be constant or non-constant. The numeral in each box represents:
0   (Robot working with two safety units),
1   (Robot working with one safety unit, the other safety unit has failed),
2   (Robot working without safety units, both safety units have failed),
3   (Robot failed with an incident),
4   (Robot failed safely),
5   (Robot failed while both safety units working normally),
6   (Robot failed due to the common-cause failure).
Note: $6_{(1)}, 6_{(2)}, \ldots, 6_{(n)}$ denote dummy or sub-states.

The following symbols are associated with the model:

j j^{th} state of the overall robot system.

i Dummy or sub-states: $i = 1, 2, \ldots, n$.

t   time

λ_s Constant failure rate of the safety units.

λ_{ri} Constant failure rate of the robot failing with an incident.

λ_{rs} Constant failure rate of the robot failing safely.

λ_r Constant failure rate of the robot.

$\lambda_{cc(i)}$ Common-cause failure rate broken into i-exponentially distributed failure rates:

                 $i = 0, 1, 2, \ldots, n$.

μ_j Constant repair rate of the safety unit; for $j = 1, 2$.

Δx Finite repair time interval.

$\mu_j(x)$ Time-dependent repair rate when the failed robot system is in state j and has an

                 elapsed repair time of x; for $j = 3, 4, 5, 6$.

$p_j(x,t)\Delta x$ The probability that at time t, the failed robot system is in state j and the elapsed

                 repair time lies in the interval $[x, x + \Delta x]$; for $j = 3, 4, 5, 6$.

pdf   Probability density function.

$q_j(x)$ *pdf* of repair time when the failed system is in state j and has an elapsed time of

                 x; for $j = 3, 4, 5, 6$.

P_j Steady-state probability that the robot system is in state j; for $j = 0, 1, \ldots, 6$.

$SSAV_{rs}$ Robot system steady state availability when the robot system is working with an

                 operating safety unit.

SSAV$_r$ Robot system steady state availability when the robot system is working with or

without a safety unit.

Using the stages method and the supplementary variables technique, the corresponding

system of integro-differential equations associated with the model described in Figure 5.3

is

$$P_0'(t) + a_0 P_0(t) = \mu_1 P_1(t) + \sum_{j-3}^{6} \int_0^\infty p_j(x,t) \mu_j(x) dx \tag{5.40}$$

$$P_1'(t) + a_1 P_1(t) = \mu_2 P_2(t) + 2\lambda_s P_0(t) \tag{5.41}$$

$$P_2'(t) + a_2 P_2(t) = \lambda_s P_1(t) \tag{5.42}$$

where

$$a_0 = 2\lambda_s + \lambda_r + \lambda_{cc0}$$
$$a_1 = \lambda_s + \lambda_r + \mu_1$$
$$a_2 = \lambda_{rl} + \lambda_{rs} + \mu_2$$

$$P_{6(1)}'(t) + \lambda_{cc1} P_{6(1)}(t) = \lambda_{cc0} P_0(t) \tag{5.43}$$

$$P_{6(2)}'(t) + \lambda_{cc2} P_{6(2)}(t) = \lambda_{cc1} P_{6(1)}(t) \tag{5.44}$$

$$P_{6(i)}'(t) + \lambda_{cc(i)} P_{6(i)}(t) = \lambda_{cc(i-1)} P_{6(i-1)}(t) \qquad (for \ i = 1, 2, , ..., n) \tag{5.45}$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_j(x) \right] p_j(x,t) = 0 \qquad (for \ j = 3,4,5,6) \tag{5.46}$$

The associated boundary conditions are:

$$p_3(0,t) = \lambda_{r1} P_2(t) \tag{5.47}$$

$$p_4(0,t) = \lambda_{r2} P_2(t) \tag{5.48}$$

$$p_5(0,t) = \lambda_r [P_0(t) + P_1(t)] \tag{5.49}$$

$$p_6(0,t) = \lambda_{cc(n)} P_{6(n)}(t) \tag{5.50}$$

At time $t = 0$, $P_0(0) = 1$ and all other initial state probabilities are equal to zero. The prime denotes differentiation with respect to time t.

5.3.1   Steady State Availability Analysis

As time approaches infinity, Equations (5.40) – (5.50) reduce to Equations (5.51) – (5.61), respectively.

$$a_0 P_0 = \mu_1 P_1 + \sum_{j=3}^{6} \int_0^\infty p_j(x)\mu_j(x)dx \tag{5.51}$$

$$a_1 P_1 = \mu_2 P_2 + 2\lambda_i P_0 \tag{5.52}$$

$$a_2 P_2 = \lambda_i P_1 \tag{5.53}$$

$$\lambda_{cc1} P_{6(1)} = \lambda_{cc0} P_0 \tag{5.54}$$

$$\lambda_{cc2} P_{6(2)} = \lambda_{cc1} P_{6(1)} \tag{5.55}$$

$$\lambda_{cc(i)} P_{6(i)} = \lambda_{cc(i-1)} P_{6(i-1)} \quad (\textit{for } i = 1, 2, \ldots, n) \tag{5.56}$$

$$\frac{d}{dx} p_j(x) = -\mu_j(x) p_j(x) \quad (\textit{for } j = 3,4,5,6) \tag{5.57}$$

$$p_3(0) = \lambda_{rt} P_2 \tag{5.58}$$

$$p_4(0) = \lambda_{rs} P_2 \tag{5.59}$$

$$p_5(0) = \lambda_r [P_0 + P_1] \tag{5.60}$$

$$p_6(0) = \lambda_{cc(n)} P_{6(n)} \tag{5.61}$$

Solving differential Equation (5.57), we get

$$p_j(x) = p_j(0) e^{-\int_0^x \mu_j(\theta)\, d\theta} \qquad (for\ j = 3, 4, 5, 6) \tag{5.62}$$

The steady state condition of the probability that due to a failure the robot system is under repair, is

$$P_j = \int_0^\infty p_j(x)\, dx \qquad (for\ j = 3, 4, 5, 6) \tag{5.63}$$

Substituting Equation (5.62) into Equation (5.63), we have

$$P_j = \int_0^\infty p_j(0) e^{-\int_0^x \mu_j(\theta)\, d\theta}\, dx \qquad (for\ j = 3, 4, 5, 6) \tag{5.64}$$

Substituting Equations (5.58) − (5.61) into Equation (5.64), the resulting expressions are

$$P_3 = \int_0^\infty \lambda_{rt} P_2\, e^{-\int_0^x \mu_3(\theta)\, d\theta}\, dx$$
$$= \lambda_{rt} P_2 E_3[x] \tag{5.65}$$

$$P_4 = \int_0^\infty \lambda_{rs} P_2\, e^{-\int_0^x \mu_4(\theta)\, d\theta}\, dx$$
$$= \lambda_{rs} P_2 E_4[x] \tag{5.66}$$

$$P_5 = \int_0^\infty \lambda_r [P_0 + P_1]\, e^{-\int_0^x \mu_5(\theta)\, d\theta}\, dx$$
$$= \lambda_r [P_0 + P_1] E_5[x] \tag{5.67}$$

$$P_6 = \int_0^\infty \lambda_{cc(n)} P_{6(n)}\, e^{-\int_0^x \mu_6(\theta)\, d\theta}\, dx$$
$$= \lambda_{cc(n)} P_{6(n)} E_6[x] \tag{5.68}$$

where

$$E_j[x] = \int_0^\infty e^{-\int_0^x \mu_j(\theta)\, d\theta}\, dx \qquad (for\ j = 3, 4, 5, 6)$$

which is the mean time to robot system repair when the failed robot system is in state j and has an elapsed repair time of x, or the expected value of x. Solving the set of Equations (5.52) – (5.56), and (5.65) – (5.68), together with

$$\sum_{j=0}^{6} P_j + \sum_{i=1}^{n} P_{6(i)} = 1 \tag{5.69}$$

leads to the following steady state probabilities:

$$P_j = \frac{\omega_j}{D} \qquad (for\quad j = 0, 1, 2) \tag{5.70}$$

$$P_j = \frac{\omega_j}{D} E_j[x] \qquad (for\ j = 3, 4, 5, 6) \tag{5.71}$$

$$P_{6(i)} = \frac{\lambda_{cc0}}{\lambda_{cc(i)}} \cdot \frac{\omega_0}{D} \qquad (for\ i = 1, 2, \dots, n) \tag{5.72}$$

$$D = \sum_{j=0}^{2} \omega_j + \sum_{j=3}^{6} \omega_j E_j[x] + \omega_0 \sum_{i=1}^{n} \frac{\lambda_{cc0}}{\lambda_{cc(i)}} \tag{5.73}$$

where

$$\omega_0 = a_1 a_2 - \mu_2 \lambda_s$$
$$\omega_1 = 2a_2 \lambda_s$$
$$\omega_2 = 2\lambda_s^2$$
$$\omega_3 = 2\lambda_{rl} \lambda_s^2$$
$$\omega_4 = 2\lambda_{rs} \lambda_s^2$$
$$\omega_5 = \lambda_r (a_1 a_2 - \mu_2 \lambda_s + 2a_2 \lambda_s)$$
$$\omega_6 = \lambda_{cc0} (a_1 a_2 - \mu_2 \lambda_s)$$

Using Equations (5.70) and (5.72), the robot system steady state availability with an operating safety unit is:

$$SSAV_{rs} = P_0 + P_1 + \sum_{i-1}^{n} P_{6(i)} = \sum_{j-0}^{1} \frac{\omega_j}{D} + \frac{\omega_0}{D} \sum_{i-1}^{n} \frac{\lambda_{cc0}}{\lambda_{cc(i)}} \qquad (5.74)$$

Similarly, using Equations (5.70) and (5.72), the robot system steady state availability with or without a working safety unit is given by

$$SSAV_r = P_0 + P_1 + P_2 + \sum_{i-1}^{n} P_{6(i)} = \sum_{j-0}^{2} \frac{\omega_j}{D} + \frac{\omega_0}{D} \sum_{i-1}^{n} \frac{\lambda_{cc0}}{\lambda_{cc(i)}} \qquad (5.75)$$

For $\lambda_{cc0} = \lambda_{cc1} = \ldots = \lambda_{cc(n)} = \lambda_{cc}$, Equations (5.73) – (5.75) become

$$SSAV_{rs} = \sum_{j-0}^{1} \frac{\omega_j}{D} + \frac{\omega_0}{D} (n) \qquad (5.76)$$

and

$$SSAV_r = \sum_{j-0}^{2} \frac{\omega_j}{D} + \frac{\omega_0}{D} (n) \qquad (5.77)$$

$$D = \sum_{j=0}^{2} \omega_j + \sum_{j=3}^{6} \omega_j E_j[x] + \omega_0(n) \tag{5.78}$$

where n is the number of stages before failure and $E_j[x]$ is the robot mean time to repair which can be expressed for various repair time distributions. The mean time to robot system repair, $E_j[x]$, represented by a **gamma** distribution is given by Equation (3.31), or

$$E_j[x] = \frac{\beta}{\mu_j} \qquad (for \ j = 3, 4, 5, 6) \tag{5.79}$$

For $\beta = 1$, the failed robot system repair times are constant and for $n = 0$ (i.e., no stages before failure), we get constant common-cause failure rate. Thus, for $\beta = 1$ and $n = 0$, Equations (5.76) and (5.77) yield steady state availabilities of a robot-safety system with constant failure and repair rates. Or,

$$SSAV_n = \sum_{j=0}^{1} \frac{\omega_j}{D} \tag{5.80}$$

$$SSAV_r = \sum_{j=0}^{2} \frac{\omega_j}{D} \tag{5.81}$$

$$D = \sum_{j=0}^{2} \omega_j + \sum_{j=3}^{6} \omega_j / \mu_j \tag{5.82}$$

For an integer $n > 0$ (i.e., 1,2, ... , and so on), Equations (5.76) and (5.77) yield robot steady state availability with non-constant common-cause failure rate.

The applicability of the stages method to the model presented in this section can be validated by using the Markov method. This is shown in the following section.

5.3.2   Markovian Representation of The Special Case Model

In the last section, the method of stages was used to represent non-exponential common-cause failure rates. n-stages were inserted between the operating and the failed states, for when there are no stages before system failure, one must obtain the same results as with the conventional Markov processes. For the sake of comparison, the following analysis is conducted using the Markov method and results are compared with those obtained in Sections 5.3.1.

The state space transition diagram of a system containing a robot with two safety units subjected to constant common-cause failure is shown in Figure 5.4.
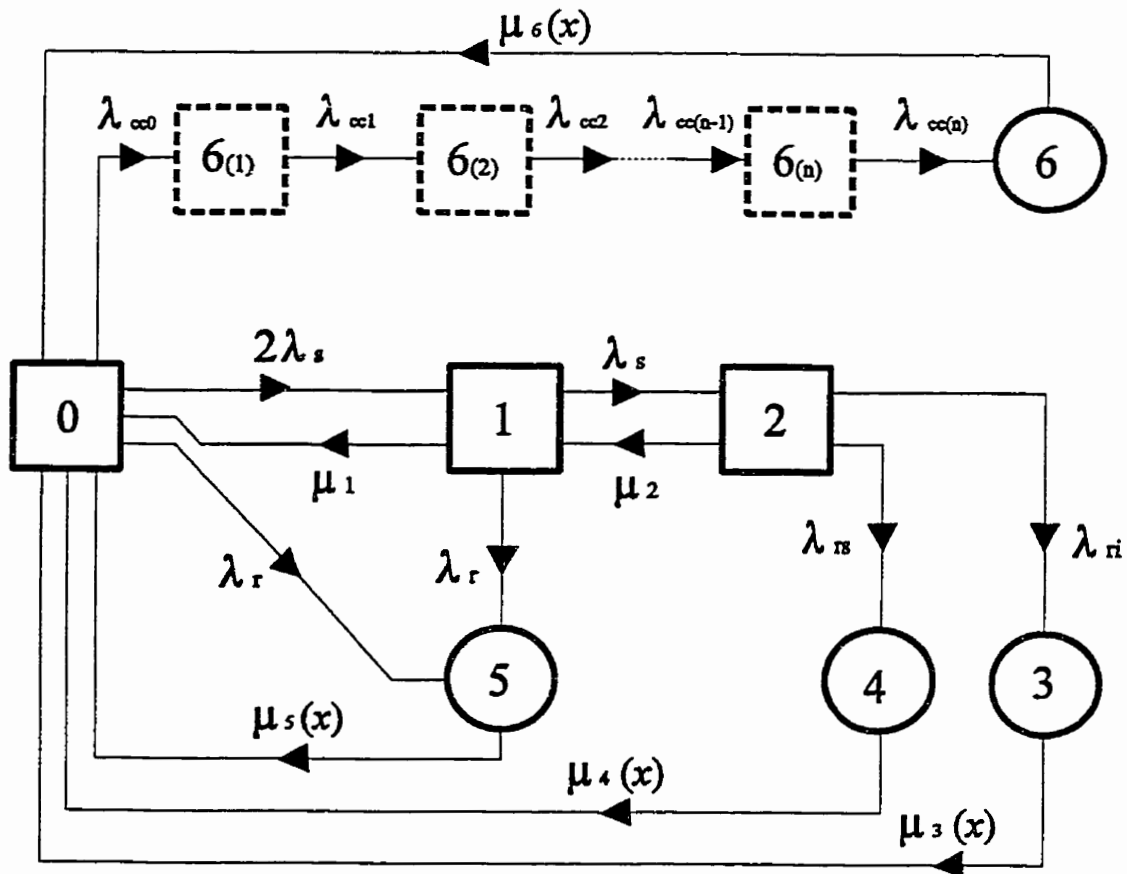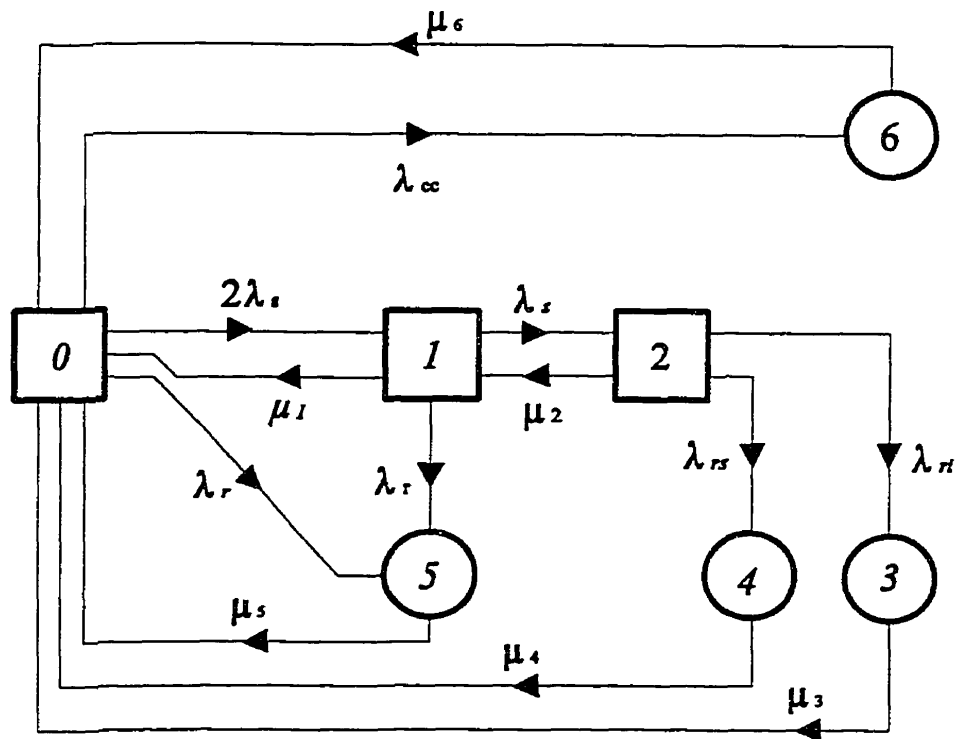

Figure 5.4.   State space transition diagram of a system containing one robot and two safety units. The robot is susceptible to **constant** common-cause failures.

Using the Markov method, the set of differential equations associated with the model given in Figure 5.4 is:

$$P_0'(t) + a_0 P_0(t) = \mu_1 P_1(t) + \sum_{j=3}^{6} \mu_j p_j(t) \tag{5.83}$$

$$P_1'(t) + a_1 P_1(t) = \mu_2 P_2(t) + 2\lambda_s P_0(t) \tag{5.84}$$

$$P_2'(t) + a_2 P_2(t) = \lambda_s P_1(t) \tag{5.85}$$

where

$$a_0 = 2\lambda_s + \lambda_r + \lambda_{cc}$$
$$a_1 = \lambda_s + \lambda_r + \mu_1$$
$$a_2 = \lambda_{rl} + \lambda_{rs} + \mu_2$$

$$P_3'(t) + \mu_3 P_3(t) = \lambda_{rl} P_2(t) \tag{5.86}$$

$$P_4'(t) + \mu_4 P_4(t) = \lambda_{rs} P_2(t) \tag{5.87}$$

$$P_5'(t) + \mu_5 P_5(t) = \lambda_r [P_0(t) + P_1(t)] \tag{5.88}$$

$$P_6'(t) + \mu_6 P_6(t) = \lambda_{cc} P_0(t) \tag{5.89}$$

At time $t = 0$, $P_0(0) = 1$ and all other initial state probabilities are equal to zero. The prime denotes differentiation with respect to time t. As time approaches infinity, Equations (5.83) – (5.89) reduce to Equations (5.90) – (5.96), respectively.

$$a_0 P_0 = 1 + \mu_1 P_1 + \sum_{j=3}^{6} \mu_j P_j \tag{5.90}$$

$$a_1 P_1 = \mu_2 P_2 + 2\lambda_s P_0 \tag{5.91}$$

$$a_2 P_2 = \lambda_s P_1 \tag{5.92}$$

$$\mu_3 P_3 = \lambda_{rt} P_2 \tag{5.93}$$

$$\mu_4 P_4 = \lambda_{rs} P_2 \tag{5.94}$$

$$\mu_5 P_5 = \lambda_r [P_0 + P_1] \tag{5.95}$$

$$\mu_6 P_6 = \lambda_{cc} P_0 \tag{5.96}$$

Solving Equations (5.91) – (5.96), together with

$$\sum_{j=0}^{6} P_j = 1 \tag{5.97}$$

result in the following steady state probabilities:

$$P_j = \frac{\omega_j}{D} \qquad (\textit{for} \quad j = 0, 1, 2) \tag{5.98}$$

$$P_j = \frac{\omega_j}{D} \cdot \frac{1}{\mu_j} \qquad (\textit{for } j = 3, 4, 5, 6) \tag{5.99}$$

$$D = \sum_{j=0}^{2} \omega_j + \sum_{j=3}^{6} \omega_j / \mu_j \tag{5.100}$$

where

$$\omega_0 = a_1 a_2 - \mu_2 \lambda_s$$
$$\omega_1 = 2 a_2 \lambda_s$$
$$\omega_2 = 2 \lambda_s^2$$
$$\omega_3 = 2 \lambda_{rt} \lambda_s^2$$
$$\omega_4 = 2 \lambda_{rs} \lambda_s^2$$
$$\omega_5 = \lambda_r (a_1 a_2 - \mu_2 \lambda_s + 2 a_2 \lambda_s)$$
$$\omega_6 = \lambda_{cc} (a_1 a_2 - \mu_2 \lambda_s)$$

Using Equation (5.98), the robot system steady state availability with an operating safety

units and the robot system with or with a working safety units are given by

$$SSAV_{rs} = \sum_{j=0}^{1} \frac{\omega_j}{D} \tag{5.101}$$

and

$$SSAV_r = \sum_{j=0}^{2} \frac{\omega_j}{D} \tag{5.102}$$

where

$$D = \sum_{j=0}^{2} \omega_j + \sum_{j=3}^{6} \omega_j / \mu_j \tag{5.103}$$

Comparing these results with those obtained in Section 5.3.1, it is clearly evident that

Equations (5.101) – (5.103) are in agreement with Equations (5.80) – (5.82), respectively.

This verifies the applicability of the stages method to the model.

5.3.3   Special Case Model Numerical Example

Setting:

$\lambda_s = 0.0005, \quad \lambda_{ri} = 0.0003, \quad \lambda_{rs} = 0.0004, \quad \lambda_r = 0.0005,$
$\mu_1 = \mu_2 = 0.0006, \qquad \mu_3 = 0.0007, \qquad \mu_4 = 0.0008, \quad \mu_5 = 0.0009, \mu_6 = 0.0005$

in Equations (5.76) and (5.77), Figure 5.5 shows plots of the robot steady state availability

(SSAV) vs common-cause failure parameter (λ_{cc}). The plots indicate SSAV for a robot

system with constant repair rate and increasing number of stages (n) before system failure.

The corresponding values associated with Figure 5.5 are given in Table 5-2. The trends

shown by these values are discussed in the concluding section of this chapter.

Table 5-2: SSAV vs λ_{cc} values for a system containing one robot with redundant safety units. Common-cause failure rate is non-constant and all other failure and repair rates are constant.

λ_{cc}	$n = 0*$		$n = 1$		$n = 2$		$n = 3$	
	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$	$SSAV_{rs}$	$SSAV_r$
0.0000	0.4353	0.6027	0.6066	0.7232	0.6981	0.7876	0.7551	0.8277
0.0004	0.3229	0.4470	0.4881	0.5820	0.5885	0.6640	0.6560	0.7191
0.0008	0.2566	0.3553	0.4084	0.4869	0.5087	0.5739	0.5799	0.6357
0.0012	0.2129	0.2948	0.3510	0.4186	0.4479	0.5054	0.5197	0.5696
0.0016	0.1819	0.2519	0.3078	0.3670	0.4001	0.4514	0.4707	0.5160
0.0020	0.1588	0.2199	0.2741	0.3268	0.3616	0.4079	0.4302	0.4716

*Exponential distribution (constant common-cause failure rate)
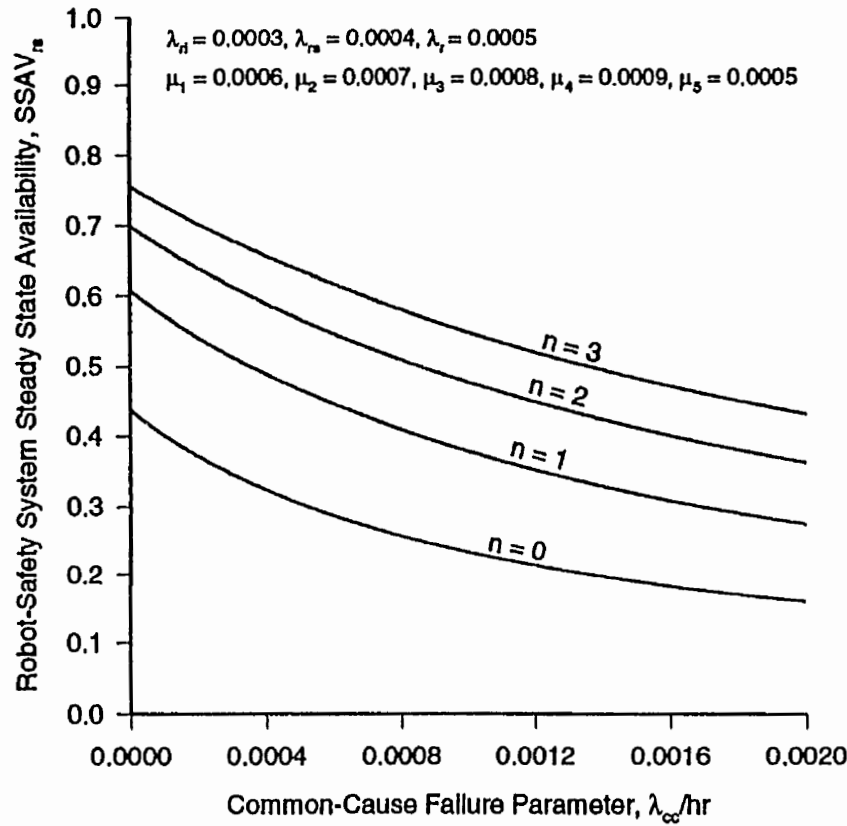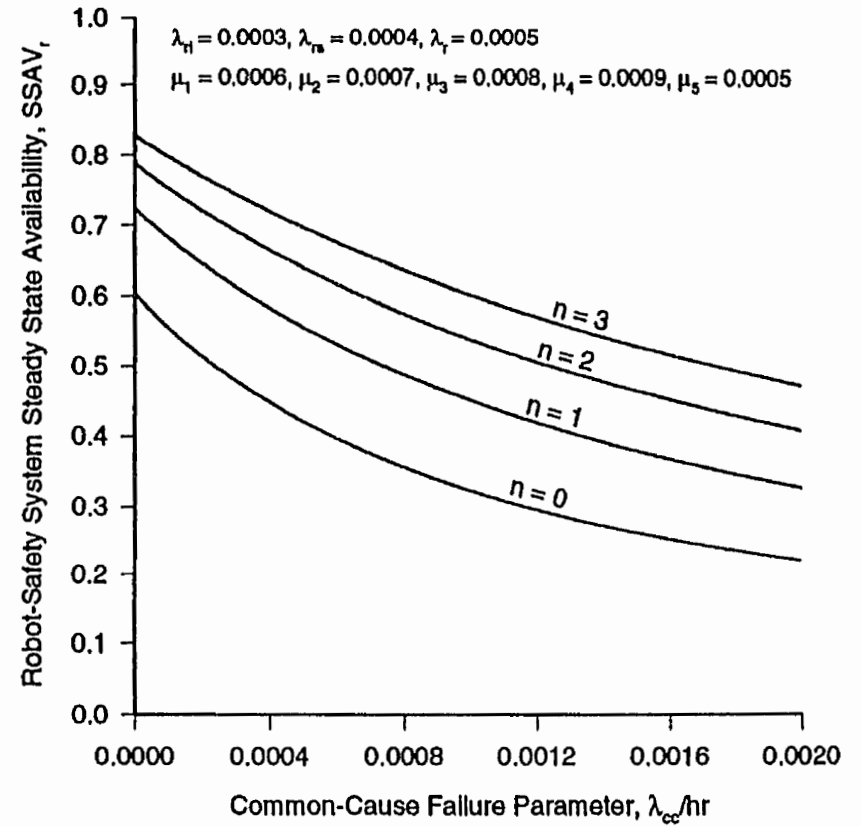
Figure 5.5.   Steady state availability vs λ_{cc} plots for a system comprised of a robot and redundant safety units. Common-cause failure rate is non-constant and all other failure and repair rates are constant; (a) robot working with an operating safety unit, (b) robot working with or without the safety unit. More specifically, the plots were obtained using Equations (5.76) and (5.77).

5.4 Redundant Robots With One Safety Unit

The objective of this section is to examine the effect of a non-constant common-cause failure rate on a system comprised of two robots with one safety unit. Similar to Section 5.3, stages method is used to obtain steady state availability expressions and its applicability is varified by means of the Markov method.

The state space transition diagram of a system containing two robots and one system unit subjected to the common-cause failure is shown in Figure 5.6. Identical robots and the associated safety unit start operating at $t = 0$ (i.e., state 0). From the operating state, the robot-safety system can move to three mutually exclusive states from their normal operating state:

- Redundant robots are operating but the safety unit has failed (state 1),

- Safety unit is functioning normally but one robot has failed (state 2),

- Robot system failed due to a common-cause failure (state 6).

From degradation state 1, the overall system is further degraded to a state having only one robot working with the failed safety unit (state 3). The system fails whenever the single operating robot fails (states 4 and 5), and fully failed system is repaired. The following assumptions are associated with the analysis presented in this section:

- The system contains one safety unit and 2 identical robots,

- The redundant robots are active or operating simultaneously,

- Statistically independent failures,

- Times to failure other than that of common-cause failures are assumed constant,

- The overall system fails only when both robots fail,

- The failed system repair times other than of safety unit repair times are non-constant,

- A repaired robot or the safety unit is as good as new.

Figure 5.6.   State space transition diagram of a system containing two robots and one safety unit. The robot system is susceptible to common-cause failures which may be constant or non-constant. The numeral in each box represents:
0   (Both robots and the safety unit are working normally),
1   (Only one robot working with a working safety unit),
2   (Both robots working while the safety unit has failed),
3   (Only one robot working with a failed safety unit),
4   (Both robots along with the safety unit have failed),
5   (Both robots failed while the safety unit is operational),
6   (Robot-safety system failed due to a common-cause failure).
Note: $6_{(1)}, 6_{(2)}, \dots, 6_{(n-1)}$ denote dummy or sub-states.

The following symbols are associated with the model:

j j^{th} state of the overall robot system.

i Dummy or sub-states: $i = 0, 1, 2, \ldots, n$.

λ_s Constant failure rate of the safety unit.

λ_r Constant failure rate of the robot.

$\lambda_{cc(i)}$ Common-cause failure rate broken into i-exponentially distributed failure rates:

               for $i = 0, 1, 2, \ldots, n$.

μ_s Constant repair rate of the safety unit.

μ_r Constant repair rate of the robot.

$\mu_j(x)$ Time-dependent repair rate when the robot is in state j and has an elapsed repair

               time of x; for $j = 4, 5, 6$.

$p_j(x,t)\Delta x$ The probability that at time t, the failed robot system is in state j and the elapsed

               repair time lies in the interval $[x, x + \Delta x]$; for $j = 4, 5, 6$.

pdf Probability density function.

$q_j(x)$ *pdf* of repair time when the robot system is in state j and has an elapsed time of

               x; for $j = 4, 5, 6$.

P_j Steady state probability that the robot is in state j at time t; for $j = 0, 1, \ldots, 6$.

$SSAV_{rs}$ Robot system steady state availability when the robot system is working with an

               operating safety unit.

$SSAV_r$ Robot system steady state availability when the robot system is working with or

               without the safety unit.

Using the stages method and the supplementary variables technique, the corresponding set

of integro-differential equations associated with the model described in Figure 5.6 is

$$P_0'(t) + a_0 P_0(t) = \mu_r P_1(t) + \mu_s P_2(t) + \sum_{j=4}^{6} \int_0^\infty p_j(x,t)\mu_j(x)dx \qquad (5.104)$$

$$P_1'(t) + a_1 P_1(t) = 2\lambda_r P_0(t) \qquad (5.105)$$

$$P_2'(t) + a_2 P_2(t) = \lambda_s P_0(t) \qquad (5.106)$$

$$P_3'(t) + a_3 P_3(t) = \lambda_s P_1(t) + 2\lambda_r P_2(t) \qquad (5.107)$$

where

$$
\begin{aligned}
a_0 &= \lambda_s + 2\lambda_r + \lambda_{cc0} \\
a_1 &= \lambda_s + \lambda_r + \mu_r \\
a_2 &= 2\lambda_r + \mu_r \\
a_3 &= \lambda_r
\end{aligned}
$$

$$P_{6(1)}'(t) + \lambda_{cc1} P_{6(1)}(t) = \lambda_{cc0} P_0(t) \qquad (5.108)$$

$$P_{6(2)}'(t) + \lambda_{cc2} P_{6(2)}(t) = \lambda_{cc1} P_{6(1)}(t) \qquad (5.109)$$

$$P_{6(i)}'(t) + \lambda_{cc(i)} P_{6(i)}(t) = \lambda_{cc(i-1)} P_{6(i-1)}(t) \qquad (for \ i = 1, 2, , ..., n) \qquad (5.110)$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_j(x)\right] p_j(x,t) = 0 \qquad (for \ j = 4,5,6) \qquad (5.111)$$

The associated boundary conditions are:

$$p_4(0,t) = \lambda_r P_3(t) \qquad (5.112)$$

$$p_5(0,t) = \lambda_r P_1(t) \tag{5.113}$$

$$p_6(0,t) = \lambda_{cc(n)} P_{6(n-1)}(t) \tag{5.114}$$

At time $t = 0$, $P_0(0) = 1$ and all other initial state probabilities are equal to zero. The prime denotes differentiation with respect to time t.

5.4.1   Steady State Availability Analysis

As time approaches infinity, Equations (5.104) – (5.114) reduce to Equations (5.115) – (5.125), respectively.

$$a_0 P_0 = \mu_r P_1 + \mu_r P_2 + \sum_{j=4}^{6} \int_0^\infty p_j(x)\mu_j(x)dx \tag{5.115}$$

$$a_1 P_1 = 2\lambda_r P_0 \tag{5.116}$$

$$a_2 P_2 = \lambda_s P_0 \tag{5.117}$$

$$a_3 P_3 = \lambda_s P_1 + 2\lambda_r P_2 \tag{5.118}$$

$$\lambda_{cc1} P_{6(1)} = \lambda_{cc0} P_0 \tag{5.119}$$

$$\lambda_{cc2} P_{6(2)} = \lambda_{cc1} P_{6(1)} \tag{5.120}$$

$$\lambda_{cc(i)} P_{6(i)} = \lambda_{cc(i-1)} P_{6(i-1)} \qquad (for \qquad i = 1, 2, , ..., n) \tag{5.121}$$

$$\frac{d}{dx} p_j(x) = -\mu_j(x) p_j(x) \qquad (for \ j = 4,5,6) \tag{5.122}$$

$$p_4(0) = \lambda_r P_3 \tag{5.123}$$

$$p_5(0) = \lambda_r P_1 \tag{5.124}$$

$$p_6(0) = \lambda_{cc(n)} P_{6(n)} \tag{5.125}$$

Solving differential Equation (5.122), we get

$$p_j(x) = p_j(0)e^{-\int_0^x \mu_j(\theta)\,d\theta} \qquad (\text{for } j = 4, 5, 6) \tag{5.126}$$

The steady state condition of the probability that due to a failure the robot system is under repair, is

$$P_j = \int_0^\infty p_j(x)dx \qquad (\text{for } j = 4, 5, 6) \tag{5.127}$$

Substituting Equation (5.126) into Equation (5.127), we have

$$P_j = \int_0^\infty p_j(0)e^{-\int_0^x \mu_j(\theta)\,d\theta}\,dx \qquad (\text{for } j = 4, 5, 6) \tag{5.128}$$

Substituting Equations (5.123) – (5.125) into Equation (5.128), the resulting expressions are

$$P_4 = \int_0^\infty \lambda_r P_3 e^{-\int_0^x \mu_4(\theta)\,d\theta}\,dx \tag{5.129}$$
$$= \lambda_r P_3 E_4[x]$$

$$P_5 = \int_0^\infty \lambda_r P_1 e^{-\int_0^x \mu_5(\theta)\,d\theta}\,dx \tag{5.130}$$
$$= \lambda_r P_1 E_5[x]$$

$$P_6 = \int_0^\infty \lambda_{cc(n)} P_{6(n)} e^{-\int_0^x \mu_6(\theta)\,d\theta}\,dx \tag{5.131}$$
$$= \lambda_{cc(n)} P_{cc(n)} E_6[x]$$

where

$$E_j[x] = \int_0^{\infty} e^{-\int_0^x \mu_j(\theta)\, d\theta}\, dx \qquad (for\ j = 4, 5, 6)$$

which is the mean time to robot system repair when the failed robot system is in state j and

has an elapsed repair time of x, or the expected value of x. Solving the set of Equations

$(5.116) - (5.121)$, and $(5.129) - (5.131)$, together with

$$\sum_{j=0}^{6} P_j + \sum_{i=1}^{n} P_{6(i)} = 1 \qquad (5.132)$$

leads to the following steady state probabilities:

$$P_j = \frac{\omega_j}{D} \qquad (for\ j = 0, 1, 2, 3) \qquad (5.133)$$

$$P_j = \frac{\omega_j}{D} E_j[x] \qquad (for\ j = 4, 5, 6) \qquad (5.134)$$

$$P_{6(i)} = \frac{\lambda_{cc0}}{\lambda_{cc(i)}} \cdot \frac{\omega_0}{D} \qquad (for\ i = 1, 2, \dots, n) \qquad (5.135)$$

$$D = \sum_{j=0}^{3} \omega_i + \sum_{j=4}^{6} \omega_j E_j[x] + \omega_0 \sum_{i=1}^{n} \frac{\lambda_{cc0}}{\lambda_{cc(i)}} \qquad (5.136)$$

where

$$\omega_0 = a_1 a_2 a_3$$
$$\omega_1 = 2\lambda_r a_2 a_3$$
$$\omega_2 = \lambda_s a_1 a_3$$
$$\omega_3 = 2\lambda_r \lambda_s (a_1 + a_2)$$
$$\omega_4 = 2\lambda_r^2 \lambda_s (a_1 + a_2)$$
$$\omega_5 = 2\lambda_r^2 a_2 a_3$$
$$\omega_6 = \lambda_{cc1} a_1 a_2 a_3$$

Using Equations (5.133) and (5.135), the robot system steady state availability with an

operating safety unit is

$$SSAV_{rs} = \sum_{j=0}^{1} P_j + \sum_{i=1}^{n} P_{6(i)} = \frac{1}{D}\left[\sum_{j=0}^{1} \omega_j + \omega_0 \sum_{i=1}^{n} \frac{\lambda_{cc0}}{\lambda_{cc(i)}}\right] \tag{5.137}$$

Similarly, using Equations (5.133) and (5.135), the robot system steady state availability

with or without a working safety unit is given by

$$SSAV_r = \sum_{j=0}^{3} P_j + \sum_{i=1}^{n} P_{6(i)} = \frac{1}{D}\left[\sum_{j=0}^{3} \omega_j + \omega_0 \sum_{i=1}^{n} \frac{\lambda_{cc0}}{\lambda_{cc(i)}}\right] \tag{5.138}$$

For $\lambda_{cc0} = \lambda_{cc1} = ... = \lambda_{cc(n)} = \lambda_{cc}$, Equations (5.137) and (5.138) become

$$SSAV_{rs} = \sum_{j=0}^{1} P_j + \sum_{i=1}^{n} P_{6(i)} = \frac{1}{D}\left[\sum_{j=0}^{1} \omega_j + \omega_0 n\right] \tag{5.139}$$

and

$$SSAV_r = \sum_{j=0}^{3} P_j + \sum_{i=1}^{n} P_{6(i)} = \frac{1}{D}\left[\sum_{j=0}^{3} \omega_j + \omega_0 n\right] \tag{5.140}$$

where

$$D = \sum_{j=0}^{3} \omega_j + \sum_{j=4}^{6} \omega_j E_j[x] + \omega_0(n) \tag{5.141}$$

where n is the number of stages before failure and $E_j[x]$ is the robot mean time to repair

which can be expressed for various repair time distributions. The mean time to robot system

repair, $E_j[x]$, represented by a **gamma** distribution is given by Equation (3.31), or

$$E_j[x] = \frac{\beta}{\mu_j} \qquad (for \ j = 4, 5, 6) \tag{5.142}$$

For $\beta = 1$, the failed robot system repair times are constant and for $n = 0$ (i.e., no stages before failure), we get constant common-cause failure rate. Thus, for $\beta = 1$ and $n = 0$, Equations (5.139) and (5.140) yield steady state availabilities of a robot-safety system with constant failure and repair rates. Or,

$$SSAV_n = \sum_{j=0}^{1} \frac{\omega_j}{D} \tag{5.143}$$

$$SSAV_r = \sum_{j=0}^{3} \frac{\omega_j}{D} \tag{5.144}$$

$$D = \sum_{j=0}^{3} \omega_j + \sum_{j=4}^{6} \omega_j / \mu_j \tag{5.145}$$

For an integer $n > 0$ (i.e., 1,2, ... , and so on), Equations (5.139) and (5.140) yield robot system steady state availability with non-constant common-cause failure rate.

The applicability of the stages method to the model presented in this section can be validated by using the Markov method. This is shown in the following section.

5.4.2   Markovian Representation of The Special Case Model

For the sake of comparison, the following analysis is conducted using the Markov method and results are compared with those obtained in Sections 5.4.1. The state space transition diagram of a system containing two robots with one safety unit subjected to constant common-cause failure is shown in Figure 5.7.
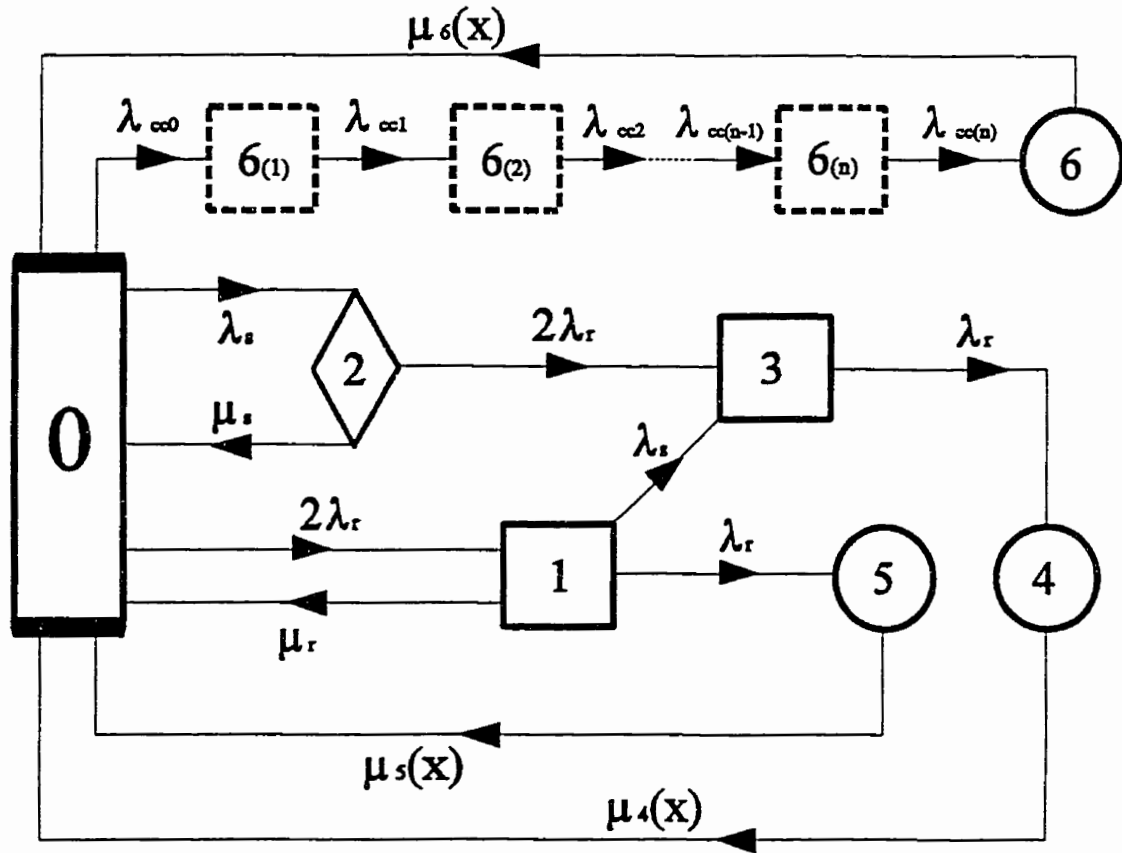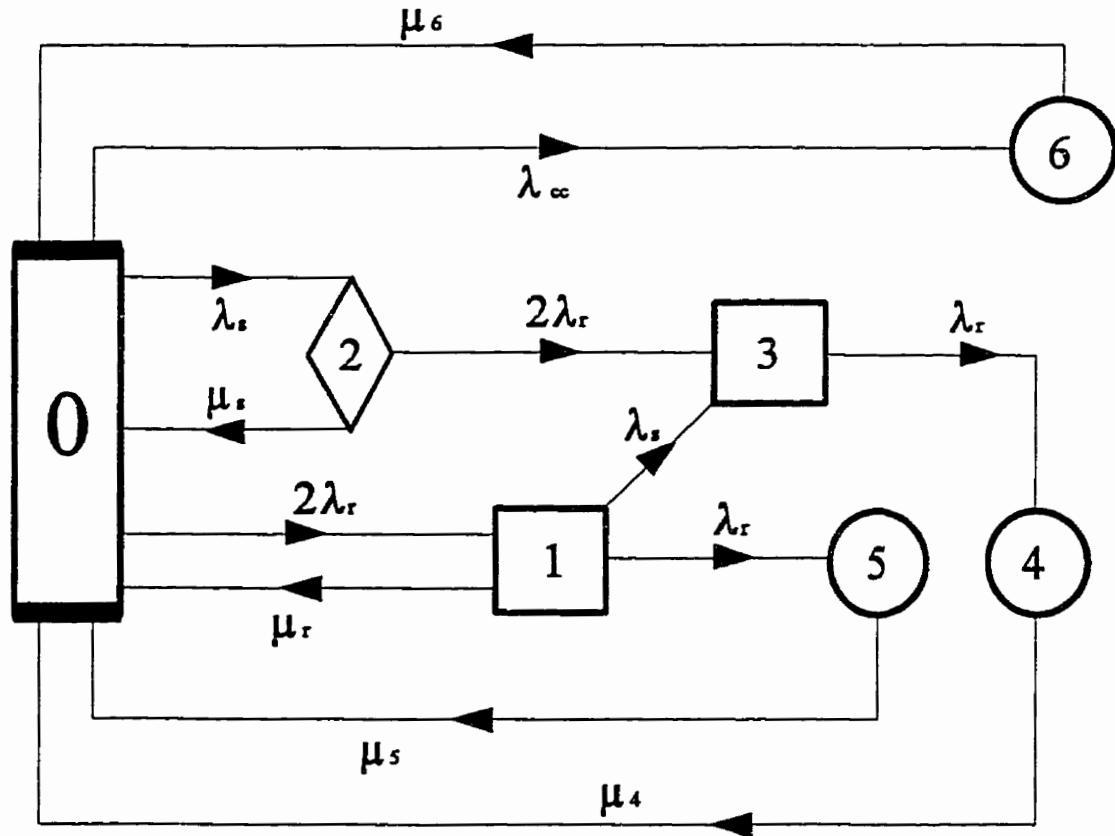
Figure 5.7.   State space transition diagram of a system containing two robots and one safety unit. The robot system is susceptible to **constant** common-cause failures. The numeral in each box represents:

0   (Both robots and the safety unit are working normally),
1   (Only one robot working with a working safety unit),
2   (Both robots working while the safety unit has failed),
3   (Only one robot working with a failed safety unit),
4   (Both robots along with the safety unit have failed),
5   (Both robots failed while the safety unit is operational),
6   (Robot-safety system failed due to a common-cause failure).

Using the Markov method, the set of differential equations associated with the model given

in Figure 5.7 is:

$$P_0'(t) + a_0 P_0(t) = \mu_r P_1(t) + \mu_s P_2(t) + \sum_{j=4}^{6} \mu_j P_j(t) \tag{5.146}$$

$$P_1'(t) + a_1 P_1(t) = 2\lambda_r P_0(t) \tag{5.147}$$

$$P_2'(t) + a_2 P_2(t) = \lambda_s P_0(t) \tag{5.148}$$

$$P_3'(t) + a_3 P_3(t) = \lambda_s P_1(t) + 2\lambda_r P_2(t) \tag{5.149}$$

where

$$a_0 = \lambda_s + 2\lambda_r + \lambda_{cc}$$
$$a_1 = \lambda_s + \lambda_r + \mu_r$$
$$a_2 = 2\lambda_r + \mu_r$$
$$a_3 = \lambda_r$$

$$P_4'(t) + \mu_4 P_4(t) = \lambda_r P_3(t) \tag{5.150}$$

$$P_5'(t) + \mu_5 P_5(t) = \lambda_r P_1(t) \tag{5.151}$$

$$P_6'(t) + \mu_6 P_6(t) = \lambda_{cc} P_0(t) \tag{5.152}$$

At time $t = 0$, $P_0(0) = 1$ and all other initial state probabilities are equal to zero. The prime denotes differentiation with respect to time t. As time approaches infinity, Equations (5.146) $-$ (5.152) reduce to Equations (5.153) $-$ (5.159), respectively.

$$a_0 P_0 = \mu_r P_1 + \mu_s P_2 + \sum_{j=4}^{6} \mu_j P_j \tag{5.153}$$

$$a_1 P_1 = 2\lambda_r P_0 \tag{5.154}$$

$$a_2 P_2 = \lambda_s P_0 \tag{5.155}$$

$$a_3 P_3 = \lambda_s P_1 + 2\lambda_r P_2 \qquad (5.156)$$

$$\mu_4 P_4 = \lambda_r P_3 \qquad (5.157)$$

$$\mu_5 P_5 = \lambda_r P_1 \qquad (5.158)$$

$$\mu_6 P_6 = \lambda_{cc} P_0 \qquad (5.159)$$

Solving Equations (5.154) – (5.159), together with

$$\sum_{j=0}^{6} P_j = 1 \qquad (5.160)$$

result in the following steady state probabilities:

$$P_j = \frac{\omega_j}{D} \qquad (for \quad j = 0, 1, 2, 3) \qquad (5.161)$$

$$P_j = \frac{\omega_j}{D} \cdot \frac{1}{\mu_j} \qquad (for \; j = 4, 5, 6) \qquad (5.162)$$

$$D = \sum_{j=0}^{3} \omega_j + \sum_{j=4}^{6} \omega_j / \mu_j \qquad (5.163)$$

where

$$\omega_0 = a_1 a_2 a_3$$
$$\omega_1 = 2\lambda_r a_2 a_3$$
$$\omega_2 = \lambda_s a_1 a_3$$
$$\omega_3 = 2\lambda_r \lambda_s (a_1 + a_2)$$
$$\omega_4 = 2\lambda_r^2 \lambda_s (a_1 + a_2)$$
$$\omega_5 = 2\lambda_r^2 a_2 a_3$$
$$\omega_6 = \lambda_{cc} a_1 a_2 a_3$$

Using Equation (5.161), the robot system steady state availability with an operating safety

units and the robot system with or with a working safety units are given by

$$SSAV_{rs} = \sum_{j=0}^{1} \frac{\omega_j}{D} \tag{5.164}$$

and

$$SSAV_r = \sum_{j=0}^{3} \frac{\omega_j}{D} \tag{5.165}$$

where

$$D = \sum_{j=0}^{3} \omega_j + \sum_{j=4}^{6} \omega_j / \mu_j \tag{5.166}$$

Comparing these results with those obtained in Section 5.4.1, it is clearly evident that Equations (5.164) – (5.166) are in agreement with Equations (5.143) – (5.145), respectively.

5.4.3   Special Case Model Numerical Example

Setting:

$\lambda_s = \lambda_r = 0.0005$,
$\mu_s = 0.0006$, $\mu_r = 0.0007$, $\mu_4 = 0.0008$, $\mu_5 = 0.0009$, $\mu_6 = 0.0005$

in Equations (5.139) and (5.140), Figure 5.8 shows plots of the robot steady state availability (SSAV) vs common-cause failure parameter (λ_{cc}). The plots indicate SSAV for a robot system with constant repair rate and increasing number of stages (n) before system failure. The corresponding values associated with Figure 5.8 are given in Table 5-3. The trends shown by these values are discussed in the concluding section of this chapter.

Table 5-3: SSAV vs λ_{cc} values for a system consist of two robots and one safety unit. Common-cause failure rate is non-constant and all other failure and repair rates are constant.

λ_{cc}	$n = 0^*$		$n = 1$		$n = 2$		$n = 3$	
	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$	$SSAV_a$	$SSAV_r$
0.0000	0.3782	0.7416	0.4978	0.9713	0.5788	0.8250	0.6373	0.8493
0.0004	0.3177	0.6229	0.4314	0.6858	0.5127	0.7307	0.5736	0.7643
0.0008	0.2739	0.5370	0.3807	0.6051	0.4601	0.6557	0.5214	0.6948
0.0012	0.2407	0.4719	0.3406	0.5414	0.4173	0.5947	0.4780	0.6369
0.0016	0.2147	0.4209	0.3082	0.4898	0.3818	0.5441	0.4412	0.5879
0.0020	0.1937	0.3798	0.2814	0.4472	0.3518	0.5014	0.4097	0.5459

*Exponential distribution (constant common-cause failure rate)
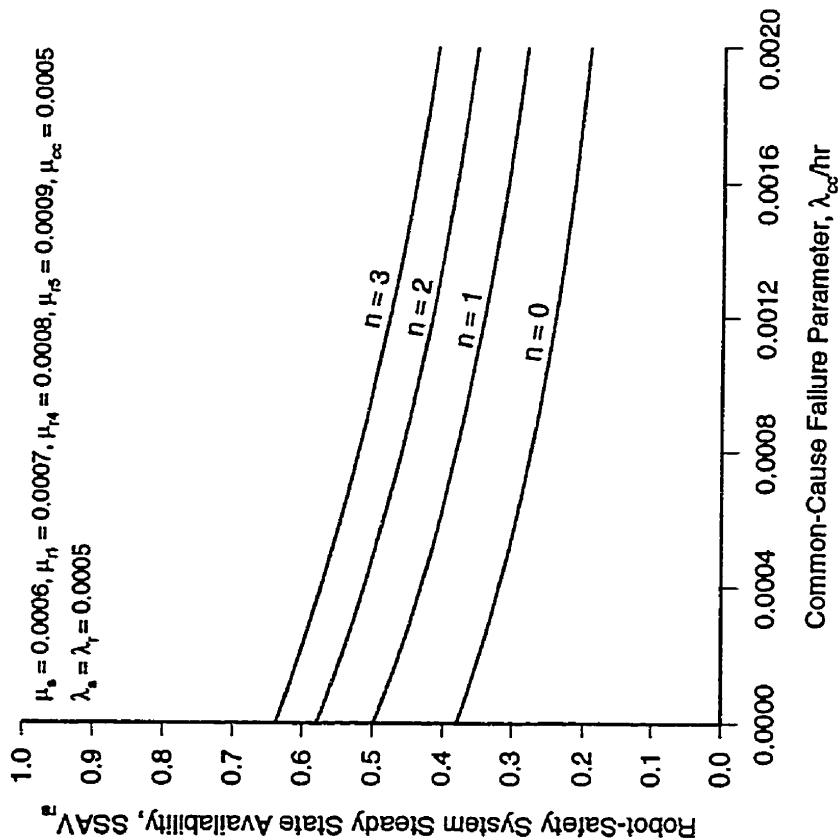
(a)
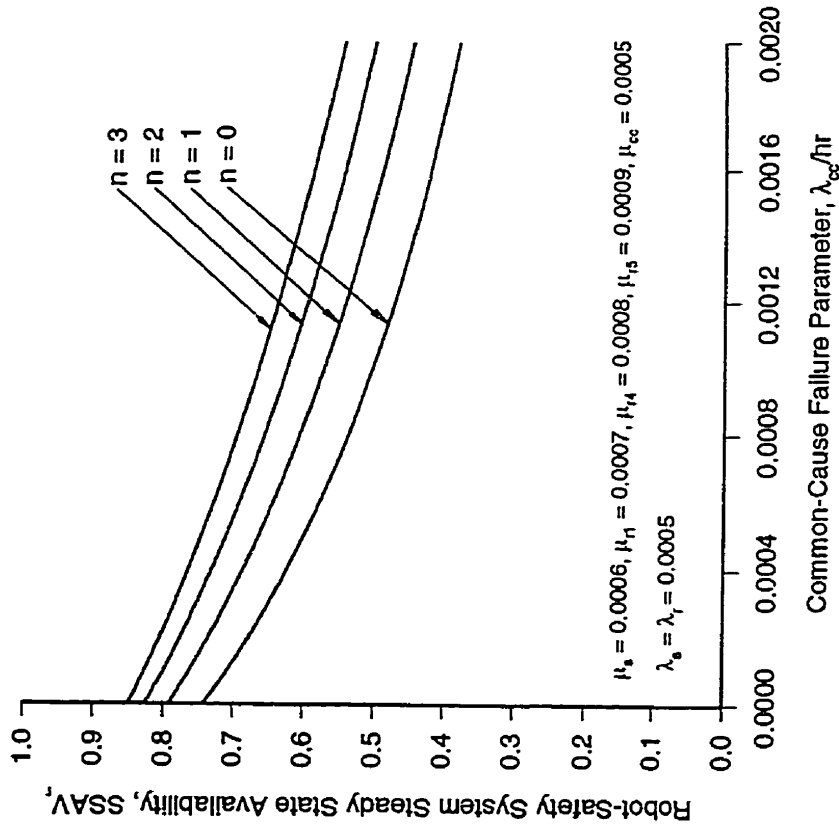
(b)

Figure 5.8.   Steady state availability vs λ_{cc} plots for a system comprised of redundant robots and common safety unit. Common-cause failure rate is non-constant and all other failure and repair rates are constant: (a) robot system working with an operating safety unit, (b) robot system working with or without the safety unit. More specifically, the plots were obtained using Equations (5.139) and (5.140).

5.5 Disscussion and Conclusions

Although the component redundancy improves system reliability, it is impossible to ensure near-perfect system reliability due to the possibility of common-cause failures. Theoretically, the higher the component redundancy, the greater the overall system reliability. In reality, the higher the equipment redundancy level, the greater the possibility of a common-cause failure.

This chapter marked two important issues with respect to the robot systems stochastic analyses. The first issue was concerned with robot systems having non-constant failure rates, and the second addressed the possibility of the occurrence of common-cause failures. A robot system with m-failure modes was modeled in Figure 5.1 and with the aid of stages method in combination with the supplementary variables technique, generalized steady state availability expressions were developed. Other performance indices such as time dependent availability, reliability, and mean time to failure were neglected because the main purpose of this chapter was to validate the applicability of the stages method to carry out non-constant failure rate analysis.

The concept of the stages method thrives from the fact that the distribution of the sum of n independent exponential random variables is gamma distributed. The sum of exponentially distributed random variables arise naturally in the theory of Markov chains, as the time required to make several transitions. This mechanism, through the insertion of dummy states, can be used to generate non-exponential transition times.

In Sections 5.3 and 5.4, the non-constant failure analysis was extended to the special case models presented in Chapters 3 and 4. Non-constant common-cause failure rates were

added to the existing models and generalized steady-state availability expressions were obtained for a system containing one robot with two identical safety units, and a system containing two identical robots with one safety unit. The main conclusion that can be drawn from this chapter can be summarized as follows:

- Stages method can be used for the stochastic analyses of systems with non-constant failure and repair rates. The basic principle associated with the stages method is the fact that through insertion of n-dummy states between the operating and the failed states, non-exponential transition times can be generated. The applicability of the stages method can be demonstrated by means of Markov models. For $n = 0$ or simply zero dummy states, one must obtain times to failure following the exponential distribution. Thus, for $n = 0$, using the stages method, one must obtain the same results as the Markov method. This fact was demonstrated in Sections 5.2 − 5.4 and results were identical.

- As the number of stages (sub-states) increase, so does the system availability. The increase in sub-states delays the time taken for the system to reach the failed state, hence, elevating the robot system availability.

6

DISCUSSIONS,
CONCLUSIONS,
AND
FUTURE DIRECTIONS

6.1 Discussions

This study presented a stochastic analysis of robot-safety systems. Literature and publications related to robot reliability and safety were collected and the most suitable reliability and safety evaluation techniques were identified.

In Chapters 3 and 4, analyses were conducted in relation to a system consisting of one robot with n-identical redundant safety units and n-identical redundant robots with one safety unit, respectively. Generalized expressions for such relevant system performance indices as the steady state availability, time dependent availability, reliability, and mean time to failure were developed. The Markov method and supplementary variables technique were utilized to obtain these performance indices. Markov method was used in models where failure and repair rates were assumed constant. The method of supplementary variables was

employed to deal with the non-Markovian models where the repair rates were assumed non-constant. Using both methods, steady state availability expressions were obtained. Various failed system repair time distributions (i.e., exponential, gamma, Weibull, Rayleigh, and log-normal distributions) were considered to fit the failed robot system repair times. Generalized time-dependent availability expressions were also developed with the aid of the supplementary variables technique. Only the gamma distribution was selected to represent the robot's failed system repair time distribution. The reason for this was the fact that gamma distribution possesses a rational Laplace transform and other distributions do not. Generalized reliability and mean time to failure (MTTF) expressions were developed by setting the robot's repair rate equal to zero. A system of first-order differential equations were obtained and with the aid of Laplace transforms, generalized reliability and MTTF expressions were developed. Special cases of the generalized model were presented and numerical analysis were performed resulting in the formation of numerical expressions and various plots demonstrating the end result.

In Chapter 5, the study was further extended to indicate the relevance of non-constant failure rates to the earlier models, particularly, non-constant common-cause failure rate. The method of stages in combination with the supplementary variables technique were used to deal with robot systems having non-constant failure and repair rates. A robot system with m-failure modes was modeled and generalized steady state availability expressions were developed. To validate the applicability of the stages method to these models, Markov method was also used to develop steady state availability expressions and the results were compared with those obtained with the stages method. Using the stages method, for $n = 0$

or simply zero stages before system failure, exponentially distributed times to failure were obtained. This meant that for $n = 0$, using the stages method, one must obtain the same results as the Markov method. This fact was demonstrated in Sections $5.2 - 5.4$ which the results were identical.

The analyses were extended to the models presented in Chapters 3 and 4 where non-constant common-cause failure rates were added to the existing models and steady state availability expressions were obtained. Again, stages method was verified by means of Markov method in which the results were identical.

Overall, this study treated the robot-safety systems by means of the stochastic processes. Although the nature of the treatment was mathematical, the intent was not to present mathematics for its own sake, but rather, its ultimate usefulness in engineering application. Stochastic processes present a valuable tool for investigating reliability and availability problems.

6.2   Conclusions

The main results associated with **Chapter 2** can be summarized as follows:

1.   The most frequent safety analysis techniques which are equally applicable in robotics are failure mode and effect analysis (FMEA) and fault-tree analysis (FTA).

2.   After considering such factors as simplicity and effectiveness, the most appropriate of the analytical methods are as follows:

    •   Failure Mode and Effects Analysis (FMEA)

    •   Fault Tree Analysis (FTA)

- Block diagram

- Combinational models (i.e., combined Fault Trees and block diagram)

- Markov and Non-Markovian Models

The following general conclusions can be made with the results associated with **Chapters 3 and 4**, unless otherwise stated:

1. When the failed robot system repair time x was represented by a gamma distribution, the steady state availability decreased as the shape parameter β, increased.

2. When failed robot system repair time x was represented by a Weibull distribution, the steady state availability increased as the shape parameter, β, increased.

3. When failed robot system repair time x was represented by a log normal distribution, the steady state availability dropped as the standard deviation, σ, increased.

4. For all failed system repair distributions, robot system steady state availability decreased as the safety unit failure rate increased.

5. For all failed system repair distributions, robot system steady state availability increased as the safety unit repair rate increased.

6. The Weibull distributed failed system repair time displayed the highest values of system steady state availability while the Erlangian distributed failed system repair time produced the least values for the system steady state availability.

7. If the robot system performance indices is correlated with a working safety unit, then its steady state availability, time dependent availability, reliability, and mean time to failure increased as the number of safety units increased. For example, system

availability increased by about 18% when two safety units were employed instead of one. This trend was applicable to Chapter 3 only.

8.   If the robot system performance indices is independent of its safety unit operating condition, then its steady state availability, time dependent availability, reliability, and mean time to failure value marginally increased as the number of safety units increased. For example, system availability increased only by about 3% when two safety units were employed instead of one. This trend was applicable to Chapter 3 only.

9.   Robot failing probability with an incident decreased as the safety mechanism repair rate increased. This trend was applicable to Chapter 3 only.

10. Robot failing probability with an incident increased as the incident failure rate, λ_{ri}, increased. This trend was applicable to Chapter 3 only.

11. Robot failing probability with an incident decreased as the number of safety units increased. For example, incident probability reduced by almost 50% when two safety units were employed instead of one. This trend was applicable to Chapter 3 only.

12. If robot system performance indices is correlated with a working safety unit, then its steady state availability, time dependent availability, reliability, and mean time to failure decreased as the number of robots increased. For example, system availability decreased by about 15% when three robots are employed instead of two. This trend was applicable to Chapter 4 only.

13. If the robot system performance indices is independent of its safety unit operating conditions, then its steady state availability, time dependent availability, reliability, and mean time to failure value marginally increased as the number of robots increased. For

example, system availability increased by about 5% when three robots are employed instead of two. This trend was applicable to Chapter 3 only.

The main conclusions that can be drawn from **Chapter 5** can be summarized as follows:

1.   Stages method is a valuable tool to deal with systems having non-constant failure or repair rates.

2.   As the number of stages (sub-states) increase, so does the system availability. The increase in sub-states delays the time taken for the system to reach the failed state, hence, elevating the robot system availability.

Finally, the failure rates selected in this study were extracted from published literature [30, 124]. The repair parameters were assumed to be slightly higher than the failure rates which may yield high mean time to system repair. This exercise would only affect the numerical results but not the expressions. The basic reason for selecting low repair parameter values was so that the resulting plots can be plotted with a good clarity. In fact, for any other parameter values, similar plots will be obtained. Appendix E presents plots for some other repair parameter values obtained from field data.

6.3 Future Directions

•   The study can be further expanded to obtain performance indices for a system containing n-redundant robots with n-redundant safety units, identical or non-identical.

•   In this study, only the common-cause failure rate was assumed non-constant, this can

also be stretched further to consider time-dependent failure rates for other sources of failures.

- Although the component redundancy improves system reliability, it is often limited by financial constraints and cost benefit concerns. Therefore, a further study can be conducted to consider the cost element. It may be practical to consider optimizing the number of safety units which is essential to satisfy not only the reliability and safety but also the cost.

- Due to computational difficulties, it is very difficult to invert resulting Laplace transform expressions from s-domain to time domain of those equations with a degree of polynomials greater than three, and consequently result in the inability to obtain general time-dependent expressions in those cases. Further studies should perhaps be conducted to deal with such difficulties. The simulation technique may be considered as an option despite its difficulties, time consumption and high costs.

- In this study, exponential, gamma, Erlangian, Weibull, Rayleigh, and log normal distributions were selected to fit the failed robot system repair times. Although these two-parameter distributions (2PD) are popular lifetime models, they are limited in their modelling capability. For example, if it is determined that the robot has a bath tub shaped hazard function, $h(x)$, none of these distributions will be appropriate unless a piecewise model over segments of the life time are used. The hazard functions of these distributions are either strictly increasing or decreasing with the exception of the log normal distribution which has an upside down bath tub shaped hazard function where $h(x)$ increases initially and then decreases. Analyses can be extended to consider other

two-parameter distributions such as muth, log logistic, inverse Gaussian, exponential power, Makeham. The exponential power distribution (EPD) may be the best choice to consider for further analysis. EPD has two properties that makes it unique. First, the hazard function increases exponentially for large t, and second, the EPD is one of the few two-parameter distributions that has a hazard function that can assume a bath tub shape. Three-parameter distributions (3PD) including Pareto, Gompertz, and generalized Pareto may also be practical since they take into account the possibility of accidental deaths by including an extra parameter. One other feature of 3PD is that they have increasing, decreasing, and bath tub shaped hazard function.

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APPENDIX A

PROBABILITY DISTRIBUTIONS: A REVIEW

A.1 Introduction

This section reviews various probability distributions, particularly, exponential, gamma, Weibull, Rayleigh, and log-normal. Except the exponential distribution, the rest of the aforementioned distributions are considered as two parameter distributions (2PD). Two parameter distributions have scale and shape parameters. Scale parameters are used to expand or contract the time axis by a factor. The shape parameter affects the shape of the probability density function and determines whether a distribution has increasing or decreasing hazard rates. Each distribution is summarized by its probabilistic properties including, density function (*p.d.f*), reliability function, hazard function, and the expected value of a density function.

A.2 Terms and Definitions

1) If X is a continuous random variable and has a density function $f(x)$, then the probability that X lies between two limits is given by

$$P(-\infty < X < \infty) = \int_{-\infty}^{\infty} f(x) \, dx = 1 \tag{A1}$$

2) If system reliability is the probability that the system operates successfully for a given period of time, then using Equation (A1), the reliability function of the system is

$$R(x) = 1 - \int_{0}^{\infty} f(x) \, dt = 1 - F(x) \tag{A2}$$

3) The hazard function which is time to system failure can be derived by using Equations (A1) and (A2)

$$\lambda(x) = \frac{f(x)}{R(x)} = -\frac{1}{R(x)} \cdot \frac{dR(x)}{dx} = \frac{f(x)}{1 - F(x)} \tag{A3}$$

4) Mathematical expectation[d] of a probability distribution function which is a measure of the location of the probability can be obtained by several different methods. Two of those methods are as follows:

I. $E[x] = \int_{0}^{\infty} x \cdot f(x) \, dx$ (A4)

II. $E[x] = Z'(0)$ (A5)

$$Z(t) = \ln[M(t)]$$

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[d]Hogg, R.V. and Tanis, E.A., *Probability and Statistical Inference*, Macmillan Publishing Inc., New York, 1977.

where

$$M(t) = \int_0^\infty e^{tx} f(x)\ dx \tag{A6}$$

Equation (A6) is also called the moment-generating function.

A.3 Exponential Distribution

A random variable X is said to have an exponential distribution if its *p.d.f* is defined by Equation (A7) [107].

$$f(x) = \lambda e^{-\lambda x} \qquad (x \geq 0,\ \lambda > 0) \tag{A7}$$

Using Equation (A3), the hazard function is constant and is given by

$$\lambda(x) = \frac{f(x)}{1 - F(x)} = \lambda \tag{A8}$$

The moment generating function is

$$M(t) = \int_0^\infty e^{tx} \lambda e^{-\lambda x}\ dx \tag{A9}$$

Solving Equation (A9), we obtain

$$M(t) = \frac{\lambda}{\lambda - t} \tag{A10}$$

Thus,

$$Z(t) = \ln \frac{\lambda}{\lambda - t}$$

$\therefore$

$$E[x] = Z'(0) = \frac{1}{\lambda} \tag{A11}$$

The probability distribution for various λ values are shown in Figure A1.
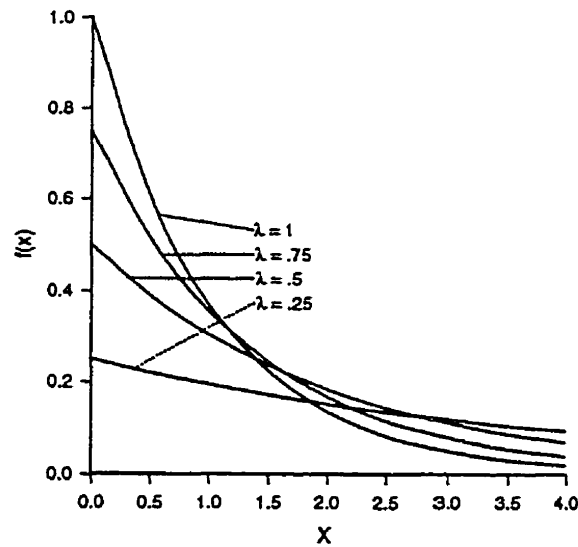

Figure A1.   Exponential probability distribution function for various λ values.

A.4 Gamma Distribution

If $f(t)$ does not follow the exponential distribution, the hazard rate is then time dependent. The random variable X has a gamma distribution if its *p.d.f* is defined by Equation (A12) [107].

$$f(x) = \frac{\lambda^{\beta} x^{\beta-1} e^{-\lambda x}}{\Gamma(\beta)} \quad ; \quad x \geq 0, \ \beta > 0 \tag{A12}$$

where β is a shape parameter, λ is a scale parameter, and $\Gamma(\beta)$ is the gamma function. Gamma distribution possesses two special cases. When $\beta = 1$, in Equation A12, the gamma distribution is equivalent to the exponential distribution which is indicated in Figure A2. When $\beta > 1$, a gamma random variable has the *Erlang* distribution which is the sum of n exponential random variables. The probability density function is illustrated for $\lambda = 1$ and
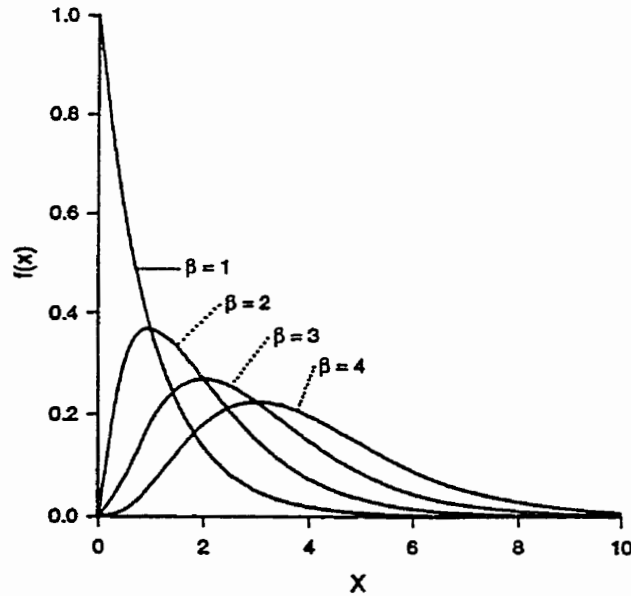
various β values in Figure A2.

Figure A2.   Gamma probability density function for $\lambda = 1$ and various β values.

The hazard function of the gamma distribution is given by

$$\lambda(x) = \frac{\lambda^{\beta} x^{\beta-1} e^{-\lambda x}}{\Gamma(\beta, \lambda x)} \quad ; \; x \geq 0, \; \beta > 0 \tag{A13}$$

where $\Gamma(\beta, \lambda x)$ is the incomplete gamma function and is defined by the integral

$$\Gamma(\beta) = \int_{\lambda x}^{\infty} y^{(\beta-1)} e^{-y} \, dy \quad , \; for \quad \beta > 0$$

For $\lambda = 1$ and various β, the hazard function increases when $\beta > 1$, decreases for $\beta < 1$, and is constant when $\beta = 1$. Using Equation A6, The moment generating function is

$$M(t) = \frac{\lambda^{\beta}}{(\lambda - t)^{\beta}} \tag{A14}$$

Thus,

$$Z(t) = \ln \frac{\lambda^{\beta}}{(\lambda - t)^{\beta}}$$

and

$$E[x] = Z'(0) = \frac{\beta}{\lambda} \tag{A15}$$

A.5 Weibull Distribution

The random variable X has a Weibull distribution if its *p.d.f* is defined by Equation (A16) [107].

$$f(x) = \lambda \beta x^{(\beta - 1)} e^{-(\lambda x)^{\beta}} \quad (\lambda, \beta > 0) \tag{A16}$$

where β and λ are shape scale parameters, respectively. Two special cases of the Weibull distribution are of interest. When $\beta = 1$ and $\beta = 2$, in Equation (A16), a Weibull random variable becomes exponential and Rayleigh distributions, respectively. Often, in engineering applications the value allocated for β is between 1 and 5. Weibull probability density function is shown in Figure A3 for $\lambda = 1$ and various β values. The hazard rate of a Weibull distribution is

$$\lambda(x) = \frac{\beta}{\lambda} x^{\beta - 1} \quad (\lambda, \beta > 0) \tag{A17}$$

The shape parameter β determines whether the hazard function is an increasing ($\beta > 1$), a constant ($\beta = 1$), or decreasing ($\beta < 1$) function of time. Plots of Equation (A17) are

shown in Figure A4. The mean of a Weibull distribution is[e]

$$E[x] = \frac{1}{\beta}(\frac{1}{\lambda})^{1/\beta}\Gamma(\frac{1}{\beta})$$
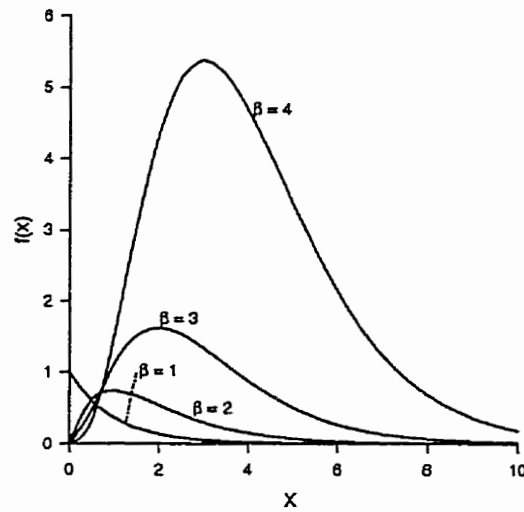
(A18)

Figure A3.   Weibull probability density function for $\lambda = 1$ and various β values.
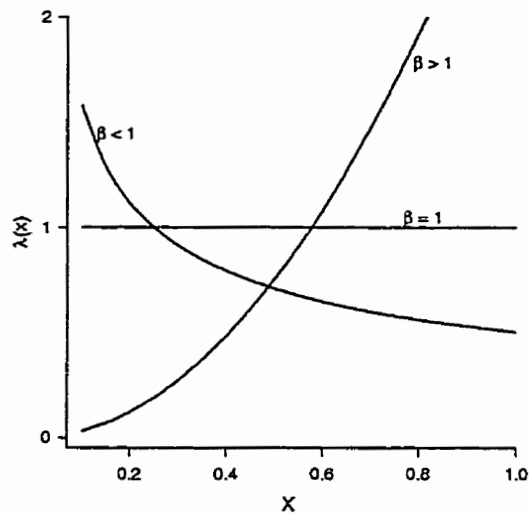

Figure A4.   Weibull hazard function for $\lambda = 1$ and various β values.

[e]Tsokos, C.P., Probability Distributions: An Introduction to Theory with Applications, Duxbury Press, Wadsworth, Ca., 1972, pp.290-291.

Although, Weibull and gamma distributions have similar probability density function plots, their differences become apparent when their hazard functions are compared. A lifetime with a gamma distribution will have an exponential tail, whereas, Weibull's hazard function possesses no boundaries and increases polynomially.

A.6 Rayleigh Distribution

As indicated in Section A.5, Rayleigh distribution is a special case of Weibull distribution for $\beta = 2$. Thus, Rayleigh distribution *p.d.f* becomes

$$f(x) = \frac{2}{\lambda} x e^{\frac{-x^2}{\lambda}} \qquad (\lambda > 0) \tag{A19}$$

The expected value of a Rayleigh distribution is

$$E(x) = \sqrt{\frac{\pi}{4\lambda}} \tag{A20}$$

A.7 Log-normal Distribution

A random variable X has a log-normal distribution if its *p.d.f* if given by

$$f(x) = \frac{1}{x\sigma_y\sqrt{2\pi}} e^{\left[\frac{-(lnx - \mu_y)^2}{2\sigma_y^2}\right]} \tag{A21}$$

where lnx is the natural logarithm of x with a mean and variance μ_y and σ_y, respectively. The conditions on parameters are given as follows:

$$\sigma_y = \ln\sqrt{1 + \left(\frac{\sigma_x}{\mu_x}\right)^2}$$

and

$$\mu_y = \ln\sqrt{\frac{\mu_x^4}{\mu_x^2 + \sigma_x^2}}$$

The expected value of a log-normal distribution is

$$E[x] = e^{\left(\mu_y + \frac{\sigma_y^2}{2}\right)} \tag{A22}$$

USEFUL
RELIABILITY
RELATIONSHIPS

B.1 General Reliability Formulas

General reliability analysis of models usually concern with the system reliability, the system mean time to failure and the system variance of time to failure. For instance, reliability of a system at any given time is the sum of all up-time state probabilities and is expressed by

$$R(t) = \sum_{i=0}^{n} [P_i(t)] = \mathscr{L}^{-1}[R(s)] = \mathscr{L}^{-1}[\sum_{i=0}^{n} P_i(s)] \tag{B1}$$

The mean time to failure or simply MTTF is the expected value, mathematically defined as the first moment of the failure time distribution and can be expressed in different forms:

$$MTTF = E(t) = \int_0^{\infty} t\, f(t)\, dt = \int_0^{\infty} R(t)\, dt \tag{B2}$$

$$MTTF = \int_0^{\infty} e^{-\int_0^{t} \lambda(x)dx}\, dt \tag{B3}$$

$$MTTF = \lim_{s \to 0} [R(s)] = \lim_{s \to 0} [\sum_{i=0}^{n} P_i(s)] \qquad (B4)$$

where R(s) is the Laplace transform of system reliability R(t).

In the above expressions, we are primarily dealing with system failure conditions. When system may be repaired on failure however, the interest is not wholly in the first time to failure, or first time to failure within a given interval, but whether the system is *available* and functioning, or is not functioning. Thus, availability at time t is the probability that the system is functioning at time t, and is the sum of the up-time probabilities, or;

$$AV(t) = \sum_{i=0}^{n} P_i(t) \qquad (B5)$$

From initial value theorem, the *point-wise* availability or system *steady-state* availability is given by

$$SSAV = \lim_{s \to 0} [s \cdot AV(s)] = \lim_{s \to 0} [s \cdot \sum_{i=0}^{n} P_i(s)] \qquad (B6)$$

APPENDIX C

MARKOV, SUPPLEMENTARY VARIABLES, AND THE DEVICE OF STAGES METHODS

C.1 Introduction

A stochastic process is a family of random variables observed at different times and defined on a specified probability space[f]. When all the random variables are exponentially distributed, the associated stochastic process is a time homogenous Markov process. The interstate transition rates are constant and availability computations are relatively simple. When, however, some of the random variables have non-exponential distributions, the interstate transition rates become functions of the time spent in states and the process becomes non-Markovian. The solution of non-Markovian models are generally difficult and

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[f]Ramakumar, R., Engineering Reliability: Fundamentals and Applications, Prentice-Hall, Inc., New Jersey, 1993.

the analytical techniques are usually of limited application in practical problems. There are few methods available to represfghent a non-exponentially distributed random variable, supplementary variables is one method and the other, the device of stages. Markov method, supplementary variables technique, and the stages method are explained in the following sections.

C.2 Markov Method

The following example demonstrates the applicability of the Markov method to a repairable robot system. The transition diagram of a repairable robot system with constant failure and repair rates is shown in Figure C1. The assumptions associated with the model in Figure C1 are

1. System failure is statistically independent.

2. System failure and repair rates are constant.

3. Fully repaired system is as good as new.

Robot constant failure rate

λ_r

Robot Operating

State 0

Robot Filed
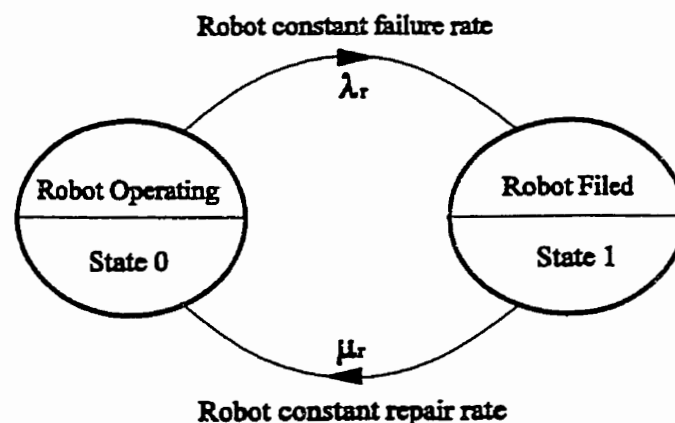
State 1

μ_r

Robot constant repair rate

Figure C1. State space transition diagram for a robot system with constant failure and repair rates.

The following equations translate the robot system diagrammatically described in Figure C1.

$$P_0(t + \Delta t) = P_0(t)(1 - \lambda_r \Delta t) + P_1(t)\mu_r \Delta t \tag{C1}$$

$$P_1(t + \Delta t) = P_1(t)(1 - \mu_r \Delta t) + P_0(t)\lambda_r \Delta t \tag{C2}$$

where

j State of the system; $j = 0$ means the robot is operating normally, $j = 1$ means robot system has failed.

$P_j(t)$ The probability that the robot is in state j at time t for $j = 0,1$.

λ_r Constant failure rate of the robot system.

$\lambda_r \Delta t$ Transitional probability of failure of the robot in finite time interval Δt.

$P_j(t + \Delta t)$ The probability that the robot system is in state j at time $t + \Delta t$ for $j = 0,1$.

$(1 - \lambda_r \Delta t)$ Probability of no failure in time interval Δt when the robot system is in state 0.

μ_r Constant repair rate of the robot system.

$\mu_r \Delta t$ Transitional probability of restoration of the robot in finite time interval Δt.

$(1 - \mu_r \Delta t)$ Probability of not restoring the robot system in time interval Δt.

Equations (C1) and (C2) can be rewritten as

$$P_0'(t) + \lambda_r P_0(t) = \mu_r P_1(t) \tag{C3}$$

$$P_1'(t) + \mu_r P_1(t) = \lambda_r P_0(t) \tag{C4}$$

Taking the Laplace transforms of Equations (C3) and (C4) and solving the resulting equations, we get

$$P_0(s) = \frac{s + \mu_r}{s^2 + (\mu_r + \lambda_r)s}$$

(C5)

$$P_1(s) = \frac{\lambda_r}{s^2 + (\mu_r + \lambda_r)s}$$

(C6)

Inverting the above equations, we can obtain time dependent probability expressions. In practice, we are usually concerned with the proportion of potential running time that the robot is available (up-time). Applying the final value theorem, the robot steady state availability (SSAV) and unavailability (SSUA) are as follows:

$$SSAV = \lim_{s \to 0} [s{\cdot}P_0(s)] = \frac{\mu_r}{\mu_r + \lambda_r}$$

(C7)

$$SSUAV = \lim_{s \to 0} [s{\cdot}P_1(s)] = \frac{\lambda_r}{\mu_r + \lambda_r}$$

(C8)

C.3 Supplementary Variables Technique

The following example demonstrates the applicability of the supplementary variables technique to repairable robot systems. The transition diagram of a repairable robot system with non-constant repair rate is shown in Figure C2. Sufficient number of supplementary variables are added to the definition of the robot's failed states which leads to the Markov processes. The assumptions associated with the model in Figure C2 are

1. System failure is statistically independent.

2. System failure and repair rates are constant.

3. Failed system repair times are arbitrary distributed.

4. Fully repaired system is as good as new.

Robot constant failure rate

λr

Robot Operating

State 0

Robot Filed

State 1
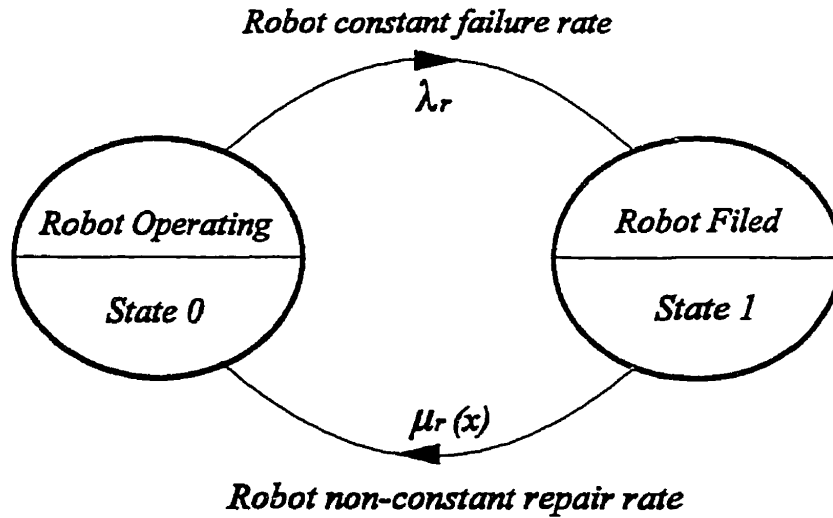
$\mu_r (x)$

Robot non-constant repair rate

Figure C2.   State space transition diagram for a repairable robot system with non-constant repair rate.

The following notation is used in the development of this model:

j denotes the jth state of the robot system; $j = 0$ (robot is operating normally), $j = 1$ (robot system failed).

$P_j(t)$ the probability that the robot is in state j at time t for $j = 0,1$.

$p_1(x, t)\Delta x$ the probability that at time t, the robot system is in state 1 and the elapsed repair time lies in the interval $[x, x + \Delta x]$.

λ_r constant failure rate of the robot system.

$\mu(x), q(x)$ repair rate and probability density of repair times, respectively, when robot system is in state 1 and has an elapsed repair time x.

Using the supplementary variables technique, the corresponding integro-differential equations associated with the Figure C2 are

$$P_0'(t) + \lambda_r P_0(t) = \int_0^\infty p_1(x,t)\mu_r(x)dx \tag{C9}$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_r(x) \right] p_r(x,t) = 0 \tag{C10}$$

The associated boundary condition is

$$p_1(0,t) = \lambda_r P_0(t) \tag{C11}$$

As time approaches infinity, the system reaches steady state and Equations (C9) – (C11) become

$$\lambda_r P_0 = \int_0^\infty p_1(x)\mu_r(x)dx \tag{C12}$$

$$\frac{d}{dx}p_1(x) = -\mu_r(x)p_1(x) \tag{C13}$$

$$p_1(0) = \lambda_r P_0 \tag{C14}$$

Using the integrating factor method and solving Equation (C13), we get

$$p_1(x) = p_1(0)e^{-\int_0^x \mu_r(\theta)\, d\theta} \tag{C15}$$

The steady state condition of the probability, P_1, that due to a failure the robot system is under repair, is

$$P_1 = \int_0^\infty p_1(x)dx \tag{C16}$$

Substituting Equation (C15) into Equation (C16), we have

$$P_1 = \int_0^\infty p_1(0)e^{-\int_0^x \mu_r(\theta)\,d\theta}\,dx \tag{C17}$$

Substituting Equation (C14) into Equation (C17), the resulting expression is

$$P_1 = \int_0^\infty \lambda_r P_0\, e^{-\int_0^x \mu_r(\theta)\,d\theta}\,dx = \lambda_r P_0 E[x] \tag{C18}$$

where

$$E[x] = \int_0^\infty e^{-\int_0^x \mu_r(\theta)\,d\theta}\,dx$$

which is the mean time to robot system repair when the failed robot system is in state 1 and

has an elapsed repair time of x, or the expected value of x. The robot mean time to repair,

$E[x]$, can be expressed for various repair time distributions which is given by

$$E[x] = \int_0^\infty x q(x)\,dx \tag{C19}$$

where $q(x)$ is the repair time probability density function (*pdf*). Using Equation (C18) and

$$P_0 + P_1 = 1 \tag{C20}$$

lead to the following steady state probability solutions:

$$P_0 = \frac{1}{1 + \lambda_r E[x]} \tag{C21}$$

$$P_1 = \frac{\lambda_r E[x]}{1 + \lambda_r E[x]} \tag{C22}$$

Using Equation (C21), robot system steady state availability is

$$SSAV = \frac{1}{1 + \lambda_r E[x]} \tag{C23}$$

If the failed robot system repair time x is gamma distributed, the probability density function (*pdf*) of the repair time is expressed by

$$q(x) = \frac{\mu_r (\mu_r x)^{\beta-1} e^{-\mu_r x}}{\Gamma(\beta)} \quad ; x \geq 0, \ \beta > 0 \tag{C24}$$

where x is the repair time, $\Gamma(\beta)$ is the gamma function, β and μ are the shape and scale parameters, respectively. Thus, the mean time to robot system repair, $E[x]$, for the gamma distribution is

$$E[x] = \int_0^\infty x q(x) \ dx = \beta / \mu_r \tag{C25}$$

Substituting Equation (C25) into Equation (C23), we get

$$SSAV = \frac{1}{1 + \lambda_r \beta / \mu_r} \tag{C26}$$

For $\beta = 1$, the repair time is constant and the probability density of repair times is exponentially distributed. Thus, Equation (C26) becomes

$$SSAV = \frac{\mu_r}{\mu_r + \lambda_r} \tag{C27}$$

This result agrees with Equation (C7). For $\beta = 2$, the repair time is non-constant and the probability density of repair times is Erlangian distributed. Therefore, Equation (C26) becomes

$$SSAV = \frac{\mu_r}{\mu_r + 2\lambda_r} \qquad (C28)$$

Steady state availability vs μ_r plots are shown in Figure C3 for various values of β. The figure indicates that as β increases steady state availability decreases, therefore, exponentially distributed failed system repair times yields the highest values for the long run availability. This behavior is expected because for $\beta > 1$, the gamma distribution has a strictly increasing hazard function, $h(x)$. The instantaneous repair rate for the system is expressed by

$$h(x) = \frac{q(x)}{1 - \int_0^t q(x)dx} \qquad (C29)$$

By substituting different values of β in Equation (C24), various probability density functions (*pdf*) of the failed system repair time can be obtained. Figure C4 shows the instantaneous repair rate for $\beta = 1, 2, 3$, and 4 as a function of time. From the figure, it is clearly evident that the instantaneous repair rate decreases as β increases, therefore, steady-state availability should be expected to decrease. Also, it is imperative to note that for all values of β, a lifetime with a gamma distribution will have an exponential tail, hence,

$$\lim_{x \to \infty} h(x) = \mu \qquad (C30)$$

where μ is the scale parameter.

Steady state availability plots vs λ_r plots for different failed system repair time distributions (i.e., Weibull, Rayleigh, and log normal) are also shown in Figure C5.
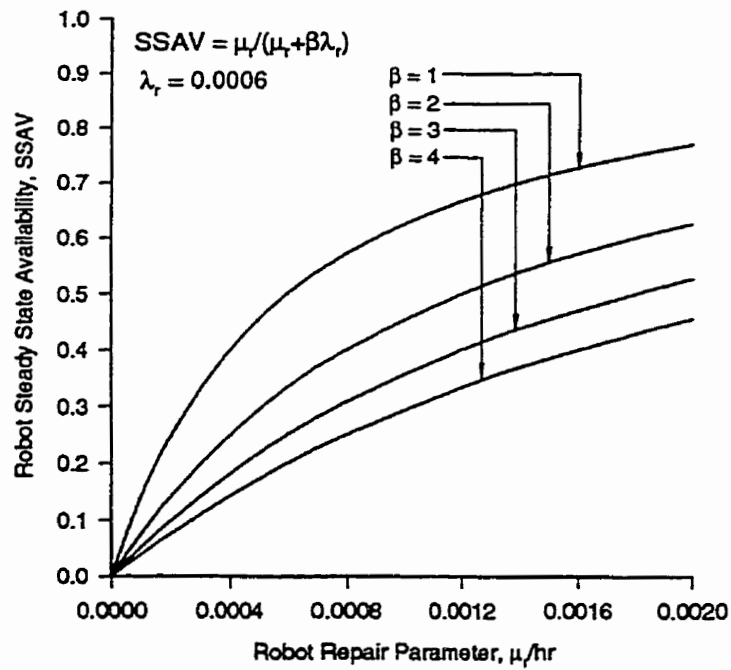
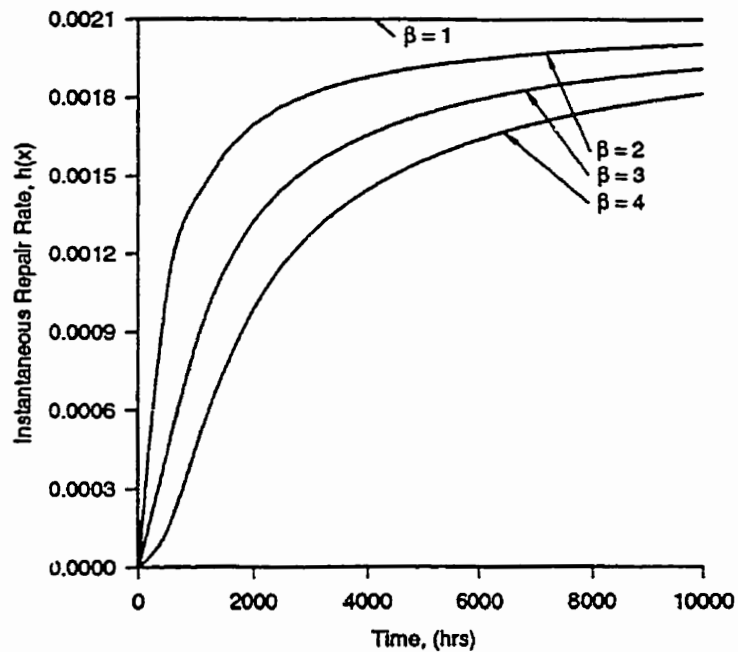Figure C3. Steady state availability vs μ_r plots for different β values.

Figure C4. Instantaneous repair rate vs time for different β values when $\mu = 0.0021$.
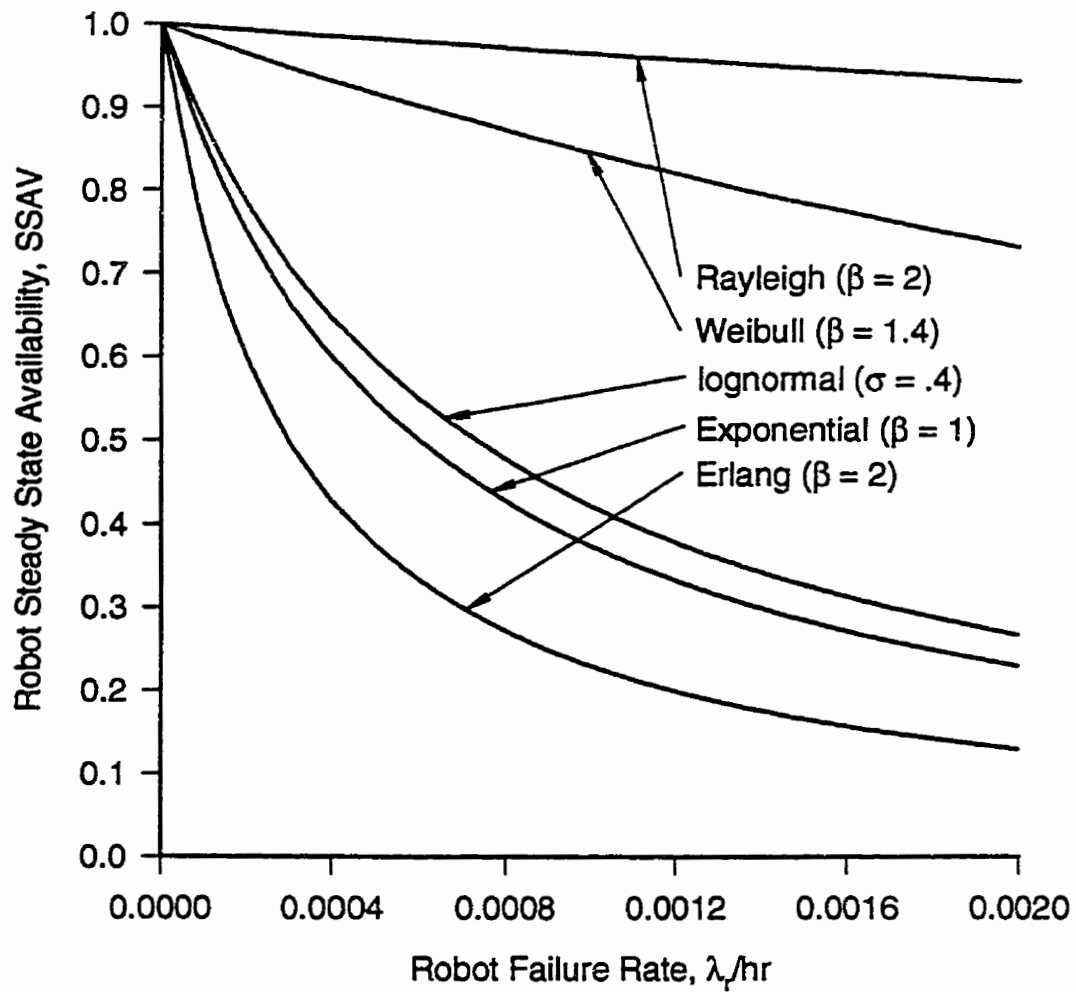
Figure C5. Steady state availability vs λ_r for various failed system repair time distributions.

C.4 Device of Stages Method

Consider the two-state (up and down) system shown in Figure C6 which is in operation over a long period of time. System's down state is non-constant and is replaced by n stages in series. The time till transition from the failed state to n stages before total repair is the sum of all transition times $\tau_{0 \to 1}, \tau_{1 \to 2}, ..., \tau_{n-1 \to n}$. The time-to-repair for each transition is assumed exponentially distributed and thus τ_n is the sum of all exponentially distributed random variables which results in non-constant system repair rate. Thus, the overall transition before system failure in Figure C6 is

$$\tau_{0 \to n} = \tau_{0 \to 1} + \tau_{1 \to 2} + ... + \tau_{n-1 \to n}$$

This procedure is called the device of stages and its application involves some approximation [234].
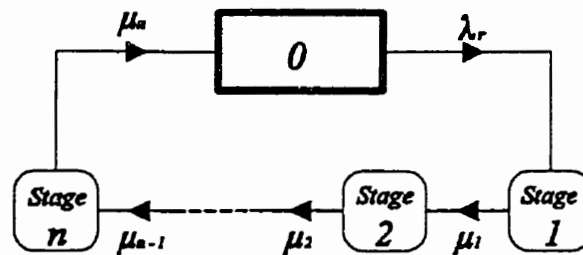
Figure C6.   State space transition diagram for a robot system with non-constant repair rate. Down state replaced by n stages in series.

The corresponding system of differential equations associated with model in Figure C6 are

$$P_0'(t) + \lambda_r P_0(t) = \mu_n P_n(t) \tag{C31}$$

$$P_1'(t) + \mu_1 P_1(t) = \lambda_r P_0(t) \tag{C32}$$

$$P_2'(t) + \mu_2 P_2(t) = \mu_1 P_1(t) \tag{C33}$$

$$P_i'(t) + \mu_i P_i(t) = \mu_{i-1} P_{i-1}(t) \qquad (for \quad i = 2, 3, \ldots, n) \tag{C34}$$

As time approaches infinity, Equations (C31) – (C34) reduce to Equations (C35) – (C38), respectively.

$$\lambda_r P_0 = \mu_n P_n \tag{C35}$$

$$\mu_1 P_1 = \lambda_r P_0 \tag{C36}$$

$$\mu_2 P_2 = \mu_1 P_1 \tag{C37}$$

$$\mu_i P_i = \mu_{i-1} P_{i-1} \qquad (for \quad i = 2, 3, \ldots, n) \tag{C38}$$

Solving Equations (C35) – (C38), together with

$$\sum_{i=0}^{n} P_i = 1 \tag{C39}$$

lead to the following system up state probability expression.

$$P_0 = \frac{1}{1 + \sum_{i=1}^{n} \lambda_r / \mu_i} \tag{C40}$$

But $SSAV = P_0$, therefore,

$$SSAV = P_0 = \frac{1}{1 + \sum_{i=1}^{n} \lambda_r / \mu_i} \tag{C41}$$

This model can be simplified by assuming that $\mu_1 = \mu_2 = ... = \mu_{n-1} = \mu_n = \mu_r$. With this assumption, all the n exponentially distributed durations will be identical. Therefore,

$$SSAV = \frac{1}{1 + n\lambda_r/\mu_r} \tag{C42}$$

When $n = 1$, we have the conventional constant hazard model with one up state and one down state, both exponetially distributed. Namely,

$$SSAV = \frac{\mu_r}{\mu_r + \lambda_r} \tag{C43}$$

This result agrees with Equations (C7) and (C27). When $n = 2$, the down state consists of two stages in series representing a special Erlangian distribution, or,

$$SSAV = \frac{\mu_r}{\mu_r + 2\lambda_r} \tag{C44}$$

By comparison, the results obtained by the method of stages agree with those by the Markov method and the supplementary variables technique.

APPENDIX D

SYMBOLS

The real and unique roots of Equations (3.115) and (3.116).

$$F_1 = \frac{H_1}{G_1} \quad , \qquad F_2 = \frac{H_2}{G_2} \quad , \qquad F_3 = \frac{H_3}{G_3}$$

$$F_4 = \frac{H_4}{G_4} \quad , \qquad F_5 = \frac{H_5}{G_5}$$

where

$H_1 = N_0(k - k_1)$

$G_1 = (k_1 - k_2)(k_1 - k_3)(k_1 - k_4)(k_1 - k_5)(k_1 - k_6)$

$H_2 = N_0(k - k_2)$

$G_2 = (k_2 - k_1)(k_2 - k_3)(k_2 - k_4)(k_2 - k_5)(k_2 - k_6)$

$H_3 = N_0(k - k_3)$

$G_3 = (k_3 - k_1)(k_3 - k_2)(k_3 - k_4)(k_3 - k_5)(k_3 - k_6)$

$H_4 = N_0(k - k_4)$

$G_4 = (k_4 - k_1)(k_4 - k_2)(k_4 - k_3)(k_4 - k_5)(k_4 - k_6)$

$H_5 = N_0(k - k_5)$

$G_5 = (k_5 - k_1)(k_5 - k_2)(k_5 - k_3)(k_5 - k_4)(k_5 - k_6)$

$$F_6 = \frac{H_6}{G_1} \quad , \qquad F_7 = \frac{H_7}{G_2} \quad , \qquad F_8 = \frac{H_8}{G_3}$$

$$F_9 = \frac{H_9}{G_4} \quad , \qquad F_{10} = \frac{H_{10}}{G_5}$$

where

$$H_6 = N_1(k = k_1) \quad , \qquad H_7 = N_1(k = k_2)$$
$$H_8 = N_1(k = k_3) \quad , \qquad H_9 = N_1(k = k_4)$$
$$H_{10} = N_1(k = k_5)$$

APPENDIX E

Miscellaneous Plots

In Chapters 3 and 4 numerical equations were obtained by inserting arbitrary repair parameter values ($\mu_1 = 0.0006$/hr, $\mu_2 = 0.0007$/hr, $\mu_3 = 0.0008$/hr, $\mu_4 = 0.0009$/hr) into the generalized steady state and time dependent availability expressions and performing numerical analysis. This section provides results for some other repair parameter values which were obtained from an actual field data.

For any system to be useful its mean time to repair must be much smaller than its mean time to failure, thus in this study, repair rates were assumed to be comparable to failure rates but greater. In practice, the values of the basic system parameters are assessed from field data. Critchlow [98] assessed the reliability of a robot comprised of components for which failure and repair rate data were available from a data base. He recommended that well-designed robots are expected to have a useful life of at least 40,000 working hours, Mean Time To Failures (MTTF) of at least 400 hours/failure($\lambda \leq 0.0025$), and a Mean Time To Repair (MTTR) of no more than 8 hours/repair ($\mu \geq 0.125$). Thus setting:

$\lambda_s = 0.0005, \quad \lambda_{ri} = 0.0003, \quad \lambda_{rs} = 0.0004, \quad \lambda_r = 0.0005$
$\mu_1 = 0.1, \quad \mu_2 = 0.105, \quad \mu_3 = 0.115, \quad \mu_4 = 0.125$

into Equations (3.98) – (3.104), we obtained robot-safety system steady state availability numerical values. Figures E1 shows plots of the robot-safety system steady state availability for various repair time distributions. These plots indicate the steady state availability as a function of safety unit (mechanism) failure rate, λ_s. The plots clearly indicate similar trend as for those obtained in Chapters 3 and 4, except that the steady state availability in this case is much higher. This was anticipated since the repair parameter values are much greater.

Inserting the same above parameter values into Equations (3.121) and (3.122), time-dependent availability plots of the robot system with an operating or a failed safety mechanism are shown in Figure E2. Comparing these plots with availability plots obtained in Chapters 3 and 4, one can observe the obvious similarities.

An increase in the robot system reliability and mean time to failure with greater robot system repair rate is also depicted by Figures 3.15 and 3.16.
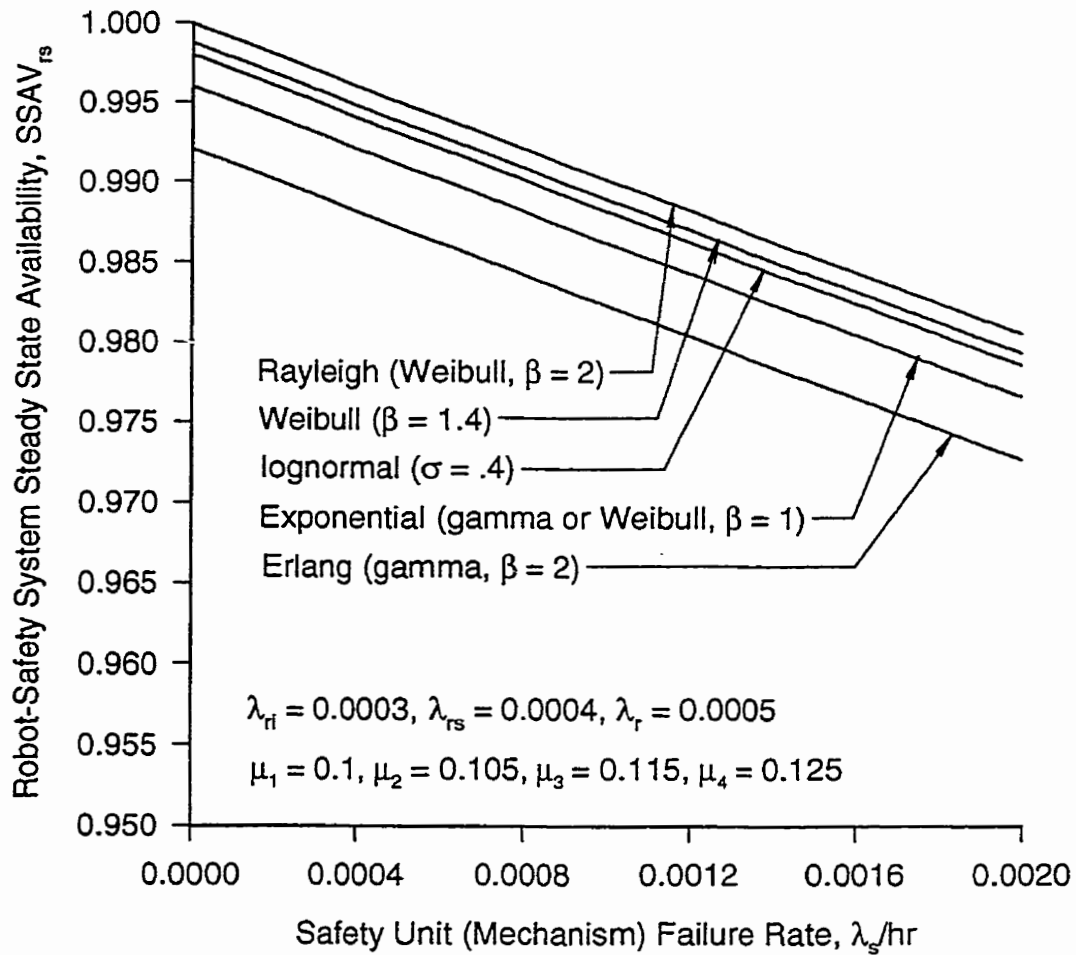
Figure E1. Steady state availability ($n = 1$) vs λ_s plots for a robot with constant failure rate and for various failed system repair time distributions. More specifically, these plots were obtained using Equations (3.98) – (3.104).
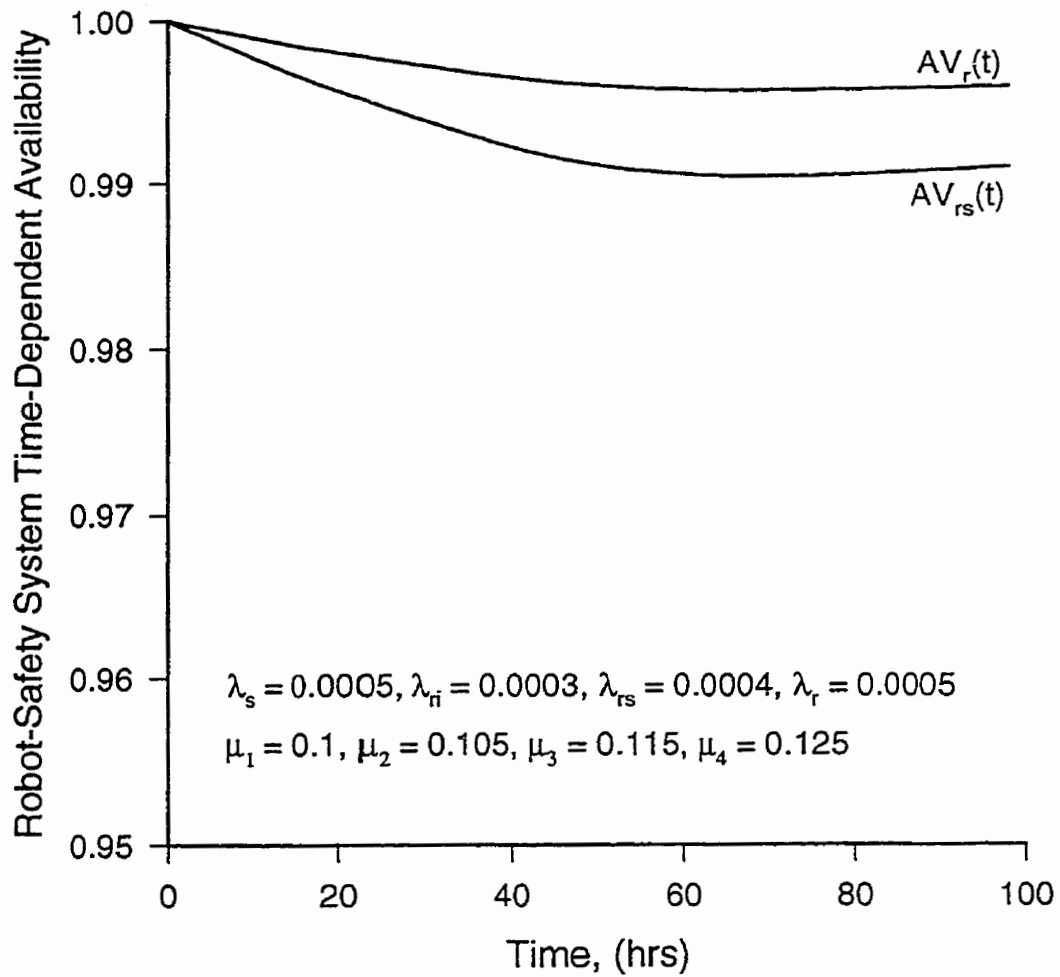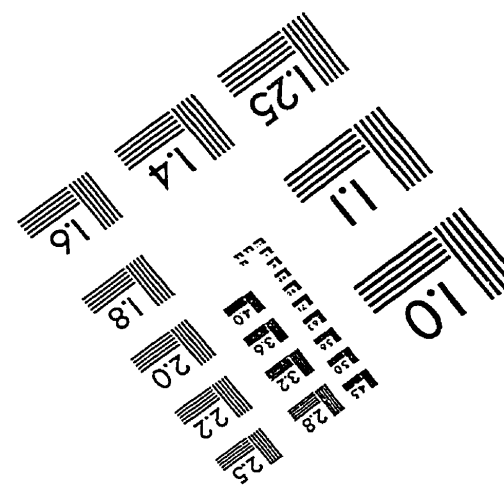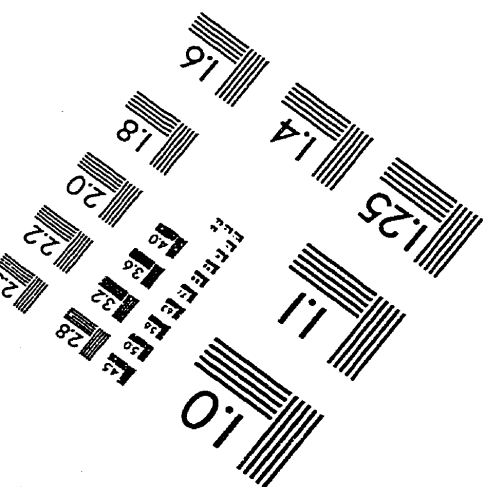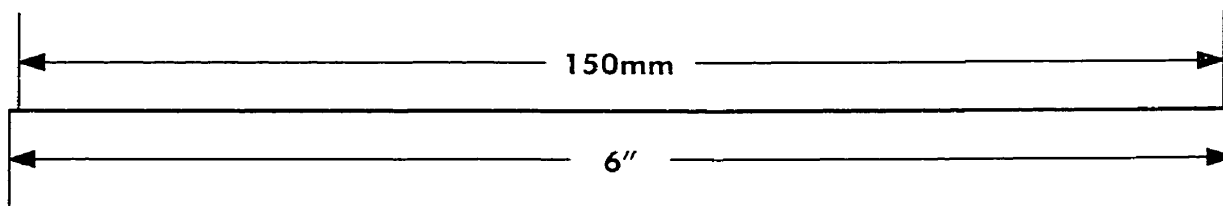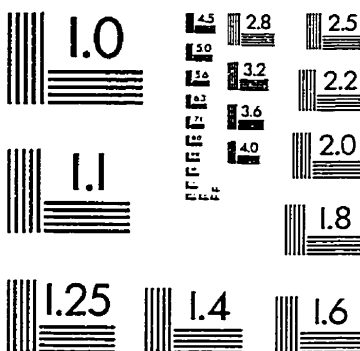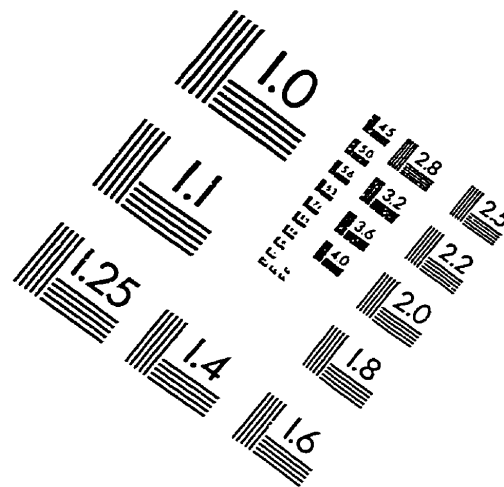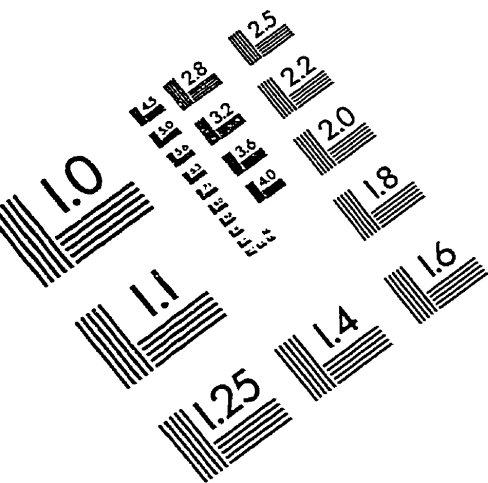
Figure E2. Availability ($n = 1$) plots for a robot with constant failure and repair rates. More specifically, the plots were obtained using Equations (3.121) and (3.122).

IMAGE EVALUATION
TEST TARGET (QA-3)

150mm

6"