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**THERMAL RESIDUAL STRESSES
IN BONDED COMPOSITE REPAIRS
ON CRACKED METAL STRUCTURES**

by

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Dipl.-Ing., Technische Universität Braunschweig, 1993

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

in

THE FACULTY OF GRADUATE STUDIES
(Department of Mechanical Engineering)

THE UNIVERSITY OF BRITISH COLUMBIA

September 1998

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0-612-34504-1

Abstract

The objective of this research is to determine the thermal residual stresses and strains in bonded composite repairs on cracked metal structures. This work is an essential contribution to a fatigue damage initiation model for bonded composite repair, where knowledge of the initial stress/strain state after an elevated temperature cure is important. Furthermore, this work is an elementary part for the development of a generic certification approach to bonded composite repairs. Accounting properly for thermal residual stresses in test specimens and in real applications will assist in determining the true feasibility of a bonded composite repair.

The objective of this work was realized in four stages of research. In the first stage, seven AMRL sandwich type composite bonded repair specimens were manufactured, of which one was instrumented by placing 44 strain gauges at eight planar locations and within five different interfaces. Residual strains at ambient temperature (including both thermal residual strains and other process induced strains) were measured during the manufacturing process. In the second stage, the stress free temperature for the repaired specimen was experimentally determined and the thermal residual strains measured as a function of operating temperature. In the third stage, a theoretical analysis was carried out to estimate the thermal residual stress and strain distributions in various bonded repairs. This analysis also addressed the effect of symmetrical disbonds around the crack. Finally, a finite element analysis was carried out to assess the limitations of the theoretical analysis as well as to provide a more detailed insight into the complex thermal residual stress and strain state of the AMRL sandwich type specimen.

During this work it was found that high thermal residual strains (reaching 15% of the yield strain) are present in the bonded repair specimen at ambient temperature. Previous

analysis schemes predicted results nearly 60% higher. The thermal residual strain versus temperature measurement showed that only very small changes in thermal residual strains occurred above 90°C leading to a defined effective stress free temperature of 85.8°C for the employed adhesive FM 73M. By utilizing an effective stress free temperature, a linear-elastic approach was used to model thermal residual stresses and strains in composite bonded repairs. Major achievements in the theoretical analysis include a linear-elastic closed form solution for tapered joints and reinforcements without the need for a numerical solution scheme, a stress field prediction ahead of the crack tip for the metal substrate of a bonded repair based on a concise complete solution of the classical fracture mechanics problem of a center crack in an infinite plate and, an extended Rose model for the prediction of the stress intensity factor of a bonded repair with symmetrical disbonds showing the severity of thermal residual stresses especially for partially disbonded composite repairs to cracked metal specimens.

The key to precise predictions of thermal residual stresses in bonded composite repairs is the knowledge of the adhesive behaviour at elevated temperatures under thermal residual stress loading. A generic type specimen is presented which allows to investigate the relevant adhesive behaviour.

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List of Symbols

<u>Symbol</u>	<u>Description</u>	<u>Dimensions</u>
a	Crack length	[mm]
A	Constant	[MPa]
b	Disbond length	[mm]
B	Constant	[MPa]
c	Geometry factor	[]
C	Integration constant	[N/mm]
d	Bolt diameter	[mm]
e	Function	[]
E	Young's modulus	[GPa]
f	Function	[]
F	Force	[N]
F_c	Function	[N/mm]
F_f	Function	[]
g	Integration constant	[mm]
G_a	Adhesive shear modulus	[MPa]
G_p	Patch shear modulus	[MPa]
G_s	Substrate shear modulus	[MPa]
G	Energy release rate	[MPa m]
h	Integration constant	[mm]
H	Function	[]
I	Current	[A]
K	Stress intensity factor	[MPa $\sqrt{m}$]
K_t	Transverse sensitivity factor	[]
l	Length of steps	[mm]
$l_{5mRef.}$	5 m Lead wire reference specimen	[mm]
n	Number of steps	[]
N	Number of cycles	[]
P	Applied load per unit length	[N/mm]
r	Distance from crack tip	[mm]
r_p^*	Plastic zone radius	[mm]
R	Resistance	[Ω]
S_G	Strain gauge factor	[]

<u>Symbol</u>	<u>Description</u>	<u>Dimensions</u>
SR	Stress ratio	[]
t	Thickness	[mm]
T	Direct stress resultant	[N/mm]
v	Displacement	[mm]
V	Voltage	[V]
W	Specimen width	[mm]
y	Axial coordinate of joint	[mm]
Y	Geometric correction factor	[]
z	Transverse coordinate of joint	[mm]
$Z()$	Complex stress function	[MPa]
α	Coefficient of thermal expansion	[$\mu\epsilon/^{\circ}\text{C}$]
δ	Constant	[]
ΔC	Function	[N/mm]
T	Temperature change	[$^{\circ}\text{C}$]
ϵ	Strain	[]
γ_a	Adhesive shear strain	[]
γ	Constant	[]
λ	Elastic shear stress distribution parameter	[1/mm]
Λ	Characteristic crack length	[mm]
ν	Poisson's ratio	[]
σ	Normal stress	[MPa]
τ	Shear stress	[MPa]
ξ	Range variable	[]
ϕ	Constant	[]
θ	Angle	[$^{\circ}$]

<u>Subscript</u>	<u>Description</u>
a	Adhesive
Al	Aluminum
$AMRL$	AMRL sandwich type specimen
B/Ep	Boron/epoxy
$corr.$	corrected
D	Double-doubler joint
Δ	Increment

<u>Subscript</u>	<u>Description</u>
Fourier	Discrete Fourier solution
G	Gauge
i	Range variable
I	Mode I
j	Range variable
LW	Lead wire
max	Maximum
min	Minimum
o	Reference
p	Patch
Preload	Preload
Ref.	Titanium silicate reference specimen
RS	Residual strain
SFT	Stress free temperature
s	Substrate
TS	Titanium silicate specimen
TSY	Transverse sensitivity
TSR	Thermal residual strain
r	Reinforced
R	Double-sided reinforcement
Shear – lag corr.	shear-lag corrected
u	Upper bound for short crack
TRS	Thermal Residual Stress
RS	Remote Stress
∞	Upper bound for long crack
$\perp$	Perpendicular
$\parallel$	Parallel

<u>Abbreviations</u>	<u>Description</u>
IAR	Institute for Aerospace Research
NRC	National Research Council of Canada
UBC	University of British Columbia
AMRL	Aeronautical and Maritime Research Laboratory
ARALL	Aramid Reinforced ALuminum Laminate
GLARE	GLASS Reinforced

Acknowledgements

I gratefully acknowledge the HSP II scholarship of the German Academic Exchange Service (DAAD) as well as the University Graduate Fellowships from the University of British Columbia. I would like to thank the National Research Council of Canada for providing funding for this research project as well as the scientific support by Mr. Donald Raizenne. In addition, I would like to thank Dr. Douglas P. Romilly as well as my research committee members, Dr. Desmond Tromans, Dr. Reza Vaziri, Dr. Henry Vaughan and Mr. Dean Leonard, for their continuing efforts and consisting guidance. Thank you also to Dr. Alan Baker for sharing some of his knowledge of bonded composite repairs. Furthermore I would like to thank Jeff Aitchison, Rudolf Seethaler, Rob Thomson and Randy Clark for the helpful ideas and discussions.

The assistance of T. Benak and other staff members of the Institute for Aerospace Research (NRC) as well as the staff of the Mechanical Engineering Machine Shop at the University of British Columbia during specimen preparation is gratefully acknowledged.

Special thanks to my parents as well as my wife Susan for their continuing support, patience and love.

Chapter 1

Introduction

Throughout the past twenty years, there has been an increasing trend towards operating commercial and military aircraft structures beyond their original fatigue design life. To achieve this, cost efficient repair schemes must be developed that will both restore the strength of the damaged structure and prevent existing cracks from becoming critical. In 1973, researchers at the Aeronautical and Maritime Research Laboratory (AMRL) in Melbourne, Australia, first proposed the repair of cracked metal aircraft structures using adhesively bonded patches fabricated from composite materials [18]. In addition to the repair of damaged structures, composite patches can be used as reinforcement to increase load carrying capability and improve fatigue resistance.

To date, a number of approaches involving analytical, numerical and experimental analysis have been undertaken to develop design schemes for composite repairs of metallic aircraft structures. One of the important areas of continuing research is the analysis of fatigue damage in composite repairs. The fatigue failure behaviour of a bonded composite repair is dependent on both the mean stress and the stress amplitude. Fatigue failure characteristics are furthermore complicated by the stress history prior to the repair and the stresses induced during the repair. Properly designed and installed bonded composite repairs can reduce or eliminate crack growth by reducing the crack opening displacement and subsequently

the stress intensity factor range. Additional retardation of the crack growth results from the difference in plastic zone size prior to and after patching due to the reduction in the peak stress intensity factor. These beneficial retardation effects may, on the other hand, be offset by thermal residual stresses.

Thermal residual stresses are induced during the repair process as a result of the elevated temperature cure which is required to achieve suitable adhesive characteristics. The magnitude of the thermal residual stresses is dependent on the difference in the coefficients of thermal expansion between the metallic substrate and the composite patch material, the local heating and support conditions of the metallic structure and the temperature differential between the stress free temperature and the operating temperature. The heating and local support conditions in the metallic structure lead to a lower effective coefficient of thermal expansion (CTE) since the unheated surrounding structure will restrain the expansion of the locally heated repair section. In addition, the supporting structure of an aircraft frame will limit the out-of-plane bending and resulting curvature of a single-sided repair, which is also caused by the difference in CTE [18, 53].

Thermal residual stresses can be quite large depending on these local heating and support conditions, as well as material characteristics, thus affecting the desired reduction in the stress intensity factor significantly. It is therefore important to realize that thermal residual stresses will vary substantially between bonded composite repair test specimens which are heated uniformly without any restraint and real structural repairs which have significant restraint (see Figure 1.1 [14]). The key is to understand the relationship between the constraint conditions of the test specimen relative to those of the damaged aircraft structure such that the thermal residual stresses can be accounted for properly in the determination of the margin of safety.

Figure 1.1: Small scale test specimen for composite repairs

Precise knowledge of the stress state in the metal substrate after the patching process including thermal residual stresses is essential in order to carry out a detailed investigation of the fatigue damage initiation characteristics for bonded composite repairs. Thermal residual stresses will also affect the fatigue crack growth rates by changing the fatigue stress ratio for the metal substrate in the bonded repair section. Baker showed an approximate 10% change in the stress intensity by accounting for the change in the stress ratio from 0.1 to 0.6 due to thermal residual stresses [18]. Thermal residual stresses have often been quoted in the literature as a possible contributing factor without really knowing their true effect on the fatigue behaviour [18].

This research addresses this shortfall by investigating the magnitude of the thermal residual stresses in detail to establish the foundation for a fatigue damage initiation model and an improved fatigue crack propagation model. This research also examines the magnitude of temperature independent process induced residual strains from effects such as anisotropic shrinkage of the patch or adhesive, temperature gradients in the specimen during processing, and/or temperature gradients due to tooling and internal resin flow [50].

Based on the need to establish the true stress/strain state after carrying out a bonded composite repair, the following research objectives were identified:

1. Determine the critical locations in a bonded composite repair;
2. Determine appropriate process specification, and manufacture suitable test specimens including an instrumented specimen;
3. Measure thermal residual stresses/strains and estimate temperature independent process induced strains;
4. Establish an appropriate thermal residual stress/strain model for bonded composite repairs, including an estimate for the stress distribution near the crack tip;
5. Determine the influence of disbonds on the stress intensity factor under thermal residual stress loading;
6. Establish the limitations of the thermal residual stress/strain model;

These objectives were achieved in four stages: In the first stage, six AMRL sandwich type composite bonded repair specimens were manufactured to provide some of the specimens required for fatigue damage initiation testing. In addition, one test specimen was instrumented by placing 44 strain gauges at eight locations and within five different interfaces of the patched component (Chapter 3). Residual strains at ambient temperature (including both thermal residual strains and other process induced strains) were measured during the manufacturing process. In the second stage, the stress free temperature of the bonded repair was experimentally determined and the thermal residual strains measured as a function of operating temperature (Chapter 4). In the third stage, a theoretical analysis was carried out to determine the thermal residual stress and strain distributions in bonded repairs and reinforcements. This analysis also addressed the effect of symmetrical disbonds around the

patched crack as well as the stress field ahead of the crack (Chapter 5). Finally, a detailed finite element analysis was carried out to assess the limitations of the theoretical analysis as well as providing a more detailed insight into the complex thermal residual stress and strain state of the AMRL sandwich type specimen (Chapter 6).

A literature review (Chapter 2) is presented next to establish the state-of-the-art of composite repair research, thus providing the necessary background and highlighting the direction for the current work.

Chapter 2

Literature Review

2.1 Introduction

The establishment of the damage tolerance approach in the airworthiness regulations allows aircraft operators to utilize their aircraft economically, well beyond their original fatigue design. An aircraft's life is now being determined by the economical balance between revenue and maintenance costs. The increasing number of aging aircraft impose a growing challenge for aircraft maintenance facilities to provide safe and economical feasible repair schemes. The classical repair scheme for metal aircraft structures are riveted repairs. If a riveted repair is not feasible, the entire component must be replaced, which can be very expensive considering parts, labour and aircraft downtime. Chartering a replacement aircraft to meet operational requirements can be quite costly. Furthermore, riveted repairs often have a limited life since they introduce new potential fatigue crack initiation sites.

In 1973, the Aeronautical and Maritime Research Laboratory (AMRL) in Australia first proposed the repair of cracked metal aircraft structures using composite materials. This technology offers an excellent alternative to riveted repairs especially for fleet wide repairs. Not introducing additional fatigue crack initiation sites in a highly loaded structure is one of the key benefits of this repair scheme over riveted repairs. Unfortunately this technology also has its downsides. The quality of the bond between the metal aircraft structure

and the composite patch is highly depended on the process specifications and quality of workmanship provided by the technician carrying out the repair. A workshop organized by the Royal Australian Air Force addressing standardization of bonded repairs focused on this problem as part of the International Conference on Composite Materials ICCM-11 in 1997. AMRL has proven that these repairs can be carried out successfully as demonstrated by their experience with Hercules C130 wing plank repairs. While over 1000 repairs have been carried out on stiffeners of the C130, no crack growth has been observed in 15 years of service [45].

Most of the composite repairs have been carried out on military aircraft [45]. In recent years, Textron has promoted the application of bonded repairs to civilian aircraft by installing bonded doublers on Federal Express 747s for evaluation [20]. Examples of ongoing research to improve the understanding and confidence in bonded repairs is AMRL's work on assessment models for damage tolerance, such as the consequences of initial flaws in bonded repairs, impacts and damage growth [11, 13]. Furthermore, AMRL is working on a generic approach to bonded repairs since this is the key to offering a cost effective solution for a broader range of repair applications. Development of smart patches which detect disbond and damage growth is also underway. The research work presented in this thesis addresses the fundamentals for a fatigue damage initiation investigation for the cracked metal component in bonded repairs. Currently bonded repairs are evaluated based on their reduction in the crack growth rate of the damaged structure. Although this is a conservative approach, it neglects a significant portion of the component life. Knowledge of the time required to reinitiate the crack would improve patched structure life predictions and increase the confidence levels of certification authorities in this repair technique. This information becomes even more critical if repairs which have an initial crack length close to the critical crack length are carried out, such as for the F-111 lower wing skin repair [97]. The first step in a life prediction is to determine the initial stress state of the bonded repair prior

to applying any external load and must include the thermal residual stresses within the patched structure. Proper experimental and theoretical determination of thermal residual strains is not only required to assess fatigue damage initiation, but also to allow for a more precise determination of the actual load experienced by the bonded repairs for the critical load cases which determine the feasibility of a repair.

This chapter presents background information on patching technology, its advantages and disadvantages as well as the relevancy of the past research with respect to the current key issues of this repair method.

2.2 Bonded Repairs Versus Riveted Repairs

Currently riveted repairs represent the main repair scheme for metal aircraft structures. Their installation is generally easy and requires only standard equipment. Additional advantages are the possibility of subsequential disassembly as well as repair installation in an uncontrolled environment. On the other hand, 'riveted repairs can substantially degrade the fatigue life if extreme care is not taken' [35, 86]. Generally, riveted repairs can restore the static strength while the fatigue strength is often neglected. Riveting produces concentrated stresses at the bearing surfaces resulting in local stress risers and fatigue crack initiation sites [18]. In most cases the maintenance engineer does not have access to original loads to carry out a thorough analysis but must rely on the Structural Repair Manual (SRM) provided by the original equipment manufacturer showing approved methods and maximum repairable sizes [35]. The installation can also create problems such as in-situ drilling during the riveted repair process causing internal damage to items such as hydraulic lines and electrical wiring. A poor mechanical fastening procedure may result in fretting damage or high residual stresses, thus encouraging fatigue cracking and/or stress-corrosion cracking [18].

An example of a poor repair was given by Fredell [35]. Figure 2.1 shows the repair to the lower wing skin of a Lockheed Electra. This particular repair was designed to reinforce the cracked section of the original blade-stiffened wing skin. Additionally, two riveted angled triplers inside the blade stiffeners were installed. Figure 2.1 [35] shows an alternating rivet pattern in the first and last rivet row. This particular rivet scheme lead to a higher bearing load in the remaining rivets in the end rows. The consequences of this riveting pattern in combination with a very stiff patch were new fatigue cracks after approximately 10000 flight hours. As a result, a complete fracture of the wing plank occurred. A previous inspection didn't reveal the new crack between location A and B since the design obstructed the inspectors view. This example shows some of the problems with riveted repairs if extreme care is not exercised in the design stage to avoid fatigue problems. In fact, one of the largest loss of life from a single aircraft accident was traced to an improperly performed structural riveted repair. In August 1985, 520 people died when the aft pressure bulkhead of a Japan

Figure 2.1: Sketch of the Lockheed Electra lower wing skin

Air Lines Boeing 747 failed due to a mistake in the riveted bulkhead repair which caused a crack to form and propagate along the repair line [35].

While riveted repair schemes are more widely accepted, bonded repair techniques certainly offer much better fatigue characteristics since no additional rivet holes are introduced during the repair process. The high stiffness of the composites allows the implementation of thin patches thus offering aerodynamic advantage over riveted repairs especially in aerodynamically important areas such as the leading edge. Tapering and conformity to existing structures can be readily achieved in comparison to riveted repairs. For lightly loaded structures, adhesive joints are more efficient as the low density materials and reduced volume provide weight saving in areas where weight is critical (e.g. control surfaces). The orthotropic characteristic of the employed composites allows tailoring to the specific load requirement of a particular repair [18]. It should be noted that bonded repairs can also lead to structural failure with possibly catastrophic consequences if they are not designed and installed properly.

Once a bonded repair scheme is established, the implementation time and required labour are, in many cases, less than the required time and labour for a riveted repair thus making it easier to implement during regular maintenance checks. This can be of significant benefit to airlines since additional downtime of an aircraft is very expensive. Fleet wide repairs can offer significant savings as shown by the Hercules C130 upper wing plank repairs carried out by AMRL (\$67 Million) as well as the C-141 weep hole repairs mainly carried out at the Warner Robins Air Logistics Center, where in total 770 repairs were performed out between September 1993 and December 1994 [83].

Bonded repairs also offer advantages with respect to the inspection of the underlying damaged structure. Eddy current can be used to monitor crack growth beneath boron/epoxy patches, i.e. the composite material most commonly used for structural repairs [18]. Crack

initiation and the growth of new cracks in riveted repairs are sometimes difficult to detect can lead to failures such as that which occurred in the Lockheed Electra example discussed previously [35]. One solution to this problem are ‘fingered’ inner doublers (see Figure 2.2 [86]) which combine long fatigue life with good damage tolerance and provide excellent external inspectability near the first row of fasteners. However, the implementation of fingered inner doublers certainly adds to the cost of the repair [35, 86].

Figure 2.2: ‘Fingered’ inner doubler for long joint fatigue life and external visual inspection

‘On the other hand, bonded repairs should not be attempted unless stringent cleaning and processing steps can be adhered to within a controlled environment’ [13]. This aspect was repeatedly emphasized during the workshop on ‘Practical Adhesive Bonding for Performance and Durability: Standards and Standardisation’ [11, 30, 31]. An example of the consequences of poor bonding is the infamous Aloha Airlines flight 243, where the upper fuselage of the Boeing 737 separated in flight causing an explosive decompression [31]. Disbonding and corrosion was caused by applying the adhesive layer in a frozen state which lead to moisture attraction inhibiting the formation of the chemical bond to the adherends during the fuselage assembly process. Adhesive bond failure subsequently overloaded the

fasteners resulting in Multi-Site Damage (MSD) [31]. Since the capabilities of assessing the quality of an adhesive bond are limited, the training and certification of maintenance engineers and technicians remains a key issue for achieving durable bonded repairs [11, 31, 83]. One of the main disadvantages of applying composite repairs to metallic structures results from the mismatch in the coefficients of thermal expansion leading to thermal residual stresses if the bonded repair is cured at an elevated temperature [18]. Currently available room temperature curing adhesives such as Flexon 241 don't have the desired material properties, especially at elevated temperatures [13].

2.3 Analysis of Bonded Repairs

Having discussed the advantages and disadvantages of riveted repairs and adhesively bonded composite repairs, the next step is to review the available theoretical models for the design of composite repairs to estimate the stress intensity factor after patching as well as the stress distribution in the adherends and adhesive under remotely applied and thermal loading. L.J. Hart-Smith's one-dimensional approach to adhesive joints based on Volkersen's solution [96] represents the foundation for the analysis of bonded repairs. To improve the estimate of the load carrying capacity of a bonded joint his theory includes the mismatch in the coefficients of thermal expansion for the adherends in different joint and reinforcement types as well as the elastic-perfectly plastic behaviour for the adhesive. He used an iteration technique to solve for the stresses in the case of plasticity in the adhesive as well as for tapered or stepped joints. This solution, with the associated numerical problems due to the high stress gradients at the steps, was implemented in the program AE4I [42] and is currently in use.

Other more complex solutions for different joint types have been presented in the literature [4, 39, 58, 68, 74, 93, 94, 100]. The complexity of these models generally requires their

implementation in a sophisticated computer program. Unfortunately these codes are not widely available thus limiting the use of adhesive bonding technology.

Rose [18] used the basic one-dimensional linear-elastic solution as part of a two step model to derive an estimate for the stress intensity factor of the cracked metal substrate in a bonded composite repair. The first step is the determination of the stress at the prospective crack location. Rose applied an inclusion analogy to determine the stress field for a repaired section with an elliptical patch over an uncracked substrate as well as its surroundings. For complicated geometries, a finite element solution can be used to determine the stress at the prospective crack location [19] while for simpler repair geometries, a one-dimensional approach gives an excellent estimate. The second step is the determination of upper bounds for the stress intensity factor where Rose utilizes the basic one-dimensional linear-elastic model for adhesively bonded joints. [78, 79]. Rose also presented a thermal residual stress estimate due to adhesive curing for an isotropic patch accounting for both the restraint of the surrounding structure due to local heating as well as the influence of bending effects for a single-sided repair on the stress intensity factor [18]. Since it is not always possible for the reinforcement to cover the entire crack, he further discusses a partial reinforcement [18, 80]. This model is based on a distributed spring model which he extended to assess the efficiency of a repair as a function of load history when progressive deterioration occurs in the spring constant due to disbonding induced by cyclic loading, ingress of moisture or a combination of such factors [80, 81]. This model offers an efficient computational approach for keeping track of progressive deterioration of the bonded repair [81].

Rose's original closed form analytical model appears to still be the preferred choice for the initial assessment of a bonded repair. It shows the influence of the adhesive and adherend material properties and thickness on the stress intensity factor. Baker also used this closed form analytical model to estimate the influence of a disbond size on patch efficiency [13]. He proposed the definition of a damage tolerant zone in the center section of a bonded

composite repair where disbonding is acceptable, thus emphasizing the analysis of disbonds as a key issue with respect to certification and inspection. It is important to provide a tool which estimates the impact of disbonds with respect to the increase in the stress intensity factor. The inspector of a bonded repair needs to decide if the bonded repair should be rejected or if the aircraft can go back into service.

Fredell [35] also implemented Rose's model in a computer code (*CalcuRep for Windows*) with further extensions to thermal influences. A peel model for this code is currently under development [35, 40]. The objective of *CalcuRep* is to cover the gap between complex finite element modeling and hand calculation. It is aimed at aircraft maintenance engineers rather than researchers allowing one to quickly study the effects of design variables and prepare a design. The complexity of a design might require a finite element analysis using the initial design data. One of the missing components in this code is the estimation of disbonds. Although the effect of thermal residual stresses is included in the code, it provides unnecessarily conservative results by using the curing temperature as the stress free temperature.

Baker presented a design procedure for minimum patch thickness which incorporates the Rose model [18]. This particular procedure is based on comparisons between the computed overlap length and the allowable length (Step 4), the peak shear strain in the patch and the allowable peak strain (Step 6), and the computed shear strain range in the adhesive and the allowable shear strain range (Step 8) as shown in Figure 2.3. The allowable peak strain (Step 5) for the patch is based on the allowable static strain corrected for strain concentration effects. The fatigue life or durability of the patch is assumed to be determined by the shear strain range (Step 7). The allowable shear strain range is dependent on humidity and temperature. Baker made no attempt at that time to design to a specified stress intensity or stress intensity factor range due to (i) the uncertainties in the estimate of ΔK_{∞} , (ii) the difficulty of accounting for the stress-ratio effects, (iii) the difficulty in allowing for overload

or crack-retardation effects and (iv) the large errors in the prediction of crack growth rates due to material variability and environmental influences [18]. Furthermore Baker didn't include disbond allowables in this initial design scheme.

Figure 2.3: Minimum patch thickness design scheme

Composite repairs have also been extensively investigated using numerical studies. Well known is the work by Ratwani, who considers an infinite plate with reinforcement [73]. Young, Cartwright, Dowrick and Rooke also authored a number of papers on model studies of repair patches using a strip patch modeling approach [33, 102, 103]. In 1979, G.L. Roderrick investigated an analytical approach with an iterative solution for the determination of crack propagation in the aluminum structure and associated disbonding at the interface between the aluminum and the infinite patch using the Paris Law [75]. Although these

models have shown good results, the main problem with these approaches is portability. Most often the complexity of numerical solution schemes and the associated implementation work prevents these approaches from becoming a common modeling approach.

In contrast, finite element approaches offering parametric studies as well as the investigation of complex geometries gained popularity due to the availability of good commercial packages and continuously increasing computing power. Thus finite element analysis approaches have become the commonly applied numerical scheme for the design and substantiation of bonded repairs [15, 18, 19, 24, 26, 46, 51, 52, 54, 55, 63, 83, 101]. Well known is the work from Jones and Callinan [24, 52, 54] who developed a two-dimensional finite element formulation for the adhesive layer in 1979. The adhesive element formulation was based on the displacement between the substrate and patch accounting also for the transverse shear stiffness in the adherends. Coupling the adhesive stiffness matrix with the composite patch stiffness matrix allowed them to use this bonded pair in conjunction with standard finite element routines. They also suggested appropriate modeling techniques for commercial FEM packages to include the transverse shear stress of the adherends in two-dimensional models. This scheme of using adhesive joint elements which account for the transverse shear deformation of the adherend was also used by other research institutions such as the Cranfield Institute of Technology [19]. Jones and Callinan also discussed the influence of local heating in combination with the supporting airframe on the magnitude of thermal residual stresses [53].

Since then the increase in computing power has allowed the use of three-dimensional finite element modeling to provide more accurate results [24], although care must be taken to ensure that the modeling is carried out correctly. Finite element analysis results are only as good as the assumptions used to create the model. Simplifications such as representing the taper in a composite patch by tapered elements or a normal stacking sequence instead of the actually used reversed stacking sequence may lead to misrepresentations depending

on the focus of the finite element analysis. The important advantage of finite element modeling especially with the available computing power and storage capacities over other numerical solution schemes is the possibility to transfer the modeling approach without the need to create a specialized code.

Rose's closed form analytical model is an essential key to obtaining a good initial guess for the design parameters thus accelerating the finite element modeling process. Not all design parameters (such as overlap length) can be readily determined by closed form analytical models or numerical approaches. Experimental testing is an important part of the determination of material behaviour and allowables as well as performance estimates for the bonded repair such as cycles to crack reinitiation, disbond growth rate and crack growth rate. It is generally assumed that the Paris law correlation still holds after the repair [35, 75, 81]. One of the main difficulties with experimental testing of bonded structures is the development of appropriate test specimens. It is important to realize the difference between results from a specific test specimen used to establish allowables and the performance of an actual repair and to account for them.

2.4 Test Specimens for Bonded Repairs

Baker's proposal [11, 12] for a generic certification approach gives an excellent overview of the required test specimens for bonded repairs. His approach includes testing of coupons, generic bonded joints, generic structural detail specimens and a representative structural detail specimen. Coupon tests are used to determine the basic adhesive and composite allowables. Knock-down factors are evaluated for hot/wet conditions and off-optimum manufacture. Generic bonded joints provide more valuable information since they simulate the actual loading conditions in a repair. The amount of coupon testing can often be reduced by generating some of the data from generic bonded joint tests. A double-doubler

joint type specimen has been commonly used as a generic bonded joint specimen [11, 18]. This type of specimen can provide the basic design data for static strength, threshold disbond growth, rate of disbond growth as well as be a platform to determine knockdown factors for hot/wet conditions, non-optimum manufacture and representative damage [11, 12].

Generic structural detail tests are used to validate the repair design approaches. Static strength, fatigue performance and damage tolerance are tested using a generic structural detail specimen such as the well known sandwich type specimen developed at the AMRL (see Figure 3.1) . This particular specimen reflects an attempt to include the influence of the supporting airframe structure in a test specimen for bonded repairs. The stiffening influence of the frame on the skin was implemented into the specimen design by bonding honeycomb between two cracked aluminum sheets. This honeycomb sandwich specimen minimizes the face-sheet curvature which occurs as a result of thermal residual stresses induced during the patching process. In addition, by using a symmetric configuration, this specimen reduces the out-of-plane deflection, which occurs on single-sided repairs due to the shift of the neutral axis under applied loading. This configuration has been proven to be representative of fighter aircraft having very stiff underlying structures. This type of specimen has also been used by the National Research Council of Canada [71], the University of British Columbia [1, 2, 3], the Aeronautical Research Laboratory/AIDC in Taiwan [67] as well as other institutions. Another type of generic structural detail test specimen, which was used by Textron Specialty Materials, is a single-sided bonded repair specimen using 0.8 mm to 2.5 mm thick 7075-T6 aluminum sheet with different patch thicknesses, ply orientations and lay-up sequences for which a large quantity of test data have been generated [20]. The final test specimen proposed by Baker is a representative structural detail test, which accounts for specific details of the repair and interrogates the differences to the generic structural detail test [11, 12].

In order to generate generic base data using the information obtained from previous tests as well as future tests, the loading conditions should be normalized with respect to geometry, material properties and manufacturing conditions. The actual load applied to a test specimen consists of three components: the remote loading, thermal residual stresses and process induced stresses. This research work addresses the magnitude of the process induced stresses and more significantly, the thermal residual stresses. It is critical to recognize that test specimens are generally free to expand during the curing process while the damaged section on an aircraft structure is typically highly restrained when heated only locally, thus reducing the 'effective' coefficient of thermal expansion. The determination of the 'effective' coefficient of thermal expansion of an aircraft component has been proven quite difficult. The analytical estimation used by AMRL [22] gave an effective coefficient of thermal expansion of $14.96 \mu\epsilon/^{\circ}\text{C}$ for the F-111 wing skin repair while the experimental measurement yielded an effective coefficient of thermal expansion of only $6 \mu\epsilon/^{\circ}\text{C}$. The difference in CTE to the unrestrained aluminum sheet, which would be applicable in a generic structural detail test specimen is $17 \mu\epsilon/^{\circ}\text{C}$. Due to the generally lower effective coefficient of thermal expansion for the damaged metallic component in a real aircraft application, test specimens with no restraint provide conservative results under identical remote loading. It is, however, important to realize that depending on the degree of restraint in the damaged real structure, the thermal residual stresses might be quite substantial. Accounting for the thermal residual stresses correctly can then make the difference between rejecting or accepting a particular repair scheme.

Having established what type of specimens are needed to allow a generic approach to bonded repairs, an overview of some of the currently available experimental data applicable to Baker's proposed generic certification approach is presented next with the focus on experimental thermal residual stress and strain measurements.

2.5 Experimental Testing of Bonded Repairs

Many design variables for adhesive joints have been evaluated during the PABST program (Primary Adhesively Bonded Structure Technology) leading to a significant gain in experience and knowledge of adhesively bonded joints which now serves as the basis for the design and fabrication of many bonded repairs [18, 44]. Recommendations for overlap length is one piece of valuable information derived from this program. For example, the overlap length for a double-lap joint should be a minimum of 30 times the thickness of the central member for subsonic aircraft made from aluminum in order to avoid creep. Hart-Smith also explains the design-philosophy behind this estimate for a double-lap joint with uniformly thick adherends [44]. The successful design of joints on the PABST fuselage, none of which have failed during four years of slow-cycle fatigue testing in a hostile hot/wet environment with the aluminum sheet stressed almost to yield, has increased the confidence in bonded structures significantly [44]. It should be noted that Baker recommends an overlap length of 6 times the load transfer length λ (see Section 5.2.1) based on AMRL's experience with bonded repairs.

The AMRL sandwich type specimen has provided the link to the experimental verification of crack growth rates for bonded repairs with respect to the applied loading. A number of results for different process specifications for the patch application, different materials and thicknesses, a variety of stress intensity factor ranges and R-ratios as well as environmental influences are available for this particular specimen [11, 18, 67, 71]. Textron has also provided a large quantity of experimental data for single-sided bonded repairs to create a generic body of test data.

Due to the lower effective coefficient of thermal expansion for the metal substrate of a real repair, it is beneficial to account for the thermal residual stresses in the absolute loading

condition for the generic bonded repair specimens and then to estimate the thermal residual stresses for the actual repair based on the true adhesive behaviour combined with the proper boundary conditions (i.e., local heating and other reinforcing structural components in the vicinity of the repair). The proper accounting of thermal residual stresses can be the deciding factor for the feasibility of a bonded repair.

Researchers at the Australian Aeronautical and Maritime Research Laboratory also investigated thermal residual stresses in bonded repairs. In 1978, they presented a paper on thermal residual stresses for CFRP - reinforced cracked specimens using X-ray diffraction procedures [17, 18]. Carbon/epoxy patches were used to allow X-ray penetration to the metal surface instead of the more common boron/epoxy patches. These measurements confirmed the high thermal stresses present in the bonded repair specimens. In addition to thermal residual stress measurements in the reinforcement, stress measurements were also taken around a machined crack with a tip radius of 0.2mm. The measured thermal residual stresses near the machined crack tip showed significant scatter (up to 19%) based on the remote thermal residual stress. The thermal residual stresses were interpreted in terms of stress intensity factors which appeared to be quite high and inconsistent with previous experimental observations by AMRL researchers on the growth of stress corrosion cracks in reinforced specimens [17]. The disadvantage of this particular measuring scheme for thermal residual strains near the crack tip is the need for a small diameter X-ray beam leading to significant data scatter in the results [17]. The measured remote thermal residual stress from the machined crack tip (40 mm ahead of a 10 mm crack) was reported up to 13% below the thermal residual stress measured in the center of a reinforced specimen giving an indication of the data scatter. Unfortunately, no data of thermal residual stresses versus temperature were presented, which would have given a better understanding of the elevated temperature behaviour of the adhesive and thus a confirmation of the theoretical relationship used to determine the thermal residual stresses.

AMRL researchers also investigated the effective coefficient of thermal expansion for large aircraft components due to the surrounding structure and local heating during the repair process. Measuring the curvature of repaired panels with different restraining conditions showed the reduction of thermal residual stresses with increasing levels of restraint [16]. All of the thermal residual stress investigations carried out at AMRL were part of a thermal fatigue study. These investigations lead to a number of recommendations by Baker to minimize thermal residual stresses in a composite repair, and included the following. The heated area should be minimized to maximize the constraint offered by the surrounding structure. The adhesive should be cured at the lowest possible temperature. If feasible, the patch structure should be pre-stressed in compression during the patch application to a level which will partially or completely nullify the residual tensile stress on cooling.

Parallel to the research presented in this thesis, AMRL conducted a study of thermal effects on composite repairs as part of a repair substantiation program for primary structure on the F-111 which was presented in 1996/1997 [22, 97]. In this study, a number of strain gauges were installed on the top of a boron/epoxy doubler on a representative wing section specimen. Thermal residual strains of $-548 \mu\epsilon$ in the boron/epoxy patch and $+531 \mu\epsilon$ in the aluminum skin were measured at 23°C with a cure temperature of 80°C [22]. Unfortunately no thermal residual strains versus temperature measurements were presented which would have allowed one to assess the reduction in thermal residual stresses by lowering the curing temperature. The key is recognition of the difference between the curing temperature and stress free temperature as well as assessing the validity of a linear-elastic relationship below the stress free temperature leading to a definition of an effective stress free temperature in this research. In addition to the thermal residual strain measurements, fatigue tests of bonded repairs were carried out at AMRL to investigate the performance of bonded repairs at temperatures of 83°C and 105°C . The experimental results with respect to crack growth indicated a lack of sensitivity to both temperature and loading frequency. Further

experiments are being conducted to fully understand the effects of test temperature and loading frequency [22].

The Delft University of Technology, in a joint program with the US Air Force investigated GLARE as a patch material due to its fatigue resistance and especially low coefficient of thermal expansion. The coefficients of thermal expansion in the longitudinal direction for GLARE-2 and GLARE-3 are $16.3 \mu\epsilon/^{\circ}\text{C}$ and $19.8 \mu\epsilon/^{\circ}\text{C}$ respectively, which are quite close to the CTE of Al 2024-T3 with $23.45 \mu\epsilon/^{\circ}\text{C}$. Considering the general restraint of an aircraft structure, a patch with a high CTE (such as GLARE) offers the benefit of crack closing forces rather than crack opening forces as caused by low CTE patches (such as boron/epoxy). This improvement, unfortunately, is traded off against significantly higher skin stresses (in comparison to a boron/epoxy patch repair) leading to possible skin cracking at the patch edge [35, 40, 95]. As part of this joint program, researchers at the Delft University of Technology presented the results of a thermal residual stress investigation around different bonded fiber metal laminate repair patches on a Fokker F28 fuselage in 1997 [95]. The temperature and thermal residual strain distribution in both longitudinal and circumferential directions were measured at four to six locations in the repaired section and at approximately 20 locations outside the repair. The aim was to verify the temperature distribution during the cure as predicted by Rose's approach as well as to determine the thermal residual stresses introduced into the airframe and patch. While the temperature distribution was within 20% of the theoretical prediction, the thermal residual strains were only predicted to the correct order of magnitude. Bending was an important influencing factor due to the flexibility of the fuselage, making isolation of the thermal residual stress component a problem [95]. Unfortunately, no strain measurements versus temperature were published, thus no information was available to quantify the adhesive behaviour. It was commented that no increase in thermal residual strains was found for the 120°C

post-cure over the 90°C pre-cure indicating that 3M's AF-163-2K adhesive might have a similar temperature independent behaviour as the FM 73 M used in this research work. Determination of the adhesive characteristics under thermal residual stress loading is an important key to proper prediction of thermal residual stresses. A simpler specimen such as a double-doubler joint specimen or the AMRL sandwich type specimen (which avoids the out-of-plane bending) is certainly more appropriate to determine this behaviour. Although the complexity of the bending in a flexible structure can be implemented using a finite element analysis for prediction of the performance of a bonded repair, knowledge of the proper material characteristics still remains the key to obtaining accurate results.

While in general the focus for the analysis of bonded repairs is generally the reduction in stress intensity, other failure modes such as substrate yielding, adhesive shear, adhesive peel stresses, patch matrix peel, patch matrix shear and patch fiber in tension or compression should be carefully investigated [40, 101]. It should also be noted that the investigation should not be limited to just the patched section as the influence in the global load distribution due to patching may lead to failures in surrounding components.

An important contribution to the experimental identification of failure modes was carried out by the National Research Council of Canada. They performed a fractographic investigation on a boron/epoxy repair for the CF116 which experienced compression induced fatigue cracking in the upper wing skin [37, 72]. The noted failure locations and modes were used to establish the locations for the strain gauges in this research effort. NRC also developed the total residual strain measurement technique which is employed in this research collaboration between the University of British Columbia and the National Research Council of Canada. This research led to an improved technique to measure thermal residual stresses versus temperature, which gives a better understanding of the overall adhesive behaviour.

Beyond the theoretical analysis and experimental verifications are certification issues and the need for standards for bonded repairs, some of which were discussed at a workshop of practical adhesive bonding for performance and durability in Brisbane in 1997.

2.6 Standards and Certification Issues for Bonded Repairs

One of the main problems with existing regulations is that they are inadequate to ensure that adhesive bonding is carried out correctly. This problem has lead to a history of failures, which has done damage to the credibility of the technology, increased maintenance costs and the loss of life. Regulations such as FAA FARs and JARs require that structures must demonstrate static strength and in some cases fatigue strength. None of these standards address the main cause of bond degradation i.e. hydration of the interface [31]. Poorly prepared surfaces can demonstrate short term strength and may even survive fatigue testing, but as the interface hydrates, disbonds gradually develop. Currently available NDI techniques allow bonds to be tested for disbonds and porosity, but they are not capable of inspecting for durability. Therefore existing regulations don't prevent inappropriate bonding practices from being used.

As a result of the history of adhesive bonding, a USAF fail-safe criterion was proposed for the safety of flight components requiring design limit load to be carried under the assumption of a 100% disbond [60]. This type of criterion would make the AMRL F-111 repair to primary structure with a remaining strength below design limit load impossible, thus limiting the used of bonded composite repair technology.

These deficiencies have lead to standards developed by the RAAF [82] and Wright Laboratories (Wright-Patterson Air Force Base) to manage adhesive bonding properly [31]. Managing adhesive bonding correctly includes the development of validated designs and processes, assuring the integrity of materials prior to processing, assuring the processes are

followed by trained personnel and non-destructively checking for disbonds in a properly controlled environment. It is not only important to train technicians but also engineers as well as ensuring engineering support for more complex problems [29, 31, 60]. It is essential to establish a better track record as demonstrated by organizations such as AMRL, the Wright Laboratories and NRC to expand the applications that benefit from the application of adhesive bonding.

Baker proposed a generic certification approach which requires standards to assure a low risk of failure due to inadequate bond durability [11]. The first step is the development of procedures to assure low risk failure due to bond environmental degradation. Significant research addressing the bond durability issue has been carried out at AMRL and has lead to improved surface treatments especially for field conditions [6]. Baker proposes a damage tolerant zone in the central region of the bonded repair where controlled crack growth and disbonding is allowed, and also defines a safe life zone in the tapered region to ensure that peel and shear strains are maintained below a damage threshold. The next step is to establish the mechanical property design allowables relevant to generic repair scenarios based on tests of both coupons and structural elements (generic bonded joints). Then the patching design procedures for generic scenarios based on design allowables are validated using generic structural details tests. The difference between the generic and actual repair has to be established by analysis and tests need to be conducted to interrogate only the expected differences. Limited tests on a specimen with representative structural details must be carried out for the final design of the repair based on the generic design process including knockdowns for variations of the actual repair. Furthermore the repaired structure or the representative component should be investigated for unexpected secondary stresses by analysis or strain survey [11].

Some relaxation of the certification requirements for the critical repair of very high cost components may be acceptable from the airworthiness perspective and may also be cost

efficient for the operator by using a 'Smart Patch' approach, where sensors monitor in-service damage [11].

2.7 Summary and Preview

This literature review gave a concise overview of the current important issues of adhesively bonded composite repairs. Lacking knowledge of the true stress state after patching is certainly a key issue due to limited knowledge of thermal residual stresses after an elevated temperature cure. Knowledge of the true stress state is important to develop and improve fatigue damage initiation and propagation models for bonded repairs. In order to create and use a generic data base, thermal residual stresses should be accounted for when assessing the performance of bonded repairs. Furthermore only little information is available about the adhesive behaviour at elevated temperatures. The question of the true benefit of lowering the curing temperature needs also to be addressed.

In order to make adhesive technology more accessible, it is necessary to have standard analysis schemes which don't require specialized numerical codes. This also simplifies the certification procedure since it allows certification authorities to verify the repair design.

The analysis of disbonds in the patch is another key issue in order to establish Baker's generic certification approach to bonded repairs. Furthermore a simple disbond model is necessary to assess partially disbonded patches in service.

Further education and implementations of standards are needed to improve the current practice of adhesively bonded repairs thus increasing the number of successful applications and repairs. Making detailed process specifications and standards available is a key issue to limit inadequate repair installations. Additional research has to be conducted to analyse occurring failure modes and to subsequently improve design practices such as the use of a normal stacking sequence versus a reversed stacking sequence for the patch.

Thus in recognition of these problems, the following was performed in the current work. This research contributes analytical tools as well as fundamental experimental data to the proposed generic approach for bonded composite repairs. In order to contribute to the existing pool of data, the AMRL sandwich type specimen was chosen for the current research. The manufacturing process will be presented in detail (Chapter 3) to provide a precise record for the interpretation of future fatigue testing as well as presenting guidelines for the precise installation of embedded strain gauges in bonded repairs. Detailed thermal residual strain measurements and an estimation of the process induced strains will be presented in (Chapter 4) thus significantly improving the current understanding of the adhesive behaviour under thermal residual stress loading. The presented measurement approach in combination with the identified adhesive characteristics will allow one to predict thermal residual stresses in bonded composite repair specimens more accurately.

In addition to the experimental measurements of thermal residual strains, an improved Rose model will be presented (Chapter 5) which accounts for a symmetrical disbond around the crack. This addresses the need for a simple tool to determine an appropriate size of a damage tolerant zone. A new solution for the linear-elastic stress distribution in the taper region of a bonded repair is also provided (Chapter 5) using a closed form solution instead of the typical numerical solution, thus reducing the requirements for specialized codes. A stress field estimation ahead of the crack tip will be presented in Chapter 5 as a tool for the placement of strain gauges for a smart patch approach as well as a guide for placing strain sensors for experimental measurements. The stress field solution is based on a new concise solution of the classical fracture mechanics problem for the stress field around a center crack. Finally new considerations for the use of reversed stacking sequences versus normal stacking sequences for bonded repairs under given remote loading conditions will be discussed (Chapter 6) based on detailed finite element calculations.

Chapter 3

AMRL Specimen Manufacturing Procedure

3.1 Introduction

The sandwich type AMRL specimen (see Figure 3.1) was chosen for the experimental determination of the stress state in bonded repairs due to the incorporation of features characteristic of composite repairs to cracked aluminum aircraft structures. Another criteria was its frequent use in the evaluation of the behaviour of bonded repairs with respect to influencing factors such as surface preparation, curing conditions, material choice and loading conditions. Although many details and results of these test are not available through the literature, it was decided to use this specimen to provide other researchers who are also using this particular specimen with valuable data.

The thermal residual stress investigation in bonded repairs on cracked metal structures represents an important part for a fatigue damage initiation study for this repair technology. It was decided not only to manufacture an instrumented specimen allowing one to measure the stress distribution in certain key locations, but also to manufacture an additional six AMRL specimens for future fatigue crack reinitiation testing. The manufacturing of these additional specimens provided excellent training for the assembly of the much more sophisticated instrumented AMRL specimen. It is also important to realize that any experimental testing beyond the thermal residual stress measurements needed to be considered at this

stage. Important choices included considerations such as the precrack loading conditions representing an influencing factor for crack reinitiation, the change in the stress distribution with increasing crack length, strain variations through the thickness of the repair and typical failure locations.

The choice of non-destructive stress or strain measuring devices was based on the availability, expense, reliability as well as applicability to composites. Strain gauges seemed to be a good compromise for this particular specimen offering availability at reasonable prices, excellent repeatability if installed correctly, as well as good averaging capabilities for composites. One of the drawbacks is the difficulty in measuring strain gradients in the load transfer zones as well as close to the crack tip, which is of special interest in the metal substrate. A number of smaller gauges installed in these regions raises significant concern about changing the strain distribution due to the large number of lead wires. Additionally, strain gauges have a given thickness which will also lead to some change in the local strain distribution, especially if a number of gauges are installed at a particular location at different interfaces of the bonded repair. Larger gauges are required to measure the average strains in composites rather the local strain around a single fiber.

The National Research Council of Canada provided important support especially with respect to proper surface preparation. All specimens were assembled under the supervision of Don Raizenne at the Institute of Aerospace Research. This chapter describes the sandwich type AMRL specimen assembly process in detail thus giving a good foundation for the reproducibility of any experimental results as well as a good resource for any researcher wishing to manufacture this particular specimen. The outlined manufacturing procedure includes constant load amplitude pre-cracking, aluminum surface preparation, boron patch fabrication, assembly and specific aspects of the instrumented AMRL specimen construction.

3.2 AMRL Specimen Specifications

The AMRL specimen consists of two edge-cracked 3.175 mm (0.125 in) thick, 2024-T3 aluminum face-sheets, which are bonded to an aluminum honeycomb core. 2024-T3 aluminum being a high strength, solution treated, cold worked and naturally aged aluminum alloy with 4.5% copper and 1.5% magnesium was chosen due to its widespread use in the aircraft industry. The edge cracks are repaired with semicircular boron patches. Figure 3.1 shows the AMRL sandwich type specimen. This particular specimen, which was developed by the Aeronautical and Maritime Research Laboratories in Australia, reflects the attempt to include the influence of the supporting airframe structure in a test specimen for bonded repairs. The stiffening influence of the frame on the skin was implemented into the specimen design by bonding honeycomb between two cracked aluminum sheets. This honeycomb sandwich specimen minimizes the face-sheet curvature, which occurs as a result of thermal residual stresses induced during the patching process. In addition, by using a symmetric configuration, this specimen reduces the out-of-plane deflection, which occurs on single-sided repairs due to the shift of the neutral axis under applied loading. This configuration has been proven to be representative of fighter aircraft having very stiff structures. In contrast, the stiffness of large commercial aircraft structures is generally lower, which leads to a different behaviour of the repair. It should be noted, however, that most cracks will emerge from rivet holes or other stress concentration points, where the local stiffness is higher due to the local supporting frame.

For the manufacture of seven AMRL specimens, a total of fourteen face-sheets were machined to the desired shape. All face-sheets were drilled and milled simultaneously to ensure repeatability of the specimen geometry. Starter notches were machined to a depth of approximately 3.0 mm (0.118 in) using an EDM machine. A copper shim having a thickness of 0.25 mm (0.010 in) was used as the electrode. Due to erosion of the electrode, the final depth of the notches varied between 2.59 mm (0.102 in) and 2.94 mm (0.116 in). Figure A.1 shows the drawing for the face-sheets complete with tolerances. The aluminum

Figure 3.1: AMRL specimen

spacer blocks (Figure A.2) were machined from AL 6061-T651. Aluminum shims were used to achieve a tight fit in the friction grips for pre-cracking. Tapered aluminum shims (Figure A.4) were bonded to the specimen for the fatigue testing to avoid fretting failure. Once prepared, the specimen face-sheets were precracked as outlined in the next section.

3.3 Constant Load Amplitude Pre-cracking Procedure

The ultimate goal of patching technology is to restore the static strength and the fatigue strength of a cracked structure. If the full fatigue strength of the test panel is restored, crack reinitiation should not occur at the maximum allowable fatigue load for the uncracked

aluminum sheet. A value of 138 MPa (20 ksi), which corresponds to the 'fatigue endurance limit' for AL 2024-T3 based on 5×10^8 cycles using a rotating beam test [88], was chosen as the characteristic maximum remote stress for the fatigue crack reinitiation testing. The literature review showed peak test loads for the AMRL sandwich type specimen range between 80 MPa (11.6 ksi) and 244 MPa (35.4 ksi) with the most common test condition of 138 MPa [12, 18], which does not include the thermal residual stresses. The experimental results of other researchers [13] indicated that at a load level of 138 MPa (20 ksi) for the AMRL sandwich type specimen, a reinitiation life of less than 100000 cycles can be found using a boron patch and FM 73M as an adhesive. Therefore this load level represents the upper limit for useful crack reinitiation testing. Any fatigue crack reinitiation testing will be dependent on the plastic zone size prior to patching. In order to minimize retardation effects, the largest allowable plastic zone size for the pre-cracking was determined by the plastic zone size due to thermal residual stresses after patching. The literature review suggested that the magnitude of the thermal residual stresses is dependent on the difference between the curing temperature and the ambient temperature, i.e. $\Delta T = 100^\circ\text{C}$ (212°F) for a curing temperature of 121°C (250°F) [18]. The corresponding estimate for the stress intensity factor K_∞ due to thermal residual stresses is $7.44 \text{ MPa}\sqrt{\text{m}}$ ($6.77 \text{ ksi}\sqrt{\text{in}}$). This value was used as a limit for the stress intensity factor for pre-cracking. Any additional fatigue loading applied on top of the thermal residual stress for the patched AMRL sandwich type specimen gives a sufficient margin to minimize any retardation effects.

The ASTM E 399 'Standard Test Method for Plane-Strain Fracture Toughness of Metallic Materials' [7] was also referred to as a guideline for pre-cracking. The ASTM E 399 standard allows a maximum stress intensity factor of up to 80% of K_{Ic} for the initial portion of the pre-cracking to avoid excessive crack growth. The stress intensity factor can be as high as 60% of K_{Ic} for the last 2.5% of the desired pre-crack length. Unfortunately, this standard does not address fatigue crack testing near the threshold stress intensity factor range. Pre-cracking has to be carried out using a stress intensity factor range above the threshold stress intensity factor range since fatigue crack growth does not occur below this value.

The stress intensity factor caused by the induced thermal residual stresses is therefore a reasonable upper limit for pre-cracking of the face sheets for the AMRL sandwich type specimen.

The face-sheets were fatigue pre-cracked from the machined 3 mm (0.118 in) notch to approximately 20 mm (0.787 in). A load shedding technique, similar to the procedure used by NRC, was employed to ensure that the plastic zone at the final crack length was realistically small [70, 71] and that the estimate for the stress intensity factor due to thermal residual stresses was not exceeded near the final crack length. Thus the mode I crack tip stress intensity factor range, ΔK_I , was reduced throughout the pre-cracking sequence from a maximum of 9.4 to 6.5 $\text{MPa}\sqrt{\text{m}}$ (8.6 to 5.9 $\text{ksi}\sqrt{\text{in}}$) at the final crack length. The final peak stress intensity factor was 7.2 $\text{MPa}\sqrt{\text{m}}$.

The ASTM E 399 defines that the stress ratio should be between -1 and 0.1. The pre-cracking of one or, simultaneously two, face-sheets without the supporting honeycomb is not well suited to compressive loading. Buckling would be the limiting case or an elaborate fixture would be required to prevent it. Thus a R-ratio of +0.1, i.e. tension at all times, was employed for this application in order to avoid early fatigue failure at the grips. The available testing equipment allowed a pre-cracking frequency of 20 Hz which fell within the ASTM E 399 allowable pre-cracking frequency of up to 100 Hz in the absence of adverse environments [7].

Harris [77] obtained the Mode I stress intensity factor for a sheet with an edge crack and bending constraints to be:

$$K_I = c \sigma \sqrt{\pi a} \quad (3.1)$$

where

$$c = \frac{5}{\sqrt{20 - 13(\frac{a}{W}) - 7(\frac{a}{W})^2}} \quad (3.2)$$

Thus the stress intensity factor range is given by:

$$\Delta K_I = c \Delta \sigma \sqrt{\pi a} \quad (3.3)$$

It should be recognized that the Harris equation, which is based on an approximate solution by Neuber, may introduce an error up to 8% [77].

Although thin face-sheets are used for this specimen, plane strain conditions can be achieved by restricting the maximum applied load. A maximum load of 37500 N (8430 lb) for each face-sheet was chosen in order to maintain plane strain conditions throughout the pre-cracking sequence. A further increase in the applied load would then lead to the change to plane stress and an increase in the plastic zone size. Plane strain conditions can be assumed when:

$$t_{Face-sheet} \leq 2.5 \left(\frac{K_{I_{max}}}{\sigma_{ys}} \right)^2 \quad (3.4)$$

For 2024-T3 aluminum, the tensile yield stress σ_{ys} is given as 310 MPa (45 ksi) [21, 91]. The highest stress intensity factor for mode I, $K_{I_{max}}$, of 10.45 MPa $\sqrt{m}$ (9.51 ksi $\sqrt{in}$) is reached during the pre-cracking sequence at a crack length of 5 mm (0.197 in) and is significantly less than the critical plane strain fracture toughness for mode I of approximately 44 MPa $\sqrt{m}$ (40 ksi $\sqrt{in}$) [47].

The remote stress range is given by:

$$\Delta\sigma = \frac{P_{max} - P_{min}}{W t} \quad (3.5)$$

Irwin gives an estimation of the plastic zone including a plastic constraint factor for plane strain, which also takes into account that plane stress conditions are present at the specimen surface [23]. The plastic zone size is given as:

$$r_p^* = \frac{1}{6\pi} \frac{K_I^2}{\sigma_{ys}^2} \quad (3.6)$$

Generally, the crack length should be modified in the case of large stress intensity factors to account for the plastic zone size [23]:

$$K = c\sigma \sqrt{\pi(a + r_p^*)} \quad (3.7)$$

Since the plastic zone size for this particular pre-cracking sequence is only approximately 1% of the crack length, the plastic zone size can be neglected for the determination of an effective crack length.

Table 3.1 shows the block loading sequence used for pre-cracking. Slight variations occurred for each specimen due to the difficulty in controlling the pre-cracking as a function of crack length.

Pre-crack Sequence	a [mm]	$\frac{a}{W}$ []	c []	P_{max} [N]	P_{min} [N]	ΔK_I [MPa $\sqrt{m}$]	r_p^* [mm]
1	3.0	0.019	1.125	37500	3750	7.256	0.036
	5.0	0.031	1.130	37500	3750	9.407	0.060
2	5.0	0.031	1.130	30000	3000	7.526	0.039
	7.5	0.047	1.136	30000	3000	9.267	0.059
3	7.5	0.047	1.136	25000	2500	7.723	0.041
	10.0	0.063	1.142	25000	2500	8.967	0.055
4	10.0	0.063	1.142	20000	2000	7.174	0.035
	12.5	0.078	1.149	20000	2000	8.067	0.044
5	12.5	0.078	1.149	17500	1750	7.058	0.034
	15.0	0.094	1.156	17500	1750	7.778	0.041
6	15.0	0.094	1.156	15000	1500	6.667	0.030
	17.5	0.109	1.163	15000	1500	7.245	0.036
7	17.5	0.109	1.163	12500	1250	6.037	0.025
	20.0	0.125	1.170	12500	1250	6.494	0.029

Table 3.1: Pre-cracking load sequence

The load calibration for the MTS 810 load frame in conjunction with the MTS 406 controller was carried out using an optimum gain for a frequency range of 0 to 2 Hz. This particular gain setting used at 20 Hz led to an approximate 10 % over- and undershoot in the load response of the system. Due to the influence of the gain setting on the calibration, the command input of the computer was adjusted by changing the set point and span on the MTS 406 controller. An initial span of 75% ensured that the specimen did not experience a higher load than intended for the pre-cracking sequence. An oscilloscope was used to

Pre-crack Sequence	$P_{\max}$ [N]	$P_{\min}$ [N]	$XDCR2_{\max}$ [V]	$XDCR2_{\min}$ [V]	$MDT_{\max}$ [V]	$MDT_{\min}$ [V]
1	37500	3750	3.049	0.305	3.049	0.305
	37500	3750	3.049	0.305	3.049	0.305
2	30000	3000	2.440	0.244	2.440	0.244
	30000	3000	2.440	0.244	2.440	0.244
3	25000	2500	2.033	0.203	2.033	0.203
	25000	2500	2.033	0.203	2.033	0.203
4	20000	2000	1.626	0.163	1.626	0.163
	20000	2000	1.626	0.163	1.626	0.163
5	17500	1750	1.423	0.142	1.423	0.142
	17500	1750	1.423	0.142	1.423	0.142
6	15000	1500	1.220	0.122	1.220	0.122
	15000	1500	1.220	0.122	1.220	0.122
7	12500	1250	1.016	0.102	1.016	0.102
	12500	1250	1.016	0.102	1.016	0.102

Table 3.2: MDT load control input

monitor the MTS 406 XDCR2 signal. The load could be adjusted within an accuracy of 2.5%, which is essentially the accuracy of the oscilloscope. Table 3.2 shows the voltages, which were entered in the MDT computer program and adjusted in the MTS 406 Controller to control the block loading sequence. Due to the design of the MDT computer card for older IBM XT/AT computers, a frequency of 33.4 Hz was entered into the program to achieve a testing frequency of 20 Hz.

3.3.1 Experimental Set-up

A computer-controlled MTS 810 Model 318.25 (Figure 3.2) with a load capacity of 250 kN (55 kips) was used for pre-cracking of the AMRL specimens. The National Research Council of Canada provided a pair of friction grips, which were manufactured by IAR. Adapters were then made at UBC to fit the grips to the MTS 810 Model 318.25.

Each of the AMRL specimen face-sheets was pre-cracked individually. This provided the opportunity to measure the crack growth rates without the interference of a second face-sheet. The installation of a face-sheet in the friction grips required pre-loading of the face-sheet with the center locating pin in place. A tensile load of 2.5 kN (562 lb) was chosen to assure proper alignment without yielding the aluminum around the center locating pin. The required torque for the grip bolts was determined using the following relationship between torque and preload [84]:

$$T = 0.20F_{Preload}d \quad (3.8)$$

Figure 3.2: MTS 810

The preload for the bolts can be calculated using the required load for the specimen and the appropriate friction coefficient for the weakest interface. In this case the critical interface is between the aluminum shims and the steel grips having the minimum friction coefficient of approximately 0.45 under clean surface conditions [66]. Therefore, based on equation (3.8), the maximum applied load of 37500 N (8430 lb) for one face-sheet requires a minimum bolt torque of 51 Nm (451 inlb). In order to reduce fretting fatigue on the face-sheets and to account for the uncertainty in the friction coefficient, a torque of 250 Nm (2213 inlb) was applied for the pre-cracking.

Two different techniques were employed to measure the crack length during the pre-cracking sequence. The first, a Fractomat Model 1078 was used to automate the pre-crack sequence. The Fractomat uses an 'indirect DC potential drop technique' to monitor crack growth. It utilizes a simple crack growth monitoring transducer known as 'Krak-gauge', which is applied in a similar manner as a conventional foil strain gauge (Figure 3.3). The overall

accuracy for measuring the crack length is 2% or better. The system has an integrated limit detector, which allows the interruption of the pre-cracking sequence as defined crack lengths are achieved.

The main advantages of the Fractomat system are [41]:

- A high linearity between the potential output and crack length
- High potential output (Volts) with very low current input (mA)
- specimen geometry independent calibration
- Independence from conductivity, size and material composition
- Infinite resolution, continuous DC output
- Suitability to fatigue testing
- Suitability to automated testing

In addition to the Fractomat, a travelling microscope (Scientific Instruments, England) was used with a 5x Leitz eyepiece (Figure 3.2). The travelling microscope could be adjusted to achieve an accuracy of $\pm 0.01\text{mm}$ ($\pm 0.0004\text{in}$). Prior to measurement, the specimen was loaded to the maximum allowable load of the particular

Figure 3.3: Indirect potential drop method

pre-crack sequence in order to enhance the visibility of the crack tip. While this optical setup is quite acceptable for pre-cracking, it is not suitable for monitoring crack growth in the patched specimen. However, due to the better accuracy of the travelling microscope, it was used as the reference for all crack length measurements.

3.3.2 Crack Growth Rates

The pre-cracking of the face-sheets offered the opportunity to record crack growth as a function of the number of cycles and thus allowed calculation of the crack growth rate. This information was used to select two face-sheets with similar crack growth rate to be combined into each AMRL specimen. The ASTM E 647 - 93, 'Standard Test Method for Measurement of Fatigue Crack Growth Rates'[8], was applied to the collected crack length versus number of cycles data. The secant method was applied to determine the crack growth rates according to:

$$\left(\frac{da}{dN}\right)_{\bar{a}} = \frac{a_{i+1} - a_i}{N_{i+1} - N_i} \quad (3.9)$$

where $\bar{a}$ is the average crack length given by:

$$\bar{a} = \frac{1}{2}(a_{i+1} + a_i) \quad (3.10)$$

The average crack length $\bar{a}$ was used to calculate the stress intensity factor range using equation (3.3). Appendix B lists all of the results from the pre-cracking. The crack growth rate was not determined, if the effect of crack initiation or retardation due to the reduction of the load from a sequence change or an accidental overload was present. Some of the crack propagation rates versus stress intensity factor range diagrams (see Figure 3.4 and Figure B.1) show a change in the slope at small stress intensity factor ranges. This corresponds to the non-linear crack growth rate versus the stress intensity factor range for low stress intensity factors in a double logarithmic plot. It is recalled that the Paris law is only applicable for larger stress intensity factor ranges where the relationship between crack growth rate and stress intensity factor range is linear in a double logarithmic plot. The crack propagation rates versus the stress intensity factor ranges are shown for each face-sheet in Figure B.1 through Figure B.14.

Due to the overall research focus being on the fatigue crack reinitiation, the average crack propagation rate for the last 2.5 mm of the pre-crack length was considered to be the

deciding factor for combining two particular face-sheets. This growth range was chosen instead of just the final crack propagation rate in order to reduce possible error due to crack length measurement accuracy. The average crack propagation rate was thus determined for the last 2.5 mm which included the initial retarded crack growth after reducing the applied load. The retardation effect is not important to consider for combining two particular face-sheets due to a relative comparison between the crack propagation rate of each face-sheet. Determination of the crack propagation rate had to be modified for face-sheet 1 due to an accidental overload at the beginning of the sequence. In this case, the crack propagation rate was determined after switching back to a maximum applied load of 12500 N (2810 lb) at a crack length of 18.65 mm (0.734 in). This ensured that a similar retardation effect was included in the determination of the average crack growth rate. The higher average stress intensity factor for this particular load block due to the larger initial crack length was neglected in this comparison.

Figure 3.4: Crack propagation rates for all face-sheets

AMRL Specimen []	Face-sheet []	Patch []	Average Crack Growth Rate (Sequence 7) [$\frac{\mu\text{m}}{\text{cycle}}$]
Instrumented Specimen	2	13	0.0077
	7	14	0.0077
1	14	1	0.0084
	6	2	0.0089
2	8	3	0.0097
	1	4	0.0100
3	12	5	0.0132
	3	6	0.0132
4	4	7	0.0163
	13	8	0.0147
5	9	9	0.0161
	10	10	0.0162
6	11	11	0.0189
	5	12	0.0180

Table 3.3: AMRL specimen component designation

Table 3.3 shows that face-sheets with very similar average crack growth rates were paired for the construction of the AMRL specimens. The only exception is AMRL specimen 4, where the crack growth rate varies by approximately 10% between the two face-sheets. Although face-sheet 4 had a higher average crack growth rate than face-sheet 9, AMRL specimen 5 was assembled using face-sheet 9 and 10 because they were adjacent in the original aluminum sheet.

All pre-cracks showed some variation in cracking direction from the ideal straight extension of the EDM notch. Figure B.15 to Figure B.28 show the different cracks for all of the face-sheets. Typically 2 or 3 small cracks would initiate at the EDM notch. After approximately 50000 cycles, one of these cracks would dominate and continue to grow faster. It was noted that crack initiation usually occurred at a slight angle from the EDM notch. This is visible in a number of pre-cracks, e.g. Figure 3.5, where the crack initially deviates significantly from the notch direction.

Figure 3.5: Pre-crack deviation (Face-sheet 3)

The effect of an accidental overload during the pre-cracking is also visible. The whitening at a crack length of approximately 12 mm (0.472 in) and 17.6 mm (0.693 in) in face-sheet 1 (Figure 3.6) reveals this clearly. While the crack tip was surrounded by a small resulting compressive stress field due to the overload, the adjacent material still remains in a high tensile stress field. The load level was returned to the sequence load after the overload, which led to visible fatigue damage in the clad layer in the form of whitening in the area which remained in a high tensile stress field. However, due to a dramatically reduced crack growth rate and increasing whitening, the load was temporarily increased to grow the crack out of the whitened area, then returned to the proper level.

Figure 3.6: Effect of pre-cracking overload (Face-sheet 1)

3.4 Aluminum Surface Preparation Procedure

Surface preparation of the aluminum prior to patching is an extremely critical element of the composite repair procedure. It controls the durability of the repair, a property which is very difficult to verify. The Boeing wedge test [9] and the lap shear test [10] are typically used to verify proper surface preparation. If the surface was prepared properly, cohesive failure should occur. Environmental testing is also used to give more insight into the durability of the bond and thus assesses the quality of the surface preparation. Attention to the details of the surface preparation procedure is therefore critical.

Two different categories of aluminum surface preparation procedures can be used: tank and non-tank procedures. Tank processes such as phosphoric acid anodizing and chromic acid anodizing allow the treatment of complete parts by immersion. For repair applications, where the part is in many cases not removable, non-tank processes must be used. The phosphoric acid anodize non-tank (PANTA or PANTACS), the chromic-sulphuric acid etch (optimized Forest Products Laboratory (FPL) procedure, ASTM D2651), PASA JELL 105 acid etch and reactive epoxy silanes process are four non-tank procedures which are currently used [18, 65, 72, 90].

The etching processes can cause potential problems such as corrosion between fasteners and the aluminum skin. Special care must be taken to isolate steel components, e.g. rivets, for the anodizing process. Anodizing is not possible if iron ions are present. The silane process was chosen for this investigation due to its simplicity and wide range of applications [72].

3.4.1 Clad Removal

Clad removal from the aluminum is the first step in the repair process. The clad material, forming a soft aluminum layer with a thickness of approximately 0.079 mm [21], provides

good corrosion protection for the aluminum skin. In the case of a composite repair it would result in a weak layer adjacent to the bond line. Unfortunately grit blasting is not very effective in removing the clad layer since the grit only tends to indent the clad layer rather than removing.

Wet 220 grit paper was therefore used to remove the clad material on both sides of the face-sheets as necessary, due to the additional bonding of the honeycomb, spacers and shims. A flat aluminum block (Figure 3.7) was used to remove the clad uniformly. A 'Kimtuff Wiper' was placed under the face-sheet to avoid scratches. Final grinding was done using wet 320 grit paper. This ensured that no scratches from the 220 grit paper were visible after the final grit blasting step and reduced the grit blasting time required to achieve a uniform surface appearance.

Figure 3.7: Clad removal

The face-sheet was then thoroughly cleaned under running water with 'Kimtuff Wipers' until no trace of contamination was visible. A water break-free surface was achieved at this stage. The face-sheets were air dried over night. The face-sheets were then wiped with MEK in only one direction with frequent changes of 'Kimtuff Wipers' to avoid the spreading of any remaining contamination. The previous thorough cleaning under water reduced the required amount of MEK significantly and is therefore preferable from an environmental point of view. The face-sheets were handled with gloves and subsequently wrapped in Kraft paper to avoid possible contamination.

3.4.2 Grit Blasting

The grit blasting, which should be carried out immediately prior to the silane process, exposes the aluminum surface for the silane coupling agent. Attention to details in this step is essential to avoid contamination. The grit blast unit had to be cleaned prior to the application (Figure 3.8). New aluminum oxide 220 grit with a 75 micron diameter was used to exclude any effects from possible prior contamination of the grit.

Nitrogen was used as the blast system gas to avoid the possibility of oil or water contamination from the regular air supply. The hose, which connects the nitrogen bottle with the grit blasting unit, was only used for nitrogen. A pressure of 690 kPa (100 psi) was supplied by the nitrogen bottle for the grit blasting process.

Figure 3.8: Grit blasting set-up

Grit blasting was carried out using parallel passes, first along the longitudinal axis of a face-sheet, and then perpendicular to the axis until a uniform dull finish was achieved. Both sides of each face-sheet were grit blasted. After completion of the abrading process, the face-sheet was cleaned using a spray nozzle and nitrogen gas at 690 kPa (100 psi). Under no circumstances should compressed air, rags or solvents be used to remove the residual abrasive. The face-sheets must be handled only with clean gloves before and after the grit blasting process, and wrapped in Kraft paper immediately after grit-blasting.

For representative testing purpose, the traveler coupon for the Boeing wedge test was also cleaned with MEK and grit blasted using the same process.

3.4.3 Preparation and Application of Silane Coupling Agent - Brush Method

Figure 3.9: Chemical bonding mechanism of the Silane treatment

The silane (γ -Glycidpropyltrimethoxysilane) provides an improved coupling between the aluminum and the adhesive. The presumed mechanism by which this coupling agent functions in chemical bonding as shown in Figure 3.9 [69]. Skin contact should be avoided with the moisture sensitive silane due to possible severe irritation and mutation. The refrigerated silane was allowed to stand at room temperature for one hour prior to opening the container. A 1% silane solution was required for the surface preparation procedure, thus 99 parts of deionized distilled water were mixed with one part silane by volume for the surface preparation procedure.

Figure 3.10: Silane solution equipment

The mixing container was carefully cleaned and rinsed with distilled water before use. The container used was polyethylene since distilled deionized water can react with glass. In total, 250 ml of 1% silane solution was required for the preparation of the two sides of each face-sheet, i.e. 247.5 ml distilled deionized water and 2.5 ml silane. The silane was poured into a small polyethylene cup to avoid possible contamination of the silane and was measured with a pipette (Figure 3.10), while a graduated cylinder was used to measure the volume of the distilled ionized water. The silane solution had to hydrolyse for one hour prior to use, thus the solution was agitated using a mixing agitator with a mixing magnet in the sealed container. Due to this time delay, preparation of the solution was carried out prior to the grit blasting of the face-sheets to minimize the risk of contamination and oxidation of the aluminum surface [72].

The face-sheets and traveler coupons were placed at a steep angle to allow the silane to run down the aluminum surface. A honeycomb block placed in a shallow glass or metal bowl allowed the entire surface to be accessed without one end sitting in the silane solution.

Figure 3.11: Silane coupling agent - brush method

A dispenser head for the mixing container was used to spray the silane on the face-sheet (Figure 3.11). The silane solution was then brushed downwards with a clean, fine haired 25mm (1 in) wide paint brush. It is essential to brush in only one direction to drive any remaining contaminants from the grit blasting into the silane solution collecting bowl.

A water-break free surface has to be visible during the silane treatment. Typically the silane treatment is continuously applied for ten minutes. In the case of the AMRL specimen, up to 4 face-sheets or 2 face-sheets and two sheets for a Boeing wedge panel were treated simultaneously. Due to the increased number of parts, the silane treatment time was increased to 15 minutes per side. After treating both sides, the face-sheets were air dried for approximately 20 to 30 minutes. The long air drying time was a result of the solution build up at the holes and the lower edge of the specimen.

The face-sheets were heated to 80°C (175°F) for 30 minutes next. A small temperature controlled oven was used to heat dry up to 4 specimens (Figure 3.12). It is important that the surface is not touched to avoid any contamination. The primer must be applied within 1 hour after finishing the silane treatment in order to ensure a lasting surface preparation.

Figure 3.12: Heat drying of the silane

3.4.4 Preparation and Application of BR127 Corrosion Inhibiting Primer - Spray Method

The BR127 primer inhibits corrosion and offers superior durability and resistance to hostile environments within the bond line. It can also be used as a protective coating outside the repair area. In addition, the primer preserves the surface preparation for up to 6 months depending on the storage condition of the specimen. The refrigerated BR127 primer was

allowed to warm to room temperature prior to opening the container. The primer was then thoroughly mixed before use to achieve a yellow coloured homogenous mixture. The primer must also be agitated during the application to prevent the chromate particles from settling out of the solution. Insufficient mixing or lack of agitation results in a shiny, nearly clear coating, while the properly agitated primer gives a matt light yellow finish after spraying [72].

The BR127 primer should be sprayed to a dry primer thickness of 0.0025 mm (0.0001 in) with a maximum thickness of 0.0075 mm (0.0003 in). The proper thickness of the primer was identified using the BR127 primer colour chip with the correct shade of yellow for optimal thickness during the spraying.

Due to space restrictions, only two face-sheets were primed at one time. Figure 3.13 shows the setup for the priming. The face-sheets were supported on clean aluminum bars on the outer three millimeters of the face-sheet. A rag was wrapped around the junction between the cup and spray gun to avoid

Figure 3.13: Setup for BR127 priming

any accidental drops due to a possible leak in the seal. The primer was sprayed using 345 kPa (50 psi) nitrogen gas to avoid any possible contamination. The optimal thickness was achieved in a single pass with the spray gun. Coverage of the whole face-sheet required three passes in an 'S' pattern with a small overlap between the passes.

After priming the first side, the face-sheets were air dried for approximately 2-3 minutes, then turned over and primed on the second side. These two face-sheets were then placed on an additional set of aluminum bars, while two additional face-sheets or the two aluminum sheets for Boeing wedge test were primed in the same manner. After spraying, the face-sheets were air dried for 30 minutes. The primer was then fully cured in a temperature controlled oven at 121°C (250°F) for 60 minutes, removed, cooled and wrapped in Kraft paper until assembly.

Table 3.4 outlines the full time sequence for the described specimen surface preparation.

Time	Task
1:00 pm	Setup for clad removal
1:30 pm	Grinding face-sheet 1 with wet 220 and 320 grit paper
2:00 pm	Grinding face-sheet 2 with wet 220 and 320 grit paper
2:30 pm	Grinding face-sheet 3 with wet 220 and 320 grit paper
3:00 pm	Grinding face-sheet 4 with wet 220 and 320 grit paper
3:30 pm	Cleaning all face-sheets under running water
Over night	Air drying of face-sheets
7:00 am	Taking BR127 primer out of freezer
7:05 am	Preparing and cleaning work area
7:30 am	Preparing of the silane solution
7:45 am	Cleaning face-sheets with MEK
8:30 am	Grit blasting face-sheets
9:10 am	Face-sheet cleaning with nitrogen blow
9:20 am	Starting silane brush procedure
9:50 am	Silane air drying
10:20 am	Oven drying silane at 80° C
10:50 am	Face-sheet cool down
11:00 am	Priming face-sheets with BR127
11:10 am	Air drying of BR127 primer
11:40 am	Curing of BR127 primer at 121° C
12:40 am	Cooling down face-sheets
13:00 am	Wrapping face-sheets in Kraft paper

Table 3.4: Schedule for surface preparation procedure

3.4.5 Boeing Wedge Test

It was essential to verify the quality of the surface preparation. Unfortunately the long term performance of the surface preparation can only be evaluated to a limited degree using the available methods. The Boeing wedge test gives some indication of poor surface preparation. The details for the Boeing wedge test are specified in ASTM D 1002 [9].

Figure 3.14: Boeing wedge test specimens

The surface prep of the two 75.24 mm x 75.24 mm (6 in x 6 in) Al 7075-T6 3.175 mm (0.125 in) thick plates for the Boeing wedge specimens was carried out exactly as on the AMRL face-sheets. Two lay-

Figure 3.15: Temperature and humidity chamber

ers of FM 73M with a density of 300 g/m^2 (0.06 psf) were used as adhesive. In addition, 3 layers of 0.08 mm (0.003 in) thick teflon foil were used in the top part of the panel as the starter notch. The edges of the panel were sealed with high temperature tape to reduce adhesive bleed and to achieve a uniform bond thickness.

The panel was cured at 80°C (176°F) for 8 hours. Then the tape and the teflon foil were removed. The panel was cut with a bandsaw and milled to five 25.4 mm (1.0 in) wide specimens (Figure 3.14). The edges were discarded.

Next, the steel wedges were inserted into the notches causing crack initiation in the adhesive. The initial crack length was then marked under the microscope and measured again after 1 hour at room temperature. No increase in crack length was noticed. There was no evidence of interfacial failure between the adhesive and the aluminum. To check the bond and surface preparation under severe environmental conditions, the specimens were placed in a temperature and humidity chamber (Figure 3.15) at 65°C (149°F) and 96% humidity for 1 hour. No crack growth was observed in this case. The specimens were then placed in the temperature and humidity chamber under the same conditions for additional 8 hours. No further crack growth was observed. One specimen was split open to check the failure mode. Complete cohesive failure in the adhesive was found. Based on this evidence it was concluded that the surface preparation used on the AMRL specimen face-sheets was carried out properly.

3.5 Boron Patch Fabrication Procedure

The patch for the AMRL specimens consists of 7 layers of 5521/4 boron prepreg and one cocured layer of FM 73M. The cocured layer of FM 73M with a density of 300 g/m² (0.06 psf) penetrates up to two layers at the tapered end and therefore improves the properties of the 5521/4 boron/epoxy prepreg by replacing the boron prepreg epoxy system with a modern toughened epoxy.

3.5.1 Cutting of Boron 5521/4 Prepreg and FM73M Adhesive

Approximately 80.0 cm (31.5 in) of 15.2 cm (6.0 in) wide 5521/4 boron prepreg tape and 377.4 cm² (58.5 in²) of FM 73M film adhesive were required for each AMRL specimen, i.e. 2 patches. The boron prepreg and the FM 73M had to unthaw in the unopened bag from the frozen storage to room temperature for 1 hour. If the bags are opened too early, moisture in the air will condense on the boron prepreg and the FM 73M film adhesive due to their surface temperatures being below the dew point. Even a very small amount of absorbed moisture will result in an increase in porosity leading to a loss in strength of the patch. As a further precaution, the boron prepreg and the FM 73M film adhesive were handled with gloves to avoid contamination [72].

The boron prepreg was unrolled with the white backing facing down (Figure 3.16). A perforated release film was placed on top of the tacky boron prepreg to allow easier template placing and removal. The nylon scrim cloth, which holds the boron fibers in place, faces the white backing paper. All plies were cut with the scrim cloth facing down. Seven 2 mm (0.079 in) thick aluminum templates with radii ranging from 57 mm (2.24 in) to 75 mm (6.73 in) in increments of 3 mm (0.12 in) were used for the precise cutting of the boron prepreg. The templates were cut 2 mm (0.079 in) wider along the straight edge due to the formation of a rounded edge of the patch during the curing process. Cutting this approximately 2 mm (0.079 in) wide strip off after curing ensured the proper thickness along the straight edge of the patch. One template at a time was clamped on the boron prepreg with a C-clamp to avoid any movement while cutting (Figure 3.16). It was important not to use excessive force during the clamping process to avoid damage to the prepreg. The straight edge of the template was lined up with the 0° direction of the prepreg. A roller type cutter (Figure 3.16) was used to cut along the perimeter of the template. The roller type cutter offered advantages over a regular pair of scissors or a razor blade due to the ability to shear the material with only limited pushing in cutting direction. This reduced the loss

of material due to distortion of the prepreg and thus allowed closer template spacing.

The FM 73M ply, which is co-cured with the boron prepreg plies, was cut using the template with a radius of 75 mm (2.95 in) in the same manner. The boron/epoxy prepreg and FM 73M film adhesive plies for each patch were then sealed in a polyethylene bag and stored below -18°C (0°F) until one

Figure 3.16: Cutting of the boron 5521/4 prepreg

hour before use. The FM 73M film adhesive ply used to bond the precured patch on to the cracked aluminum sheet was then cut using an additional template with a radius of 77.5 mm (3.05 in) and stored in a separate bag in the same manner.

3.5.2 Lay-up of Boron 5521/4 Prepreg and FM73M Adhesive

A reversed stacking sequence was chosen for the lay-up procedure of the patch to reduce peel stresses at the tapered end. In addition the reversed stacking sequence allowed easier placement of the plies during the lay-up.

As a first step, a transparent template with a scale of 1:1 was taped on to a flat surface. If a laser printer is used for the template manufacturing, it is important to ensure that the dimensions are printed properly and not distorted or scaled. Time consuming program calibration may be necessary to achieve a good template to meet the specifications. One layer of perforated release film was placed over the template and secured (Figure 3.17). Then the perforated release film, which was placed during the cutting process, was removed from

the smallest boron prepreg ply. Using the template as a guide, the ply was placed carefully on the template with the white backing paper facing up. A teflon roller (Figure 3.17) was used to roll the ply down. This made the removal of the white backing significantly easier as well as it consolidated the lay-up. Otherwise the scrim cloth had the tendency to stick to the backing and got significantly distorted during the backing removal. A knife was carefully used to lift one corner of the backing. The remaining six boron prepreg plies were placed in the same manner.

After placing the last boron ply, two layers of peel ply were cut to the dimensions of the largest boron prepreg ply using the same technique as for cutting the boron/epoxy prepreg. One peel ply was then carefully placed over the last boron prepreg ply. Perforated release film, which covered the template on the layup table,

Figure 3.17: Lay-up of the boron prepreg

was then cut around the patch. Next, the patch was turned upside down and the remaining perforated release film was removed. A single layer of FM 73M was then placed on the bottom side of the patch. Finally the second prepared peel ply was placed over the adhesive.

3.5.3 Vacuum Bagging of Boron 5521/4 Prepreg and FM 73M Adhesive

The seven ply boron patch and one layer of FM 73M were cocured prior to bonding to the aluminum face-sheet in order to reduce the complexity of the assembly procedure. This

cocuring step also allowed the trimming of the edges of the patch, especially the straight edge, which ensured that the proper patch thickness was achieved along the face-sheet edge. Figure 3.18 illustrates the vacuum bagging procedure prior to the autoclave curing.

Figure 3.18: Vacuum bagging procedure

The vacuum bagging procedure was carried out using a 6.35 mm (0.25 in) thick aluminum plate. One layer of non-perforated release film was placed on the aluminum to avoid any bonding to the aluminum as well as to facilitate easy removal of the patch after curing. The patch, consisting of one layer of peel ply on each side of the patch, one layer of FM 73M and seven layers of boron/epoxy prepreg, was then placed on the non-perforated release film. Two short fiberglass roving strands were positioned at the corners of the patch to allow for an additional breather path during the curing of the patch. A perforated layer of release film was placed on top of the patch to allow excessive resin and adhesive to flow away from the patch. The three consecutively stacked layers of fiberglass absorbed any epoxy flowing through the perforated release film. Two layers of thin breather cloth, which were extended beyond the patch for the placement of the vacuum fitting, were placed on top of the bleeder cloth to ensure full vacuum pressure over the entire patch as well as to remove any volatiles. A hole was then cut in the vacuum bag to mount the vacuum fitting. Next, the vacuum bag was positioned over the entire area and sealed with sealant tape around its edge.

It was important that none of the layers surrounding the patch had a wrinkle in the patch area as these can cause wrinkles in the cured patch leading to a reduction in strength. The bag was properly sealed and a vacuum of 100 kPa (29.5 in Hg) achieved. The bag was then carefully checked for leaks since the loss of vacuum during certain times of the autoclave cycle may result in a poorly processed laminate.

3.5.4 Cocure Cycle of Boron 5521/4 Prepreg and FM 73M Adhesive

The factory recommended cure cycle for the boron 5521/4 prepreg was used for the cocuring of the boron prepreg and the FM 73M (Figure 3.19).

Figure 3.19: Recommended boron 5521/4 cure cycle

Prior to starting the cure cycle in the autoclave, the bagged patches were hooked up to the autoclave vacuum system. The vacuum system should be operating prior to hook up in order to avoid a loss of vacuum in the bag. After connecting the vacuum source, the vacuum control system was switched to the automatic mode. The elapsed time before the

vacuum system restarted gave a good indication about the quality of the vacuum bag. If this time period was at least 2 minutes, the autoclave was closed.

The curing cycle started with raising the pressure to 344.7 kPa (50 psi). Then the autoclave temperature was increased at a rate of 2.8 °C/min (5.0 °F/min). The 'Baron' Autoclave (see Figure 3.20), was operated in the manual mode for the curing cycle. The temperature set point was read-

Figure 3.20: Autoclave

justed every 2 minutes thus providing a fairly close temperature profile to the optimal ramp up. At 112.8 °C (235.0 °F) the vacuum was released. The temperature ramp up was finished at 121.1 °C (250.0 °F), which was maintained for 60 minutes.

After this hold period, the temperature was reduced at a rate of 2.8 °C/min (5.0 °F/min). The cooling rate decreased at approximately 65.6 °C (150 °F) due reaching the maximum cooling rate of the autoclave. The pressure was dropped at approximately 50.0 °C (122.0 °F) after which the bagged patches were removed from the autoclave. The aluminum plate, on which the patches were bagged, served as an additional heat source to maintain cooling of the patches near the recommended rate. The vacuum bag was opened when the aluminum plate reached room temperature. Figure 3.21 shows the cocured patches on nonperforated release film after removing them from the vacuum bag.

The cured patch was trimmed to the desired size. The excessive adhesive was ground

off to the edge of the boron along the circular boundary. The straight edge of the semicircular patch was then cut using a diamond coated blade on a table saw.

The resulting cured patch was found to be slightly curved on the bottom side due to the difference in the CTE between the FM 73M film adhesive and boron/epoxy prepreg in combination with adhesive shrinkage. Radial contraction of the boron fibers during the cooling is small in comparison to

Figure 3.21: Cured patches

the epoxy used as a matrix for the FM 73M which resulted in the slightly curved shape.

Prior to bonding the patch to the face-sheet, the bottom peel ply of the patch was removed and grit blasted with clean 220 aluminum oxide grit using nitrogen as the blast system. Afterwards the surface was cleaned with a nitrogen blast using a spray nozzle. The grit blasting removed a significant amount of the cocured film adhesive. The patches were then wrapped in Kraft paper until bonding to the face-sheets.

3.5.5 Ultrasonic Inspection of the Cocured Patches

All boron/epoxy patches were inspected for flaws by ultrasound. Amplitude C-scans were carried out using the ARIUS system at the Institute for Aerospace Research (NRC) in Ottawa. A 15 Mhz focused transducer was used in pulse-echo mode. The patches were immersed in water and placed on two brass bars. One bar was placed along the straight edge while the other one was positioned parallel to the first supporting the tapered end. It

was important to place the center part of the semicircular patch as flat as possible to avoid angled reflection of the signal causing a decrease in the signal amplitude.

Figure 3.22: C-scan of a cocured boron/epoxy patch

Full scale amplitude is indicated by white, with decreasing amplitude indicated by red, yellow, green, blue, and black, which is the minimum amplitude. Figure 3.22 gives an example of a patch C-scan. The center area of the patch shows white, red and yellow. The tapered end shows a significant change in colour. This change is due to the angle of the taper, which reduces the amplitude of the signal received by the probe. The patch taper itself is not flat, which leads to an additional change in the amplitude of the signal shown by the periodical change between yellow and green. A few very small defects, indicated in blue, are visible in the tapered section. These are usually steeper local gradients in the taper right next to a ply drop off or possibly tiny air bubbles which are enclosed at a ply drop off. Changes in colour can also be noticed close to the supporting bars due to partial signal reflection off the bar.

Appendix C shows the NDI image of all cocured boron/epoxy patches. Only the C-scan of patch 1 (Figure C.1) reveals a small defect left of the center of the patch. Visual inspection

also revealed a small area of discolouration, which may be related to poor bonding or the use of a damaged ply.

3.6 AMRL Specimen Assembly Procedure

3.6.1 Fixture Design

The design of the fixture for the AMRL Specimen (Figure A.5) was dictated by the process specification:

- Temperature cycle from room temperature to 121 °C (250 °F)
- Unrestricted thermal expansion of the face-sheets
- Vacuum bagging
- Autoclave pressure of 344.7 kPa (50 psi)

The large temperature change and associated thermal strains of the face-sheets were taken into account by manufacturing the entire fixture from aluminum. Thus equal thermal strains in the face-sheets and fixture were achieved. Alignment of the two face-sheets was maintained during the cure cycle by using aluminum center alignment bolts.

Due to the local thickness change of the specimen in the patch area, the typical aluminum support plate used in the autoclave process was not applicable for this specific specimen. Thus the fixture required complete vacuum bag enclosure. This also ensured proper application of positive autoclave pressure on the patch and the honeycomb/face-sheet interface. Support in the grip area of the face-sheets was required to avoid bending of the face-sheets and possible damage to the honeycomb. This was achieved by machining the fixture to the exact thickness of the honeycomb. The parallel sides of the fixture combined with the tight machining tolerances required tapping the face-sheets with wooden block and a hammer to remove the cured specimen. A small gap was left between the honeycomb

and the fixture to avoid accidental bonding between the fixture and the face-sheets. The risk of bonding the face-sheets to the fixture was significantly reduced by avoiding vacuum or positive pressure on the adhesive film between the honeycomb and face-sheet. Only the honeycomb/face-sheet interface itself was properly stressed through the vacuum and autoclave pressure. The film adhesive without any pressure did not flow significantly.

The fixture was also designed with rounded corners and dimensions exceeding the face-sheet area to avoid any possible ruptures of the vacuum bag at the sharp face-sheet corners during the curing process. From a manufacturing point of view, it was preferred to cut the honeycomb after the curing process along the edges due to the difficulty in precisely machining the honeycomb. Therefore the edges of the honeycomb needed sufficient protection to avoid damage during the curing process. This led to the two part design of the fixture, which enclosed the honeycomb completely and allowed for removal of the specimen after curing. The exposed honeycomb edges exceeding the face-sheet width were covered with thin aluminum sheets which had the same contour as the face-sheets on one side. As a further precaution, the alignment bolts (Figure A.6) were designed with tapered heads to avoid punctures in the bag.

3.6.2 Preparation of the Honeycomb

The fixture design allowed large tolerances for the different sides of the honeycomb, which makes the cutting process significantly simpler. A reasonably precise cut without any significant distortions was achieved by clamping the honeycomb between two aluminum plates and cutting slowly with a sharp knife. Figure 3.23 shows the setup.

The honeycomb was solvent flushed with MEK prior to the bonding process [90]. The honeycomb core was then oven dried at 50°C (122°F) for 30 minutes. After drying, the honeycomb core was protected by wrapping it in Kraft paper.

Figure 3.23: Setup for honeycomb cutting

3.6.3 Assembling the AMRL Specimen

The assembly procedure for the AMRL specimen is directly linked to the fixture design. The first part of the assembly deals with alignment of the two face-sheets and the honeycomb core. The second step is concerned with the proper installation of the patches. During this process it is essential to avoid any contamination of the clean surfaces of the patch

Figure 3.24: AMRL specimen assembly (Face-sheet)

Figure 3.25: AMRL specimen assembly (Face-sheet mounting)

and face-sheets. As the initial step, the adhesive films, which bond the honeycomb to the face-sheet, were firmly rolled onto the face-sheets using a teflon roller. The use of a heat gun helped to keep the adhesive sticking to the face-sheet. Then a small strip of high temperature tape was placed approximately 0.5 mm (0.02 in) adjacent to the adhesive as a small barrier towards the grip area (Figure 3.24).

Figure 3.26: AMRL specimen assembly (Honeycomb mounting)

Then the first face-sheet was placed on the fixture and precisely located using the center alignment pins (Figure 3.25). Next, the backing of the FM 73M film adhesive was removed and the honeycomb placed on top of the adhesive. A vacuum bag foil was placed over the honeycomb and a teflon roller was used to press the honeycomb against the adhesive film and face-sheet. The fixture design provided light clamping of the honeycomb which eliminated any possible lateral movement (Figure 3.26). After installing the honeycomb, the second face-sheet was placed on the fixture after removing the film adhesive backing. The nuts for the center alignment bolts were then placed and tightened (Figure 3.27).

Figure 3.27: AMRL specimen assembly (2. Face-sheet mounting)

At this point, the area between the face-sheet and the fixture was covered with thin aluminum sheets (Figure 3.28), which had the same contour on one side as the face-sheet. High temperature tape was applied to the top of the aluminum sheet to avoid accidental bonding during the curing process. Each aluminum sheet was held in place by two pieces of high temperature tape. One layer of FM 73M film adhesive with a radius of 77.5 mm (3.05 in) was carefully marked in the center. The center point was then placed at the edge

Figure 3.28: AMRL specimen assembly (Aluminum sheets)

of the face-sheet over the pre-cracking notch. The adhesive film was rolled down with the teflon roller while carefully applying heat using a heat gun (Figure 3.29).

In order to achieve a nice fillet and avoid adhesive loss through bleeding, high temperature tape pieces were pre-cut using the 77.5 mm (3.05 in) template (Figure 3.30). The tape

Figure 3.29: AMRL specimen assembly (FM 73M film adhesive)

was placed as close as possible to the adhesive and firmly pressed down. This allowed to formation of a half circular barrier around the adhesive.

The precured boron patch was then centered on the adhesive and firmly pressed down. A second layer of high temperature tape was placed along the circular edge of the patch, therefore partially covering the patch, the adhesive and the previously placed high temperature tape on the face-sheet (Figure 3.30). In addition, high temperature tape was placed on the 3.175 mm (0.125 in) wide edge of the face-sheet and then folded onto the patch to seal in the adhesive. Much care was taken to firmly attach the tape to the thin edge. Vacuum sealant tape was then placed along this edge to ensure that the autoclave pressure was applied to this surface. This procedure proved to be very successful for avoiding adhesive bleeding. The bleeding only occurred through the precrack notch. The patch was securely held in place by two or three additional pieces of high temperature tape across the patch (Figure 3.31).

Figure 3.30: AMRL specimen assembly (High temperature tape barrier)

Figure 3.31: AMRL specimen assembly (Patch fixation)

The patch mounting procedure was then identically carried out on the other side of the face-sheet. The whole fixture was subsequently covered in one layer of perforated release film and one layer of bleeder cloth. In addition, one layer of breather cloth, which extended beyond the fixture on one side by approximately 15 cm (5.9 in) for the placement of the vacuum fitting, was wrapped around the fixture and affixed with high temperature tape.

Figure 3.32: AMRL specimen assembly (Vacuum bag)

The covered fixture was then placed in a vacuum bag. Folds of the vacuum bag were avoided in the patch areas (Figure 3.32). The vacuum bag was then connected to the autoclave system.

The cure cycle for the AMRL specimen was based on the FM 73M film adhesive. This cure cycle is similar to the one for the boron prepreg except that the vacuum was maintained throughout the curing process (Figure 3.33). The cure cycle started by raising the pressure to 344.7 kPa (50 psi). The temperature was then increased at a rate of $2.8^{\circ}\text{C}/\text{min}$ ($5.0^{\circ}\text{F}/\text{min}$). The 'Baron' Autoclave was operated in the manual mode for the curing cycle. The set point was adjusted every 2 minutes during the ramp up and down. The temperature ramp up was finished at 121.1°C (250.0°F), which was maintained for 60 minutes. After this time period, the temperature was ramped down at a rate of $2.8^{\circ}\text{C}/\text{min}$ ($5.0^{\circ}\text{F}/\text{min}$). The cooling rate was slowed at approximately 65.6°C (150°F) due to reaching the maximum cooling rate of the autoclave. The pressure was then dropped at approximately 45.0°C (113.0°F) and the bagged AMRL specimen removed from the autoclave. The vacuum

Figure 3.33: Cure cycle for the AMRL specimen

bag was opened when the fixture and AMRL specimen reached room temperature. For future work it is recommended to drop the vacuum after the initial temperature ramp up (see Figure 3.19) to avoid the possibility of 'adhesive boiling' at the honeycomb/face-sheet interface since this interface doesn't experience positive pressure.

The top part of the fixture and the center alignment pins were unscrewed and removed. The wide bottom section of the fixture was clamped in a vice and a wooden block with a hammer was used to tap the specimen out of the fixture. Touching the grip area of the specimen was avoided in order to maintain adequate surface preparation for gluing the spacers and shims. The grip area was then covered with Kraft paper. Small scratches on the fixture, which occurred close to the inner edge of the fixture, were removed using 220 wet grinding paper.

The shims (Figure A.4) and spacers (Figure A.2) were bonded to the specimen using the room temperature curing Hysol EA 9396 QT System. The spacers were machined to a tight tolerance specification to ensure proper fit. The adhesive was spread in a thin layer on each bonding surface prior to inserting the spacer. Alignment was achieved using the center alignment pins and a pin in one of the bolt holes on each side. Special care was taken to achieve a uniform bondline between the face-sheet and the shims. Piano wires with a diameter of 0.25 mm (0.01 in) were placed in the bondline. Piano wires were chosen due to their uniform thickness and allowed better adhesive flow throughout the mounting process. The face-sheet section outside the grip area was masked off with tape to avoid undesired adhesive spill. The wires were cut and placed first on one side of the specimen in the grip area. Then the adhesive was spread slightly thicker than the wires. The shims were aligned with the two pins on each grip side and firmly pressed down. Any excessive adhesive was then removed. Tape was placed along the tapered section of the shims and onto the face-sheets. This avoided adhesive being squeezed out between the face-sheet and

the shim when the grip section was clamped onto a 25.4 mm (1 in) thick flat aluminum plate after the same process was repeated with the opposite side. Two 12.78 mm (0.5 in) thick aluminum plates with slightly larger holes than the alignment pins were placed on top of the shim, which were facing up, to ensure uniform clamping pressure. The clamping pressure was applied by four C-clamps. Excessive adhesive was removed after the clamping.

The required thickness of 25.27 mm (0.995 in) in the grip area was achieved by milling half of the excessive amount on each side off. The mounting on a thick aluminum plate ensured that one side was perfectly flat in both grip areas, which simplified the mounting on the milling table. Flatness of the face-sheets was also verified prior to milling. No cutting fluids were used to avoid degradation of the patches and bonds. In addition the patch was protected against chips during the milling process.

3.6.4 Ultrasonic Inspection of the Boron Patch Repair

Submersion of the AMRL specimen for ultrasonic inspection was not chosen due to the possibility of water getting into the honeycomb, which could lead to deterioration of the bond if all of the water could not be fully removed after the NDI inspection. Instead, a Panametrics Epoch III Model 2300 ultrasonic system including a Nova NDT Instruments (D11P) hand-held probe was used in combination with a diagnostic ultrasound scanning couplant. The probe was fitted with a plexiglas delay shoe for signal damping and noise reduction at the interface.

The instrument settings were chosen based on the signal reflection of the back surface of a 3.175 mm (0.125 in) aluminum plate. The reflected signal of the composite repair remained approximately at 80% in comparison to the reflecting signal for the plain aluminum plate. The taper area was also carefully checked. As the probe was moved slowly towards the tapered end, the signal moved to the left on the time axis indicating a thinner specimen. In

addition, the signal decayed due to the angle of the taper. It is important that the transition of the signal is smooth. If a local disbond is present, the signal would be significantly reduced or even disappear. The AMRL specimens did not show any evidence of local disbonds in the patched area.

3.7 Instrumented AMRL Specimen Construction

One AMRL specimen was instrumented with strain gauges to provide experimental verification of the strain distribution in the patch and the cracked aluminum. The intent was to measure the strain distribution in the patched area as well as the strain distribution through the thickness of the repair in several critical locations. To accomplish this task, 350 Ω strain gauges were chosen in single element and 90° rosette configurations since high resistance gauges offer the advantage of reducing heat effects and therefore providing better stability.

The locations of the strain gauges were selected based predominantly on previously reported failure locations in the literature [13, 37, 70]. Strain gauges were therefore placed near the crack tip and at the tapered end. In addition, strain gauges were located in the center area of the patch close to the beginning of the taper. This gave a good reference location due to the reduced influence of the crack and taper on the magnitude of the strains at these locations. Figure A.11 shows most of the gauges in a superimposed drawing. Two reference gauges were mounted on the aluminum outside the patch. This allows for the normalization of the strains in comparison to a remote strain to allow for better applicability for other repair applications.

A number of strain gauges were installed at each of the above described locations. The gauges were placed between the different layers through the patch thickness and on top of

the patch. The through thickness locations were based on the critical points of the load transfer, where failures have been reported [13, 37, 70]. Therefore gauges were typically mounted on the aluminum, on top of the adhesive and on top of the first boron/epoxy layer. In addition some gauges were mounted on top of the patch. Unfortunately the strain gauges do not allow measurement of the shear strain in the adhesive. A main concern regarding placing strain gauges through the thickness of the patch is the presence of a local increase in thickness. For this reason the number of gauges was limited to four through the thickness of the repair. The gauge on top of the patch has limited influence by increasing the local stiffness only slightly, however the gauges embedded in the patch do increase the thickness of the patch introducing some error in the measured strains at the desired location. This error will be discussed in detail in conjunction with the finite element results in Chapter 6.

In order to minimize the increase in thickness, special attention was given to the types of gauges, their orientation, lead attachment and placement procedure. The 90° rosette configurations, where the gauges were arranged side by side, were a thinner choice for biaxial measurements than a stacked rosette configuration. These gauges were placed on the face-sheet and on top of the patch. Single element gauges were chosen for the placement on top of the adhesive and on top of the first boron/epoxy layer in order to reduce the number of lead wires in the patch. The measurement of biaxial strains on the aluminum was favoured due to the easier placement of the wires in the adhesive layer. All lead wires were preferably aligned with the boron fibers to minimize strain distortion. All gauges were placed with the lead wires initially facing different directions to allow for the smallest possible disturbance of the strain distribution especially along the fiber direction. The 34 gauge (0.16 mm or 0.006 in) lead wires, having a slightly larger diameter than the boron prepreg thickness of 0.13 mm (0.005 in), were attached with minimal solder to allow the wires to be easily embedded in the boron prepreg. The lead wires, which were connected to the gauges on the face-sheet, were entirely covered in the adhesive layer (approximately

0.25mm (0.010 in) thick) and thus didn't add additional thickness. Only the lead wires embedded in the adhesive or placed on top of the patch were oriented perpendicular to the loading direction and fiber orientation close to the gauge. Figure A.11 shows the complex arrangement of lead wires in the patch. The concern of changing the strain distribution significantly by using a large number of gauges and the associated leadwires led also to the decision not to place a row of very small strain gauges in the critical load transfer zones especially around the crack tip. Note that strain gauges mounted on composite material require a certain minimum size to get averaged strain readings across a number of fibers rather than measuring the local distribution around a fiber.

In addition, two strain gauge rosettes were placed on the back of the aluminum face-sheet. These particular gauges offer the option to measure destructively the residual strains in the aluminum. This allows a comparison with the measured residual strains using the non-destructive methods as described in Chapter 4, if the specimen is not required for any additional measurements.

3.7.1 Instrumented Boron Patch Fabrication Procedure

Manufacturing of the instrumented cocured boron/epoxy patch was divided into three steps:

1. Preparing the gauges,
2. Lay-up of the patch including placing the gauges and
3. Curing the patch.

Only single element strain gauges (CEA-13-062UW-350) were placed in the patch. Unfortunately, these gauges were unavailable with sufficiently long preattached leads. Therefore

150 mm (5.91 in) long lead wires were soldered prior to placing the gauges on the prepreg material. Special care was taken to minimize the solder build up at the tabs. Stripper X was used to remove the polyimide cover on the wire. The exposed solder tabs with the attached copper wire required a protective coating due to the conductivity of the tungsten core of the boron fibers. M-Coat D was chosen due to its excellent electrical isolation properties. The white colour offered easy verification of complete coverage. The solder joint and coating were inspected under the microscope. The coating was allowed to cure in 24 hours at room temperature.

The placement of the gauges and terminals was based on an accurate template of the desired gauge locations. The template was fixed on an aluminum plate and covered with a clear plastic sheet. In addition, a perforated release film was placed on top. A wooden frame was built around this setup. The next step dealt with marking the x and y locations of the gauges on the wooden frame. Nails were then placed in the frame in order to attach thin wires (38 gauge), which crossed at the center of each gauge (Figure 3.34). Then the FM 73M film adhesive layer was placed. Earlier trials showed that the handling of wires after the patch curing process is extremely difficult and in most cases led to damage to the wire or its coating. Therefore all wires including the terminals were embedded in the larger FM 73M adhesive layer. The terminal had to be cleaned after the curing process. The only drawback to this procedure was that the face-sheet was now slightly reinforced through the larger adhesive layer. The transparency of the FM 73M film adhesive allowed placement of gauges and terminals using the template underneath. Prior to placing gauges on the adhesive, all terminals were placed at their predefined locations. Once a gauge was placed on the adhesive, removal was extremely difficult. It also proved to be very difficult to place the wires along the desired straight lines. Due to the viscoelastic behaviour of the adhesive prior to curing, only limited pulling of the wires was permitted to straighten them out. Excessive tension led to movement of the gauge. The wires adhered only temporarily

to the adhesive after pressing them down with a flat spatula. Then the wires were cut to the correct length and the coating removed. Soldering to the terminal had to be done quickly due to the significant change in viscosity of the adhesive with temperature, i.e. the adhesive began to flow.

After placing the desired six gauges on the film adhesive, the first boron/epoxy prepreg ply was placed. The wire crossings were used to position the additional four gauges on this boron/epoxy ply at the proper locations. The lead wires were placed and connected as before. Then the remaining six

Figure 3.34: Placement of the strain gauges in the patch

boron/epoxy plies were positioned. The gauges on the top of the patch were not installed at this point to avoid any additional distortion in the boron/epoxy prepreg. A peel ply was added on each side. The peel ply on the top ensured acceptable handling qualities of the extended adhesive layer of the precured patch which protected the lead wires during the mounting procedure on the cracked face-sheet. The bagging and curing of the lay-up was carried out identically to the procedure outlined previously for the regular patches with the exception that the adhesive which exceeded the boron/epoxy prepreg was covered with high temperature tape to avoid excessive bleed. It should be noted that the placement of the 10 gauges required approximately 8 hours.

The patch was inspected after the autoclave curing process with both X-ray (Figure 3.35) and C-scan (Figure C.14) techniques. The X-ray film shows the precise placement of the

gauges on top of each other while the C-scan did not indicate the gauges clearly. In addition, the C-scan showed some distortion of the prepreg caused by the lead wires.

After trimming the edges of the patch, the gauge locations were accurately measured on a milling table. These coordinates, which differed by a maximum of 0.5 mm (0.02 in) from the desired values, were then used for the placement of the gauges on the face-sheet and on top of the patch. The draw-

Figure 3.35: X-ray of the instrumented patch

ings for the gauge locations were then modified to reflect the true positions.

3.7.2 Instrumented AMRL Specimen Assembly Procedure

The first step prior to assembling the instrumented AMRL specimen was the mounting of the six 90° rosettes (CEA-13-062UT-350) on the the aluminum face-sheet, which were later covered by the patch. In addition, two residual strain gauge rosettes (EA-13-062RE-120) were installed on the backside of the face-sheet. The terminals for these gauges were placed on the aluminum just outside the extended adhesive layer of the patch. The strain gauge adhesive system had to withstand the 121 °C (250 °F) curing temperature of the specimen, thus M-Bond 610 was chosen as an appropriate adhesive. Only two gauges were bonded simultaneously due to the difficulty of using spring clamps in the patch area. The two component, solvent-thinned, epoxy-phenolic adhesive was cured at 121 °C (250 °F) for 2.5

hours according to its specifications. The face-sheet surface was protected by a vacuum bag sheet against accidental touching and other possible contamination.

The lead wires were positioned as per Figure A.7 and Figure A.12. The wires were held in place using M-Coat D. The residual strain gauges on the backside of the face-sheet were coated with 3 layers of M-Coat A. The clear M-Coat A was chosen to allow for proper placement of the drill for the possibility of destructive residual strain measurements. The solder tabs with the attached wires were protected with M-Coat D.

The honeycomb which extended past the face-sheet edges also required modification to avoid cutting through the lead wires on the backside of the face-sheet during the autoclave cycle. The honeycomb was carefully indented, which led to small distortions. Small strips of FM 73M were placed in the indented section of the honeycomb with the exception of the gauge area to achieve some degree of bonding between the face-sheet and the honeycomb in this modified area.

The larger rectangular FM 73M film adhesive was placed carefully over the gauges on the topside of the face-sheet. The FM 73M layer exceeded the patch including the large rectangular cocured adhesive film by 2.5 mm. The gauges on the backside of the face-sheet were not covered with the film adhesive as these sections were carefully cut out of the film adhesive prior to the placement. The remaining assembly and curing procedure for the bonding of the patches, aluminum face-sheets and the honeycomb of this instrumented specimen were identical to the previously outlined procedure.

In total, 28 gauges were installed at this stage. After removal and clean-up, spacers and shims were installed in the grip region using the room curing adhesive HYSOL EA 9396. The last step in the instrumented specimen preparation process was the installation of eight, two-element (90°) strain gauges on top of the instrumented patch and outside the repair

section (see Figure A.10). A maximum of two gauges were installed at any one time using M-Bond 610, resulting in the need for four curing cycles. To limit the influence of thermal cycling, the strain gauge glue was cured at 100°C . A final post-cure was then carried out at 107°C , which was the highest temperature anticipated for the thermal experiments. This completed the installation of the 44 strain gauges.

The described manufacturing process is a significant achievement in itself, which wouldn't have been possible without the help from NRC especially Don Raizenne. One of the biggest challenges was the collection of all details required to build this specimen type. Many of the details are not revealed in the literature to the extent necessary. NRC provided missing details with respect to the surface preparation procedure. No jig specifications were provided in the literature either. The designed jig is characterized by simplicity and low manufacturing cost. Installing the large number of strain gauge required patience and skill as well as extensive planning to minimize the impact on the strain distribution. It is important to keep in mind that any mistakes in the manufacturing process, such as poor surface preparation leading to a poor bond or disbonds, is detrimental for any kind of experimental work.

Having outlined the construction of an instrumented AMRL specimen in detail, the following chapter will deal with the challenges of carrying out thermal residual strain measurements and the total residual strain measurement. The total residual strains are a combination of mainly thermal residual strains and some processed induced strains such as adhesive shrinkage. Their measurement was integrated in the instrumented AMRL specimen assembly procedure based on measuring the strain state in the patch and face-sheet prior and after assembling the specimen.

Chapter 4

Residual and Thermal Residual Strain Measurements

4.1 Introduction

This chapter discusses the challenges involved in measuring residual strains induced by the elevated temperature cure in bonded repairs. A significant amount of new experimental data is provided giving the opportunity to verify theoretical estimates of the residual stress and strain state, as well as establishing the foundation for a fatigue damage initiation model for bonded repairs, where knowledge of the initial stress/strain state of the repair is important. Although thermal residual strains are the primary focus of this research, residual strains due to the elevated temperature cure of any bonded repair consists of both thermal and process induced residual strains, the latter resulting from effects such as anisotropic shrinkage of the patch or adhesive, temperature gradients in the specimen during processing, and/or temperature gradients due to tooling and internal resin flow [50]. While thermal strains are affected by subsequent changes in temperature, process induced residual strains may not be affected.

The residual strain measurement procedure, which was carried out during the assembly stage of the instrumented AMRL specimen, is outlined first. This particular method was only carried out for measuring the total residual stresses at room temperature, while the thermal residual stresses were continuously measured from the stress free temperature (101.2°C) to extremely low temperatures representing typical aircraft operating temper-

atures (-56.5°C). The stress free temperature was experimentally measured rather than assuming that the curing temperature corresponds to the stress free temperature as is commonly done. The results emphasized the importance of this measurement. The continuous measurement of the thermal residual strains revealed additional characteristics about the adhesive at elevated temperatures giving a better understanding for the estimation of thermal residual strains. Process induced strains were assessed based on the difference between the measured residual strains and the thermal residual strains. In order to simplify the estimation of thermal residual strains for other material combinations especially with respect to the utilized adhesive, a suggestion for a simple test specimen rather than using the complex AMRL specimen is presented focusing on the behaviour of the adhesive near the stress free temperature.

4.2 Residual Strain Measurement Procedure

4.2.1 Approach and Experimental Set-up

To determine the magnitude of the residual strains introduced during curing requires knowledge of the strain state both prior to and after completion of the thermal curing cycle. The strain state prior to the curing process was obtained using two Vishay 2310 signal conditioning amplifiers,

Figure 4.1: Initial patch strain measurement

one each for the 120 Ω and 350 Ω quarter bridge configurations. At this point, 120 Ω and 350 Ω precision resistors, respectively, were used in the quarter bridge to zero the two signal conditioning amplifiers. The precision resistors were then removed and replaced by the mounted strain gauges, one gauge at a time, with the indicated resistance change (i.e. strain) being recorded (see Figure 4.1). Since the cocuring process of the one layer of FM 73M with the seven boron/epoxy plies had introduced residual stresses in the patch itself (as indicated by the slightly curved patch), the patch was vacuum-bagged against a flat aluminum plate for the initial strain recordings to provide a repeatable and representative measurement condition both pre- and post-cure. While this flat patch geometry was representative of the pre-curing setup in the autoclave, it should be noted that the peel ply on the bottom of the patch was not removed for this measurement to protect the bonding surface from contamination. After curing, this measurement condition (including temperature conditions) was maintained and the strain measurement recording procedure repeated.

4.2.2 Data Analysis

The residual strains induced during the curing cycle were then determined by calculating the difference in the pre- and post-cure strain measurements. The residual strain values were then corrected for the resistance of the lead wires [28]:

$$\epsilon_{LW \text{ corr. RS}} = \frac{\epsilon_{RS}}{1 - \rho} \quad (4.1)$$

where

$$\rho = \frac{\frac{2R_{Long \text{ LW}} + 2R_{Short \text{ LW}}}{R_G}}{1 + \frac{2R_{Long \text{ LW}} + 2R_{Short \text{ LW}}}{R_G}} \quad (4.2)$$

In addition the residual strain values of the strain gauge rosettes were adjusted for transverse sensitivity as follows [61].

For 90° rosettes parallel and perpendicular to the fiber direction:

$$\epsilon_{\parallel \text{TSY corr. RS}} = \frac{\epsilon_{\parallel \text{LW corr. RS}}(1 - \nu_o K_{t\parallel}) - K_{t\parallel} \epsilon_{\perp \text{LW corr. RS}}(1 - \nu_o K_{t\perp})}{1 - K_{t\parallel} K_{t\perp}} \quad (4.3)$$

and

$$\epsilon_{\perp \text{TSY corr. RS}} = \frac{\epsilon_{\perp \text{LW corr. RS}}(1 - \nu_o K_{t\perp}) - K_{t\perp} \epsilon_{\parallel \text{LW corr. RS}}(1 - \nu_o K_{t\parallel})}{1 - K_{t\parallel} K_{t\perp}} \quad (4.4)$$

For gauges 17 and 20 (see residual strain rosettes in Figure A.12):

$$\begin{aligned} \epsilon_{\text{G17 TSY corr. RS}} = & \frac{\epsilon_{\text{G17 LW corr. RS}}(1 - \nu_o K_{t\text{G17}})}{1 - K_{t\text{G17}}} \\ & - \frac{K_{t\text{G17}}(\epsilon_{\text{G18 LW corr. RS}}(1 - \nu_o K_{t\text{G18}})(1 - K_{t\text{G19}})}{(1 - K_{t\text{G18}} K_{t\text{G19}})(1 - K_{t\text{G17}})} \\ & - \frac{K_{t\text{G17}}(\epsilon_{\text{G19 LW corr. RS}}(1 - \nu_o K_{t\text{G19}})(1 - K_{t\text{G18}})}{(1 - K_{t\text{G18}} K_{t\text{G19}})(1 - K_{t\text{G17}})} \end{aligned} \quad (4.5)$$

and

$$\begin{aligned} \epsilon_{\text{G20 TSY corr. RS}} = & \frac{\epsilon_{\text{G20 LW corr. RS}}(1 - \nu_o K_{t\text{G20}})}{1 - K_{t\text{G20}}} \\ & - \frac{K_{t\text{G20}}(\epsilon_{\text{G21 LW corr. RS}}(1 - \nu_o K_{t\text{G21}})(1 - K_{t\text{G22}})}{(1 - K_{t\text{G21}} K_{t\text{G22}})(1 - K_{t\text{G20}})} \\ & - \frac{K_{t\text{G20}}(\epsilon_{\text{G22 LW corr. RS}}(1 - \nu_o K_{t\text{G22}})(1 - K_{t\text{G21}})}{(1 - K_{t\text{G21}} K_{t\text{G22}})(1 - K_{t\text{G20}})} \end{aligned} \quad (4.6)$$

where

$$\nu_o = 0.285 \quad (4.7)$$

Correcting for the transverse sensitivity of the single element gauges was considered unnecessary as the maximum error introduced was calculated to be less than $4\mu\epsilon$ due to the low residual strains perpendicular to the fiber direction. The error estimate is based on

the change in the measured residual strains due to transverse sensitivity from the rosettes mounted on the aluminum and on top of the patch.

An additional small error in the strain gauge readings was introduced by the applied hydrostatic pressure (0.1 MPa (29.5 inHg)) during the initial strain measurement of the boron/epoxy patch. The magnitude of the error for the gauges installed in the boron/epoxy prepreg was estimated using Hooke's law:

$$\epsilon_{\parallel} = \frac{1}{E_{\parallel}}((-p) - 2\nu_{\parallel}(-p)) \quad (4.8)$$

and

$$\epsilon_{\perp} = \frac{1}{E_{\perp}}((-p) - 2\nu_{\perp}(-p)) \quad (4.9)$$

Using the material properties provided in Appendix D, the estimated strain parallel to the fiber orientation was calculated to be $-0.3\mu\epsilon$ and $-3.8\mu\epsilon$ perpendicular to the fiber orientation. The total residual strains were not corrected for this small error due to the lack of a suitable instrumented boron/epoxy specimen with a flat bottom surface for experimental verification. In order to experimentally determine the magnitude of these strains, the instrumented AMRL specimen was vacuum-bagged. Strains between 0 and $3\mu\epsilon$ were measured in the boron/epoxy patch.

4.2.3 Results

Table 4.1 lists the results for the residual strain measurement. The residual strains are also shown in Figure 4.16 at their measured locations. Having measured the total residual strains, the next step was the experimental determination of the thermal residual strains as a function of temperature which exclude any process induced strains. The stress free temperature was required for the material combination used in the AMRL specimen in order to carry this measurement out.

Location []	Gauge No. []	Orientation [°]	Gauge Resistance [Ω]	Initial Strain Reading [μϵ]	Final Strain Reading [μϵ]	Residual Strain [μϵ]	Corrected Residual Strain [μϵ]
Face-sheet (Front)	1	90	350	608	580	-28	-28
Face-sheet (Front)	2	0	350	871	868	-3	-3
Face-sheet (Front)	3	0	350	981	1674	693	693
Face-sheet (Front)	4	90	350	1092	951	-141	-147
Face-sheet (Front)	5	0	350	1413	1926	513	513
Face-sheet (Front)	6	90	350	246	107	-139	-143
Face-sheet (Front)	7	0	350	1227	1560	333	334
Face-sheet (Front)	8	90	350	606	488	-118	-121
Face-sheet (Front)	9	90	350	158	-6	-164	-168
Face-sheet (Front)	10	0	350	1677	2163	486	487
Face-sheet (Front)	11	90	350	1221	1176	-45	-50
Face-sheet (Front)	12	0	350	1335	1932	597	596
Face-sheet (Back)	17	45	120	1767	1938	171	168
Face-sheet (Back)	18	90	120	1587	1383	-204	-211
Face-sheet (Back)	19	0	120	1170	1722	552	554
Face-sheet (Back)	20	45	120	2244	2409	165	165
Face-sheet (Back)	21	90	120	2112	1854	-258	-263
Face-sheet (Back)	22	0	120	730	1164	434	437
Patch (Adhesive)	23	0	350	1458	1686	228	228
Patch (Adhesive)	24	0	350	1665	975	-690	-690
Patch (Adhesive)	25	0	350	1656	926	-730	-730
Patch (Adhesive)	26	0	350	1494	660	-834	-835
Patch (Adhesive)	27	0	350	1266	450	-816	-817
Patch (Adhesive)	28	0	350	2121	1443	-678	-678
Patch (1 st B/ep Ply)	29	0	350	1278	540	-738	-738
Patch (1 st B/ep Ply)	30	0	350	1782	1059	-723	-724
Patch (1 st B/ep Ply)	31	0	350	1686	910	-776	-777
Patch (1 st B/ep Ply)	32	0	350	1473	651	-822	-823

Table 4.1: Residual Strains

Note: The 0° orientation corresponds to the fiber orientation.

4.3 Stress Free Temperature Measurement Procedure

Determination of the thermal residual strains at any temperature requires knowledge of the stress free temperature at which no thermal residual strains are present. In order to determine the stress free temperature for the current application, an additional identical patch was bonded to a 152.4 mm x 152.4 mm aluminum alloy sheet using one layer of FM 73M. After curing and returning to room temperature the residual strains caused a significant curvature in the aluminum sheet. To determine the stress free temperature, the patched aluminum test sheet was heated until it returned to its initial straight condition in the fiber direction.

This measurement was carried out using a Blue M industrial oven equipped with a Microtrol controller. The patched aluminum test sheet was supported with a flat steel bar and a pin (see Figure 4.2). A halogen light was used to clearly indicate the presence of any gap existing between the curved aluminum sheet and the flat steel bar.

Figure 4.2: Stress free temperature measurement setup

Both the oven and aluminum sheet temperatures were monitored using thermocouples. The temperature measurement of the aluminum was carried out by clamping a thermocouple

between a silicon rubber pad and the aluminum sheet using a spring clamp. The welded joint of the thermocouple was flattened to the wire thickness prior to clamping. At 101.2°C , with both the specimen and oven in thermal equilibrium, the single-side patched aluminum sheet was straight along the supported edge. The temperature was held for over 30 minutes without monitoring any changes in the shape. This experimental set-up had an estimated potential error of 1.5°C based on the stated assumptions (i.e. potential resolution error of 0.5°C + potential temperature measurement error of $< 1^{\circ}\text{C}$ using listed device specifications). Thus the stress free temperature for the boron/epoxy patch-aluminum sheet combination was deemed to be $101.2 \pm 1.5^{\circ}\text{C}$.

4.4 Thermal Residual Strain Measurement Procedure

4.4.1 Approach

To quantify the thermal residual strains (TRS) within the AMRL specimen, it was decided to heat the specimen to the stress free temperature (SFT). However, accurate determination of the thermal residual strains in the bonded composite repair required the measurement of not only ther-

Figure 4.3: Titanium silicate bar with installed gauges

mal strains in the AMRL specimen, but also measurement of the thermal strains for an unbonded separate aluminum and a boron/epoxy material specimens as required by equa-

tions (4.10) and (4.11). The thermal strains in the boron/epoxy were measured in the fiber direction as well as the transverse direction. In addition, the thermal strains in the aluminum were distinguished between those measured in rolling direction and those perpendicular to the rolling direction. It should be noted that the fiber orientation in the patch corresponds with the rolling direction of the aluminum.

$$\epsilon_{Al\ TRS}|_T = \epsilon_{Al}|_{T=SFT} - \epsilon_{AMRL}|_{T=SFT} - \epsilon_{Al}|_T + \epsilon_{AMRL}|_T \quad (4.10)$$

$$\epsilon_{B/Ep\ TRS}|_T = \epsilon_{B/Ep}|_{T=SFT} - \epsilon_{AMRL}|_{T=SFT} - \epsilon_{B/Ep}|_T + \epsilon_{AMRL}|_T \quad (4.11)$$

To allow separation of the true thermal strain from the indicated strains, one identical gauge of each installed strain gauge type was mounted on a titanium silicate bar (see Figure 4.3) having a near zero coefficient of thermal expansion (i.e. $0.03 \pm 0.03 \cdot 10^{-6}/^{\circ}C$) as shown in Figure 4.4 [62].

Figure 4.4: Thermal expansion characteristics of titanium silicate

By accounting for the small thermal strains of the titanium silicate, the actual thermal strains at the strain gauged locations on the aluminum, boron/epoxy as well as on the AMRL specimen are given by equation (4.12):

$$\epsilon_{Specimen\ Thermal}|_T = \epsilon_{Specimen\ (Indicated)}|_T - (\epsilon_{TS\ (Indicated)}|_T - \epsilon_{TS\ (Ref.)}|_T) \quad (4.12)$$

4.4.2 Experimental Setup

The measurements of the thermal residual strains were carried out inside a Blue M industrial oven equipped with a Microtrol controller. The instrumented AMRL specimen was supported on a frame consisting of steel rods and two small aluminum sheets. Thin

teflon tape was used in the contact areas between the specimens and the frame to reduce friction. To minimize thermal shock effects, the instrumented AMRL specimen together with the titanium silicate bar and the lead wire resistance measurement specimens were wrapped in breather cloth and covered with a thin aluminum foil (see Figure 4.5). The same protection was provided for the instrumented aluminum and the boron/epoxy specimens. This setup led to the very small scatter found in the recorded data. The temperature of the AMRL specimen and the titanium silicate bar were both measured using bonded thermocouples. A Schlumberger Orion datalogger utilizing a high stability, dual current technique allowed simultaneous acquisition of temperature, wire resistance and strain gauge data. To avoid any potential heating effects due to power dissipation at the strain gauge sites, only gauges within a single interface (i.e. ply, layer, etc.) were measured during any single test.

The difference in thermal conductivity and mass between the AMRL and titanium silicate specimens caused a significant difference in the rate of temperature change. Therefore it was

decided to establish thermal equilibrium between the specimens and the oven temperature at specific temperatures, i.e. 21°C (room temperature), 50°C, 80°C and 107°C for the elevated temperature cycle (see Figure 4.6).

Figure 4.6: Regular and high heating/cooling rate cycles

Measuring to temperatures slightly above the stress free temperature showed that the strain change with temperature remained essentially constant in this region. The strain measurements were carried out both during the heating and cooling portions of the thermal cycles. Approximately two hours were required to achieve thermal equilibrium at each of the specified temperatures. To reduce testing time, the oven temperature was initially raised above each equilibrium temperature by 20°C and then returned to the equilibrium temperature when the AMRL specimen was close to reaching this temperature. Due to the indication of stress relaxation by an open hysteresis while holding at elevated temperatures, this measurement procedure was repeated for 12 gauges to check if the measured strains were repeatable. In addition, data was also obtained using an approximate 2 to 3 fold increase in heating and cooling rates (see Figure 4.6) to investigate the influence of heating and cooling rates on the hysteresis caused by stress relaxation.

4.4.3 Data Analysis

Figure 4.7 shows the indicated strains with temperature for the regular and the high heating/cooling rate cycles for gauge 3 which was mounted on the aluminum and aligned with the fiber direction.

Figure 4.7: Indicated strains (Gauge 3) for different temperature cycles

It should be noted that Figure 4.7 shows compressive strains instead of the expected tensile strains for the aluminum which are caused by using an aluminum temperature compensated strain gauge on the boron/epoxy restrained aluminum sheet, i.e. the strain gauge material expands more than the restrained aluminum sheet. The true thermal strains determined in the data analysis are tensile strains.

The indicated strain versus temperature for this gauge location remains roughly linear up to approximately 80°C. Between 80°C and 90°C, a slope change occurs to a near zero strain versus temperature gradient. The initial change of indicated strain between ambient temperature and approximately 80°C reflects the combined expansion of the bonded repair, while above 90°C a reduction in shear modulus combined with the visco-elastic properties of the FM 73M film adhesive allow nearly unrestrained expansion of the patch

and the aluminum. Studies by Jurf and Vinson [56] indicated that the creep compliance of FM 73M changes significantly above 80°C. The visco-elastic behavior of FM 73M is also clearly visible at 80°C in Figure 4.7 (see location B-B' and D-D'), where stress relaxation occurred while holding the specimen at this temperature for an extended period of time (see Figure 4.6). Hysteresis develops mainly in the temperature range between 80 and 90°C. The indicated strains between ambient temperature and 80°C appear to be only slightly affected by the visco-elastic behavior. The strain measurements obtained for higher heating and cooling rates (see Figure 4.7) also show the development of hysteresis between 80°C and 90°C with the difference that no drop in the indicated strain was present at 80°C. This measurement case represents more closely the cooling of the specimen from the curing temperature to room temperature at the end of the autoclave cycle.

Due to these observations it was decided to correct the measured data for stress relaxation while holding the specimen at an elevated temperature, e.g. shifting point B' on to B and D' on to D. This procedure is also shown in Figure 4.10. This procedure will not eliminate stress relaxation effects entirely, which will also occur during heating and cooling periods the AMRL specimen, but it allows better comparison between measurements.

Figure 4.8: Origin and stress relaxation corrected indicated strains (Gauge 3)

Essential for the determination of the thermal residual strains is the strain measured at the stress free temperature (101.2°C). This particular strain represents the reference value for zero thermal residual strain. Up to three measurements for the heating and cooling cycles were carried out for each strain gauge. Due to the agreement of the indicated strains in the temperature range between ambient temperature and approximately 80°C for the repeated measurements, the data from the heating and cooling segments were separated and the indicated strains from the cooling cycle were shifted to $0\mu\epsilon$ at ambient temperature. The result of correcting for stress relaxation while holding at elevated temperatures and shifting of the measured strains during the cooling cycle is shown in Figure 4.8. It should be noted that the resulting curve is now much closer to the strain measurement with the high heating and cooling rate (see Figure 4.7).

Figure 4.9: Superposition of all corrected indicated strains (Gauge 3)

This procedure has the advantage that the hysteresis effect is projected to the stress free temperature, where now a certain scatter between the heating and cooling cycle as well as repeated measurements is visible (see Figure 4.9). The scatter of the corrected indicated strains at the stress free temperature appears rather large, but it is only 2.5% in comparison to the true thermal strain at this temperature.

Figure 4.10: Indicated strains for elevated temperature range (Gauge 38)

The amount of stress relaxation while holding at an elevated temperature is not only dependent on the time but also on the location and orientation within the patch. The largest change in indicated strain was found at gauge 38 at the tapered edge of the boron/epoxy patch oriented transverse to the fiber direction (see location B-B' and C-C'). It is interesting to note that no stress relaxation occurred during the cooling cycle while holding at 80°C (see location D-D'). The previously outlined method of adjusting for stress relaxation and shifting the cooling cycle to 0 $\mu\epsilon$ at ambient temperature shows acceptable agreement between the indicated strain measured using the regular temperature cycle and the high heating/cooling rate cycle. Although the hysteresis seems quite large when evaluating Figure 4.10, the true thermal strain at this location is close to 1900 $\mu\epsilon$ at the stress free temperature, i.e. the hysteresis is less than 6% of the largest thermal strain for the origin and stress relaxation corrected measurement.

Only negligible strain hysteresis response were found for the strain gauges mounted either inside or on top of the boron/epoxy patch for the gauges oriented in the fiber direction. Figure 4.11 shows the uncorrected indicated strains for gauge 24 for three repeated measurements.

Figure 4.11: Indicated strains (Gauge 24)

Due to the low thermal conductivity of the titanium silicate, long holding periods were required to achieve accurate readings at the equilibrium temperatures. In-between these equilibrium points, large hystereses are present (see Figure 4.12).

Figure 4.12: Indicated thermal strains of titanium silicate (Gauge 45)

Only the indicated strains at the equilibrium temperatures of the gauges mounted on the titanium silicate bar were used in the subsequent analysis due to these large hystereses. A second order polynomial regression analysis which uses different polynomials depending on

the location on the data curve was employed to approximate the indicated thermal strains for the entire temperature range.

Generally the indicated strains for the low temperature range (see Figure 4.13) show only minor hysteresis effects in comparison to the elevated temperature range. Figure 4.13 displays the uncorrected indicated strains for gauge 38 which has a large hysteresis for the elevated temperature range. The data analysis of the low temperature range includes a shift of the indicated thermal strains during the heating cycle to $0\mu\epsilon$ at 21°C . This projects any hysteresis to the lowest temperature.

Figure 4.13: Indicated strains for low temperature range (Gauge 38)

All recorded strain measurements were subsequently corrected for errors due to the resistance change with temperature of the lead wires connecting the strain gauges to the terminals (referred to subsequently as short lead wires), the lead wire resistance itself, the gauge factor change with temperature and the transverse strain sensitivity. A specific order for correcting errors was maintained to avoid introducing unnecessary small errors. As a first step, a polynomial regression using a localized regression scheme [92] was carried out to determine the indicated strains at defined temperatures allowing subsequent mathematical operations. Next the recorded strains were converted to the ratio of the resistance changes

to the gauge resistance:

$$\frac{\Delta R_{\text{indicated}}}{R_G} = S_G \epsilon_{\text{indicated}} \quad (4.13)$$

Three lead wire configurations providing thermal compensation were installed between the terminal blocks mounted on the specimens and the data acquisition system (referred to subsequently as long lead wires). Using a two lead wire system between the terminals and strain gauges introduces a significant error due to the lack of thermal compensation in the circuit. In order to correct the measured data, the resistance change per °C and per mm of lead wire length between the strain gauge and terminal blocks was obtained from two lead wire resistance specimens. The resistance values for both a 5.01 m long and a 0.01 m long lead wire specimen were measured as a function of temperature using four wire resistance measurement setups. These resistances values were determined only after stabilizing at specified temperatures within the measurement range. Employing the difference of these two measured resistances in the analysis provides the resistance for a 5 m long lead wire section and compensates for possible small thermoelectric effects. Due to the approximately linear relationship of the resistance change with temperature for a metal conductor within the investigated temperature range [57],

$$R|_T = R|_{T=21^\circ\text{C}}(1 + \alpha(T - 21^\circ\text{C})) \quad (4.14)$$

the resistance change versus temperature was determined using a linear regression analysis (see Figure 4.14).

The resistance change per mm of lead wire and °C was determined to be:

$$\frac{\Delta R_{\text{Short LW}}}{\Delta T l_{5\text{mRef. Short LW}}} = 3.46 \frac{\mu\Omega}{^\circ\text{C mm}} \quad (4.15)$$

with a standard error of $0.036 \mu\Omega/^\circ\text{C mm}$. Note that the resistance change of a 120Ω strain gauge with a gauge factor of 2 is $240 \mu\Omega$ for an applied strain of $1 \mu\epsilon$. For some gauges having up to 230 mm long lead wires and a measurement temperature range between

Figure 4.14: Lead wire resistance change with temperature

−56°C and 101.2°C, it was found that the introduced error is significant (up to 1000 $\mu\epsilon$) and cannot be neglected. The 350 Ω gauges have a significant advantage over the 120 Ω gauges in this respect.

The superimposed resistance change of the two short lead wires on the strain gauge resistance change is:

$$\left. \frac{\Delta R_{\text{Short LW}}}{R_G + R_{\text{Long LW}} + 2R_{\text{Short LW}}} \right|_T = \frac{2l_{\text{Short LW}} \frac{\Delta R_{\text{Short LW}}}{\Delta T l_{5m\text{Ref. Short LW}}} (T - 21^\circ\text{C})}{R_G + R_{\text{Long LW}} + 2R_{\text{Short LW}}} \quad (4.16)$$

Thus the ratio of the corrected resistance change due to the resistance change of the lead wire to the resistance of the strain gauge including the lead wires is:

$$\left. \frac{\Delta R_{G \text{ Short LW corr.}}}{R_G + R_{\text{Long LW}} + 2R_{\text{Short LW}}} \right|_T = \left. \frac{\Delta R_{G \text{ indicated}}}{R_G + R_{\text{Long LW}} + 2R_{\text{Short LW}}} \right|_T - \left. \frac{\Delta R_{\text{Short LW}}}{R_G + R_{\text{Long LW}} + 2R_{\text{Short LW}}} \right|_T \quad (4.17)$$

Adjusting now for the total lead wire resistance gives the ratio of resistance change to gauge resistance corrected for all lead wire errors:

$$\left. \frac{\Delta R_{G \text{ LW corr.}}}{R_G} \right|_T = \frac{\left. \frac{\Delta R_{G \text{ Short LW corr.}}}{R_G + R_{\text{Long LW}} + 2R_{\text{Short LW}}} \right|_T}{1 - \rho_{\text{AMRL}}} \quad (4.18)$$

where

$$\rho_{AMRL} = \frac{\frac{R_{Long\ LW}}{R_G} + \frac{2R_{Short\ LW}}{R_G}}{1 + \frac{R_{Long\ LW}}{R_G} + \frac{2R_{Short\ LW}}{R_G}} \quad (4.19)$$

Note that only the resistance for one long lead wire is used in this equation due to the three wire system employed between the terminals and the data acquisition system. The resistance for both short lead wires is required based on the two wire system between the terminals and the strain gauges.

Although the three lead wire system connecting the terminals to the data acquisition unit offers temperature compensation, the absolute resistance of the lead wire increases with temperature. In equation (4.19) only the lead wire resistance at room temperature was used and thus introduces a small second order error. In order to maintain temperature compensation, special attention was brought to the arrangement of the long lead wires at the oven exit. All long lead wires were cut to an identical length (2.25m) and arranged to have an identical length inside the oven.

Based on the information provided by the strain gauge manufacturer, the gauge factor was adjusted with respect to the temperature based on the following:

$$S_G|_T = S_G|_{T=24^\circ C} + \frac{dS_G}{dT}(T - 24^\circ C) \quad (4.20)$$

The strain at the strain gauged location was then determined using:

$$\epsilon_{Specimen}|_T = \frac{1}{S_G|_T} \left(\frac{\Delta R_{G_{Specimen\ LW\ corr.}}}{R_G} \Big|_T - \frac{\Delta R_{G_{TS\ LW\ corr.}}}{R_G} \Big|_T \right) \quad (4.21)$$

Note that the indicated thermal strains from the titanium silicate specimen were corrected for the true thermal strain of the titanium silicate. The next step included the correction for transverse sensitivity thus giving the true thermal strains. The transverse sensitivity

correction for the rosettes is carried out using (4.3) through (4.6) identically to the correction used for the total residual strains. The transverse sensitivity correction for single element gauges mounted inside the boron/epoxy patch or at the interface between the film adhesive and the boron/epoxy were corrected using the transverse strains measured on top of the patch. This approach is reasonable due to the similar magnitude of thermal strains measured on the aluminum sheet and on top of the patch perpendicular to the fiber orientation. The maximum difference in thermal strains between the aluminum and the transverse direction of the boron/epoxy was $175 \mu\epsilon$ at -56.5°C thus resulting in a possible error in the transverse thermal strains of only $2\mu\epsilon$. Finally the thermal residual strains for the gauges mounted on the aluminum and on the boron/epoxy are as follows:

$$\epsilon_{\text{AL TRS}}|_T = \epsilon_{\text{AI}}|_{T=\text{SFT}} - \epsilon_{\text{AMRL}}|_{T=\text{SFT}} - \epsilon_{\text{AI}}|_T + \epsilon_{\text{AMRL}}|_T \quad (4.22)$$

$$\epsilon_{\text{B/EP TRS}}|_T = \epsilon_{\text{B/EP}}|_{T=\text{SFT}} - \epsilon_{\text{AMRL}}|_{T=\text{SFT}} - \epsilon_{\text{B/EP}}|_T + \epsilon_{\text{AMRL}}|_T \quad (4.23)$$

Error estimates indicate that the maximum variation due to hysteresis during these tests was $31 \mu\epsilon$ with an average of only $9 \mu\epsilon$, which does not include stress relaxation effects while holding at elevated temperatures during the experiments. It was noted that an error of 1% in the gauge factor leads to a change of up to $8 \mu\epsilon$ in the thermal residual strain while the maximum uncertainty of 10 mm in the length of the lead wires connecting the strain gauges with the terminals will result in an error of only $4 \mu\epsilon$. An error in the measured resistance change with temperature of the lead wires connecting the strain gauges to the terminals has a significant effect on the thermal strain of the 120Ω gauges. An error of up to $30 \mu\epsilon$ was observed for a 6% change in the resistance change with temperature. The maximum error for the 350Ω was only $5 \mu\epsilon$ due to the higher gauge resistance and the shorter lead wires. Errors regarding the absolute resistance of all lead wires can be neglected. An error of 2°C in the stress free temperature was found to have only a small effect (approximately $1 \mu\epsilon$).

4.4.4 Results

Figure 4.15 shows the location of the installed strain gauges as well as their numbers. The complete record of the corrected results obtained for the thermal and thermal residual strains is presented in Appendix E. The thermal strains were plotted in addition to the thermal residual strains to show the relative magnitude between them. Figure 4.16 shows the thermal residual strains based on the temperature difference between the measured stress free temperature (101.2°C) and ambient temperature (21°C). The more severe case with respect to thermal residual strains for an operating temperature of -56.5°C is summarized in Figure 4.17, which is equivalent to the altitude range between 11050m and 20000m based on the ICAO standard atmosphere [49].

The thermal residual strains presented in Figure 4.16 and Figure 4.17 are averaged results based on data obtained from the heating and cooling cycles as well as repeated measurements. Holding for extended periods of time at elevated temperatures caused detectable strain change due to stress relaxation in the bonded repair, especially perpendicular to the fiber direction (on average $< 20\ \mu\epsilon$ with maximum of $90\ \mu\epsilon$ at the tapered end in 55 minutes at 80°C). All of the displayed data were corrected for any stress relaxation which occurred while holding at elevated temperatures

Appendix E contains plots of the measured thermal strains as well as thermal residual strains for all strain gauges. As an example, the typical thermal residual strain characteristic versus temperature is discussed for gauges mounted at one strain gauged location. Figure 4.18 shows the thermal residual strains versus the temperature at specimen location 'B' (see Figure 4.16) close to the crack tip. The curves show the thermal residual strains in both the longitudinal and transverse directions of the specimen for the aluminum top surface and for the top surface of the boron/epoxy patch in the fiber direction.

Figure 4.15: Gauge locations

Figure 4.16: Thermal residual and total residual strains at ambient temperature due to an elevated temperature cure

Figure 4.17: Thermal residual strains at -56.5°C due to an elevated temperature cure

The thermal residual strains for the aluminum in the longitudinal direction decrease nearly linearly between -56.5°C and approximately 83°C . Above 90°C the change in the thermal residual strain with temperature is small, i.e. the boron/epoxy patch and the aluminum expand nearly at their unrestrained rates.

Figure 4.18: Thermal residual strain versus temperature

The thermal residual strain for the aluminum in transverse direction is nearly constant between -56.5°C and -20°C and then slowly increases up to approximately 15°C . The thermal residual strain gradient remains approximately linear beyond this point up to 83°C . The thermal residual strains for the aluminum in the transverse direction remain near zero above 90°C .

The thermal residual strain in the fiber direction versus temperature curve for the top surface of the boron/epoxy is initially linear from -56.5°C to approximately 60°C . At this temperature the slope of the curve changes to a slightly steeper gradient before the slope changes significantly at approximately 83°C . Above 87°C the residual thermal strain in the boron/epoxy changes only slightly with trends corresponding to those of the aluminum.

The cause for the change of the thermal residual strain gradient with temperature of the boron/epoxy patch above approximately 60°C is not conclusive based on the obtained measurements. It should be noted that for other strain gauged locations in or on the boron/epoxy patch, the slope above $60 - 65^{\circ}\text{C}$ changes in a slightly different fashion. Figure 4.19 shows a superposition plot of the gauges mounted in the central section of the patch, i.e. locations B, C, D and H. Location A and G being near the free edge of the patch were excluded due to their higher sensitivity to creep. All plots for the thermal residual strains versus temperature for the boron/epoxy except for gauge 25 seem to merge between 60°C and 70°C before some spread is visible at higher temperatures. A possible cause might be the matrix behaviour of the boron/epoxy prepreg with temperature which will also experience a change in shear modulus as well as increased creep with high temperatures. The situation is additionally complicated by the penetration of FM 73 into the boron/epoxy, resulting in the presence of two epoxies with different thermal properties in the patch. It should be noted that the thermal residual strain versus temperature plot for gauge 25 shows a nearly constant offset (see Figure 4.19) in comparison to the general bottom envelope of the other curves. Since the thermal residual strain versus temperature gradient change near 60°C seems both location and load dependent, as well as of reasonable small magnitude, the slight non-linearity at elevated temperatures has been neglected with respect to the development of a predictive model for thermal residual stresses. Additional research is required to determine the thermal behaviour of boron/epoxy under shear load transfer at elevated temperatures. The focus of this research work is on the development of an improved, simple to use predictive tool as well as experimentally established guidelines for the estimation of thermal residual stresses allowing composite patch designers to include this effect as part of a generic design and analysis procedure for composite repairs on metallic structures. Some discrepancy between the measured and predicted thermal residual strains should be expected due to this simplification.

Figure 4.19: Thermal residual strain versus temperature for strain gauges mounted in or on the boron/epoxy patch at locations B, C, D and H

The thermal residual strain versus temperature for the transverse direction on top of the boron/epoxy patch at location B is nearly constant between -56.5°C and 60°C . The curve slowly approaches zero thermal residual strains with increasing temperature. Only the strain gauges oriented in the transverse direction of the specimen near the free edges show a more linear behaviour as well as an increase in magnitude (see Figure E.38 and Figure E.44). This behaviour can be attributed to the high thermal residual strains in fiber orientation in combination with in-plane bending at these particular locations. The overall small magnitude of transverse thermal residual strains can be attributed to the small difference in the coefficients of thermal expansion between the aluminum and the boron/epoxy in transverse direction, i.e. around $2\ \mu\epsilon/^{\circ}\text{C}$.

It should be noted that earlier published results [2, 3] show small deviations (on average $7\ \mu\epsilon$) in comparison to the data shown in Figure 4.16, mainly due to the implementation of additional experimental results of residual strain measurements for the temperature range between 21°C and -56.5°C . The resistance change with temperature for the lead wires connecting the strain gauges with the terminals changed by 6% based on a regression analysis over the entire temperature range. The largest change of up to $30\ \mu\epsilon$ occurred

for the thermal residual strains for the rosettes mounted on the backside of the aluminum. For most of the other strain gauges, the change is significantly less as indicated by the average given above. In addition, the analysis was improved by using a localized polynomial regression analysis [92] instead of the localized linear interpolation, an implementation of the true thermal strains of the titanium silicate as well as distinguishing between the thermal strains for the aluminum in rolling and transverse directions.

4.5 Discussion and Estimation of the Process Induced Strains

The thermal residual strains at ambient temperature for the aluminum within the center section of the repair (specimen locations B, C, D and H in Figure 4.16) vary in fiber direction of the patch between $491 \mu\epsilon$ and $686 \mu\epsilon$ with an average of $541 \mu\epsilon$ (standard deviation of 14%). The residual strains due to the elevated temperature cure are very close to this result with an average of $546 \mu\epsilon$ (standard deviation of 16%), which indicates that the process induced strains present are essentially negligible in this direction. In contrast, the average residual strain due to the elevated temperature cure measured in the aluminum perpendicular to the fiber direction of the patch at these same locations ($-164 \mu\epsilon$ with a standard deviation of 44%) is significantly higher in compression than the thermal residual strain average of $-111 \mu\epsilon$ (with a standard deviation of 20%) indicating that process induced strains are substantially higher in this direction. Unfortunately, the possibility of residual strain relaxation during the first few thermal cycles cannot be investigated here.

The thermal residual strains for the boron/epoxy patch at these four locations decline from an average of $-638 \mu\epsilon$ at the patch adhesive interface to $-572 \mu\epsilon$ on top of the patch (standard deviations $< 6\%$). It is expected that the local increase in patch thickness due to the placement of strain gauges is one of the influencing factors in the measured through thickness strain gradient. The finite element analysis in Chapter 6 provides a

better understanding of the implications of placing a number of strain gauges at a particular location. It is important to note that the initial strain state of the boron/epoxy patch cocured with one layer of FM 73M was not identical to the seven ply boron/epoxy specimen which was used for the thermal residual strain measurement thus not allowing identification of any process induced strains. The peel ply was removed after the initial measurement and the bottom of the patch grit blasted, both of which will have an effect on the initial strain state. The cause for the very large discrepancy in the measured thermal residual strains and residual strains due to the elevated temperature cure in the boron/epoxy at specimen location **A** has not been identified. A measurement error cannot be ruled out.

The maximum thermal residual strains found at the strain gauged locations at room temperature were close to $690 \mu\epsilon$ at the aluminum/adhesive interface (see specimen location **B**), which is approximately 15% of the aluminum yield strain and should therefore be carefully considered for the determination of the margin of safety of a bonded repair. The thermal residual strains in the boron/epoxy exceeded $-740 \mu\epsilon$ (see specimen location **G**) which may be a concern for repairs under compression loading. These thermal residual strains are increasing to $1500 \mu\epsilon$ for the aluminum in the fiber direction at low temperatures (associated with high altitude flight), which is about 33 % of the yield strain. The thermal residual strains for the boron/epoxy reach $-1720 \mu\epsilon$ at -56.5°C (location **G**) while the peak value $-1808 \mu\epsilon$ of for the boron/epoxy is reached at location **A**.

In summary, the results provided here for a bonded repair utilizing FM 73M indicate that using the difference between the curing and ambient temperature as input to a thermal residual strain model may substantially over-estimate the thermal residual strains present at ambient temperature in the repaired structure. Approximately $855 \mu\epsilon$ are predicted using a conventional closed form solution for the uncracked metallic component of a reinforced specimen [18] based on a temperature differential between curing temperature and ambient

temperature, which is nearly 60 % higher than the measured average of $541 \mu\epsilon$ for the cracked aluminum within the center section of the AMRL specimen repair. Unfortunately, the more general question of the real benefit of a lower temperature cure (80°C) for FM 73M cannot be answered at this point since more detailed experiments are required to show the thermal residual strain versus temperature behavior. Very recently Walker [97] provided some basic information about the thermal residual strains measurement at AMRL on the F-111 wing skin test specimen. Unfortunately no plots for the thermal residual strain versus temperature behaviour are available for this at 80°C cured specimen. Furthermore, the effect of temperature on crack growth retardation in the aluminum must also be considered in any comparison to assess the actual benefits of lower curing temperatures.

Finding this significant difference between the measured thermal residual strains and the basic one-dimensional analysis, an improved experimentally determined 'effective stress free temperature' is discussed as a reference temperature instead of the commonly used curing temperature in the following section.

4.6 Effective Stress Free Temperature

The experimental results suggest that the thermal residual strains can be effectively estimated using a linear relationship for the temperature range of interest. Non-linear effects are mainly limited to temperatures above 80°C , where the thermal residual strains are close to zero. The temperature range above approximately $70 - 80^\circ\text{C}$ should preferably be avoided as an operating range for real bonded repairs utilizing a curing temperature of 121°C due to a significant reduction in the shear modulus of the adhesive. Current research at AMRL [22, 97] is focused on the performance of bonded repairs at temperatures above 80°C . It should be noted that the AMRL has been investigating dynamic loading at frequencies of 0.5 Hz and 5 Hz, while the thermal residual strain measurements and related

adhesive characteristics were carried out quasi static in comparison. Based on the experimental evidence that the adherends expand with temperature at nearly their unconstrained rate above 87°C to 90°C , the definition of an 'effective stress free temperature' based on a linear regression analysis for the temperature range between 21°C and 70°C (Figure 4.20) allows one to employ elastic theory, which permits easy modification of the current models and avoids the need for determination of visco-elastic properties and shear modulus characteristics at elevated temperature for currently used film adhesives. Note that only the shear modulus at the operating temperature is required assuming elastic behaviour for the adhesive.

Using the outlined approach, an effective stress free temperature of 85.8°C with a standard deviation of 2.1°C was determined using all gauges oriented in fiber direction except gauges near the free edge. The small standard deviation reflects the quality of the measurement as well as the strong nature of investigated effect.

Figure 4.20: Effective stress free temperature

The difference between the measured stress free temperature and effective stress free temperature is characterized by the significant change in the adhesive character with temperature. A large nearly linear change in thermal residual strains with temperature is dominant

up to approximately 80°C. The significant reduction in shear modulus around 80°C seems to be the major cause for the small gradient of the thermal residual strains with temperature above 80°C. In addition, significant strain relaxation, especially at the tapered edges, is occurring above 80°C. Note that the stress free temperature measurement was held for approximately 30 minutes at 101.2°C which should have allowed for strain relaxation to occur. Similarly the AMRL specimen was held at 80°C and 107°C for extended periods of time during which strain relaxation was measured.

More experimental work needs to be carried out to improve the understanding of the adhesive behaviour under thermal residual stress loading at elevated temperatures close to the curing temperature. Research at AMRL is currently being carried out to investigate FM 73 adhesive characteristics above 80°C especially under dynamic loading. This research has established that a linear-elastic approach to thermal residual strains below 80°C is acceptable for the utilized material combinations using an effective stress free temperature instead of the commonly used cure temperature.

A more generic specimen might be desirable since the sandwich type AMRL specimen is not the ideal specimen for determining adhesive behaviour due to its complexity. In order to eliminate any non-linear behaviour of the composite patch, a tungsten patch should be chosen instead of the boron/epoxy patch. Tungsten also has a coefficient of thermal expansion of 4.5 PPM/°C (i.e. very similar to the boron/epoxy) and is available in a 1 mm thick foil. A 6.35 mm thick aluminum plate can be used as the center part of the specimen as outlined in Figure 4.21.

Figure 4.21: Effective stress free temperature test specimen

The adhesive behaviour can be monitored in the autoclave by bonding strain gauges to the adherends prior to assembling the test specimen. Due to isotropic adherends, the number of strain gauges can be reduced to one gauge mounted on the aluminum and one on the boron/epoxy. The gauges should face the film adhesive on different sides of the aluminum core therefore reducing the disturbance of the strain distribution in the adhesive. Curing the strain gauge glue at 150°C will allow proper strain readings during the entire autoclave process. In order to measure thermal residual strains, identical strain gauges need to be placed on an unrestrained aluminum and tungsten piece. Additionally identical strain gauges are required on a titanium silicate specimen to determine the true strains in the adhesively bonded specimen. This measurement setup should allow a more refined determination of the effective stress free temperature than presented here for the AMRL specimen.

If measurement during the actual cure (autoclave, vacuum bag or positive pressure) of the specimen shown in Figure 4.21 is not possible, the measurement setup as used for the AMRL specimen can be employed. In this case, a second single sided specimen is required to determine the stress free temperature. The use of a tungsten plate instead of the boron/epoxy is also suggested in this case to avoid influences from the composite material (see Figure 4.22).

Figure 4.22: Stress free temperature test specimen

The establishment of an 'effective stress free temperature' presents the link to an improved theoretical model for thermal residual strains. It is now possible to use a linear-elastic

model, which should correlate significantly better than previous models due to a better understanding of the adhesive behaviour. Note that not only the theoretical approach (as discussed in the following chapter) but also any previously developed model will benefit from the thermal residual strain measurements carried out as part of this research work leading to the proposed effective stress free temperature. The attention to details in the experimental work has proved again to be the key to good results.

Chapter 5

Theoretical Analysis of Thermal Residual Stresses in Bonded Repair Specimens

5.1 Introduction

This chapter presents a closed form analytical approach for the the stress distribution in bonded repair specimens. In order to be cost effective, data from small test specimens rather than from full scale tests are used to predict the behaviour of real repairs. However, while bonded repair applications on aluminum aircraft structures are usually characterized by significant restraints, bonded repair specimens may differ substantially due to having an unrestrained substrate during the curing cycle. The effect of thermal residual strains is significant in patching applications on unrestrained substrates, where the effective coefficient of thermal expansion of the substrate varies largely from the patch. The objective of the current work is to provide a closed form analytical approach to allow the designer to relate data from small test specimens to real applications. It will be evident that the the use of test data from unrestrained specimens is conservative, but the key here is to determine the true stress intensity factor under which the specimen is tested and therefore give a better estimate of the margin of safety for a real repair.

First, a one-dimensional linear-elastic analysis is presented for bonded reinforcements and double-doubler joints with uniform thickness based on Hart-Smith's work [42]. A significant

achievement in the current research was the development of an algorithm which allows the direct determination of the elastic stress distribution for a tapered patch without the usual requirement for a specialized numerical code, thus making this approach more accessible. A correction for the shear-lag in the adherends as presented by AMRL researchers [18, 52] was included to improve Hart-Smith's one-dimensional analysis.

The Rose model was then used to determine the stress intensity factor due to thermal residual stresses. The model, as presented by Rose [78, 79], was modified to determine stress intensity factor for the AMRL specimen due to the thermal residual stresses for a highly anisotropic patch. A two-dimensional approach was taken to estimate the error of the simpler one-dimensional model. Additionally the model was modified to account for an edge crack.

In order to improve the theoretical predictions near the crack tip, a stress field model was introduced for the determination of the stresses in the crack tip region. One of the key achievements is the development of a concise solution for the stress field of a center crack in an infinite plate which can be used as a predictive tool for bonded repairs. The complex stress field approach solves this classical fracture mechanics problem in a unique way replacing the requirement to use a large number of boundary conditions to determine a sufficient number of coefficients for an accurate result, or the more widely employed approximations with their limitations.

An additional contribution to the analysis of bonded repairs was achieved by developing a more generalized version of the Rose model, which accounts for disbonds based on Baker's initial approach to the problem [13]. The model can account for all disbond lengths including a complete disbond. This extension shows most clearly the severity of thermal residual stresses and should become an important tool for the initial assessment of bonded repairs.

The generalized Rose model maintains the simplicity of the original Rose model as one of its features. This approach was further explored by accounting for the tapered edge of the patch, which was achieved by using the direct solution for tapered patches developed in the first part of this chapter.

An excellent model which characterizes the impact of thermal residual stresses on bonded repairs especially for test specimens was developed by combining all these aspects. A detailed finite element analysis as presented in Chapter 6 will generally be required to study the stress distributions for most repairs in more detail, while the presented theory based on the work of L.J. Hart-Smith and researchers at AMRL gives a good estimate on the feasibility of the repair.

5.2 One-dimensional Analysis of Tapered Double-doubler Joints and Reinforcements

One of the problems associated with adhesive bonding is the complexity of the stress analysis of utilized joints and reinforcements, and has lead to the development of specialized computer programs addressing a number of joint configurations [4, 39, 58, 68, 74, 93, 94, 100]. Unfortunately these codes are not widely available thus limiting the use of adhesive bonding technology. In an effort to reduce the dependence on these numerical solutions, this section contains a linear-elastic analysis of a number of joint and reinforcement configurations based on Hart-Smith's one-dimensional approach without the need of such programs thus giving a basic tool for the determination of the stress distribution in bonded repairs. A double symmetric bonded repair can be reduced to sections which can be represented as an idealized double-doubler joint, i.e. where the patch bridges the crack, or a double-sided reinforcement in order to get a reasonable estimate of the stress distribution.

Hart-Smith extended the classical elastic solution for double-lap joints first published by Volkersen [96] in 1938. Volkersen's solution, as well as the similar one by de Bruyne [32], included only adherend stiffness imbalance but not thermal mismatch. Hart-Smith's [42] derived relationships accounted for both adhesive plasticity and thermal mismatch thus allowing a better prediction of the load carrying capacity of adhesively bonded joints. He also analyzed the shear stress distribution in both scarf and stepped-lap joints [43] commonly used in aircraft structures. The numerical solution for stepped-lap joints proved to be difficult due to the non-uniformities in the load transfer especially in the outer three steps of this joint type [43].

One of the achievements presented in this section are the easy-to-use linear-elastic solutions for tapered double-sided reinforcements and tapered double-doubler joints as well as stepped-lap joints. The derivation of the adhesive shear stress and the adherend normal stress distribution for uniform thickness double-doubler joints and double-sided reinforcements based on Hart-Smith's one-dimensional approach is reiterated in detail thus providing the foundation for the investigation of tapered double-sided reinforcements and tapered double-doubler joints. In this analysis, stepped-lap joints are treated as tapered double-doubler joints with changing thickness of the substrate. A correction for shear-lag in the adherends is also given, which can be important when using composite adherends with a low shear modulus.

It is important to note that an elastic-plastic analysis such as given by Hart-Smith [42] is required for the determination of the load bearing capacity of a joint or repair, while for many problems such as patch design within the elastic limits or the determination of thermal residual stresses and strains due to the curing of the adhesive at elevated temperatures, the one-dimensional linear-elastic theory will generally be sufficiently accurate.

5.2.1 General Approach for Double Symmetric Joints and Reinforcements with Uniform Thickness

The linear-elastic stress analysis for double-sided reinforcements and double-doubler joints having uniform thickness substrate and patch materials (see Figure 5.1) represents the foundation for the evaluation of double symmetric tapered reinforcements and double-doubler joints. Note that the double-doubler joint can be represented by two double-lap joints. The presented derivation is an extension of Hart-Smith's elastic solution [42]. The equilibrium

Figure 5.1: Reinforcement (a) and double-doubler joint (b) with uniform substrate and patch thickness

equations and boundary conditions, including stiffness imbalance and thermal mismatch of the adherends, are derived using the free body diagram in Figure 5.2 in conjunction with the chosen coordinate system as shown in Figure 5.1. For these configurations, the two differential equations for the horizontal force equilibrium are given by:

$$\frac{dT_p(y)}{dy} - \tau_a(y) = 0 \quad \text{and} \quad \frac{dT_s(y)}{dy} + 2\tau_a(y) = 0 \quad (5.1)$$

where

$$T_p(y) = \sigma_p(y)t_p \quad \text{and} \quad T_s(y) = \sigma_s(y)t_s . \quad (5.2)$$

The compatibility equations for the adherends and the adhesive can be expressed as:

$$\epsilon_p(y) = \frac{dv_p(y)}{dy} = \frac{T_p(y)}{E_p t_p} + \alpha_p \Delta T \quad (5.3)$$

$$\epsilon_s(y) = \frac{dv_s(y)}{dy} = \frac{T_s(y)}{E_s t_s} + \alpha_s \Delta T \quad (5.4)$$

$$\tau_a(y) = G_a \gamma_a(y) \approx \frac{G_a}{t_a} (v_p(y) - v_s(y)) . \quad (5.5)$$

Note that the Young's modulus along the axial coordinate of the joint or reinforcement is used in the case of composite materials. Differentiating equation (5.5) with respect to y and substituting equation (5.3) and (5.4) yields:

$$\begin{aligned} \frac{d\tau_a(y)}{dy} &= \frac{G_a}{t_a} \left(\frac{dv_p(y)}{dy} - \frac{dv_s(y)}{dy} \right) \\ &= \frac{G_a}{t_a} \left(\frac{T_p(y)}{E_p t_p} - \frac{T_s(y)}{E_s t_s} - (\alpha_s - \alpha_p) \Delta T \right) . \end{aligned} \quad (5.6)$$

The differential equation governing the adhesive shear stress distribution is derived by differentiation of equation (5.6) and substitution of the equilibrium equations (5.1) thus obtaining:

$$\frac{d^2 \tau_a(y)}{dy^2} = \frac{G_a}{t_a} \left(\frac{\frac{dT_p(y)}{dy}}{E_p t_p} - \frac{\frac{dT_s(y)}{dy}}{E_s t_s} \right) = \frac{G_a}{t_a} \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right) \tau_a(y) . \quad (5.7)$$

Figure 5.2: Force equilibrium in a bonded joint

The elastic shear stress distribution parameter λ is then defined as:

$$\lambda^2 = \frac{G_a}{t_a} \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right) \quad (5.8)$$

where the characteristic load transfer length of a joint is defined as λ^{-1} . Using this parameter, equation (5.7) is rewritten as:

$$\frac{d^2 \tau_a(y)}{dy^2} - \lambda^2 \tau_a(y) = 0. \quad (5.9)$$

The general solution for this characteristic differential equation is:

$$\tau_a(y) = A \sinh(\lambda y) + B \cosh(\lambda y). \quad (5.10)$$

The unknown coefficients A and B are then evaluated using boundary conditions of the stress resultants $T_p(y)$ and $T_s(y)$, which are given through integration of the equilibrium equations (5.1).

$$T_p(y) = \int \tau_a(y) dy = \frac{A}{\lambda} \cosh(\lambda y) + \frac{B}{\lambda} \sinh(\lambda y) + C_p \quad (5.11)$$

$$T_s(y) = \int -2\tau_a(y) dy = -\frac{2A}{\lambda} \cosh(\lambda y) - \frac{2B}{\lambda} \sinh(\lambda y) + C_s \quad (5.12)$$

The two integration constants C_s and C_p can be determined using the overall force equilibrium:

$$P = 2T_p(y) + T_s(y) = 2C_p + C_s \quad (5.13)$$

and the first derivative of the shear stress $\tau_a(y)$ with respect to y, which can be equated to equation (5.6):

$$\begin{aligned} \frac{d\tau_a(y)}{dy} &= A\lambda \cosh(\lambda y) + B\lambda \sinh(\lambda y) \\ &= \frac{G_a}{t_a} \left(\frac{T_p(y)}{E_p t_p} - \frac{T_s(y)}{E_s t_s} - (\alpha_s - \alpha_p) \Delta T \right). \end{aligned} \quad (5.14)$$

Substituting the relationships for the stress resultants (5.11 and 5.12) and equation (5.13) into this equation yields:

$$\begin{aligned} A\lambda \cosh(\lambda y) + B\lambda \sinh(\lambda y) &= \frac{G_a}{t_a \lambda} \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right) (A \cosh(\lambda y) + B \sinh(\lambda y)) \\ &\quad + \frac{G_a}{t_a} \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right) C_p \\ &\quad - \frac{G_a}{t_a} \left(\frac{P}{E_s t_s} + (\alpha_s - \alpha_p) \Delta T \right). \end{aligned} \quad (5.15)$$

Solving for C_p gives

$$C_p = \frac{\frac{P}{E_s t_s} + (\alpha_s - \alpha_p) \Delta T}{\left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right)} \quad (5.16)$$

while C_s can be determined using the overall force equilibrium equation (5.13).

$$C_s = \frac{\frac{P}{E_p t_p} - 2(\alpha_s - \alpha_p) \Delta T}{\left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right)} . \quad (5.17)$$

Note that these two integration constants are independent of the actual boundary conditions and thus apply for both double-doubler joints and double-sided reinforcements. Using these derived relationships, the equations associated with each joint type are thus generated based on their specific boundary conditions.

5.2.2 Double-sided Reinforcements with Uniform Thickness

The boundary conditions for the stress resultants of a symmetric double-sided reinforcement having a constant thickness substrate are evaluated at the center of the substrate and at the edge of the patch as follows:

Boundary condition at the center

$$T_s^R(y) = T_s^R(-y) \quad (5.18)$$

Boundary conditions at the edges

$$T_p^R(y=l) = \frac{A^R}{\lambda} \cosh(\lambda l) + \frac{B^R}{\lambda} \sinh(\lambda l) + C_p = 0 \quad (5.19)$$

and

$$T_p^R(y=-l) = \frac{A^R}{\lambda} \cosh(-\lambda l) + \frac{B^R}{\lambda} \sinh(-\lambda l) + C_p = 0 . \quad (5.20)$$

Since the boundary condition in the center as given by equation (5.18) can only be fulfilled if

$$B^R = 0 \quad (5.21)$$

equations (5.19) and (5.20) representing the boundary condition at the edge of the patch must be identical due to symmetry and thus allows determination of the second coefficient A^R for all y :

$$A^R = -C_p \frac{\lambda}{\cosh(\lambda l)} . \quad (5.22)$$

The shear stress and adherend stress distributions for a constant thickness double-sided reinforcement can then be written as:

$$\tau_a^R(y) = -\frac{\frac{P}{E_s t_s} + (\alpha_s - \alpha_p) \Delta T}{\left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right)} \lambda \frac{\sinh(\lambda y)}{\cosh(\lambda l)} \quad (5.23)$$

$$\sigma_p^R(y) = \left(\frac{\frac{P}{E_s t_s} + (\alpha_s - \alpha_p) \Delta T}{t_p \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right)} \right) \left(1 - \frac{\cosh(\lambda y)}{\cosh(\lambda l)} \right) \quad (5.24)$$

$$\begin{aligned} \sigma_s^R(y) = & 2 \left(\frac{\frac{P}{E_s t_s} + (\alpha_s - \alpha_p) \Delta T}{t_s \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right)} \right) \frac{\cosh(\lambda y)}{\cosh(\lambda l)} \\ & + \frac{\frac{P}{E_p t_p} - 2(\alpha_s - \alpha_p) \Delta T}{t_s \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right)} \end{aligned} \quad (5.25)$$

where

$$-l \leq y \leq l . \quad (5.26)$$

Having determined the shear stress and adherend stress distributions for constant thickness double-sided reinforcements, the stress distributions for double-doubler joints can be derived in a similar manner.

5.2.3 Double-doubler Joints with Uniform Thickness

The boundary conditions for the stress resultants of a double-doubler joint are evaluated at the center of the substrate and at the edges of the patch as follows:

Boundary condition at the center

$$T_s^D(y=0) = -\frac{2A^D}{\lambda} + C_s = 0 \quad (5.27)$$

(Note that this boundary condition assumes that the two substrate sections don't come into contact, which might occur under remote compression loading or if $(\alpha_s - \alpha_p)\Delta T > 0$. Under these two conditions the double-doubler joint would act like a double-sided reinforcement.)

Boundary condition at the positive edge

$$T_p^D(y=l) = \frac{A^D}{\lambda} \cosh(\lambda l) + \frac{B^D}{\lambda} \sinh(\lambda l) + C_p = 0 \quad (5.28)$$

The boundary condition in the center (equation (5.27)) can be used to solve for A^D :

$$A^D = \frac{\lambda}{2} C_s. \quad (5.29)$$

The second coefficient B^D can now easily be determined by substituting A^D in equation (5.28):

$$B^D = -\frac{\lambda}{2} \frac{C_s}{\tanh(\lambda l)} - \lambda \frac{C_p}{\sinh(\lambda l)}. \quad (5.30)$$

The shear stress and adherend stress distributions for the positive section of a uniform thickness double-doubler joint are now expressed as:

$$\begin{aligned} \tau_a^D(y \geq 0) = & \frac{\lambda}{2} \left(\frac{\frac{P}{E_p t_p} - 2(\alpha_s - \alpha_p)\Delta T}{\left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s}\right)} \right) \left(\sinh(\lambda y) - \frac{\cosh(\lambda y)}{\tanh(\lambda l)} \right) \\ & - \lambda \left(\frac{\frac{P}{E_s t_s} + (\alpha_s - \alpha_p)\Delta T}{\left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s}\right)} \right) \frac{\cosh(\lambda y)}{\sinh(\lambda l)} \end{aligned} \quad (5.31)$$

$$\begin{aligned} \sigma_s^D(y \geq 0) = & \left(\frac{\frac{P}{E_p t_p} - 2(\alpha_s - \alpha_p)\Delta T}{t_s \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right)} \right) \left(1 - \cosh(\lambda y) + \frac{\sinh(\lambda y)}{\tanh(\lambda l)} \right) \\ & + 2 \left(\frac{\frac{P}{E_s t_s} + (\alpha_s - \alpha_p)\Delta T}{t_s \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right)} \right) \frac{\sinh(\lambda y)}{\sinh(\lambda l)} \end{aligned} \quad (5.32)$$

$$\begin{aligned} \sigma_p^D(y \geq 0) = & \frac{1}{2} \left(\frac{\frac{P}{E_p t_p} - 2(\alpha_s - \alpha_p)\Delta T}{t_p \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right)} \right) \left(\cosh(\lambda y) - \frac{\sinh(\lambda y)}{\tanh(\lambda l)} \right) \\ & + \left(\frac{\frac{P}{E_s t_s} + (\alpha_s - \alpha_p)\Delta T}{t_p \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right)} \right) \left(1 - \frac{\sinh(\lambda y)}{\sinh(\lambda l)} \right). \end{aligned} \quad (5.33)$$

The stresses for the negative section of the double-doubler joint can also be generated using additional boundary conditions:

$$\tau_a^D(-y) = -\tau_a^D(y) \quad (5.34)$$

$$\sigma_p^D(-y) = \sigma_p^D(y) \quad (5.35)$$

$$\sigma_s^D(-y) = \sigma_s^D(y) \quad (5.36)$$

where

$$0 \leq y \leq l. \quad (5.37)$$

5.2.4 Interface Conditions between Steps for Tapered Joints and Reinforcements

The relationship governing the stresses in both tapered double-doubler joints and double-sided reinforcements as well as stepped-lap joints can now be derived as an extension of the basic theory for double symmetric joints and reinforcements with uniform thickness. All of the equations ((5.1) - (5.17)) derived in Section 5.2.1 are valid for any step. For each step it is assumed that the elastic moduli for the substrate and patch are identical, while the thickness and length of the substrate, patch and adhesive, as well as the shear modulus

of the adhesive can be individually specified for each step. These parameters are indexed with the step number in the subsequent analysis. The stepped-lap joint can be treated as a special case of a single side of a double-doubler joint with changing substrate thickness.

The step specific indexed coordinate systems for the reinforcements and investigated tapered joints are shown in Figure 5.3 and Figure 5.4. Note that origins of the positive and negative coordinates of the outer steps are separated by the inner step lengths for tapered joints and reinforcements. In order to reduce computational time, the system of equations was developed only for positive coordinates. The stresses for negative coordinates are provided based on symmetry.

Each step adds two interface conditions to the center and edge boundary conditions. These additional conditions are based on the horizontal force equilibrium of the patch and the shear stress:

$$T_{p_{j+1}}(y_{j+1}=0) = T_{p_j}(y_j=l_j) \quad (5.38)$$

$$\frac{A_{j+1}}{\lambda_{j+1}} = \frac{A_j}{\lambda_j} \cosh(\lambda_j l_j) + \frac{B_j}{\lambda_j} \sinh(\lambda_j l_j) + (C_{p_j} - C_{p_{j+1}}) \quad (5.39)$$

$$\tau_{a_{j+1}}(y_{j+1}=0) = \tau_{a_j}(y_j=l_j) \quad (5.40)$$

$$\frac{B_{j+1}}{\lambda_{j+1}} = \frac{A_j}{\lambda_j} \frac{\lambda_j}{\lambda_{j+1}} \sinh(\lambda_j l_j) + \frac{B_j}{\lambda_j} \frac{\lambda_j}{\lambda_{j+1}} \cosh(\lambda_j l_j) \quad (5.41)$$

where

$$1 \leq j \leq n - 1. \quad (5.42)$$

Generally these interface conditions are combined with the boundary conditions at the outer edges ($y_n = \pm l_n$) and at the center ($y_1 = 0$) and solved numerically. The chosen approach in this work is based on rewriting the interface conditions between steps so that A_n and B_n are functions of A_1 and B_1 :

$$\frac{A_n}{\lambda_n} = f_{AA} \frac{A_1}{\lambda_1} + f_{AB} \frac{B_1}{\lambda_1} + \Delta C_A \quad (5.43)$$

and

$$\frac{B_n}{\lambda_n} = f_{BA} \frac{A_1}{\lambda_1} + f_{BB} \frac{B_1}{\lambda_1} + \Delta C_B. \quad (5.44)$$

In order to evaluate f_{AA} , f_{AB} , f_{BA} and f_{BB} , a substitution scheme based on equations (5.39) and (5.41) is utilized. Using $j = 1$ in equation (5.39) and (5.41) provides the desired form of equation (5.43) and (5.44) for $n = 2$. This particular form is achieved for additional steps by evaluating equations (5.39) and (5.41) for $j = 1$ and substituting them in equations (5.39) and (5.41) with $j = 2$ thus giving the relationship between A_3 , A_1 and B_1 as well as between B_3 , A_1 and B_1 as follows:

$$\begin{aligned} \frac{A_3}{\lambda_3} = & \left(\cosh(\lambda_1 l_1) \cosh(\lambda_2 l_2) + \frac{\lambda_1}{\lambda_2} \sinh(\lambda_1 l_1) \sinh(\lambda_2 l_2) \right) \frac{A_1}{\lambda_1} \\ & + \left(\sinh(\lambda_1 l_1) \cosh(\lambda_2 l_2) + \frac{\lambda_1}{\lambda_2} \cosh(\lambda_1 l_1) \sinh(\lambda_2 l_2) \right) \frac{B_1}{\lambda_1} \\ & + ((C_{p_1} - C_{p_2}) \cosh(\lambda_2 l_2) + (C_{p_2} - C_{p_3})) \end{aligned} \quad (5.45)$$

$$\begin{aligned} \frac{B_3}{\lambda_3} = & \left(\frac{\lambda_2}{\lambda_3} \cosh(\lambda_1 l_1) \sinh(\lambda_2 l_2) + \frac{\lambda_1}{\lambda_2} \frac{\lambda_2}{\lambda_3} \sinh(\lambda_1 l_1) \cosh(\lambda_2 l_2) \right) \frac{A_1}{\lambda_1} \\ & + \left(\frac{\lambda_2}{\lambda_3} \sinh(\lambda_1 l_1) \sinh(\lambda_2 l_2) + \frac{\lambda_1}{\lambda_2} \frac{\lambda_2}{\lambda_3} \cosh(\lambda_1 l_1) \cosh(\lambda_2 l_2) \right) \frac{B_1}{\lambda_1} \\ & + \left((C_{p_1} - C_{p_2}) \frac{\lambda_2}{\lambda_3} \sinh(\lambda_2 l_2) \right). \end{aligned} \quad (5.46)$$

This substitution scheme is continued until $j = n - 1$ and equations (5.43) and (5.44) are generated. It should be noted that with each additional step the number of terms in f_{AA_n} , f_{AB_n} , f_{BA_n} , f_{BB_n} , ΔC_{A_n} and ΔC_{B_n} approximately double. Furthermore, each of these terms is multiplied by up to two additional coefficients for each step.

Repeating patterns of $(\lambda_j/\lambda_{j+1})$, $\sinh(\lambda_j l_j)$ and $\cosh(\lambda_j l_j)$ in each of the terms which are combined to f_{AA_n} , f_{AB_n} , f_{BA_n} , f_{BB_n} , ΔC_{A_n} and ΔC_{B_n} can be found. Three functions (equations (5.49) - (5.51)) were derived for these patterns returning a value of 1 if the particular coefficient $(\lambda_j/\lambda_{j+1})$, $\sinh(\lambda_j l_j)$ or $\cosh(\lambda_j l_j)$ exists, otherwise a value of 0 is returned. These functions were based on an expression which is 0, unless δ is a multiple of

$2^{\gamma+1}$, thereby becoming 1. This relationship is:

$$e(\gamma, \delta) = \prod_{\xi=1}^{\gamma+1} \left[\frac{1 + \cos \left(\frac{\delta}{2^{\xi-1}} \pi \right)}{2} \right]. \quad (5.47)$$

Alternatively the discrete Fourier solution shown below can be used:

$$e_{\text{Fourier}}(\gamma, \delta) = \frac{1 + \cos(\delta\pi)}{2^{\gamma+1}} + \sum_{\xi=1}^{2^{\gamma}-1} \left[\frac{1}{2^{\gamma}} \cos \left(\frac{\delta}{2^{\gamma}} \xi \pi \right) \right]. \quad (5.48)$$

These expressions can be shifted by a value of ζ in the δ domain by replacing δ with $\delta - \zeta$ in the previous equations. The desired patterns of 1's and 0's were then achieved by adding the shifted basic expressions for certain values of ζ leading to $e_{sc}(\gamma, \delta)$, $e_{\lambda}(\gamma, \delta)$ and $e_{\Delta C}(\gamma, \delta)$.

$$e_{sc}(\gamma, \delta) = \sum_{\zeta=1+2^{\gamma-1}}^{3 \cdot 2^{\gamma-1}} \left[\prod_{\xi=1}^{\gamma+1} \left[\frac{1 + \cos \left(\frac{\delta - \zeta}{2^{\xi-1}} \pi \right)}{2} \right] \right] \quad (5.49)$$

$$e_{\lambda}(\gamma, \delta) = \sum_{\zeta=1+2^{\gamma}}^{2^{\gamma+1}} \left[\prod_{\xi=1}^{\gamma+1} \left[\frac{1 + \cos \left(\frac{\delta - \zeta}{2^{\xi-1}} \pi \right)}{2} \right] \right] \quad (5.50)$$

$$e_{\Delta C}(\gamma, \delta) = \prod_{\xi=1}^{\gamma+1} \left[\frac{1 + \cos \left(\frac{\delta - 1}{2^{\xi-1}} \pi \right)}{2} \right] \quad (5.51)$$

These functions are then used as an exponent for the $(\lambda_j / \lambda_{j+1})$, $\sinh(\lambda_j l_j)$ and $\cosh(\lambda_j l_j)$ coefficients or as a factor for the change in the integration constants from one step to the next, i.e. $(C_{p_j} - C_{p_{j+1}})$.

The terms f_{AA} , f_{AB} , f_{BA} and f_{BB} can actually be expressed using a single function $f(\phi)$ with different values of ϕ . The function $\Delta C(\phi)$ can similarly be used to express ΔC_A and ΔC_B . Thus equations (5.43) and (5.44) can be rewritten as:

$$\frac{A_n}{\lambda_n} = f(\phi=-1) \frac{A_1}{\lambda_1} + f(\phi=0) \frac{B_1}{\lambda_1} + \Delta C(\phi=0) \quad (5.52)$$

where

$$\frac{B_n}{\lambda_n} = f(\phi=2^{n-1}-1) \frac{A_1}{\lambda_1} + f(\phi=2^{n-1}) \frac{B_1}{\lambda_1} + \Delta C(\phi=2^{n-1}). \quad (5.53)$$

It can be proven that the functions $f(\phi)$ and $\Delta C(\phi)$ are:

$$f(\phi) = \sum_{\theta=1}^{2^{n-2}} \left[\prod_{j=1}^{n-1} \left[\left(\frac{\lambda_j}{\lambda_{j+1}} \right)^{e_{\lambda}(j, 2\theta+\phi)} \cdot (\sinh(\lambda_j l_j))^{e_{sc}(j, 2\theta+\phi)} \cdot (\cosh(\lambda_j l_j))^{1-e_{sc}(j, 2\theta+\phi)} \right] \right] \quad (5.54)$$

$$\Delta C(\phi) = \sum_{\theta=1}^{2^{n-1}} \left[\sum_{j=1}^{n-1} \left[e_{\Delta C}(j, \theta+\phi) (C_{p_j} - C_{p_{j+1}}) \cdot \frac{\prod_{\chi=j}^{n-1} \left[\left(\frac{\lambda_{\chi}}{\lambda_{\chi+1}} \right)^{e_{\lambda}(\chi, \theta+\phi)} \right]}{\left(\frac{\lambda_j}{\lambda_{j+1}} \right)^{e_{\lambda}(j, \theta+\phi)}} \cdot \frac{\prod_{\chi=j}^{n-1} \left[(\sinh(\lambda_{\chi} l_{\chi}))^{e_{sc}(\chi, \theta+\phi)} \right]}{(\sinh(\lambda_j l_j))^{e_{sc}(j, \theta+\phi)}} \cdot \frac{\prod_{\chi=j}^{n-1} \left[(\cosh(\lambda_{\chi} l_{\chi}))^{1-e_{sc}(\chi, \theta+\phi)} \right]}{(\cosh(\lambda_j l_j))^{1-e_{sc}(j, \theta+\phi)}} \right] \right] \quad (5.55)$$

Equations (5.52) through (5.55) represent the required additional equations to determine the stress distributions for the various stepped or tapered joints and reinforcements.

5.2.5 Tapered Double-sided Reinforcements

Figure 5.3: Tapered reinforcement

The boundary conditions for a tapered double-sided reinforcement as shown in Figure 5.3 are similar to the conditions for a constant thickness double-sided reinforcement as given

by equations (5.18) through (5.20) but must address the presence of multiple steps (i.e. n) as shown below.

Boundary condition at the center

$$T_{s1}^R(y_1) = T_{s1}^R(-y_1) \quad (5.56)$$

Boundary condition at the positive edge ($y_n = l_n$)

$$T_{pn}^R(y_n = l_n) = \frac{A_n^R}{\lambda_n} \cosh(\lambda_n l_n) + \frac{B_n^R}{\lambda_n} \sinh(\lambda_n l_n) + C_{pn} = 0 \quad (5.57)$$

The boundary condition in the center can only be fulfilled if:

$$B_1^R = 0. \quad (5.58)$$

A_1^R can be determined by substituting equation (5.52) and (5.53) into equation (5.57) and then simplified using $B_1^R = 0$:

$$A_1^R = \frac{-\lambda_1 (C_{pn} + \Delta C_{(\phi=0)} \cosh(\lambda_n l_n) + \Delta C_{(\phi=2^{n-1})} \sinh(\lambda_n l_n))}{f_{(\phi=-1)} \cosh(\lambda_n l_n) + f_{(\phi=2^{n-1}-1)} \sinh(\lambda_n l_n)} \quad (5.59)$$

Having determined A_1^R and B_1^R , seeded iteration with the seed vector $(A_1^R \ B_1^R)^T$ is then employed to determine the remaining coefficients A_{j+1}^R and B_{j+1}^R based on equation (5.39) and (5.41):

$$\begin{bmatrix} A_{j+1}^R \\ B_{j+1}^R \end{bmatrix} = \begin{bmatrix} \lambda_{j+1} \left[\frac{A_j^R}{\lambda_j} \cosh(\lambda_j l_j) + \frac{B_j^R}{\lambda_j} \sinh(\lambda_j l_j) + (C_{pj} - C_{pj+1}) \right] \\ A_j^R \sinh(\lambda_j l_j) + B_j^R \cosh(\lambda_j l_j) \end{bmatrix}. \quad (5.60)$$

Thus the stresses for the positive sections of the reinforcement are:

$$\tau_{a_i}^R(y_i \geq 0) = A_i^R \sinh(\lambda_i y_i) + B_i^R \cosh(\lambda_i y_i) \quad (5.61)$$

$$\sigma_{pi}^R(y_i \geq 0) = \frac{1}{t_{pi}} \left(\frac{A_i^R}{\lambda_i} \cosh(\lambda_i y_i) + \frac{B_i^R}{\lambda_i} \sinh(\lambda_i y_i) + C_{pi} \right) \quad (5.62)$$

$$\sigma_{s_i}^R(y_i \geq 0) = \frac{1}{t_{s_i}} \left(-\frac{2A_i^R}{\lambda_i} \cosh(\lambda_i y_i) - \frac{2B_i^R}{\lambda_i} \sinh(\lambda_i y_i) + C_{s_i} \right) \quad (5.63)$$

where

$$1 \leq i \leq n. \quad (5.64)$$

The stresses for the negative sections of the reinforcement can then be determined using the additional boundary conditions:

$$\tau_a^R(-y_i) = -\tau_a^R(y_i) \quad (5.65)$$

$$\sigma_p^R(-y_i) = \sigma_p^R(y_i) \quad (5.66)$$

$$\sigma_s^R(-y_i) = \sigma_s^R(y_i) \quad (5.67)$$

where

$$0 \leq y_i \leq l_i. \quad (5.68)$$

Having determined the shear stress and adherend stress distributions for tapered double-sided reinforcements, the stress distributions for tapered double-doubler joints can be derived in a similar manner.

5.2.6 Tapered Double-doubler Joints

Figure 5.4: Tapered double-doubler joint

The boundary conditions for a tapered double-doubler joint as shown in Figure 5.4 are similar to the conditions for a constant thickness double-doubler joint as given by equations

(5.27) and (5.28) modified to account for multiple steps (i.e. n) as shown below.

Boundary condition at the center

$$T_{s_1}^D(y_1=0) = -\frac{2A_1^D}{\lambda} + C_{s_1} = 0 \quad (5.69)$$

Boundary condition at the positive edge

$$T_{p_n}^D(y_n=l_n) = \frac{A_n^D}{\lambda_n} \cosh(\lambda_n l_n) + \frac{B_n^D}{\lambda_n} \sinh(\lambda_n l_n) + C_{p_n} = 0 \quad (5.70)$$

The boundary condition in the center can be used to solve for A_1^D :

$$A_1^D = \frac{\lambda_1}{2} C_{s_1} . \quad (5.71)$$

B_1^D can now be calculated by substituting equation (5.52) and (5.53) into equation (5.70) and then simplified using equation (5.71):

$$B_1^D = -\frac{\lambda_1 \frac{C_{s_1}}{2} (f(\phi=-1) \cosh(\lambda_n l_n) + f(\phi=2^{n-1}-1) \sinh(\lambda_n l_n))}{f(\phi=0) \cosh(\lambda_n l_n) + f(\phi=2^{n-1}) \sinh(\lambda_n l_n)} - \frac{\lambda_1 (C_{p_n} + \Delta C(\phi=0) \cosh(\lambda_n l_n) + \Delta C(\phi=2^{n-1}) \sinh(\lambda_n l_n))}{f(\phi=0) \cosh(\lambda_n l_n) + f(\phi=2^{n-1}) \sinh(\lambda_n l_n)} . \quad (5.72)$$

A seeded iteration with the seed vector $(A_1^D \ B_1^D)^T$ is also used to determine the remaining coefficients A_{j+1}^D and B_{j+1}^D :

$$\begin{bmatrix} A_{j+1}^D \\ B_{j+1}^D \end{bmatrix} = \begin{bmatrix} \lambda_{j+1} \left[\frac{A_j^D}{\lambda_j} \cosh(\lambda_j l_j) + \frac{B_j^D}{\lambda_j} \sinh(\lambda_j l_j) + (C_{p_j} - C_{p_{j+1}}) \right] \\ A_j^D \sinh(\lambda_j l_j) + B_j^D \cosh(\lambda_j l_j) \end{bmatrix} . \quad (5.73)$$

Thus the stresses for the positive sections of the tapered double-doubler joint are:

$$\tau_{a_i}^D(y_i \geq 0) = A_i^D \sinh(\lambda_i y_i) + B_i^D \cosh(\lambda_i y_i) \quad (5.74)$$

$$\sigma_{p_i}^D(y_i \geq 0) = \frac{1}{t_{p_i}} \left(\frac{A_i^D}{\lambda_i} \cosh(\lambda_i y_i) + \frac{B_i^D}{\lambda_i} \sinh(\lambda_i y_i) + C_{p_i} \right) \quad (5.75)$$

$$\sigma_{s_i}^D(y_i \geq 0) = \frac{1}{t_{s_i}} \left(-\frac{2A_i^D}{\lambda_i} \cosh(\lambda_i y_i) - \frac{2B_i^D}{\lambda_i} \sinh(\lambda_i y_i) + C_{s_i} \right) . \quad (5.76)$$

The stresses for the negative sections of the double-doubler joint can then be determined utilizing the additional boundary conditions:

$$\tau_a^D(-y_i) = -\tau_a^D(y_i) \quad (5.77)$$

$$\sigma_p^D(-y_i) = \sigma_p^D(y_i) \quad (5.78)$$

$$\sigma_s^D(-y_i) = \sigma_s^D(y_i) \quad (5.79)$$

where

$$0 \leq y_i \leq l_i. \quad (5.80)$$

The presented one-dimensional analysis provides a good estimation for the stress distribution in the discussed joints and reinforcements. Shortfalls in the one-dimensional analysis include the neglect of peel stresses and shear-lag in the adherends. The shear-lag in the adherends can be included in the results of the one-dimensional analysis by carrying out the following analysis.

5.2.7 Correction for Shear-lag in the Adherends

The importance of including shear-lag effects of the adherends when determining the gap opening displacement between the two substrate sections of a double-doubler joint was shown by researchers at the Aeronautical and Maritime Research Laboratory (AMRL) in Australia [18, 52]. The one-dimensional analysis shows significant error for joint configurations with low shear modulus adherends such as composites. Boron/epoxy has a shear modulus of 7.24 GPa versus 27.2 GPa for 2024-T3 aluminum. This correction to the one-dimensional analysis is provided for double-sided reinforcements and double-doubler joints with uniform thickness, but it can also be readily adapted for tapered/stepped joints or reinforcements.

It is reasonable to assume that the shear stresses, which develop in the adhesive, are continuous across the adhesive-adherend interfaces. In addition, equilibrium requires that

the shear stress $\tau_{(y,z)}$ be zero at the free surface as well as at the plane of symmetry. Due to the general thin adherends, it can be assumed that the shear stress $\tau_{(y,z)}$ is linear through the substrate and patch thickness. The assumed distribution is shown in Figure 5.5 [52].

Figure 5.5: Distribution of the shear stress through the thickness of a double-sided joint or reinforcement

Note that the nomenclature of the previously determined shear stress in the adhesive $\tau_a(y)$ is kept. In general the shear stress in a bonded joint or reinforcement $\tau_{(y,z)}$ is given by:

$$\tau_{(y,z)} = G\gamma_{(y,z)} \approx G \frac{\partial v_{(y,z)}}{\partial z} . \quad (5.81)$$

Evaluating the shear stress for the patch gives:

$$\tau_p(y,z) = \frac{1}{t_p} \left(\frac{t_s}{2} + t_a + t_p - z \right) \tau_a(y) = G_p \frac{\partial v_p(y,z)}{\partial z} . \quad (5.82)$$

The in-plane displacement $v_p(y,z)$ is then determined through integration

$$\begin{aligned} v_p(y,z) &= \int \frac{\partial v_p(y,z)}{\partial z} dz \\ &= \frac{z}{G_p t_p} \left(\frac{t_s}{2} + t_a + t_p - \frac{z}{2} \right) \tau_a(y) + h(y) \end{aligned} \quad (5.83)$$

with the integration constant $h(y)$ being determined as a function of the midplane displacement $v_p(y, z = \frac{t_s}{2} + t_a + \frac{t_p}{2})$ of the patch, i.e.

$$h(y) = v_p(y, z = \frac{t_s}{2} + t_a + \frac{t_p}{2}) - \frac{1}{G_p t_p} \left(\frac{t_s}{2} + t_a + \frac{t_p}{2} \right) \left(\frac{t_s}{4} + \frac{t_a}{2} + \frac{3}{4} t_p \right) \tau_a(y) . \quad (5.84)$$

Substituting $h(y)$ into equation (5.83) and evaluating the in-plane displacement $v_p(y,z)$ at the patch/adhesive interface yields:

$$v_p(y, z = \frac{t_a}{2} + t_a) = v_p(y, z = \frac{t_a}{2} + t_a + \frac{t_p}{2}) - \frac{3}{8} \frac{t_p}{G_p} \tau_a(y) . \quad (5.85)$$

The in-plane displacement $v_s(y,z)$ at the substrate/adhesive interface can be determined in a similar manner. Based on Figure Figure 5.5, the shear stress at any point in the substrate is evaluated as:

$$\tau_s(y,z) = \frac{2z}{t_s} \tau_a(y) = G_s \frac{\partial v_s(y,z)}{\partial z} . \quad (5.86)$$

The in-plane displacement of the substrate $v_s(y,z)$ can also be determined through integration as:

$$\begin{aligned} v_s(y,z) &= \int \frac{\partial v_s(y,z)}{\partial z} dz \\ &= \frac{z^2}{G_s t_s} \tau_a(y) + g(y) \end{aligned} \quad (5.87)$$

where the integration constant $g(y)$ can be assessed as a function of the midplane displacement $v_s(y, z=0)$ of the substrate, i.e.

$$g(y) = v_s(y, z=0) . \quad (5.88)$$

Substituting $g(y)$ in equation (5.87) and evaluating the in-plane displacement $v_s(y,z)$ at the substrate/adhesive interface gives:

$$v_s(y, z = \frac{t_a}{2}) = v_s(y, z=0) - \frac{1}{4} \frac{t_s}{G_s} \tau_a(y) . \quad (5.89)$$

Applying the assumption of constant shear stress through the adhesive yields:

$$\tau_a(y) = \frac{G_a}{t_a} (v_p(y, z = \frac{t_a}{2} + t_a) - v_s(y, z = \frac{t_a}{2})) . \quad (5.90)$$

The relationship between the midplane displacements of the adherends and the adhesive shear stress can be obtained by substituting equations (5.85) and (5.89) into equation (5.90)

to give:

$$\tau_a(y) = \frac{(u_p(y, z = \frac{t_a}{2} + t_a + \frac{t_p}{2}) - u_s(y, z = 0))}{\left(\frac{1}{4} \frac{t_s}{G_s} + \frac{t_a}{G_a} + \frac{3}{8} \frac{t_p}{G_p}\right)}. \quad (5.91)$$

This result can easily be incorporated into the basic one-dimensional analysis by modifying the elastic shear stress distribution parameter λ . If equation (5.5) is replaced by equation (5.91), the elastic shear stress distribution parameter λ for a constant thickness double-sided reinforcement or a double-doubler joint yields:

$$\lambda_{\text{Shear-lag}} = \sqrt{\frac{\left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s}\right)}{\left(\frac{1}{4} \frac{t_s}{G_s} + \frac{t_a}{G_a} + \frac{3}{8} \frac{t_p}{G_p}\right)}}. \quad (5.92)$$

The remaining one-dimensional analysis is unaffected by accounting for the shear-lag effect in the adherends. Thus the elastic shear stress distribution parameter λ_i for tapered or stepped joints and reinforcements is obtained simply by indexing the appropriate variables.

5.2.8 Results

This section describes the application of the derived equations associated with the various joint and reinforcement types for a specific set of composite patch and metal substrate material properties representing cross sections of the sandwich type AMRL specimen. The honeycomb which is used in this particular specimen to eliminate out-of-plane bending is neglected in this case and the two face sheets are combined as the substrate in the analysis. The stress distributions for uniform thickness adherends versus tapered edges of the patch are then directly compared. Boron/epoxy 5521/4 was chosen as patch material, aluminum 2024-T3 as substrate and FM73M as film adhesive with the following properties:

Boron/epoxy 5521/4 (0°-Orientation):

$$E_p = 210 \text{ GPa} \quad G_p = 7.24 \text{ GPa} \quad \alpha_p = 4.61 \mu\epsilon/^\circ\text{C}$$

Aluminum 2024-T3:

$$E_s = 72.4 \text{ GPa} \quad G_s = 27.2 \text{ GPa} \quad \alpha_p = 23.45 \mu\epsilon/^\circ\text{C}$$

FM73M Film Adhesive

$$G_a = 844 \text{ MPa} .$$

The applied loads for the reinforcements and the double-doubler joint were chosen identically as thermal loads representing thermal residual stress loading of the AMRL specimen after the curing cycle. The temperature differential is given as the difference between the effective stress free temperature and ambient temperature. Note that the shear modulus at ambient temperature was used in the subsequent calculations.

Applied Load:

$$\Delta T = -64.8^\circ\text{C} \quad P = 0 \frac{\text{N}}{\text{mm}}$$

Double-sided reinforcement - Uniform versus tapered

To compare the effect of tapering for a double-sided reinforcement, the following geometric parameters were employed.

Double-sided reinforcement with uniform thickness:

$$t_s = 6.35 \text{ mm} \quad t_a = 0.25 \text{ mm} \quad t_p = 0.924 \text{ mm}$$

$$l = 75 \text{ mm}$$

Double-sided reinforcement with tapered edges

$$t_{s_{1 \leq i \leq 7}} = 6.35 \text{ mm} \quad t_{a_{1 \leq i \leq 7}} = 0.25 \text{ mm} \quad t_{p_{1 \leq i \leq 7}} = 1.056 \text{ mm} - i \cdot 0.132 \text{ mm}$$

$$l_1 = 57 \text{ mm} \quad l_{2 \leq i \leq 7} = 3 \text{ mm}$$

Equations (5.23) through (5.25) as well as equation (5.92) were utilized to calculate the stress distribution for double-sided reinforcements with uniform thickness. To determine

the stress distribution for tapered double-sided reinforcement, Equations (5.16), (5.17), (5.49)-(5.51), (5.54), (5.55), and (5.58) - (5.68) were applied.

Figure 5.6 through Figure 5.8 show the stress distributions for the double-sided reinforcements with uniform thickness as well as with a taper at the patch edges. High shear stresses occur at the outer edges of the reinforcement while they drop to zero in the center of the reinforcement as shown in Figure 5.6. The taper leads to a significant reduction in the shear stress at the edge of the patch. This reduction in the adhesive shear stress at the tapered edge at the expense of a higher compressive stress in the patch is clearly indicated by comparing Figure 5.6 and Figure 5.7. The peak compressive stress in the tapered patch is reached in the outer step and has a magnitude of approximately twice the compressive stress reached in the center of the uniform thickness patch (see Figure 5.7). The sharp jumps in the stress distribution for the tapered patch are caused by the change in patch thickness. The thermal load leads to significant tensile stresses in the substrate by applying the patch as indicated in Figure 5.8. The patch taper leads to a reduced stress gradient in the substrate when approaching the outer edge.

Figure 5.6 also indicates a shortfall of the one-dimensional analysis. The shear stress should be zero at the free edge. Although a more complex analysis will be able to predict zero

Figure 5.6: Adhesive shear stress in double-sided reinforcements with uniform thickness and tapered edges

Figure 5.7: Normal stress in the patch of double-sided reinforcements with uniform thickness and tapered edges

Figure 5.8: Normal stress in the substrate of double-sided reinforcements with uniform thickness and tapered edges

shear stress at the free edge and the peak shear stress being reached approximately $1/2$ adhesive layer thickness from the edge, the peak shear stress is predicted quite accurately by the one-dimensional analysis.

Double-doubler joint - Uniform versus tapered

To compare the effect of tapering for a double-doubler joint, the following geometric parameters were employed.

Double-doubler joint with uniform thickness:

$$t_s = 6.35 \text{ mm} \quad t_a = 0.25 \text{ mm} \quad t_p = 0.924 \text{ mm}$$

$$l = 75 \text{ mm}$$

Double-doubler joint with tapered edges

$$\begin{aligned}
 t_{s_{1 \leq i \leq 7}} &= 6.35 \text{ mm} & t_{a_{1 \leq i \leq 7}} &= 0.25 \text{ mm} & t_{p_{1 \leq i \leq 7}} &= 1.056 \text{ mm} - i \cdot 0.132 \text{ mm} \\
 l_1 &= 57 \text{ mm} & l_{2 \leq i \leq 7} &= 3 \text{ mm}
 \end{aligned}$$

The stress distribution for a double-doubler joint with uniform thickness was found using equations (5.31) through (5.37) in combination with equation (5.92). To determine the stress distribution for tapered double-doubler joints, equations (5.16), (5.17), (5.49)-(5.51), (5.54), (5.55) and (5.71) - (5.80) were employed.

The stress distributions for double-doubler joints with uniform thickness as well as tapered edges of the patch are presented in Figure 5.9 through Figure 5.11. Peak shear stresses are reached at the joint center and at the edge of the patch (see Figure 5.9) for the applied thermal loading condition. The advantage of the tapering is clearly shown by the reduction in shear stress at the tapered edge. The high shear stress at the joint center is unaffected by the taper. The peak stress in the patch for a uniform thickness joint is reached between the joint center and edge while the peak compressive stress for a tapered patch is reached within the outer step of the patch for this particular loading condition (see Figure 5.10). As for the tapered reinforcement, the stress distribution shows sharp jumps due to the thickness change of the patch. Figure 5.11 shows that the highest tensile stress in the substrate is reached between the outer edge and the center of the joint while zero tensile

Figure 5.9: Adhesive shear stress in double-doubler joints with uniform thickness and tapered edges

Figure 5.10: Normal stress in the patch of double-doubler joints with uniform thickness and tapered edges

stress is predicted at the joint center and at the edge. The taper leads to a reduced stress gradient in the substrate when approaching the outer edge. It should be emphasized that the discussed results are specific for the given loading condition. The critical location of the shear stress and the stress in the adherends will be dependent on the applied combination of thermal and mechanical loads.

The presented analysis provides an effective tool to determine the stress distribution in double-doubler joints and double-sided reinforcements. This analysis scheme can now be used to get a first estimate of the stress distribution in bonded repairs applications on cracked structures. The repaired section can be divided into two regions, one containing the

Figure 5.11: Normal stress in the substrate of double-doubler joints with uniform thickness and tapered edges

crack and the other being essentially a reinforcement. Typically the composite material will be oriented perpendicular to the crack giving a clear line at the crack tip which represents the boundary between the two regions. The region containing the crack is treated as a double-doubler joint, while the second region is analysed as a double-sided reinforcement. In the application of one-sided bonded repairs to cracked aluminum aircraft structures it is generally assumed that the supporting substructure is sufficiently stiff to prevent out-of-plane bending thus allowing the repair to be treated as a double symmetric case.

The one-dimensional model does not account for the stress concentrations at the ply drop-offs in the tapered region which will lead to a localized error in the stress and strain prediction for the patch. Additionally minor errors will be introduced in the adhesive and substrate. The finite element analysis on the other hand will overpredict this stress concentration due to modeling this ply drop off as a sharp corner. It is generally prohibitive to model the true microstructure of a composite due to the size of such a model. In order to improve the one dimensional analysis, the influence of the crack tip stress field has to be included. Therefore the next step is the determination of the stress intensity factor of the patched crack followed by an estimation of the resulting stress field in the cracked substrate.

5.3 Application of the Rose Model for Thermal Residual Stress Loading in Unrestrained and Uniformly Heated Specimens

Dr. L.F.R. Rose developed a model for the estimation of the stress intensity factor reduction due to patching. The basic model, which consists of two stages, is well explained in his original papers [78, 79]. The first stage determines the stress at the prospective crack location for a bonded reinforcement over the undamaged structure. The second stage estimates two upper boundaries of the stress intensity factor based on the stress at the

prospective crack location. The stress intensity factor reduction for any given crack length is then determined using an interpolation between these two upper boundaries.

5.3.1 Stage I: Stress at the Prospective Crack Location

In addition to the determination of the stress at the prospective crack location due to remote stress loading using the inclusion analogy [18, 78], Rose discusses the determination of thermal residual stress in an uncracked substrate below an isotropic reinforcement as well [18]. The efforts with respect to thermal residual stresses are focused on accounting for the restraining effect of the surrounding structure. In this particular investigation the analyses considered the thermal residual stress at the prospective crack location of a restrained isotropic circular plate with an isotropic reinforcement [18] with identical Poisson's ratios ($\nu = \nu_s = \nu_p$). The bond was assumed to be rigid. Note that this particular solution as given by equation (5.93) overestimates the thermal residual stress by approximately a factor of $1/(\nu - 1)$ for a highly orthotropic patch with no restraints and uniform heating. The thermal residual stress for an unrestrained isotropic plate and double-sided reinforcement with identical Poisson's ratios $\nu = \nu_s = \nu_p$ and uniform heating/cooling using Rose's approach is:

$$\sigma_o^T = -\frac{2(\alpha_s - \alpha_p)\Delta T}{(1 - \nu)t_s \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right)} \quad (5.93)$$

Additional work using the Rose model was carried out by R. Fredell who presents a two-dimensional solution for the thermal stress and thermal residual stress distributions near the prospective crack location incorporating the anisotropic characteristics of the reinforcement [35]. This approach deals with the thermal stress and thermal residual stress at the prospective crack location for a fuselage section in which the expansion of the skin is restrained at the stringer and frame locations. In applying the approach as presented in [35],

care needs to be exercised with respect to the employed coordinate system and orthotropic material definitions of the patch.

The sandwich type AMRL specimen, which can be idealized as an unrestrained repair with a uniform temperature distribution throughout the curing process, represents a special case of Fredell's approach. Neglecting that it is only a partial reinforcement across the width and length of the specimen, the thermal residual stress at the prospective crack location can be easily determined as follows.

The stress and strain relationship for the isotropic substrate is given by:

$$\epsilon_{sx} = \frac{1}{E_s}(\sigma_{sx} - \nu_s \sigma_{sy}) + \alpha_s \Delta T \quad (5.94)$$

$$\epsilon_{sy} = \frac{1}{E_s}(\sigma_{sy} - \nu_s \sigma_{sx}) + \alpha_s \Delta T \quad (5.95)$$

where

$$E_s = E_{sx} = E_{sy} \quad \text{and} \quad \nu_s = \nu_{sx} = \nu_{sy} . \quad (5.96)$$

The stress and strain relationships for the orthotropic reinforcement with $E_{py} > E_{px}$ and ν_{pyx} being the major Poisson's ratio:

$$\epsilon_{px} = \frac{1}{E_{px}}(\sigma_{px} - \nu_{pxy} \sigma_{py}) + \alpha_{px} \Delta T \quad (5.97)$$

$$\epsilon_{py} = \frac{1}{E_{py}}(-\nu_{pyx} \sigma_{px} + \sigma_{py}) + \alpha_{py} \Delta T . \quad (5.98)$$

To simplify the estimation of the thermal residual stress, a rigid bond is assumed based on the small load transfer length in comparison to the overlap length ($1/\lambda \ll l$) such that

$$\epsilon_{sx} = \epsilon_{px} \quad \text{and} \quad \epsilon_{sy} = \epsilon_{py} . \quad (5.99)$$

The equilibrium requirements for a double-sided reinforcement then yields:

$$\sigma_{sx} t_s + 2\sigma_{px} t_p = 0 \quad (5.100)$$

$$\sigma_{sy} t_s + 2\sigma_{py} t_p = 0 . \quad (5.101)$$

Substituting the stress-strain relationships (equations (5.94) through (5.98)) into the rigid bond assumptions (5.99) and employing the equilibrium conditions to eliminate the stress components in the patch yields the following linear system of equations.

$$-\sigma_{sx} \left(\frac{1}{E_{px}t_p} + \frac{2}{E_s t_s} \right) + \sigma_{sy} \left(\frac{\nu_{pyx}}{E_{py}t_p} + \frac{2\nu_s}{E_s t_s} \right) = \frac{2}{t_s} (\alpha_s - \alpha_{px}) \Delta T \quad (5.102)$$

$$\sigma_{sx} \left(\frac{\nu_{pyx}}{E_{py}t_p} + \frac{2\nu_s}{E_s t_s} \right) - \sigma_{sy} \left(\frac{1}{E_{px}t_p} + \frac{2}{E_s t_s} \right) = \frac{2}{t_s} (\alpha_s - \alpha_{py}) \Delta T \quad (5.103)$$

The solution of this linear system of equations represents the thermal residual stresses in a rigidly bonded reinforcement, i.e.

$$\sigma_{sx} = - \frac{(\alpha_s - \alpha_{px}) \Delta T \left(\frac{1}{E_{py}t_p} + \frac{2}{E_s t_s} \right) + (\alpha_s - \alpha_{py}) \Delta T \left(\frac{\nu_{pyx}}{E_{py}t_p} + \frac{2\nu_s}{E_s t_s} \right)}{\frac{t_s}{2} \left(\left(\frac{1}{E_{px}t_p} + \frac{2}{E_s t_s} \right) \left(\frac{1}{E_{py}t_p} + \frac{2}{E_s t_s} \right) - \left(\frac{\nu_{pyx}}{E_{py}t_p} + \frac{2\nu_s}{E_s t_s} \right)^2 \right)} \quad (5.104)$$

$$\sigma_{sy} = - \frac{(\alpha_s - \alpha_{px}) \Delta T \left(\frac{\nu_{pyx}}{E_{py}t_p} + \frac{2\nu_s}{E_s t_s} \right) + (\alpha_s - \alpha_{py}) \Delta T \left(\frac{1}{E_{px}t_p} + \frac{2}{E_s t_s} \right)}{\frac{t_s}{2} \left(\left(\frac{1}{E_{px}t_p} + \frac{2}{E_s t_s} \right) \left(\frac{1}{E_{py}t_p} + \frac{2}{E_s t_s} \right) - \left(\frac{\nu_{pyx}}{E_{py}t_p} + \frac{2\nu_s}{E_s t_s} \right)^2 \right)} \quad (5.105)$$

Using the following material property characteristics (which can be observed for the typical material combination of unidirectional Boron/epoxy 5521/4 and aluminum 2024-T3)

$$\left(\frac{\nu_{pyx}}{E_{py}t_p} + \frac{2\nu_s}{E_s t_s} \right)^2 \ll \left(\frac{1}{E_{px}t_p} + \frac{2}{E_s t_s} \right) \left(\frac{1}{E_{py}t_p} + \frac{2}{E_s t_s} \right) \quad (5.106)$$

$$(\alpha_s - \alpha_{px}) \Delta T \left(\frac{\nu_{pyx}}{E_{py}t_p} + \frac{2\nu_s}{E_s t_s} \right) \ll (\alpha_s - \alpha_{py}) \Delta T \left(\frac{1}{E_{px}t_p} + \frac{2}{E_s t_s} \right) \quad (5.107)$$

The thermal residual stresses can be expressed as:

$$\sigma_{sx} \cong - \frac{2(\alpha_s - \alpha_{px}) \Delta T}{t_s \left(\frac{1}{E_{px}t_p} + \frac{2}{E_s t_s} \right)} - \frac{2(\alpha_s - \alpha_{py}) \Delta T \left(\frac{\nu_{pyx}}{E_{py}t_p} + \frac{2\nu_s}{E_s t_s} \right)}{t_s \left(\frac{1}{E_{px}t_p} + \frac{2}{E_s t_s} \right) \left(\frac{1}{E_{py}t_p} + \frac{2}{E_s t_s} \right)} \quad (5.108)$$

$$\sigma_{sy} \cong - \frac{2(\alpha_s - \alpha_{py}) \Delta T}{t_s \left(\frac{1}{E_{py}t_p} + \frac{2}{E_s t_s} \right)} \quad (5.109)$$

Note that equation (5.109) is identical to the result from the one-dimensional solution at the center of a double-sided reinforcement with $1/\lambda \ll l$. The loss in accuracy by reducing the two-dimensional analysis to a one-dimensional problem for the given material combination

is well below 1% for the thermal residual stress in the substrate along the fiber direction (i.e. σ_{s_y}) which is the main concern with respect to the performance of the bonded repair. In addition, the one-dimensional analysis of a double-sided reinforcement allows one to easily include the influence of the adhesive and the associated load transfer zone using equation (5.25). Thus the stress in the fiber direction at the prospective crack location in the center of the reinforced region due to the elevated temperature cure is given by:

$$\sigma_o = -\frac{2(\alpha_s - \alpha_{py})\Delta T}{t_s \left(\frac{1}{E_{py}t_p} + \frac{2}{E_s t_s} \right)} \left(1 - \frac{1}{\cosh(\lambda l)} \right) \quad (5.110)$$

5.3.2 Stage II: Upper Bounds for K_r

Having determined the stress at the prospective crack location σ_o , the crack is introduced in the second stage of the analysis. The load which was transmitted across the crack is now partly transferred into the patch and partly redistributed in the plate. The relative magnitudes of this redistribution primarily depend on the crack length, i.e. the longer the crack, the more load is transferred into the reinforcement [78].

The stress intensity factor K_r for a crack with a crack length a in a bonded repair is estimated by interpolation between two upper bounds, K_u and K_∞ for short and long cracks respectively. An upper bound for K_r for short cracks can be derived on the assumption that no load is transferred into the reinforcement [23]:

$$K_u = Y\sigma_o\sqrt{\pi a} \quad (5.111)$$

Rose gave the equations for a center crack, where $Y = 1$, while the AMRL specimen contains an edge crack. Based on the assumption that the edge crack is small in comparison to the width of the specimen, the correction factor for this geometry is $Y = 1.12$.

Rose showed that the relationship between energy release rate G and stress intensity factor K also holds for a reinforced cracked structure, i.e. that the body force field corresponding to the shear tractions does not affect the relation between G and K [78]. The crack in the isotropic substrate will not always be under plane stress conditions as it might be expected in a repair of an aircraft skin. The crack in a bonded repair under only thermal residual stress loading will generally be under plane strain conditions while combined with high external loading it will be under plane stress conditions.

The relationships between the energy release rate and the stress intensity factor are given for the crack in the isotropic substrate under plane stress and plane strain conditions as:

$$G = \frac{K^2}{E} \quad (\text{Plane stress}) \quad (5.112)$$

$$G = \frac{K^2}{E} (1 - \nu_s^2) \quad (\text{Plane strain}) \quad (5.113)$$

Thus the corresponding energy release rate G_u can be determined using equation (5.112) or (5.113). This upper bound will be close to the correct value for short cracks and represents an asymptote with first order contact in the limit $a \rightarrow 0$ for K_r of an infinite patched plate with a crack [78]. Note that the energy release rate G_u is linearly increasing with the crack length, although G_u cannot exceed the energy release rate G_∞ for a semi-infinite crack. Rose showed that the limiting value for the energy release rate G_∞ can be adequately estimated using one-dimensional modeling which ignores the stress variation across the thickness [79]. This upper value for the energy release rate can be estimated by the change in potential energy when introducing a cut in the substrate of a double-sided reinforced plate converting the specimen to a double-doubler joint. Thus G_∞ is the work extracted (per unit thickness and per unit crack growth) on the stress σ_o at the prospective crack location relaxed to zero through the displacement v_o on both crack faces [79]:

$$G_\infty = 2\left(\frac{1}{2}\sigma_o v_o\right). \quad (5.114)$$

The stress at the prospective crack location σ_o was determined using equation (5.3.1) in stage I of the Rose model using a one-dimensional model of a double-sided reinforcement. The displacement v_o as a result of introducing a cut in the double-sided reinforcement can be easily determined using superposition. According to Figure 5.12, the displacement v_o is

Figure 5.12: Illustration of the superposition principle

given as the displacement of one joint or crack face for a pressurized cut in a double-doubler joint. The general analysis as given in Section 5.2.1 has to be slightly altered to account for the different boundary conditions. The compatibility equations for the adherends and the adhesive are now expressed as:

$$\epsilon_p(y) = \frac{dv_p(y)}{dy} = \frac{T_p(y)}{E_p t_p} \quad (5.115)$$

$$\epsilon_s(y) = \frac{dv_s(y)}{dy} = \frac{T_s(y)}{E_s t_s} \quad (5.116)$$

$$\tau_a(y) = G_a \gamma_a(y) \approx \frac{G_a}{t_a} (v_p(y) - v_s(y)) \quad (5.117)$$

Differentiating equation (5.117) with respect to y and substituting equation (5.115) and

(5.116) yields:

$$\begin{aligned}\frac{d\tau_a(y)}{dy} &= \frac{G_a}{t_a} \left(\frac{dv_p(y)}{dy} - \frac{dv_s(y)}{dy} \right) \\ &= \frac{G_a}{t_a} \left(\frac{T_p(y)}{E_p t_p} - \frac{T_s(y)}{E_s t_s} \right)\end{aligned}\quad (5.118)$$

The characteristic differential equation remains identical. The two integration constants C_s and C_p can be determined using the overall force equilibrium

$$2T_p(y) + T_s(y) = 2C_p + C_s = 0 \quad (5.119)$$

and the first derivative of the shear stress $\tau_a(y)$ with respect to y , which can be equated to equation (5.118) as shown below:

$$\begin{aligned}\frac{d\tau_a(y)}{dy} &= A\lambda \cosh(\lambda y) + B\lambda \sinh(\lambda y) \\ &= \frac{G_a}{t_a} \left(\frac{T_p(y)}{E_p t_p} - \frac{T_s(y)}{E_s t_s} \right)\end{aligned}\quad (5.120)$$

Substituting the relationships for the stress resultants (5.11, 5.12) and equation (5.119) yields

$$\begin{aligned}A\lambda \cosh(\lambda y) + B\lambda \sinh(\lambda y) &= \frac{G_a}{t_a \lambda} \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right) (A \cosh(\lambda y) + B \sinh(\lambda y)) \\ &\quad + \frac{G_a}{t_a} \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right) C_p.\end{aligned}\quad (5.121)$$

Solving for C_p and subsequently for C_s gives

$$C_p = C_s = 0. \quad (5.122)$$

The boundary conditions for the stress resultants of an internally pressurized double-doubler joint are evaluated at the center of the sheet and at the edge of the patch as follows.

Boundary condition at the center

$$T_s^P(y=0) = -\sigma_o t_s = -\frac{2A}{\lambda} \quad (5.123)$$

Boundary conditions at the positive edge

$$T_p^P(y=l) = \frac{A^P}{\lambda} \cosh(\lambda l) + \frac{B^P}{\lambda} \sinh(\lambda l) = 0 \quad (5.124)$$

The boundary condition in the center as given by equation (5.123) is fulfilled if:

$$A^P = \lambda \sigma_o \frac{t_s}{2} \quad (5.125)$$

The second coefficient B^P can now easily determined by substituting A^P in equation (5.124):

$$B^P = -\lambda \sigma_o \frac{t_s}{2} \frac{1}{\tanh \lambda l} \quad (5.126)$$

Thus the shear stress distributions for an internally pressurized double-doubler joint can be written as:

$$\tau_a^P(y) = -\lambda \sigma_o \frac{t_s}{2} \frac{\cosh(\lambda(l-y))}{\sinh(\lambda l)} \quad (5.127)$$

where

$$0 \leq y \leq l \quad (5.128)$$

The displacement v_o at the center of the joint can be evaluated using equation (5.117). Note that the displacement for the patch is zero at this location. In order to determine the displacement v_o also for the case where shear-lag in the adherends is included, equation (5.91) has to be used. Both cases can be readily combined by using the stress distribution parameter λ and the elastic moduli and thickness of the adherends instead of the shear moduli and thickness of the adhesive.

$$v_o = v_s(y=0) = -\frac{t_a}{G_a} \tau_a^P(y=0) = \frac{\sigma_o t_s}{2} \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right) \frac{1}{\lambda \tanh(\lambda l)} \quad (5.129)$$

Thus the upper limit for the energy release rate can be expressed for plane stress and plane strain conditions as follows [18].

Plane stress conditions

$$G_\infty = \sigma_o v_o = \frac{Y^2 \sigma_o^2}{E_s} \pi \Lambda \quad (5.130)$$

where

$$\Lambda = \frac{1}{\pi Y^2} \left(1 + \frac{E_s t_s}{2 E_p t_p} \right) \frac{1}{\lambda \tanh(\lambda l)} \quad (5.131)$$

Plane strain conditions

$$G_\infty = \sigma_o v_o = \frac{Y^2 \sigma_o^2}{E_s} \pi \Lambda (1 - \nu_s^2) \quad (5.132)$$

where

$$\Lambda = \frac{1}{\pi Y^2 (1 - \nu_s^2)} \left(1 + \frac{E_s t_s}{2 E_p t_p} \right) \frac{1}{\lambda \tanh(\lambda l)} \quad (5.133)$$

Based on the validity of equations (5.112) and (5.113), the stress intensity factor K_∞ for this upper bound is:

$$K_\infty = \sqrt{E_s G_\infty} = Y \sigma_o \sqrt{\pi \Lambda} \quad (\text{Plane stress}) \quad (5.134)$$

$$K_\infty = \sqrt{\frac{E_s G_\infty}{(1 - \nu_s^2)}} = Y \sigma_o \sqrt{\pi \Lambda} \quad (\text{Plane strain}) . \quad (5.135)$$

Note that the differentiation between plane stress and plane strain conditions for G_∞ was introduced due to the definition of Λ in K_∞ . Furthermore, Rose showed that this result is also the first term in the asymptotic expansion of K_r for $\Lambda \ll a$. He gives an analytical approximation by interpolating between the asymptotes K_u and K_∞ which is generalized for different crack shapes below [18]. Rose verified the validity of K_r using the numerical results from other researchers such as Ratwani, Jones and Callinan [78], i.e.

$$K_r = Y \sigma_o \sqrt{\frac{\pi a \Lambda}{a + \Lambda}} \quad (5.136)$$

with the corresponding crack extension force

$$G_r = \frac{K_r^2}{E_s} \quad (\text{Plane stress}) \quad (5.137)$$

$$G_r = \frac{K_r^2}{E_s} (1 - \nu_s^2) \quad (\text{Plane strain}) . \quad (5.138)$$

The influence of the adherend shear stiffnesses can be readily implemented by using $\lambda_{\text{Shear-lag}}$ as given in equation (5.92) instead of λ . The characteristic crack length Λ can be derived from physical parameters of the repair without reference to the crack length a

or the remote applied loading as long as the crack length is small in comparison to the extent of the specimen. Otherwise the specimen width to crack ratio has to be accounted for as typically done in metal fracture specimens. Due to the long overlap length in the case of the AMRL specimen, the influence of the taper can be neglected for an investigation focused around the crack without a loss in accuracy. Thus the characteristic crack length, including accounting for the shear stiffness of the adherends, can be determined under typical thermal residual stress loading as $\Lambda = 4.0$ mm using the AMRL sandwich type specifications. This crack length justifies the previously made assumption that the characteristic crack length in comparison to the specimen width is small. The introduced error in the geometric correction factor Y is less than 0.25% within the valid range of a between $0 \text{ mm} \leq a \leq \Lambda$ for K_u . If Λ gets large, the geometric correction factor becomes a function of the crack length in which case Λ needs to be solved numerically. Figure 5.13 shows the variation of K_r with crack length as well as the two asymptotes K_u and K_∞ for the investigated AMRL specimen. K_r has reached 91% of K_∞ at a crack length of

Figure 5.13: Estimation and upper bounds for the stress intensity factor

$a = 20$ mm as shown in Figure 5.13. This clearly shows the usefulness of determining the boundary value K_∞ . Based on the estimation of K_r , the patched 20 mm long edge crack

is acting like an unpatched edge crack with a crack length of:

$$a_{\text{patched}}^{\text{EC}} = \frac{K_r^2}{Y^2 \sigma_o^2 \pi} = 3.4 \text{ mm} \quad (5.139)$$

Having determined the stress intensity factor for the patched specimen, it is now necessary to find the corresponding stress distribution in the substrate. This will allow comparison of the experimentally measured strains with results of the theoretical analysis.

5.4 Estimation of the Stress Field around the Crack Tip

An estimation of the stress field of a patched cracked plate can be obtained by calculating the stress field for a uniaxially loaded center crack in an infinite plate using the stress intensity factor estimation K_r of the patched crack with the load σ_o at the prospective crack location as given in equation (5.3.1) for the thermal residual stress due to an elevated temperature cure. While this approach gives a good estimation of the stress field ahead of the crack, it is not suitable to predict the stress field behind the crack tip and close to the crack faces since localized load transfer effects due to the stress singularity at the crack tip are not addressed. Another approximation in this approach is the use of the center crack solution for an edge crack problem. The boundary conditions for the stress along the fiber direction, which is of main concern, are met, while the boundary conditions for the stress perpendicular to the fiber orientation are not properly fulfilled. The boundary value problem is additionally complicated by the use of an effective smaller crack length a_{patched} in comparison to the real crack length a .

Currently available solutions for the two-dimensional stress field around a crack in an infinite plate are typically based on an infinite power series using a complex variable approach as discussed by Westergaard [98], Sih [85] and Liebowitz [59], or an eigenfunction expansion with an infinite number of terms as devised by Williams [99]. Most researchers neglect the higher order terms and include only the singularity term, which describes the stress

Figure 5.14: Biaxial loaded center crack in an infinite plate

field close to the crack tip quite well. Unfortunately these approximated solutions show significant error when estimating the far field stresses for a crack in an infinite plate.

The chosen approach for the stress field solution for crack patching is based on Westergaard's [98] complex variable analysis. In previous work, Sih [85] corrected Westergaard's original analysis accounting for a missing integration coefficient. Eftis and Liebowitz [34] gave the resulting Mode I stress field equations for biaxial loading (Figure 5.14 [23]) using only the singularity term as:

$$\sigma_x \cong \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) - (1 - k)\sigma_0 \quad (5.140)$$

$$\sigma_y \cong \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \quad (5.141)$$

$$\tau_{xy} \cong \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} . \quad (5.142)$$

Note that neither σ_x nor σ_y converge to the remote stresses $k\sigma_0$ or σ_0 respectively. An improved stress field solution using the complete series solution for the stress field was

given by Theocaris and Gdoutos [38, 89] as shown below:

$$\begin{aligned} \frac{\sigma_x}{\sigma_o} = & \left(\frac{2r}{a}\right)^{-\frac{1}{2}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2}\right) + \frac{3}{4\sqrt{2}} \left(\frac{r}{a}\right)^{\frac{1}{2}} \cos \frac{\theta}{2} \left(1 + \sin^2 \frac{\theta}{2}\right) \\ & + \left(\frac{r}{2a}\right)^{\frac{1}{2}} \left[\sum_{n=1}^{\infty} (-1)^n \frac{[1 \cdot 3 \cdot 5 \cdots (2n-1)](2n+3)}{2^{2(n+1)}(n+1)!} \right. \\ & \cdot \left. \left(\frac{r}{a}\right)^n \left[\cos \frac{(2n+1)\theta}{2} - \frac{(2n+1)}{2} \sin \frac{(2n-1)\theta}{2} \sin \theta \right] \right] - (1-k)\sigma_o \end{aligned} \quad (5.143)$$

$$\begin{aligned} \frac{\sigma_y}{\sigma_o} = & \left(\frac{2r}{a}\right)^{-\frac{1}{2}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2}\right) + \frac{3}{4\sqrt{2}} \left(\frac{r}{a}\right)^{\frac{1}{2}} \cos \frac{\theta}{2} \left(1 - \sin^2 \frac{\theta}{2}\right) \\ & + \left(\frac{r}{2a}\right)^{\frac{1}{2}} \left[\sum_{n=1}^{\infty} (-1)^n \frac{[1 \cdot 3 \cdot 5 \cdots (2n-1)](2n+3)}{2^{2(n+1)}(n+1)!} \right. \\ & \cdot \left. \left(\frac{r}{a}\right)^n \left[\cos \frac{(2n+1)\theta}{2} + \frac{(2n+1)}{2} \sin \frac{(2n-1)\theta}{2} \sin \theta \right] \right] \end{aligned} \quad (5.144)$$

$$\begin{aligned} \frac{\tau_{xy}}{\sigma_o} = & \left(\frac{2r}{a}\right)^{-\frac{1}{2}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2} - \frac{3}{4\sqrt{2}} \left(\frac{r}{a}\right)^{\frac{1}{2}} \cos^2 \frac{\theta}{2} \sin \frac{\theta}{2} \\ & + \left(\frac{r}{2a}\right)^{\frac{1}{2}} \left[\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{1 \cdot 3 \cdot 5 \cdots (2n+3)}{2^{2(n+3)}(n+1)!} \right. \\ & \cdot \left. \left(\frac{r}{a}\right)^n \sin \frac{(2n-1)\theta}{2} \sin \theta \right] \end{aligned} \quad (5.145)$$

Figure 5.15 presents the results using these approximations for the stress in the loading direction for a center crack with $a = 4.22$ mm and $\sigma_o = 40.43$ MPa along $\theta = 0^\circ$. The approximate series solutions as found in the literature are compared to the correct elastic solution which is derived in this section. The one term approximation using equation (5.141) shows a significant error of 53% at a distance of $2a$ from the crack tip. The error reduces to 18% if a two term approximation is used. Excellent agreement with respect to the correct elastic solution up to a distance of $2a$ from the crack tip can be observed when a larger number of terms is used.

Unfortunately, the series approximation is not applicable for locations beyond a distance of $2a$ from the crack tip. The determined stress diverges rapidly beyond this point as

Figure 5.15: Approximate crack tip stress field solution

shown in Figure 5.15. This approximate solution is inadequate for the estimation of the stress field in the substrate of a bonded repair due to the small effective crack length a_{patched} . Significant interest is currently focused on ‘smart patches’ where crack growth is monitored using strain sensors, however knowledge of the stress distribution is essential for their proper placement. Figure 5.15 indicates that the approximate solution would give false stress values only a short distance away from the crack tip. Therefore it is important to use a stress field solution which more accurately (or better) fulfills the remote stress conditions. The concise solution discussed in this section, which satisfies all of the required boundary conditions for a center crack in an infinite plate, is achieved by carrying out the full complex variable analysis without using the approximation of including only the singularity term ($1/\sqrt{r}$) or an infinite power series substitution.

5.4.1 Theoretical Derivation

Sih’s [85] correction of the original Westergaard approach [98] is derived using the complex representation of the general solution of the equations based on plane theory of elasticity as given by Muskhelishvili [64]. If the external loads are symmetric with respect to the

x-axis along which the crack is situated, the general complex representation of the stress field yields [34, 85]:

$$\sigma_x = \operatorname{Re}[Z(z)] - y \operatorname{Im}\left[\frac{dZ(z)}{dz}\right] - A \quad (5.146)$$

$$\sigma_y = \operatorname{Re}[Z(z)] + y \operatorname{Im}\left[\frac{dZ(z)}{dz}\right] + A \quad (5.147)$$

$$\tau_{xy} = -y \operatorname{Re}\left[\frac{dZ(z)}{dz}\right]. \quad (5.148)$$

Westergaard's [98] derivation did not include the constant A due to an oversight in MacGregor's work [34]. The boundary conditions for a center crack in an infinite plate using the coordinate system shown in Figure 5.14 are [85]:

$$\sigma_y(-2a < x < 0, y=0) = 0 \quad (5.149)$$

$$\tau_{xy}(-2a < x < 0, y=0) = 0 \quad (5.150)$$

$$\sigma_x(\sqrt{x^2+y^2} \rightarrow \infty) = k\sigma_o \quad (5.151)$$

$$\sigma_y(\sqrt{x^2+y^2} \rightarrow \infty) = \sigma_o \quad (5.152)$$

$$\tau_{xy}(\sqrt{x^2+y^2} \rightarrow \infty) = 0. \quad (5.153)$$

Sih gave the corrected Westergaard type stress function, which fulfills the above stated boundary conditions as [85]:

$$Z(z) = \frac{\sigma_o(z+a)}{\sqrt{(z+a)^2 - a^2}} - A \quad (5.154)$$

where

$$A = (1-k) \frac{\sigma_o}{2}. \quad (5.155)$$

The derivative of $Z(z)$ with respect to z can be easily determined as:

$$\frac{dZ(z)}{dz} = -\frac{\sigma_o a^2}{(z^2 + 2az)^{\frac{3}{2}}}. \quad (5.156)$$

In the second step of the analysis, the real and imaginary part of $Z(z)$ and the derivative $dZ(z)/dz$ have to be determined. It is convenient to switch to polar coordinates using:

$$z = re^{i\theta} \quad (5.157)$$

where

$$r = \sqrt{x^2 + y^2} \quad (5.158)$$

$$e^{i\theta} = \cos \theta + i \sin \theta . \quad (5.159)$$

Thus $Z(z)$ can be rewritten as:

$$\begin{aligned} Z(z) &= \frac{\sigma_o(z+a)}{\sqrt{(z+a)^2 - a^2}} - (1-k)\frac{\sigma_o}{2} \\ &= \frac{\sigma_o(re^{i\theta} + a)}{\sqrt{r^2e^{i2\theta} + 2are^{i\theta}}} - (1-k)\frac{\sigma_o}{2} \\ &= \frac{\sigma_o(re^{i\theta} + a)}{\sqrt{(r^2 \cos 2\theta + 2ar \cos \theta) + i(r^2 \sin 2\theta + 2ar \sin \theta)}} \\ &\quad - (1-k)\frac{\sigma_o}{2} \\ &= \frac{\sigma_o(re^{i\theta} + a) \sqrt{(r^2 \cos 2\theta + 2ar \cos \theta) - i(r^2 \sin 2\theta + 2ar \sin \theta)}}{\sqrt{(r^2 \cos 2\theta + 2ar \cos \theta)^2 + (r^2 \sin 2\theta + 2ar \sin \theta)^2}} \\ &\quad - (1-k)\frac{\sigma_o}{2} . \end{aligned} \quad (5.160)$$

In order to allow the separation of $Z(z)$ in real and imaginary parts as well as to reduce the size of the expression, $r^*(r, \theta)$ and $\theta^*(r, \theta)$ are defined as:

$$\begin{aligned} r^*(r, \theta) &= [(r^2 \cos 2\theta + 2ar \cos \theta)^2 + (r^2 \sin 2\theta + 2ar \sin \theta)^2]^{\frac{1}{4}} \\ &= [r^4 + 4a^2r^2 + 4ar^3 \cos \theta]^{\frac{1}{4}} \end{aligned} \quad (5.161)$$

$$\theta^*(r, \theta) = \arccos\left(\frac{r^2 \cos 2\theta + 2ar \cos \theta}{r^*(r, \theta)^2}\right) . \quad (5.162)$$

Note that the $\arccos()$ is used to maintain a unique solution for $0 \leq \theta \leq \pi$. Dealing with quarter symmetry, it is important to keep in mind that r is limited to:

$$r_{max} = \frac{a}{-\cos \theta} \quad \text{for} \quad \frac{\pi}{2} < \theta \leq \pi \quad (5.163)$$

Based on this expression it is resolved that $\theta^*(r, \theta)$ reaches its maximum value of π along the line $x = -a$, $y \geq 0$ and at the crack face. The parameter $\theta^*(r, \theta)$ has its minimum value of

0 along the line $x > 0$, $y = 0$. Using the relationships for $r^*(r, \theta)$ and $\theta^*(r, \theta)$, $Z(z)$ yields:

$$\begin{aligned}
 Z(z) &= \frac{\sigma_o(r e^{i\theta} + a)r^*(r, \theta)e^{-i\frac{1}{2}\theta^*(r, \theta)}}{r^*(r, \theta)^2} - (1 - k)\frac{\sigma_o}{2} \\
 &= \sigma_o \left(\frac{r}{r^*(r, \theta)} e^{i(\theta - \frac{1}{2}\theta^*(r, \theta))} + \frac{a}{r^*(r, \theta)} e^{-i\frac{1}{2}\theta^*(r, \theta)} \right) - (1 - k)\frac{\sigma_o}{2} \\
 &= \sigma_o \left(\frac{r}{r^*(r, \theta)} \cos(\theta - \frac{1}{2}\theta^*(r, \theta)) + \frac{a}{r^*(r, \theta)} \cos(\frac{1}{2}\theta^*(r, \theta)) \right) \\
 &\quad - (1 - k)\frac{\sigma_o}{2} \\
 &\quad + i \sigma_o \left(\frac{r}{r^*(r, \theta)} \sin(\theta - \frac{1}{2}\theta^*(r, \theta)) + \frac{a}{r^*(r, \theta)} \sin(\frac{1}{2}\theta^*(r, \theta)) \right) .
 \end{aligned} \tag{5.164}$$

Finally $\text{Re}[Z(z)]$ and $\text{Im}[Z(z)]$ can be expressed as:

$$\begin{aligned}
 \text{Re}[Z(z)] &= \sigma_o \left(\frac{r}{r^*(r, \theta)} \cos(\theta - \frac{1}{2}\theta^*(r, \theta)) + \frac{a}{r^*(r, \theta)} \cos(\frac{1}{2}\theta^*(r, \theta)) \right) \\
 &\quad - (1 - k)\frac{\sigma_o}{2}
 \end{aligned} \tag{5.165}$$

$$\text{Im}[Z(z)] = \sigma_o \left(\frac{r}{r^*(r, \theta)} \sin(\theta - \frac{1}{2}\theta^*(r, \theta)) + \frac{a}{r^*(r, \theta)} \sin(\frac{1}{2}\theta^*(r, \theta)) \right) . \tag{5.166}$$

The evaluation of the real and imaginary part of $dZ(z)/dz$ is carried out in a similar fashion:

$$\begin{aligned}
 \frac{dZ(z)}{dz} &= -\frac{\sigma_o a^2}{(z^2 + 2az)^{\frac{3}{2}}} \\
 &= -\frac{\sigma_o a^2}{(r^2 e^{i2\theta} + 2ar e^{i\theta})^{\frac{3}{2}}} \\
 &= -\frac{\sigma_o a^2}{[(r^2 \cos 2\theta + 2ar \cos \theta) + i(r^2 \sin 2\theta + 2ar \sin \theta)]^{\frac{3}{2}}} \\
 &= -\frac{\sigma_o a^2 [(r^2 \cos 2\theta + 2ar \cos \theta) - i(r^2 \sin 2\theta + 2ar \sin \theta)]^{\frac{3}{2}}}{[(r^2 \cos 2\theta + 2ar \cos \theta)^2 + (r^2 \sin 2\theta + 2ar \sin \theta)^2]^{\frac{3}{2}}} \\
 &= -\sigma_o \frac{a^2}{r^*(r, \theta)^3} e^{-i\frac{3}{2}\theta^*(r, \theta)} \\
 &= -\sigma_o \frac{a^2}{r^*(r, \theta)^3} \left(\cos(\frac{3}{2}\theta^*) - i \sin(\frac{3}{2}\theta^*(r, \theta)) \right) .
 \end{aligned} \tag{5.167}$$

Thus the real and imaginary part of $dZ(z)/dz$ are:

$$\text{Re}\left[\frac{dZ(z)}{dz}\right] = -\sigma_o \frac{a^2}{r^*(r, \theta)^3} \cos(\frac{3}{2}\theta^*(r, \theta)) \tag{5.168}$$

$$\text{Im}\left[\frac{dZ(z)}{dz}\right] = \sigma_o \frac{a^2}{r^*(r, \theta)^3} \sin(\frac{3}{2}\theta^*(r, \theta)) . \tag{5.169}$$

Having determined the real and imaginary parts of $Z(z)$ and $dZ(z)/dz$, the stresses σ_x , σ_y and τ_{xy} can be expressed in a concise form as:

$$\sigma_x(r, \theta) = \sigma_o \left[\frac{r}{r^*(r, \theta)} \cos\left(\theta - \frac{1}{2}\theta^*(r, \theta)\right) + \frac{a}{r^*(r, \theta)} \cos\left(\frac{1}{2}\theta^*(r, \theta)\right) - \frac{a^2 r}{r^{*3}(r, \theta)} \sin \theta \sin\left(\frac{3}{2}\theta^*(r, \theta)\right) - (1 - k) \right] \quad (5.170)$$

$$\sigma_y(r, \theta) = \sigma_o \left[\frac{r}{r^*(r, \theta)} \cos\left(\theta - \frac{1}{2}\theta^*(r, \theta)\right) + \frac{a}{r^*(r, \theta)} \cos\left(\frac{1}{2}\theta^*(r, \theta)\right) + \frac{a^2 r}{r^{*3}(r, \theta)} \sin \theta \sin\left(\frac{3}{2}\theta^*(r, \theta)\right) \right] \quad (5.171)$$

$$\tau_{xy}(r, \theta) = \sigma_o \left[\frac{a^2 r}{r^{*3}(r, \theta)} \sin \theta \cos\left(\frac{3}{2}\theta^*(r, \theta)\right) \right] \quad (5.172)$$

where $r^*(r, \theta)$ and $\theta^*(r, \theta)$ are given by equation (5.161) and (5.162) previously.

As previously discussed, θ and r are limited to $0 \leq \theta \leq \pi$ and $0 \leq r \leq -a/\cos(\theta)$ for $\pi/2 < \theta \leq \pi$ due to the use of θ^* . Symmetry can be readily used to determine the full stress field around the crack. As in the simplified Westergaard solution (equation (5.140)-(5.142)), the Mode I stress intensity factor K_I can be introduced by replacing the crack half length a with:

$$a = \frac{1}{\pi} \frac{K_I^2}{\sigma_o^2}. \quad (5.173)$$

The solution for the stress field can also be expressed as a function of x and y with the coordinate system origin at the crack tip (Figure 5.14) by replacing r and θ using:

$$r = \sqrt{x^2 + y^2} \quad (5.174)$$

$$\theta = \arccos\left(\frac{x}{\sqrt{x^2 + y^2}}\right) = \arcsin\left(\frac{y}{\sqrt{x^2 + y^2}}\right). \quad (5.175)$$

Thus

$$\sigma_x(x,y) = \sigma_o \left[\frac{x+a}{r^*(x,y)} \cos\left(\frac{1}{2}\theta^*(x,y)\right) + \frac{y}{r^*(x,y)} \sin\left(\frac{1}{2}\theta^*(x,y)\right) - \frac{a^2 y}{r^{*3}(x,y)} \sin\left(\frac{3}{2}\theta^*(x,y)\right) - (1-k) \right] \quad (5.176)$$

$$\sigma_y(x,y) = \sigma_o \left[\frac{x+a}{r^*(x,y)} \cos\left(\frac{1}{2}\theta^*(x,y)\right) + \frac{y}{r^*(x,y)} \sin\left(\frac{1}{2}\theta^*(x,y)\right) + \frac{a^2 y}{r^{*3}(x,y)} \sin\left(\frac{3}{2}\theta^*(x,y)\right) \right] \quad (5.177)$$

$$\tau_{xy}(x,y) = \sigma_o \left[\frac{a^2 y}{r^{*3}(x,y)} \cos\left(\frac{3}{2}\theta^*(x,y)\right) \right] \quad (5.178)$$

where

$$r^*(x,y) = [(x^2 - y^2 + 2ax)^2 + (2xy + 2ay)^2]^{\frac{1}{4}} \quad (5.179)$$

$$\theta^*(x,y) = \arccos\left(\frac{x^2 - y^2 + 2ax}{r^{*2}(x,y)}\right). \quad (5.180)$$

Note that x and y are limited in this case to $-a \leq x \leq \infty$ and $0 \leq y \leq \infty$ due to the use of θ^* . This solution simplifies slightly by moving the origin of the Cartesian coordinate system to the center of the crack. Furthermore, it is easier to determine the full stress field around the crack since symmetry conditions are now applied with respect to the center of the crack. The stress field solution with the origin placed at the crack center is given by:

$$\sigma_x(x_c, y_c) = \sigma_o \left[\frac{x_c}{r^*(x_c, y_c)} \cos\left(\frac{1}{2}\theta^*(x_c, y_c)\right) + \frac{y_c}{r^*(x_c, y_c)} \sin\left(\frac{1}{2}\theta^*(x_c, y_c)\right) - \frac{a^2 y_c}{r^{*3}(x_c, y_c)} \sin\left(\frac{3}{2}\theta^*(x_c, y_c)\right) - (1-k) \right] \quad (5.181)$$

$$\sigma_y(x_c, y_c) = \sigma_o \left[\frac{x_c}{r^*(x_c, y_c)} \cos\left(\frac{1}{2}\theta^*(x_c, y_c)\right) + \frac{y_c}{r^*(x_c, y_c)} \sin\left(\frac{1}{2}\theta^*(x_c, y_c)\right) + \frac{a^2 y_c}{r^{*3}(x_c, y_c)} \sin\left(\frac{3}{2}\theta^*(x_c, y_c)\right) \right] \quad (5.182)$$

$$\tau_{xy}(x_c, y_c) = \sigma_o \left[\frac{a^2 y_c}{r^{*3}(x_c, y_c)} \cos\left(\frac{3}{2}\theta^*(x_c, y_c)\right) \right] \quad (5.183)$$

where

$$r^*(x_c, y_c) = [(x_c^2 - y_c^2 - a^2)^2 + (2x_c y_c)^2]^{\frac{1}{4}} \quad (5.184)$$

$$\theta^*(x_c, y_c) = \arccos\left(\frac{x_c^2 - y_c^2 - a^2}{r^{*2}(x_c, y_c)}\right). \quad (5.185)$$

Note that x_c and y_c are limited for the case of the coordinate system placed in the center of the crack to $0 \leq x_c \leq \infty$ and $0 \leq y_c \leq \infty$ due to the use of θ^* .

In addition to the analyzed stress field for symmetric loading conditions, the displacement field can be solved using the same approach by utilizing the displacement equations provided by Sih [85]. Furthermore, the screw symmetric problem can also be solved in a similar fashion.

5.4.2 Results

The derived system of equations for σ_x , σ_y and τ_{xy} does not readily indicate the $(1/\sqrt{r})$ dependence. By substituting r^* and θ^* back into the equations for σ_x , σ_y and τ_{xy} , the dependence on $(a/r)^\alpha$ is revealed, where α represents different fractions. The singularity characteristic at the crack tips can then be readily shown by plotting the stress field. A double-sided reinforced infinite plate with a center crack is used to present an estimation for the sandwich type AMRL specimen. Using equations (5.182) through (5.185), the stress distributions for uniaxial remote tension of $\sigma_o = 40.43$ MPa applied perpendicular to a center crack with the half crack length of $a_{\text{patched}}^{\text{cc}} = 4.22$ mm are presented in Figure 5.16 through Figure 5.18. The crack length for the stress field estimation is based on determining an equivalent center crack length for a given stress intensity factor. Therefore the crack length was determined using the stress intensity factor $K_r = 4.66 \text{ MPa}\sqrt{\text{m}}$ given by Rose's model for the sandwich type AMRL specimen containing an edge crack with a length of $a = 20$ mm and thermal residual stresses on the order of $\sigma_o = 40.43$ MPa caused by cooling from an effective stress free temperature of 85.8°C to ambient temperature (21°C). Note that the singularities are not perfectly displayed due to the employed discrete mesh. The singularities can be better illustrated with decreasing mesh size. The stress distribution σ_x parallel to the crack is plotted in Figure 5.16. The solution reveals that along the crack

Figure 5.16: Stress parallel to a crack ($-4.22\text{mm} \leq x \leq 4.22\text{mm}$, $y = 0\text{mm}$) in an infinite plate with uniaxial loading of 40.43 MPa perpendicular to the crack

face, σ_x drops to $-\sigma_0$. The crack tip singularities are clearly present and the boundary condition of no stress in the x-direction far away from the crack is satisfied. The stress distribution of σ_y as presented in Figure 5.17 fulfills the boundary conditions of a stress free crack face at an applied remote stress of σ_0 . Here also the singularities are very pronounced at the crack tips. The shear stress is shown in Figure 5.18 with the singularities visible. Note that the z-axis scale has been changed to emphasize the singularities. Other features to note are that zero shear stress is attained both at the symmetry lines and far away from the crack tips.

The presented analysis offers an estimate of the stress field in the isotropic substrate for bonded repairs based on Rose's model. The results show the stress distributions for the sandwich type AMRL specimen under thermal residual stress loading. In order to compare

Figure 5.17: Stress perpendicular to a crack ($-4.22\text{mm} \leq x \leq 4.22\text{mm}$, $y = 0\text{mm}$) in an infinite plate with uniaxial loading of 40.43 MPa perpendicular to the crack

the experimentally measured strains with the theoretical predictions, the corresponding strains from the predicted stress field have to be determined. It is important to account for the stresses perpendicular to the fiber orientation close to the crack tip, although they can be neglected further away due to their small magnitude. If only the thermal residual stresses due to cooling from an effective stress free temperature to ambient temperature or operating temperature are considered, the thermal residual strains along the fiber direction being the main concern are given by:

$$\epsilon_y^{\text{TRS}} = \frac{1}{E_s} \left(\sigma_y^{\text{TRS}} - \nu_s \sigma_x^{\text{TRS}} \right) \quad (\text{close to the crack tip}) \quad (5.186)$$

$$\epsilon_y^{\text{TRS}} = \frac{1}{E_s} \sigma_y^{\text{TRS}} \quad (\text{far away from the crack tip}) . \quad (5.187)$$

Having derived an approximation for the thermal residual strain field for a patched crack,

Figure 5.18: Shear stress around a crack ($-4.22\text{mm} \leq x \leq 4.22\text{mm}$, $y = 0\text{mm}$) in an infinite plate with uniaxial loading of 40.43 MPa perpendicular to the crack

it is now possible to compare the theoretically derived thermal residual strains with the experimentally measured values at the strain gauged locations.

5.5 Comparison between Closed Form Theoretical and Experimental Results

Two practically important temperatures were chosen for a comparison between the experimentally measured thermal residual strains and the theoretically determined thermal residual strains: ambient temperature (21°C) and the temperature at cruising altitude (-56.5°C). The theoretical evaluation of the thermal residual strains was based on the

experimentally determined effective stress free temperature of (85.8°C) as discussed in the previous Section 4.6 giving a temperature differential of -64.8°C and -142.3°C.

One of the concerns is the use of the adhesive shear modulus in the theoretical estimation of the thermal residual stresses and strains. The shear modulus of the adhesive changes significantly from near zero during the initial part of the curing process to approximately 905 MPa for the low temperature at cruising altitude. A significant change in shear modulus for FM 73M can be identified from the experimental measurements occurring, when the slope of thermal residual strains versus temperature changes significantly at a temperature of approximately 85°C. The creep rate associated with the experimental testing sequence might be the reason for the variation to the glass transition temperature. This theory is supported by the measured creep which occurred during testing for extended periods at elevated temperatures as well as by the viscoelastic measurements carried out by Jurf and Vinson [56] .

A concern arises when considering which shear modulus should be used to determine the thermal residual stresses and strains at a specific temperature. Many researchers utilize an incremental approach in the determination of thermal residual and other process induced stresses and strains in which the shear modulus gradually changes from near zero at curing temperature to the shear modulus corresponding to the temperature at the end of the process [24, 50]. Although this seems to be a good model for the initial curing phase due to the change in cross-linking between the molecules, it is important to note that this is not an elastic modeling approach. This model assumes that a temperature change leads to an incremental stress/strain change caused by the mismatch in the CTE of the adherends which is independent of the actual stress and strain state. Curve A in Figure 5.19 shows the shear stress behaviour at the edge of a long reinforcement with four temperature changes utilizing this type of approach. Alternatively an elastic model can be assumed which is represented in Curve B.

Figure 5.19: Shear stress versus shear strain for the adhesive layer of a reinforcement during the cooling cycle

A simple experiment can be used to investigate the true behaviour of the adhesive. A double-doubler joint consisting of identical materials for the adherends can be loaded using a dead weight at elevated temperature but below a temperature where creep is significant during the cooling cycle from the curing temperature. Measuring the displacement as a function of temperature should reveal if an elastic approach is reasonable. If the non-elastic behaviour is true such as shown in curve **A**, no change in displacement should occur with change in temperature due to a constant load. A similar experiment with increasing temperature can be carried out. Here it seems very likely that the reduction in shear modulus will lead to an increase in the displacement.

The experimental thermal residual strain measurements using the AMRL specimen are unfortunately not conclusive enough to allow for determination of the appropriate shear modulus. Most gauges were not placed in the region of very high adhesive shear stress gradients with the exception of location **A** and to a limited extent location **B**. Because of the integrating character of strain gauges and the gauge size involved in the thermal residual strain measurement using the sandwich type AMRL specimen, no clear indication can be made from the measured data as to which of the above described models should

be used to justify the adhesive shear modulus change in the analysis. The true behaviour, which might very well be a combination of both models depending on temperature, needs to be determined in future experiments.

The attempt made in this research work is to simplify the prediction of thermal residual stresses to an elastic model by using definitions such as an effective stress free temperature to create readily available engineering tools. Based on the speculated outcome of the above described experiment, the shear modulus change is assumed elastic for this theoretical evaluation, i.e. the adhesive shear modulus is evaluated at the desired temperature. Based on the elastic assumption, adhesive shear moduli of 844 MPa and 905 MPa are used at 21°C and -56.5°C, respectively.

Unfortunately, the one-dimensional analysis is not suitable to predict through thickness changes in the adherends. Thus, in order to compare the theoretical estimates with the experimental measurements, average through thickness values of the thermal residual strains for the adherends were used. One of the disadvantages of using a through thickness average value is the neglect of errors caused by embedding strain gauges in the bonded repair with the associated small local thickness change. These thickness variations can result in bending stresses thus affecting the indicated strains. The finite element analysis gives a better tool to determine the cause of through thickness strain variations such as shear-lag, out-of-plane bending or embedding strain gauges.

It should also be recalled that the one-dimensional analysis focuses on the fiber direction being the main loading direction with respect to thermal residual strains and generally of main concern due to their magnitude. It is estimated that the locations of the strain gauges were measured to an accuracy of approximately ± 0.25 mm. This error is accounted for in the subsequent analysis. No other uncertainties such as the moduli of the adherends are included in this comparison.

A rough estimation for the thermal residual stress perpendicular to the fiber direction in the center region of the patch can be achieved using equations (5.104) and (5.105) in conjunction with equations (5.100) and (5.101). It becomes evident that significant in-plane bending is present by checking the strain gauge readings of locations **E** and **F**, thus making identification of the transverse strains below $140\mu\epsilon$ at the strain gauged locations using this type of analysis unsatisfactory. In addition, the influence of the crack creates significant additional transverse stresses in the crack tip region. The finite element analysis, being readily able to include multi-dimensional influences, is more suitable for the investigation of the transverse stresses and strains.

Table 5.1 shows the comparison between theoretically determined thermal residual strains in the fiber direction and the experimental measurements at ambient temperature. Table 5.2 presents a comparison at the typical operating temperature of aircraft structures (-56.5°C). The theoretical evaluation is carried out first using the one-dimensional model for double-sided reinforcements and double-doubler joints. Additionally the thermal residual strains are determined using the Rose model combined with the presented stress field estimation for applicable gauge locations.

Excellent agreement between the theoretical analysis and the experimental measurements for the thermal residual stresses in the aluminum at locations **C**, **D** and **H** are achieved. The fracture mechanics analysis predicts higher thermal residual strains at the strain gauged locations than the experimental measurements with a maximum difference of $35\mu\epsilon$ at ambient temperature and $81\mu\epsilon$ at -56.5°C . The measured thermal residual stresses in the aluminum at location **B** are approximately 25% higher than the strains predicted by the fracture mechanics analysis. A possible explanation for such an increase would be a large disbond. Using the analysis presented in Section 5.6, a disbond length of $2b \approx 60\text{ mm}$ would be required to increase the thermal residual strain to such a high level. A disbond of such an extent would also increase the thermal residual strains in other locations, espe-

Location	Material	Gauge No.	Experimental Measurement	Experimental Measurement Average	1-D Analysis	Fracture Mechanics Analysis
A	Al	2	5 $\mu\epsilon$	5 $\mu\epsilon$	84 $\pm$ 11 $\mu\epsilon$	N/A
	B/Ep	23	-607 $\mu\epsilon$	-636 $\mu\epsilon$	-696 $\pm$ 89 $\mu\epsilon$	N/A
	B/Ep	43	-665 $\mu\epsilon$			
B	Al	3	696 $\mu\epsilon$	696 $\mu\epsilon$	488 $\pm$ 3 $\mu\epsilon$	518 $\pm$ 12 $\mu\epsilon$
	B/Ep	24	-598 $\mu\epsilon$	-574 $\mu\epsilon$	-579 $\pm$ 4 $\mu\epsilon$	N/A
	B/Ep	30	-593 $\mu\epsilon$			
	B/Ep	33	-532 $\mu\epsilon$			
C	Al	12	566 $\mu\epsilon$	566 $\mu\epsilon$	558 $\pm$ 1 $\mu\epsilon$	585 $\pm$ 2 $\mu\epsilon$
	B/Ep	25	-688 $\mu\epsilon$	-633 $\mu\epsilon$	-661 $\pm$ 1 $\mu\epsilon$	N/A
	B/Ep	31	-627 $\mu\epsilon$			
	B/Ep	42	-585 $\mu\epsilon$			
D	Al	10	508 $\mu\epsilon$	518 $\mu\epsilon$	541 $\pm$ 1 $\mu\epsilon$	550 $\pm$ 1 $\mu\epsilon$
	Al	22	528 $\mu\epsilon$			
	B/Ep	28	-638 $\mu\epsilon$	-616 $\mu\epsilon$	-641 $\pm$ 1 $\mu\epsilon$	N/A
	B/Ep	29	-641 $\mu\epsilon$			
	B/Ep	40	-568 $\mu\epsilon$			
G	Al	7	327 $\mu\epsilon$	327 $\mu\epsilon$	134 $\pm$ 1 $\mu\epsilon$	N/A
	B/Ep	27	-720 $\mu\epsilon$	-734 $\mu\epsilon$	-1038 $\pm$ 1 $\mu\epsilon$	N/A
	B/Ep	37	-748 $\mu\epsilon$			
H	Al	5	532 $\mu\epsilon$	527 $\mu\epsilon$	558 $\pm$ 1 $\mu\epsilon$	562 $\pm$ 1 $\mu\epsilon$
	Al	19	521 $\mu\epsilon$			
	B/Ep	26	-632 $\mu\epsilon$	-634 $\mu\epsilon$	-661 $\pm$ 1 $\mu\epsilon$	N/A
	B/Ep	32	-647 $\mu\epsilon$			
	B/Ep	35	-622 $\mu\epsilon$			

Table 5.1: Comparison of the thermal residual strains at 21°C between experimental results, the one-dimensional analysis and the fracture mechanics analysis

cially location C where the thermal residual strains would increase to approximately 660 $\mu\epsilon$, which is certainly not the case. The thermal residual strains are predicted at location C quite well assuming no disbond. Therefore a disbond can be ruled out as a likely cause. Neglecting that the patch covers only 47% of the specimen width leads to some inaccuracy in the estimation of the stress at the prospective crack location. The stress estimate at the

Location	Material	Gauge No.	Experimental Measurement	Experimental Measurement Average	1-D Analysis	Fracture Mechanics Analysis
A	Al	2	38 $\mu\epsilon$	38 $\mu\epsilon$	184 $\pm$ 23 $\mu\epsilon$	N/A
	B/Ep	23	-1469 $\mu\epsilon$	-1639 $\mu\epsilon$	-1528 $\pm$ 195 $\mu\epsilon$	N/A
	B/Ep	43	-1808 $\mu\epsilon$			
B	Al	3	1504 $\mu\epsilon$	1504 $\mu\epsilon$	1065 $\pm$ 6 $\mu\epsilon$	1122 $\pm$ 26 $\mu\epsilon$
	B/Ep	24	-1262 $\mu\epsilon$	-1224 $\mu\epsilon$	-1261 $\pm$ 7 $\mu\epsilon$	N/A
	B/Ep	30	-1258 $\mu\epsilon$			
	B/Ep	33	-1152 $\mu\epsilon$			
C	Al	12	1215 $\mu\epsilon$	1215 $\mu\epsilon$	1212 $\pm$ 1 $\mu\epsilon$	1268 $\pm$ 3 $\mu\epsilon$
	B/Ep	25	-1456 $\mu\epsilon$	-1364 $\mu\epsilon$	-1435 $\pm$ 1 $\mu\epsilon$	N/A
	B/Ep	31	-1343 $\mu\epsilon$			
	B/Ep	42	-1293 $\mu\epsilon$			
D	Al	10	1145 $\mu\epsilon$	1150 $\mu\epsilon$	1177 $\pm$ 2 $\mu\epsilon$	1194 $\pm$ 1 $\mu\epsilon$
	Al	22	1154 $\mu\epsilon$			
	B/Ep	28	-1399 $\mu\epsilon$	-1356 $\mu\epsilon$	-1395 $\pm$ 2 $\mu\epsilon$	N/A
	B/Ep	29	-1386 $\mu\epsilon$			
	B/Ep	40	-1282 $\mu\epsilon$			
G	Al	7	758 $\mu\epsilon$	758 $\mu\epsilon$	291 $\pm$ 1 $\mu\epsilon$	N/A
	B/Ep	27	-1602 $\mu\epsilon$	-1661 $\mu\epsilon$	-2261 $\pm$ 1 $\mu\epsilon$	N/A
	B/Ep	37	-1719 $\mu\epsilon$			
H	Al	5	1161 $\mu\epsilon$	1138 $\mu\epsilon$	1210 $\pm$ 1 $\mu\epsilon$	1219 $\pm$ 1 $\mu\epsilon$
	Al	19	1115 $\mu\epsilon$			
	B/Ep	26	-1371 $\mu\epsilon$	-1396 $\mu\epsilon$	-1434 $\pm$ 1 $\mu\epsilon$	N/A
	B/Ep	32	-1417 $\mu\epsilon$			
	B/Ep	35	-1399 $\mu\epsilon$			

Table 5.2: Comparison of the thermal residual strains at -56.5°C between experimental results, the one-dimensional analysis and the fracture mechanics analysis

prospective crack location can be improved over the one-dimensional analysis by carrying out a finite element analysis of the AMRL specimen without a crack. The adhesive does not need to be modeled in this particular analysis since virtually no load transfer occurs at the prospective crack location.

The finite element analysis of the AMRL specimen as discussed in the following chapter

gives additional insight into the complex stress distribution in the vicinity of the crack thus giving a better understanding of the cause for the high tensile residual strains in the aluminum at location **B**. The stress and strain distributions as caused by the crack tip stress intensity factor are affected by the load transfer into the patch due to the crack as shown in the transverse shear stress plot for the AMRL specimen (see Figure 6.32). This influence is not accounted for in the fracture mechanics approach.

The predictions at location **G** are not accurate either ($\Delta\epsilon = 193 \mu\epsilon$ for the aluminum). The explanation for the discrepancy is given by the gradient of the thermal residual strains in the fiber direction through the tapered section. Additionally, in-plane bending has to be expected. The finite element model gives an excellent prediction for the thermal residual strains in the aluminum in this particular region. The thermal residual strains at the tapered location **A** are predicted reasonably well. After accounting for position uncertainty of the strain gauge, the thermal residual stresses are well predicted for the boron/epoxy. The thermal residual stresses in the aluminum are somewhat overpredicted ($84 \mu\epsilon$ at ambient temperature and $184 \mu\epsilon$ at an operating temperature of -56.5°C). The prediction of the thermal residual strains in the boron/epoxy patch at location **B**, **C**, **D** and **H** are quite good. The average discrepancy is only $20 \mu\epsilon$ at ambient temperature and $46 \mu\epsilon$ at an operating temperature of -56.5°C .

The thermal residual stress field in the bonded repair will change with crack growth when the test specimen is subjected to fatigue loading. In addition to crack growth, disbands around the crack face are quite common. Initial flaws such as inadequate surface preparation can also lead to large disbands. The influence of disbands on the stress and strain distribution can be quite large. A more generalized Rose model, which can account for disbands, is therefore developed in the following section and represents one of the major achievements of this research work.

5.6 Generalization of the Rose Model for Partially Disbonded Patches

The accurate prediction of the effect of a disbond on crack propagation in the metallic substrate requires elaborate calculations [5, 73, 76, 81]. Baker [13] presented the development of a useful estimate of the influence of disbond size by adapting the Rose model [79] giving a ratio between the stress intensity factor for a repair without a disbond and the stress intensity accounting for a symmetric disbond. The approach in this section expands on Baker's initial formulation by adding an additional term to improve the determination of the crack opening displacement as well as accounting for an imbalance in the stiffness of the patches and substrate and their thermal mismatch. The approach is first carried out for uniform thickness patches and then extended to tapered patches. The analysis presents a significantly improved estimate for the change of the stress intensity factor in the presence of a disbond.

Figure 5.20: Illustration of the superposition principle for a composite repair with disbonds

As discussed in Section 5.3, the stress at the prospective crack location in the presence of a disbond has to be determined first using the stage I analysis of the Rose model. The previously outlined method to determine the stress at the prospective crack location for a repair without a disbond utilizing a double-sided reinforcement can be used for a disbonded patch only if the two patches and substrate have identical stiffnesses and equal coefficients of thermal expansion. The reinforcement would be modeled from the free edge to the edge of the disbond. The requirement of identical stiffness and CTE of the adherends fulfills the requirement for zero shear stress at the edge of the disbond thus being equivalent to the center of a reinforcement without a disbond. The requirement for zero shear stress at the edge of the disbond can also be fulfilled for specific combinations of adherend stiffnesses and CTE mismatches. In any other case, it is necessary to derive a one-dimensional model, which takes the disbond as a boundary condition into account. The half crack opening displacement v_o as a result of introducing a cut in the substrate of the double-sided reinforcement can then be assessed using superposition.

Superposition can be used to determine the joint opening displacement v_o^d in the presence of a disbond which is required to determine the upper bound K_∞ similar to the approach presented in Section 5.3. Figure 5.20 shows the superposition scheme with symmetric disbonds.

5.6.1 Double-sided Uniform Thickness Repair with Disbonds

Stage I: Stress at the prospective crack location

The first step of the Rose model requires the determination of the stress at the prospective crack location. Figure 5.21 shows the free body diagram for half of a reinforcement with a disbond. In order to be able to use the system of equations derived for a reinforcement without a disbond, two coordinate systems were introduced.

The y - coordinate starts at the end of the disbond, while the y^* - coordinate originates in the center of the reinforcement. The following approach is directed toward deriving the influence of a disbond as a boundary condition. The disbond generally results in a shear stress in the adhesive at $y^* = b$ in comparison to zero shear stress in the center of a reinforcement without a disbond. The length changes in the patch and substrate due to both a direct stress resultant and a temperature change in the disbonded area and are given by:

$$\Delta l_p = \epsilon_p b = \frac{T_{po}^{Rd}}{E_p t_p} b + \alpha_p \Delta T b \quad (5.188)$$

$$\Delta l_s = \epsilon_s b = \frac{T_{so}^{Rd}}{E_s t_s} b + \alpha_s \Delta T b . \quad (5.189)$$

Using the definition of the elastic shear stress distribution parameter λ and the force equilibrium given in equation (5.13), the shear stress at $y = 0$ can be expressed as:

$$\tau_a^{Rd}(y=0) = \frac{\lambda^2}{\left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s}\right)} (v_p(y=0) - v_s(y=0)) \quad (5.190)$$

$$= \frac{\lambda^2}{\left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s}\right)} (\Delta l_p - \Delta l_s) \quad (5.191)$$

$$= \lambda^2 b \left(T_{po}^{Rd} - \frac{\frac{P}{E_s t_s} + (\alpha_s - \alpha_p) \Delta T}{\left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s}\right)} \right) \quad (5.192)$$

Figure 5.21: Free body diagram for a reinforcement with a disbond

The shear stress expression simplifies even further using the integration constant C_p as given in equation (5.16).

$$\tau_a^{Rd}(y=0) = \lambda^2 b (T_{p_o}^{Rd} - C_p) \quad (5.193)$$

A proper boundary condition can now be established by substituting $T_{p_o}^{Rd}$ and $\tau_a^{Rd}(y=0)$. The direct stress resultant for the patch T_{p_o} can be evaluated using equation (5.11) as

$$T_{p_o}^{Rd} = T_p^{Rd}(y=0) = \frac{A^{Rd}}{\lambda} + C_p. \quad (5.194)$$

The shear stress $\tau_a^{Rd}(y=0)$ is then determined using equation (5.10).

$$\tau_a^{Rd}(y=0) = B^{Rd} \quad (5.195)$$

The boundary condition for the disbonded reinforcement is given by substituting equation (5.194) and (5.195) into equation (5.193) thus yielding:

$$B^{Rd} = A^{Rd} \lambda b. \quad (5.196)$$

Note that this boundary condition for $b = 0$ is identical to the boundary at the center of a double-sided reinforcement without a disbond. The second boundary condition for the double-sided partly disbonded reinforcement is evaluated at the edge of the reinforcement ($y = l$) as:

$$T_p^{Rd}(y=l) = \frac{A^{Rd}}{\lambda} \cosh(\lambda l) + \frac{B^{Rd}}{\lambda} \sinh(\lambda l) + C_p = 0 \quad (5.197)$$

where A^{Rd} can be determined by substituting B^{Rd} in equation (5.197) yielding:

$$A^{Rd} = -\frac{C_p}{\frac{1}{\lambda} \cosh(\lambda l) + b \sinh(\lambda l)} \quad (5.198)$$

$$= -\frac{\frac{P}{E_s t_s} + (\alpha_s - \alpha_p) \Delta T}{\left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right) \left(\frac{1}{\lambda} \cosh(\lambda l) + b \sinh(\lambda l) \right)}. \quad (5.199)$$

The second coefficient B^{Rd} and then be calculated using equation (5.196):

$$B^{\text{Rd}} = -\frac{C_p}{\frac{1}{\lambda^2 b} \cosh(\lambda l) + \frac{1}{\lambda} \sinh(\lambda l)} \quad (5.200)$$

$$= -\frac{\frac{P}{E_s t_s} + (\alpha_s - \alpha_p) \Delta T}{\left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right) \left(\frac{1}{\lambda^2 b} \cosh(\lambda l) + \frac{1}{\lambda} \sinh(\lambda l) \right)} \quad (5.201)$$

Having determined A^{Rd} and B^{Rd} , the stress distributions for the adhesive and adherends can be readily determined. The stress at the prospective crack location in the presence of a disbond is given as:

$$\sigma_o^d = \frac{1}{t_s} \left(-\frac{2A^{\text{Rd}}}{\lambda} + C_s \right) \quad (5.202)$$

$$= 2 \left(\frac{\frac{P}{E_s t_s} + (\alpha_s - \alpha_p) \Delta T}{t_s \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right)} \right) \frac{1}{\cosh(\lambda l) + b \lambda \sinh(\lambda l)} \quad (5.203)$$

$$+ \frac{\frac{P}{E_p t_p} - 2(\alpha_s - \alpha_p) \Delta T}{t_s \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right)}.$$

Stage II: Upper bounds K_r

After determining the stress at the prospective crack location σ_o^d , the crack is introduced into the second stage of the analysis. The analysis is, for the most part, identical with the approach given in Section 5.3.2. The main difference is the contribution of three components to the half crack opening v_o^d rather than one. The strains in the disbanded sections of the patches and substrate lead to increased displacements, in addition to the crack opening due to shear in the adhesive.

An upper bound for the stress intensity factor for short cracks K_u^d assuming no load transfer into the reinforcement is given as [23]:

$$K_u^d = Y \sigma_o^d \sqrt{\pi a}. \quad (5.204)$$

The corresponding energy release rate G_u^d can then be determined based on plane stress or plane strain conditions:

$$G_u^d = \frac{K_u^{d2}}{E_s} \quad (\text{Plane stress}) \quad (5.205)$$

$$G_u^d = \frac{K_u^{d2}}{E_s} (1 - \nu_s^2) \quad (\text{Plane strain}) . \quad (5.206)$$

This upper bound will be close to the correct value for short cracks and represents an asymptote with first order contact in the limit $a \rightarrow 0$ for K_r^d of an infinite patched plate with a crack [78]. Note that the energy release rate G_u^d is linearly increasing with crack length, although G_u^d cannot exceed the energy release rate G_∞^d for a semi-infinite crack. The limiting value for the energy release rate G_∞^d is again modeled using the one-dimensional approach which ignores the stress variation across the thickness [79]. This upper value for the energy release rate is estimated by the change in potential energy when introducing a cut in the substrate of a double-sided reinforced plate with disbonds, therefore converting the specimen to a double-doubler joint with disbonds centered around the joint. Thus G_∞^d as given in equation (5.114) is the work extracted (per unit thickness and per unit crack growth) on the stress at the prospective crack location σ_o^d to relax to zero through the displacement v_o^d on both crack faces [79].

The stress at the prospective crack location σ_o^d was determined in equation (5.203) as stage I of the Rose model using a one-dimensional model of a double-sided partially disbonded reinforcement. The displacement v_o^d as a result of introducing a cut in the double-sided partially disbonded reinforcement can then be easily determined using superposition as shown in Figure 5.20.

According to Figure 5.20, the displacement v_o^d is given as the displacement of one joint or crack face for a pressurized cut in a partially disbonded double-doubler joint. The general analysis of the shear stress in the adhesive as given in Section 5.3.2 is still applicable in contrast to the partially disbonded double-sided reinforcement.

Thus the shear stress at the edge of the disbond for an internally pressurized partially disbonded double-doubler joint can be written as:

$$\tau_a^{pd}(y=0) = -\lambda \sigma_o^d \frac{t_s}{2 \tanh(\lambda l)} . \quad (5.207)$$

The displacement v_{oi}^d due to the shear displacement in the adhesive can be evaluated using equation (5.117). Therefore the displacement v_{oi}^d is given by equation (5.129) as:

$$v_{oi}^d = \frac{\sigma_o^d t_s}{2} \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right) \frac{1}{\lambda \tanh(\lambda l)} . \quad (5.208)$$

As stated above, it is necessary to include the displacements due to the strain in the disbonded section of the patch and substrate. These additional displacements are given as:

$$v_{oII}^d = \frac{\sigma_o^d t_s}{2} \frac{1}{E_p t_p} b \quad (5.209)$$

$$v_{oIII}^d = \frac{\sigma_o^d}{E_s} b \quad (5.210)$$

which sum to provide the total half crack opening displacement as:

$$v_o^d = v_{oi}^d + v_{oII}^d + v_{oIII}^d \quad (5.211)$$

$$= \frac{\sigma_o^d t_s}{2} \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right) \frac{1}{\lambda \tanh(\lambda l)} + \frac{\sigma_o^d t_s}{2} \frac{1}{E_p t_p} b + \frac{\sigma_o^d}{E_s} b \quad (5.212)$$

$$= \frac{\sigma_o^d t_s}{2} \left(\frac{1}{E_p t_p} + \frac{2}{E_s t_s} \right) \left(\frac{1}{\lambda \tanh(\lambda l)} + b \right) . \quad (5.213)$$

Thus the upper limit for the energy release rate can be expressed for plane stress and plane strain conditions as follows [18].

Plane stress

$$G_\infty^d = \sigma_o^d v_o^d = \frac{Y^2 \sigma_o^{d2}}{E_s} \pi \Lambda \quad (5.214)$$

where

$$\Lambda = \frac{1}{\pi Y^2} \left(1 + \frac{E_s t_s}{2 E_p t_p} \right) \left(\frac{1}{\lambda \tanh(\lambda l)} + b \right) \quad (5.215)$$

Plane strain

$$G_\infty^d = \sigma_o^d v_o^d = \frac{Y^2 \sigma_o^{d2}}{E_s} \pi \Lambda (1 - \nu_s^2) \quad (5.216)$$

where

$$\Lambda = \frac{1}{\pi Y^2(1 - \nu_s^2)} \left(1 + \frac{E_s t_s}{2E_p t_p} \right) \left(\frac{1}{\lambda \tanh(\lambda l)} + b \right) \quad (5.217)$$

Based on the validity of equation (5.112) and (5.113), the stress intensity factor K_∞^d for this upper bound is:

$$K_\infty^d = \sqrt{E_s G_\infty^d} = Y \sigma_o^d \sqrt{\pi \Lambda} \quad (\text{Plane stress}) \quad (5.218)$$

$$K_\infty^d = \sqrt{\frac{E_s G_\infty^d}{(1 - \nu_s^2)}} = Y \sigma_o^d \sqrt{\pi \Lambda} \quad (\text{Plane strain}) . \quad (5.219)$$

As before, this result can be used as the first term in the asymptotic expansion of K_r^d for $\Lambda \ll a$. The Rose model suggests an analytical approximation by interpolating between the asymptotes K_u^d and K_∞^d which is generalized for different crack shapes below [18]

$$K_r^d = Y \sigma_o^d \sqrt{\frac{\pi a \Lambda}{a + \Lambda}} \quad (5.220)$$

with the corresponding crack extension force

$$G_r^d = \frac{K_r^{d2}}{E_s} \quad (\text{Plane stress}) \quad (5.221)$$

$$G_r^d = \frac{K_r^{d2}}{E_s} (1 - \nu_s^2) \quad (\text{Plane strain}) . \quad (5.222)$$

The accuracy of the analysis can also be improved by accounting for the shear stiffness of the adherends. This can be readily implemented by using $\lambda_{\text{Shear-lag}}$ as given in equation (5.92) instead of λ .

The results of this analysis scheme are presented for three different cases:

- Thermal residual stress loading ($\Delta T = -64.8^\circ\text{C}$)
- Remote stress loading ($\sigma_\infty = 40 \text{ MPa}$)
- Combined thermal residual stress and remote stress loading ($\Delta T = -64.8^\circ\text{C}$, $\sigma_\infty = 40 \text{ MPa}$)

Note that the stress intensity factors are determined for all cases using the plane stress solution to allow for a better comparison between the three cases. Plane stress conditions are typically present under a combination of high external loading and thermal residual stresses in a real application. The material and geometric data used in this analysis are given below:

Substrate Material

Young's modulus	$E_s = 72.4 \text{ GPa}$
Shear modulus	$G_s = 27.2 \text{ GPa}$
Coef. of Therm. Expan.	$\alpha_s = 23.45 \frac{\mu\epsilon}{^\circ\text{C}}$
Thickness	$t_s = 6.35 \text{ mm}$
Edge crack length	$a = 20 \text{ mm}$
Correction factor	$Y = 1.12$

Patch Material

Young's modulus ()	$E_p = 210 \text{ GPa}$
Shear modulus	$G_p = 7.24 \text{ GPa}$
Coef. of Therm. Expan.	$\alpha_p = 4.61 \frac{\mu\epsilon}{^\circ\text{C}}$
Thickness	$t_p = 6.35 \text{ mm}$
Length	$l_p = 75.0 \text{ mm}$

Adhesive

Shear modulus	$G_a = 0.844 \text{ GPa}$
Thickness	$t_a = 0.25 \text{ mm}$

The stress intensity factors K_∞^d , K_u^d and K_r^d were determined for each of the loading conditions for the entire range of b , i.e. from no disbond ($b=0 \text{ mm}$) to completely disbanded ($b=75 \text{ mm}$). Note that K_u^d is dependent on the chosen crack length and will therefore shift up or down for different crack lengths. K_r^d indicates for which disbond length K_∞^d or K_u^d is dominant.

Figure 5.22: Stress intensity factor dependence on disbond length for a bonded repair under thermal residual stress loading ($\Delta T = -64.8^\circ\text{C}$)

Figure 5.22 through Figure 5.24 show some interesting features when looking at large disbonds. The stress intensity factors drop to zero for the case of thermal residual stress loading when approaching a complete disbond, which should be expected. This effect seems to be dominant only when the remaining bonded length is approximately equal to the load transfer length, which in this case is 6.5 mm. The opposite behaviour can be seen for the

Figure 5.23: Stress intensity factor dependence on disbond length for a bonded repair under remote stress loading ($\sigma_\infty = 40\text{ MPa}$)

Figure 5.24: Stress intensity factor dependence on disbond length for a bonded repair under combined thermal residual stress and remote stress loading ($\Delta T = -64.8^\circ\text{C}$, $\sigma_\infty = 40\text{ MPa}$)

case of a remotely applied stress (Figure 5.23). The stress intensity factor K_∞^d converges to infinity, which is to be expected due to using a pressurized double-doubler joint model, i.e. the substrate would separate. K_u^d reaches its limiting value of an unpatched specimen, which is also properly reflected by K_r^d . The behavior of the stress intensity factors for the combined loading case (Figure 5.24) also satisfies expectations. K_∞^d is again converging to infinity caused by the remaining remote stress after the complete disbond, while K_u^d is again limited by the unpatched scenario. K_r^d reflects the change between the limiting stress intensity factors well.

Rose's patching efficiency definition [78] leading to zero patching efficiency for the unpatched crack and 1.0 for elimination of the crack i.e. a reduction in stress intensity to 0 $\text{MPa}\sqrt{\text{m}}$ shows the problem of disbonds clearly:

$$R_K = 1 - \frac{K_r^d}{K_u^d(b=l_p)} \quad (5.223)$$

This particular definition cannot be used for the case where only thermal residual stresses are present, i.e. $K_u^d(b=l_p)$ equal to zero. But the definition reflects the patch efficiency

quite well for partially disbonded patches with a loading combination of thermal residual stresses and remote stresses, which is most common for test specimens (Figure 5.25). Under the given boundary and loading conditions, the patching leads only to a reduction in the absolute stress intensity factor to a disbonding length of approximately 19.1 mm. Larger disbonds represent a worse case than the unrepaired specimen due to the thermal residual stresses. Note that this particular disbond length shifts to a higher value if the crack length increases. For a crack length of 50 mm the disbond can be up to approximately 58.7 mm before patching will lead to an increase in the stress intensity factor. The influence of thermal residual stresses on the stress intensity factor for real bonded repair applications with disbonds is generally not as severe as shown here for an unrestrained specimen due to the lower effective coefficient of thermal expansion of the substrate. The patching efficiency is much closer to the case of remote stress loading (Figure 5.25).

Figure 5.25: Patch efficiency for partially disbonded patches

The results show how readily an idealized disbond can be included in the analysis. In comparison to the basic analysis carried out in Section 5.3, the revised results for the determination of an upper bound for the stress intensity factor including disbonds have

only the additional disbond length term b . If $b = 0$, the original solution without disbonds is obtained. In this respect, the presented solution is a more generalized estimation for the performance of bonded repairs and represents one of the main achievements of this research work.

The solution can be improved further by accounting for plastic strain in the adhesive although the influence of the adhesive shear deformation towards the crack opening displacement is reduced from 100% of the total crack opening displacement for a fully bonded repair to only 10% for a repair containing large disbonds using the fully elastic analysis. Even for a significant increase in the shear displacement by accounting for plastic strain, the change in the stress intensity factor will be relatively small making the elastic solution an excellent first order approximation. Additional experimental work as well as higher order analytical work is required to determine the overall performance of the method.

5.6.2 Double-sided Tapered Repair with Disbonds

The stress intensity factor change with disbond growth can also be determined for a repair involving a patch with a tapered edge. The direct closed form method for determining the stress distribution for tapered reinforcements and double-doubler joints was presented in Section 5.2. This method can be easily adapted for analyzing the stress at the prospective crack location using a partially disbonded reinforcement model and the shear stress at the edge of the disbond in a pressured double-doubler joint. Note that this approach is only valid until the disbond reaches the last step adjacent to the edge. Then the model reduces to a simpler analysis similar to the uniform thickness patch but must include the thickness change of the patch within the disbonded region. Using this two step model, the influence of a taper on disbond tolerance can be investigated for thermal, mechanical and

Figure 5.26: Free body diagram for a tapered reinforcement with a disbond

a combination of both load types. The model is fully elastic and can be extended using a numerical approach for elastic-plastic behaviour in the adhesive.

Stage I: Stress at the prospective crack location

Determination of the stress at the prospective crack location is carried out in a manner similar to that of the uniform thickness repair with a disbond. In the case of a tapered edge, the boundary condition for determining the stress at the prospective crack location must consider the change in thickness over the partial disbond. Figure 5.26 shows the free body diagram for this particular case. Note that there is no need for an interface condition between steps when only the last step l_n remains bonded, thus simplifying the analysis.

In order to use the previously developed system of equations for tapered reinforcements, the notation for the different steps in the disbonded section is indexed $2-d, \dots, -1, 0, 1$ where 1 indicates the identical geometry as the first section of the remaining bonded area and d is the number of disbonded steps. The stress at the prospective crack location for a disbond with a fully bonded last step is determined in the first part of the stage I analysis of the Rose model.

As before, the change in length in the patch and substrate can be evaluated using:

$$\Delta l_p = \sum_{k=2-d}^l \epsilon_{p_k} b_k = T_{p1}^{Rd} \sum_{k=2-d}^l \frac{b_k}{E_{p_k} t_{p_k}} + \alpha_p \Delta T b \quad (5.224)$$

$$\Delta l_s = \epsilon_s b = T_{s1}^{Rd} \frac{b}{E_s t_s} + \alpha_s \Delta T b \quad (5.225)$$

where

$$b = \sum_{k=2-d}^l b_k. \quad (5.226)$$

Using the definition of the elastic shear stress distribution parameter λ and the force equilibrium given in equation (5.13), the shear stress at $y_1 = 0$ can be expressed as:

$$\tau_{a1}^{Rd}(y_1=0) = \frac{\lambda_1^2}{\left(\frac{1}{E_{p1} t_{p1}} + \frac{2}{E_s t_s}\right)} (v_p(y_1=0) - v_s(y_1=0)) \quad (5.227)$$

$$= \frac{\lambda_1^2}{\left(\frac{1}{E_{p1} t_{p1}} + \frac{2}{E_s t_s}\right)} (\Delta l_p - \Delta l_s) \quad (5.228)$$

$$= \lambda_1^2 b \left(T_{p1}^{Rd} \frac{\sum_{k=2-d}^l \frac{b_k}{b} \left(\frac{1}{E_{p_k} t_{p_k}} + \frac{2}{E_s t_s}\right)}{\left(\frac{1}{E_{p1} t_{p1}} + \frac{2}{E_s t_s}\right)} - \frac{\frac{P}{E_s t_s} + (\alpha_s - \alpha_p) \Delta T}{\left(\frac{1}{E_{p1} t_{p1}} + \frac{2}{E_s t_s}\right)} \right). \quad (5.229)$$

The shear stress expression can be simplified by defining the constant H as

$$H = \frac{\sum_{k=2-d}^l \frac{b_k}{b} \left(\frac{1}{E_{p_k} t_{p_k}} + \frac{2}{E_s t_s}\right)}{\left(\frac{1}{E_{p1} t_{p1}} + \frac{2}{E_s t_s}\right)} \quad (5.230)$$

and using the integration constant C_p from Section 5.2 as given by equation (5.16):

$$\tau_{a1}^{Rd}(y_1=0) = \lambda_1^2 b \left(T_{p1}^{Rd} H - C_{p1} \right). \quad (5.231)$$

A proper boundary condition can now be established by substituting T_{p1}^{Rd} and $\tau_{a1}^{Rd}(y_1=0)$.

The direct stress resultant for the patch can be evaluated using equation (5.11).

$$T_{p1}^{Rd} = T_{p1}^{Rd}(y_1=0) = \frac{A_1^{Rd}}{\lambda_1} + C_{p1} \quad (5.232)$$

The shear stress is determined using equation (5.10)

$$\tau_{a_1}^{\text{Rd}}(y_1=0) = B_1^{\text{Rd}} \quad (5.233)$$

thus the boundary condition yields:

$$B_1^{\text{Rd}} = A_1^{\text{Rd}} \lambda_1 b H + C_{p_1} \lambda_1^2 b (H - 1) . \quad (5.234)$$

The additional equation to solve for A_1^{Rd} and B_1^{Rd} is given by the boundary condition at the edge, thus:

$$T_{p_n}(y_n=l_n) = \frac{A_n}{\lambda_n} \cosh(\lambda_n l_n) + \frac{B_n}{\lambda_n} \sinh(\lambda_n l_n) + C_{p_n} = 0 . \quad (5.235)$$

Using equation (5.52) and (5.53) developed in Section 5.2, the boundary condition at the edge yields:

$$A_1^{\text{Rd}} = -B_1^{\text{Rd}} \frac{f(\phi=0) \cosh(\lambda_n l_n) + f(\phi=2^{n-1}) \sinh(\lambda_n l_n)}{f(\phi=-1) \cosh(\lambda_n l_n) + f(\phi=2^{n-1}-1) \sinh(\lambda_n l_n)} - \lambda_1 \frac{\Delta C(\phi=0) \cosh(\lambda_n l_n) + \Delta C(\phi=2^{n-1}) \sinh(\lambda_n l_n) + C_{p_n}}{f(\phi=-1) \cosh(\lambda_n l_n) + f(\phi=2^{n-1}-1) \sinh(\lambda_n l_n)} \quad (5.236)$$

$$= -B_1^{\text{Rd}} F_f - \lambda_1 F_c \quad (5.237)$$

where

$$F_f = \frac{f(\phi=0) \cosh(\lambda_n l_n) + f(\phi=2^{n-1}) \sinh(\lambda_n l_n)}{f(\phi=-1) \cosh(\lambda_n l_n) + f(\phi=2^{n-1}-1) \sinh(\lambda_n l_n)} \quad (5.238)$$

$$F_c = \frac{\Delta C(\phi=0) \cosh(\lambda_n l_n) + \Delta C(\phi=2^{n-1}) \sinh(\lambda_n l_n) + C_{p_n}}{f(\phi=-1) \cosh(\lambda_n l_n) + f(\phi=2^{n-1}-1) \sinh(\lambda_n l_n)} . \quad (5.239)$$

Thus A_1^{Rd} and B_1^{Rd} can be determined by solving the linear system of equations given by the boundary conditions at the edge of the disbond and at the patch edge, i.e.

$$A_1^{\text{Rd}} = -\frac{\lambda_1 F_c + \lambda_1^2 b C_{p_1} F_f (H - 1)}{1 + \lambda_1 b F_f H} \quad (5.240)$$

$$B_1^{\text{Rd}} = \lambda_1^2 b \frac{-H F_c + C_{p_1} (H - 1)}{1 + \lambda_1 b F_f H} . \quad (5.241)$$

Finally, the stress at the prospective crack location for a disbond leaving the last step fully bonded can be determined using:

$$\sigma_o^d = \frac{1}{t_s} \left(-\frac{2A_1^{\text{Rd}}}{\lambda_1} + C_{s1} \right) \quad (5.242)$$

$$= \frac{1}{t_s} \left(\frac{2F_c + 2\lambda_1 b C_{p1} F_f (H - 1)}{1 + \lambda_1 b F_f H} + C_{s1} \right) . \quad (5.243)$$

The stress at the prospective crack location for a disbond extending underneath the last step of the patch can be easily determined using the analysis for uniform patch thickness as well as the analysis presented for the tapered patch excluding the last step. For this particular case the index of the tapered analysis is $1 = i = n$. Equation (5.234) and (5.235) give the two equations required to determine A_1^{Rd} as

$$A_1^{\text{Rd}} = -\frac{C_{p1} \lambda_1 + \lambda_1^2 b C_{p1} (H - 1) \sinh(\lambda_1 l_1)}{\cosh(\lambda_1 l_1) + H \lambda_1 b \sinh(\lambda_1 l_1)} . \quad (5.244)$$

Thus the stress at the prospective crack location for a repair where the disbond extends underneath the last step of the patch can be evaluated using equation (5.242) yielding

$$\sigma_o^d = \frac{1}{t_s} \left(-\frac{2C_{p1} + 2\lambda_1 b C_{p1} (H - 1) \sinh(\lambda_1 l_1)}{\cosh(\lambda_1 l_1) + H \lambda_1 b \sinh(\lambda_1 l_1)} + C_{s1} \right) . \quad (5.245)$$

Stage II: Upper bounds K_r

Having determined the stress at the prospective crack location σ_o^d , the crack is introduced in the second stage of the analysis. The analysis is, for the most part, identical with the approach for double-sided uniform thickness repairs with disbonds given in Section 5.6.1. The half crack opening v_o consists again of three components. Crack opening displacements are caused by the shear strain in the adhesive and the strains in the disbanded sections of the patches and substrate.

An upper bound for the stress intensity factor for short cracks K_u^d and the corresponding

energy release rate G_u^d assuming no load transfer into the reinforcement can be evaluated using equations (5.204), (5.205) and (5.206) as presented for the case of partially disbonded uniform thickness repairs. The second upper bound for the stress intensity and energy release rate as given already by equations (5.218), (5.219), (5.214) and (5.216) is estimated by the change in potential energy when introducing a cut in the substrate of a double-sided reinforced plate with disbonds, thereby converting the specimen to a double-doubler joint with disbonds adjacent to the joint.

The stress at the prospective crack location σ_o^d was determined in equation (5.243) and (5.245) depending on the disbond size in stage I of the Rose model using a one-dimensional model of a tapered, double-sided, partially disbonded reinforcement. The displacement v_o^d as a result of introducing a cut in the tapered, double-sided, partially disbonded reinforcement can be easily determined using superposition as shown in Figure 5.20.

According to Figure 5.20, the displacement v_o^d is given as the displacement of one joint or crack face for a pressurized cut in a partially disbonded double-doubler joint. The general analysis of the shear stress in the adhesive as given in Section 5.3.2 is still applicable in contrast to the partially disbonded double-sided reinforcement. In the case of accounting for the influence of the tapered edges of the patch, the stress analysis for tapered patches as outlined in Section 5.2 can be applied. It is again necessary to distinguish between disbonds extending underneath the last step of the patch and those having a shorter disbond length. Tapered bonded repairs with disbond length not extending underneath the last step are considered first. The boundary conditions for this case are given below.

Boundary condition at the disbond edge:

$$T_s^{Pd}(y_1=0) = -\frac{2A_1^{Pd}}{\lambda_1} = -\sigma_o^d t_s \quad (5.246)$$

Thus A_1^{Pd} yields:

$$A_1^{Pd} = \sigma_o^d \lambda_1 \frac{t_s}{2} \quad (5.247)$$

Boundary condition at the edge:

$$T_p^{Pd}(y_n=l_n) = \frac{A_n^{Pd}}{\lambda_n} \cosh(\lambda_n l_n) + \frac{B_n^{Pd}}{\lambda_n} \sinh(\lambda_n l_n) = 0 \quad (5.248)$$

The fundamental relationships between A_1^{Pd} and A_n^{Pd} as well as between B_1^{Pd} and B_n^{Pd} are given in equations (5.52) and (5.53). For pressurized joints it turns out that $\Delta C(\phi)$ is zero thus simplifying the relationships to:

$$\frac{A_n^{Pd}}{\lambda_n} = f(\phi=-1) \frac{A_1^{Pd}}{\lambda_1} + f(\phi=0) \frac{B_1^{Pd}}{\lambda_1} \quad (5.249)$$

$$\frac{B_n^{Pd}}{\lambda_n} = f(\phi=2^{n-1}-1) \frac{A_1^{Pd}}{\lambda_1} + f(\phi=2^n-1) \frac{B_1^{Pd}}{\lambda_1} . \quad (5.250)$$

Substituting equations (5.247), (5.249) and (5.250) in the edge boundary condition equation yields the constant B_1^{Pd} as:

$$B_1^{Pd} = -\sigma_o^d \lambda_1 \frac{t_s}{2} \frac{f(\phi=-1) \cosh(\lambda_n l_n) + f(\phi=2^{n-1}-1) \sinh(\lambda_n l_n)}{f(\phi=0) \cosh(\lambda_n l_n) + f(\phi=2^n-1) \sinh(\lambda_n l_n)} . \quad (5.251)$$

Having determined the two constants A_1^{Pd} and B_1^{Pd} , the shear stress at the edge of the disbond is given by:

$$\tau_{a_1}^{Pd}(y_1=0) = B_1^{Pd} \quad (5.252)$$

$$= -\sigma_o^d \lambda_1 \frac{t_s}{2} \frac{f(\phi=-1) \cosh(\lambda_n l_n) + f(\phi=2^{n-1}-1) \sinh(\lambda_n l_n)}{f(\phi=0) \cosh(\lambda_n l_n) + f(\phi=2^n-1) \sinh(\lambda_n l_n)} \quad (5.253)$$

$$= -\sigma_o^d \lambda_1 \frac{t_s}{2} \frac{1}{F_f} . \quad (5.254)$$

The corresponding half crack opening displacement v_{oI}^d is then calculated using equation (5.8) and (5.117):

$$v_{oI}^d = \frac{\sigma_o^d t_s}{2} \left(\frac{1}{E_{p1} t_{p1}} + \frac{2}{E_s t_s} \right) \frac{1}{\lambda_1 F_f} . \quad (5.255)$$

The additional displacements due to strains in the substrate and patch are given as:

$$v_{oII}^d = \frac{\sigma_o^d t_s}{2} \sum_{k=2-d}^1 \frac{1}{E_{p_k} t_{p_k}} b_k \quad (5.256)$$

$$v_{oIII}^d = \frac{\sigma_o^d}{E_s} b \quad (5.257)$$

which are then used to calculate the total half crack opening displacement as follows:

$$v_o^d = v_{oI}^d + v_{oII}^d + v_{oIII}^d \quad (5.258)$$

$$= \frac{\sigma_o^d t_s}{2} \left(\frac{1}{E_{p_1} t_{p_1}} + \frac{2}{E_s t_s} \right) \frac{1}{\lambda_1 F_f} + \frac{\sigma_o^d t_s}{2} \sum_{k=2-d}^1 \frac{1}{E_{p_k} t_{p_k}} b_k + \frac{\sigma_o^d}{E_s} b \quad (5.259)$$

$$= \frac{\sigma_o^d t_s}{2} \left(\frac{1}{E_{p_1} t_{p_1}} + \frac{2}{E_s t_s} \right) \left(\frac{1}{\lambda_1 F_f} + \frac{\sum_{k=2-d}^1 b_k \left(\frac{1}{E_{p_k} t_{p_k}} + \frac{2}{E_s t_s} \right)}{\left(\frac{1}{E_{p_1} t_{p_1}} + \frac{2}{E_s t_s} \right)} \right). \quad (5.260)$$

As previously discussed, it is necessary to determine the half crack opening for a disbond extending underneath the last step of the patch separately. The half crack opening due to the shear displacement in the adhesive can be determined using equation (5.208) from the uniform thickness patch analysis while the displacements due to the strain in the substrate and patch are given in equation (5.256) and (5.257). Thus the total half crack opening displacement for a disbond extending underneath the last step of the patch is given by:

$$v_o^d = \frac{\sigma_o^d t_s}{2} \left(\frac{1}{E_{p_1} t_{p_1}} + \frac{2}{E_s t_s} \right) \left(\frac{1}{\lambda_1 \tanh(\lambda_1 l_1)} + \frac{\sum_{k=2-d}^1 b_k \left(\frac{1}{E_{p_k} t_{p_k}} + \frac{2}{E_s t_s} \right)}{\left(\frac{1}{E_{p_1} t_{p_1}} + \frac{2}{E_s t_s} \right)} \right). \quad (5.261)$$

The upper limit for the stress intensity factor under plane stress conditions can be expressed for the partially disbonded tapered repair as

$$K_{\infty}^d = Y \sigma_o^d \sqrt{\pi \Lambda} \quad (5.262)$$

where Λ is given for disbonds not extending underneath the last step of the patch as

$$\Lambda = \frac{1}{\pi Y^2} \left(1 + \frac{E_s t_s}{2E_{p1} t_{p1}} \right) \left(\frac{1}{\lambda_1 F_f} + \frac{\sum_{k=2-d}^1 b_k \left(\frac{1}{E_{pk} t_{pk}} + \frac{2}{E_s t_s} \right)}{\left(\frac{1}{E_{p1} t_{p1}} + \frac{2}{E_s t_s} \right)} \right) \quad (5.263)$$

and for disbonds extending underneath the last step of the patch as

$$\Lambda = \frac{1}{\pi Y^2} \left(1 + \frac{E_s t_s}{2E_{p1} t_{p1}} \right) \left(\frac{1}{\lambda_1 \tanh(\lambda_1 l_1)} + \frac{\sum_{k=2-d}^1 b_k \left(\frac{1}{E_{pk} t_{pk}} + \frac{2}{E_s t_s} \right)}{\left(\frac{1}{E_{p1} t_{p1}} + \frac{2}{E_s t_s} \right)} \right). \quad (5.264)$$

A factor of $1/(1 - \nu_s^2)$ needs to be included in Λ in the case of plane strain conditions. Using K_∞^d and K_u^d as asymptotes, as suggested by the original Rose model, the stress intensity factor K_r^d and the corresponding energy release rate G_r^d for different crack lengths are then derived by interpolation using equation (5.220) and (5.221). It is noted that the accuracy of this analysis was improved by accounting for the shear stiffness of the adherends by using $\lambda_{\text{Shear-lag}}$ (equation (5.92)) instead of λ .

The results of this analysis scheme are presented for the same three cases as presented in Section 5.6.1. i.e.:

- Thermal residual stress loading ($\Delta T = -64.8^\circ\text{C}$)
- Remote stress loading ($\sigma_\infty = 40 \text{ MPa}$)
- Combined thermal residual stress and remote stress loading ($\Delta T = -64.8^\circ\text{C}$, $\sigma_\infty = 40 \text{ MPa}$)

The only difference between the uniform thickness patch analysis and the tapered patch analysis is the thickness change of the patch near the edge. All other material parameters remain unchanged.

The patch thickness is defined as:

$$t_{p_{1 \leq i \leq 7}} = 1.056 \text{ mm} - i 0.132 \text{ mm} \quad (5.265)$$

with step lengths of:

$$l_1 = 57 \text{ mm} \quad \text{and} \quad l_{2 \leq i \leq 7} = 3 \text{ mm} . \quad (5.266)$$

Note that the above given length and thickness data are indexed for a specimen with a disbond length not exceeding the first step. For larger disbonds, the index changes as presented in Figure 5.26. The stress intensity factors K_∞^d , K_u^d and K_r^d were determined for each of the loading conditions for the entire range of b from, i.e. from no disbond ($b=0$ mm) to completely disbonded ($b=75$ mm). Note that K_u^d is dependent on the chosen crack length and will therefore shift up or down for different crack lengths. The parameter K_r^d shows for which disbond length K_∞^d or K_u^d is dominant. In order to highlight the change in the behaviour of the stress intensity factors due to introducing a tapered edge, the results for the uniform thickness patch have been plotted in each graph. The observed behaviour of

Figure 5.27: Stress intensity factor dependencies on disbond length for a bonded tapered and uniform thickness patches under thermal residual stress loading ($\Delta T = -64.8^\circ\text{C}$)

Figure 5.28: Stress intensity factor dependencies on disbond length for bonded tapered and uniform thickness patches under remote stress loading ($\sigma_\infty = 40$ MPa)

partially disbonded uniform thickness patches is very similar to the results determined for partially disbonded tapered patches. It becomes evident that only a relatively small change in the stress intensity factor is introduced by designing a bonded repair with a tapered edge. The stress intensity factors for the tapered patch changes less rapidly when approaching a complete disbond (Figure 5.27 through Figure 5.29). The change in the stress intensity factors is quite small for a combined thermal residual stress and remote stress loading. This is due to an increase in K for the remote loaded situation and a decrease in K for the thermal residual stress loaded situation, thus leading to a partial cancellation of the individual effects.

As done previously, the results can also be shown in the form of patch efficiency plots (Figure 5.30) for the case of remote stress loading (noted RS) and combined thermal residual stress and remote stress loading (noted RS&TRS) using equation (5.223) as before. The small change in the efficiency leading to a gentler transition when approaching complete disbonding is also confined to the tapered region. Realistic disbond sizes measure typically $2b \approx 11$ mm with large disbonds reaching $2b \approx 27$ mm [13]. From these experimental

observations it will generally not be necessary to include the taper in the stress intensity analysis since using the same patch length with uniform thickness will provide predictions with sufficient accuracy as long as the disbond does not enter the tapered region. The efficiency of a bonded repair test specimen will still have a significant improvement over the unrepaired specimen due to the development of generally small disbonds with crack growth. Larger disbonds will most likely be the cause of very poor surface preparation or poor design.

If a more accurate solution is required for determining the stress intensity factor of a partially disbonded tapered patch, the analysis can be improved further by accounting for plastic strain in the adhesive. Note that in the case of partially disbonded tapered patches, the influence of the shear deformation of the adhesive towards the crack opening displacement is reduced from 100% of the total crack opening displacement for a fully bonded repair to only 10% for large disbonds in the repair using the outlined elastic analysis. Even for a significant increase in the shear displacement by accounting for plastic strain,

Figure 5.29: Stress intensity factor dependencies on disbond length for bonded tapered and uniform thickness patches under combined thermal residual stress and remote stress loading ($\Delta T = -64.8^\circ\text{C}$, $\sigma_\infty = 40 \text{ MPa}$)

Figure 5.30: Patch efficiency for partially disbonded patches with tapered edges under remote stress loading (RS) and combined thermal residual stress and remote stress loading (TRS&RS)

the change in the stress intensity factor will still be relatively small making the elastic solution an excellent first order approximation.

Based on these results, additional experimental work as well as higher order analytical work can be focused on uniform thickness patches rather than tapered patches to determine the overall performance of the method for estimating the stress intensity factor for partially disbonded patches in the crack vicinity.

5.7 Summary and Discussion

The influence of thermal residual stresses on the stress intensity factor in bonded repairs was theoretically investigated and presented in detail. A number of improvements over previous theoretical models have been established. One of the most significant achievements was the stress field estimation in the isotropic substrate based on a new concise solution for the classic fracture mechanics problem of a center crack in an infinite plate. This stress field

estimation is a useful tool for experimental work such as the placement of strain gauges near the crack tip. This model can also be used for the development of 'smart patches', where strain sensing devices are used to detect crack growth.

Another significant advance is the disbond model for bonded repairs based on the Rose model which gives a tool for estimating the impact of disbonds on the stress intensity factor. The model considers all disbond lengths up to full patch lengths and has also been extended for tapered patches. The model indicates the severity of thermal residual stresses on unrestrained specimens with relatively short disbonds. Special attention should therefore be brought to real applications where the effective coefficient of thermal expansion of the substrate is only slightly lower than the unrestrained specimen and quite different from the patch material. The effectiveness of the bonded repair with respect to the reduction in the stress intensity factor is also dependent on the ratio of the thermal residual stresses compared to the applied loading. The importance of such a theoretical tool is emphasized by the necessity to assess what disbond size around the crack is permissible prior to rejecting a repair. Significant experimental testing will be required in the future to validate this type of model and gain confidence in its performance.

Additionally a new theoretical approach has been presented for determining the stresses in tapered joints and reinforcements as well as stepped joints. This approach avoids the typical numerical solution scheme requiring specialized computer codes, which are not readily available. Any mathematical spreadsheet such as MathCad, MatLab or Excel can be used to determine the stress distribution in tapered or stepped joints with a limited number of steps which will be sufficient for many applications. Therefore, this theoretical approach provides engineers with a simple tool to understand the behaviour of adhesive joints better. Emphasis in this chapter was placed on providing an easy to follow derivation of the basic equations for the stress distribution in uniform thickness as well as tapered joints and reinforcements, thus giving the complete background rather than a 'black box'.

The presented models have been evaluated for the thermal residual stresses and strains present in the AMRL sandwich type specimen. Generally good agreement has been found with the experimental measurements within the limitations of the model such as in-plane bending. The results of a detailed finite element analysis representing the next step in this research work will reveal the strength and weaknesses of these models in more detail than it was possible with the experimental measurements using strain gauges.

Chapter 6

Finite Element Analysis of the AMRL Specimen

6.1 Introduction

The finite element analysis represents an important part of this research work as it provides better insight into the stress distribution of bonded repairs. A significant number of research papers have been published discussing finite element modeling approaches for bonded repairs. Well known is the work from Jones and Callinan [24, 52, 54], who developed a two-dimensional finite element formulation for the adhesive layer in 1979. The adhesive element formulation was based on the displacement between the substrate and patch accounting also for the transverse shear stiffness in the adherends. Coupling the adhesive stiffness matrix with the composite patch stiffness matrix allowed them to use this bonded pair in conjunction with standard finite element routines. The authors also suggested appropriate modeling techniques for commercial FEM packages to include the transverse shear stress of the adherends in two-dimensional models. This scheme of using adhesive joint elements which account for the transverse shear deformation of the adherend was also used by other research institutions such as the Cranfield Institute of Technology [19].

Since 1979 the increase in computing power has allowed the development of three-dimensional finite element modeling thus providing a more accurate results tool which can better ac-

count for the transverse shear stress through the adherends and adhesive [24]. Additionally the three-dimensional representation is desirable as it allows for a better comparison with the experimental thermal residual strain data measured at a number of locations through the thickness. A two-dimensional approach utilizing plate elements for the substrate and patch cannot model the effect of the transverse shear stress at different through thickness locations in the adherends properly. Plate elements generally assume zero shear stress at the top and bottom surface. A three-dimensional approach was chosen based on the inadequacy of the two-dimensional approach to predict the strains properly at the strain gauged locations. In this model, 20 noded brick elements were chosen as the main element type in order to get a linear stress prediction capability sufficient to model the transition zones with high stress gradients, such as the taper and crack region with less elements.

Submodeling can be a useful option for modeling details of bonded repairs, such as the crack region or the tapered section, depending on the available computing power. Such an approach was recently presented by the Y. Xiong and M.D. Raizenne [101]. Similarly, Callinan uses a number of models with refinements in different critical regions [24]. These approaches are certainly an excellent option when plastic deformations are investigated or the computing power is limited. A Pentium II-266 PC with 128 MB RAM and 12 GB of swap/storage space in conjunction with the finite element program MSC/Nastran for Windows was available for this research, thus allowing creation of a linear-elastic model of the AMRL sandwich type specimen in sufficient detail without the need for submodeling.

Due to the complexity of bonded repairs, a number of verification models were created to investigate each critical section in detail. These models allowed one to determine a sufficiently fine mesh ensuring confidence in the results while reducing the overall degrees of freedom as well as the required solution time for an entire model of the AMRL sandwich type specimen significantly.

The following material properties were employed for the verification models and the AMRL model:

Boron/epoxy 5521/4 Laminate:

$$E_{p_x} = E_{p_z} = 25 \text{ GPa}$$

$$E_{p_y} = 210 \text{ GPa}$$

$$G_{p_{xy}} = G_{p_{yz}} = 7.24 \text{ GPa}$$

$$G_{p_{xz}} = 1 \text{ GPa}$$

$$\nu_{p_{xy}} = 0.025$$

$$\nu_{p_{yz}} = 0.21$$

$$\nu_{p_{xz}} = 0.17$$

$$\alpha_{p_x} = \alpha_{p_z} = 25.87 \text{ } \mu\epsilon/^{\circ}\text{C}$$

$$\alpha_{p_y} = 4.61 \text{ } \mu\epsilon/^{\circ}\text{C}$$

2024-T3 Aluminum:

$$E_{s_x} = E_{s_y} = E_{s_z} = 72.4 \text{ GPa}$$

$$G_{s_{xy}} = G_{s_{yz}} = G_{s_{xz}} = 27.2 \text{ GPa}$$

$$\nu_{s_{xy}} = \nu_{s_{yz}} = \nu_{s_{xz}} = 0.33$$

$$\alpha_{s_x} = \alpha_{s_z} = 23.29 \text{ } \mu\epsilon/^{\circ}\text{C}$$

$$\alpha_{s_y} = 23.45 \text{ } \mu\epsilon/^{\circ}\text{C}$$

FM 73M:

$$E_{a_x} = E_{a_y} = E_{a_z} = 2.19 \text{ GPa}$$

$$G_{a_{xy}} = G_{a_{yz}} = G_{a_{xz}} = 844 \text{ MPa}$$

$$\nu_{a_{xy}} = \nu_{a_{yz}} = \nu_{a_{xz}} = 0.3$$

$$\alpha_{a_x} = \alpha_{a_y} = \alpha_{a_z} = 50 \text{ } \mu\epsilon/^{\circ}\text{C}$$

Note: The y-direction is aligned with the boron fiber and aluminum grain orientation.

6.2 Verification Models

In total, five verification models were created. The initial model representing a uniform thickness reinforcement was used to determine the mesh size in regions of high shear stress. This model served as a basic reference case due to the availability of a good theoretical model for comparison. A tapered reinforcement was chosen as a second model to optimize the required number of elements in the tapered region. Both normal and reversed stacking sequence were investigated to assess the importance of detailed representation of the actual structure.

Next, a model of the substrate in the cracked section was investigated. The focus of this model was placed on the stress intensity factor prediction. The mesh for the crack region had to be sufficiently fine along the crack face to predict the adhesive shear stress properly once the adhesive and patch were added to the model, i.e. the next step. The results of this model were then compared to the developed closed form approximation for the stress field around a patched crack. The AMRL specimen exhibits additional in-plane bending due to the partial patching across the specimen width which are not considered in the theoretical approach making this patched crack model an assessment tool for the theoretical crack tip stress field model.

The half circular geometry of the patch also complicated the finite element analysis. It was required to check if the aspect ratios found for the tapered reinforcement were acceptable for a circular patch due to the change in the material orientation with respect to the element orientation.

Finally the entire AMRL specimen was modeled using the mesh specifications derived by the different verification models. The number of elements required to get an accurate representation of the AMRL specimen was so large that an attempt to verify the mesh size in the full model was too time consuming and resources were quickly exhausted.

6.2.1 Constant Thickness Double-sided Reinforcement

This first model was intended to confirm the input data into MSC/Nastran as well as compare the results with the proven one-dimensional model of L.J. Hart-Smith. In order to avoid any stress concentration effects, the model had identical areas for the substrate, adhesive and patch. In-plane quarter symmetry as well as symmetry through the constant thickness model were used. The material thickness were selected to represent the inner section of the AMRL specimen: 3.175 mm thick aluminum face sheet, 0.25 mm thick adhesive layer and the 0.924 mm thick seven layers of boron/epoxy. This investigation was also useful for determining the required element size for the reinforced crack region in the AMRL specimen since the shear stress distribution away from the crack tip can be approximated by the shear stress distribution in a double-doubler joint. It should be noted that the shear stress distribution near the center of a double-doubler joint under thermal loading is identical to the shear stress distribution near the free edge of reinforcement considering a sufficiently long overlap length.

The aluminum, adhesive and patch were modeled initially using two elements through the thickness of each material. Five elements were used across the half-width of 12.5 mm. The specimen half-length was 75 mm. The element size was reduced from 5 mm in the center of the specimen with zero shear stress to 0.25 mm, 0.0625 mm and 0.03125 mm respectively along the length to the specimen edge with the highest shear stresses. The last four elements adjacent to the free edge had the same element length when an element length of 0.25 mm and 0.0625 mm was used. Eight elements with equal length were chosen for the model with an element length of 0.03125 mm. The chosen element lengths were based on fractions of the adhesive thickness between t_a and $t_a/8$ covering Callinan's recommendation of a maximum element length of $t_a/5$ near the edge [24]. A temperature change of -64.8°C was applied as a loading condition representing the cooling stage from the stress free temperature to room temperature.

Figure 6.1 shows the shear stress in the mid-plane of the adhesive along the centerline and close to the end of the specimen for the three FEM analyses and Hart-Smith's theoretical solution. The theoretical analysis shows excellent agreement up to 0.5 mm away from the free edge. This one-dimensional approach cannot account for the drop to zero shear stress at the free edge. The coarse FEM model shows the highest shear stress (21.4 MPa) 0.25 mm away from the free edge while the two finer meshes show a peak shear stress (20.0 MPa) at approximately 74.85 mm from the center of the specimen. It should be noted that the one-dimensional analysis predicts nearly the same value (19.7 MPa). Changing from an element length equivalent of 25% to 12.5% of the adhesive thickness results in only minor differences in the peak shear stress and its location.

Figure 6.1: Adhesive shear stress for a uniform thickness reinforcement using element lengths between t_a and $t_a/8$

The mesh with an element length of 12.5% of the adhesive thickness shows a shear stress of -5.0 MPa at the free edge while the finer mesh drops to -6.1 MPa. Both meshing schemes show a significant error which is not surprising considering the large stress gradient near the free edge. The shear stress prediction for an element length equivalent to 1/4 of the adhesive thickness shows the better result for this particular finite element verification model.

In addition, a model with a larger number of elements through the thickness was tested. The mesh with the smallest element length of $(12.5\%) t_a$ was used with 6 elements through the thickness for each material. Figure 6.2 shows the variation in shear stress results for 2 and 6 elements through the thickness.

Figure 6.2: Adhesive shear stress for a uniform thickness reinforcement with a different number of elements through the thickness

The position of the peak shear stress shifted by only 0.04 mm and the peak stress was reduced by 0.5 MPa. The shear stress at the free edge was less than 0.1 MPa, i.e. nearly the theoretical value of 0 MPa. The larger number of elements through the thickness would be preferred near the free edge, but it would increase the total number of elements since a transition should not be made in sections with high stress gradients. Based on the importance of predicting the peak shear stress and its location, it was decided to use an element length equivalent to 25% of the adhesive thickness. Generally, two elements through the thickness were used for each material except in the aluminum near the crack where four elements through the thickness were employed to improve the prediction of the crack opening.

6.2.2 Tapered Reinforcement with a Reversed Stacking Sequence

A tapered reinforcement was chosen as the second verification model. It was important to verify that a sufficient number of elements has been used for each step to determine the thermal residual strains in the adherends and the adhesive shear stress properly. The smallest number of elements was desirable to reduce the overall degrees of freedom for the AMRL sandwich type model. In the past, tapered edges have been modeled using tapered elements or a normal stacking sequence, not necessarily representing the real lay-up sequence [19, 24]. Specially tapered elements reduce the required number of elements but they do not represent the true structural details of the actual taper. As part of this verification model, a normal stacking sequence was compared to the actual reversed stacking sequence of the AMRL specimen (Figure 6.3) showing the importance of proper structural representation and design choices under given loading conditions.

Figure 6.3: Reverse and normal stacking sequence

The reversed stacking sequence is significantly more complex to model than the normal stacking sequence if the change in material orientation in the thickness direction is included. The size of the adhesive fillet was measured using a precured patch where the peel ply was not removed. The fillet shows up as a smooth region versus the typical fabric texture of the peel ply. The size of the fillet varies with respect to the location along the outer edge in the case of a half circular patch as used for the AMRL sandwich type specimen. The fillet length is approximately 1 to 1.25 mm for boron/epoxy if the taper is aligned with the fiber orientation, while it is only 0.25 mm where the fiber orientation is parallel

to the taper. The carrier fabric of the boron/epoxy prepreg causes a small fillet where the fibers are perpendicular to the taper direction while the larger fillet is a result of the stiffness of the boron fibers where the fibers are parallel to the taper orientation. In order to reduce the complexity and the required number of elements to account for the fillet properly, a triangular fillet with a length of 1 mm was chosen regardless of the location in the patch. This approach can be justified by the relatively long distance from the locations of the locally thicker adhesive to the regions of high shear stress in case where the fiber orientation is not parallel to the taper orientation, thus not affecting the results in a critical area.

Figure 6.4 through Figure 6.9 show results of the convergence study which was carried out for the reversed stacking sequence. The shear stress in the mid-plane of the adhesive as well as the thermal residual strains at the top and bottom surface of the patch and substrate were evaluated. The effect of smaller elements as well as more elements through the thickness are shown in these figures.

Figure 6.4: Transverse shear stress at the adhesive mid-plane in the tapered region for different FEM meshes

Typically seven elements through the thickness of the boron/epoxy were chosen to simplify the modeling while the adhesive and aluminum were represented by 2 elements through the thickness as shown in Figure 6.5.

Figure 6.5: Detailed view of the finite element mesh in the tapered section-reversed stacking

The number of elements was doubled to study the element number dependence through the thickness. These figures show the extreme cases of this convergence study for element length along the taper with the smallest element size of both 0.125 mm and 0.03125 mm at the ply drop-off. Furthermore, the number of elements was reduced along the taper for the

Figure 6.6: Thermal residual strains at the top surface of the patch in the tapered region for different FEM meshes

case of seven elements through the thickness of the boron/epoxy, two elements through the adhesive and two elements through the aluminum thickness. The smallest element length

Figure 6.7: Thermal residual strains at the bottom surface of the patch in the tapered region for different FEM meshes

was 0.125 mm in comparison to the curve for the model with twice as many elements through the thickness. The labels in the plots identify the number of elements for each component and the smallest element size at the ply drop-offs.

Figure 6.8: Thermal residual strains at the top surface of the substrate in the tapered region for different FEM meshes

Figure 6.9: Thermal residual strains at the bottom surface of the substrate in the tapered region for different FEM meshes

All curves in Figure 6.4 through Figure 6.9 show good agreement, especially the thermal residual strains at the aluminum substrate bottom (see Figure 6.9). Location A in Figure 6.6 shows the effect of reducing the number of elements along the taper in some non-critical regions. The smaller element length at the ply drop-offs emphasizes the singularities in the thermal residual strains in the boron/epoxy (see location B in Figure 6.7). The mesh with seven elements through the patch and two elements through the thickness of the adhesive as well as the substrate and an element length of 0.125 mm at the ply drop-off was chosen to model the tapered region of the AMRL specimen based on the good agreement with the finer meshes.

A comparison between the normal and reverse stacking sequence was carried out next. In addition, the results of the one-dimensional analysis are plotted. Figure 6.10 shows how well the one-dimensional analysis can predict the shear stresses in the mid-plane of the adhesive especially for the normal stacking sequence. The general trend is predicted properly, even for the reversed stacking sequence but the finite element analysis shows clearly the influence of local increases in the adhesive thickness. The peak adhesive shear stress is overpredicted

Figure 6.10: Transverse shear stress at the adhesive mid-plane for reversed and normal stacking sequence

by the one-dimensional analysis, which is caused by extending the adhesive layer past the last boron/epoxy layer. Figure 6.1 shows how well the one-dimensional theory predicts the peak adhesive shear stress if the patch and adhesive end at the same location. It should be noted that the mid-plane does not show the highest shear stress using the reversed stacking sequence. The singularities at the ply drop-offs lead to an increase in the shear stress close to the patch/adhesive interface.

Figure 6.11 and Figure 6.12 show the sensitivity of the thermal residual strains in the patch towards the choice of stacking sequence. A large discrepancy between the finite element results and the one-dimensional analysis is visible. The magnitude of the thermal residual strains predicted by the one-dimensional analysis compares only reasonably well to the strains determined in the FEM analysis at the patch/adhesive interface with a patch utilizing the normal stacking sequence.

The large spikes in the finite element results for the thermal residual strains using either stacking sequence are caused by the stress singularities at the ply drop-offs. This effect is not included in the theoretical one-dimensional analysis. A significant reduction of over

Figure 6.11: Thermal residual strains at the top surface of the patch for reversed and normal stacking sequence

500 $\mu\epsilon$ in the peak compressive strain through the thickness of the patch is evident for the reversed stacking sequence. In addition the location of the peak compressive strain also shifts from near the ply drop-off at the bottom of the patch to the end of the adhesive fillet at the top surface of the patch. For the normal stacking sequence for the patch lay-up,

Figure 6.12: Thermal residual strains at the bottom surface of the patch for reversed and normal stacking sequence

the reduction in compressive thermal residual strains through the thickness of the patch is even more significant (up to $700\mu\epsilon$) for this particular loading case.

Figure 6.13 and Figure 6.14 show the results for the thermal residual strains in the substrate. The effect of the stacking sequence on the aluminum substrate is not significant especially at the bottom side of the 3.175 mm thick aluminum sheet where the deviation in thermal residual strains between the normal and reversed stacking sequence is less than $15\mu\epsilon$. The result of the one-dimensional analysis is also quite close to the FEM analysis with generally less than $30\mu\epsilon$ variation regardless of the stacking sequence at this location. It should also be noted that the thermal residual stresses in the substrate don't drop to zero at the patch edge due to the extended adhesive layer.

The effect of the reversed stacking sequence on the thermal residual strains in the substrate are more pronounced at the substrate/adhesive interface. The thermal residual strains are reduced in the regions of increased adhesive thickness while they are in excellent agreement in the mid-section of a tapered step.

Figure 6.13: Thermal residual strain at the top surface of the substrate for reversed and normal stacking sequence

Figure 6.14: Thermal residual strain at the bottom surface of the substrate for reversed and normal stacking sequence

One of the most important findings are the high peel stresses in the adhesive due to the reversed stacking sequence (see Figure 6.15). The main cause is the orientation change of the upper plies passing over a ply drop-off resulting in peel forces. The peel stresses develop at different locations depending on tension or compression in the patch. The peak peel stress in the adhesive mid-plane occurs at the ply drop off in case of compressive stresses in the

Figure 6.15: Peel stresses in the adhesive mid-plane for the reversed stacking sequence

patch (see location **A** in Figure 6.16) while for tensile stresses the peak occurs at location **B**. Tensile loading of the bonded repair will therefore generally change the location and reduce the magnitude of the peel stresses introduced by the thermal residual stresses, while compressive loading will reverse the peel stresses and thus may contribute to the failure of the bonded repair. A fractographic investigation by the National Research Council showed a change in failure mode at the ply drop using a reversed stacking sequence. This failure mode change was not present when using a normal stacking sequence [37]. Based on the findings in the finite element analysis, peel stresses can be associated with this failure mode change.

Figure 6.16: Critical location for peel stresses in the adhesive mid-plane for the reversed stacking sequence

Two conclusions can be drawn from the prediction of high peel stresses: The stresses and strains in a tapered patch with a reversed stacking sequence should not be analysed by approximating the taper using a normal stacking sequence nor a tapered element. Additionally special attention should be paid to the peel stress in the tapered region when using a reversed stacking sequence under compressive loading of the bonded repair.

The stresses in the substrate and in the adhesive are generally the main concern for bonded repairs. Most crack patching applications arise from cracking in sections loaded in tension. The one-dimensional analysis predicts the thermal residual strains for the aluminum substrate and the adhesive shear stress very well. Small discrepancies arise due to specific de-

sign choices, such as extending the adhesive layer beyond the patch. Patch design choices such as the stacking sequence become far more important for reinforcements or cracked sections of components loaded in compression based on the high peel stresses found in the taper using a reversed stacking sequence.

6.2.3 Crack Mesh Verification

The crack mesh verification model was the first step in generating an appropriate finite element model for the cracked region of the AMRL specimen. This model deals only with the metal substrate due to the availability of a closed form solution allowing the verification of the finite element results rather than a convergence study. A large number of crack meshes were investigated. One of the main difficulties in this investigation was the small element length requirement perpendicular to the crack in order to allow for the proper determination of the adhesive shear stresses and thermal residual stresses in the adherends. This requirement was complicated by the lack of a crack tip element in MSC/Nastran for Windows. For this reason three approaches were investigated. First, a finite element mesh was generated using only parabolic elements. In a second approach, additional bulk data cards were submitted as part of the analysis sequence creating MSC/Nastran CRAC3D crack tip elements which are not supported in the user interface. The third approach utilized the typical quarter point method where the mid-side nodes of the elements surrounding the crack tip are moved to the quarter point location.

This third approach was ruled out due to the number of warnings issued by MSC/Nastran with respect to the determination of the stresses, i.e. crack tip stress reached 10000 GPa. The MSC/Nastran user guide noted that this approach might also lead to an early abortion of the calculation. Another significant disadvantage of moving the mid-side is the connec-

tivity to the bonded patch. If the mid-side node in the patch is moved to the quarter point location, the stress distribution will be highly distorted. The other option is to make a transition in the aluminum or adhesive which will lead to uncertain answers due to the unknown element behaviour. Significant verification work would be required to study such element behaviour.

The advantage of using the regular parabolic elements is a better spatial stress distribution. The shape of the constant stress contour lines are closer to the theoretical estimation especially for elements with high through thickness aspect ratios (25:1). In order to get a good spatial distribution with the MSC/Nastran CRAC3D elements, a through thickness aspect ratio of 6:1 was required, thus using significantly more elements through the thickness. The use of the CRAC3D elements is significantly more complex and the implementation is time consuming. In addition, the elements cannot be visually checked and results cannot be displayed although they were available in the .f06 output file. It should also be noted that the computed stress intensity factor by MSC/NASTRAN varied by approximately 13% depending on the use of the full modeling option or the symmetry option. The listed element stresses for these two modeling options varied as well based on different natural coordinates. The stresses, strains and displacements for the surrounding elements were compared and found identical despite the differences in the output of the crack tip element. The stress intensity predictions given by MSC/NASTRAN seemed rather low in comparison to extrapolation of stress intensity factor plots based on the crack opening displacement. The theoretical stress intensity factor for this particular verification model can be calculated using [23]:

$$K_I = Y\sigma\sqrt{\pi a} \quad (6.1)$$

where

$$Y = 1.12 - 0.23 \left(\frac{a}{W} \right) + 10.55 \left(\frac{a}{W} \right)^2 - 21.71 \left(\frac{a}{W} \right)^3 + 30.38 \left(\frac{a}{W} \right)^4 \quad (6.2)$$

and was determined to be $K_I = 21.12 \text{ MPa}\sqrt{\text{m}}$ for a specimen containing a 20 mm long crack and a width of 50 mm under 40 MPa of remote tensile loading. This theoretical value was used to assess the quality of the finite element crack tip mesh by comparing it to the stress intensity factor calculated from the finite element results. The finite element model length was chosen to be 100 mm. It was decided to use a sheet thickness of 1.5875 mm to achieve plane stress conditions under the chosen remote loading. The choice of plate thickness also improved the aspect ratio of the elements in the crack tip region to approximately 1:6 by utilizing symmetry through the thickness and with respect to the crack. Two elements were placed to represent the half sheet thickness. The crack opening displacements determined by the finite element analysis were used to calculate the stress intensity factor as given by [25]:

$$K_{I_{FEM}} = \sqrt{\frac{2\pi}{r} \frac{E}{4} v} \quad (6.3)$$

Note that the equation for $K_{I_{FEM}}$ is shown for plane stress conditions. Figure 6.17 shows the stress intensity factors calculated for the finite element crack opening displacements utilizing the different crack tip modeling approaches. The results for the stress intensity

Figure 6.17: Stress intensity factors for different crack tip elements

factor based on the crack opening displacements are within 1% of each other for the different crack tip modeling approaches for locations which are more than 10% of the crack length away from the crack tip. Larger deviations occur closer to the crack tip. The MSC/Nastran CRAC3D element shows a spike after the second element from the crack tip. The reason for this spike is possibly the different displacement formulation used by MSC/Nastran for the crack tip element in comparison to the parabolic elements. The quarter point approach results also in a spike in the stress intensity factor at the interface between the crack tip element and the adjacent parabolic elements. The model with only parabolic elements shows a gentle decrease in the stress intensity factor close to the crack tip.

Chan et.al.'s [25] approach of using a linear extrapolation is applied rather than calculating $K_{I_{FEM}}$ just from the crack tip elements. Good linear sections of the stress intensity factor versus r are found from approximately 15% to 50% of the crack length. The extrapolated stress intensity factors are within 0.5% of the theoretical estimation for the three modeling procedures representing excellent results. This can be largely contributed to the small element size of approximately 0.3% of the crack length in the crack tip vicinity.

The stress in the loading direction was investigated as a second parameter to check the performance of the crack tip modeling. Figure 6.18 shows the stress in the loading direction (σ_y) ahead of the crack along the through thickness center line. Negative coordinates refer in this case to the cracked section. The nodal stress for the CRAC3D element are not provided in the post-processor thereby not allowing nodal stress averaging with the adjacent elements. Generally this is not a concern with a fine finite element mesh, but the large stress gradient present at a crack tip it will distort the results. Therefore the nodal stresses in the elements next to the crack tip element were not displayed.

It is apparent that the stresses for the model with the CRAC3D element are higher ahead of the crack tip than the results given by the two other modeling approaches. It should

Figure 6.18: Stress perpendicular to the crack (σ_y) for different crack tip elements

be noted that this difference only becomes evident for stresses exceeding the yield stress of the material (310 MPa) very close to the crack tip (≈ 0.35 mm). The modeling approach using the quarter point method and no crack tip element have excellent agreement ahead of the crack tip except that the stresses for the quarter node element approach generate extremely large values (> 10000 GPa) in the crack tip element while the parabolic element adjacent to the crack tip reaches nine times the yield stress of the material.

A point of verification is that the stress along the crack face should be zero. All modeling approaches reach this value within 1-1.5% of the total crack length from the crack tip. The parabolic element shows the highest over shoot in the element adjacent to the crack tip, which is not surprising considering the steep gradient and the element formulation.

It should be noted that the crack tip displacements the in x-direction (ahead of the crack tip) using the CRAC3D element were not satisfactory. The crack front through the thickness appeared jagged and possibly unreliable.

Having investigated these different modeling approaches, it was decided to use parabolic elements at the crack tip. All three methods can be used to determine the stress intensity factor quite accurately utilizing Chan's [25] extrapolation technique. The quarter node method was rejected due to the difficulties in connectivity to the patch. The CRAC3D element approach showed generally good results but required a tedious input as well as a larger number of elements through the thickness to avoid distortion of the in-plane stress field. Additionally the jagged displacements found along the crack front were unrealistic. The behaviour of the parabolic elements was quite good due to the small element size in the crack tip vicinity. The parabolic element also handled higher aspect ratios through the thickness better. The only short fall was the approximately 10% overshoot in comparison to the peak stress along the crack front. Having established a good mesh scheme using parabolic elements for the crack region of the substrate, the adhesive layer and the patch were added to the model as shown in Figure 6.19.

6.2.4 Patched Crack Model

The patched crack verification model was chosen to evaluate the theoretical crack tip stress field model for bonded repairs. The previous crack mesh verification model provided confidence in a proper mesh for the substrate. A constant thickness patch of 0.924 mm as well as the 0.25 mm thick adhesive layer were added to the crack tip verification model after increasing the thickness of the substrate to 3.175 mm thereby representing the center section of AMRL sandwich type specimen. The honeycomb used in the AMRL specimen is neglected since the theoretical model doesn't account for it. A uniform thickness patch over the entire substrate was chosen to avoid any additional bending due to partial coverage of the patch across the width of the model. The model length of 100 mm was chosen long enough to avoid an interaction between the load transfer zone at the free edge of the model and the high stress region around the crack tip. No mesh refinement was carried out near

the free edge due to the focus on the stress distribution in the vicinity of the crack tip. Two elements were used through the thickness of the adhesive, while four elements through the thickness of the seven ply patch were applied. The first layer of elements was used to represent the boron/epoxy layer next to the adhesive. The three remaining element layers for the patch always represented two boron/epoxy plies each. The mesh for the aluminum substrate was left identical to the previous verification model. Figure 6.19 shows the finite element mesh used in this investigation:

One of the most difficult decisions to be made in this finite element approach was the modeling of the adhesive along and across the crack. Modeling the adhesive continuously across the crack implies that no crack opening occurs at the adhesive/substrate interface based on no crack opening in the initial unstressed state. In order to model the adhesive continuously across the crack using a discrete modeling approach such as finite elements, a small crack opening displacement must be present and the adhesive must be extended past the crack face towards the symmetry line of the repair. A plasticity analysis can then be carried out to get a proper result for the crack opening displacement in the substrate.

The approach taken in this research work is based on a discontinuous adhesive layer across the crack. Its advantage is that the analysis can be carried out using a linear-elastic

Figure 6.19: Finite element mesh for the patched crack verification model

approach thus reducing the computational requirements. Additionally it presents an upper bound approach for the crack opening displacement. Failure of the adhesive in tension on top of the crack is a realistic possibility. Planned fractographic investigations of the region around the crack after fatigue failure will hopefully give a better indication about the true failure sequence. Unfortunately, papers dealing with the finite element modeling of this critical region have not discussed this problem in detail. The approach taken in this research work is similar to finite element models of bonded repairs developed by other researchers using spring elements to represent the adhesive [19]. A spring model implies that the adhesive is discontinuous across the crack. These models have been successfully used to predict the crack growth rate in bonded repairs.

The finite element stress distributions for σ_x , σ_y and τ_{xy} along $\theta = 0^\circ$ and 90° are presented and compared to the closed form theoretical analysis in Figure 6.20 through Figure 6.25. All graphs show the theoretical results for plane stress and plane strain conditions. The stress intensity factor caused by the thermal residual stresses alone will generally lead to plane strain conditions for the crack analysis while combined with other external loading plane stress conditions at the crack tip are common. The finite element results for the substrate are displayed at the bottom and top surface of the aluminum face-sheet. The substrate bottom surface is the critical case due to the larger crack opening displacement. The adhesive restricts the crack opening displacement at the top surface of the substrate to a significant degree. The difference in the crack opening displacement between the top and bottom surface of one face-sheet at the edge of the AMRL specimen is approximately 22% which compares well with the 19% given by Baker for the FEM analysis of a simple double-doubler joint [18].

Figure 6.20 shows excellent agreement with the theoretical stresses in the loading direction along $\theta = 0^\circ$ for plane strain conditions and the finite element results for the substrate bottom. Note that the substrate bottom coincides with the through thickness symmetry

plane due to neglecting the honeycomb. The difference between the fracture mechanics analysis for plane stress and plane strain conditions is approx. 5% for stresses below 150 MPa. Significant difference is visible between the finite element results for the top and bottom surface of the substrate. Good agreement between the closed form theoretical analysis and the FEM analysis for the symmetry plane can be attributed to accounting for the shear-lag in the adherends. The discrepancy in the stresses at locations more than 17 mm from the crack tip is due to modeling an edge crack in the FEM analysis and using the center crack assumption in the theoretical approach.

The estimation of the stress intensity factor for the substrate using the finite element results imposes some difficulties. Typically the crack opening displacement is used to estimate the stress intensity factor for a cracked sheet. The questions that arises is, can this approach be applied to bonded repairs? Inspecting the stress field behind the crack tip ($\theta > 90^\circ$) for a bonded repair reveals differences in comparison to the typical stress field of a crack in an unreinforced sheet due to the load transfer into the patch rather than through the remaining

Figure 6.20: Stress in loading direction (σ_y) for the patched substrate along $\theta = 0^\circ$

uncracked section. This finding makes the application of the crack opening displacement method based on an unreinforced sheet questionable. On the other hand, the stress field observed ahead of the crack tip for a bonded repair shows the typical stress distribution. It seems therefore more appropriate to use a stress extrapolation method to determine the stress intensity factor for different through thickness locations in the substrate. This approach is also supported due to the stresses ahead of the crack tip being the driving force for the crack rather than the crack opening displacement. The stress extrapolation method can be based on the usual one term approximation if data points are selected close to the crack tip. The corresponding stress intensity factor for a given stress in loading direction at a distance r from the crack tip along $\theta = 0^\circ$ can be determined using:

$$K_I = \sqrt{2\pi r} \sigma_y \quad (6.4)$$

Alternatively the correct elastic solution for a center crack in an infinite plate can be used to determine the stress intensity factor. In addition to the local stress σ_y , the remote stress σ_o has to be provided:

$$K_I = \sigma_o \sqrt{\pi r \left(\frac{\sigma_y^2 - \sigma_o^2}{\sigma_o^2} + \frac{\sigma_y}{\sigma_o^2} \sqrt{\sigma_y^2 - \sigma_o^2} \right)} \quad (6.5)$$

The extrapolation of both equations yield stress intensity factors which are within 0.25% of each other. Using equation (6.5), the stress intensity factor is determined as $5.02 \text{ MPa}\sqrt{\text{m}}$ for the symmetry plane of the substrate and $2.72 \text{ MPa}\sqrt{\text{m}}$ for the top surface. The determination of the stress intensity factor using the crack opening displacement method yields $5.25 \text{ MPa}\sqrt{\text{m}}$ for the symmetry plane and $4.61 \text{ MPa}\sqrt{\text{m}}$ for the top surface of the substrate. Comparing the results between these extrapolation methods, a large discrepancy (70%) is present at the top surface, while the stress intensity factor matches quite well for the symmetry plane of the substrate (5%). This clearly shows the problem using the existing stress and displacement extrapolation methods for the stress intensity factor. One option for the determination of the average stress intensity factor for the substrate could be established using the change in strain energy with an increase in crack length. Using Rose's proof,

the stress intensity factor can be calculated applying the standard relationship between the energy release rate and the stress intensity factor. Unfortunately the maximum stress intensity factor at the symmetry plane of the substrate cannot be established. More research is needed to identify the proper determination methods of the stress intensity factor for bonded repairs using finite element results.

Additionally these stress intensity factors can be compared to Rose's one-dimensional model. Assuming plane strain conditions, the upper bound K_{∞} yields $5.11 \text{ MPa}\sqrt{\text{m}}$. The stress intensity factor K_r can be calculated as $4.66 \text{ MPa}\sqrt{\text{m}}$ by applying the interpolation scheme between K_u and K_{∞} . It is at first surprising that K_r is approximately 7% below the highest stress intensity factor at the symmetry plane of the substrate. Verification work carried out by an AMRL researcher [18] shows similar findings. Their FEM analysis of double-doubler joints shows higher opening displacements of up to 13% between the two substrate pieces for comparison to the one-dimensional analysis. The opening displacement is directly related to the stress intensity factor K_{∞} in the Rose model. This discrepancy between the one-dimensional analysis and the FEM analysis can be attributed to the three-dimensional effects which are accounted for in the solid finite element model. In addition, the shear-lag correction for the adherends in Rose's model uses a linear relationship. In reality, a quadratic approach is more likely thus introducing some error in the theoretical analysis.

The stress in loading direction along $\theta = 90^\circ$ is quite well predicted close to the crack tip ($r < 2.5 \text{ mm}$) and shows good agreement with the finite element results for the symmetry plane of the substrate (see Figure 6.21). The FEM results show higher stresses for $r > 2.5 \text{ mm}$. This can be explained by the load transfer into the patch close to the crack rather than redistribution through the remaining section, and shows clearly the limitations of the developed theoretical approach. The theoretical approach provides good results ahead of the crack tip but the model does not account for the local load transfer into the patch.

Figure 6.21: Stress perpendicular to crack (σ_y) in the patched substrate for $\theta = 90^\circ$

Thus care should be exercised when using the model close to the load transfer zone. Note that the finite element results are only shown for $r \leq 12.5$ mm due to the lack of nodes along that line. Unfortunately MSC/Nastran for Windows does not offer postprocessing along arbitrary lines.

The stresses in the x-direction are also quite well predicted by the closed form analysis especially close to the crack tip as shown in Figure 6.22 and Figure 6.23. It should be noted that the finite element analysis accounts for the small difference in the coefficient of thermal expansion between the patch and substrate in the x-direction. This influence was neglected in the theoretical analysis. The predicted stress parallel to the crack using the theoretical model is in-between the FEM results for the top and bottom surface of the substrate along $\theta = 0^\circ$ and $\theta = 90^\circ$ and close to the crack tip. The drop into compressive stresses for σ_x along $\theta = 90^\circ$ near the crack tip is also properly indicated. The remote stresses are also in good agreement. The finite element analysis generally predicts higher stresses for $2 \text{ mm} < r < 10 \text{ mm}$. The stress σ_x along the crack face cannot really be

Figure 6.22: Stress parallel to the crack (σ_x) in the patched substrate for $\theta = 0^\circ$

predicted using the theoretical model. The theoretical model is based on a center crack and an effective crack length for the bonded repair which is only a fraction of the real crack.

Figure 6.23: Stress parallel to the crack (σ_x) in the patched substrate for $\theta = 90^\circ$

The theoretical prediction for the in-plane shear stresses, τ_{xy} , show excellent agreement with the FEM analysis. The shear stress along $\theta = 0^\circ$ must be zero. Figure 6.24 shows clearly the error in the FEM analysis which must be expected at the singularity considering the use of parabolic elements and the high stress gradient near the crack tip. Figure 6.25 shows again just slightly better agreement between FEM results for the substrate bottom and the theoretical approach.

Figure 6.24: Shear stress (τ_{xy}) in the patched substrate for $\theta = 0^\circ$

In general the developed closed form theoretical model for the stress field in a bonded repair gives a good estimate for the stresses in the substrate considering the limitations of the model. The most important limitation is imposed by the load transfer across the crack from the substrate into the patch. The load transfer itself is accounted for in the reduction of the stress intensity factor but it is not accounted for in the stress field prediction. Thus the stress field estimate especially for the stress in loading direction, is certainly not well applicable for $\theta > 80^\circ$ to 90° due to the local load transfer as well as modeling an effective crack length instead of the real crack length with the restraint of the patch.

Figure 6.25: Shear stress (τ_{xy}) in the patched substrate for $\theta = 90^\circ$

The stress field ahead of the crack tip is predicted quite well although it doesn't account for the localized load transfer into the patch around the stress singularity. In this case the load transfer is linked to the stress increase due to the singularity thus limiting the distortion of the stress field as predicted for a cracked sheet without a reinforcement. The reduction in the stress intensity factor due to the load transfer caused by the stress rise ahead of the crack tip is accounted for in Rose's model. The finite element results also showed that the transverse shear stress ahead of the crack tip is quite low i.e. less than 1 MPa.

The through thickness variation of the in-plane stresses is not addressed in the stress field approach. The Rose model accounts for the shear-lag in the adherends using a linear relationship thus showing generally better agreement with the stresses at the substrate bottom, which is the through thickness symmetry plane of the model. Some improvement should be expected if a higher order approach is taken for the transverse shear distribution. A significant improvement in the stress field prediction should be achieved if proper stress functions are used which address the interaction with the patch. The developed theoretical

model can be used for preliminary design work and experimental work such as the placement of strain gauges.

6.2.5 Half Circular Patch Model

The half circular patch model is the last requirement prior to modeling the sandwich type AMRL specimen. The objective is the establishment of the number of elements needed along the perimeter of the patch. The model consists of a two ply patch with one step using the reversed stacking sequence. Only two plies were chosen to reduce the modeling time. The substrate is modeled with the same in-plane dimensions as the longest ply, i.e. 75 mm. No crack was introduced in this model and symmetry conditions were applied to reduce the model to a 90° patched section. To check for convergence, runs of 8, 16 and 32 elements were used along the 90° patched section.

Figure 6.26 shows the mesh used for the half circular patch verification model with 32 elements along the circular perimeter. Figure 6.27 shows the through thickness meshing

Figure 6.26: FEM mesh for circular patch verification model

scheme at the tapered edge. The two bottom element layers represent the aluminum. The adjacent two element layers are used to model the adhesive. In addition, the fillet in the tapered step has adhesive material characteristics. The top element layer to the right of the ply drop-off and the two top layers left of the ply drop-off represent the boron/epoxy composite layers. The adhesive fillet can be better seen in Figure 6.34.

Figure 6.27: Detailed view of the tapered edge in the circular patch verification model

The transverse shear stress τ_{yz} at a distance of 0.125 mm away from the free edge was chosen as the convergence parameter. Figure 6.28 shows the results of this convergence study. The highest shear stress occurs for $\theta = 90^\circ$ as expected, while it is nearly zero for $\theta = 0^\circ$. Less than a 0.2% difference in the shear stress prediction is visible between 20 and 90 degrees using different number of elements along the perimeter. The error increases to approximately 1% for smaller angles which is an excellent result.

Based on these results, 20 elements were chosen along the perimeter of the AMRL specimen patch. In order to account better for free edge effects, the element size was reduced for four elements near the free edge. The same refinement was used along the symmetry line at $y = 0$ mm to ensure accurate results for the outer taper step with the shortest fiber length. Element sizes 13% larger than the presented case of 16 elements for the 90° section were chosen for the section between $6^\circ < \theta < 78^\circ$.

Figure 6.28: Shear stress (τ_{yz}) at $R = 74.875$ mm of a circular patch with a radius of $R = 75$ mm for different angles θ

6.3 AMRL Sandwich Type Specimen Model

By utilizing the results of the previously discussed refinement models, a finite element model for the AMRL specimen can now be created without the need of further mesh refinements. Figure 6.29 shows the final mesh for the AMRL sandwich type specimen using double-symmetry. A representation of the honeycomb was added in order to get a better comparison with the experimental data. Although the honeycomb limits the out-of-plane bending, it does not eliminate it. The honeycomb was modeled as an isotropic material with an elastic modulus equivalent to the compressive modulus of the honeycomb in thickness direction (1.517 GPa [48]) and a coefficient of thermal expansion identical to the CTE of the aluminum face-sheet. Although the assumed in-plane elastic modulus of the honeycomb is only 2% of the aluminum face-sheet elastic modulus, it is still somewhat higher than the true moduli. An attempt to use significantly lower values for the in-plane modulus

Figure 6.29: FEM mesh for AMRL sandwich type specimen

of the honeycomb caused numerical problems, i.e. pivot ratio of the stiffness matrix was not within the specifications of the finite element solver. Accounting for the honeycomb in the finite element model leads to changes of up to 9% in the thermal residual strains at the strain gauged locations. It should be noted that the maximum out-of-plane deflection does not occur in the center of a patch but near the tapered region for this particular honeycomb. The AMRL sandwich type specimen is modeled using 18806 elements with 254670 degrees of freedom.

The purpose of the finite element analysis was to provide a better understanding of the detailed stress and strain distributions while the experimental work gave a good foundation for the analysis assumptions. This goal has been fulfilled as illustrated by the following results. Figure 6.30 and Figure 6.31 show the thermal residual strain fields for the mid-plane of the substrate and the top layer of the boron/epoxy patch in longitudinal direction giving examples for the complexity of the stress and strain fields in this particular application. Additionally, the transverse shear stress in the mid-plane of the adhesive caused by the mismatch in the coefficients of thermal expansion in the adherends is shown in Figure 6.32 since it is one of the important design variables. The peel stresses caused by the thermal loading are presented due to their importance with respect to disbonding. The top surface of the adhesive layer was chosen for this representation based on the magnitude of the peel stress at this interface. Finite element results are only presented for the cooling cycle from the effective stress free temperature to ambient temperature, i.e. $\Delta T = 64.8^\circ$, since the stress and strain distributions don't change assuming the coefficient of thermal expansion remains constant. The measured changes in these material coefficients with temperature are not large enough to cause any significant changes in the stress and strain distributions.

Figure 6.30 shows the thermal residual strains parallel to the fiber orientation in the mid-plane of the aluminum face-sheet. The stress field ahead of the crack tip shows the typical stress field solution of a center crack in an infinite plate. The stress field gets more dis-

Figure 6.30: Thermal residual strains parallel to the fiber orientation (y-direction) for the AMRL sandwich type specimen in $[\mu\epsilon]$ at the mid-plane of the substrate

torted with increasing distance from the crack tip as shown by the $500 \mu\epsilon$ contour line in Figure 6.30. The load transfer from the substrate into the patch near the crack face occurs approximately parallel to the crack and does not show the typical distribution of an unpatched crack where the load is carried by the remaining section ahead of the crack. The bending influence of the specimen is clearly visible for $x > 117$ mm. An increase in the strains outside the patched region is found along the center line of the specimen although its magnitude is not critical.

Figure 6.31: Thermal residual strains parallel to the fiber orientation (y-direction) for the AMRL sandwich type specimen in $[\mu\epsilon]$ in the top-surface of the patch

The thermal residual strain distribution for the top layer of the boron/epoxy patch as presented in Figure 6.31 shows high strain gradients in the tapered region and near the crack. The thermal residual strain distribution in the tapered section along the y-axis in Figure 6.31 is very similar to the results of the tapered reinforcement verification model as presented in Figure 6.11. Moving from the y-axis to the x-axis, the thermal residual strain distribution changes initially just slightly but past 45° more significant changes occur in the

tapered region. The high gradients are significantly reduced as the symmetry line of the patch is approached. Due to the large element size in tangential direction, some inaccuracy in the tangential extent of contour lines should be expected. The convergence study for the circular patch showed that the chosen element size leads only to relatively small changes in the transverse shear stress distribution near the free edge.

Large thermal residual strain changes can be identified near the crack face, where the load is transferred from the substrate into the patch. The compressive thermal residual strains decrease from approximately $-700 \mu\epsilon$ ($y = 22.5 \text{ mm}$) to $-200 \mu\epsilon$ ($y = 8 \text{ mm}$). Within 2 mm of the crack face, an additional increase in compressive stresses can be found at the top surface of the patch. Note that at the bottom surface of the patch, tensile strains are present leading to a high through thickness strain gradient in the patch over the crack.

Figure 6.32 shows the transverse shear stress τ_{yz} in the mid-plane of the adhesive. The load transfer zone from the substrate into the patch extends approximately 30 mm perpendicular from the crack face. The transverse shear stress drops to near zero at the crack face fulfilling the boundary condition. It is important to note that the transverse shear stress ahead of the crack tip and up to θ angles of 80° measured from the crack tip against the x-axis is nearly zero, i.e. below 1 MPa for this particular load case. This substantiates the assumption of negligible interaction between the patch and substrate ahead of the crack made for the estimation of the stress field. The increase in transverse shear stresses past approximately 80° shows the limitations in the stress field approach in the cracked section since the local load transfer is not accounted for in the theoretical approach. The transverse shear stress increases up to 15.5 MPa in the tapered section near the free edge of the patch, as also shown in Figure 6.10 for the tapered reinforcement verification model. Note that very low transverse shear stresses are present in the 2.5 mm section extending beyond the patch. The overall highest transverse shear stress is found close to the crack faces at the free edge

Figure 6.32: Transverse shear stress τ_{yz} for the AMRL sandwich type specimen in [MPa] in the mid-plane of the adhesive

of the specimen. The transverse shear stress reaches nearly -20 MPa in the mid-plane of the adhesive at this location leading to the typical disbond pattern as presented by Baker [13].

One of the most important findings of the finite element analysis were the high tensile peel stresses in the adhesive at each tapered step caused by the reversed stacking sequence for the patch. The most critical locations are at the interface between the adhesive and the

Figure 6.33: Adhesive peel stress σ_z for the AMRL sandwich type specimen in [MPa] at the adhesive/patch interface

patch. Figure 6.33 shows the peel stress distribution and the critical locations in red on the top surface of the adhesive layer. Note that this view includes the adhesive fillets at the taper steps. A detailed isometric view of two tapered steps along the free edge of the specimen ($R = 66$ mm and $R = 69$ mm) is shown in Figure 6.34. This view also shows the modeling of the adhesive fillets well. Note that only one element is used through the

Figure 6.34: Detailed view of the adhesive peel stress σ_z for the AMRL sandwich type specimen in [MPa] for the ply drop-offs at $R = 66$ mm and $R = 69$ mm

thickness. Increasing the number of elements does not necessarily improve the accuracy since additional elements in the adjacent composite may lead to a misrepresentation. A more accurate determination of the stress and strain field would require modeling the actual fibers and matrix of the composite material. The highest peel stresses in the taper are found at the tip of the adhesive fillet in the current representation (see Figure 6.34). The overall

highest peel stresses are found near the crack face at the interface between the adhesive and patch.

Figure 6.35: Fractographic investigation patch/adhesive interface at the ply drop-off

The high peel stresses for the reversed stacking sequence at the steps in the tapered section could be the cause for the change in failure mode at a ply drop-off documented by the National Research Council of Canada [37]. Figure 6.35 [37] shows the finding in the fractographic investigation. This particular repair was subjected to high compressive loading which increases the peel stresses at the ply drop-off in addition to the peel stresses caused by the elevated temperature cure. Most bonded repairs are subjected to tensile loading which acts to reduce the peel stresses caused by the thermal residual loading, thus reducing the risk of failure initiation at this site. Based on this finding, it should be clear that the design of the tapered region of composite patches must account for the stacking sequence induced stresses at the tapered edge under combined thermal residual and applied loading.

Table 6.1 shows the finite element results for the thermal residual strains at the strain gauged locations. The finite element results were obtained by averaging over the strain

gauge area using the contour plots for the thermal residual strains. By zooming in on the strain gauge area or sections of it, contour lines were plotted for every $2 \mu\epsilon$ (or less) thereby allowing the determination the average thermal residual strains quite accurately. An error of less than $20 \mu\epsilon$ is estimated for locations where the strain distribution varies in a highly non-linear fashion, e.g. locations A and G (see Figure 4.15). An error of less than $5 \mu\epsilon$ is estimated for locations with nearly linear thermal residual strain gradients, e.g. locations B-F and H.

Generally good agreement between the finite element results and the experimental measurements has been achieved considering the assumptions made for the numerical modeling as well as the limitations of the experimental measurements. The two important assumptions made in the finite element analysis are 1.) a uniform thickness adhesive layer exists under the patch except for the fillets in the tapered section and 2.) zero thickness of the strain gauges. It is known that some adhesive bleeding occurred at the free edge of the bonded repair, through the crack and starter notch during the curing process, however this is not believed to have had any significant effect on the stress distribution. Additionally, a local reduction in the adhesive thickness should be expected near the tapered edge. It is also known that the thickness of the patch was also been altered by embedding up to three strain gauges at one location. The placement of the gauges themselves were carefully planned to minimize any effect, but some increase in thickness cannot be avoided. It is assumed that the two gauges placed at the adhesive interface to the adherends were embedded into the adhesive thus reducing the increase in overall thickness. This has certainly had some limited effect on the local adhesive thickness which may also explain some of the discrepancy between the experimental and finite element results at locations with high transverse shear stresses, such as location A.

The thermal residual strains for the aluminum substrate are quite well predicted for location A. The maximum discrepancy is only $31 \mu\epsilon$, a good result considering the high strain

Location	Material	Orient.	Gauge No.	FEM	Experimental Measurement	Experimental Measurement Average	1-D Analysis	Fracture Mechanics Analysis
A	Al		2	-24 $\mu\epsilon$	5 $\mu\epsilon$	5 $\mu\epsilon$	84 $\pm$ 11 $\mu\epsilon$	N/A
	Al	$\perp$	1	-24 $\mu\epsilon$	7 $\mu\epsilon$	7 $\mu\epsilon$	N/A	N/A
	B/Ep		23	-746 $\mu\epsilon$	-607 $\mu\epsilon$	-636 $\mu\epsilon$	-696 $\pm$ 89 $\mu\epsilon$	N/A
	B/Ep		43	-889 $\mu\epsilon$	-665 $\mu\epsilon$	-636 $\mu\epsilon$	-696 $\pm$ 89 $\mu\epsilon$	N/A
	B/Ep	$\perp$	44	129 $\mu\epsilon$	176 $\mu\epsilon$	176 $\mu\epsilon$	N/A	N/A
B	Al		3	606 $\mu\epsilon$	696 $\mu\epsilon$	696 $\mu\epsilon$	488 $\pm$ 3 $\mu\epsilon$	518 $\pm$ 12 $\mu\epsilon$
	Al	$\perp$	4	-174 $\mu\epsilon$	-130 $\mu\epsilon$	-130 $\mu\epsilon$	N/A	N/A
	B/Ep		24	-623 $\mu\epsilon$	-598 $\mu\epsilon$	-574 $\mu\epsilon$	-579 $\pm$ 4 $\mu\epsilon$	N/A
	B/Ep		30	-618 $\mu\epsilon$	-593 $\mu\epsilon$	-574 $\mu\epsilon$	-579 $\pm$ 4 $\mu\epsilon$	N/A
	B/Ep		33	-594 $\mu\epsilon$	-532 $\mu\epsilon$	-574 $\mu\epsilon$	-579 $\pm$ 4 $\mu\epsilon$	N/A
C	B/Ep	$\perp$	34	34 $\mu\epsilon$	-94 $\mu\epsilon$	-94 $\mu\epsilon$	N/A	N/A
	Al		12	512 $\mu\epsilon$	566 $\mu\epsilon$	566 $\mu\epsilon$	558 $\pm$ 1 $\mu\epsilon$	585 $\pm$ 2 $\mu\epsilon$
	Al	$\perp$	11	-43 $\mu\epsilon$	-78 $\mu\epsilon$	-78 $\mu\epsilon$	N/A	N/A
	B/Ep		25	-681 $\mu\epsilon$	-688 $\mu\epsilon$	-633 $\mu\epsilon$	-661 $\pm$ 1 $\mu\epsilon$	N/A
	B/Ep		31	-682 $\mu\epsilon$	-627 $\mu\epsilon$	-633 $\mu\epsilon$	-661 $\pm$ 1 $\mu\epsilon$	N/A
D	B/Ep		42	-693 $\mu\epsilon$	-585 $\mu\epsilon$	-633 $\mu\epsilon$	-661 $\pm$ 1 $\mu\epsilon$	N/A
	B/Ep	$\perp$	41	116 $\mu\epsilon$	-105 $\mu\epsilon$	-105 $\mu\epsilon$	N/A	N/A
	Al		10	496 $\mu\epsilon$	508 $\mu\epsilon$	518 $\mu\epsilon$	541 $\pm$ 1 $\mu\epsilon$	550 $\pm$ 1 $\mu\epsilon$
	Al		22	519 $\mu\epsilon$	528 $\mu\epsilon$	518 $\mu\epsilon$	541 $\pm$ 1 $\mu\epsilon$	550 $\pm$ 1 $\mu\epsilon$
	Al	$\perp$	09	-164 $\mu\epsilon$	-139 $\mu\epsilon$	-126 $\mu\epsilon$	N/A	N/A
E	Al	$\perp$	21	-164 $\mu\epsilon$	-112 $\mu\epsilon$	-126 $\mu\epsilon$	N/A	N/A
	B/Ep		28	-682 $\mu\epsilon$	-638 $\mu\epsilon$	-616 $\mu\epsilon$	-641 $\pm$ 1 $\mu\epsilon$	N/A
	B/Ep		29	-679 $\mu\epsilon$	-641 $\mu\epsilon$	-616 $\mu\epsilon$	-641 $\pm$ 1 $\mu\epsilon$	N/A
	B/Ep		40	-677 $\mu\epsilon$	-568 $\mu\epsilon$	-616 $\mu\epsilon$	-641 $\pm$ 1 $\mu\epsilon$	N/A
	B/Ep	$\perp$	39	5 $\mu\epsilon$	-95 $\mu\epsilon$	-95 $\mu\epsilon$	N/A	N/A
F	Al		13	36 $\mu\epsilon$	96 $\mu\epsilon$	96 $\mu\epsilon$	N/A	N/A
	Al	$\perp$	14	-32 $\mu\epsilon$	-32 $\mu\epsilon$	-32 $\mu\epsilon$	N/A	N/A
G	Al		16	-87 $\mu\epsilon$	-96 $\mu\epsilon$	-96 $\mu\epsilon$	N/A	N/A
	Al	$\perp$	15	35 $\mu\epsilon$	49 $\mu\epsilon$	49 $\mu\epsilon$	N/A	N/A
H	Al		7	317 $\mu\epsilon$	327 $\mu\epsilon$	327 $\mu\epsilon$	134 $\pm$ 1 $\mu\epsilon$	N/A
	Al	$\perp$	8	-77 $\mu\epsilon$	-62 $\mu\epsilon$	-62 $\mu\epsilon$	N/A	N/A
	B/Ep		27	-866 $\mu\epsilon$	-720 $\mu\epsilon$	-734 $\mu\epsilon$	-1038 $\pm$ 1 $\mu\epsilon$	N/A
	B/Ep		37	-860 $\mu\epsilon$	-748 $\mu\epsilon$	-734 $\mu\epsilon$	-1038 $\pm$ 1 $\mu\epsilon$	N/A
	B/Ep	$\perp$	38	130 $\mu\epsilon$	235 $\mu\epsilon$	235 $\mu\epsilon$	N/A	N/A
I	Al		5	484 $\mu\epsilon$	532 $\mu\epsilon$	527 $\mu\epsilon$	558 $\pm$ 1 $\mu\epsilon$	562 $\pm$ 1 $\mu\epsilon$
	Al		19	477 $\mu\epsilon$	521 $\mu\epsilon$	527 $\mu\epsilon$	558 $\pm$ 1 $\mu\epsilon$	562 $\pm$ 1 $\mu\epsilon$
	Al	$\perp$	6	-130 $\mu\epsilon$	-117 $\mu\epsilon$	-104 $\mu\epsilon$	N/A	N/A
	Al	$\perp$	18	-132 $\mu\epsilon$	-91 $\mu\epsilon$	-104 $\mu\epsilon$	N/A	N/A
	B/Ep		26	-739 $\mu\epsilon$	-632 $\mu\epsilon$	-634 $\mu\epsilon$	-661 $\pm$ 1 $\mu\epsilon$	N/A
	B/Ep		32	-739 $\mu\epsilon$	-647 $\mu\epsilon$	-634 $\mu\epsilon$	-661 $\pm$ 1 $\mu\epsilon$	N/A
J	B/Ep		35	-740 $\mu\epsilon$	-622 $\mu\epsilon$	-634 $\mu\epsilon$	-661 $\pm$ 1 $\mu\epsilon$	N/A
	B/Ep	$\perp$	36	31 $\mu\epsilon$	30 $\mu\epsilon$	30 $\mu\epsilon$	N/A	N/A

Table 6.1: Comparison of the thermal residual strains at 21°C between finite element results, experimental results, the one-dimensional analysis and the fracture mechanics analysis

gradient and the uncertainty in the shear modulus behaviour with temperature. A discrepancy of up to 25% between the finite element thermal residual strain prediction and the experimental measurements is present for the boron/epoxy patch at this location. Possible

causes are the position inaccuracy of the gauges, unaccounted changes in the local adhesive thickness especially in a high strain gradient area as well as local thickness increase due to the embedded gauges. The error for position inaccuracy is certainly large ($\approx 90 \mu\epsilon$) as presented in Table 5.1. The uncertainty in the shear modulus behaviour with temperature might also be a contributing factor.

The finite element analysis provided improved insight as to why high tensile thermal residual strains for gauge 3 mounted on the aluminum face sheet in fiber orientation at location B were measured. The transverse shear distribution showed that the fracture mechanics model should only be applied to an angle θ of approximately 80° or less to limit the influence of the local load transfer for this particular load case. Gauge 3 is located in an area with increasing transverse shear stress. The increasing transverse shear stress indicates a distortion of the stress and strain distribution of an unreinforced crack which was assumed in the theoretical fracture mechanics analysis. In addition to the local increase in transverse shear stress in the adhesive, the placement of the strain gauges at the adhesive interfaces leads to a thinner adhesive layer, which will also affect the local stress distribution in comparison the uniform thickness adhesive layer assumed in the finite element model. The thermal residual strain in the aluminum face-sheet at gauge 4 is reasonable well predicted with an discrepancy of only $44 \mu\epsilon$ (25%). The thermal residual strains measured in the fiber direction in the boron/epoxy patch near the adhesive interface shows excellent agreement with the finite element results with a discrepancy of only 4%. The thermal residual strains for the gauge mounted on top of the patch show larger differences between the numerical and experimental result which is likely caused by the local increase in thickness due to embedding strain gauges. This local section experiences some bending which decreases the strains in fiber orientation and increases the strains perpendicular to the fiber orientation under the given thermal residual loading conditions.

The quality of the finite element prediction at location C is quite similar. The gauges placed in or on top of the patch show again the effect of the local increase in thickness. The finite element analysis and the experimental measurement agree very well at the bottom surface of the patch (within $\approx 1\%$). The thermal residual strains for gauge 31, which is mounted on the first boron/epoxy ply, show a discrepancy of 8% which is quite high for this particular location. A probable cause is the thickness of the underlying gauges including the solder connections. The highest difference was found for the thermal residual strains in the transverse direction on top of the patch. Two factors may have contributed to this result. The local increase in thickness leads to bending stresses under the applied tension due to the transverse mismatch in the coefficients of thermal expansion. Furthermore, the experimental strain measurements in the transverse direction show significantly more creep than the gauges in the fiber orientation at this location i.e. $60\ \mu\epsilon$ vs. $10\ \mu\epsilon$ in the fiber direction. It should be noted that the measured thermal residual strains are significantly smaller perpendicular in comparison to parallel to the fiber orientation thus making the inaccuracy due to creep more significant in the transverse direction. The thermal residual strains for the aluminum in fiber direction show a difference of just over 9% and perpendicular to the fiber orientation a discrepancy of $35\ \mu\epsilon$ is found.

The thermal residual strains are very well predicted for the aluminum in fiber direction at location D. The discrepancy between the prediction and measurement is less than $12\ \mu\epsilon$. The change through the thickness of the aluminum was predicted by including the honeycomb in the finite element analysis. The thermal residual strains in the transverse direction show larger discrepancies of $25\ \mu\epsilon$ and $52\ \mu\epsilon$. This larger difference might be due to a combination of the long lead wires whose effect was numerically corrected and the lower resistance of this gauge. The difference between the predicted and measured thermal residual strains for the boron/epoxy in fiber orientation shows again the effect of the local increase in thickness. The discrepancy for the two gauges close to the bottom of the patch

is approximately 6% increasing to 16% at the top. The thermal residual strains for the boron/epoxy in transverse direction show a similar trend as discussed for location B and C.

The thermal residual strains measured at location E outside the patched area show a significant discrepancy of $60 \mu\epsilon$ in longitudinal direction between the FEM prediction and the experimental measurement while the transverse direction indicates perfect agreement. A possible cause might be the influence of the grip arrangement which was not modeled in the theoretical analysis. Furthermore, a small difference in the coefficients of thermal expansion might be present between the Al 6061-T6, which was used for the reinforcements and spacers in the grip section, and the Al 2024-T3 used for the face-sheets.

The experimental measurements and finite element prediction for the thermal residual strains at location F show only a discrepancy of less than $14 \mu\epsilon$, which is excellent. The magnitude certainly shows the in-plane bending generated by the thermal residual stresses.

The thermal residual strains in the aluminum for the longitudinal and transverse direction at location G show excellent agreement between the finite element results and the experimental measurements ($< 15 \mu\epsilon$). This is a significant improvement over the one-dimensional analysis previously used to get an estimate for the thermal residual strains. The finite element analysis also provides a better prediction of the thermal residual strain distribution for the boron/epoxy patch but still overpredicts their magnitude. Possible causes include the uncertainty in the shear modulus assumptions with temperature, the presence of creep especially for the transverse direction ($150 \mu\epsilon$), or possibly local increases in thickness as well as inaccuracy in the gauge position.

The finite element predictions don't correspond to the experimental measurements as well as expected at location H. The through thickness change in thermal residual strains for

the aluminum in the longitudinal direction is well predicted due the inclusion of the honeycomb in the finite element model, but they disagree in the absolute magnitude by about 10%. The thermal residual strain in the transverse direction on top of the face-sheet is well predicted (discrepancy of only $13 \mu\epsilon$) but for the bottom surface they disagree by $41 \mu\epsilon$. Possible explanations are the long lead wires and the low resistance of the strain gauge installed at the bottom surface of the face-sheet combined with the lower measured strain. The discrepancy in the thermal residual strains between the experimental measurements and finite element results for the boron/epoxy patch is between 12 and 16%. Again the local increase in thickness due to embedded gauges is probably one reason for these disagreements. Other factors include misalignment of gauges, local reduction the adhesive thickness and the simplification of not modeling the grip section thus affecting the in-plane bending behaviour of the specimen.

Overall, the thermal residual strains are well predicted for the aluminum in both longitudinal and transverse directions. The average discrepancy between the finite element results and the experimental results are only $36 \mu\epsilon$ and $27 \mu\epsilon$, respectively. Although these magnitudes are small, it should be noted that the finite element analysis underpredicted the thermal residual strains for the aluminum in longitudinal direction which is not particularly desirable from a design viewpoint. One recommendation would be to increase the effective stress free temperature slightly such that a conservative finite element prediction is achieved. The thermal residual strains determined by the finite element analysis for the boron/epoxy are, on average, $88 \mu\epsilon$ higher than the experimental results. The most important cause is the local increase in thickness by embedding strain gauges, which is also applicable to the transverse thermal residual strains for the boron/epoxy patch. The average discrepancy of even $100 \mu\epsilon$ in transverse direction can also be attributed to creep. The through thickness variation of the thermal residual strains in the patch will be influenced by the particular placement of each gauge as well as the solder connection to the lead wires.

Considering the influencing factors which were discussed in this section, good agreement between the experimental results and the finite element results has been achieved.

These measurements suggest that for future experimental work, only a single gauge should be embedded at any given location. The finite element analysis revealed that significant through thickness effects are only present very close to the crack and in the tapered section. The use of strain gauges is limited in these regions due to their size. Very small gauges may be applied to the metallic substrate but are inadequate for composites since they would be affected by the local interaction between fibers and matrix. The other important achievement is the actual embedding of gauges in a bonded repair with reliable connections. Embedded gauges offer a good option for 'smart patches' since the gauge is protected from environmental influences. AMRL started to investigate the feasibility of 'smart patches' [36] by monitoring surface strains of the patch using an AMRL sandwich type specimen. The challenge is to find a location for the strain gauge where the gauge installation doesn't lead to disbonds around the gauge under fatigue loading while at the same time can indicate disbonds around the crack tip and/or crack propagation. A possible solution would be the placement of a gauge underneath the top layer of the patch. The optimum in-plane location needs to be determined by a number of finite element representations which model the crack growth and disbond growth as observed by AMRL [13]. Their surface strain measurements should give an excellent verification tool as well as an excellent initial guess for an appropriate location.

Chapter 7

Discussion and Conclusion

7.1 Introduction

As outlined in Chapter 1, the main goals of this research were to identify the magnitude of thermal residual stresses in bonded composite repairs and to establish an appropriate thermal residual stress/strain model. These research goals were achieved through four stages: The first stage involved manufacturing six AMRL sandwich type composite bonded repair specimens to provide some of the specimens required for fatigue damage initiation testing in addition to one test specimen which was instrumented by placing 44 strain gauge at eight locations and at up to five different interfaces. Residual strains at ambient temperature combining thermal residual strains and other process induced strains were measured during the manufacturing process. In stage two, the stress free temperature of the bonded repair was experimentally determined and thermal residual strains were measured as a function of operating temperature. For stage three, a theoretical analysis was carried out to determine the thermal residual stress and strain distributions in bonded repairs which also addressed the effect of symmetrical disbonds around the crack as well as the stress field ahead of the crack. As final stage four, a detailed finite element analysis was carried out to assess the limitations of the theoretical analysis as well as providing a more detailed insight in the complex thermal residual stress and strain state of the AMRL sandwich type specimen.

The conclusions and contributions of this research work are highlighted in the following sections.

7.2 AMRL Specimen Manufacturing Procedure

The objective of this first stage included the selection of a suitable test specimen for thermal residual stress or strain measurements as well as for future fatigue damage initiation testing. Appropriate process specifications had to be determined as part of the manufacturing procedure in order to create durable and representative specimens. The instrumentation locations had to be determined based on critical failure locations as reported in the literature.

The conclusions and contributions of this portion of the research are outlined below:

- A representative specimen was selected, constructed and instrumented for determining the residual strain distribution in a bonded repair.

The sandwich type AMRL specimen (see Figure 3.1) was chosen for the experimental determination of the stress state in bonded repairs due to its ability to represent real aircraft structures and frequent use in the evaluation of bonded repairs. Improving the understanding of the stress distribution in the AMRL specimen is valuable for all researchers interpreting experimental results obtained from this specimen.

- An effective manufacturing process for the AMRL sandwich type specimen has been developed and presented in detail, including jigs and fixtures to prepare this specimen.

Proper process specifications represent the single most critical element of bonded composite repairs since they control the fatigue characteristics of the repair including the occurring failure modes. Due to the lack of detailed process specifications and guidelines in the literature, the manufacturing process for the AMRL sandwich type

specimen was developed and documented in great detail to provide other researchers with the opportunity to build this specimen. The National Research Council of Canada provided the surface preparation procedure as part of a research collaboration.

- An instrumentation technique has been developed for obtaining the thermal residual strain distribution as a function of temperature in a patched repair specimen.

Strain gauges were chosen as non-destructive measuring devices based on availability, expense, reliability as well as applicability to composites. Techniques for embedding strain gauges and lead wire routing in the patch are presented. The innovative solutions provided for instrumented AMRL sandwich type specimen manufacturing procedures can be applied in smart patches.

7.3 Residual and Thermal Residual Strain Measurements

Although thermal residual strains were the primary objective of this research phase, the residual strains due to the elevated temperature cure of a bonded repair consist of both thermal and process induced residual strains, the latter resulting from effects such as anisotropic shrinkage of the patch or adhesive, temperature gradients in the specimen during processing, and/or temperature gradients due to tooling and internal resin flow [50]. While thermal residual strains are affected by subsequent changes in temperature, process induced residual strains may not be affected.

The contributions and conclusion of the research performed include the following:

- The total residual strains were obtained during the assembly stage of the AMRL sandwich type specimen.

The key finding from these measurements were the 60% lower residual strains in the fiber direction in comparison to standard thermal residual strain estimate which is based on the temperature differential between curing and ambient temperature.

- The stress free temperature for FM73M was experimentally measured to be 101.2°C.

This type of measurement was not been reported in the literature for bonded repairs. The difference between the stress free temperature and the curing temperature of 19.8°C shows the importance of this measurement.

- Thermal strains were measured in the AMRL sandwich type specimen and the thermal residual strains were evaluated based on the previously determined stress free temperature.

This experimental work represents the key measurement of this research work since neither thermal strains versus temperature nor thermal residual strains versus temperature have been documented in the literature. The experimental data represent an important element to improve current estimates of the stress state after patching as well as the the stress intensity factor and crack growth rates.

- An effective stress free temperature of 85.8°C was defined based on the experimental thermal residual strain data in order to establish a linear-elastic thermal residual stress/strain model.

An important finding was the significant change in the slope of the thermal residual strain versus temperature plots at approximately 80°C to 90°C instead of the expected cure temperature. Near linear behaviour was found below 80°C. Therefore an effective stress free temperature of 85.8°C was defined using a linear regression analysis of the thermal residual strains versus temperature plots. The error in the temperature

differential employed in standard calculations for the thermal residual stresses and strains is 35.2°C based on the experimental measurements making this definition an excellent improvement over current practices.

- A simple test scheme is proposed to evaluate the effective stress free temperature for different adhesives.

It cannot be assumed that the same thermal behaviour is true for other adhesives. The presented experimental data and conclusions are only valid for the use of FM 73M as adhesive material. Therefore the proposed specimen presents an excellent tool for determining the the effective stress free temperature for other adhesives.

- Process induced strains were assessed based on the difference between the measured residual strains and thermal residual strains and found to be negligible for FM73M.

Based on the good agreement between the average residual strains (541 $\mu\epsilon$ with a standard deviation of 14%) and the thermal residual strains (546 $\mu\epsilon$ with a standard deviation of 16%) for the aluminum at ambient temperature, only negligible temperature independent process induced strains are present in the fiber direction. In contrast, the residual strain average due to the elevated temperature cure measured in the aluminum perpendicular to the fiber direction of the patch at the same locations (-164 $\mu\epsilon$ with a standard deviation of 44%) is significantly higher in compression than the thermal residual strain average of -111 $\mu\epsilon$ (with a standard deviation of 20%) indicating that process induced strains are substantially higher in this direction (see Table 6.1). The magnitude of the estimated temperature independent process induced strains shows that they can generally be neglected in future analysis, which has not been previously shown for bonded repairs.

7.4 Theoretical Analysis of Thermal Residual Stresses in Bonded Repair Specimens

The research objectives for the third phase were to establish an appropriate thermal residual stress/strain model for bonded repairs, an estimate for the stress distribution near the crack tip and the influence of disbonds on the stress intensity factor under thermal residual stress loading. All of these objectives were achieved with a focus on practical use thus avoiding the need of specialized computer programs. The conclusions and contribution of the presented theoretical approach are provided below:

- A solution algorithm allowing the direct determination of the elastic stress distribution for a tapered patch was developed without the usual requirement for a specialized numerical code.

This solution algorithm makes the linear-elastic approach from L.J. Hart-Smith for tapered and stepped joints more accessible for more engineers to get a basic understanding of joint design. Furthermore this approach avoids the difficulties encountered in the numerical solution scheme as outlined by Hart-Smith. A correction for the shear-lag in the adherends as presented by AMRL researchers [18, 52] was included to improve Hart-Smith's one-dimensional analysis.

- A stress field model was introduced for the determination of the stresses in the crack tip region in order to improve the theoretical predictions near the crack tip.

One of the key achievements was the development of a concise solution for the stress field of a center crack in an infinite plate representing a superior solution for this classic fracture mechanics problem. A complex stress field approach was used to solve this problem in a unique way replacing the need to use a large number of boundary conditions to determine a sufficient number of coefficients for an accurate result, or the more widely employed approximations with their limitations. The

developed solution was then employed as a predictive tool for bonded repairs for the stress and strain field near the crack tip in bonded repairs. Applications for this type of approach include the determination of sensor locations for smart patches as well as other experimental work.

- A generalized version of the Rose model was developed, which accounts for disbonds.

The model can account for all disbond lengths including a complete disbond. The generalized Rose model maintains the simplicity of the original Rose model as one of its features. It was found that thermal residual stresses in combination with a partial disbond can lead to an increase in the stress intensity factor beyond the stress intensity factor of the unpatched specimen depending on the degree of restraint. This effect was found to be most severe for unrestrained specimens where a disbond length of less than 20 mm eliminated the benefit of the patching. Further experimental work is required to verify the predictive capabilities of the extended Rose model for disbonds.

- Generally good agreement has been found between the experimental measurements and theoretical predictions using the one-dimensional as well as the stress field model within the limitations of these models.

The presented models have been evaluated for the thermal residual stresses and strains present in the AMRL sandwich type specimen. Effects such as in-plane bending due to the smaller patch in comparison to the substrate and out-of-plane bending due to the limited stiffness of the honeycomb in case of the AMRL specimen have to be considered when assessing the predictive capabilities of the models. The key to the good agreement between the closed form analytical solution and the experimental data was the definition of the stress free temperature as determined through the experimental work.

7.5 Finite Element Analysis of the AMRL Specimen

The objectives for the last phase of research were to establish the limitations of the theoretical model as well as to obtain a better insight into the complex thermal residual stress and strain distribution of the AMRL specimen. The stress distribution around the crack tip and the taper were the main focus. Due to the complexity of bonded repairs, a number of verification models were created to investigate each critical section in detail. This approach allowed one to determine a sufficiently fine mesh ensuring confidence in the results while reducing the overall degrees of freedom (and required solution time) for an entire model of the AMRL sandwich type specimen. A linear-elastic three-dimensional model for AMRL specimen was created utilizing 18806 elements with 254670 degrees of freedom.

The conclusions and contribution of this final portion of the research are outlined below:

- The stress distribution in both normal and reversed stacking sequence patch design was investigated and compared with respect to potential failure modes.

High peel stresses were found at the ply drop-off due to the incline of the boron/epoxy plies using a reverse stacking sequence. The most critical case for the reversed stacking sequence with respect to peel stresses was identified as a remote compressive loading combined with compressive thermal residual stresses in the patch. This finding was substantiated by experimental observations from the National Research Council of Canada where a change in failure mode at the ply drop off was demonstrated under these loading conditions. Therefore the magnitude of the peel stresses should be carefully examined whenever compressive thermal residual stresses are combined with compressive remote loading for a patch with a reversed stacking sequence. As a general design rule, a normal stacking sequence should be used with remote compressive loading and a reversed stacking sequence with tensile remote loading for low CTE patches.

- Overall the finite element analysis predictions compare well with the experimentally measured thermal residual strains in the AMRL sandwich type specimen especially for the aluminum.

The average discrepancy between the finite element results and the experimental results at room temperature for the aluminum in fiber orientation as well as perpendicular to the fiber orientation are only $36 \mu\epsilon$ (with a standard deviation of $27 \mu\epsilon$) and $27 \mu\epsilon$ (with a standard deviation of $16 \mu\epsilon$), respectively. The thermal residual strains determined by the finite element analysis for the boron/epoxy are on average $88 \mu\epsilon$ higher than the experimental results (with a standard deviation of $56 \mu\epsilon$). The main cause for this discrepancy is presumed to be the local increase in thickness by embedding strain gauges. This is also applicable to the transverse thermal residual strains for the boron/epoxy patch in combination with creep leading to an average discrepancy of even $100 \mu\epsilon$ (with a standard deviation of $74 \mu\epsilon$). The through thickness variation of the thermal residual strains in the patch will be influenced slightly by the particular placement of each gauge as well as the solder connection to the lead wires. Considering the influencing factors good agreement between the experimental results and the finite element results was achieved.

- A finite element verification model utilizing a uniform thickness patch over the entire substrate region showed good agreement with the presented theoretical stress field approach ahead of the crack tip.

The theoretical model has certain limitations for the stress field behind the crack tip since no load transfer mechanism into the patch is included into the stress field model. Furthermore, the effective crack length predicted by the Rose model is significantly smaller than the physical crack length. The theoretical stress field predictions for the AMRL sandwich type specimen shows additional discrepancies to the finite element

results and experimental measurements since it does not account for in-plane bending and deflection of the honeycomb.

- The finite element analysis substantiated that embedded strain gauges can be used to obtain excellent strain measurements in bonded repairs.

Due to the overall good agreement between theoretical predictions and experimental measurements, it is concluded that embedded gauges offer a good option for 'smart patches' since the gauge is protected from environmental influences. Further experimental work is required to test the influence of damage initiation around embedded strain gauges. The finite element analysis also showed the influence of embedding of strain gauges at multiple interfaces. The local increase in thickness is considered to be the cause for the decrease in compressive residual strains towards the top surface. It is recommended that for future experimental strain measurements, the number of embedded strain gauges be limited to one gauge per location.

7.6 Implementation in a Design Procedure

The experimental and theoretical achievements can readily be included in patch design procedures such as Baker's [18] design procedure for minimum patch thickness as presented in Chapter 2. Figure 7.1 shows the extension to Baker's original design scheme based on the results of the current work, and are discussed below.

The design procedure remains unchanged through Step 5. Then the peak strain in the patch (Step 6) occurs typically in the tapered section or over the crack. The derived direct linear-elastic stress/strain solution for tapered joints and reinforcements provides a simple tool to substantiate the strain at both locations instead of just checking over the crack as presented by Baker [18]. Strain concentration factors, which can be evaluated using finite element models should be included in the peak strain estimate.

Choosing the effective stress free temperature as determined in this research work instead of the curing temperature will give an accurate estimate of the thermal residual stress for Step 7. This improvement will also lead to a better prediction of the peak stress intensity factor (Step 9).

Figure 7.1: Minimum patch thickness design scheme

The design approach can be extended to design for a particular stress intensity factor and/or stress intensity factor range (Step 10 & 11). An improvement in the prediction of the stress intensity factor range can also be addressed by including the stress ratio. The true stress ratio for a bonded repair can now be determined based on the experimental and theoretical determination of thermal residual stresses in this research work. The design to a particular stress intensity factor or stress intensity factor range should include allowable disbond sizes. The presented generalized Rose model allows the determination of the stress intensity factor with symmetrical disbonds in the bonded repair which also applies to Baker's proposed generic certification approach [11, 12]. The availability of an increasing database of experimental fatigue crack propagation data for different materials as well as environmental conditions allows the development of a predictive model for the crack propagation rate. Crack retardation effects due to overloads will generally be impossible to include in any certification approach since the stress history is unknown in most real repairs.

It should be noted that Figure 7.1 shows an idealized schematic which is based on a reduction of the crack opening displacement for an increase in patch thickness. This is not always guaranteed since the stress intensity factor can increase under thermal loading with increasing patch thickness.

7.7 Recommendations for Future Work

The following recommendations can be made for future work in the area of thermal residual stresses:

- Measuring the residual strains during the curing cycle would be quite valuable to improve our understanding of the adhesive behaviour during this stage. The proposed generic thermal residual strain specimen can be used for this research.

- One of the remaining important questions is the magnitude of thermal residual stresses when a 80°C temperature cure is used. Based on Boykett's and Walker's estimated thermal residual strains and the true adhesive behaviour for FM 73M investigated in this research, it appears that the reduction in thermal residual strains due to lowering the curing temperature from 121°C and 80°C is quite small. The suggested generic thermal residual strain specimen is also well suited for this test.
- The thermal residual stresses as a function of temperature should be evaluated for the different adhesives in use. Again the generic thermal residual strain specimen can be used for this application.
- Future thermal residual strain measurements and other strain measurements utilizing embedded strain gauges should limit the number of gauges preferably to one gauge at a single location to minimize changes in the local strain distribution due to the increase in thickness.
- In order to assess the true thermal residual stresses for repair, the coefficient of thermal expansion should be determined at the repair location experimentally. The measurement can be carried out during the thermal survey for placing heat blankets. The strain gauge should be installed using an adhesive suitable for elevated temperature measurements.

The following recommendations can be made for future work in the general area of composite repairs:

- Experimental evaluation of the presented disbond model is suggested since its simplicity offers the potential for an excellent tool to evaluate the influence of disbonds in situ.
- A detailed finite element analysis is suggested to evaluate an improvement of the normal stacking sequence by using a final composite layer which extends over the previously stacked plies. This final layer acts as a protective layer for the exposed

edges of the tapered section. This study should consider different materials for this layer to minimize peel stresses.

- The presented research work should be continued by carefully investigating damage initiation since the true stress state of the repair is now available. This investigation should focus on reinitiation of the crack as well as disbond initiation over the crack and possibly near the free edge. This research would allow one to identify the damage initiation life of a bonded repair, which can represent a major portion of the fatigue life of a bonded repair.
- One of the key challenge for the widespread application of adhesive bonding is the need for a non-destructive durability test for adhesive joints. In addition, a non-destructive inspection technique is needed to verify proper surface preparation. While these issues can be somewhat addressed under laboratory conditions, any NDI techniques must be suitable for field conditions.
- Further research is required to investigate the adhesive behaviour under elevated temperatures (incl. temperatures above the curing temperature). A strong frequency dependency is suspected since tests at AMRL showed the FM 73 performs well under cyclic loading (0.5 and 5 Hz) at temperatures above 100°C, while quasi static tests during this research showed that the patch and substrate can expand nearly unrestrained at temperatures above 90°C.
- Further research is required to establish smart patches as a optional design feature to simplify certification in critical repairs. Embedding sensors can be easily carried out as shown in this research. Early fatigue damage around the sensor is a concern which needs to be addressed. Furthermore, the development of suitable instrumentation is necessary. AMRL is currently working on the development of smart patch technology [11].

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Appendix A

AMRL-Specimen Drawings

Figure A.1: AMRL specimen (Face-sheets)

Figure A.2: AMRL specimen (Spacer block)

Figure A.3: AMRL specimen (Honeycomb)

Figure A.4: AMRL specimen (Shims)

Figure A.7: Instrumented AMRL specimen (Aluminum)

Figure A.8: Instrumented AMRL specimen (Patch-adhesive interface)

Figure A.9: Instrumented AMRL specimen (1st boron/epoxy ply)

Figure A.10: Instrumented AMRL specimen (Top boron/epoxy ply)

Figure A.11: Instrumented AMRL specimen (Superimposed drawing)

Figure A.12: Instrumented AMRL specimen (Backside of face-sheet 7)

Figure A.13: Instrumented AMRL specimen (Gauge locations)

Figure A.14: Instrumented AMRL specimen (Backside gauge locations)

Appendix B

Pre-cracking Documentation

B.1 Pre-crack Data

Sequence []	N []	a [mm]	P _{min} [N]	P _{max} [N]	$\bar{a}$ [mm]	$\Delta\sigma$ [MPa]	c []	ΔK [MPa $\sqrt{m}$]	$\frac{da}{dN}$ [$\frac{\mu m}{cycle}$]
1	0	2.94	3750	37500	Crack Initiation				
	50000	4.74	3750	37500	5.01	66.44	1.130	9.41	0.0990
	55351	5.27	3750	37500	Sequence Change				
2	55352	5.27	3000	30000	Retardation				
	95603	7.62	3000	30000	Sequence Change				
3	95604	7.62	2500	25000	Retardation				
	100000	7.84	2500	25000	8.63	44.29	1.139	8.30	0.0416
	137762	9.41	2500	25000	9.70	44.29	1.141	8.82	0.0454
	150315	9.98	2500	25000	Sequence Change				
4	150316	9.98	2000	20000	Retardation				
	200000	11.58	2000	20000	11.84	35.43	1.147	7.84	0.0389
	213098	12.09	Overload		Retardation				
	229329	12.42	2000	20000	Retardation				
	500000	12.42	2000	20000	Retardation				
	512500	13.10	2828	28280	Increased Load to overcome Retardation				
5	512501	13.10	1750	17500	Retardation				
	600000	14.32	1750	17500	14.71	31.00	1.155	7.70	0.0253
	630490	15.09	1750	17500	Sequence Change				
6	630491	15.09	1500	15000	Retardation				
	650000	15.42	1500	15000	15.94	26.57	1.158	6.89	0.0206
	700000	16.45	1500	15000	16.88	26.57	1.161	7.10	0.0249
	734099	17.30	1500	15000	17.42	26.57	1.162	7.23	0.0219
	744615	17.53	1500	15000	Sequence Change				
7	744616	17.53	1250	12500	Retardation				
	750000	17.59	Overload		Retardation				
	765000	17.97	1250	12500	Retardation				
	780000	18.07	1250	12500	Retardation				
	795000	18.17	1250	12500	Retardation				
	810000	18.18	1250	12500	Retardation				
	875000	18.23	1250	12500	Retardation				
	931000	18.37	1750	17500	Increased Load to overcome Retardation				
	938000	18.57	1500	15000	Retardation				
	950000	18.65	1250	12500	Retardation				
	1000000	19.19	1250	12500	19.22	22.15	1.168	6.35	0.0033
	1015000	19.24	1250	12500	19.36	22.15	1.168	6.38	0.0153
	1030000	19.47	1250	12500	19.52	22.15	1.168	6.41	0.0067
	1045000	19.57	1250	12500	19.66	22.15	1.169	6.43	0.0120
	1060000	19.75	1250	12500	19.83	22.15	1.169	6.46	0.0100
	1075000	19.90	1250	12500	19.92	22.15	1.170	6.48	0.0044
	1084100	19.94	1250	12500	20.00	22.15	1.170	6.49	0.0186
	1090000	20.05	1250	12500					

Table B.1: Pre-crack data for face-sheet 1

Sequence []	N []	a [mm]	P _{min} [N]	P _{max} [N]	$\bar{a}$ [mm]	$\Delta\sigma$ [MPa]	c []	ΔK [MPa $\sqrt{m}$]	$\frac{da}{dN}$ [$\frac{\mu m}{cycle}$]
1	0	2.86	3750	37500	Crack Initiation				
	50000	3.69	3750	37500	3.84	66.44	1.127	8.22	0.0561
	55351	3.99	3750	37500	4.44	66.44	1.128	8.85	0.0224
	95603	4.89	3750	37500	Sequence Change				
2	95604	4.89	3000	30000	Retardation				
	114580	5.57	3000	30000	6.44	53.15	1.133	8.57	0.0647
	141462	7.31	3000	30000	7.46	53.15	1.136	9.24	0.0591
	146366	7.60	3000	30000	Sequence Change				
3	146367	7.60	2500	25000	Retardation				
	191528	9.88	2500	25000	10.02	44.29	1.142	8.97	0.0452
	197503	10.15	2500	25000	Sequence Change				
4	197504	10.15	2000	20000	Retardation				
	225600	10.83	2000	20000	11.32	35.43	1.146	7.65	0.0398
	250000	11.80	2000	20000	12.07	35.43	1.148	7.92	0.0312
	267611	12.34	2000	20000	12.45	35.43	1.149	8.05	0.0402
	272787	12.56	2000	20000	Sequence Change				
5	272788	12.56	1750	17500	Retardation				
	300000	13.18	1750	17500	13.50	31.00	1.152	7.35	0.0299
	321218	13.82	1750	17500	14.26	31.00	1.154	7.57	0.0311
	350000	14.71	1750	17500	14.86	31.00	1.155	7.74	0.0322
	359318	15.01	1750	17500	Sequence Change				
6	359319	15.01	1500	15000	Retardation				
	375000	15.38	1500	15000	15.57	26.57	1.157	6.80	0.0152
	400000	15.76	1500	15000	16.36	26.57	1.159	6.99	0.0240
	450000	16.96	1500	15000	17.18	26.57	1.162	7.17	0.0287
	465000	17.39	1500	15000	17.51	26.57	1.163	7.25	0.0563
	469088	17.62	1500	15000	Sequence Change				
7	469089	17.62	1250	12500	Retardation				
	480000	17.63	1250	12500	17.74	22.15	1.163	6.08	0.0053
	521223	17.85	1250	12500	17.98	22.15	1.164	6.13	0.0100
	546198	18.10	1250	12500	18.22	22.15	1.165	6.17	0.0071
	580000	18.34	1250	12500	18.45	22.15	1.165	6.21	0.0056
	617581	18.55	1250	12500	18.72	22.15	1.166	6.26	0.0111
	648339	18.89	1250	12500	18.97	22.15	1.167	6.31	0.0046
	681273	19.04	1250	12500	19.12	22.15	1.167	6.34	0.0077
	701941	19.20	1250	12500	19.32	22.15	1.168	6.37	0.0090
	727472	19.43	1250	12500	19.53	22.15	1.169	6.41	0.0089
	750000	19.63	1250	12500	19.68	22.15	1.169	6.44	0.0067
	765000	19.73	1250	12500	19.81	22.15	1.169	6.46	0.0189
	772955	19.88	1250	12500	19.95	22.15	1.170	6.48	0.0185
	780000	20.01	1250	12500					

Table B.2: Pre-crack data for face-sheet 2

Sequence []	N []	a [mm]	P _{min} [N]	P _{max} [N]	a [mm]	Δσ [MPa]	c []	ΔK [MPa√m]	$\frac{da}{dN}$ [$\frac{\mu m}{cycle}$]
1	0	2.97	3750	37500	Crack Initiation				
	30000	3.32	3750	37500	3.49	66.44	1.126	7.83	0.0330
	40000	3.65	3750	37500	3.92	66.44	1.127	8.31	0.0540
	50000	4.19	3750	37500	4.61	66.44	1.129	9.03	0.0840
	60000	5.03	3750	37500	Sequence Change				
2	60001	5.03	3000	30000	Retardation				
	85750	5.93	3000	30000	6.55	53.15	1.134	8.64	0.0589
	106619	7.16	3000	30000	7.41	53.15	1.136	9.21	0.0799
	112873	7.66	3000	30000	Sequence Change				
3	112874	7.66	2500	25000	Retardation				
	120000	7.98	2500	25000	8.55	44.29	1.139	8.26	0.0575
	139637	9.11	2500	25000	9.26	44.29	1.140	8.61	0.0280
	150000	9.40	2500	25000	9.72	44.29	1.142	8.84	0.1028
	156223	10.04	2500	25000	Sequence Change				
4	156224	10.04	2000	20000	Retardation				
	162500	10.06	2000	20000	11.06	35.43	1.145	7.56	0.0379
	215000	12.05	2000	20000	12.23	35.43	1.148	7.97	0.0360
	225000	12.41	2000	20000	12.47	35.43	1.149	8.05	0.0788
	226396	12.52	2000	20000	Sequence Change				
5	226397	12.52	1750	17500	Retardation				
	250000	12.62	1750	17500	13.18	31.00	1.151	7.26	0.0297
	287688	13.74	1750	17500	14.29	31.00	1.154	7.58	0.0367
	317400	14.83	1750	17500	14.98	31.00	1.156	7.77	0.1027
	320322	15.13	1750	17500	Sequence Change				
6	320323	15.13	1500	15000	Retardation				
	357438	15.70	1500	15000	16.11	26.57	1.159	6.93	0.0293
	385389	16.52	1500	15000	16.99	26.57	1.161	7.13	0.0293
	417519	17.46	1500	15000	17.49	26.57	1.163	7.24	0.0148
	420904	17.51	1500	15000	Sequence Change				
7	420905	17.51	1250	12500	Retardation				
	450000	17.91	1250	12500	18.05	22.15	1.164	6.14	0.0134
	470118	18.18	1250	12500	18.42	22.15	1.165	6.21	0.0157
	500000	18.65	1250	12500	18.75	22.15	1.166	6.27	0.0118
	516885	18.85	1250	12500	19.14	22.15	1.167	6.34	0.0125
	562500	19.42	1250	12500	19.47	22.15	1.168	6.40	0.0080
	575000	19.52	1250	12500	19.61	22.15	1.169	6.42	0.0136
	587500	19.69	1250	12500	19.75	22.15	1.169	6.45	0.0096
	600000	19.81	1250	12500	19.92	22.15	1.170	6.48	0.0191
	610971	20.02	1250	12500					

Table B.3: Pre-crack data for face-sheet 3

Sequence []	N []	a [mm]	P _{min} [N]	P _{max} [N]	$\bar{a}$ [mm]	$\Delta\sigma$ [MPa]	c []	ΔK [MPa $\sqrt{m}$]	$\frac{da}{dN}$ [$\frac{\mu m}{cycle}$]
1	0	2.78	3750	37500	Crack Initiation				
	30000	3.09	3750	37500	3.50	66.44	1.126	7.85	0.0410
	50000	3.91	3750	37500	4.39	66.44	1.128	8.80	0.1238
	57757	4.87	3750	37500	4.97	66.44	1.130	9.38	0.0440
	62303	5.07	3750	37500	Sequence Change				
2	62304	5.07	3000	30000	Retardation				
	85000	5.25	3000	30000	6.24	53.15	1.133	8.43	0.0566
	120000	7.23	3000	30000	7.32	53.15	1.135	9.15	0.0641
	122810	7.41	3000	30000	7.49	53.15	1.136	9.26	0.1122
	124236	7.57	3000	30000	Sequence Change				
3	124237	7.57	2500	25000	Retardation				
	146688	8.38	2500	25000	8.49	44.29	1.138	8.23	0.0615
	150101	8.59	2500	25000	9.33	44.29	1.141	8.65	0.0660
	172366	10.06	2500	25000	Sequence Change				
4	172367	10.06	2000	20000	Retardation				
	175000	10.13	2000	20000	11.12	35.43	1.145	7.58	0.0396
	225000	12.11	2000	20000	12.31	35.43	1.148	8.00	0.0343
	236357	12.50	2000	20000	Sequence Change				
5	236358	12.50	Overload		Retardation				
	249185	12.92	1750	17500	Retardation				
	300000	14.73	1750	17500	14.87	31.00	1.155	7.74	0.0452
	305979	15.00	1750	17500	Sequence Change				
6	305980	15.00	1500	15000	Retardation				
	325000	15.43	1500	15000	15.82	26.57	1.158	6.86	0.0308
	350000	16.20	1500	15000	16.59	26.57	1.160	7.04	0.0308
	375000	16.97	1500	15000	17.25	26.57	1.162	7.19	0.0327
	391819	17.52	1500	15000	Sequence Change				
7	391820	17.52	1250	12500	Retardation				
	422459	17.95	1250	12500	18.18	22.15	1.165	6.16	0.0163
	450000	18.40	1250	12500	18.57	22.15	1.166	6.23	0.0132
	475000	18.73	1250	12500	18.95	22.15	1.167	6.30	0.0172
	500000	19.16	1250	12500	19.26	22.15	1.168	6.36	0.0160
	512500	19.36	1250	12500	19.49	22.15	1.168	6.40	0.0200
	525000	19.61	1250	12500	19.75	22.15	1.169	6.45	0.0224
	537500	19.89	1250	12500	19.95	22.15	1.170	6.48	0.0172
	543882	20.00	1250	12500					

Table B.4: Pre-crack data for face-sheet 4

Sequence []	N []	a [mm]	P _{min} [N]	P _{max} [N]	$\bar{a}$ [mm]	$\Delta\sigma$ [MPa]	c []	ΔK [MPa $\sqrt{m}$]	$\frac{da}{dN}$ [$\frac{\mu m}{cycle}$]
1	0	2.82	3750	37500	Crack Initiation				
	40000	3.15	3750	37500	3.21	66.44	1.125	7.51	0.0120
	50000	3.27	3750	37500	3.47	66.44	1.126	7.81	0.0390
	60000	3.66	3750	37500	4.11	66.44	1.128	8.51	0.0450
	80000	4.56	3750	37500	4.86	66.44	1.129	9.27	0.0760
	87763	5.15	3750	37500	Sequence Change				
2	87764	5.15	3000	30000	Retardation				
	110524	6.19	3000	30000	6.55	53.15	1.134	8.64	0.0490
	125000	6.90	3000	30000	7.16	53.15	1.135	9.05	0.1137
	129575	7.42	3000	30000	7.48	53.15	1.136	9.25	0.1082
	130684	7.54	3000	30000	Sequence Change				
3	130685	7.54	2500	25000	Retardation				
	150000	8.32	2500	25000	8.84	44.29	1.139	8.41	0.0629
	166528	9.36	2500	25000	9.66	44.29	1.141	8.80	0.0696
	175000	9.95	2500	25000	10.03	44.29	1.142	8.98	0.0951
	176683	10.11	2500	25000	Sequence Change				
4	176684	10.11	2000	20000	Retardation				
	200000	10.66	2000	20000	11.16	35.43	1.145	7.60	0.0400
	225000	11.66	2000	20000	12.10	35.43	1.148	7.93	0.0519
	241759	12.53	2000	20000	Sequence Change				
5	241760	12.53	1750	17500	Retardation				
	250000	12.67	1750	17500	13.29	31.00	1.151	7.29	0.0248
	300000	13.91	1750	17500	14.16	31.00	1.153	7.54	0.0245
	320000	14.40	1750	17500	14.74	31.00	1.155	7.71	0.0423
	336069	15.08	1750	17500	Sequence Change				
6	336070	15.08	1500	15000	Retardation				
	350000	15.43	1500	15000	15.73	26.57	1.158	6.84	0.0236
	375000	16.02	1500	15000	16.44	26.57	1.160	7.00	0.0332
	400000	16.85	1500	15000	17.03	26.57	1.161	7.14	0.0333
	410825	17.21	1500	15000	17.40	26.57	1.162	7.22	0.0380
	420564	17.58	1500	15000	Sequence Change				
7	420565	17.58	1250	12500	Retardation				
	450000	17.98	1250	12500	18.29	22.15	1.165	6.18	0.0155
	490000	18.60	1250	12500	18.80	22.15	1.166	6.28	0.0152
	516383	19.00	1250	12500	19.16	22.15	1.167	6.34	0.0235
	530000	19.32	1250	12500	19.51	22.15	1.168	6.41	0.0274
	543893	19.70	1250	12500	19.77	22.15	1.169	6.45	0.0213
	550000	19.83	1250	12500	19.92	22.15	1.170	6.48	0.0310
	555808	20.01	1250	12500					

Table B.5: Pre-crack data for face-sheet 5

Sequence []	N []	a [mm]	P _{min} [N]	P _{max} [N]	$\bar{a}$ [mm]	$\Delta\sigma$ [MPa]	c []	ΔK [MPa $\sqrt{m}$]	$\frac{da}{dN}$ [$\frac{\mu m}{cycle}$]
1	0	2.74	3750	37500	Crack Initiation				
	40000	3.43	3750	37500	3.56	66.44	1.126	7.91	0.0260
	50000	3.69	3750	37500	3.85	66.44	1.127	8.23	0.0320
	60000	4.01	3750	37500	4.30	66.44	1.128	8.71	0.0580
	70000	4.59	3750	37500	4.80	66.44	1.129	9.21	0.0805
	75092	5.00	3750	37500	Sequence Change				
2	75093	5.00	3000	30000	Retardation				
	95000	5.72	3000	30000	6.28	53.15	1.133	8.45	0.0540
	115562	6.83	3000	30000	7.07	53.15	1.135	8.99	0.0550
	124105	7.30	3000	30000	7.41	53.15	1.136	9.21	0.1528
	125545	7.52	3000	30000	Sequence Change				
3	125546	7.52	2500	25000	Retardation				
	150000	8.42	2500	25000	8.94	44.29	1.140	8.46	0.0515
	170015	9.45	2500	25000	9.62	44.29	1.141	8.79	0.0661
	175004	9.78	2500	25000	9.91	44.29	1.142	8.92	0.0602
	179155	10.03	2500	25000	Sequence Change				
4	179156	10.03	2000	20000	Retardation				
	200000	10.56	2000	20000	10.89	35.43	1.145	7.50	0.0327
	219850	11.21	2000	20000	11.31	35.43	1.146	7.65	0.0369
	225000	11.40	2000	20000	11.76	35.43	1.147	7.81	0.0360
	245014	12.12	2000	20000	12.21	35.43	1.148	7.97	0.0361
	250000	12.30	2000	20000	12.42	35.43	1.149	8.04	0.0391
	255875	12.53	2000	20000	Sequence Change				
5	255876	12.53	1750	17500	Retardation				
	275000	13.04	1750	17500	13.33	31.00	1.151	7.30	0.0232
	300000	13.62	1750	17500	13.84	31.00	1.152	7.45	0.0265
	316237	14.05	1750	17500	14.31	31.00	1.154	7.58	0.0319
	332544	14.57	1750	17500	14.83	31.00	1.155	7.73	0.0346
	347588	15.09	1750	17500	Sequence Change				
6	347589	15.09	1500	15000	Retardation				
	387131	15.79	1500	15000	16.09	26.57	1.159	6.92	0.0212
	415000	16.38	1500	15000	16.55	26.57	1.160	7.03	0.0227
	430000	16.72	1500	15000	16.86	26.57	1.161	7.10	0.0158
	447712	17.00	1500	15000	17.21	26.57	1.162	7.18	0.0284
	462500	17.42	1500	15000	17.47	26.57	1.163	7.24	0.0203
	466926	17.51	1500	15000	Sequence Change				
7	466927	17.51	1500	15000	Retardation				
	500000	17.84	1250	12500	17.93	22.15	1.164	6.12	0.0075
	522783	18.01	1250	12500	18.13	22.15	1.164	6.15	0.0085
	550000	18.24	1250	12500	18.42	22.15	1.165	6.21	0.0077
	596529	18.60	1250	12500	18.72	22.15	1.166	6.26	0.0072
	629665	18.84	1250	12500	18.92	22.15	1.167	6.30	0.0053
	659800	19.00	1250	12500	19.31	22.15	1.168	6.37	0.0154
	700000	19.62	1250	12500	19.69	22.15	1.169	6.44	0.0052
	725000	19.75	1250	12500	19.88	22.15	1.170	6.47	0.0108
	748217	20.00	1250	12500					

Table B.6: Pre-crack data for face-sheet 6

Sequence []	N []	a [mm]	P _{min} [N]	P _{max} [N]	$\bar{a}$ [mm]	$\Delta\sigma$ [MPa]	c []	ΔK [MPa $\sqrt{m}$]	$\frac{da}{dN}$ [$\frac{\mu m}{cycle}$]
1	0	2.72	3750	37500	Crack Initiation				
	40000	3.18	3750	37500	3.36	66.44	1.126	7.68	0.0350
	50000	3.53	3750	37500	3.68	66.44	1.127	8.05	0.0300
	60000	3.83	3750	37500	4.09	66.44	1.128	8.49	0.0520
	70000	4.35	3750	37500	4.68	66.44	1.129	9.09	0.0650
	80000	5.00	3750	37500	Sequence Change				
2	80001	5.00	3000	30000	Retardation				
	100000	5.70	3000	30000	6.60	53.15	1.134	8.67	0.0611
	129273	7.49	3000	30000	Sequence Change				
3	129274	7.49	2500	25000	Retardation				
	150000	8.18	2500	25000	8.81	44.29	1.139	8.39	0.0504
	175000	9.44	2500	25000	9.60	44.29	1.141	8.78	0.0429
	182233	9.75	2500	25000	9.89	44.29	1.142	8.92	0.1729
	183852	10.03	2500	25000	Sequence Change				
4	183853	10.03	2000	20000	Retardation				
	225000	10.92	2000	20000	11.31	35.43	1.146	7.65	0.0390
	245006	11.70	2000	20000	12.12	35.43	1.148	7.93	0.0425
	264530	12.53	2000	20000	Sequence Change				
5	264531	12.53	1750	17500	Retardation				
	275000	12.72	1750	17500	13.09	31.00	1.150	7.23	0.0292
	300000	13.45	1750	17500	13.83	31.00	1.152	7.45	0.0304
	325000	14.21	1750	17500	14.68	31.00	1.155	7.69	0.0387
	349060	15.14	1750	17500	Sequence Change				
6	349061	15.14	1500	15000	Retardation				
	370126	15.53	1500	15000	15.85	26.57	1.158	6.87	0.0214
	400000	16.17	1500	15000	16.42	26.57	1.160	7.00	0.0200
	425000	16.67	1500	15000	16.96	26.57	1.161	7.12	0.0228
	450000	17.24	1500	15000	17.41	26.57	1.162	7.22	0.0342
	459931	17.58	1500	15000	Sequence Change				
7	459932	17.58	1250	12500	Retardation				
	525000	17.92	1250	12500	18.36	22.15	1.165	6.20	0.0087
	625682	18.80	1250	12500	18.90	22.15	1.167	6.29	0.0078
	650000	18.99	1250	12500	19.06	22.15	1.167	6.32	0.0070
	670000	19.13	1250	12500	19.25	22.15	1.168	6.36	0.0095
	695310	19.37	1250	12500	19.43	22.15	1.168	6.39	0.0075
	710000	19.48	1250	12500	19.58	22.15	1.169	6.42	0.0080
	735000	19.68	1250	12500	19.84	22.15	1.169	6.47	0.0077
	776829	20.00	1250	12500					

Table B.7: Pre-crack data for face-sheet 7

Sequence []	N []	a [mm]	P _{min} [N]	P _{max} [N]	$\bar{a}$ [mm]	$\Delta\sigma$ [MPa]	c []	ΔK [MPa $\sqrt{m}$]	$\frac{da}{dN}$ [$\frac{\mu m}{cycle}$]
1	0	2.85	3750	37500	Crack Initiation				
	30000	3.16	3750	37500	3.26	66.44	1.126	7.56	0.0190
	40000	3.35	3750	37500	3.45	66.44	1.126	7.78	0.0190
	50000	3.54	3750	37500	3.77	66.44	1.127	8.15	0.0460
	60000	4.00	3750	37500	4.14	66.44	1.128	8.54	0.0270
	70000	4.27	3750	37500	4.63	66.44	1.129	9.05	0.0720
	80000	4.99	3750	37500	Sequence Change				
2	80001	4.99	3000	30000	Retardation				
	100000	5.60	3000	30000	6.51	53.15	1.133	8.62	0.0611
	129785	7.42	3000	30000	7.46	53.15	1.136	9.24	0.1015
	130573	7.50	3000	30000	Sequence Change				
3	130574	7.50	2500	25000	Retardation				
	175000	9.38	2500	25000	9.72	44.29	1.142	8.83	0.0553
	187125	10.05	2500	25000	Sequence Change				
4	187126	10.05	2000	20000	Retardation				
	200000	10.25	2000	20000	10.83	35.43	1.144	7.48	0.0316
	236358	11.40	2000	20000	11.71	35.43	1.147	7.79	0.0391
	252235	12.02	2000	20000	12.33	35.43	1.148	8.01	0.0477
	265223	12.64	2000	20000	Sequence Change				
5	265224	12.64	1750	17500	Retardation				
	275000	12.78	1750	17500	13.19	31.00	1.151	7.26	0.0328
	300000	13.60	1750	17500	14.00	31.00	1.153	7.50	0.0357
	322439	14.40	1750	17500	14.73	31.00	1.155	7.70	0.0357
	340654	15.05	1750	17500	Sequence Change				
6	340655	15.05	1500	15000	Retardation				
	350000	15.26	1500	15000	15.75	26.57	1.158	6.84	0.0194
	400000	16.23	1500	15000	16.51	26.57	1.160	7.02	0.0200
	427468	16.78	1500	15000	17.08	26.57	1.161	7.15	0.0266
	450000	17.38	1500	15000	17.44	26.57	1.162	7.23	0.0273
	454396	17.50	1500	15000	Sequence Change				
7	454397	17.50	1250	12500	Retardation				
	475000	17.67	1250	12500	17.76	22.15	1.163	6.08	0.0068
	500000	17.84	1250	12500	18.14	22.15	1.164	6.16	0.0079
	575146	18.43	1250	12500	18.69	22.15	1.166	6.26	0.0095
	628607	18.94	1250	12500	19.35	22.15	1.168	6.38	0.0113
	700000	19.75	1250	12500	19.82	22.15	1.169	6.46	0.0186
	707545	19.89	1250	12500	19.96	22.15	1.170	6.49	0.0182
	715226	20.03	1250	12500					

Table B.8: Pre-crack data for face-sheet 8

Sequence []	N []	a [mm]	P _{min} [N]	P _{max} [N]	$\bar{a}$ [mm]	$\Delta\sigma$ [MPa]	c []	ΔK [MPa $\sqrt{m}$]	$\frac{da}{dN}$ [$\frac{\mu m}{cycle}$]
1	0	2.86	3750	37500	Crack Initiation				
	30000	3.26	3750	37500	3.38	66.44	1.126	7.70	0.0230
	40000	3.49	3750	37500	3.68	66.44	1.127	8.04	0.0370
	50000	3.86	3750	37500	4.07	66.44	1.128	8.47	0.0420
	60000	4.28	3750	37500	4.50	66.44	1.129	8.91	0.0440
	70000	4.72	3750	37500	4.95	66.44	1.130	9.36	0.0790
	75822	5.18	3750	37500	Sequence Change				
2	75823	5.18	3000	30000	Retardation				
	80000	5.31	3000	30000	5.91	53.15	1.132	8.19	0.0397
	110012	6.50	3000	30000	7.02	53.15	1.135	8.96	0.0635
	126379	7.54	3000	30000	Sequence Change				
3	126380	7.54	2500	25000	Retardation				
	150000	8.56	2500	25000	9.29	44.29	1.140	8.63	0.0625
	173211	10.01	2500	25000	Sequence Change				
4	173212	10.01	2000	20000	Retardation				
	200000	10.58	2000	20000	11.03	35.43	1.145	7.55	0.0359
	225081	11.48	2000	20000	12.01	35.43	1.148	7.90	0.0403
	251137	12.53	2000	20000	Sequence Change				
5	251138	12.53	1750	17500	Retardation				
	285892	13.65	1750	17500	14.06	31.00	1.153	7.51	0.0338
	309877	14.46	1750	17500	14.79	31.00	1.155	7.72	0.0415
	325770	15.12	1750	17500	Sequence Change				
6	325771	15.12	1500	15000	Retardation				
	350000	15.65	1500	15000	16.04	26.57	1.159	6.91	0.0253
	380431	16.42	1500	15000	16.77	26.57	1.161	7.08	0.0353
	400000	17.11	1500	15000	17.32	26.57	1.162	7.20	0.0399
	410523	17.53	1500	15000	Sequence Change				
7	410524	17.53	1250	12500	Retardation				
	447205	17.96	1250	12500	18.18	22.15	1.165	6.16	0.0155
	475002	18.39	1250	12500	18.58	22.15	1.166	6.24	0.0148
	500002	18.76	1250	12500	18.99	22.15	1.167	6.31	0.0180
	525000	19.21	1250	12500	19.37	22.15	1.168	6.38	0.0203
	540308	19.52	1250	12500	19.62	22.15	1.169	6.43	0.0196
	550000	19.71	1250	12500	19.87	22.15	1.170	6.47	0.0200
	566001	20.03	1250	12500					

Table B.9: Pre-crack data for face-sheet 9

Sequence []	N []	a [mm]	P _{min} [N]	P _{max} [N]	$\bar{a}$ [mm]	$\Delta\sigma$ [MPa]	c []	ΔK [MPa $\sqrt{m}$]	$\frac{da}{dN}$ [$\frac{\mu m}{cycle}$]
1	0	2.67	3750	37500	Crack Initiation				
	30000	3.04	3750	37500	3.14	66.44	1.125	7.43	0.0200
	40000	3.24	3750	37500	3.54	66.44	1.126	7.89	0.0590
	50000	3.83	3750	37500	4.17	66.44	1.128	8.58	0.0680
	60000	4.51	3750	37500	4.79	66.44	1.129	9.20	0.0884
	66224	5.06	3750	37500	Sequence Change				
2	66225	5.06	3000	30000	Retardation				
	108775	7.63	3000	30000	Sequence Change				
3	108776	7.63	2500	25000	Retardation				
	125000	8.15	2500	25000	8.84	44.29	1.139	8.41	0.0552
	150000	9.53	2500	25000	9.63	44.29	1.141	8.79	0.0597
	153352	9.73	2500	25000	9.89	44.29	1.142	8.92	0.0788
	157413	10.05	2500	25000	Sequence Change				
4	157414	10.05	2000	20000	Retardation				
	175000	10.65	2000	20000	11.06	35.43	1.145	7.56	0.0328
	200000	11.47	2000	20000	11.93	35.43	1.147	7.87	0.0467
	219470	12.38	2000	20000	12.44	35.43	1.149	8.05	0.0372
	222700	12.50	2000	20000	Sequence Change				
5	222701	12.50	1750	17500	Retardation				
	250000	13.22	1750	17500	13.70	31.00	1.152	7.41	0.0380
	275000	14.17	1750	17500	14.49	31.00	1.154	7.63	0.0497
	287685	14.80	1750	17500	14.93	31.00	1.155	7.76	0.0322
	295449	15.05	1750	17500	Sequence Change				
6	295450	15.05	1500	15000	Retardation				
	321872	15.80	1500	15000	16.24	26.57	1.159	6.96	0.0248
	357004	16.67	1500	15000	16.96	26.57	1.161	7.12	0.0322
	375000	17.25	1500	15000	17.39	26.57	1.162	7.22	0.0305
	383855	17.52	1500	15000	Sequence Change				
7	383856	17.52	1250	12500	Retardation				
	417915	17.91	1250	12500	18.19	22.15	1.165	6.16	0.0171
	450000	18.46	1250	12500	18.63	22.15	1.166	6.25	0.0187
	467615	18.79	1250	12500	18.83	22.15	1.166	6.28	0.0095
	475000	18.86	1250	12500	19.07	22.15	1.167	6.33	0.0168
	500000	19.28	1250	12500	19.41	22.15	1.168	6.39	0.0208
	512500	19.54	1250	12500	19.66	22.15	1.169	6.43	0.0184
	525000	19.77	1250	12500	19.84	22.15	1.169	6.47	0.0153
	534156	19.91	1250	12500	19.97	22.15	1.170	6.49	0.0250
	538561	20.02	1250	12500					

Table B.10: Pre-crack data for face-sheet 10

Sequence []	N []	a [mm]	P _{min} [N]	P _{max} [N]	$\bar{a}$ [mm]	$\Delta\sigma$ [MPa]	c []	ΔK [MPa $\sqrt{m}$]	$\frac{da}{dN}$ [$\frac{\mu m}{cycle}$]
1	0	2.77	3750	37500	Crack Initiation				
	30000	3.10	3750	37500	3.26	66.44	1.126	7.56	0.0310
	40000	3.41	3750	37500	3.77	66.44	1.127	8.14	0.0710
	50000	4.12	3750	37500	4.46	66.44	1.128	8.87	0.0670
	60000	4.79	3750	37500	4.90	66.44	1.130	9.31	0.1570
	61338	5.00	3750	37500	Sequence Change				
2	61339	5.00	3000	30000	Retardation				
	70000	5.31	3000	30000	6.08	53.15	1.132	8.31	0.0574
	96633	6.84	3000	30000	7.06	53.15	1.135	8.98	0.0746
	102529	7.28	3000	30000	7.41	53.15	1.136	9.21	0.0670
	106262	7.53	3000	30000	Sequence Change				
3	106262	7.53	2500	25000	Retardation				
	120000	8.08	2500	25000	8.95	44.29	1.140	8.46	0.0577
	150000	9.81	2500	25000	9.91	44.29	1.142	8.92	0.0818
	152324	10.00	2500	25000	Sequence Change				
4	152325	10.00	2000	20000	Retardation				
	175000	10.57	2000	20000	11.41	35.43	1.146	7.69	0.0422
	214848	12.25	2000	20000	12.38	35.43	1.149	8.02	0.0360
	221784	12.50	2000	20000	Sequence Change				
5	221785	12.50	1750	17500	Retardation				
	250000	13.35	1750	17500	13.78	31.00	1.152	7.43	0.0344
	275000	14.21	1750	17500	14.66	31.00	1.155	7.68	0.0444
	295063	15.10	1750	17500	Sequence Change				
6	295064	15.10	1500	15000	Retardation				
	325000	15.72	1500	15000	16.23	26.57	1.159	6.96	0.0270
	362796	16.74	1500	15000	17.13	26.57	1.162	7.16	0.0296
	389144	17.52	1500	15000	Sequence Change				
7	389145	17.52	1250	12500	Retardation				
	415703	17.94	1250	12500	18.18	22.15	1.165	6.16	0.0153
	446372	18.41	1250	12500	18.64	22.15	1.166	6.25	0.0198
	469082	18.86	1250	12500	19.11	22.15	1.167	6.33	0.0191
	495230	19.36	1250	12500	19.66	22.15	1.169	6.43	0.0257
	518558	19.96	1250	12500	19.98	22.15	1.170	6.49	0.0270
	520041	20.00	1250	12500					

Table B.11: Pre-crack data for face-sheet 11

Sequence []	N []	a [mm]	P _{min} [N]	P _{max} [N]	$\bar{a}$ [mm]	$\Delta\sigma$ [MPa]	c []	ΔK [MPa $\sqrt{m}$]	$\frac{da}{dN}$ [$\frac{\mu m}{cycle}$]
1	0	2.79	3750	37500	Crack Initiation				
	30000	3.03	3750	37500	3.20	66.44	1.125	7.49	0.0330
	40000	3.36	3750	37500	3.50	66.44	1.126	7.85	0.0280
	50000	3.64	3750	37500	3.82	66.44	1.127	8.20	0.0350
	60000	3.99	3750	37500	4.29	66.44	1.128	8.70	0.0600
	70000	4.59	3750	37500	4.93	66.44	1.130	9.33	0.1234
	75430	5.26	3750	37500	Sequence Change				
2	75431	5.26	3000	30000	Retardation				
	118697	6.98	3000	30000	7.26	53.15	1.135	9.11	0.0725
	126417	7.54	3000	30000	Sequence Change				
3	126418	7.54	2500	25000	Retardation				
	157619	8.59	2500	25000	9.11	44.29	1.140	8.54	0.0539
	176726	9.62	2500	25000	9.91	44.29	1.142	8.92	0.1072
	182041	10.19	2500	25000	Sequence Change				
4	182042	10.19	2000	20000	Retardation				
	200000	10.29	2000	20000	11.12	35.43	1.145	7.58	0.0330
	250000	11.94	2000	20000	12.23	35.43	1.148	7.97	0.0387
	264716	12.51	2000	20000	Sequence Change				
5	264717	12.51	1750	17500	Retardation				
	300000	13.64	1750	17500	14.32	31.00	1.154	7.59	0.0308
	344220	15.00	1750	17500	Sequence Change				
6	344221	15.00	1500	15000	Retardation				
	373940	15.46	1500	15000	15.67	26.57	1.157	6.82	0.0224
	392269	15.87	1500	15000	16.17	26.57	1.159	6.94	0.0222
	418803	16.46	1500	15000	16.83	26.57	1.161	7.09	0.0278
	445077	17.19	1500	15000	17.37	26.57	1.162	7.21	0.0348
	455123	17.54	1500	15000	Sequence Change				
7	455124	17.54	1250	12500	Retardation				
	475000	17.58	1250	12500	17.88	22.15	1.164	6.11	0.0127
	521582	18.17	1250	12500	18.33	22.15	1.165	6.19	0.0136
	545192	18.49	1250	12500	18.71	22.15	1.166	6.26	0.0120
	581820	18.93	1250	12500	19.32	22.15	1.168	6.37	0.0180
	624711	19.70	1250	12500	19.86	22.15	1.169	6.47	0.0174
	642532	20.01	1250	12500					

Table B.12: Pre-crack data for face-sheet 12

Sequence []	N []	a [mm]	P _{min} [N]	P _{max} [N]	$\bar{a}$ [mm]	$\Delta\sigma$ [MPa]	c []	ΔK [MPa $\sqrt{m}$]	$\frac{da}{dN}$ [$\frac{\mu m}{cycle}$]
1	0	2.87	3750	37500	Crack Initiation				
	30000	3.29	3750	37500	3.56	66.44	1.126	7.91	0.0530
	40000	3.82	3750	37500	4.20	66.44	1.128	8.61	0.0760
	50000	4.58	3750	37500	4.86	66.44	1.129	9.27	0.1269
	54334	5.13	3750	37500	Sequence Change				
2	54335	5.13	3000	30000	Retardation				
	75000	5.90	3000	30000	6.31	53.15	1.133	8.47	0.0557
	89553	6.71	3000	30000	7.18	53.15	1.135	9.06	0.0900
	100000	7.65	3000	30000	Sequence Change				
3	100001	7.65	2500	25000	Retardation				
	120863	8.42	2500	25000	8.76	44.29	1.139	8.37	0.0533
	133617	9.10	2500	25000	9.48	44.29	1.141	8.72	0.0605
	146021	9.85	2500	25000	9.94	44.29	1.142	8.94	0.0483
	149749	10.03	2500	25000	Sequence Change				
4	149750	10.03	2000	20000	Retardation				
	177096	10.72	2000	20000	11.06	35.43	1.145	7.56	0.0375
	195245	11.40	2000	20000	11.72	35.43	1.147	7.80	0.0488
	208364	12.04	2000	20000	12.28	35.43	1.148	7.99	0.0445
	218916	12.51	2000	20000	Sequence Change				
5	218917	12.51	1750	17500	Retardation				
	238809	13.25	1750	17500	13.77	31.00	1.152	7.43	0.0365
	267029	14.28	1750	17500	14.41	31.00	1.154	7.61	0.0326
	275000	14.54	1750	17500	14.82	31.00	1.155	7.73	0.0431
	287980	15.10	1750	17500	Sequence Change				
6	287981	15.10	1500	15000	Retardation				
	309564	15.57	1500	15000	16.05	26.57	1.159	6.91	0.0282
	343578	16.53	1500	15000	17.04	26.57	1.161	7.14	0.0317
	375749	17.55	1500	15000	Sequence Change				
7	375750	17.55	1250	12500	Retardation				
	436513	18.19	1250	12500	18.27	22.15	1.165	6.18	0.0119
	450000	18.35	1250	12500	18.52	22.15	1.166	6.23	0.0146
	472526	18.68	1250	12500	18.87	22.15	1.167	6.29	0.0236
	488216	19.05	1250	12500	19.12	22.15	1.167	6.33	0.0095
	501939	19.18	1250	12500	19.40	22.15	1.168	6.39	0.0206
	523300	19.62	1250	12500	19.83	22.15	1.169	6.46	0.0191
	544815	20.03	1250	12500					

Table B.13: Pre-crack data for face-sheet 13

Sequence []	N []	a [mm]	P _{min} [N]	P _{max} [N]	$\bar{a}$ [mm]	$\Delta\sigma$ [MPa]	c []	ΔK [MPa $\sqrt{m}$]	$\frac{da}{dN}$ [$\frac{\mu m}{cycle}$]
1	0	2.59	3750	37500	Crack Initiation				
	30000	3.28	3750	37500	3.38	66.44	1.126	7.70	0.0190
	40000	3.47	3750	37500	3.56	66.44	1.126	7.91	0.0180
	50000	3.65	3750	37500	3.89	66.44	1.127	8.27	0.0470
	60000	4.12	3750	37500	4.43	66.44	1.128	8.84	0.0610
	70000	4.73	3750	37500	4.92	66.44	1.130	9.33	0.1436
	72576	5.10	3750	37500	Sequence Change				
2	72577	5.10	3000	30000	Retardation				
	100000	6.07	3000	30000	6.79	53.15	1.134	8.80	0.0576
	125000	7.51	3000	30000	Sequence Change				
3	125001	7.51	2500	25000	Retardation				
	150000	8.36	2500	25000	8.75	44.29	1.139	8.36	0.0524
	164899	9.14	2500	25000	9.29	44.29	1.140	8.63	0.0287
	175000	9.43	2500	25000	9.76	44.29	1.142	8.85	0.0725
	184106	10.09	2500	25000	Sequence Change				
4	184107	10.09	2000	20000	Retardation				
	219067	10.50	2000	20000	10.95	35.43	1.145	7.52	0.0329
	246451	11.40	2000	20000	11.70	35.43	1.147	7.79	0.0425
	260580	12.00	2000	20000	12.27	35.43	1.148	7.99	0.0414
	273639	12.54	2000	20000	Sequence Change				
5	273640	12.54	2000	20000	Retardation				
	300000	13.22	2000	20000	13.55	35.43	1.152	8.42	0.0243
	327178	13.88	2000	20000	14.22	35.43	1.153	8.64	0.0410
	343529	14.55	2000	20000	14.79	35.43	1.155	8.82	0.0329
	358117	15.03	2000	20000	Sequence Change				
6	358118	15.03	1500	15000	Retardation				
	375000	15.24	1500	15000	15.47	26.57	1.157	6.78	0.0159
	403933	15.70	1500	15000	16.08	26.57	1.159	6.92	0.0163
	450000	16.45	1500	15000	16.69	26.57	1.160	7.06	0.0188
	475000	16.92	1500	15000	17.21	26.57	1.162	7.18	0.0208
	502941	17.50	1500	15000	Sequence Change				
7	502942	17.50	1250	12500	Retardation				
	546800	17.82	1250	12500	18.02	22.15	1.164	6.13	0.0081
	596065	18.22	1250	12500	18.40	22.15	1.165	6.20	0.0067
	650000	18.58	1250	12500	18.73	22.15	1.166	6.26	0.0079
	688092	18.88	1250	12500	19.09	22.15	1.167	6.33	0.0089
	735506	19.30	1250	12500	19.53	22.15	1.169	6.41	0.0130
	770135	19.75	1250	12500	19.91	22.15	1.170	6.48	0.0086
	807262	20.07	1250	12500					

Table B.14: Pre-crack data for face-sheet 14

B.2 Pre-crack Diagrams

Figure B.1: Crack propagation rates for face-sheet 1

Figure B.2: Crack propagation rates for face-sheet 2

Figure B.3: Crack propagation rates for face-sheet 3

Figure B.4: Crack propagation rates for face-sheet 4

Figure B.5: Crack propagation rates for face-sheet 5

Figure B.6: Crack propagation rates for face-sheet 6

Figure B.7: Crack propagation rates for face-sheet 7

Figure B.8: Crack propagation rates for face-sheet 8

Figure B.9: Crack propagation rates for face-sheet 9

Figure B.10: Crack propagation rates for face-sheet 10

Figure B.11: Crack propagation rates for face-sheet 11

Figure B.12: Crack propagation rates for face-sheet 12

Figure B.13: Crack propagation rates for face-sheet 13

Figure B.14: Crack propagation rates for face-sheet 14

B.3 Pre-crack Pictures

Figure B.15: Face-sheet 1 pre-crack

Figure B.16: Face-sheet 2 pre-crack

Figure B.17: Face-sheet 3 pre-crack

Figure B.18: Face-sheet 4 pre-crack

Figure B.19: Face-sheet 5 pre-crack

Figure B.20: Face-sheet 6 pre-crack

Figure B.21: Face-sheet 7 pre-crack

Figure B.22: Face-sheet 8 pre-crack

Figure B.23: Face-sheet 9 pre-crack

Figure B.24: Face-sheet 10 pre-crack

Figure B.25: Face-sheet 11 pre-crack

Figure B.26: Face-sheet 12 pre-crack

Figure B.27: Face-sheet 13 pre-crack

Figure B.28: Face-sheet 14 pre-crack

Appendix C

Ultrasonic C-Scans

C.1 Ultrasonic C-Scans of the Precured Patches

Figure C.1: C-scan of patch 1

Figure C.2: C-scan of patch 2

Figure C.3: C-scan of patch 3

Figure C.4: C-scan of patch 4

Figure C.5: C-scan of patch 5

Figure C.6: C-scan of patch 6

Figure C.7: C-scan of patch 7

Figure C.8: C-scan of patch 8

Figure C.9: C-scan of patch 9

Figure C.10: C-scan of patch 10

Figure C.11: C-scan of patch 11

Figure C.12: C-scan of patch 12

Figure C.13: C-scan of patch 13

Figure C.14: C-scan of patch 14

Appendix D

Material Properties

D.1 Mechanical Properties of Boron/epoxy 5521/4 Laminates

The following material properties for Textron Specialty Materials Boron 5521/4 for dry environmental condition were used in this research work [45, 52, 87].

Thickness:	$t_{p_1} = 0.132 \text{ mm (0.0052 in)}$
Longitudinal Elastic Modulus:	$E_{p_y} = 210 \text{ GPa (30.5 ksi)}$
Transverse Elastic Modulus:	$E_{p_x} = E_{p_z} = 25 \text{ GPa (3.6 ksi)}$
Shear Moduli:	$G_{p_{xy}} = G_{p_{yz}} = 7.24 \text{ GPa (1050 ksi)}$ $G_{p_{xz}} \approx 1 \text{ GPa (145 ksi)}$
Longitudinal Poisson's Ratio:	$\nu_{p_{xy}} = 0.025$
Transverse Poisson's Ratios:	$\nu_{p_{yz}} = 0.21$ $\nu_{p_{xz}} = 0.17$
Longitudinal CTE @21°C:	$\alpha_{p_y} = 4.61 \mu\epsilon/^{\circ}\text{C (2.56 } \mu\epsilon/^{\circ}\text{F)}$
Transverse CTE @21°C:	$\alpha_{p_x} = \alpha_{p_z} = 25.87 \mu\epsilon/^{\circ}\text{C (14.37 } \mu\epsilon/^{\circ}\text{F)}$
Longitudinal CTE @-56.5°C:	$\alpha_{p_y} = 4.16 \mu\epsilon/^{\circ}\text{C (2.31 } \mu\epsilon/^{\circ}\text{F)}$
Transverse CTE @-56.5°C:	$\alpha_{p_x} = \alpha_{p_z} = 25.40 \mu\epsilon/^{\circ}\text{C (14.11 } \mu\epsilon/^{\circ}\text{F)}$

The following coefficients of thermal expansion were measured as part of this research work:

Figure D.1: Coefficient of thermal expansion for boron/epoxy 5521/4 parallel to the fiber direction based on different reference temperatures

Figure D.2: Coefficient of thermal expansion for boron/epoxy 5521/4 perpendicular to the fiber direction based on different reference temperatures

D.2 Mechanical Properties of Aluminum 2024-T3

The following material properties for 2024-T3 aluminum were used in this research work [91]:

Thickness:	$t_s = 3.175 \text{ mm (0.125 in)}$
Elastic Moduli:	$E_{s_x} = E_{s_y} = E_{s_z} = 72.4 \text{ GPa (10.5 msi)}$
Shear Moduli:	$G_{s_{xy}} = G_{s_{yz}} = G_{s_{xz}} = 27.2 \text{ GPa (3.95 msi)}$
Poisson's Ratio:	$\nu_{s_{yx}} = \nu_{s_{zy}} = \nu_{s_{xz}} = 0.33$
Longitudinal CTE @21°C:	$\alpha_{s_y} = 23.45 \text{ } \mu\epsilon/^{\circ}\text{C (12.94 } \mu\epsilon/^{\circ}\text{F)}$
Transverse CTE @21°C:	$\alpha_{s_x} = \alpha_{s_z} = 23.29 \text{ } \mu\epsilon/^{\circ}\text{C (12.93 } \mu\epsilon/^{\circ}\text{F)}$
Longitudinal CTE @-56.5°C:	$\alpha_{s_y} = 22.77 \text{ } \mu\epsilon/^{\circ}\text{C (12.65 } \mu\epsilon/^{\circ}\text{F)}$
Transverse CTE @-56.5°C:	$\alpha_{s_x} = \alpha_{s_z} = 22.70 \text{ } \mu\epsilon/^{\circ}\text{C (12.61 } \mu\epsilon/^{\circ}\text{F)}$
Tensile Yield Stress:	$\sigma_{s_{yt}} = 331 \text{ MPa (48 ksi)}$
Compressive Yield Stress:	$\sigma_{s_{yc}} = 276 \text{ MPa (40 ksi)}$

The following coefficients of thermal expansion were measured as part of this research work:

Figure D.3: Coefficient of thermal expansion for 2024-T3 aluminum parallel to the rolling direction based on different reference temperatures

Figure D.4: Coefficient of thermal expansion for 2024-T3 aluminum perpendicular to the rolling direction based on different reference temperatures

D.3 Mechanical Properties of FM 73M

The following material properties for Cytec's FM 73M with no environmental prebond or postbond exposure were used in this research work [27].

Thickness:	$t_a = 0.25 \text{ mm (0.010 in)}$
Elastic Moduli @21°C:	$E_{ax} = E_{ay} = E_{az} = 2.19 \text{ GPa (318 ksi)}$
Shear Modulus @21°C:	$G_{axy} = G_{ayz} = G_{axz} = 844 \text{ MPa (122 ksi)}$
Elastic Moduli @-56.5°C:	$E_{ax} = E_{ay} = E_{az} = 2.35 \text{ GPa (341 ksi)}$
Shear Modulus @-56.5°C:	$G_{axy} = G_{ayz} = G_{axz} = 905 \text{ MPa (131 ksi)}$
Poisson's Ratio:	$\nu_{axy} = \nu_{ayz} = \nu_{axz} = 0.3$
CTE:	$\alpha_{ax} = \alpha_{ay} = \alpha_{az} \approx 50 \mu\epsilon/^{\circ}\text{C (27.78 } \mu\epsilon/^{\circ}\text{F)}$
Elastic Shear Stress:	$\tau_{ay} = 17.3 \text{ MPa (2510 psi)}$

Note: - The fiber orientation is parallel to the y-axis.

- The rolling direction of the aluminum coincides with the fiber direction for the AMRL specimen.
- Approximate values were used where no data were found in the literature or experimentally measured (indicated with $\approx$).
- The transverse Poisson's ratio for the boron/epoxy ν_{pzy} was calculated based on the elastic moduli and the major Poisson's ratio.
- The FM73M material properties were chosen for the fillets at the ply drop offs.

Appendix E

Measured Thermal and Thermal Residual Strains

Figure E.1: Gauge 1 - Thermal and thermal residual strains [Aluminum $\perp$] Figure E.2: Gauge 2 - Thermal and thermal residual strains [Aluminum $\parallel$] Figure E.3: Gauge 3 - Thermal and thermal residual strains [Aluminum $\parallel$]

Figure E.4: Gauge 4 - Thermal and thermal residual strains [Aluminum ⊥]

Figure E.5: Gauge 5 - Thermal and thermal residual strains [Aluminum ||]

Figure E.6: Gauge 6 - Thermal and thermal residual strains [Aluminum ⊥]

Figure E.7: Gauge 7 - Thermal and thermal residual strains [Aluminum ||]

Figure E.8: Gauge 8 - Thermal and thermal residual strains [Aluminum $\perp$]

Figure E.9: Gauge 9 - Thermal and thermal residual strains [Aluminum $\perp$]

Figure E.10: Gauge 10 - Thermal and thermal residual strains [Aluminum ||]

Figure E.11: Gauge 11 - Thermal and thermal residual strains [Aluminum $\perp$]

Figure E.12: Gauge 12 - Thermal and thermal residual strains [Aluminum ||]

Figure E.13: Gauge 13 - Thermal and thermal residual strains [Aluminum ||]

Figure E.14: Gauge 14 - Thermal and thermal residual strains [Aluminum $\perp$] Figure E.15: Gauge 15 - Thermal and thermal residual strains [Aluminum $\perp$]

Figure E.16: Gauge 16 - Thermal and thermal residual strains [Aluminum ||]

Figure E.17: Gauge 17 - Thermal and thermal residual strains [Aluminum 45 $^{\circ}$] Figure E.18: Gauge 18 - Thermal and thermal residual strains [Aluminum $\perp$]

Figure E.19: Gauge 19 - Thermal and thermal residual strains [Aluminum ||]

Figure E.20: Gauge 20 - Thermal and thermal residual strains [Aluminum 45 $^{\circ}$] Figure E.21: Gauge 21 - Thermal and thermal residual strains [Aluminum $\perp$]

Figure E.22: Gauge 22 - Thermal and thermal residual strains [Aluminum ||]

Figure E.23: Gauge 23 - Thermal and thermal residual strains [Boron/epoxy ||]

Figure E.24: Gauge 24 - Thermal and thermal residual strains [Boron/epoxy ||]

Figure E.25: Gauge 25 - Thermal and thermal residual strains [Boron/epoxy ||]

Figure E.26: Gauge 26 - Thermal and thermal residual strains [Boron/epoxy ||]

Figure E.27: Gauge 27 - Thermal and thermal residual strains [Boron/epoxy ||]

Figure E.28: Gauge 28- Thermal and thermal residual strains [Boron/epoxy ||]

Figure E.29: Gauge 29 - Thermal and thermal residual strains [Boron/epoxy ||]

Figure E.30: Gauge 30 - Thermal and thermal residual strains [Boron/epoxy ||]

Figure E.31: Gauge 31 - Thermal and thermal residual strains [Boron/epoxy III]

Figure E.32: Gauge 32 - Thermal and thermal residual strains [Boron/epoxy III]

Figure E.33: Gauge 33 - Thermal and thermal residual strains [Boron/epoxy III]

Figure E.34: Gauge 34 - Thermal and thermal residual strains [Boron/epoxy ⊥]

Figure E.35: Gauge 35 - Thermal and thermal residual strains [Boron/epoxy ||]

Figure E.36: Gauge 36 - Thermal and thermal residual strains [Boron/epoxy ⊥]

Figure E.37: Gauge 37 - Thermal and thermal residual strains [Boron/epoxy ||]

Figure E.38: Gauge 38 - Thermal and thermal residual strains [Boron/epoxy $\perp$] Figure E.39: Gauge 39 - Thermal and thermal residual strains [Boron/epoxy $\perp$]

Figure E.40: Gauge 40 - Thermal and thermal residual strains [Boron/epoxy ||]

Figure E.41: Gauge 41 - Thermal and thermal residual strains [Boron/epoxy $\perp$]

Figure E.42: Gauge 42 - Thermal and thermal residual strains [Boron/epoxy ||]

Figure E.43: Gauge 43 - Thermal and thermal residual strains [Boron/epoxy ||]

Figure E.44: Gauge 44 - Thermal and thermal residual strains [Boron/epoxy ⊥]

IMAGE EVALUATION TEST TARGET (QA-3)

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