

**WATER DISTRIBUTION NETWORK
REHABILITATION:
Selection and Scheduling of Pipe Rehabilitation Alternatives**

by

Yehuda Kleiner

**A thesis submitted in conformity with the requirements
for the degree of Doctor of Philosophy
Graduate Department of Civil Engineering
University of Toronto**

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ABSTRACT

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The most expensive component of a water supply system is the distribution network. Aging water supply infrastructure, coupled with the continuous stress placed on the pipes by operational and environmental conditions, have led to system deterioration which manifests itself in increased operation and maintenance costs, water losses, reduction in the quality of service and reduction in the quality of water supplied.

In this thesis, a methodology is proposed to select, for each pipe in an existing network, the rehabilitation alternative and the time of its implementation, so as to minimize the cost of the rehabilitation investment and all maintenance costs over a pre-defined time horizon and subject to certain constraints. The methodology explicitly considers the deterioration over time of both the structural integrity and the hydraulic capacity of every pipe in the system. The cost associated with each pipe in the network is calculated as the present value of an infinite stream of costs. The proposed method, named the multistage network rehabilitation analysis procedure (MNRAP) is based on a dynamic programming approach combined with partial and (sometimes) implicit enumeration schemes.

A computer program was developed to implement the methodology. Subsequently, the validity of the MNRAP was established in two ways:

1. Some simple systems were exhaustively enumerated for all possible combinations of rehabilitation alternatives selection and scheduling, and their minimum cost sequences were compared to those obtained by applying MNRAP. An excellent agreement was demonstrated between the two sets of results.

2. A study was conducted in which six water utility managers from the Greater Toronto Area and the Region of Waterloo (the participants) were presented with a sample water distribution system including all pertinent data that would typically be available when preparing a long-term rehabilitation program. The participants were required to implement their best engineering judgment and analysis tools in determining an optimal rehabilitation policy, subject to stated constraints. These policies were then compared to that obtained by the MNRAP. The comparison demonstrated the advantages of MNRAP over existing analysis practices; (a) in its comprehensiveness and (b) in the explicit, robust manner with which the economics and the hydraulics of the water distribution system are simultaneously considered over a planning time horizon.

In conclusion, the MNRAP promises to provide a valuable decision support system for engineers and decision makers in devising long-term network rehabilitation plans.

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1. INTRODUCTION

1.1 General

A safe, reliable and efficient water supply system is essential to any community - urban or rural. In a water supply system that includes the supply source works, treatment plant, distribution system (water mains) and storage facilities, the most expensive component is the distribution system. In North America, the first piped water supply system was built in Boston, in 1652 (Mays, in the ASCE - Task Committee on Risk and Reliability Analysis of Water Distribution Systems, 1989). Since then, millions of miles of pipes have been laid, and it is not uncommon (especially in older cities) to have pipes 100 years and older still in service.

The aging of water supply infrastructure systems, coupled with the continuous stress placed on these systems by operational and environmental conditions, have led to their deterioration which manifests itself in the following:

- Increased rate of pipe breakage due to deterioration in pipe structural integrity. This in turn causes increased operation and maintenance costs, increased loss of (treated) water and social costs such as loss of service, disruption of traffic, disruption of business and industrial processes, disruption of residential life and loss of landscape vegetation. In addition, special provisions (e.g. backup tanks) associated with low-reliability water distribution systems may be required.
- Decreased hydraulic capacity of pipes in the systems, which results in increased energy consumption and disrupts the quality of service to the public (Adams and Heinke. 1987).
- Deterioration of water quality in the distribution system due to the condition of inner surfaces of pipes which may result in taste, odour and aesthetic problems in the supply water and even public health problems in extreme cases.

It has been reported that the distribution system often involves 80% of the total expenditure in drinking water supply systems (Clark and Gillian, 1977). Given the reality of scarce capital resources, it is imperative that a comprehensive methodology be developed to assist planners and decision makers in finding the best (most cost-effective) rehabilitation policy that addresses the issues of safety, reliability, quality and efficiency.

The objective of this thesis is to present a method for the selection and scheduling of pipe rehabilitation alternatives that minimizes system rehabilitation investment and maintenance costs over a pre-selected lifetime horizon, subject to certain physical and hydraulic constraints. The proposed method is based on a dynamic programming approach combined with partial and (sometimes) implicit enumeration schemes.

1.2 Problem Scope and Thesis Scope

In the present context, to optimize a system means to maximize its net benefits to society. In order to measure net benefits, one has to evaluate the benefits and costs associated with the system. The benefits of a water distribution system are vast since modern urban centers could not exist without them; thus, the fundamental need for a water distribution system has been taken for granted. Consequently, the benefits will be measured by the system's performance and quality of service that the public receives. The performance of a water distribution system may be measured by the degree to which the following objectives are accomplished:

- The system should be economically efficient.
- The system should provide the demand for water at an acceptable residual pressure during an acceptable portion of the time. The exact specification of acceptable levels of pressure and time (e.g., provide demand flow at a minimum 30m pressure for 99% of the time) are often subject to regulations and may vary with locale.
- The system should be capable of providing emergency flows (e.g., for fire fighting) at an acceptable pressure.
- The system should have an acceptable level of reliability.

- The system should provide safe drinking water.
- The system should provide water that is acceptable to the consumer in terms of aesthetics, odour and taste.

The cost of a water distribution system is comprised of all direct, indirect and social costs that are associated with:

- Capital investment in system design, installation and upgrading.
- System operation – energy cost, materials, labour, monitoring, etc.
- System maintenance – inspection, breakage repair, rehabilitation, etc.

Thus, a water distribution system has multiple objectives, of which all but one (economic efficiency) are either difficult to quantify (e.g., reliability, water quality and aesthetics) or difficult to evaluate in monetary terms (e.g., level of service) or both. The situation is somewhat simpler concerning the cost aspects, where direct and indirect costs can be evaluated with relative ease, and social costs can (with some effort) be assessed, albeit with less certainty.

There are various methods to handle multiple objectives and criteria which generally are non-commensurable and are expressed in different units. However, some of these techniques involve explicit (or implicit) assignment of monetary values to all objectives (e.g., linear scoring method, goal programming) which may introduce biases (e.g., trying to assign a monetary value to reliability or to water quality). On the other hand, other multi-objective evaluation techniques are suitable for only a moderate number of alternatives (e.g., surrogate worth tradeoff method, utility matrices) which makes them unsuitable for handling the vast number of rehabilitation measures and scheduling alternatives. An alternative approach is to formulate the problem as a “traditional” optimization problem (see discussion of optimization techniques in Appendix 2) in which the optimization criterion is minimum cost, while all the components that cannot be assigned monetary values are taken into consideration as constraints. A general formulation for the full scope of the problem could then be expressed as follows:

Minimize: {capital investment + operation costs + maintenance costs + rehabilitation costs }

Subject to:

- physical/hydraulic constraints (e.g., mass conservation, continuity equations, etc.)
- network topology constraints (e.g., layout of water sources, streets, tanks, etc.)
- supply pressure head boundaries (i.e., minimum and maximum residual pressure head)
- minimal level of reliability constraints
- minimal level of water quality constraints
- available equipment constraints (e.g., treatment plants, pumps, pipes, appurtenances, etc.)

All costs and constraints are considered for the entire life of the system. (The “life of a system” is a somewhat vague idea which is further discussed in section 3.5.)

It appears at present that any attempt to handle the full scope of the problem would be overly ambitious in light of current knowledge and computational tools available.

The objective of this thesis is to develop a methodology to select, for each pipe in an existing network, the rehabilitation alternative and its implementation timing, so as to minimize the cost of the rehabilitation investment and all subsequent maintenance costs (see problem statement in section 3) over a pre-defined time horizon and subject to certain constraints. The decision variables are therefore (1) the type of rehabilitation alternative (e.g., replace with the same diameter pipe, replace with a larger diameter pipe, reline, etc.) and (2) the timing of its implementation, for every pipe in the distribution network. Every constraint in the system must include at least one decision variable in a mathematical formulation that depicts the effect that this variable has on the constrained activity.

Although it is evident that pipe condition affects water quality, there is no available research that suggests any mathematical relationship to the decision variables in the proposed methodology (see section 2.5 – water quality in distribution systems). Consequently, although water quality is

not explicitly considered, the formulation allows for the incorporation of a pertinent constraint should the effect of pipe condition on water quality be quantified in the future.

Defining the reliability of a water supply system is by itself a major undertaking. Reliability criteria may include frequency of failure, duration of down time, severity of the consequences of failure, etc. Many of the methodologies that have been suggested for evaluating system reliability are either limited to certain types of systems, or pertain to network topology (which is constant for rehabilitation issues), or are computationally intensive (a discussion of reliability issues is provided in section 2.4). Consequently, distribution system reliability is also not explicitly considered in the proposed methodology. However, here too, due to the formulation of this methodology, there is no theoretical obstacle to incorporating such a constraint.

The supply pressure and flow rates in a water distribution system change constantly during normal operation. In addition, the hydraulic capacity of the network typically diminishes over time. Consequently, in order to calculate system energy cost over a time horizon, it is necessary to integrate the energy usage over the entire time horizon, taking into account both demand patterns throughout the year and headloss increases from year to year. It is assumed here that supply pressure (at the pumps) is not significantly influenced by small incremental changes occurring in the distribution system's hydraulic capacity due to deterioration on one hand and rehabilitation on the other (see discussion on cost of energy in section 3.8).

Consequently, energy cost is not explicitly considered in the proposed methodology.

However, it should be noted that the computational burden of numerous extended time simulations is the only obstacle to incorporating energy cost into the methodology.

It should be emphasized that the complexity of the problem lies not in the number of the various constraints but rather that (a) due to deterioration, system hydraulic capacity diminishes over time and maintenance and repair costs increase over time; and (b) any change in any pipe in a distribution network causes a redistribution of flows throughout the network. Consequently, the operational mode of any pipe in the system depends on the characteristics

of all other pipes. As a result, the rehabilitation of any pipe in the network will influence the choice of pipes to be rehabilitated in the future, as well as the timing and type of future rehabilitation projects. Furthermore, knowledge of rehabilitation projects to be implemented in the future is required to make an optimal choice of rehabilitation project in the present.

1.3 Thesis Outline

The relevant literature is reviewed in Chapter 2. The topics covered include existing water distribution systems rehabilitation models (with and without optimization), reliability of distribution systems, and water quality in distribution pipes. An overview of rehabilitation technologies and of commonly used optimization techniques is provided (for reference) in Appendices 1 and 2, respectively.

The problem scope is defined in Chapter 3. A cost function is formulated for the objective function, a pipe friction deterioration model is adopted and the complete mathematical formulation of the problem is presented.

The solution methodology, which is a dynamic programming approach which incorporates partial and (sometimes) implicit enumeration schemes (branch and bound) is presented in Chapter 4. The methodology is named the Multistage Network Rehabilitation Analysis Procedure MNRAP.

Chapter 5 comprises a detailed example which is presented step-by-step with all intermediate results.

Chapter 6 provides a discussion that utilizes this example to highlight some key issues in the MNRAP.

Chapter 7 contains a description of two procedures that were applied to test the validity of the results obtained from the MNRAP.

In Chapter 8, the computer implementation (written in C language) of the MNRAP is presented.

Final discussions, conclusions and suggestions for future research are presented in Chapter 9.

2. LITERATURE REVIEW

2.1 Introduction

A comprehensive list of criteria to be considered in deciding whether a pipe should be replaced was outlined by Stacha (1978). These criteria include comparison of costs (maintenance and capital), evaluation of hydraulic carrying capacity of the pipe, effect of pipe condition on water quality, risks of pipe condition to the safety of people and property, evaluation of system performance in predicted future demands, and frequency of failure.

While some of these criteria are explicitly quantifiable (e.g., maintenance cost, capital investment, hydraulic carrying capacity at present and for future demands), others like risk, safety, reliability and some components of social costs associated with failure may require surrogate or implicit evaluation techniques to be quantified.

To date, there has been no attempt to explicitly incorporate all these criteria into a single model that would enable a comprehensive analysis of water main rehabilitation strategies. It seems that such an undertaking would be far too ambitious at present since some of the phenomena (e.g. water quality deterioration in the distribution system) are not fully understood and since such a model would likely require tremendous computing resources.

In this literature review, special attention is given to existing rehabilitation models and an extensive reference list is provided. The issues of water quality in distribution systems and reliability of distribution systems are discussed on a more introductory level. The purpose of this discussion is to illustrate the breadth of each of these topics, and to provide the reader with some idea about the complexity of integrating all these issues. Some of the more specific points that are pertinent to the proposed methodology are discussed (and references are provided) in the relevant section, rather than in the literature review. For the reader not familiar with all of the terminology used in the subsequent sections, overviews are provided

about existing pipe rehabilitation technologies and practices in Appendix 1 and about optimization methods in Appendix 2.

In addition, since the proposed methodology uses the respective ages of pipes as one of the decision variables in the model, a separate section is provided in which the premise that pipe age may be an effective measure of its condition is discussed.

2.2 Existing Rehabilitation Models

Shamir and Howard (1979) used regression analysis to obtain a pipe break prediction model that relates a pipe's breakage to its age. This model is subsequently used to analyze the cost of pipe replacement in terms of the present value of both break repair and capital investment. This analysis shows that a time exists at which the total cost is minimized.

Some limitations of Shamir and Howard's approach are:

- The data for their regression model were derived from pools of data that were grouped by similar characteristics. Hence, it was argued by some, that the results are suitable for drawing conclusions about groups of pipes rather than individual pipes in the network.
- The analysis neglects the cost of break repair after pipe replacement. This may not be the case for low discount rates and for older pipes where the optimal (minimum cost) replacement time is in the near future (it is shown later that the model can be reformulated to include breakage repair costs after replacement).
- It was argued in later research that break prediction based on pipe age only is quite limited, as other factors such as pipe diameter, length, pressure, type, soil corrosivity and stability, land development, number of previous breaks and time to the first break are contributing factors and hence may be used as predictive variables in addition to pipe age (Marks, 1985).
- This model does not give any consideration to the hydraulic capacity of the pipe.

Walski and Pelliccia (1982) proposed an approach very similar to that of Shamir and Howard, i.e. an exponential dependency of breakage rate on the age of a pipe. They, however, observed that age is not the only determining factor in the prediction of breakage rate, and consequently included two other factors in their model. One factor accounted for known previous breaks in the pipe, based on an observation that once a pipe broke it was more likely to break again. The second factor accounted for observed differences in breakage rates in larger diameter pipes. Walski and Pelliccia also elaborated much more on the cost of break repair and how to account for all its components. The cost analysis they used for determining optimal pipe replacement is similar to that of Shamir and Howard. Although Walski and Pelliccia addressed some of the limitations of the Shamir and Howard model, they too do not consider repair costs past replacement time and they do not address the hydraulic capacity of the pipe.

Walski (1987) suggested replacement rules for water mains based on the assumption of exponential dependency of breakage rates on the age of a pipe. In this model the author incorporated the cost of water losses through leaking pipes and the cost of broken valve replacement. The annual water losses and valve breaks are also assumed to increase exponentially with time.

Clark et al. (1982) proposed another regression model based on the observation that a lag period occurs between the pipe's installation and the first break. Two equations were developed – one to predict the time elapsed until the first break occurred and the second to predict the number of subsequent breaks. Similar to previously described models, here too an exponential growth was assumed for the breakage rate (after the first break). Repair costs after replacement were not considered and hydraulic capacity was not addressed.

Marks (1985) led a project that encompassed the water distribution systems in New Haven, Connecticut and Cincinnati, Ohio. Multiple regression techniques were used to determine variables that can predict pipe breakage rate. The variables found to affect breakage rate were pipe diameter, length of pipe section (node to node), age, pressure, type, soil corrosivity, intensity of land development (which may be viewed as a surrogate measure for external loads on

pipes), number of previous breaks, time to the second break and period of installation¹. The cost analysis for determining optimal pipe replacement is similar to that of Shamir and Howard. The authors discuss some considerations in formulating a comprehensive approach to pipe replacement scheduling and prioritizing (e.g., minimum cost, relative importance of the pipe, proximity to other high risk pipes, hydraulic capacity, combining pipe replacement with the public works facilities of other utilities, etc.). These considerations however, are mentioned in a general discussion only are not translated into a course of action.

Elstad, Byer and Adams (1987) analyzed the energy costs associated with a pipeline and proposed a model in which the decision to rehabilitate a pipe depends on its current coefficient of friction. This model however does not consider maintenance costs and is limited to water mains which are directly connected to a pump and are not part of a looped network.

Goulter, together with various co-authors (Goulter and Kazemi, 1988; 1989; Goulter, Davidson and Jacobs, 1993) observed temporal and spatial clustering of water main failures. An explanatory model was developed as follows: (a) a process termed “cross-referencing scheme” was used for identifying clusters in certain types of pipes (e.g. cast iron); (b) a nonlinear regression analysis was used to derive parameters from these clusters; and (c) the parameters were applied to a non-homogeneous Poisson distribution that predicted the probability of subsequent breaks in a pipe, given that the first break already occurred.

The model was verified with data from the City of Winnipeg, and the authors reported a good agreement between the observed and predicted distribution of breakage events. This agreement however diminished as the mean number of subsequent breakage events increased. No economical nor hydraulic capacity considerations were introduced in this model. In addition, the model was restricted to predicting only subsequent failures, and could not be used to predict the occurrence of a first failure.

¹ It appears that pipes installed in different time periods, especially within the 20th century, display substantially different deterioration rates probably due to differences in materials, manufacturing technologies and construction practices of pipelines.

Woodburn et al. (1987) proposed a more comprehensive model designed to determine the minimum cost rehabilitation schedule of components in a water supply network. The model employed a nonlinear programming procedure (GRG2) with a hydraulic simulation program (KYPIPE) to determine which pipes should be replaced, rehabilitated or left alone.

The objective function was to minimize cost, where the cost function included the cost of rehabilitated pipes, replaced pipes, break repair in existing and rehabilitated pipes (replaced pipes were assumed to have negligible repair cost), cost of expanded pumping stations and the cost of pumping energy. The nodal residual pressure head was treated in two optional ways – one as a constraint (minimum residual pressure head), and the other in the form of a penalty function for residual pressure heads below a pre-determined threshold value.

Each reach (pipe) in the network was divided into three sections – rehabilitated, replaced and intact. The respective lengths of these sections were the decision variables in the model. The constraints in the model included conservation of mass and energy in the supply network, non-negativity constraints and constraints requiring the sum of the decision variables (in each reach) being equal to the physical length of the reach. If the option of treating residual pressure head as a constraint was chosen, an additional set of constraints was required depicting the minimum residual pressure head at every node.

The model of Woodburn et al. was the first attempt to optimize rehabilitation/replacement of pipes in a distribution network in a comprehensive manner, encompassing capital and maintenance cost as well as hydraulic carrying capacity of the pipes and energy costs.

A major limitation of the proposed model is its “flatness” with respect to time. The model can not schedule rehabilitation/replacement at different times during a specified time horizon, but rather the results indicate only whether a pipe segment should be rehabilitated or replaced in the present time. In effect, the model handles a distribution system with fixed parameters rather than as a dynamic entity whose characteristics (namely hydraulic carrying capacity) deteriorate with time.

Su and Mays (1988) proposed an alternative to the Woodburn et al. model in which the pipe roughness coefficient, the nodal demand and nodal residual pressure requirements are considered as random variables. The respective constraints are then expressed in a probabilistic form. A chance constrained approach is used to transform the probabilistic model into a deterministic equivalent. The deterministic model is then decomposed into a master problem which enumerates combinations of rehabilitated/replaced/intact pipes, and a subproblem which uses a non-linear programming solver (GRG2) to optimize diameters within each combination. The network solver KYPIPE was used here as well to solve the hydraulic constraint equations. After the subproblem is solved, the enumerated (optimized) combinations are then compared to each other, and the one with the least cost is selected.

It seems that the probabilistic approach in this model provided some insight into the uncertainties inherent in a real-life distribution system. However, aside from the fact that this model has the same limitations as the Woodburn et al. model (namely no time dimension), explicit enumeration of all possible combinations is impractical even for systems of moderate size. Subsequently, another model was later developed (see Kim and Mays, 1994, following) in which a process of implicit enumeration (branch and bound) is employed in order to reduce the number of combinations from which the optimal solution is to be identified.

Kim and Mays (1994) proposed a model which combines a master problem that generates possible combinations of pipes to rehabilitate, replace or leave as is. These combinations are then solved for optimal diameters as a subproblem and a network solver is used to verify conformity to hydraulic constraints.

The objective function in this model is to minimize cost, comprising capital cost for rehabilitation/replacement, pipe repair cost and energy cost. All costs are taken at present value. The decision variables are integer (binary) variables assigned to each pipe to represent yes/no replacement, yes/no rehabilitation or yes/no leave as is.

The constraints in the model include hydraulic constraints (e.g., conservation of mass and energy in the network, conformity to minimum or maximum residual nodal pressure head, pump characteristics, etc.) and binary constraints ensuring that only one option (rehabilitate/replace/do nothing) is selected for every pipe and that the appropriate set of parameters (C values, breakage rates) is assigned the respective selection.

The solution methodology separates the general problem into a master problem and subproblems. The master problem employs an implicit enumeration technique to generate combinations of possible solutions (represented as binary values). Every such possible solution is then sent to the subproblem solver. The subproblem solver employs a nonlinear programming procedure (GRG2) to find optimal pipe diameters and pump horsepower for this condition. At this stage, a network solver (KYPIPE) is used to ensure conformity to the hydraulic constraints. When the subproblem is solved, the (optimized) cost of the possible solution at hand is sent back to the master problem solver. The master problem solver then uses a branch and bound process to compare the optimized solutions, to bound the nonfeasible ones and to branch the feasible ones to generate further combinations of possible solutions. The process proceeds until no further improvement is found.

The authors claim that the process of branch and bound that is used in this model substantially reduces the number of trial solutions before arriving at the optimal solution.

This model has the same limitations as the Woodburn et al. model described previously, i.e. it is “flat” with respect to time. The results may only indicate whether a pipe should be replaced, rehabilitated or left intact at the time of analysis. No optimal scheduling of rehabilitation/replacement into the future is possible. No consideration is given to the fact that the hydraulic performance of a distribution system deteriorates over time.

Arulraj and Suresh, 1995 introduced the concept of significance index (SI) which is an optimality criterion that can be applied both in prioritizing rehabilitation measures for pipes and designing new water distribution networks. SI is a dimensional index defined for each pipe

as $LQ/(CD)$ where L =pipe length, Q =flow in the pipe, C =Hazen Williams friction coefficient and D =pipe diameter. The authors also defined the critical pipe in a network as the pipe in which the replacement/rehabilitation would result in the maximum change (increase) in nodal heads throughout the network.

The authors found empirically that the pipe with the highest SI is the critical pipe in the network. They then outline a procedure in which critical pipes are replaced until a desired nodal head criterion is achieved. When applied to the design of new systems, SI is modified to include the cost of pipe per unit length in the denominator.

The significance index approach is appealing as a "common sense" approach, however, the following limitations should be pointed out: (a) it is not certain that the pipe with the highest SI is indeed the critical pipe; (b) it is not certain that replacing the critical pipe is the most cost-effective policy; (c) pipes can be replaced with the same diameter pipe or a different ones, in which case critical pipe status can change; (d) pipe deterioration in time is not considered; and (e) maintenance costs are not considered.

2.3 Pipe Age as a Decision Variable

As described in the previous section, several researchers observed that pipe age cannot generally be considered as the only factor affecting its condition. Diameter, material type, soil corrosively and stability and operating conditions are also contributing factors.

O'Day (1982) discussed two distribution systems in the Severn-Trent district of England (reported by Newport) and New York City. In both cities, it was observed that pipe condition could be predicted quite accurately from age, if geographical location was considered as one of the main contributing factors. Clark and Goodrich (1989) supported this observation (of geographical location as a contributing factor) after analyzing data from Cincinnati, Ohio and New Haven, Connecticut. Habibian (1992) supported this observation as well, based on

experience accumulated in establishing a comprehensive data base for the water utility of the Washington Suburban Sanitary Commission (WSSC) in Maryland.

Jacobs and Karney (1994) in consideration of the clustering phenomenon in pipe breaks that was observed in Winnipeg (Goulter and Kazemi, 1988; Goulter and Kazemi, 1989; Goulter, Davidson and Jacobs, 1993) defined independent breaks as breaks that occur more than 90 days after and/or more than 20 metres from a previous break. By their definition, an independent break is often the first in a cluster of breaks. They found a linear relation between independent breakage rate and pipe length, i.e. clusters of pipe breaks are uniformly distributed. In addition, they found that adding pipe age into the regression model improved the predictive power of the model marginally for relatively new pipes and significantly for old pipes (age was taken as a linear factor in the regression vs. exponential factor in the models of Shamir and Howard, 1979 and Walski and Pelliccia, 1982). The authors attributed this correlation with age to different manufacturing, installation and operation practices that are typical of different pipe age groups. The authors further observed that these differences can be classified geographically and that a pipe's age may be a convenient surrogate for a number of more basic parameters which may be gathered and managed by a Geographic Information System (GIS).

The methodology presented here is based on the premise that pipe age can serve as an effective surrogate measure for pipe condition. The pipe break prediction model of Shamir and Howard (with some modification) is adopted, but the parameters for the model (namely initial breakage rate and growth of breakage rate) are assigned on an individual pipe basis without any bundling or grouping of pipes. Consequently, deterioration rates are considered on an individual pipe basis, subject to variable basic conditions within the distribution system. These basic conditions are, to a large extent, geographically dependent (as was suggested by Jacobs and Karney, 1994) and the parameters (initial breakage rate and growth of breakage rate) should be derived by regression techniques applied to pipes that are subject to similar basic conditions.

All references cited in this section discuss observations in the broader context of the need to develop, maintain and use a comprehensive data base on the distribution system. It should be emphasized that for any model or methodology, the upper boundary for its quality and its applicability is the quality of the data from which its parameters are derived, hence the great importance of a good water distribution system data base. It seems that given the rapid evolution of computer capabilities, GIS is the most promising avenue for implementing a comprehensive data base for any infrastructure system including water utilities.

2.4 Distribution System Reliability

A water supply system may be conceptually divided into five major components: (1) source(s), (2) pumping station(s), (3) transmission pipe(s), (4) storage tank(s) and reservoir(s), and (5) distribution system. Each of these major components may be further divided into subcomponents and sub-subcomponents and so on. The level of the sub-division depends upon the level of detail of the required analysis.

In this section, a brief description is provided of recent work that has been conducted in the area of reliability of the water distribution system only. Since the proposed methodology does not include a reliability component, this section merely illustrates the complexity of the reliability issue and the difficulty in explicitly incorporating reliability considerations into a comprehensive rehabilitation scheduling model at the present time.

A discussion concerning distribution system reliability must start with a definition of failure of the system and its causes. Bouchart and Goulter (1991) define failure as the inability of the network to supply demand at minimum pressure. They observe two types of events that can contribute to failure occurrence: (a) the demand is greater than that for which the system was designed, and (b) a component (pipe, pump, valve, etc.) that leads to inadequate hydraulic capacity. If the above definition of failure is accepted, it seems that a third contributor to failure should be considered, namely the deterioration in the hydraulic capacity of pipes in the

network. This phenomenon occurs in all pipes simultaneously and if appropriate measures are not taken, failure (as defined above) will occur.

The following short glossary of terminology (extracted from the various references cited later in this section) is useful to understand the subsequent discussion:

Reliability of a component – the probability that the component will not fail during a given time interval.

Availability of a component – the probability that a component is in an operating condition at time t , given that the component was as good as new at time 0. This measure is more appropriate than reliability, for components that are repairable.

Path-set – a set of elements (pipes, pumps, valves, etc.) that connect (any) two nodes in a network.

Minimal path-set – if a path-set P_i does not contain a sub-set S_i which by itself is a path-set, P_i is called a minimum path-set.

Cut-set – a set of elements which, if it fails, causes the system to fail regardless of the condition of the other elements in the system.

Minimal cut-set – if a cut-set C_i does not contain a sub-set S_i which by itself is a cut-set, C_i is called a minimal cut-set.

Distribution system redundancy – a measure of the number of independent alternatives that exist to perform a task. In a water distribution system this may be expressed in terms of the number of independent hydraulic paths (no common nodes beside initial and final nodes) that exist between a source node and every demand node and have sufficient capacity to provide the demand flow at the required pressure.

Wagner et al. (1988a) introduced two terms: (a) “reachability of a demand node” – a demand node is reachable if it is connected (through a continuous set of pipe links) to at least one source node; (b) “connectivity” – the situation in which every demand node in the network is

connected to at least one source node. After assuming a known probability of failure for each link (pipe) in the network, they define the reliability of a distribution system in terms of its reachability and connectivity. In addition, they propose an algorithm to avoid enumeration of all the possibilities and efficiently solve the composite probabilities. In this algorithm, some of the network links are reduced by a series-parallel scheme to a tree structure. For systems that are not series-parallel reducible, the authors point to available heuristics for network reduction that exist for various network configurations. The authors acknowledge that since this model relies on some simplifying assumptions (e.g., independent probabilities of pipe failure) that may not be realistic, it is best suited for preliminary analysis and screening purposes.

Wagner et al. (1988b) introduced a simulation based method for evaluating system reliability. Recognizing the limitations of the analytical approach presented in their companion paper (Wagner et al., 1988a), this method is said to be capable of evaluating a much broader range of reliability measures, at the expense of greater computer resources and results that are more difficult to generalize. The methodology involves an event generator which generates failure and repair events according to a specified probability distribution, and a network solver to calculate the flow distribution in the pipes for a given scenario. The authors defined three operating states for each node – normal, in which demand is fully supplied; reduced service, in which the pressure falls below a threshold value (service head) but is still above a minimum value (and as a result demand flow is not supplied in full) and failure mode, in which the pressure falls below a specified minimum and supply is assumed shut off. Similarly, three operational states are defined for the entire system; normal, when all nodes are normal; failure, when one or more nodes are in a failure state; and reduced mode, when one or more nodes are in reduced service state but no node is in a failure state.

During a simulation session, various outcomes are continuously recorded. Events such as the time duration in which each node is in any operational mode and total demand shortfall (the amount not supplied in various reduced service modes) are accounted for and relevant statistics are subsequently computed. It appears that simulation is the only way in which so

much detail can be gathered without over simplification of the system and its underlying assumptions.

Cullinane et al. (1989) proposed a model in which a continuous hydraulic availability function is evaluated in a cut-set framework. The concept of a continuous availability function was suggested in contrast to previous work by others, in which failure was considered a step function (fail/no-fail). The availability at a node was modeled as an increasing function of nodal pressure (a curve similar to a cumulative normal distribution curve was used). The authors proposed two measures of reliability – cut-set reliability of a node and availability of a node. The system reliability and availability was taken as the average nodal cut-set reliability and availability, respectively. The authors then proceed to develop an optimization framework for the system based on these reliability measures.

Goulter and Bouchart (1990) proposed a procedure which uses a chance constrained technique¹ to combine probabilities of pipe failure and probabilities of nodal demand exceeding design values into a single reliability measure. This reliability measure is defined as the probability of no node failure which is equal to the product of the probability of no node isolation (node isolation = failure of all pipes incident on a node) and the probability of no demand failure (when demand exceeds available flow). The authors then proceed to develop an optimization network in which the nodal design flow is increased incrementally until a satisfactory level of reliability is achieved.

Fujiwara and De Silva (1990), in their procedure for reliability-based optimal design of water networks, proposed a method to calculate the minimum shortfall of supply quantities during a pipe failure and then defined reliability in terms of the complement of the ratio between this shortfall and the total demand.

¹ The chance constrained method is a technique developed by Charnes and Stedry (1966) to transform a probabilistic constraint into a deterministic constraint using an acceptable threshold value for the variable at hand, its inverse probability distribution and its variance.

Bao and Mays (1990) proposed a methodology to evaluate system reliability that was based on Monte Carlo simulation. The nodal demand flows and pipe roughness were considered random variables with known probability distributions and nodal pressure heads are dependent random variables which are calculated using a network simulator (KYPIPE). Nodal reliability was defined as the probability that a given node receives sufficient flow rate at the required pressure head, which mathematically means the joint probability of flow rate and pressure head being satisfied at a given node (minimum required pressure may be taken as a random or a deterministic value). The authors provided three optional definitions for system reliability – (1) minimal nodal reliability, (2) the arithmetic mean of nodal reliability, and (3) weighted average (by nodal demand flow) of nodal reliability. Ten different probability distributions were tested for nodal demand flows and pipe roughness although it is not clear how the parameters for these distributions were obtained. Each distribution was tested with 500 repetitions¹. Nodal and system reliabilities were calculated and summed after every repetition of the simulation.

Ormsbee and Kessler (1990) suggested system redundancy as a measure of reliability. They defined two types of redundancy, topologic and hydraulic. Topologic redundancy ensures the existence of a physical path from the source to any demand node in the event of a single random component failure. Hydraulic redundancy ensures the capacity of a redundant path to provide adequate pressure at all the demand nodes for a specific load condition. The authors then proposed a procedure to design/check a network for redundancy. The topologic redundancy procedure is based on elements from graph theory and includes transforming the system topologically to a series of “two node connected graphs” and then decomposing every two node connected graphs into two overlapping spanning trees. The hydraulic redundancy then ensures that each of these trees provide a hydraulic path to every node in the system. It should be noted that this procedure is suitable for a system with only one source node, and that only level one redundancy (failure of no more than one pipe at the time) is ensured.

¹ The authors found that nodal and system reliabilities varied from one repetition to the next because of variabilities in the computerized random number generator. Average nodal and system reliabilities varied as well for a small number of repetitions. However, beyond 500 repetitions the average reliabilities remained constant.

Quimpo and Shamsi (1991) proposed a strategy for prioritizing decisions for the maintenance of a water distribution system based on reliability considerations. They suggested a reliability measure that is calculated using the concept of node-pair reliability (the probability that a specified node – source – can communicate with another specified node – demand) which is similar to the connectivity concept of Wagner et al. (1988a). To calculate node-pair reliabilities, the network is first reduced to series, parallel, mixed series-parallel and other reliability blocks. Next, either cut-sets or all path-sets have to be enumerated for every node-pair. Next, reliability is calculated as the complement of the probability of a failed cut-set or as the complement of the probability of a union failure of all path-sets. The next step in the methodology is to create a map with iso-reliability contours for the analyst to identify problem areas.

Awumah et al. (1991) adopted a concept of entropy (used in other types of networks) as a surrogate measure for system redundancy. The authors define nodal redundancy as a mathematical function in which redundancy is maximized when all the links incident on the node carry equal flow. In addition, system redundancy is defined as the (weighted) sum of all nodal redundancies and a measure of how well the flow in the network is distributed among all the demand nodes.

Bouchart and Goulter (1991) proposed a procedure to improve the reliability of water distribution networks. In this procedure, they explicitly considered the fact that demand actually occurs along the pipes and not at the nodes, thus the number and location of isolation valves has direct impact on the demand shortfall that results from a pipe failure. The reliability measure they proposed was defined in terms of demand shortfall due to pipe failure (the segment of pipe that is isolated during repair supplies zero demand) and to demand exceeding available flows (demand is considered a random variable).

The complexity of the reliability issue has been demonstrated in this section. It should be noted that if a surrogate measure for reliability is developed, which is both reasonably accurate and

quick to calculate, it could be incorporated into the rehabilitation model proposed in this thesis as an additional constraint.

2.5 Water Quality in the Distribution System

Several researchers observed the phenomenon of water quality deterioration in the distribution system. Although some of the underlying causes have been identified, no attempt has been made to model the degree of water quality deterioration as a function of system age although (as the following review demonstrates) pipe condition which is a function of age can have significant impacts on water quality. The following is a brief summary of select papers published on this issue. As stated in the introduction to this chapter (section 2.1), the purpose of the following summary is to illustrate the complexity of integrating water quality considerations into a rehabilitation model.

LeChevallier et al. (1990) conducted research in which they found that coliforms in the distribution system originated from pipeline biofilm. They observed that coliform levels increased as the water moved from the treatment plant through the distribution system at a rate that was higher than the coliforms' ability to naturally reproduce. This indicated transfer of populations from biofilm on the pipe inner surfaces to the water column.

In addition, they observed that free chlorine at 1–2 mg/L was not sufficient to eliminate or even reduce these occurrences of coliform bacteria. In some of the trunk lines, free chlorine levels as high as 4.3 mg/L did not eliminate the coliforms¹. The authors noted that continuously maintaining high chlorine residuals in the distribution system would have adverse effects on the consumer, including excessive trihalomethane (THM) formation², taste and odour problems and increased corrosion. In comparing different disinfection agents, the

¹ The authors recite the well known incidents at Muncie and Seymour in which chlorine levels were boosted to 15 mg/L in some instances to control the coliforms.

² Some THM are proven carcinogenic substances (e.g. chloroform) while others are suspected as carcinogenic substances.

authors found that monochloramine was the most effective in controlling biofilm growth on iron pipes; however, doses of 4 mg/L were required. The authors further observed that pipe internal corrosion has a distinct impact on the disinfection efficiency. It was observed that rough inner pipe surfaces protected attached bacteria from inactivation by a free chlorine residual. It was subsequently hypothesized that corrosion by-products interfere with the efficiency of biofilm disinfection. Application of corrosion inhibitors (e.g. polyphosphate, zinc orthophosphate) resulted in a substantial improvement in biofilm disinfection. In a later publication by LeChevallier et al. (1993), this phenomenon was further investigated and corroborated.

LeChevallier (1990) expanded on the topic of bacterial regrowth in the distribution system. Analysis of inner pipe surfaces (both iron and cement-lined iron) revealed that tubercles were composed predominantly of iron. The surface of the tubercles was covered with crevices, which provided increased surface area and protection for microorganism growth. Another factor the author lists in affecting bacterial regrowth is the hydraulics of the system. Increased flow velocities cause a greater flux of nutrients across the pipe surface, greater transport of disinfectants and greater shearing of biofilm from the pipe's inner surface. Stagnant water in pipes cause increased microbial growth as a result of loss of disinfectant residual.

Herson et al. (1991) conducted studies that proved that coliform and indigenous noncoliform organisms are able to accumulate on the inner surfaces of pipelines. Consequently, large differences in microbial numbers may exist between the pipe inner surfaces and the bulk water phase. The high densities of bacteria on the inner surfaces cannot be detected by standard water quality determination procedures which are based on samples taken from the bulk phase only. As hydraulic conditions change (e.g., transient pressures, increased flow velocities), these bacterial accumulations may shear off the surfaces thus increasing their concentrations in the bulk phase. These findings support the conclusions LeChevallier (1990) and LeChevallier et al. (1993) reported previously.

Clark et al. (1993) and Clark et al. (1994) report on an experiment conducted to verify mathematical models of contaminant and chlorine propagation in distribution systems. One of their findings is evidence that the chlorine demand of pipes was much higher than the chlorine decay in the bulk water phase (assuming first order decay). This led them to conclude that the distribution system components exert chlorine demand due to the existence of biofilm on their inner surfaces. Another finding pertains to the adverse impact of distribution storage on water quality.

2.6 Proposed Methodology and How It Advances Knowledge

In this thesis, a methodology is proposed to select for each pipe in an existing network, the rehabilitation alternative and its implementation timing, so as to minimize the cost of the rehabilitation investment and all subsequent maintenance costs subject to certain constraints. The proposed methodology provides a decision support system that facilitates long term planning of pipe rehabilitation in a water distribution network. The methodology advances knowledge compared to other models by taking this long term approach while explicitly and simultaneously considering the deterioration over time of both structural integrity and hydraulic capacity of every pipe in the system throughout a user-defined analysis period. The proposed method is based on a dynamic programming approach combined with partial and (sometimes) implicit enumeration schemes.

3. MODEL FORMULATION

3.1 Problem Statement

Consider a water distribution network with p pipes (links) and n nodes. Every pipe in the network may be rehabilitated by one of R rehabilitation alternatives including relining or replacement with the same or with a larger diameter pipe.

For the purpose of formulating the proposed approach, all rehabilitation techniques may be classified as those which only improve pipe hydraulic capacity, and those which provide added structural integrity as well. The first class of techniques are considered here “reline alternative”, meaning that only the pipe’s friction coefficient changes and possibly its inside diameter. The techniques in the second class are equivalent in their effect to pipe replacement, hence, no special type is needed to distinguish them (Appendix 1 contains a review of current rehabilitation technologies and practices).

For a given time horizon of H years, the objective is to minimize the present value of the total cost of maintaining and rehabilitating the pipe network (total cost = maintenance cost + capital investment cost) subject to the following:

- Mass conservation of flow in all nodes (continuity equations).
- Energy conservation of flow in all pipes (links).
- Residual supply pressure in every node is above a stated minimum¹.
- As the network ages, the hydraulic carrying capacities of the pipes diminish.

¹ The network operates between two extreme states; maximum flow demands and minimum flow demands. Rehabilitation of pipelines is required, among other reasons, for the purpose of restoring the network’s diminishing hydraulic capacity. This diminishing capacity manifests itself in low residual pressures at maximum flow conditions. At minimum flow conditions, friction losses are minimal and residual pressure in the network is at its highest. The designers and operators of the network have to provide measures (e.g., pressure reducing valves) to protect sections of the network (e.g., nodes at low elevation) from excess residual pressure. It is assumed that such measures have been provided and therefore compliance to maximum residual pressure is not examined in this procedure.

- As the network ages, the breakage rates of the pipes increase.
- A pipe can be relined only once during the analysis time period (a limitation discussed later).
- A pipe can be replaced more than once during the analysis time period, but all subsequent replacements are identical to the initial replacement (a limitation discussed later).
- When a pipe is relined, it is assumed that it will be replaced at a later time; thus, consecutive relinings are not considered for a given analysis period.

Although the number of rehabilitation alternatives ($j=1,\dots,R$) is not restricted by the model, in most cases it is sufficient to consider five rehabilitation alternatives as follows:

- 1) reline pipe,
- 2) replace with same diameter pipe,
- 3) replace with a pipe one (nominal) diameter larger,
- 4) replace with a pipe two (nominal) diameters larger, or
- 5) do nothing.

The “do nothing” alternative in fact means “do nothing within the specified time horizon”, and does not have to be explicitly considered for the following reason: If a pipe is scheduled to be rehabilitated at year n , the hydraulic affect of this rehabilitation is presumed to commence at year $(n+1)$ ¹. Therefore, if a pipe is scheduled for rehabilitation at the last year of the analysis time period, it is hydraulically equivalent to a “do nothing” alternative.

3.2 Cost of Capital Investment

The present value of the capital investment to rehabilitate any pipe i in a network with rehabilitation alternative j is:

¹ This generalization is applied for simplicity. While in small rehabilitation projects it may be an over statement, large projects often take the better part of a year. The time step of one year however, is taken here for convenience only.

$$CAP(T_{ij}) = Cr_{ij} \cdot L_i \cdot e^{-rT_{ij}} \quad i = 1, 2, \dots, p; \quad j = 1, 2, \dots, R \quad (1)$$

where Cr_{ij} = cost of rehabilitation measure j in pipe i (\$ /km)
 L_i = length of pipe i (km)
 r = discount rate
 T_{ij} = time from present that pipe i is rehabilitated with alternative j

3.3 Cost of Maintenance

The cost of maintenance comprises primarily the present value of repairing all pipe breaks throughout the analysis period since other maintenance cost components are relatively constant and change relatively little with the pipe age. The breakage rate N of a pipe i at any given year t increases with pipe age as follows:

$$N(t)_i = N(t_0)_i \cdot e^{A_i(t-g_i)} \quad (2)$$

where t = time elapsed (from present) in years
 $N(t)_i$ = number of breaks per unit length per year in pipe i ($\text{km}^{-1} \text{ year}^{-1}$)
 $N(t_0)_i$ = $N(t)_i$ at the year of installation of pipe i (i.e., when the pipe is new)
 g_i = age of pipe i at the present time
 A_i = coefficient of breakage rate growth in pipe i (year^{-1})

This model for the prediction of pipe breakage rate is based on that of Shamir and Howard (1979). Other models exist, as described in the literature review. The Dynamic Programming approach presented herein can accommodate any such model, as long as the total cost function (capital cost + maintenance) has one minimum with respect to the time variable or is a non-declining function of time.

The present value of breakage repairs in pipe i for the years elapsed from the present to the year of rehabilitation with alternative j is given by:

$$C_M(T_{ij}) = \int_0^{T_{ij}} L_i \cdot Cb_i \cdot N(t_0)_i \cdot e^{A_i(t-t_0)} \cdot e^{-rt} dt = \frac{L_i \cdot Cb_i \cdot N(t_0)_i \cdot e^{A_i T_{ij}} (e^{-(A_i-r)T_{ij}} - 1)}{A_i - r} \quad (3)$$

(A_i ≠ r)

where Cb_i = cost of a single breakage repair in pipe i (\$)

T_{ij} = year of rehabilitation of pipe i with alternative j

3.4 Total Cost

3.4.1 Total Cost for Replacement Alternatives

Previous work (e.g., Shamir and Howard, 1979; Walski and Pelliccia, 1982) assumed negligible breakage repair costs after pipe replacement, which subsequently would result in a total cost comprising the sum of the costs depicted in equations (1) and (3). However, this assumption of negligible costs after pipe replacement introduces biases into the cost analysis of multiple pipes, in favour of older pipes that need to be replaced in the near future.

To alleviate this problem, a steady state approach is taken in which it is assumed that a pipe is replaced periodically to infinity. It is assumed further that if a certain size and type of pipe was used in the initial replacement, all subsequent replacements will utilize the same type and size, i.e., the pipe breakage rate is the same for all subsequent replacement pipes to infinity¹. Subsequent analysis is partially based on principles similar to those used by Manne (1967) in analyzing steady state capacity expansion.

¹ This assumption introduces a limitation in considering demand growth over time, especially in urban centers that are in the midst of their expansion period and expect substantial future demand increase. This limitation can be overcome by the manner with which the proposed methodology is implemented in practice. This issue is discussed further in section 6.6 and 6.7.

The schematic in Figure 1 describes the stream of costs due to maintenance (annual break repair) and periodic replacement capital cost.

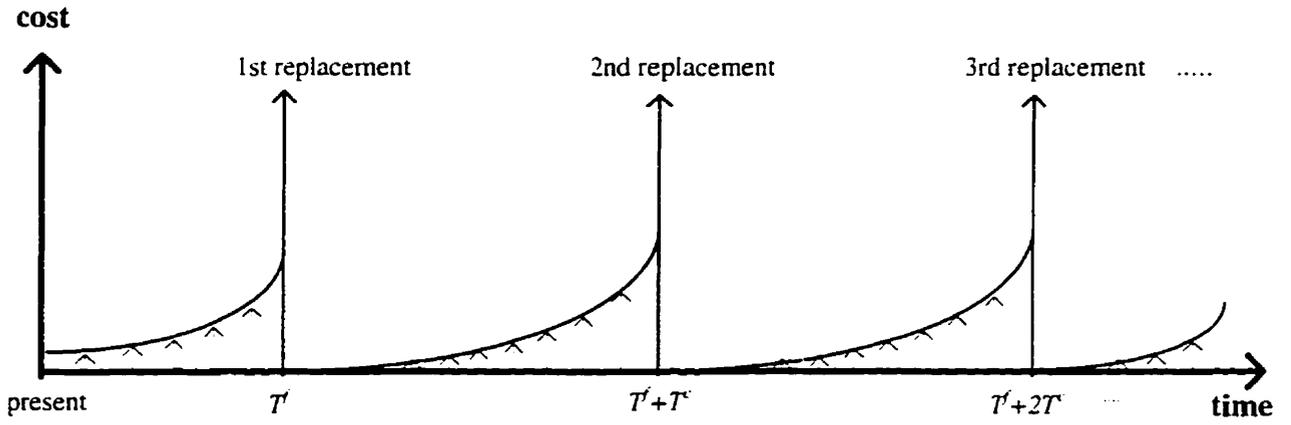


Figure 1. Stream of costs (steady state) associated with a single pipe

T^f is the time of the first replacement and T^c is the duration of a cycle of all subsequent streams of costs (a cycle is the duration from immediately after one replacement until immediately after the next replacement).

In the subsequent calculations (through equation 15c) all costs pertain to a unit length of pipe. It should also be noted that the notations used in equations 4 through 8 hereafter are consistent with the notations used in equations 1 through 3 before, except that the indices i, j (for pipe and rehabilitation alternative) were temporarily omitted in the interest of compactness and ease of reading.

To find the optimal T^c consider the period from T^f to infinity. The cost of capital investment for replacement in the first (full) cycle discounted to time T^f is

$$CAP(T^c) = Cr \cdot e^{-rT^c} \quad (4)$$

The cost of break repair for the period of the cycle discounted to time T^f is

$$C_M(T^c) = \int_0^{T^c} Cb \cdot N(t_0) \cdot e^{(A-r)t} dt \quad (5)$$

The total cost in one cycle, discounted to the beginning of the cycle is

$$C^{tot}(T^c) = CAP(T^c) + C_M(T^c) = Cr \cdot e^{-rT^c} + \int_0^{T^c} Cb \cdot N(t_0) \cdot e^{(A-r)t} dt \quad (6)$$

Consider an infinite series of costs (each equals C^{tot}) occurring at times $T^f + mT^c$ ($m=0, 1, 2, \dots, \infty$).

The value of this series discounted to time T^f is

$$C_{inf}(T^c) = \sum_{m=0}^{\infty} C^{tot}(T^c) e^{-mrT^c} = \frac{C^{tot}(T^c)}{1 - e^{-rT^c}} \quad (7)$$

Define T^{**} equal to T^c that minimizes C_{inf} . To find T^{**} , solve $\frac{\partial C_{inf}(T^c)}{\partial T^c} = 0$ for T^c as follows:

....(8)

$$\frac{\partial C_{inf}}{\partial T^c} = \frac{\partial}{\partial T^c} \left[\frac{Cr \cdot e^{-rT^c} + \int_0^{T^c} Cb \cdot N(t_0) e^{(A-r)t} dt}{1 - e^{-rT^c}} \right] = \frac{\partial}{\partial T^c} \left[\frac{Cr \cdot e^{-rT^c} + \frac{Cb \cdot N(t_0)}{A-r} (e^{(A-r)T^c} - 1)}{1 - e^{-rT^c}} \right] = 0$$

$$\frac{\partial C_{inf}}{\partial T^c} = \frac{(1 - e^{-rT^c})[-r \cdot Cr \cdot e^{-rT^c} + Cb \cdot N(t_0) e^{(A-r)T^c}] - r \cdot e^{-rT^c} [Cr \cdot e^{-rT^c} + \frac{Cb \cdot N(t_0)}{A-r} (e^{(A-r)T^c} - 1)]}{(1 - e^{-rT^c})^2} = 0$$

For $r \neq 0$, $T^c \neq 0$ and $A \neq r$, multiply by $(1 - e^{-rT^c})^2$ and divide by e^{-rT^c} to yield

$$(1 - e^{-rT^*})[-r \cdot Cr + Cb \cdot N(t_0)e^{AT^*}] - r[Cr \cdot e^{-rT^*} + \frac{Cb \cdot N(t_0)}{A-r}(e^{(A-r)T^*} - 1)] = 0$$

and rearranging yields

$$-r \cdot Cr + Cb \cdot N(t_0)e^{AT^*} - Cb \cdot N(t_0)e^{(A-r)T^*} - \frac{r}{A-r} Cb \cdot N(t_0)(e^{(A-r)T^*} - 1) = 0$$

Dividing by $Cb \cdot N(t_0)$ and rearranging yields

$$e^{AT^*}(1 - e^{-rT^*}) + \frac{r}{A-r}(1 - e^{(A-r)T^*}) = \frac{r \cdot Cr}{Cb \cdot N(t_0)} \quad (8a)$$

T^{**} can now be easily calculated (e.g. by Newton's method).

Note: The first and second derivative of (8a) with respect to T^c exist, and are continuous for $A \neq 0$, $r \neq 0$ and $A \neq r$.

Equation (8a) may be rewritten for the general case in which pipe i is to be replaced by alternative j as follows:

$$e^{A_y T_y^{**}}(1 - e^{-r T_y^{**}}) + \frac{r}{A_y - r}(1 - e^{(A_y - r) T_y^{**}}) = \frac{r \cdot Cr_y}{Cb_y \cdot N(t_0)_y} \quad (9)$$

Consider now the stream of costs from the present to infinity. The present value of all costs associated with a single pipe from the present to infinity is given by:

$$C^{tot}(T_y^j) = Cr_y \cdot e^{-r T_y^j} + \int_0^{T_y^j} Cb_i \cdot N(t_0)_i e^{A_i(t+x_i)} e^{-rt} dt + C_{inf}(T_y^{**}) e^{-r T_y^j} \quad (10)$$

where Cb_i = cost of break repair in existing pipe i

$N(t_0)_i$ = initial breakage rate in existing pipe i

A_i = break growth rate in existing pipe i

g_i = age of existing pipe i

Define T_{ij}^* equal to T_{ij}^f that minimizes C^{tot} . To find T_{ij}^* solve $\frac{\partial C^{tot}(T_{ij}^f)}{\partial T_{ij}^f} = 0$ for T_{ij}^f as

follows:

$$\frac{\partial C^{tot}(T_{ij}^f)}{\partial T_{ij}^f} = \frac{\partial}{\partial T_{ij}^f} \left[[Cr_{ij} + C_{inf}(T_{ij}^{**})]e^{-rT_{ij}^f} + Cb_i \cdot N(t_0)_i e^{A_i} \int_0^{T_{ij}^f} e^{(A_i - r)t} dt \right] = 0 \quad (11)$$

$$-r[Cr_{ij} + C_{inf}(T_{ij}^{**})]e^{-rT_{ij}^f} + Cb_i \cdot N(t_0)_i e^{A_i} \cdot e^{(A_i - r)T_{ij}^f} = 0$$

Dividing by $e^{-rT_{ij}^f}$ and rearranging yields;

$$e^{A_i(T_{ij}^f - g_i)} = \frac{r(Cr_{ij} + C_{inf}(T_{ij}^{**}))}{Cb_i \cdot N(t_0)_i} \quad (12)$$

$$T_{ij}^f = -g_i + \frac{1}{A_i} \ln \left[r \frac{Cr_{ij} + C_{inf}(T_{ij}^{**})}{Cb_i \cdot N(t_0)_i} \right] \equiv T_{ij}^*$$

Graphically, the total cost function of a single pipe vs. year of first replacement may be represented as in Figure 2:

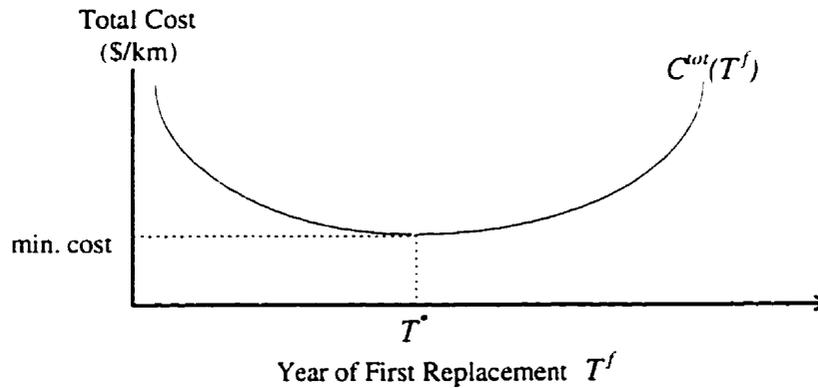


Figure 2. Total cost vs. replacement time.

It should be noted that given the physical/economical terms of the problem. C''' is always convex in the vicinity of T_{ij}^* , hence a minimum always exists there. To prove this assertion, the second derivative of C''' with respect to T_{ij}^f for $T_{ij}^f = T_{ij}^*$ is shown to be positive as follows:

$$\frac{\partial^2 C'''(T_{ij}^f)}{\partial T_{ij}^{f2}} = r^2 [Cr_{ij} + C_{inf}(T_{ij}^{**})]e^{-rT_{ij}^f} + (A_i - r)Cb_i \cdot N(t_0)_i e^{A_i \kappa_i} e^{-(A_i - r)T_{ij}^f} > 0 \quad (13)$$

Dividing by $e^{-rT_{ij}^f}$ and rearranging yields

$$\frac{\partial^2 C'''(T_{ij}^f)}{\partial T_{ij}^{f2}} = r^2 [Cr_{ij} + C_{inf}(T_{ij}^{**})] + (A_i - r)Cb_i \cdot N(t_0)_i e^{A_i \kappa_i - rT_{ij}^f} > 0 \quad (14)$$

As can be seen, for $A_i > r$ equation (14) is true in all T_{ij}^f . To account for both $A_i > r$ and $A_i < r$, equation (14) is further manipulated and then evaluated for $T_{ij}^f = T_{ij}^*$. Dividing both sides of equation (14) by $(Cb_i \cdot N(t_0)_i r)$ the following is obtained:

$$r \frac{Cr_{ij} + C_{inf}(T_{ij}^{**})}{Cb_i \cdot N(t_0)_i} + \frac{A_i - r}{r} e^{A_i \kappa_i - rT_{ij}^f} > 0 \quad (14a)$$

Substituting into (14a) the expression $r \frac{Cr_{ij} + C_{inf}(T_{ij}^{**})}{Cb_i \cdot N(t_0)_i}$ from (12) which defines $T_{ij}^f = T_{ij}^*$ yields

$$e^{A_i \kappa_i - rT_{ij}^f} + \frac{A_i - r}{r} e^{A_i \kappa_i - rT_{ij}^f} > 0 \quad (14b)$$

After dividing by the exponent and rearranging,

$$A_i > 0 \quad (14c)$$

Expression (14c) must always be true for the terms of the problem (i.e., an actual deterioration with age in pipe breakage rate is assumed) which means that a minimum exists for all $T_{ij}^f = T_{ij}^*$.

As for the existence of a minimum for $C_{inf}(T_{ij}^{**})$, a similar procedure is applied. The second derivative of C_{inf} with respect to T^c is obtained after manipulation as follows:

.....(15)

$$\frac{\partial^2 C_{inf}}{\partial T^c} = \frac{Cb_{ij} \cdot N(t_0)_{ij} e^{A_i T} \left[e^{2rT} (A_i - 1)^2 + e^{rT} (r^2 + 2A_i r - 2A_i^2) + A_i^2 \right] + r^2 e^{rT} (e^{rT} + 1) \left[Cr_{ij} (A_i - r) - Cb_{ij} \cdot N(t_0)_{ij} \right]}{(A_i - r)(e^{rT} - 1)^3}$$

Equation (15) must be positive in the vicinity of $T^c = T_{ij}^{**}$. Dividing by $(Cb_{ij} N(t_0)_{ij})$ and rearranging yields:

$$\frac{e^{A_i + 2rT} (A_i - 1)^2 + e^{A_i + rT} (r^2 + 2A_i r - 2A_i^2) + A_i^2 e^{A_i T} + r e^{rT} (e^{rT} + 1) \left[\frac{r \cdot Cr_{ij} (A_i - r)}{Cb_{ij} \cdot N(t_0)_{ij}} - r \right]}{(A_i - r)(e^{rT} - 1)^3} > 0 \quad (15a)$$

Substituting into (15a) the value of $\frac{r \cdot Cr_{ij} (A_i - r)}{Cb_{ij} \cdot N(t_0)_{ij}}$ from (8a) which defines $T^c = T^{**}$ and

rearranging gives:

.....(15b)

$$\frac{e^{A_i + 2rT} (A_i - 1)^2 + e^{A_i + rT} (r^2 + 2A_i r - 2A_i^2) + A_i^2 e^{A_i T} + r \cdot e^{rT} (e^{rT} + 1) \left[(A_i - r) e^{A_i T} - A_i e^{A_i - rT} + r - r \right]}{(A_i - r)(e^{rT} - 1)^3} > 0$$

Simplifying and rearranging yields:

$$\frac{A_i e^{A_i + 2rT} (A_i - r) + 2A_i e^{A_i + rT} (r - A_i) + A_i e^{A_i T} (A_i - r)}{(A_i - r)(e^{rT} - 1)^3} > 0$$

Dividing by $\frac{A_i e^{A_i}}{(e^{rT} - 1)^3}$ and rearranging results in the following:

$$(e^{2rT} - 1)^2 > 0 \quad (15c)$$

Expression (15c) is positive for all T^c ; hence, C_{inf} is convex and $C_{inf}(T_{ij}^{**})$ is always a minimum.

It should also be noted that some relining technologies provide, in addition to improved hydraulic performance, some structural improvement as well (especially in larger pipes). The cost of these rehabilitation alternatives is analyzed the same way as a replacement alternative with a set of pertinent parameters. Other relining technologies provide no structural improvement, in which case the following analysis is applied.

3.4.2 Total Cost for Relining Alternatives

When pipe relining is considered with the assumption that no structural improvement occurs, the steady state approach is modified. From an engineering point of view, it is economically feasible to reline a pipe whose hydraulic capacity deteriorates much faster than its structural integrity. Since maintenance costs of a relined pipe rise with age, it is assumed that the relined pipe will have to be replaced some time in the future. Consequently, if the hydraulic capacity of a pipe needs to be improved well before this time in the future, relining should be considered as a feasible alternative. The schematic in Figure 3 is used to illustrate this concept.

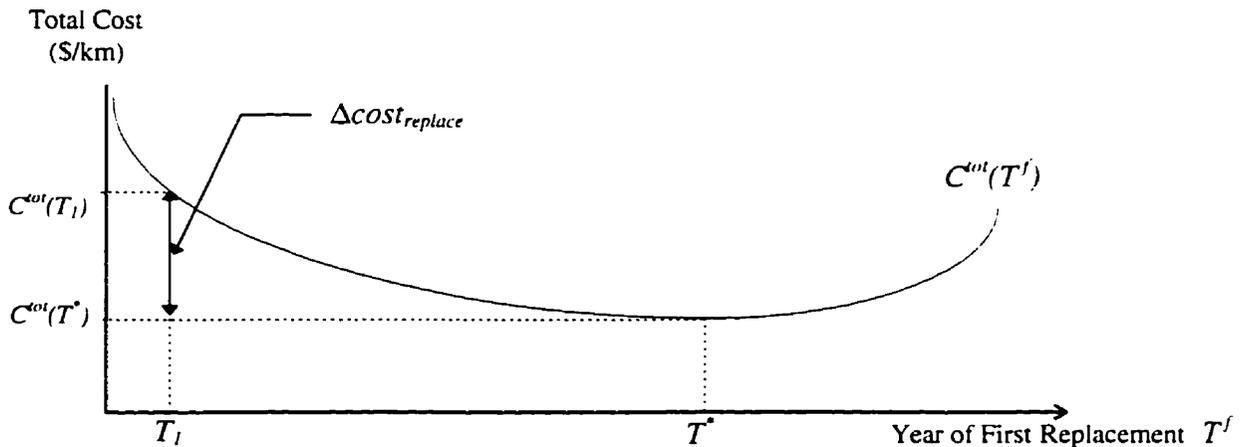


Figure 3. The feasibility of relining.

As a first approximation, it is initially assumed that the pipe in question will be replaced by the same diameter pipe at time T^* . If rehabilitation has to be implemented at time T_j (due to early deterioration in hydraulic capacity), the relining alternative is compared with an early replacement (with same diameter pipe) alternative. An early replacement incurs an additional cost $\Delta cost_{replace}$ above the cost of replacement at T^* , whereas relining at time T_j incurs an additional cost equal to the capital investment for relining or CAP_{reline} . Subsequently, if $\Delta cost_{replace} > CAP_{reline}$, then the reline alternative is chosen and vice versa.

It should be noted that (a) if a pipe is relined, it is assumed to be subsequently replaced at time T^* and then every T^{**} to infinity; (b) if early replacement is selected, subsequent replacements are assumed every T^{**} to infinity; and (c) this approach cannot consider two consecutive relinings. Consequently, the total cost of the reline alternative is

$$C_{rel}^{tot}(T_{ij}, T_{ik}) = CAP_{reline} e^{-rT_j} + C^{tot}(T_{ik}^*) \quad (16)$$

where j is the relining alternative ($j \in R$) and k denotes the alternative of replacement with the same diameter pipe ($k \in R$).

3.5 Total Cost and Time Horizon

Although the proposed methodology considers an infinite stream of costs for every pipe and every rehabilitation alternative in the network, from a hydraulic point of view the system is not in steady state because the hydraulic integrity of the system is ensured only for a specified time horizon. From a practical point of view it seems unnecessary to even attempt to consider a steady state hydraulic analysis because a water distribution system is a dynamic entity with requirements and operational conditions that are constantly changing. The premise of the proposed methodology is that there is a time horizon (say 30 to 60 years) for which any water utility is capable (and required) of making some forecasting about future operational

requirements of the distribution system. For this time horizon, it is therefore feasible and necessary to devise a strategy for watermain rehabilitation.

The approach of infinite stream of costs for individual pipes derived in equations (4) through (16) above, facilitates the capability of the proposed model to account for costs of multiple rehabilitations in any given pipe within the selected time horizon, while explicitly considering only the timing of the pipe's initial rehabilitation. In addition, an infinite stream of costs approach minimizes biases that may exist due to the fact that the system's residual capacity (beyond the time horizon) is not considered. These key issues are further elaborated after the model is presented in full.

3.6 Deterioration of Pipe Friction Coefficient

The head loss h in any pipe i is calculated (following the Hazen-Williams Equation) by

$$h_i = 10.653 \left(\frac{Q_i}{C_i^{HW}} \right)^{1.852} D_i^{-4.87} L_i \quad (17)$$

where Q_i = flow rate in pipe i (m^3/s)
 C_i^{HW} = Hazen Williams hydraulic conductivity coefficient in pipe i
 D_i = diameter of pipe i (m)
 L_i = length of pipe i (m)

The coefficient of conductivity deteriorates with the age of the system. The rate of deterioration will vary according to the type of pipe, the quality of supply water and operation and maintenance practices. Colebrook and White (1937) were among the first to address the issue of pipe roughness changing over time. They found a close to linear growth rate of pipe roughness with time. Hudson (1966) investigated the variations of C^{HW} with time in seven distribution networks in the United States. Hudson's results are presented in Figure 4.

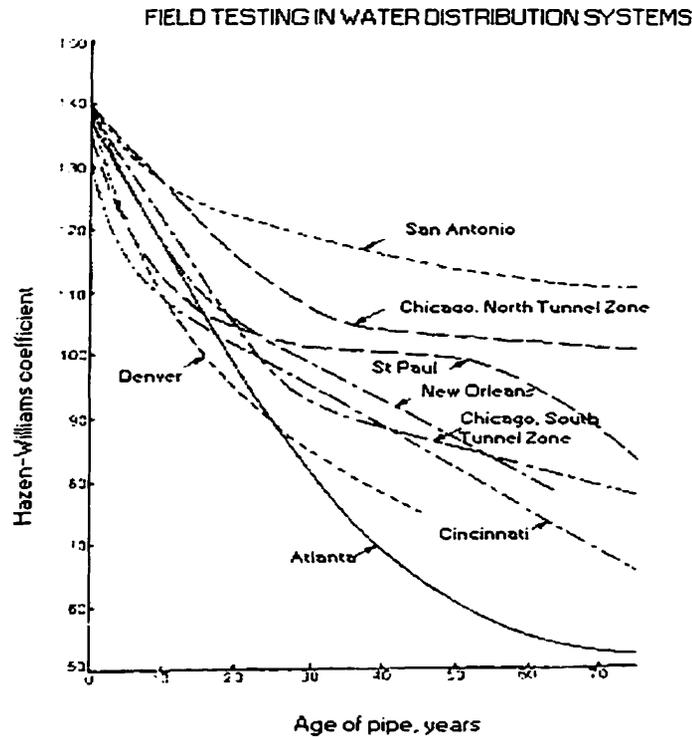


Figure 4. Deterioration of C^{HW} with time (after Hudson, 1966)

Lamont (1981) tied the roughness growth rate to the degree of calcium carbonate saturation of the supply water. Sharp and Walski (1988) proposed a methodology that was essentially based on the Colebrook and White findings, in which roughness values were converted to equivalent C^{HW} values.

The equation of Sharp and Walski is used in this paper to model the effect that aging has on the carrying capacity of pipes in the distribution network¹. The equation can be written in two forms, one for the pipe before rehabilitation and one after rehabilitation as follows:

¹ The suitability of this model is demonstrated with anecdotal data collected from the City of Scarborough in Ontario, Canada, and used for the validation of the thesis results in Chapter 6 and the relevant Appendix.

$$C_i^{HW}(t) = 18.0 - 37.2 \log\left(\frac{e_{0i} + a_i(t + g_i)}{D_i}\right) \quad (18a)$$

- where $C_i^{HW}(t)$ = Hazen-Williams friction coefficient in pipe i at year t
- e_{0i} = initial roughness (ft) in pipe i at the time of installation when it was new
- a_i = roughness growth rate (ft/yr) in pipe i
- D_i = diameter (ft) of pipe i
- g_i = age (in years) of pipe i at the present time (time of analysis)
- t = time (years) elapsed from present time to future periods ($t < T_{ij}$)

After rehabilitation with alternative j , the HW coefficient C^{HW} in pipe i at year t is

$$C_y^{HW}(t) = 18.0 - 37.2 \log\left(\frac{e_{0y} + a_y(t - T_{ij})}{D_{ij}}\right) \quad (18b)$$

- where e_{0y} = initial roughness (ft) in pipe i at the time of implementing rehabilitation alternative j
- a_y = roughness growth rate (ft/yr) in pipe i with rehabilitation alternative j
- D_{ij} = diameter (ft) of pipe i with rehabilitation alternative j
- T_{ij} = year of rehabilitation alternative j in pipe i (time elapsed from present time)
- t = time (years) elapsed from present time to future periods ($t > T_{ij}$)

It should be noted that Sharp and Walski's equation was originally derived for unlined metal pipes. In the absence of a more general model, since the parameters e_0 and a are derived experimentally (by regression) for the specific pipe used, it is rationalized that using this equation for pipes other than unlined metal, would yield reasonably accurate results where the characteristics of different types of pipe are reflected in the set of parameters (e_0 and a) that are derived for each pipe. However, it should be emphasized that the methodology developed

in this thesis is not restricted to any one model, and should a more suitable equation for predicting C^{HW} be developed in the future, it could easily be incorporated into the proposed methodology.

3.7 Additional Constraints

In the proposed model, the DP process of searching for a least cost rehabilitation policy is subject to the supply system's conformity to all physical/hydraulic laws and to adequate supply criteria as outlined below.

i) Conservation of mass

$$Q_{in,y} = Q_{out,y} \quad \text{for all nodes } y \in \{1, 2, \dots, n\} \text{ in the network} \quad (19)$$

where Q_{in} = flow rate into a node
 Q_{out} = flow rate out of a node

ii) Conservation of energy

$$\sum h_l = \text{constant} \quad \text{for all paths that either form closed loops in the network or connect two nodes with fixed hydraulic grade (e.g. reservoir, etc.)} \quad (20)$$

where h = head changes in a pipe or a component in the network
 l = all pipes and components included in a path
 $\text{constant} = 0$ for a path that is a closed loop

iii) Minimum supply pressure

$$P_{vt} \geq P_{min,y} \quad \text{for all nodes } y \in \{1, 2, \dots, n\} \text{ in the network and all years } t \in \{0, 1, 2, \dots, H\} \text{ in the analysis period} \quad (21)$$

where $P_{y,t}$ = residual supply pressure at node y in year t

$P_{min,y}$ = minimum residual pressure allowed at node y in the system

The conformity of every stage in the DP process to these constraints is achieved through the implementation of a network simulation program.

3.8 Cost of Energy

Energy costs are not explicitly considered in the proposed methodology because evaluating the cost of energy in a system over an analysis period which is typically 30 to 60 years long, involves numerous extended time simulations that would make the model computationally prohibitive.

The omission of the proposed methodology to consider energy costs (using current computer technology) may at first glance seem like a significant drawback. Some practical engineering considerations, however, suggest that this is not so.

Pumping stations in water distribution systems typically comprise a number of pumps (sometimes of various sizes) that can be operated in various configurations (in series, parallel, etc.) in order to achieve greater efficiency in supplying the demands which typically fluctuate in daily and seasonal cycles. Furthermore, an increasing number of water utilities use variable speed pumps to even further enhance the efficiency of pumping in variable demand conditions. The flexibility of pumping stations in supplying variable demand conditions enhances the importance of explicitly considering energy costs in a general optimization procedure.

On the other hand, the operation of a real life water distribution system takes place within the bounds of a pressure head envelope typically between 30 and 70 metres of water column (maximum operating pressure is required to minimize breaks and leaks and because of limited

pressure rating of appurtenances and home appliances). That leaves only 40 metres of head for topographical variations and friction head losses in the distribution system (or in a pressure zone within a distribution system). Since distribution systems on terrain that is completely flat are rare, the result is a narrow envelope of operating heads. Furthermore, a water distribution network is typically fed simultaneously from several sources including pumps, elevated storage tanks, etc. The elevation of the tanks is typically governed by the minimum required supply pressure at peak demand. Consequently, much of the operating pressure of the pumps in the system is determined by the elevation difference (static head) between the pumps and the elevated storage tanks.

Another issue is the cost that pipe leaks incur both in term of energy cost (leaks are energy dissipaters in a distribution system due to the energy required to pump leaked water and to increased headlosses in the system due to “inflated demand”) and the cost invested in treating the lost water. This issue is not addressed by the proposed methodology because a model that ties leaks to pipe age does yet exist and it seems that here too extended simulations will be required to account for long term energy costs.

In summary, it is posited that as the system’s carrying capacity diminishes, the possibility of compensating by increasing pumping pressure is limited (because of maximum pressure considerations) and the result is mainly a reduction in residual nodal pressure rather than increased energy costs.

3.9 Mathematical Statement of the Problem

The following is the formal mathematical statement of the problem described in sections 3.1 – 3.8 :

$$\text{minimize: } \sum_{\substack{i=1 \\ j \in R}}^p CAP(T_{ij}) + C_M(T_{ij}) + C_{int}(T_{ij}^{**})e^{-rT_{ij}} \quad (22)$$

where $CAP(T_{ij}) = L_i \cdot Cr_{ij} \cdot e^{-rT_{ij}}$

and $C_M(T_{ij}) = \int_0^{T_{ij}} L_i \cdot Cb_i \cdot N(t_{ij})_i e^{A_i(t-x)} \cdot e^{-rt} dt$ for a replacement alternative

and $C_{int}(T_{ij}^{**}) = \sum_{m=0}^{\infty} \left[e^{-mrT_{ij}^{**}} \cdot L_i \left(Cr_{ij} \cdot e^{-rT_{ij}^{**}} + \int_0^{T_{ij}^{**}} Cb_{ij} \cdot N(t_{ij})_i e^{A_i(t-x)} \cdot e^{-rt} dt \right) \right]$ for a replacement alternative

or if j is a relining alternative

$C_M(T_{ij}) = \int_0^{T_{ik}^*} L_i \cdot Cb_i \cdot N(t_{ij})_i e^{A_i(t-x)} \cdot e^{-rt} dt$ (k denotes the alt. of replacement

and $C_{int}(T_{ij}^{**})e^{-rT_{ij}} = CAP(T_{ik}^*) + C_{int}(T_{ik}^{**})e^{-rT_{ij}}$ with the same diameter pipe)

subject to

(a) $Q_{in,y} = Q_{out,y} \quad y \in \{1, 2, \dots, n\}$ flow continuity, see equation (19)

(b) $\sum h_i = \text{constant}$ energy conservation, see equation (20)

where $h_i = 163 \left(\frac{Q_{ij}}{C_{ij}^{HW}} \right)^{1.852} D_{ij}^{-4.87} L_i \quad i \in p, \quad j \in R$ friction headloss, see equation (17)

and $C_{ij}^{HW}(t) = 18.0 - 37.2 \log\left(\frac{e_{o,i} + a_{ij}(t - T_{ij})}{D_{ij}}\right)$ for a pipe i after rehabilitation j

or $C_i^{HW}(t) = 18.0 - 37.2 \log\left(\frac{e_{o,i} + a_i(t + g_i)}{D_i}\right)$ for a pipe i before rehabilitation

for $t \in \{1, 2, \dots, H\}$

(c) $P_{i,t} \geq P_{\min}, \quad y \in \{1, 2, \dots, n\}, \quad t \in \{1, 2, \dots, H\}$ min. supply pressure, see equation (21)

Notation

Comments

- T_{ij} - Decision variables - timing of rehabilitation alternative j in pipe i .
- $CAP(T_{ij})$ - Capital cost of implementing rehabilitation alternative j in pipe i at time T_{ij} , see equation (1).
- $C_M(T_{ij})$ - Cost of maintenance associated with pipe i and alternative j from the present to time T_{ij} . See equation (3) for replacement and equation (16) for a relining alternative.
- $C_{int}(T_{ij}^{**})$ - Total cost stream (capital + maintenance) of replacement cycles (with duration T^{**}) from T_{ij} to infinity. See equations (6) and (7) for replacement alternatives and for a relining alternative see also equation (16).
- $C_{ij}^{HW}(t)$ - The Hazen-Williams coefficient in pipe i , alternative j at time t , see equations (18a) and (18b).

4. THE OPTIMIZATION PROCEDURE

4.1 Introduction

In a water supply network, the operational mode of every pipe depends simultaneously on the distribution of demand flows at the demand nodes, the relation of pressure versus flow in the source nodes (boundary conditions), and the hydraulic characteristics of all other pipes in the network. With demand and supply flows assumed constant, any change in any pipe in the distribution network generally causes a redistribution of flows in *all* pipes in the network and consequently a change in the residual supply pressure at all nodes. As described previously in section 3.6, the hydraulic capacity of every pipe in the system deteriorates throughout its lifetime. It follows that the residual nodal pressures in the system change constantly over time, even for an assumed steady state supply mode. Rehabilitation of any pipe in the network imposes an additional change in the system by increasing the hydraulic carrying capacity of the rehabilitated pipe.

Because the operational mode of any pipe in the system depends on the characteristics of all other pipes, the rehabilitation of any pipe in the network influences the choice of other pipes to be rehabilitated in the future, as well as the timing and type of future rehabilitation choices. Moreover, the optimal choice of the rehabilitation project to be implemented at present, requires knowledge of the rehabilitation projects to be implemented in the future. The result is a multi-stage process where the optimum decision in any stage depends on the decisions made in earlier stages and the decisions to be made in later stages.

Consider a water supply network with p pipes, R rehabilitation alternatives for every pipe and an analysis period of H years (the analysis period may be discretized into single or multiple year timesteps, however, throughout this work a single year timestep is assumed unless otherwise noted). Enumeration of all possible rehabilitation and scheduling combinations would comprise $[R+1]^{p \cdot H}$ alternatives. Even for a very small system with 3 pipes, a 30 year analysis period and 3 rehabilitation alternatives, the number of combination exceeds 10^{54} .

The proposed optimization model utilizes a Dynamic Programming (DP) approach in which an implicit enumeration scheme is incorporated. The proposed approach was inspired by earlier work done by Erlenkotter and Rogers (1975) on optimized sequencing of competing projects. The proposed approach incorporates a branch and bound process in which suboptimal sequences are identified and discarded as the search for an optimal sequence progresses.

4.2 Conceptual Approach

The proposed procedure considers two types of deterioration in the pipes of a water distribution system: hydraulic capacity and structural integrity. The deterioration in the hydraulic capacity of pipes results in a reduction of the supply pressure, and the deterioration of the structural integrity causes increased breakage rates which result in increased pipe maintenance costs. The principles of jointly considering the network hydraulics and the individual pipe economics to determine the timing and sequencing of rehabilitation projects are as follows:

- (a) The network hydraulics determines the latest time to implement the next rehabilitation project. It is the time at which, due to the hydraulic deterioration of the pipes, the hydraulic capacity of the network becomes inadequate.
- (b) The individual pipe economics determines the actual timing of the next rehabilitation project within the upper boundary set by the network hydraulics.
- (c) The process of systematically considering all possible sequences of rehabilitation alternatives is based on Dynamic Programming in which a partial (and sometimes implicit) enumeration scheme is incorporated.

The concepts of the procedure are explained with the aid of a simple example. Consider the two pipe water distribution “system” illustrated in Figure 5, with one source node and one demand node.

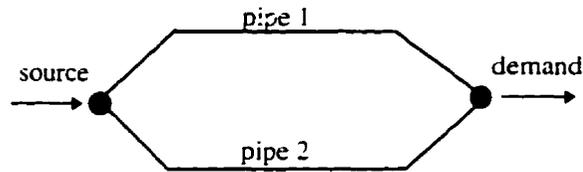


Figure 5. A simple water distribution system

- **Network hydraulics**

The residual pressure at the demand node is determined by the demand flow rate, the supply pressure at the source node, the demand node elevation (constant) and the hydraulic capacity of the “distribution network” which comprises in this case, two pipes in parallel. Assuming constant demand and supply pressure at the source node, the residual pressure at the demand node depends only on the hydraulic capacity of the “distribution network”. Since the hydraulic capacity of the pipes diminishes over time, so does the residual pressure at the demand node. Suppose that P_{min} is the minimum residual pressure required at the demand node to maintain adequate service. It is clear that the diminished hydraulic capacity in one of the pipes will have to be increased to avoid the residual pressure at the demand node dropping below P_{min} . This increase in pipe hydraulic capacity is achieved by implementing a rehabilitation project.

Possible projects could be;

- (1) reline the pipe,
- (2) replace the pipe with a new pipe of the same diameter, or
- (3) replace the pipe with a larger diameter pipe.

If the goal is to maintain adequate pressure at the demand node during a planning period of, say, 30 years, it may be necessary to implement rehabilitation projects for both pipes (possibly even more than once), depending on the deterioration rates of both the existing pipes and the rehabilitated pipes. This process is graphically illustrated in figure 6.

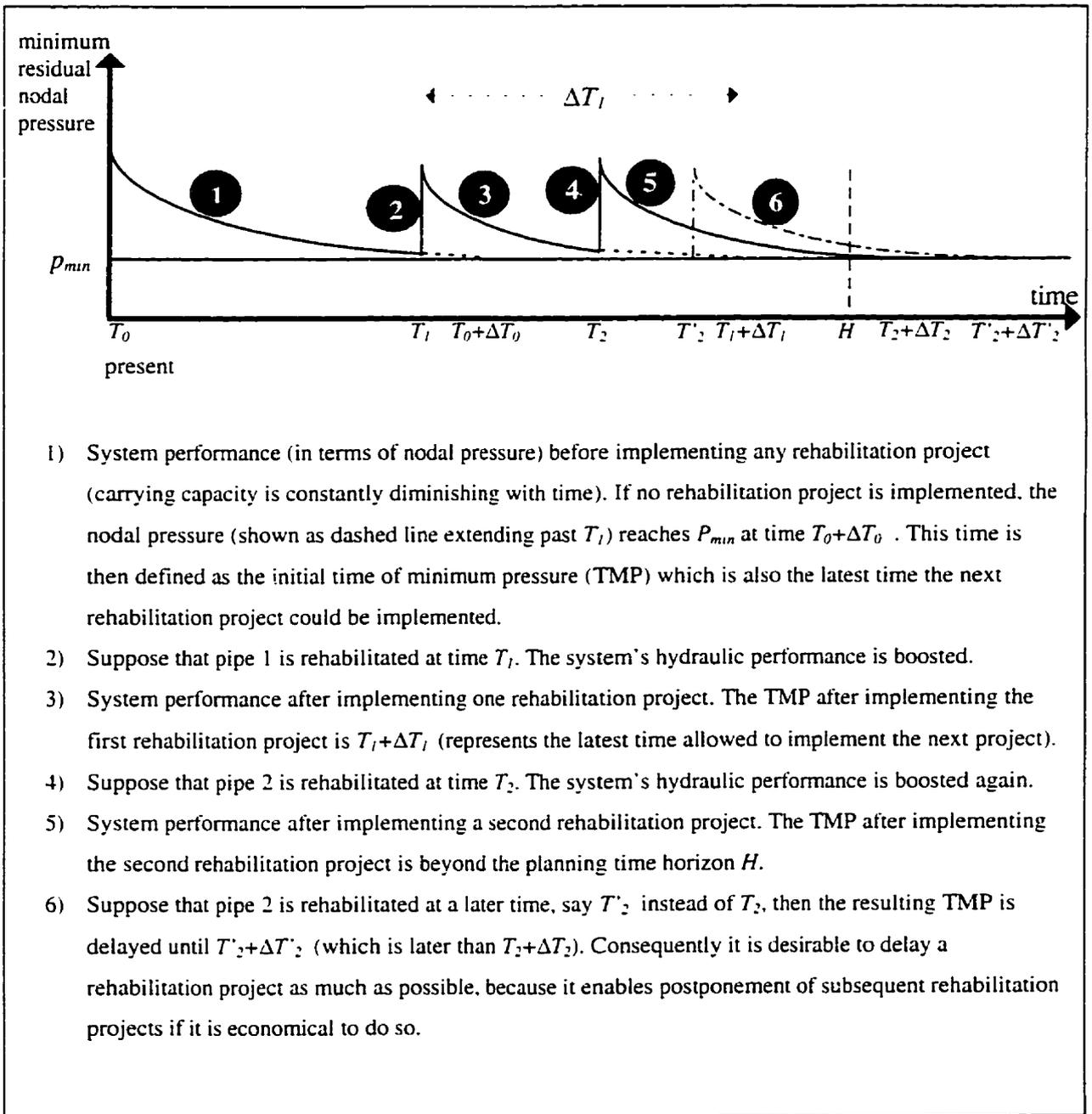


Figure 6. The hydraulic behavior of a water distribution network

The hydraulic framework described above determines the latest implementation time that is allowed for any rehabilitation project that is one in a sequence of projects. It should be emphasized that the hydraulic behavior of the network is determined simultaneously by all its pipes and the interactions between them because any change in the hydraulic properties of any pipe in the network causes a redistribution of flows in all pipes in the network.

- **Individual pipe economics**

Within the hydraulic framework, the actual timing of the rehabilitation project is determined using economic considerations as well. The total cost of a pipe comprises (a) the maintenance costs of the existing pipe until it is initially replaced, (b) the cost of the first replacement, and (c) the costs of maintenance and replacement in subsequent replacement cycles to infinity. This total cost function, is convex with respect to the timing of the first replacement, and has a minimum cost point depicting a time at which if the first replacement of a pipe is indeed implemented, the total cost of this pipe is minimized. This minimum cost replacement timing (MCRT)¹ is a function of only the structural properties of the existing pipe and the replacement pipe; hence, every pipe in the network has a unique MCRT for every replacement alternative. For example, suppose that for pipe 1 two replacement alternatives are considered e.g., same diameter and a larger diameter pipe, each alternative has its own MCRT. Pipe relining is handled somewhat differently (when the pipe is initially relined, the relining cost is added to the total cost) as described later in this section.

Consequently, in determining the timing of a replacement project there are two “motivations”: (a) implement it as late as possible (without violating hydraulic integrity) in order to delay the TMP as much as possible, and (b) implement the replacement as close as possible to the pertinent MCRT in order to reduce cost.

To demonstrate these concepts, the two pipe system is used with a numerical example as illustrated in Figure 7.

¹ MCRT is denoted by T^* in sections 3.2 through 3.4, where the mathematical formulations for the total cost are developed.

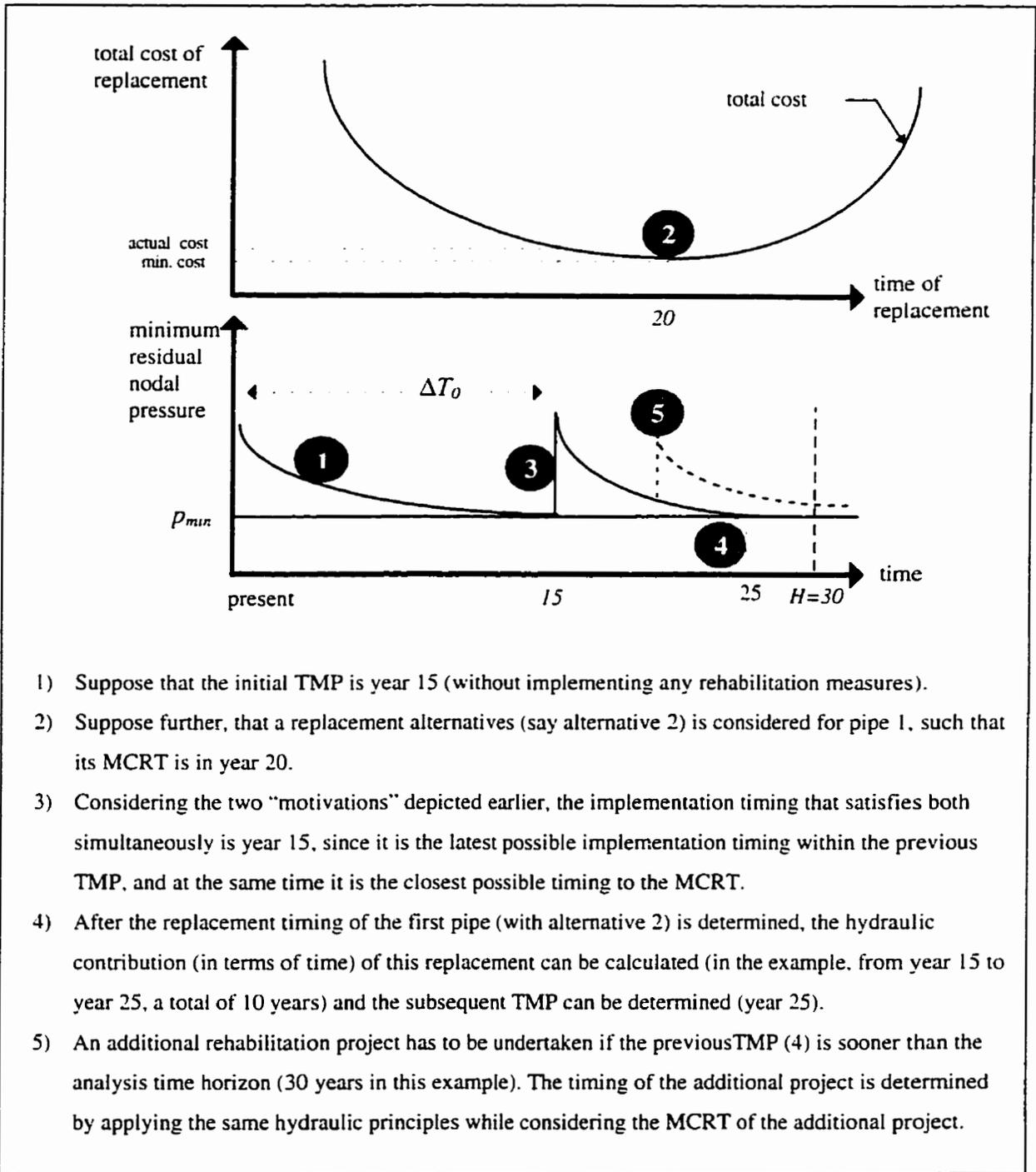


Figure 7. Example determination of replacement timing

In this numerical example, the MCRT of replacement alternative 2 (year 20) happens to be later than the previous TMP (year 15). This is not always the case as other situations may exist (e.g., if the MCRT is in year 14 or earlier). Rules of how to determine replacement

timing when the MCRT of the next replacement alternative happens to be before the previous TMP, are provided in the detailed discussion in the next section.

The relining of a pipe is assumed to be hydraulically equivalent to replacing it with the same diameter pipe, although this is only an approximation. From a structural integrity viewpoint however, the relining alternative is different than replacement because it provides no reduction in breakage rate. It is consequently assumed that if a pipe is relined, it will eventually be replaced, and this time of (eventual) replacement is assumed to be the MCRT of the replacement with same diameter alternative. From this point on, a series of replacement cycles to infinity is assumed. This topic is presented in more detail in section 3.4.2, including the criterion for the economical feasibility of pipe relining as a rehabilitation alternative.

- **Dynamic programming**

Next, a procedure of systematically considering all possible sequences of rehabilitation alternatives is applied. This procedure uses an algorithm that integrates Dynamic Programming and a partial enumeration scheme. In order to describe this procedure a few terms are defined:

Stage - the number of pipes that have been rehabilitated (e.g., stage 0 is the initial stage in which no pipe has yet been rehabilitated; stage 1 is the stage at which one pipe has been rehabilitated; etc.).

State - a unique sequence of rehabilitated pipes in a given stage (e.g., in the example above, possible states in stage 1 could be pipe 1 rehabilitated with alternative 1 at year 15, or pipe 2 rehabilitated with alternative 5 at year 5, etc.). A state has three main attributes:

- (a) the unique sequence of pipes it comprises
- (b) the state cost, consisting of the costs of all individual pipes in the sequence
- (c) the TMP resulting from the state, e.g., in the example, the state (in stage 1) comprising pipe 1 with rehabilitation alternative 2 at year 15, has a TMP equal to year 25.

Transition - adding a pipe to the states in one stage to generate states in the next stage.

Inferior state - if in a given stage there are two states that comprise the same pipes in their sequences and if one of these states has an earlier TMP yet a higher cost than the other, it is inferior to the other and can be discarded.

The procedure is presented in a conceptual flow chart in Figure 8. After the MCRTs for all combinations of pipes and rehabilitation alternatives in the network are calculated, the procedure is initialized by determining the TMP at stage zero (which is the latest time allowed for implementing the next rehabilitation project). The transition to stage 1 involves adding to the (empty at this stage) sequence all possible combinations of pipes and rehabilitation alternatives with timings that are determined using the principles of network hydraulics and individual pipe economics (as discussed earlier in this section). This operation establishes the states in the first stage. These states are then examined, respective total costs and TMPs are compared, and the inferior states are identified and discarded.

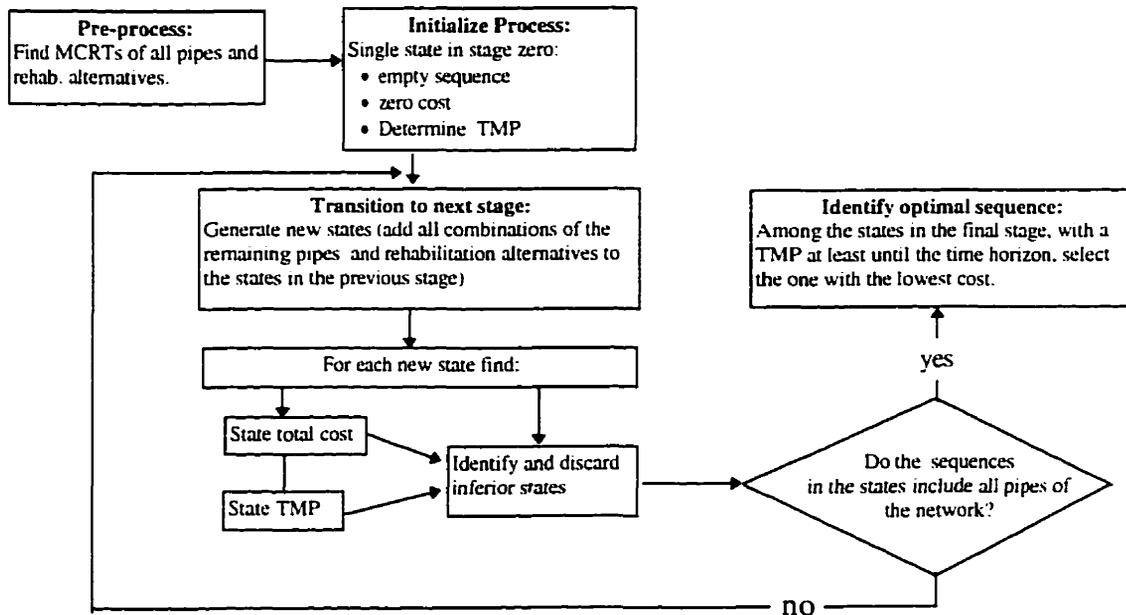


Figure 8. A conceptual description of the optimization procedure

The remaining states are now a starting point for generating new states in the transition to the next stage. This transition involves adding all possible combinations of the remaining pipes

and rehabilitation alternatives, thus generating the states in the next stage. The transition process is repeated until all pipes are considered in the sequences. The last stage comprises states whose respective sequences consist of all the pipes in the network. Among all the states whose TMP extends at least until the pre-defined time horizon, the one with the least cost is selected as the optimal sequence of rehabilitated pipes. The detailed mathematical formulation is provided in the following sections.

4.3 Relative Timing of Rehabilitation Measures

Consider a water distribution network with p pipes (links) and n nodes. A rehabilitation project is defined as (a) relining a pipe and subsequently implementing replacement cycles to infinity, or (b) replacing a pipe pipe and subsequently implementing replacement cycles to infinity. The maximum number of rehabilitation projects that can be equals the number of pipes (P) in the network. T_i is the time at which pipe i is rehabilitated ($0 \leq T_i \leq H$). For every pipe there are R alternatives for the type of rehabilitation. The DP approach does not restrict R ; however, for this presentation R will be restricted to 3 ($R=1$ reline pipe, $R=2$ replace pipe, $R=3$ replace with a pipe one nominal diameter larger). T_{ij} is the time at which pipe i is rehabilitated with rehabilitation alternative j ($0 \leq T_{ij} \leq H$ for $j \in \{1, 2, \dots, R\}$). In addition, the “do nothing” alternative is defined as a rehabilitation alternative which is implemented at or after the end of the analysis period ($T_{ij} \geq H$). This is because it is assumed that a rehabilitation project implemented at (or after) the analysis time horizon does not affect the hydraulic performance of the network during the analysis period. Consequently it can be assumed that the number of rehabilitation projects that are undertaken for the network within the analysis period is exactly p .

Let s denote an arbitrary subset of projects that comprises p' rehabilitated pipes. Let i denote the indices of all the pipes in s ($i = \{1, 2, \dots, p'\}$). For every pipe i in s there is a corresponding year of rehabilitation T_{ij} ($j \in \{1, 2, \dots, R\}$) where j is the rehabilitation alternative that was implemented for pipe i . Assume that $i=p'$ is the index of the pipe that was rehabilitated in the

latest year among all pipes in s ($T_{p'} = \max\{T_{ij} : i \in s\}$). Assume further that the system's conformity to the minimum nodal residual pressure constraints (equations (22)) is maintained in the period between the present (time of analysis) and $T_{s+1} = T_{p'} + \Delta t_{p'}$ where $\Delta t_{p'}$ is an additional time increment beyond the time of the rehabilitation of pipe p' (T_{s+1} is referred to as TMP in section 4.2). The total cost of s is $C^{tot}(s) = \sum C_{ij}^{tot}$ ($i \in s, j \in R$) where j can take only one value for every i . Assume that $C^{tot}(s)$ is minimized. The objective is to add projects to s until all pipes are rehabilitated and total cost is kept minimal; i.e.,

$$\text{minimize } C_{ij}^{tot}(p) = \sum_{\substack{i=1 \\ j \in R}}^p C_{ij}^{tot} \quad (23)$$

Assume next that another project m (rehabilitated pipe) is to be added to subset s ($m \notin s$). Let T_{mj} denote the time of implementing rehabilitation alternative j in pipe m . Since the system's conformity to constraint (21) (minimum residual nodal pressure) without m is maintained in the time interval $[0, T_{s+1}]$, it follows that in order not to violate this conformity, m must be implemented at $T_{mj} \in [0, T_{s+1}]$. Furthermore, if the sequence order of the $p'+1$ projects is important, project p' was already defined as the latest in s with implementation time of $T_{p'}$ and m is the next in sequence, it follows that $T_{mj} \in [T_{p'}, T_{s+1}]$. Graphically, this is represented in Figure 9.

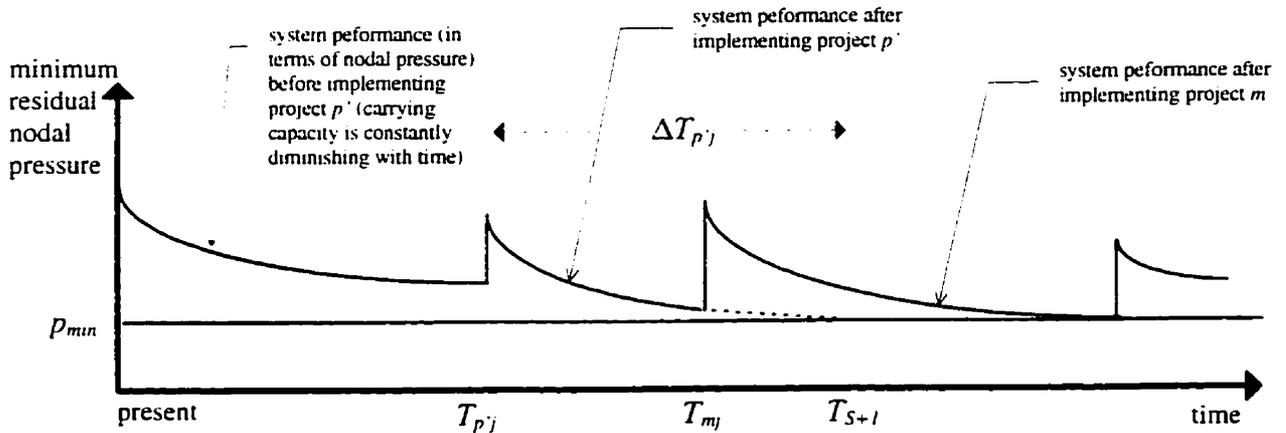


Figure 9. Timing of next project in sequence.

As previously stated, T_{mj} can be at any point between $T_{p'j}$ and T_{s+l} without violating conformity to constraint (21). In order to determine the optimal value for T_{mj} , the total cost of project m must be taken into consideration. It was already depicted in equation (12) that $C_{mj}^{tot}(T_{mj})$ is minimized for $T_{mj}=T_{mj}^*$ (referred to as MCRT in section 4.2). Observation reveals that there are three possible values which T_{mj}^* can assume relative to the interval $[T_{p'j}, T_{s+l}]$:

- (i) $T_{mj}^* > T_{s+l}$ (MCRT of the next project is later than the current TMP)
- (ii) $T_{mj}^* \in [T_{p'j}, T_{s+l}]$ (MCRT is between the timing of previous project and the current TMP)
- (iii) $T_{mj}^* < T_{p'j}$ (MCRT is before the implementation timing of the previous project)

In the first situation, since T_{mj} must be smaller than T_{s+l} (to maintain conformity to constraint (21)), and since $C_{mj}^{tot}(T_{mj} < T_{mj}^*)$ decreases with time, it follows that selecting the time of replacement for project m , $T_{mj} = T_{s+l}$ (i.e., replacement timing is the current TMP) would minimize the total cost of project m , or mathematically stated

$$\min[C_{mj}^{tot}(T_{mj})] = C_{mj}^{tot}(T_{mj} = T_{s+l}) \quad \text{for} \quad T_{mj}^* > T_{s+l} \quad (24)$$

The first situation is graphically represented in Figure 10.

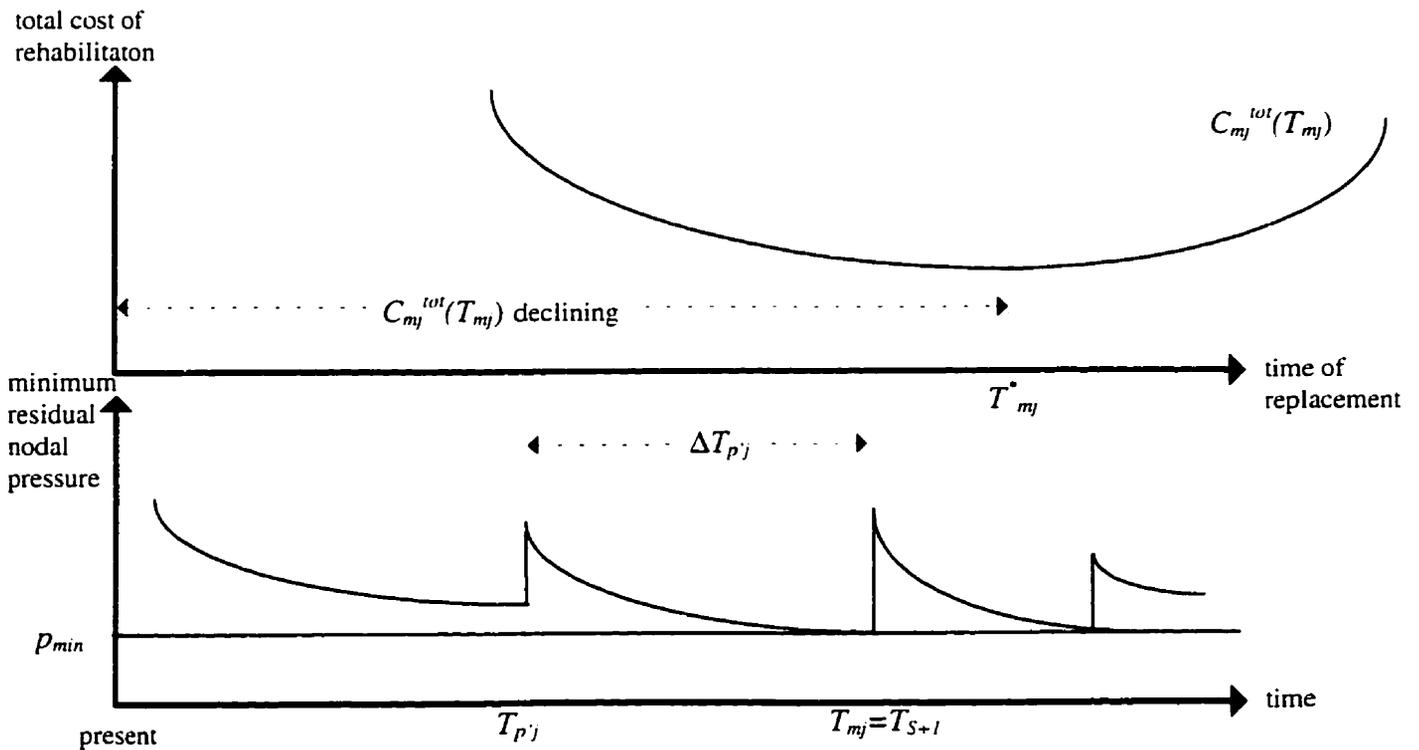


Figure 10. Selection of rehabilitation time when $T_{mj}^* > T_{s+l}$.

Since it was assumed that ΣC_{ij}^{tot} is minimized for $i \in s$, adding $C_{mj}^{tot}(T_{m,l}=T_{s'+1})$ will keep ΣC_{ij}^{tot} at minimum for $i \in s \cup m$. Hence, for case (i) the following rule applies:

$$T_{mj} = T_{s'+1} \quad \text{for} \quad T_{mj}^* \geq T_{s'+1} \quad (25)$$

In words: When the MCRT of the next replacement is later than the current TMP, select the current TMP as the time to implement the next replacement. Applying the first case to the relining alternative is discussed later.

The second case is $T_{mj}^* \in [T_{p'}, T_{s'+1}]$. To analyze this case it is necessary to divide the interval into: $[T_{p'}, T_{mj}^*]$ and $(T_{mj}^*, T_{s'+1}]$. In the first interval, $C_{mj}^{tot}(T_{m,l})$ is a decreasing function of time; hence, the least cost would occur at $T_{mj}=T_{mj}^*$ (the next MCRT). In the second interval, $C_{mj}^{tot}(T_{m,l})$ is an increasing function of time hence, from a single pipe viewpoint, the least cost would occur at $T_{mj} \rightarrow T_{mj}^*$. However, there is another factor that should be considered at this point, namely how the selection of T_{mj} may affect the cost of the next project in sequence. Let s' denote a subset of projects consisting of $s \cup m$. Then s' comprises $p'+1$ projects. Let u denote the next project in sequence ($u \notin s'$). Consistent with previous notation, T_{uj} denotes the time of implementing project u ; T_{uj}^* is the MCRT of project u (at which C_{uj}^{tot} is minimized); Δt_{mj} is an additional time increment beyond the time of the implementation of project m ; $T_{s'+1} = T_{mj} + \Delta t_{mj}$ and $T_{uj} \in [T_{mj}, T_{s'+1}]$.

It is clear that the later T_{mj} is, the later $T_{s'+1}$ becomes. If $T_{uj}^* > T_{s'+1}$, then the latest time of rehabilitation for project u is $T_{uj} = T_{s'+1}$, and project u is now in what was previously described as case (i). Since in case (i) $C_{uj}^{tot}(T_{uj})$ is a declining function of T_{uj} , it follows that an increase in $T_{s'+1}$ would result in a decrease of C_{uj}^{tot} . However, in order to increase $T_{s'+1}$, T_{mj} has to be increased and when $T_{mj} \in (T_{mj}^*, T_{s'+1}]$, an increase in T_{mj} results in an increase in C_{mj}^{tot} . In other words, postponing the implementation of a project m_j beyond its MCRT T_{mj}^* will, on one hand, result in a cost increase of project m_j , but on the other hand, this postponement will delay the resulting TMP which may enable postponement of subsequent projects and thereby, possibly reducing their costs.

In a situation in which project u is the final project to enter the sequence, it is relatively easy

to solve this problem by solving the equation $\frac{\partial C_{m_j}^{tot}(T_{m_j})}{\partial T_{m_j}} = \frac{\partial C_{u_j}^{tot}(T_{m_j} + \Delta T_{m_j})}{\partial T_{m_j}}$ for T_{m_j} .

However, when project u is not the final project, postponement of u may result in possible postponement of the project subsequent to u , and then the next project after that and so forth. Since it is impossible at the time T_{m_j} to analytically account for all of these possible future effects, it is concluded that for case (ii) the following rule applies:

$$T_{m_j} \in [T_{m_j}^*, T_{S+1}] \quad \text{for} \quad T_{m_j}^* \in [T_{p^j}, T_{S+1}] \quad (j \neq \text{reline}) \quad (26)$$

The second situation is graphically represented in Figure 11.

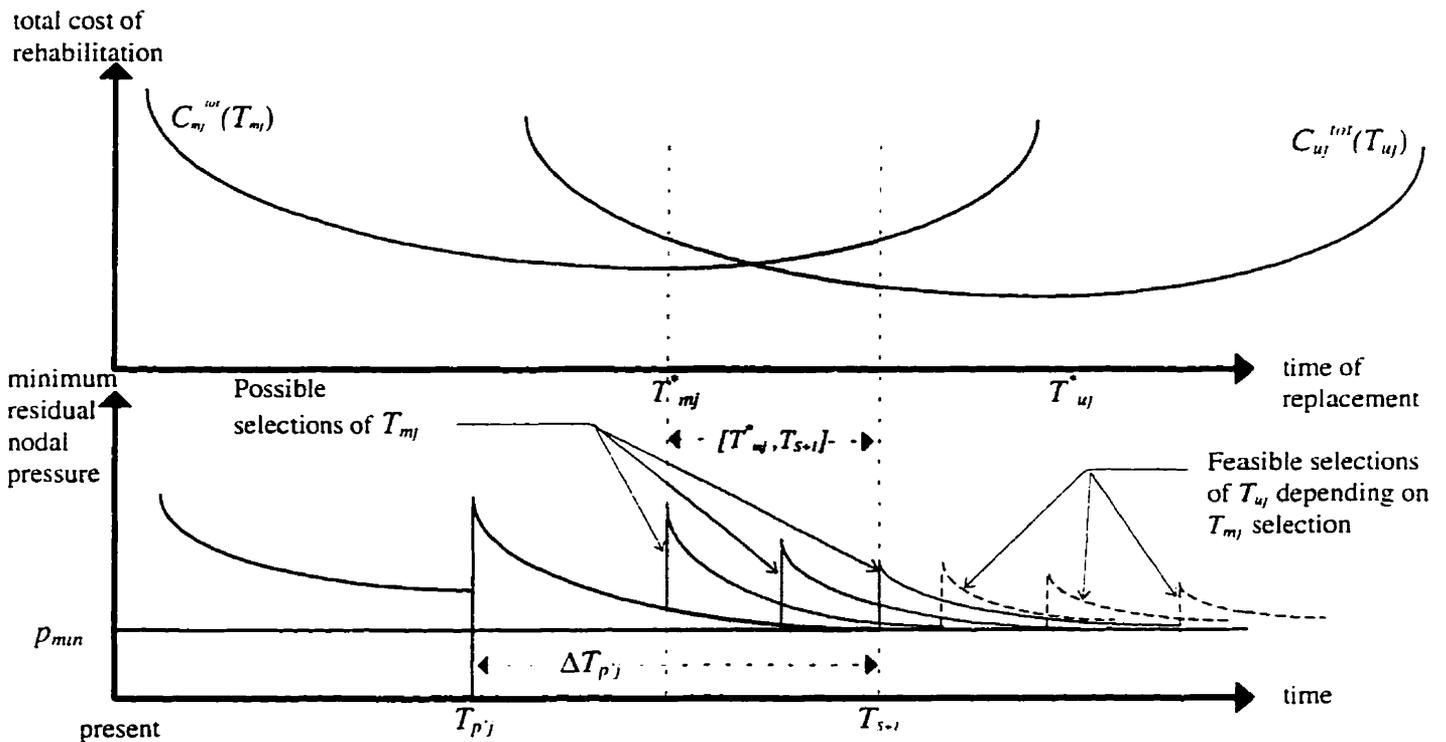


Figure 11. Selection of rehabilitation time when $T_{m_j}^* \in [T_{p^j}, T_{S+1}]$.

The third case occurs when $T_{mj}^* < T_{p'j}$. T_{mj} can be in the interval $[T_{p'j}, T_{s+1}]$. In this interval, $C_{uj}^{tot}(T_{uj})$ is an increasing function; hence, the total cost $C_{uj}^{tot}(T_{uj})$ is minimized for $T_{mj} = T_{p'+1}$. However, similar to what was described in case (ii), the choice of T_{mj} may affect the cost of future projects (and thus the total cost of all projects $\sum C_{ij}^{tot} : i=\{1,2,\dots,p\}$). The following rule is therefore applied to case (iii):

$$T_{mj} \in [T_{p'j}, T_{s+1}] \quad \text{for} \quad T_{mj}^* < T_{p'j} \quad (j \neq \text{reline}) \quad (27)$$

The third situation is graphically represented in Figure 12.

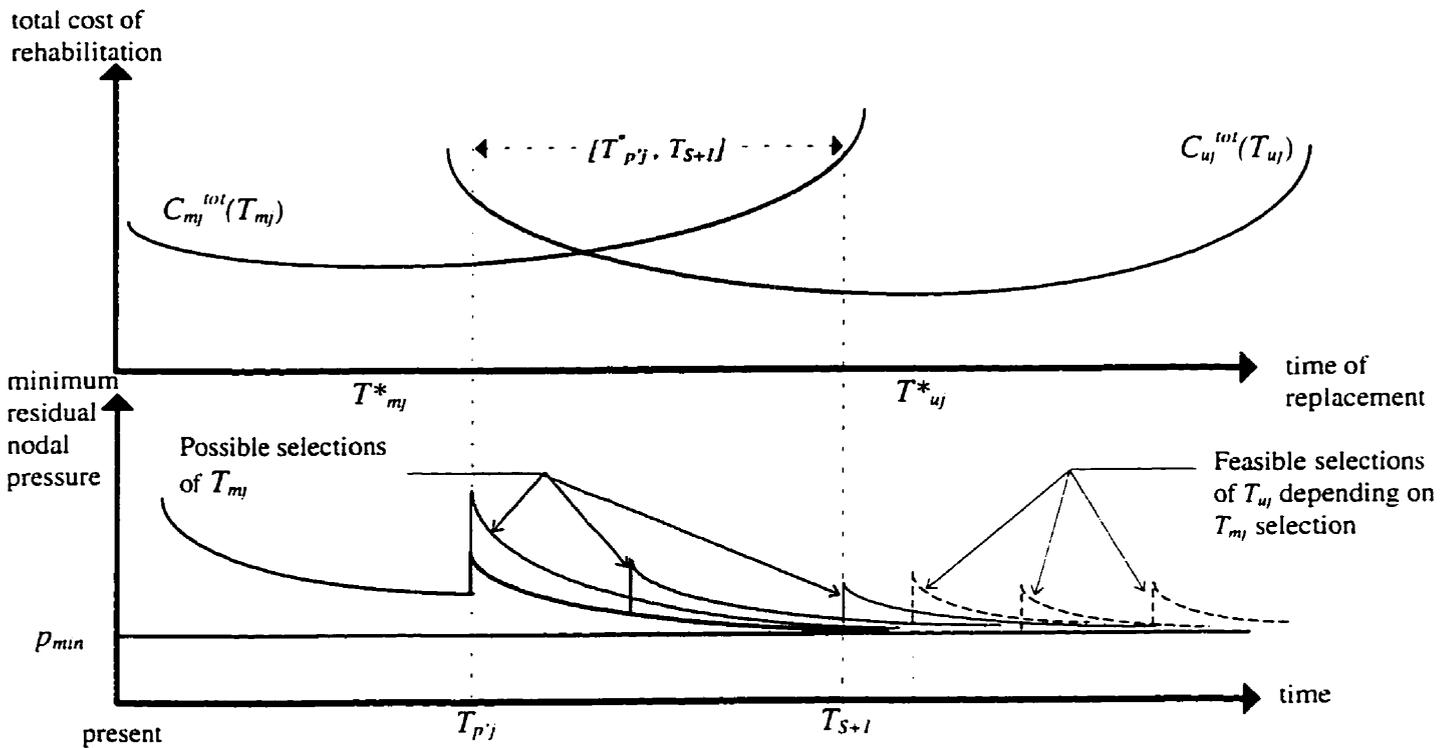


Figure 12. Selection of rehabilitation time when $T_{mj}^* < T_{p'j}$.

When the rehabilitation alternative under consideration is pipe relining, the three rules discussed above are applied differently. Recall from section 3.4.2 and Figure 3 that a relining alternative is considered only when its implementation time is well before the T^* (or MCRT) of a replacement with same diameter alternative. Furthermore, relining is selected only if $CAP_{reline}(T_m) < [C_{tot}(T_{mk}) - C_{tot}(T_{mk}^*)]$ where k denotes the alternative of replacement with the same diameter pipe. Consequently, relining a pipe is considered only if the first situation [case (i)] exists for alternative k . Since in the first situation both $CAP_{reline}(T_m)$ and $C_{tot}(T_{mk})$ are declining functions of the time of implementation T_m , it follows that equation (25) applies to the relining alternative as well as to replacement alternatives, and rule (i) may be extended as follows:

$$T_{mj} = T_{m,relined} = T_{s+1} \quad \text{for} \quad T_{mj}^* \geq T_{s+1} \quad (j=1,2,\dots,R) ; (k=\text{replace with same diameter})$$

if $j=k$ and $CAP_{reline}(T_{s+1}) < [C_{tot}(T_{mk}) - C_{tot}(T_{mk}^*)]$ (28)
 $(T_{mk}=T_{s+1})$
then reline is a superior alternative to k and k should not be considered

if $j=k$ and $CAP_{reline}(T_{s+1}) > [C_{tot}(T_{s+1}) - C_{tot}(T_{mk}^*)]$
then reline is an inferior alternative to k and should not be considered

Conceptually, the process of pipe rehabilitation scheduling may be decomposed into two sub-processes: sequencing the projects (order of implementation)¹ and timing their implementation within a given sequence. A Dynamic Programming (DP) approach combined with implicit enumeration and branch and bound elements is proposed for this task.

¹ In a network that comprises p pipes, the total number of different sequencing orders is $(p!)$. However, every project within the sequencing order can take any one of R rehabilitation alternatives so the actual total number of different sequencing orders is $(R^p * p!)$ which in a minimal network of 3 pipes and 3 rehabilitation alternatives amounts to 162.

4.4 The Dynamic Programming Approach

The dynamic programming approach is based on the premise that at any stage ($f+1$), whatever decisions were made previously, the remaining decisions must constitute an optimal sequence with regard to the states that are found in stage f (see Appendix 2).

The stages in the proposed DP are defined as the number of pipes that have already been rehabilitated. The states in the DP are defined as unique sequences in a given stage. Let f denote the stage and let $S_{f,s,k,j,T}$ denote a state in stage f . The five indices of S denote the following:

- f – denotes the stage to which S belongs
- s – denotes the subset of pipes that S comprises
- k – denotes the pipe that was rehabilitated last in state S
- j – denotes the rehabilitation alternative implemented on pipe k
- T – denotes the time at which pipe k was rehabilitated (elapsed from the present)

For example, stage $f=2$ is a stage where two pipes have been rehabilitated. A state $S_{2,s,6,1,5}$ is a sequence comprising $f=2$ projects of which the last one was the rehabilitation (alternative 1) of pipe #6 which was implemented in year 5. Transition from one stage to the next means adding to s an additional rehabilitation project mj ($m \notin s$ and $j \in R$) at time T' so that $S_{f+1,s',k',j',T'} = S_{f,s \cup mj,m,j,T_{mj}}$. For example, in a network of 6 pipes, if $S_{2,s,4,3,5}$ comprises pipes 2 and 4, ($s = \{2,4\}$, where pipe 4 was rehabilitated last, with alternative 3, at year 5), transition to stage $f=3$ means adding project mj to the sequence where m can be pipe 1, 3, 5 or 6, j can be rehabilitation alternative 1, 2, 3 or 4, the new state $S_{3,s',k',j',T'}$ comprises 3 pipes and T' is selected according to the three rules described previously.

The cost of a state comprises the cost of all the projects in the state $C_{f,s,k,j,T}^{tot} = \sum_{\substack{\text{all } i \in s \\ j \in R}} C_{ij}^{tot}$.

The cost of a state after transition is $C_{f+1,s',k',j',T'}^{tot} = C_{f,s,k,j,T}^{tot} + C_{mj}^{tot}(T_{mj})$.

For state $S_{f,s,k,j,T}$, hydraulic integrity (equation 22) is sustained (or in other words, the system's TMP is extended) until time $T_{S_{f+1,s,k,j,T+1}} = T_{S_{f,s,k,j,T}} + \Delta t_{S_{f,s,k,j,T}}$ where $\Delta t_{S_{f,s,k,j,T}}$ is the additional time increment in which hydraulic integrity is sustained beyond the time T (at which project k was implemented) due to the increase in system capacity that resulted from adding project k into the set at the previous stage.

Since T_{mj} (the time for implementing project m with rehabilitation alternative j), is determined according to three rules described previously (equations 24-26), the following applies:

- In case (i) $T_{mj}^* \geq T_{S_{f+1,s,k,j,T+1}} \Rightarrow T_{mj} = T_{S_{f+1,s,k,j,T+1}}$; i.e. one possible value for T_{mj} .
- In case(ii) $T_{mj}^* \in [T_{S_{f,s,k,j,T}}, T_{S_{f+1,s,k,j,T+1}}] \Rightarrow T_{mj} \in [T_{mj}^*, T_{S_{f+1,s,k,j,T+1}}]$; i.e., several possible values for T_{mj} .
- In case (iii) $T_{mj}^* < T_{S_{f,s,k,j,T}} \Rightarrow T_{mj} \in [T_{S_{f,s,k,j,T}}, T_{S_{f+1,s,k,j,T+1}}]$; i.e., several possible values for T_{mj} .

Consequently, in case (i) the transition from stage f to stage $f+1$ yields a single, definite state in stage $f+1$ for every state in stage f . In contrast, transitions in cases (ii) or (iii) are not definite because T_{mj} may take on several possible values. Since the best value cannot be determined, all possible values for T_{mj} are enumerated and new corresponding states are generated in stage $f+1$. Let T_{mj_y} denote all possible values for T_{mj} . In case (i) $y=\{1\}$, in case (ii) $y=\{1,2,\dots,w\}$ where $w = (1 + T_{S_{f+1,s,k,j,T+1}} - T_{mj}^*)$ and in case (iii) $y=\{1,2,\dots,w\}$ where $w = (1 + T_{S_{f+1,s,k,j,T+1}} - T_{S_{f,s,k,j,T}})$. It follows that in cases (ii) and (iii) a single state in stage f may generate multiple feasible states in stage $f+1$. This phenomenon of one state producing several states upon transition, increases the dimensionality of the DP problem and may result in a computationally prohibitive method. Fortunately, there are ways to reduce this dimensionality.

It is apparent in all previous formulations and examples that the time discretization used in the proposed methodology is a single year (or a similar time period)¹. If two states of the same stage comprise the same partial set of pipes, then one is superior to the other if its cost is lower *and* it extends system hydraulic integrity to a later time. Mathematically stated:

$$\begin{aligned}
 S_{f,s,k,j,T} \text{ is superior to } S_{f,s',k',j',T} & \text{ if } s = s' & \text{ and} \\
 C_{f,s,k,j,T}^{tot} < C_{f,s',k',j',T}^{tot} & \text{ and} \\
 T_{S_{f,s,k,j,T}+1} \geq T_{S_{f,s',k',j',T}+1} &
 \end{aligned} \tag{29}$$

The full transition process from stage f to stage $f+1$ may now be defined as follows²:

- (a) For each state $S_{f,s,k,j,T}$ in stage f , identify the TMP³ $T_{S_{f,s,k,j,T}+1} = T_{S_{f,s,k,j,T}} + \Delta t_{S_{f,s,k,j,T}}$.
- (b) To every state $S_{f,s,k,j,T}$ add every project m_j ($m \notin s$) to obtain a state (or states) in the next stage $S_{f+1,s \cup m, m, j, T_{m_j}}$. T_{m_j} (the time of implementing project m_j) is determined using the three rules. If more than one T_{m_j} exists (cases (ii) and (iii)) multiple states $S_{f+1,s \cup m, m, j, T_{m_j}}$ are created in stage $f+1$ from a single state in stage f .
- (c) For all states in $f+1$ calculate total cost $C_{f+1,s \cup m, m, j, T_{m_j}}^{tot}$.
- (d) For all states in $f+1$ identify and discard all the non-superior states using equation (29).

A numerical example best illustrates these steps:

¹ The rationale for this is the fact that (a) the typical time to implement a project is measured in months, and (b) allocation of funds for the projects is planned on an annual basis. The DP process allows, however, for any time increment desired. A smaller time increment may result in somewhat better accuracy in the results, but the computational time will increase substantially. In contrast, a longer time increment may be used to get a (relatively) quick result while sacrificing some accuracy.

² Having calculated for all possible projects m_j ($i=\{1,2,\dots,p\}$ and $j=\{1,2,\dots,R\}$) the value of $T^*_{i,j}$

³ Whenever $T_{S_{f,s,k,j,T}+1}$ is greater than time horizon H , it is truncated to H . Further discussion about this practice is given later.

4.5 An Illustration of the DP Approach

The rehabilitation sequencing of a network of 6 pipes (enumerated 1 through 6) is analyzed for a time horizon of $H=20$ years. A state $S_{2,3,4,1,5}$ exists where $s=\{2,4\}$ (i.e., a state in stage 2, pipes 2 and 4 were rehabilitated, pipe 4 was the latest to be rehabilitated with alternative 1 at year 5). It was found that $\Delta t_{S_{2,3,4,1,5},T} = 6 \text{ years}$, thus $T_{S_{2,3,4,1,5},T+1} = T_{S_{2,3,4,1,5},T} + \Delta t_{S_{2,3,4,1,5},T} = 5 + 6 = 11$ (i.e., after rehabilitating pipe 4 at year 5, the hydraulic integrity of the system is sustained until year 11 without the rehabilitation of any additional pipes).

In transition to stage $f+1=3$, the rehabilitation of pipe $m=6$ is considered. The following was computed for pipe $m=6$:

$T_{6,2}^* = 4$ i.e., for pipe 6, rehabilitation alternative $j=2$ (replace with pipe of the same diameter) minimum cost occurs when rehabilitation is implemented at year 4.

$T_{6,3}^* = 8$ i.e., for pipe 6, rehabilitation alternative $j=3$ (replace with pipe one nominal diameter larger) minimum cost occurs when rehabilitation is implemented at year 8.

$T_{6,4}^* = 13$ i.e., for pipe 6, rehabilitation alternative $j=4$ (replace with pipe two nominal diameters larger) minimum cost occurs when rehabilitation is implemented at year 13.

$T_{6,j=1}$ is a relining alternative for pipe 6.

- Clearly, adding project $mj=6,4$ constitutes case (i) since $T_{6,4}^* \geq T_{S_{2,3,4,1,5},T+1}$ ($13 > 11$). Consequently, according to the first rule, $T_{6,4}$ is assigned the value of $T_{S_{2,3,4,1,5},T+1} = 11$

- Adding project $mj=6,3$ constitutes case (ii) since $T_{6,3}^* \in [T_{S_{2,3,4,1,5},T}, T_{S_{2,3,4,1,5},T+1}]$ ($8 \in [5, 11]$). Consequently, according to the second rule $T_{6,3} \in [T_{6,3}^*, T_{S_{2,3,4,1,5},T+1}] = [8, 11]$. Enumerating all possible values yields $w=(1+11-8)=4$ values for $T_{6,3}$ i.e. $T_{6,3,v} = \{8, 9, 10, 11\}$.

- Adding project $mj=6,2$ constitutes case (iii) since $T_{6,2}^* \leq T_{S_{2,3,4,1,5}}$ ($4 < 5$). Consequently, according to the third rule $T_{6,2} \in [T_{S_{2,3,4,1,5}}, T_{S_{2,3,4,1,5}-1}] = [5,11]$. Enumerating all possible values yields $w=(1+11-5)=7$ values for $T_{6,2}$ i.e. $T_{6,2}=\{5,6,7,8,9,10,11\}$.
- Before adding project $mj=6,1$ it is noted that for project $mj=6,2$ case (iii) exists. Consequently, relining pipe 6 is not considered (to consider relining a pipe case (i) has to exist for j =replace with same diameter - see equation 28).

The transition above yielded 13 states in stage $f=3$. Next, calculate the total cost of each state, determine $T_{S_{f,s,k,j,T-1}} = T_{S_{f,s,k,j,T}} + \Delta r_{S_{f,s,k,j,T}}$ for each state and then identify and discard all the non-superior states. The following table summarizes this process for the example:

Stat	Indices					Cost		Remarks
	f	s	k	j	T_S			
1	3	2,4,6	6	2	8	12	94	⇐superior (among all $T_{S+1}=12$)
2	3	2,4,6	6	2	9	12	96	
3	3	2,4,6	6	2	10	13	97	⇐superior (among all $T_{S+1}=13$)
4	3	2,4,6	6	2	11	14	98	
5	3	2,4,6	6	3	5	14	93	⇐superior (among all $T_{S+1}=14$)
6	3	2,4,6	6	3	6	14	95	
7	3	2,4,6	6	3	7	15	97	⇐superior (among all $T_{S+1}=15$)
8	3	2,4,6	6	3	8	15	99	
9	3	2,4,6	6	3	9	15	101	
10	3	2,4,6	6	3	10	16	103	⇐superior (among all $T_{S+1}=16$)
11	3	2,4,6	6	3	11	16	105	
12	3	2,4,6	6	4	11	20	108	⇐superior T_{i+1} truncated from 22

In real applications, a higher degree of state reduction is achieved since states with equal s comprise all $S_{2,3,4,j,T}$ for $j=2,3$ and 4 as well. The steps above are repeated for subsequent transitions until the last transition is achieved.

In a system of p pipes there are $p+1$ stages and p transitions, where stage 0 is defined as the initial stage in which no pipe has yet been rehabilitated. At the end of stage p , all states $S_{p,s,k,j,T}$

comprise exactly p rehabilitation projects for p pipes, i.e., same s for all states. Also, all $T_{S_{p,i,k,j,T+1}} \leq H$. Since the system is required to perform adequately for the entire design period, only those states for which $T_{S_{p,i,k,j,T+1}} = H$ are considered. In compliance with equation (29), the state with the lowest cost for which $T_{S_{p,i,k,j,T+1}} = H$ is the optimal sequence of projects.

4.6 Summary of the DP Process

The dynamic programming process can be summarized by the following recursive expression:

$$C_{f+1,s \cup m,m,j,T}^{tot} = \min_{m \in s} \{ C_{f,s,k,j,T}^{tot} + C_{mj}^{tot} \} \quad \text{for any given subset } s, \quad (30)$$

$$\text{subject to } T_{S_{f+1,s \cup m,m,j,T+1}} \geq T_{S_{f+1,s \cup m,m,j,T_y+1}} \quad \text{for } y = 1, 2, \dots, w$$

- where
- f = stage number ($f \in \{0, 1, 2, \dots, p-1\}$)
 - s = subset of projects contained in a state
 - m = index of the pipe that is rehabilitated next ($m \in \{0, 1, 2, \dots, p-1\}$)
 - j = rehabilitation alternative ($j \in \{0, 1, 2, \dots, R\}$)
 - k = index of the pipe that was the latest to be rehabilitated in a sequence
 - T = time of rehabilitation of the last pipe in a sequence ($T \in \{0, 1, 2, \dots, H\}$)
 - $C_{f,s,k,j,T}^{tot}$ = total cost of a state that is defined by indices f, s, k, j, T
 - C_{mj}^{tot} = total cost of rehabilitating pipe m with alternative j (time of rehabilitation is determined following the three rules)
 - $T_{S_{p,i,k,j,T+1}}$ = the TMP for a state S that is defined by the given indices
 - w = number of possible states that can be generated from a given state in the previous stage

The flow chart in Figure 13 summarizes the process graphically.

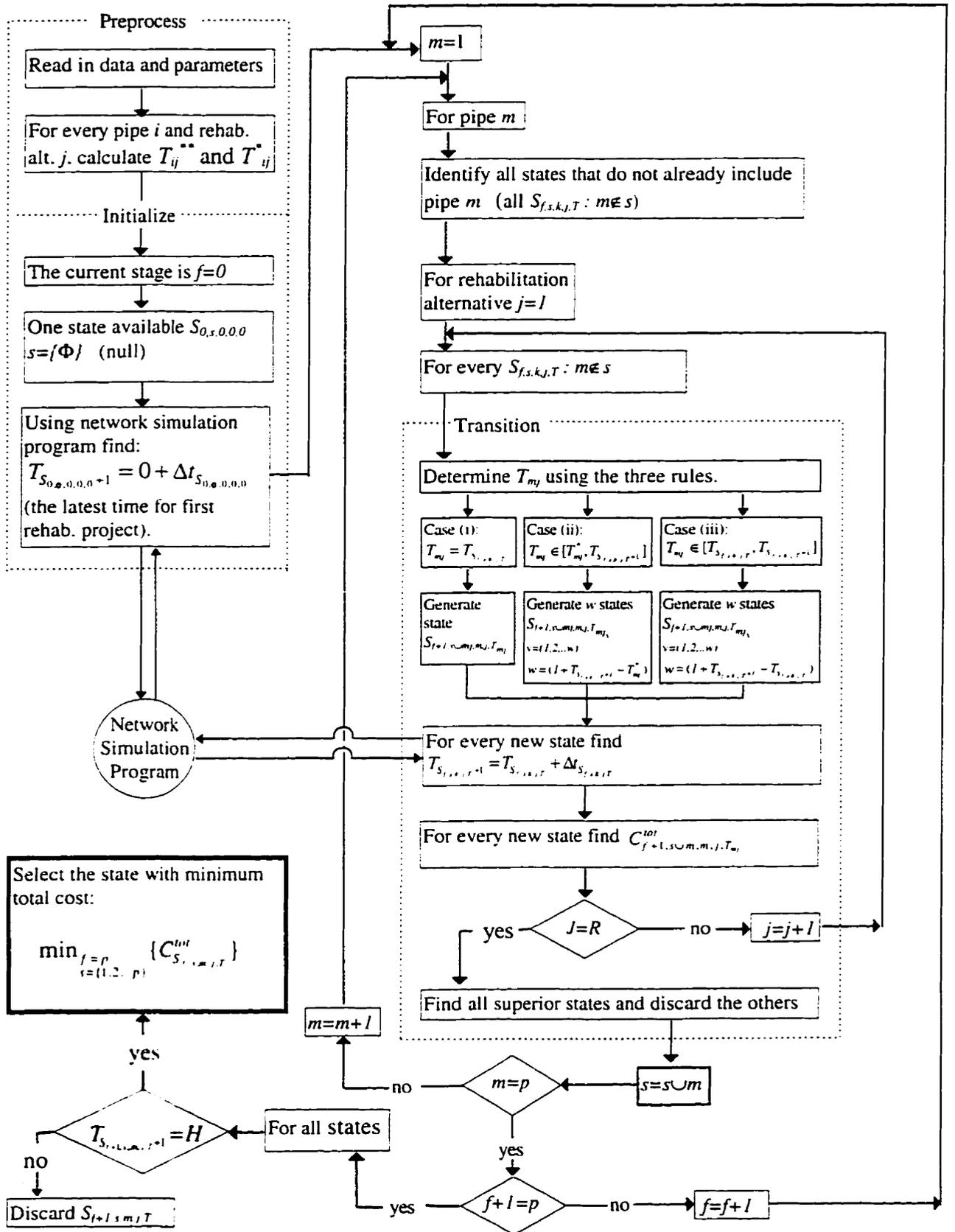


Figure 13. Flow Chart for the proposed methodology.

4.7 Additional State Reduction

Since the dimensionality of the problem is the single most important factor in determining the execution time of the analysis, it is important to reduce it as much as possible. The following method pertains only to pipe replacement alternatives because it is based on the fact that in certain situations some inferior states may be identified for an individual pipe due to the relationship between pipe breakage rate and diminishing hydraulic capacity rate in the replacement cycle.

Recall Section 3.5 (Total Cost and Time Horizon), where a discussion is presented on the single pipe, steady-state formulation within the framework of a defined analysis period. Since the total cost of pipe replacement is taken in consideration of a steady state stream of costs (equations 4 through 12), it is constant with respect to H (time horizon). Consequently, it is assumed that within the analysis period, if pipe i was initially replaced ($j=2,3$ or 4) at time T_{ij} , it is subsequently replaced at times $T_{ij}+Z \cdot T_{ij}^{**}$, where $Z=(1,2,\dots)$ and T_{ij}^{**} is defined in equation (8). The total cost per unit length of a pipe as a function of its initial replacement time is given in equation (10). After integration

$$C_{ij}^{tot}(T_{ij}) = (Cr_{ij} + C_{inf}(T_{ij}^{**})) e^{-rT_{ij}} + \frac{Cb_i N(t_0)_i e^{A_i s_i}}{A_i - r} \left[e^{(A_i - r)T_{ij}} - 1 \right] \quad (31)$$

Consider two cases as follows: Case 1 in which initial pipe (i) replacement (j) is implemented at time T_{ij} . The total pipe cost in case 1 is $C_{ij}^{tot}(T_{ij})$. In case 2, the same pipe is initially replaced in year $T_{ij} = T_{ij} + T_{ij}^{**}$. The total cost of this pipe in case 2 is $C_{ij}^{tot}(T_{ij}')$. The hydraulic capacity of the pipe in case 1 is the same as in case 2 for the period between T_{ij}' and H . However, in the period between the present and T_{ij}' , the pipe in case 1 has a greater hydraulic capacity than in case 2 (in case 1 the pipe is “newer” than in case 2 for this period). If the total cost of the pipe in case 1 is lower than the total cost of the pipe in case 2, then case 1 is clearly superior to case 2. Since $T_{ij}' \geq T_{ij}^*$ then $C_{ij}^{tot}(T_{ij}')$ is an increasing function of time¹.

¹ The reason: $T_{ij} + T_{ij}^{**} \geq T_{ij}^*$ for every T_{ij} . The equality exists only for $T_{ij} = 0$, i.e. only when the pipe is brand new at the time of analysis (at present). Hence, if $T_{ij}' \geq T_{ij} + T_{ij}^{**}$ it follows that $T_{ij}' \geq T_{ij}^*$.

It follows that case 1 is superior to all cases in which replacement is implemented at $T'_{ij} \geq T_{ij} + T_{ij}^{**}$.

Mathematically stated:

$$S_{f,s,k,j,T_{k_j}} \text{ is superior to } S_{f,s,k,j,T'_{k_j}} \text{ if } C_{k_j}^{tot}(T_{k_j}) \leq C_{k_j}^{tot}(T'_{k_j}) \text{ and } T_{k_j} + T_{k_j}^{**} \leq T'_{k_j} \quad (32)$$

It should be noted that the non-superior states found with equation (30) can be found and discarded a priori, before the transition procedure is started. Let $T_{k_j \text{ first}}$ denote the earliest T_{k_j} that renders equation (30) true. All states comprising pipe k , alternative j and implementation time $T'_{k_j} \geq (T_{k_j \text{ first}} + T_{k_j}^{**})$ are inferior. This is graphically illustrated in Figure 14.

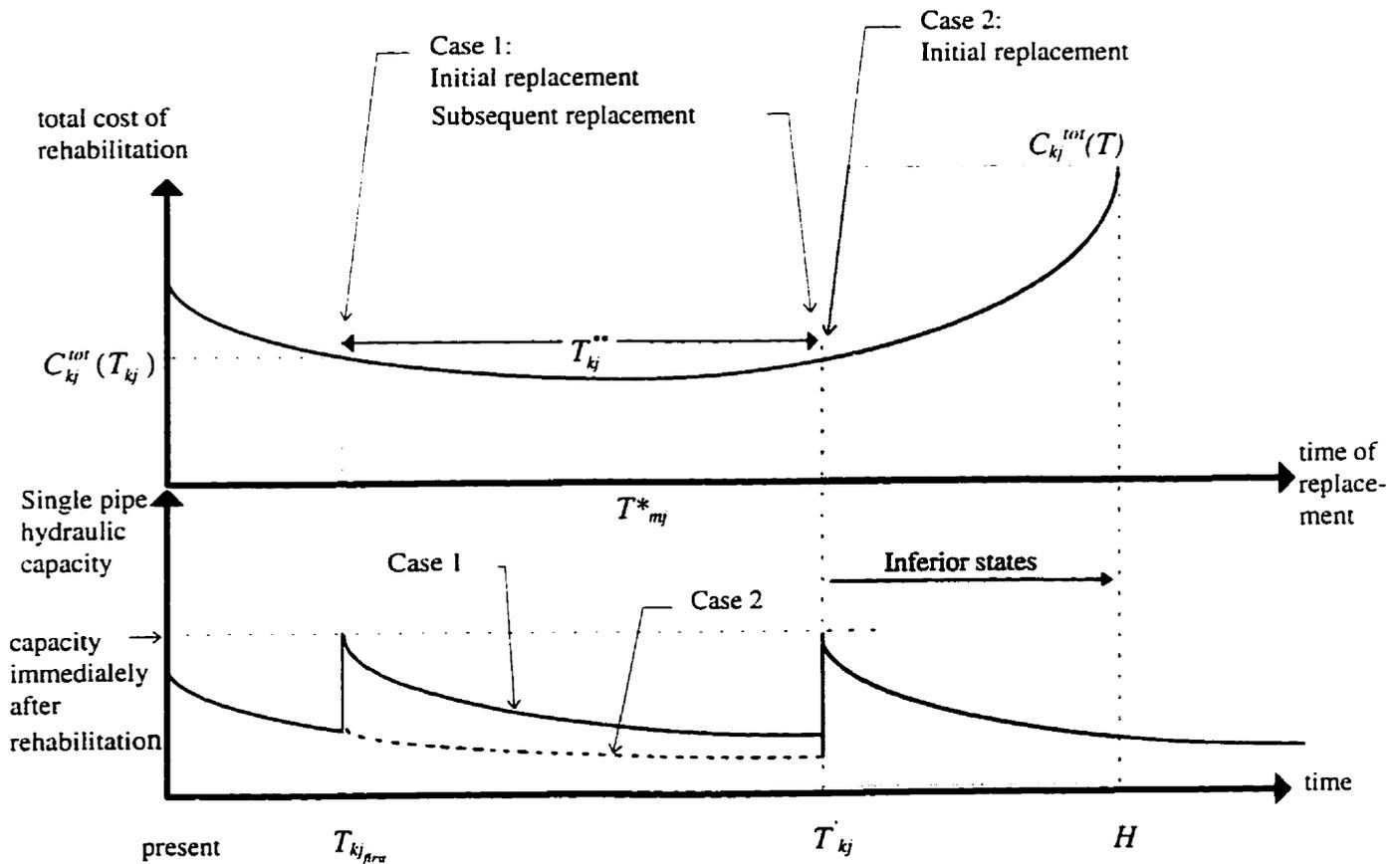


Figure 14. Additional state reduction.

Further state reductions may be achieved while stepping through the DP process (these are applicable only to replacement alternatives) as follows:

- If at a stage f , a subset s is identified so that $T_{S_f, \dots, T} + \Delta t_{S_f, \dots, T} \geq H$ then subsequent projects need not be considered in the regular fashion but rather a shortcut may be implemented. If the next project falls in the category of case (i), then by definition the year of its implementation will be year H . If the next project falls in the category of case (ii), then the time of its implementation will be its T^* year (no need to check beyond that because the system is already hydraulically adequate to the end of the analysis period). If the next project falls in the category of case (iii), then the time of its implementation will be the year the previous project in s was implemented; i.e., at time T . The above is valid providing the next project indeed enhances the system's hydraulic capacity for the whole period from the time of its implementation to the end of the time horizon (a situation may exist in which $e_{ij} > e_i$ such that towards the year H , the system's hydraulic capacity with the new project becomes inadequate).
- When adding a project m_{ij} to a subset s in stage f , a new state $S_{f+1, s \cup m, m, j, T}$ is created and the system's hydraulic integrity is extended by $\Delta t_{S_{f+1, s \cup m, m, j, T}}$. If this time extension is zero (i.e. the addition of project m_{ij} does not contribute to increasing the system's hydraulic capacity), then the newly created state $S_{f+1, s \cup m, m, j, T}$ may be discarded unless $T = T_{ij}^*$.

5. EXAMPLE

The following is a numerical example that illustrates the concepts described in the previous sections. The example network (Figure 15) consists of three pipes, one supply node and two demand nodes.

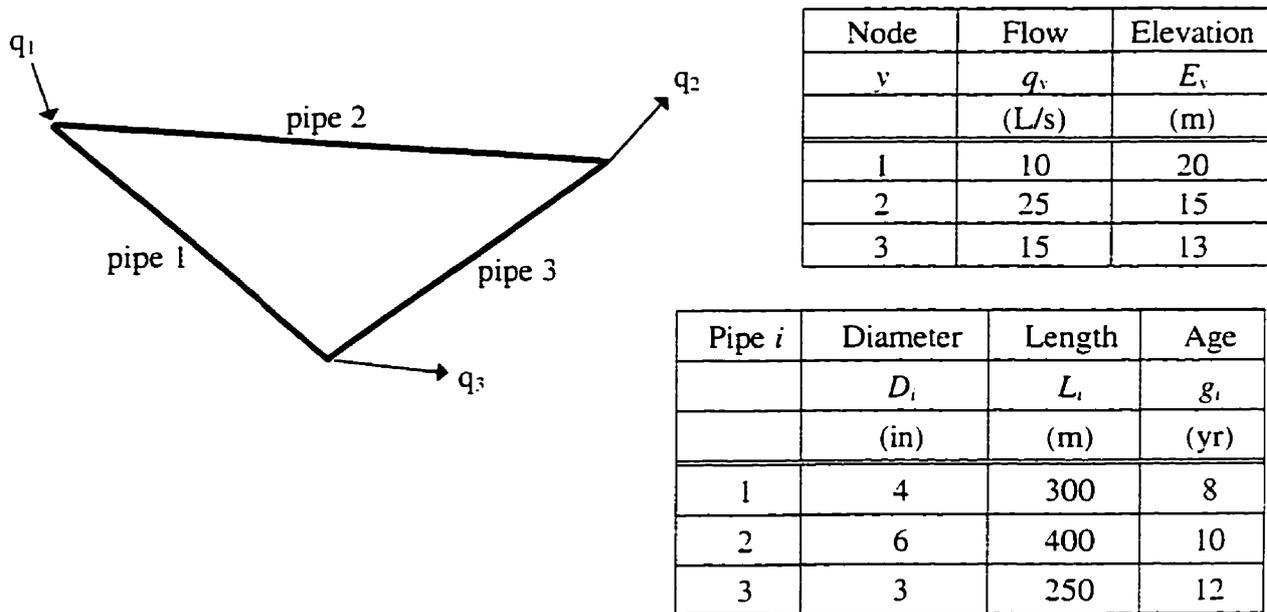


Figure 15. Example network.

The source node is assumed to have a total head of 70m (i.e., a reservoir with a fixed water level at 70m elevation). The demand flows are peak demands and are assumed constant throughout the design period. Minimum residual head required at any node is 40.0m. The analysis period is 40 years and the discount rate used is 0.05. Other data and costs are presented in the following tables.

The rehabilitation alternatives considered are relining, replacement with same diameter pipe, and replacement with a pipe one nominal diameter larger (alternatives $j=1,2,3$, respectively). The network simulator used for calculations of nodal residual pressures is EPANET by the United States Environmental Protection Agency, Rossman (1993).

The least cost combination of rehabilitation alternative selection and scheduling for each pipe was found in this example to be:

stage	pipe	Rehabilitation alternative	Year of implementation	Cost	Cumulative cost	Hydraulic capacity adequate until year (or TMP)
f	i	j	T_{ij}	$C_{wr}(T_{ij})$	$\sum C_{wr}(T_{ij})$	$T_{S_{f,i,j,T}} + \Delta t_{S_{f,i,j,T}}$
1	3*	2	5	40,917	40,917	10
2	1**	1	10	32,685	73,602	23
3	2	2	23	38,220	111,822	40

* Pipe 3 is replaced again at year $T_{3,2} + T_{3,2}^{**} = 5 + 24 = 29$

** Pipe 1 is relined at year 10 and subsequently replaced at year $T_{1,2}^* = 28$

The tables in the following pages demonstrate the entire process of reaching this least cost combination. The first table contains all the parameters for the existing pipes as well as the rehabilitation alternatives in the example. In addition, the first table also demonstrates the a priori state reduction based on equation (30). The second table shows the initial stage in the process, while the subsequent tables show the subsequent transitions up to stage 5 which is the final stage. Specific explanations are provided within the tables.

Table 1. Pipe parameters and initial calculations.

Item	Symbol	Units	Values	Pipe 1			Pipe 2			Pipe 3						
				Exist	1	2	3	Exist	1	2	3	Exist	1	2	3	
# of pipes	p		5													
# of nodes	n		4													
Horizon Year	H	year	40													
Break Repair Cost	C_b	\$/break	2000													
Discount Rate	r	fraction	0.05													
Demand in node i	q_i	m ³ /h														
Length of pipe i	L_i	m	300													
Age of pipe i	e_i	years	5													
Diameter pipe i alt. j	D_{ij}	inch		4	4	4	6	6	6	8	8	8	10	10	10	12
Initial roughness pipe i alt. j	e_{ij}	ft		0.0005	0.0003	0.0003	0.00045	0.0004	0.00045	0.0006	0.0005	0.00025	0.00025	0.00025	0.0003	0.0003
Roughness growth rate pipe i alt. j	g_{ij}	ft/year		0.0003	0.00025	0.00025	0.00025	0.0003	0.00025	0.00025	0.0003	0.00025	0.00025	0.00025	0.00025	0.00025
Break rate (new) pipe i alt. j	$N(i)_{ij}$	breaks/yr/km		0.25	0.25	0.114	0.25	0.3	0.3	0.25	0.35	0.35	0.3	0.3	0.3	0.3
Break rate (old) pipe i alt. j	A_{ij}	breaks/yr/km		0.08	0.08	0.125	0.08	0.07	0.07	0.051	0.051	0.15	0.15	0.11	0.11	0.11
Rehab. cost pipe i alt. j	$C_{r_{ij}}$	\$/km		40,000	110,000	150,000		55,000	150,000	180,000		30,000	90,000	90,000	110,000	
Optimal Cycle for Rehabilitation (years) T_{ij}					29	39			58	61			24	26		
Year of first rehab. to minimize individual pipe cost T_{ij}^*					28	31			29	32			5	6		
$\Delta T_{ij, 200}$ (years)																
Latest year of first rehabilitation	$T_{ij, 200}^*$															

A priori Reduction of States

(values pertain to total cost as a function of rehabilitation year. "N/A" and/or dark cells denote inferior options)

Year of Rehab	Pipe 1			Pipe 2			Pipe 3			
	0	1	2	0	1	2	3	0	1	2
1	36,821	50,142	62,515	57,780	72,907	84,943	N/A	N/A	42,701	49,360
2	36,264	48,008	59,778	56,759	69,873	81,322	N/A	N/A	41,903	48,237
3	35,735	46,002	57,198	55,789	67,022	77,912	N/A	N/A	41,334	47,359
4	35,231	44,118	54,768	54,865	64,344	74,703	N/A	N/A	41,002	46,733
5	34,752	42,351	52,481	53,987	61,833	71,687	N/A	N/A	40,917	46,369
6	34,296	40,696	50,332	53,151	59,481	68,854	N/A	N/A	41,092	46,278
7	33,863	39,148	48,314	52,356	57,283	66,197	N/A	N/A	41,541	46,474
8	33,450	37,703	46,423	51,600	55,226	63,707	N/A	N/A	42,281	46,973
9	33,058	36,357	44,651	50,881	53,310	61,377	N/A	N/A	43,330	47,793
10	32,685	35,105	42,995	50,197	51,527	59,201	N/A	N/A	44,709	48,954
11	32,330	33,945	41,450	49,546	49,871	57,171	N/A	N/A	46,442	50,481
12	31,992	32,872	40,011	N/A	48,337	55,281	N/A	N/A	48,557	52,398
13	31,671	31,883	38,674	N/A	46,920	53,525	N/A	N/A	51,083	54,738
14	N/A	30,975	37,435	N/A	45,614	51,898	N/A	N/A	54,056	57,532
15	N/A	30,145	36,290	N/A	44,417	50,393	N/A	N/A	57,512	60,819
16	N/A	29,391	35,236	N/A	43,322	49,007	N/A	N/A	61,495	64,640
17	N/A	28,709	34,269	N/A	42,326	47,733	N/A	N/A	66,052	69,044
18	N/A	28,097	33,386	N/A	41,424	46,569	N/A	N/A	71,235	74,081
19	N/A	27,554	32,585	N/A	40,615	45,508	N/A	N/A	77,104	79,811
20	N/A	27,076	31,861	N/A	39,892	44,547	N/A	N/A	83,724	86,299
21	N/A	26,662	31,214	N/A	39,255	43,682	N/A	N/A	91,167	93,616
22	N/A	26,310	30,640	N/A	38,698	42,910	N/A	N/A	99,513	101,843
23	N/A	26,018	30,136	N/A	38,220	42,227	N/A	N/A	108,852	111,068
24	N/A	25,784	29,702	N/A	37,818	41,629	N/A	N/A	119,282	121,391
25	N/A	25,607	29,334	N/A	37,489	41,114	N/A	N/A	N/A	132,919
26	N/A	25,486	29,031	N/A	37,230	40,678	N/A	N/A	N/A	145,775
27	N/A	25,420	28,792	N/A	37,039	40,319	N/A	N/A	N/A	N/A
28	N/A	25,406	28,614	N/A	36,914	40,034	N/A	N/A	N/A	N/A
29	N/A	25,445	28,496	N/A	36,853	39,821	N/A	N/A	N/A	N/A
30	N/A	25,535	28,437	N/A	36,854	39,677	N/A	N/A	N/A	N/A
31	N/A	25,675	28,435	N/A	36,915	39,600	N/A	N/A	N/A	N/A
32	N/A	25,864	28,490	N/A	37,034	39,589	N/A	N/A	N/A	N/A
33	N/A	26,102	28,600	N/A	37,210	39,640	N/A	N/A	N/A	N/A
34	N/A	26,388	28,765	N/A	37,442	39,753	N/A	N/A	N/A	N/A
35	N/A	26,722	28,983	N/A	37,727	39,926	N/A	N/A	N/A	N/A
36	N/A	27,103	29,254	N/A	38,065	40,156	N/A	N/A	N/A	N/A
37	N/A	27,531	29,577	N/A	38,454	40,444	N/A	N/A	N/A	N/A
38	N/A	28,006	29,951	N/A	38,893	40,786	N/A	N/A	N/A	N/A
39	N/A	28,526	30,377	N/A	39,382	41,182	N/A	N/A	N/A	N/A
40	N/A	29,093	30,854	N/A	39,919	41,631	N/A	N/A	N/A	N/A

Table 2. Transition from stage 0 to stage 1.

1st in sequence	Pipe	Alt	Year	Latest Year for Next Rehab	Total Cost of 1st State	Comments
1	1	7	20		33,863	Case (1) for rehab (equations 25,28). T^* year 28 (see Table 1)
	2	N/A (a repair - inferior)				Between rehab and replace with same diameter rehab is superior (equation 28)
	3	7	40	48,314		Rule for case (1) (equation 25). T^* year 31 (see Table 1)
2	1	7	13		52,356	T^* year 29
	2	N/A (a repair - inferior)				Same considerations as pipe 1 above
	3	7	26		66,196	T^* year 32
3	1	N/A (a repair - inferior)				Between rehab and replace with same diameter replace is superior (equation 28)
	2	5	10		40,917	According to the rule for case (a) (equation 26). T^* year 5 (see table 1)
	6	10		41,092	inferior	Same time of hydraulic integrity at a higher cost (equation 29)
	7	10		41,541	inferior	same as above
3	6	13		46,277		According to the rule for case (a) (equation 26). T^* year 6 (see table 1)
	7	13		46,474	inferior	Same time of hydraulic integrity at a higher cost (equation 29)

The latest year that hydraulic capacity is adequate, after implementing project m_j at time T_{m_j}

Table 3. Transition from stage 1 to stage 2.

Values (years) of $T_{S_{2,j,k \cup m,m,j},T_{m_j}+1}$
 Circles denote superior states

First in sequence								
Pipe	j	Year	1	2	3	4	5	
All	j	Year	1	2	3	4	5	
Year	j	Year	1	2	3	4	5	
Latest Next Year $T_{S_{2,j,k \cup m,m,j},T_{m_j}+1}$			20	40	13	26	10	13
2nd in sequence								
Pipe	m	Year						
1	1	10					23	
1	2	10		40			23	
1	1	13			15		24	
1	2	26			40		40	
1	1	10				40	40	
1	2	26			40	40	40	
2	1	10					17	
2	2	10					17	
2	1	13					14	
2	2	20	40					
2	1	29		40				
2	2	30		40				
2	1	31		40				
2	2	32		40				
2	1	33		40				
2	2	34		40				
2	1	35		40				
2	2	36		40				
2	1	37		40				
2	2	38		40				
2	1	39		40				
2	2	40		40				
1	1	10				40		
1	2	11					40	
1	1	20	40					
1	2	12		40				
1	1	13		40				
1	2	14		40				
1	1	15		40				
1	2	16		40				
1	1	17		40				
1	2	18		40				
1	1	19		40				
1	2	20		40				
1	1	21		40				
1	2	22		40				
1	1	23		40				
1	2	24		40				
1	1	7	21	40	17	40		
1	2	8	21	40	17	40		
1	1	9	21	40	17	40		
1	2	10	21	40	17	40		
1	1	11	21	40	18	40		
1	2	12	21	40	18	40		
1	1	13	21	40	18	40		
1	2	14	21	40	17			
1	1	15	21	40	17			
1	2	16	21	40	18			
1	1	17	21	40	18			
1	2	18	21	40	19			
1	1	19	21	40	19			
1	2	20	21	40	40			
1	1	21		40	40			
1	2	22		40	40			
1	1	23		40	40			
1	2	24		40	40			
1	1	7	20	40	28	40		
1	2	8	20	40	28	40		
1	1	9	20	40	28	40		
1	2	10	20	40	40	40		
1	1	11	20	40	40	40		
1	2	12	20	40	40	40		
1	1	13	20	40	40	40		
1	2	14	20	40	40	40		
1	1	15	19	40	40			
1	2	16	19	40	40			
1	1	17	19	40	40			
1	2	18	19	40	40			
1	1	19	19	40	40			
1	2	20	19	40	40			
1	1	21		40	40			
1	2	22		40	40			
1	1	23		40	40			
1	2	24		40	40			
1	1	25		40	40			
1	2	26		40	40			

This table contains the year until which the hydraulic capacity is adequate in the network, providing a pipe from the list in the left column (next state) was rehabilitated at the specified year, and was thus added to the existing projects (previous state) that are listed in the top row

Values (\$) of $C_{S_{2,j,k \cup m,m,j},T_{m_j}}^{tot}$

First in sequence								
Pipe	j	Year	1	2	3	4	5	
All	j	Year	1	2	3	4	5	
Year	j	Year	1	2	3	4	5	
Latest Next Year $T_{S_{2,j,k \cup m,m,j},T_{m_j}+1}$			20	40	13	26	10	13
2nd in sequence								
Pipe	m	Year						
1	1	10					73,602	
1	2	10		64,027			77,949	
1	1	13			64,219		78,161	
1	2	26			91,681		81,912	
1	1	10			91,030		84,951	
1	2	26			95,228			
2	1	10					91,114	
2	2	10					92,444	
2	1	13					91,198	
2	2	20	73,755					
2	1	29		85,167				
2	2	30		85,168				
2	1	31		85,229				
2	2	32		85,349				
2	1	33		85,525				
2	2	34		85,756				
2	1	35		86,042				
2	2	36		86,379				
2	1	37		86,769				
2	2	38		87,208				
2	1	39		87,696				
2	2	40		88,233				
1	1	10					100,118	
1	2	11					99,803	
1	1	20	78,410					
1	2	12		87,903				
1	1	13		87,955				
1	2	14		88,068				
1	1	15		88,240				
1	2	16		88,471				
1	1	17		88,758				
1	2	18		89,100				
1	1	19		89,497				
1	2	20		89,945				
1	1	7	75,404	89,856	91,897	107,718		
1	2	8	76,144	90,596	94,617	108,478		
1	1	9	77,192	91,644	95,686	109,526		
1	2	10	78,571	93,023	97,065	110,905		
1	1	11	80,305	94,757	98,798	112,619		
1	2	12	82,419	96,871	100,913	114,764		
1	1	13	84,946	99,398	102,419	117,280		
1	2	14	87,918	102,370		120,252		
1	1	15	91,375	105,827		123,709		
1	2	16	95,157	109,809		127,692		
1	1	17	99,914	114,366		132,248		
1	2	18	105,098	119,550		137,432		
1	1	19	110,967	125,419		143,301		
1	2	20	117,586	132,038		149,921		
1	1	21		139,481		157,361		
1	2	22		147,827		165,710		
1	1	23		157,166		175,049		
1	2	24		169,705		187,587		
1	1	7	80,337	94,789	98,830	112,671		
1	2	8	80,836	95,288	99,329	113,170		
1	1	9	81,656	96,108	100,149	113,990		
1	2	10	82,817	97,269	101,310	115,151		
1	1	11	84,343	98,795	102,837	116,677		
1	2	12	86,261	100,713	104,755	118,595		
1	1	13	88,600	103,052	107,094	120,914		
1	2	14	91,394	105,846		123,729		
1	1	15	94,681	109,133		127,015		
1	2	16	98,503	112,955		130,837		
1	1	17	102,906	117,358		135,240		
1	2	18	107,944	122,396		140,278		
1	1	19	113,674	128,126		146,008		
1	2	20	120,162	134,611		152,496		
1	1	21		141,931		159,813		
1	2	22		150,158		168,040		
1	1	23		159,383		177,265		
1	2	24		169,705		187,587		
1	1	25		181,234		199,116		
1	2	26		194,089		211,972		

This table contains the total cost associated with the new states (corresponding to the new states in the table on the left). The new states, at this point (after the transition from stage 1 to stage 2), comprise subsets of two projects (a subset of one project in the top row and an additional rehabilitation project from the left column)

These are the non-interior states from the previous stage

Example: At the end of stage 2 this state contains a subset of two rehabilitation projects: the first is pipe #3 with alternative #2 implemented at year 5 (which is one of the superior states from the previous stage), and the second is pipe #1 with rehabilitation alternative #1 implemented at year 10. These two projects extend the hydraulic capacity of the system until year 23, and their total cost is \$73,602. This state is a non-interior state because it carries the smallest cost among all the states in stage 2 that contain projects involving pipes #3 and #1 and extend the hydraulic capacity of the system to year 23 or earlier.

Table 4. Transition from stage 2 to stage 3.

Values (years) of $T_{S_{3,s,k \cup m,m,j,T_m}+1}$

Circles denote superior states

1st in sequence			1	3	1	3	3	3
Pipe								
All	<i>k</i>	<i>j</i>	1	2	2	2	1	1
Year	<i>T</i>		7	5	5	5	6	6
2nd in sequence								
Pipe	<i>k</i>	<i>j</i>	2	1	1	2	2	2
All	<i>j</i>		2	1	3	1	2	3
Year	<i>T</i>		20	10	10	10	13	13
Latest Next Year $T_{S_{2,s,k,j,T}+1}$			40	23	40	17	34	40
3rd in sequence								
Pipe	<i>m</i>	<i>j</i>						
All	<i>j</i>							
Year	<i>T_m</i>							
1	2	17						
		28		40		40	40	
		29				40	40	
		30				40	40	
		31				40	40	
		32				40	40	
	3	33				40	40	
		34				40	40	
		35				40	40	
		36				40	40	
		37				40	40	
		38				40	40	
2	1	17						
		11		40		40	40	
		32				40	40	
		33				40	40	
		34				40	40	
		35				40	40	
	3	36				40	40	
		37				40	40	
		38				40	40	
		39				40	40	
		40				40	40	
		23		40				
3	2	29		40				
		30		40				
		31		40				
		32		40				
		33		40				
		34		40				
	3	35		40				
		36		40				
		37		40				
		38		40				
		39		40				
		40		40				
3	2	23		40				
		32		40				
		33		40				
		34		40				
		35		40				
		36		40				
	3	37		40				
		38		40				
		39		40				
		40		40				
		20		40				
		21						
3	2	22						
		23						
		24						
		20		40				
		21						
		22						
	3	23						
		24						
		25						
		26						

Values (\$) of $C_{S_{3,s,k \cup m,m,j,T_m}^{tot}}$

1st in sequence			1	3	1	3	3	3
Pipe								
All	<i>k</i>	<i>j</i>	1	2	2	2	1	1
Year	<i>T</i>		7	5	5	5	6	6
2nd in sequence								
Pipe	<i>k</i>	<i>j</i>	2	1	1	2	2	2
All	<i>j</i>		2	1	3	1	2	3
Year	<i>T</i>		20	10	10	10	13	13
Latest Next Year $T_{S_{2,s,k,j,T}+1}$			40	23	40	17	34	40
3rd in sequence								
Pipe	<i>m</i>	<i>j</i>						
All	<i>j</i>							
Year	<i>T_m</i>							
1	2	17						
		28				119,823		
		29					118,604	125,209
		30					118,643	125,248
		31					118,733	125,338
		32					118,873	125,478
	3	33					119,062	125,667
		34					119,300	125,905
		35					119,586	126,191
		36						126,525
		37						126,906
		38						127,334
1	1	17						
		31					121,633	128,238
		32					121,688	128,293
		33					121,798	128,403
		34					121,963	128,568
		35						128,786
	3	36						129,057
		37						129,380
		38						129,754
		39						130,180
		40						130,657
		23					111,822	
2	2	29					120,765	
		30					120,766	
		31					120,827	
		32					120,946	
		33					121,122	
		34					121,354	
	3	35					121,639	
		36					121,977	
		37					122,366	
		38					122,805	
		39					123,294	
		40					123,831	
3	2	23					115,829	
		32					123,501	
		33					123,532	
		34					123,665	
		35					123,838	
		36					124,068	
	3	37					124,356	
		38					124,698	
		39					125,094	
		40					125,543	
		20		157,479				
		21						
3	2	22						
		23						
		24						
		20		160,054				
		21						
		22						
	3	23						
		24						
		25						
		26						

These are the non-interior states from the previous stage.

This is the superior state in the final stage

These tables depict the transition from the 2nd to the 3rd (and final) stage. The superior sequence (state) is that which provides hydraulic integrity until the end of the analysis period (year 40) and at the same time carries the lowest cost.

6. DISCUSSION

6.1 General

The numerical example presented in the previous section, demonstrates the implementation of the proposed methodology henceforth referred to as the Multistage Network Rehabilitation Analysis Procedure (MNRAP).

The proposed MNRAP provides a great deal of flexibility in the characterization of the analyzed network. In addition to assigning different deterioration rates (of both structural integrity and hydraulic capacity) to every pipe, other elements may be (relatively) easily incorporated into the procedure. These elements include among others:

- Demand increase (or any demand change patterns) over time.
- Different demand change patterns at different demand nodes.
- Different values for minimum residual supply pressure at different nodes.

6.2 Minimum Residual Pressure

The issue of minimum residual nodal pressure has been deliberated extensively in the literature, mainly in the context of network reliability. While some treated it as a rigid constraint (e.g., Kim and Mays, 1994, Wagner et al., 1988a), others acknowledged the fact that lower residual nodal pressures reflect lower levels of service and do not necessarily mean failure (e.g., Wagner et al., 1988b, Woodburn et al., 1987). In the proposed MNRAP, the objective is to minimize cost. The only way to incorporate the residual pressure into the objective function (rather than treat it as a rigid constraint) would necessitate the development of some form of a penalty function for residual pressures below a pre-determined threshold value. This practice gives rise to a new set of problems. One problem is what cost should be assigned to deviations from the threshold (in the absence of market value for supply pressure

it is in fact a social cost that is difficult to quantify). Another problem is that reduced supply pressures may have different social costs in different parts of the system, depending on who is being serviced at a particular location.

If the MNRAP is applied to a network with a given value (say 40m) for minimum residual pressure head P_{min} , the total cost $C_{P_{min}=40}^{tot}$ of that network is found for the optimal sequence order. If then the process is repeated for $P_{min}=39m$ and $C_{P_{min}=39}^{tot}$ is found, the difference ($C_{P_{min}=40}^{tot} - C_{P_{min}=39}^{tot}$) depicts the true cost savings in reducing minimum supply pressure from 40m to 39m. This does not eliminate the need to evaluate the social costs associated with reduced supply pressure; however, it eliminates the need to incorporate them into the optimization process. Once the decision makers know how much can be saved by reducing supply pressure, they still need to assess whether or not these savings outweigh the social costs. In the numerical example from the previous section, the result illustrated in Figure 16 was obtained.

P_{min} (m)	PIPE			Total Cost (\$)	
	1	2	3		
37.0	alt	2	2	2	104,844
	year	20	29	5*	
38.0	alt	2	2	2	107,102
	year	17	29	7*	
39.0	alt	1	2	2	109,439
	year	13**	29	5*	
40.0	alt	1	2	2	111,821
	year	10**	23	5*	
41.0	alt	1	3	2	122,596
	year	7**	17	4*	
42.0	alt	1	3	2	135,605
	year	3**	11	1	
43.0	alt	3	2	2	140,284
	year	1***	29	5*	

- * Pipe 3 is replaced again 26 years later
- ** Pipe 1 is replaced at year 29
- *** Pipe 1 is replaced again 29 years later

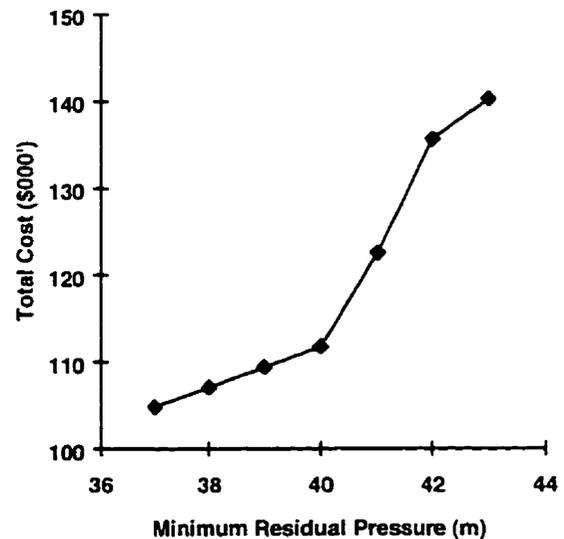


Figure 16. Sensitivity of total cost to P_{min} .

These results demonstrate the dependency of the total cost on P_{min} . It should be noted that the extent of this dependency (the slope of the curve) is unique to the individual network and cannot be predicted before applying MNRAP. However, in general, the larger and more elaborate the system is, the smaller and smoother this slope should become.

6.3 The Size of the Analysis Timestep

The MNRAP can be applied with various size timesteps. With a single year timestep, the system is hydraulically evaluated every year within the specified analysis period, and new states are generated with implementation times of one year increments. With a timestep of two years, the same operations are applied at two year increments, and so forth. In general, the shorter the timestep, the more likely the results are to be closer to the true minimum cost solution, but the longer it takes to arrive at this solution. The MNRAP was applied to the example in Chapter 5 with different timesteps (Appendix 3 contains a listing of some results obtained from applying MNRAP to the example at different timesteps). The results obtained are given in the following table.

time step (year)	PIPE			Total Cost (\$)	
	1	2	3		
1	alt	1	2	2	111,821
	year	10**	23	5*	
2	alt	1	2	3	116,967
	year	12	22	6*	
3	alt	2	2	3	120,019
	year	28	12	6*	
4	alt	1	2	2	114,343
	year	8**	20	4*	
5	alt	1	3	2	113,493
	year	10**	20	5*	

* Pipe 3 is replaced again 26 years later

** Pipe 1 is replaced at year 29

These results indicate that sometimes a larger timestep does not necessarily result in a higher cost solution. The reason in this example is the manner in which the MNRAP treats the timestep computationally. The MNRAP considers all actions executed in a timestep as if executed at the end of the timestep. Consequently, if for instance with a 5-year timestep, the system requires rehabilitation at timestep 2, then the cost of this rehabilitation is incurred at year 10, possibly reducing it if in the transition case (i) occurs, or increasing it if cases (ii) or (iii) occur (recall the three cases described in section 4.2). Furthermore, the fact that rehabilitation is implemented at the end of the timestep, may introduce a small bias in two ways: (1) the hydraulic capacity of the system is not checked within the time step (and it may possibly drop below P_{min} , without being detected) and (2) the system's hydraulic integrity is maintained for a longer period within the analysis period.

Since the occurrence of all these phenomena (with contradicting effects) is largely random, the statement about the general tendency of the total cost to increase as the timestep increases, seems to be valid with possible exceptions in very small systems.

It should be noted that the runtime durations for 1,2,3,4 and 5 year timesteps were 38, 17, 20, 11 and 10 seconds respectively, (see Chapter 8 - The Computer Code). This indicates that running the system with longer timesteps could be useful to screen out inferior alternatives, thus reducing runtimes of shorter timesteps.

6.4 Time Horizon, Infinite Cost Stream and Residual Capacity

The way the proposed MNRAP incorporates an infinite stream of costs of the individual pipe with a finite analysis period for the entire network is a key issue. It should be stressed that the results provided by MNRAP do not necessarily indicate a steady state system. Rather, as was stated in section 3.5, the infinite cost stream analysis of the individual pipe facilitates the capability of the procedure to handle more than one replacement of any given pipe, within the

specified time horizon. Any other method to achieve this capability would result in a dramatic increase in problem dimensionality (see Chapter 7 - Model Validation).

Since the results do not indicate a system in steady state, the issue of residual capacity of the system at the time horizon has to be discussed. In order to consider the residual capacity of the system (at the time horizon) in the cost function, it is necessary to (a) estimate the magnitude of this residual capacity and (b) assign a dollar value to it. It is relatively easy (albeit computationally expensive) to estimate the residual capacity in terms of how many years beyond the time horizon the system is still hydraulically adequate without implementing any additional (first time) rehabilitation projects. However, assigning an objective cost value to this capacity is (at best) impractical. Furthermore, the impact of residual capacity considerations is typically not too great because whatever monetary value is assigned to this capacity it will be substantially discounted by H years (typical values of H are 30 to 60 years). The infinite stream of costs consideration further reduces the impact of the residual capacity by perpetually “replacing” the pipes every T^* years, to infinity. Residual capacity is therefore not directly addressed by MNRAP but rather is left to the analyst and the decision maker to evaluate.

The most informed way to evaluate the residual capacity of a system is to solve it for an incrementally increasing time horizon until the least cost combination changes. In the previous example, the residual capacity equals six years; i.e., for $H = 40$ through 46 years the least cost combination of rehabilitation measures does not change, while for $H = 47$ years pipe #2 is no longer replaced by the same diameter pipe but rather by a larger diameter pipe (still at year 23) and the total cost increases from \$111,821 to \$115,827.

6.5 The Discount Rate

The MNRAP considers only costs (not the benefits) that are associated with a water distribution system; hence, the discount factor r affects only those costs. In general, the higher the discount rate the greater the economic drive to delay expenditure as much as possible and

vice versa. The choice of an appropriate discount rate for public investment is a debated issue that has been researched extensively. Steiner (1969) mentions two major approaches to this issue: (a) the discount rate should reflect some kind of marginal cost of capital, and (2) the discount rate should reflect some kind of explicit measure of social time preference (hence the term “social discount rate”). A brief discussion of the social and economical significance of choosing a discount rate is provided below.

Hufschmidt et al. (1983) offer three explanations to justify the practice of discounting in making investment decisions: (a) individuals value future consumption less than that in the present, (b) the theory of capital productivity (opportunity cost of capital), and (c) the discount rate is an instrument of government policy which reflects the government’s attitude towards public investment versus private investment. Swartzman (1982) perceives the discount rate as a means to distribute available resources over time, or in his words “...discounting acts to distribute benefits today, paid for tomorrow”. Consequently, he concludes that the selection of the discount rate reflects the political and ethical attitudes of the decision maker.

In the context of investment in the rehabilitation of a water distribution system (or any infrastructure system for that matter), the question is how important is it to the decision maker to invest in the present, in order to provide an adequate, manageable system in the future. However, when using the MNRAP to plan the infrastructure rehabilitation, the impact of different discount rates is greatly buffered. This may be illustrated using the following numerical example, derived from the example problem in Chapter 5.

Pipe <i>i</i>	Minimum cost replacement time (MCRT) T_{ij}^* (years)						Optimal duration of replacement cycles T_{ij}^{**} (years)					
	1		2		3		1		2		3	
Alt. <i>j</i>	2	3	2	3	2	3	2	3	2	3	2	3
$r=0.02$	25	27	24	26	4	5	27	35	51	53	23	24
$r=0.04$	27	29	28	30	4	5	29	37	55	58	24	25
$r=0.05$	28	31	29	32	5	6	29	39	58	61	24	26
$r=0.06$	29	32	31	33	5	6	30	40	60	63	25	26
$r=0.08$	31	34	34	37	6	7	31	42	64	68	26	27

T^* (or MCRT as it is referred to in section 4.2) is a reference point in time, relative to which MNRAP assigns the timing of the first implementation of every rehabilitation alternative, according to the three rules depicted in section 4.3. It is evident from the table above that the smaller the discount rate, the smaller T^* becomes. This is a result of the fact that the relative weight of the future cost components increases, making it more economical to incur costs earlier. Consequently the timing of the first implementation of the rehabilitation alternatives obtained by the optimal solution can be expected to be closer to the present time. Similarly, subsequent implementations will occur in shorter cycles, since T^{**} becomes shorter as the discount rate decreases.

The optimal results for the different discount rates are presented in the following table. The total costs in the right column are the present value of the respective ORPs, calculated at a discount rate of 0.05. Stated differently, these numbers reflect what the 'true' costs would be if the 'true' discount rate is 0.05 but MNRAP was 'mistakenly' applied with the 'incorrect' rate.

Discount Rate r	Pipe i	Rehabilitation Alternative j	Implementation Year T_{ij}	Total Cost	Total Cost (normalized to 5%) C_{ij}^{tot}
0.02	1	2	10	\$284,604	\$115,040
	2	2	23		
	3	2	4		
0.04	1	1	10	\$141,461	\$112,016
	2	2	23		
	3	2	4		
0.05	1	1	10	\$111,822	\$111,822
	2	2	23		
	3	2	5		
0.06	1	1	10	\$91,719	\$111,970*
	2	2	23		
	3	2	5		
0.08	1	1	10	\$66,096	\$112,457
	2	2	23		
	3	2	6		

* The The normalized cost of the 6% ORP is slightly higher than that of the 5% ORP (although both comprise the same alternatives and the same timing) is attributed to the fact that in the 6% ORP pipe 1 is replaced (after initial relining) at year $T^*=29$, whereas in the 5% ORP it is replaced at year $T^*=28$.

The optimal solutions for the various discount rates does not change much. Only at the 2% level is a rehabilitation alternative different (in pipe 1 it changed from alternative 1 to alternative 2). In all other discount rates the only change is a small shift in the timing of one of the rehabilitation alternatives in one pipe. Consequently, the “penalties” for deviating from the “true” discount rate (± 3 percentage points) seem to be relatively small. Although this stability with respect to the discount factor can be partially attributed to the limited size of the example, it nevertheless seems, that the infinite replacement cycles approach has a buffering effect that reduces the sensitivity of MNRAP results are not very sensitive to the discount rate selected. The reason for this is likely due to a that the has on the process, with respect to the discount rate.

6.6 System Hydraulic Capacity

It should be noted that MNRAP contains two implicit assumptions about the hydraulic behaviour of a water distribution network:

- (a) The minimum residual pressure of a water distribution network is non-increasing with respect to time due to the deterioration in the Hazen-Williams coefficient of all pipes.
- (b) Increasing the Hazen-Williams coefficient in any pipe in a water distribution network will never cause a decrease in the minimum residual pressure of the network (i.e., the system’s hydraulic capacity improves after implementing a rehabilitation measure).

Although these assumptions are generally true for systems operating under peak demand conditions (which is what MNRAP assumes), it should be noted that in some specific cases (e.g., extreme topographical variations in a distribution network in which some parts are oversized while others are undersized) these assumptions may not be true for the full range of the demand loadings on the network. It should further be noted that in such specific

cases, good design practices usually dictate incorporation of devices such as pressure reducing or pressure sustaining valves to prevent these situations.

These assumptions may give rise to another limitation if and when a pipe is considered to be replaced by a smaller diameter pipe. In such (rare) cases, under certain circumstances, the MNRAP results may be biased.

The definition of system hydraulic capacity as the nodal residual pressure at the critical node during peak demand, may introduce a limitation to the MNRAP in cases where critical pressure situations occur under conditions other than peak demand. For instance, in a situation where the hydraulic capacity of a pipe leading to an elevated tank has deteriorated to the extent that it can not fill up adequately during low demand conditions, the result may be reduced pressure at adjacent nodes during subsequent peak demand, due to insufficient static head at the tank. Furthermore, in the manner with which MNRAP simulates long term performance there is an implicit assumption that the distribution system's boundary conditions (e.g., water level in a tank or a reservoir) do not change over time because of the deterioration in the system's hydraulic capacity. This assumption may not always be accurate as demonstrated by the tank example. Applying MNRAP in various demand loadings may be a way to address this problem, however, the aspect of incorporating the results of a multiple loadings into one optimal rehabilitation policy has to be further studied. Extended time simulations are the most comprehensive way to address issues involving time-changes of boundary condition, however, the limitations in using them is discussed in section 3.8 - Cost of Energy.

Technically, there may be several solutions to the problem of insufficient hydraulic capacity to fill up an elevated tank. For instance, besides (or in addition to) rehabilitating the inlet pipe, a pump can be added locally to facilitate adequate filling conditions. This gives rise to a whole range of additional considerations namely the optimization of pump configuration and location, as well as tank operation. Incorporating these considerations into MNRAP is beyond

the scope of this thesis, however, it seems that such an attempt would dramatically increase the dimensionality of the problem to be computationally prohibitive.

An additional issue that deserves mention is the limitation introduced by the assumption of identical replacement cycles to infinity. In urban centers experiencing expansion period, the design maximum flow in a given pipe may increase substantially over a period equal to T^{**} of the pipe. Consequently, it is reasonable to assume that in many instances, while the initial pipe replacement will be implemented with a certain diameter pipe, subsequent replacements will be with larger diameter pipes. If these instances were to be explicitly accommodated in the MNRAP, the dimensionality of the problem would increase to magnitudes that would make it computationally prohibitive. As a result, the only way that these instances can be accommodated is in the manner with which MNRAP is to be implemented in practice. This is discussed in the following section.

6.7 Using MNRAP

MNRAP is essentially a decision support system (DSS) intended to aid engineers and decision makers in devising long term plans for water distribution system rehabilitation. MNRAP synthesizes the economic aspects and the hydraulic behavior of a water distribution network over a pre-defined planning period, and provides the user with an operational framework for this period. Prior to the practical implementation of MNRAP, the following should be remembered:

- MNRAP employs some simplified models that use historical data to predict the deterioration rates of the structural integrity and hydraulic capacity of the pipes in the network. These models are by nature prone to errors due to (a) natural variation of the derived parameters, (b) possible oversimplification of the model, and (c) the availability and the quality of the historical data.

- A typical water distribution network is a dynamic entity whose operational conditions change constantly due to changing demographics, changes in consumption habits, development and rezoning, changes in pipe installation and maintenance practices, etc. While only some of these changes can be forecasted, all should be accounted for in a long-term rehabilitation plan. MNRAP can address these issues (subject to some additional programming that is required, see section 8.4).
- Open-trench waterworks (e.g., 'traditional' pipe replacement) should typically be implemented in conjunction with other infrastructure works in the same location in order to save costs. Another way to save costs is to combine several pipe rehabilitation projects into one, in order to benefit from economies of scale. These considerations are not addressed by MNRAP.

The optimal rehabilitation policy (ORP) obtained by MNRAP should therefore not be regarded as a policy cast in stone, but rather as a framework within which fine tuning may (and should) be implemented to accommodate special conditions and considerations that MNRAP does not consider.

Consequently, the MNRAP could be applied in the following manner. The ORP obtained by the MNRAP for the entire analysis period may be divided into a short-term segment (comprising say, three to six years) and the long-term segment comprising the rest of the analysis period. The rehabilitation measures depicted in the short-term segment are then considered for actual implementation. Towards the end of the short-term segment the MNRAP is reapplied to the water distribution network, utilizing up-to-date data pertaining to pipes as well as demand flows. A new and revised ORP is then obtained, which again is divided into a short-term and a long-term segment, and so on.

Organizing and managing the water supply network data through a computerized system such as a GIS (see section 2.3) seems to be the most efficient way of implementing such a process. Social costs associated with the geographical location of specific pipes (e.g., high costs associated with breakage repair or pipe replacement on a main street of an urban center) can

be incorporated into the MNRAP since both breakage repair costs and pipe replacement costs are assigned on an individual pipe basis. In addition, zones in the network that are prone to either accelerated corrosion rates or excessive pressure surges (both resulting in higher breakage rates) can be identified. The MNRAP can then be applied with higher class pipes (pipes with greater wall thickness resulting in smaller breakage rates but are more expensive to install and repair) as alternative replacement pipes for these zones. The economical feasibility of higher class pipes could then be examined (provided their deterioration parameters are available).

Experience shows that sometimes watermains display a relatively high breakage rate immediately after installation, then at a certain time (typically a few years after installation) the breakage rate drops abruptly at which point a slow “natural” deterioration process begins. This phenomenon (known also as a “bathtub” distribution) typically indicates problems with improper installation or faulty pipes or joints. Once the installation problems are corrected, or the faulty component replaced, the deterioration becomes gradual. In deriving parameters for the pipe breakage rate model, the breakage rate data should be examined, and breakage records attributed to the “bathtub” effect should be excluded from the regression analysis.

7. MODEL VALIDATION

7.1 Introduction

Validation of MNRAP was a challenging task on its own, since there is no equivalent procedure in existence, with results that can be compared with the MNRAP results. Consequently, two approaches were taken towards validating the MNRAP procedure as follows:

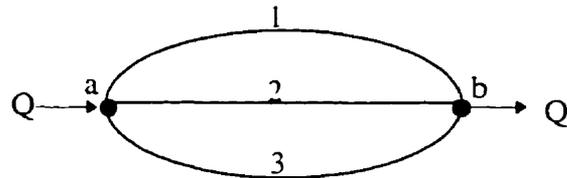
- (a) For a given distribution network, a least cost combination of rehabilitation alternatives and scheduling (herein termed the Optimal Rehabilitation Policy or ORP) is identified by exhaustively enumerating all the feasible combinations. The MNRAP is applied to the same distribution network to obtain the ORP. The ORPs obtained from the two processes are then compared. This approach can be implemented for only a very simple system because of the vast number of combinations involved.
- (b) Large systems cannot be exhaustively enumerated with available computing resources. It was consequently rationalized that if MNRAP can find an ORP that is less costly than any policy arrived at by existing professional practices, its potential contribution to the existing analysis practices would be apparent. A sample distribution system was prepared including network layout, pipe characteristics, pipe history, costs, etc. This sample was sent to seven water utility managers in Southern Ontario who were willing to participate in the experiment (eventually the number of participants dropped to six)¹. The participants were required to apply their existing analysis and decision making tools to devise an ORP while conforming to stated hydraulic constraints. The respective results were then compared to the results obtained by applying MNRAP to the sample system.

¹ Including Etobicoke, Markham, North York, Region of Peel, Scarborough, Toronto and Waterloo.

7.2 Exhaustive Enumeration

For the exhaustive enumeration, the system illustrated in Figure 17 was analyzed.

	Node a	Node b
Demand (L/s)	-83.33	83.33
Elevation (m)	10.0	15.0
HGL	70.0	
Min. residual pressure head (m)		various values



Existing Pipes

Pipe	Length (m)	Diameter (inch)	e (ft)	a (ft/year)	N(to) brk/yr/km	A (1/year)	Cb (\$/break)
1	400	6	0.0004	0.0004	0.30	0.10	2000
2	300	4	0.0030	0.0006	0.25	0.08	1800
3	500	8	0.001	0.0004	0.30	0.07	2300

Figure 17. Distribution network for exhaustive

An unrestricted exhaustive enumeration for such a system with $p=3$ pipes, $R=3$ rehabilitation alternatives for each pipe, and time horizon $H=30$ years, comprises $(R+1)^{p \cdot H}$ combinations which is in the order of magnitude of 10^{54} (if H is taken at a yearly timestep).

This simple network was selected so that headlosses may be directly calculated in order to avoid the need for a network simulation program. It should be noted that the hydraulic complexity of the network has no implication on the validity of the MNRAP results since the hydraulic calculations are merely a means to ascertain the distribution system's conformity to the hydraulic constraints at every stage of the dynamic programming process. Since exhaustively enumerating all the possibilities is computationally prohibitive, two restricted enumeration schemes were applied using computer codes written specifically for this purpose.

7.2.1 Restricted Enumeration Scheme #1

In the first enumeration scheme pipe, parameters were assigned so that the replacement cycles were long enough to ensure no more than one rehabilitation per pipe per analysis period (i.e., in the enumeration, if a pipe was rehabilitated at any time T during the analysis period, it cannot be considered for an additional rehabilitation before the end of the analysis period) . This scheme reduces the number of combinations to $(R \cdot H + 1)^p$ or about 800,000 (for a time horizon of 30 years where the analysis timestep is one year). The cost of each combination was calculated in exactly the same manner as in the MNRAP; i.e., the total cost comprises the present value (PV) of breakage repair costs from the present to first replacement, the PV of capital cost of first replacement and the PV of an infinite stream of cost cycles from the first replacement to infinity according to equation (22).

A total of 144 scenarios were enumerated in this manner (it took between 1 and 1.5 hours of CPU time for each scenario) and the ORP in each test was identified and recorded. The scenarios that were tested consisted of random combinations of pipe parameters taken at (uniformly distributed) time horizons of 30 and 40 years, discount rates of 3% and 6%, minimum residual pressure head at node 2 varying (uniformly) between 40 and 50 meters and ages of existing pipes varying (uniformly) between 10 and 30 years. Subsequently, MNRAP was applied to the same 144 scenarios and the resulting ORPs compared to those obtained by the enumeration scheme. Since the procedure of calculating the costs in this enumeration scheme is completely compatible with that used in MNRAP, the results are expected to be almost identical in both. The comparison yielded the following:

- Out of 144 scenarios, 122 ORPs identified by MNRAP were identical to those identified by the exhaustive enumeration scheme.
- In the remaining 22 scenarios the least cost combinations identified by MNRAP were the same as those identified by exhaustive enumeration with respect to the optimal rehabilitation

alternatives selected for each pipe. However, the scheduling of these rehabilitation alternatives in MNRAP differed from those identified by the exhaustive enumeration by no more than one year (i.e., a pipe in an ORP identified by MNRAP would be scheduled for rehabilitation up to one year earlier or later than the same pipe in the ORP identified by exhaustive enumeration). An analysis of these differences revealed that they could be attributed to differences in the level of precision of the hydraulic calculations in the MNRAP computer program (described in detail in Chapter 8) and the exhaustive enumeration program. The exhaustive enumeration program calculates pipe headlosses directly and with a high precision, whereas the MNRAP program uses a network simulator which solves the equations numerically in an iterative process until an acceptable level of convergence is achieved. When applying the MNRAP program to some of those 22 scenarios with P_{min} at node b adjusted a few centimeters up or down (as the case may be) to compensate for the difference in precision, the ORPs obtained for the adjusted scenarios were identical to those obtained from the exhaustive enumeration program obtained for the unadjusted P_{min} .

In conclusion, it can be safely stated that under the stated restrictions of the first enumeration scheme (i.e., one rehabilitation per pipe per analysis period and all the assumptions in the cost calculations) MNRAP is capable of identifying the least cost rehabilitation policy.

7.2.2 Restricted Enumeration Scheme #2

The second enumeration scheme allowed for the consideration of more than one rehabilitation measure per pipe per the analysis period; however, it did not allow more than one rehabilitation measure per timestep. In this scheme, the total number of combination is $(Rp+1)^H$. The 3-pipe system was enumerated for the same three rehabilitation alternatives, and for an analysis period of 32 years divided into 8 timesteps of 4 years each. This brought the number of combinations to 10^8 (which takes about 25 to 30 hours to enumerate). The cost of each combination was calculated following the same principles as MNRAP, but applied somewhat differently, as described below.

For the specified analysis period, all costs are explicitly calculated by accumulating all PVs of breakage repair costs from timestep to timestep and all capital costs for rehabilitation as follows:

$$COST_H = \sum_{i=1}^{i=H} \sum_{t=1}^{t=p} \left[\int_{t-1}^t Cb_i \cdot N(t_0)_i e^{A_i(t-g_{it})} e^{-r \cdot t} dt + Cr_i \cdot e^{-r \cdot t} \cdot z_{it} \right] \quad (33)$$

where g_{it} = age of pipe i at year t

z_{it} = a binary variable ($z=1$ if pipe i is rehabilitated in timestep t , otherwise $z=0$). All the parameters with index i pertain to the pipe that is the existing pipe in timestep t (i.e. if a pipe was replaced in timestep $t-1$ or earlier, the parameters of the replacement pipe are in effect at timestep t).

At the end of the analysis period a system exists comprising a set of pipes with their respective ages and parameters (much like the situation at the beginning of the analysis period). At this point the cost function depicted in equations (10) or (16) (for a replacement or a relining alternative, respectively), is applied to each pipe, where time zero (i.e., the present time) is time H . The total cost of the entire system then becomes

$$COST = COST_H + Cr_y \cdot e^{-rT_y^*H} + \int_H^{T_y^{**}} Cb_i \cdot N(t_0)_i e^{A_i(t-g_{iH})} e^{-r \cdot t} dt + C_{inf}(T_y^{**}) e^{-rT_y^*H} \quad (34)$$

where g_{iH} = age of pipe i at year H

T_y^{**} = time of first replacement after the time horizon H , which minimizes the total stream of costs from year H to eternity (equivalent to T^* that is calculated in equation (12), except calculated from year H onward rather than from year zero onward)

The ORP identified by the second enumeration scheme cannot be expected to be identical to the one obtained by MNRAP because (1) the costs are calculated somewhat differently, (2) the timestep used was 4 years, and (3) activities are restricted to only one per timestep.

Several scenarios were enumerated and their ORPs recorded. MNRAP was applied to the same scenarios and the resulting ORPs were compared to those obtained by the enumeration scheme. Four such comparisons are presented in Appendix 5. It is evident that the ORPs obtained by the two different procedures are remarkably close. The comparisons also validate in a very convincing way the concept of the replacement cycles (where the least cost timing of the second replacement of a pipe is T^{**} years after the first replacement – see section 3.4.1) and the concept that pipe replacement (at year T^*) will follow the initial relining of the pipe under certain conditions (see section 3.4.2).

7.3 Comparison to Existing Analysis Practices

The system that the survey participants were required to analyze is a 12-pipe system with one source node and several demand nodes as illustrated in Figure 18.

Node Number	Flow (L/s)	Elevation (m)
1	-80	45
2	0	25
3	15	23
4	0	21
5	22	26
6	0	22
7	25	19
8	0	21
9	18	23

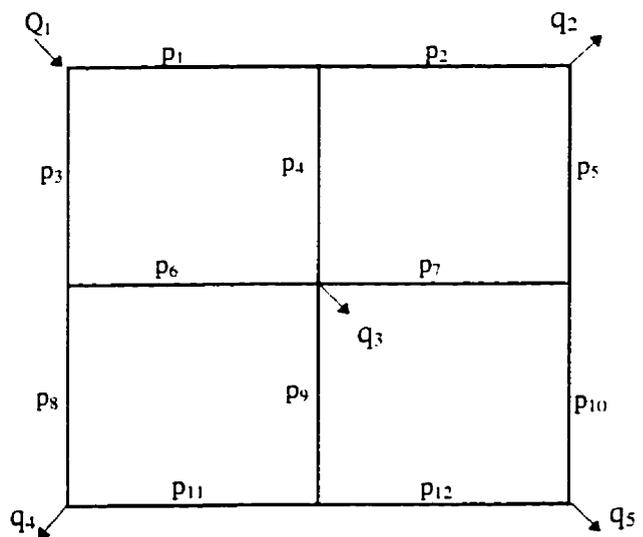


Figure 18. Distribution network for comparison to existing analysis practices.

The source node (node #1) is an elevated tank with a minimum water level of 25m above grade. The demand flows are peak demands and are assumed unchanged throughout the analysis period. The minimum residual head required at each node (except the source node) is

35m. The data for the existing pipes was not arbitrary but rather was based upon typical data obtained from the distribution system in Pressure District 5S of the City of Scarborough, in Ontario, Canada. The diameters and lengths of the pipes were provided (see Appendix 4 for the complete data set provided to each participant). Pipe installation dates range between the years 1945 and 1965. Available data on Hazen-Williams coefficients were provided based on a 1987 survey conducted in Scarborough. In addition, available breakage records for each pipe were provided. The participants were required to analyze three rehabilitation alternatives (reline, replace with same diameter and replace with a larger diameter), and propose an ORP. Costs and parameters for rehabilitated pipes were provided.

A direct comparison on an equal basis of all ORPs was not possible due to the fact that most participants made additional assumptions necessary for their respective analysis processes, and some even deviated from the guidelines. Consequently, the ORPs are presented (see Appendix 6) as submitted by the participants, followed by comments that highlight strengths, weaknesses and/or deficiencies identified in the ORPs.

The ORP obtained by the MNRAP is also presented in Appendix 6 together with a sensitivity analysis of the ORP to P_{min} (the minimum nodal residual pressure allowed) and to extending the time horizon from 30 to 40 years. The process of extracting the pertinent parameters from the raw data is presented as well, together with appropriate graphs that illustrate the “goodness of fit” of the parameters to the raw data. As was stated previously, a direct comparison of the cost of the ORP obtained by the MNRAP to any other ORP would be useless because some of the ORPs were derived based on different assumptions, while others did not conform to all the stated constraints. In order to gain a full appreciation of the differences between the various responses, it is necessary to read through Appendix 6. In doing so, the reader may distinguish between three different approaches to the problem of long term planning of water distribution network rehabilitation:

- (a) The first approach is a point rating evaluation that is typically developed based on some professional publications combined with local experience and practices. The point scores

are typically assigned according to age, breakage history, pipe material, location and operational conditions. This method is used to obtain a priority list for pipe rehabilitation. The actual timing of rehabilitation is determined by budget availability and planned roadwork (or other public works) construction. This approach, if combined with some breakage prediction practices, can become a more comprehensive analysis tool in which rehabilitation timing is considered more explicitly. This method, however, does not explicitly consider costs, nor does it consider hydraulic deterioration rate.

- (b) The second approach systematically considers pipe breakage rate (but not the deterioration thereof) and some form of hydraulic deterioration. It seems, however, that while the single pipe behavior is generally understood, the network behavior is handled intuitively, without a robust systematic process.
- (c) The third approach is reactive, in which no long term planning is implemented, but rather all rehabilitation measures are contingent upon other scheduled public works. It seems that this approach would be more frequent in relatively new municipalities, in which water distribution network rehabilitation has not yet become a major item in the budget.

The survey results clearly demonstrate the advantages of MNRAP over existing analysis practices in (a) its ability to explicitly consider the deterioration over time of both the structural integrity and the hydraulic capacity of the pipes in the water distribution network, (b) its ability (by considering rehabilitation cycles to infinity) to compare projected cost streams that are independent of the selected analysis period, and (c) its ability to consider the economics and performance of the entire network while regarding each pipe as a separate entity with its own characteristics and parameters.

8. THE COMPUTER CODE

A computer program was developed for the implementation of MNRAP. The program was developed in a portable C code that can be compiled for personal computer applications (under DOS and Windows) as well as for UNIX machines. The code consists of about 2600 lines. The program emulates the process in a manner that is very similar to the flow-chart presented in Figure 9, section 4.5. The program interfaces with the network simulator EPANET which was modified to run as a subroutine of the main program.

8.1 Data Structure

As can be seen in the model formulation, and as is demonstrated in the example, the number of states varies from stage to stage in a manner that cannot be predicted prior to run-time. Consequently, the use of arrays to hold the data is highly inefficient (as huge sections of memory would have to be committed to store the data) if not virtually impossible. After exploring several possible data structures, the “doubly-linked lists” scheme was selected and adopted to this application.

In doubly-linked lists, every data item is a link in a list. The links are connected to each other by pointers. New links can be added to the list at run-time through dynamic memory allocation, or deleted from the list if required; thus, a list can expand and contract to dynamically fit the size of data. In its application to MNRAP, at every stage a list is created to store the states at this stage. Every link (state) in this list contains the necessary information that is relevant to the state, and information (pointers) about the location (in memory) of the previous link (state) and the next link. The relevant information of every state must also include the subset of pipes that have already been rehabilitated in the previous stages. This subset expands as the stages advance, until it contains a full set in the last stage. In the interest of efficiency in the usage of computer resources, this sublist is held in a (singly) linked lists

that “branches” from every link (state) in the main list¹. The following is a graphical representation of the data structure:

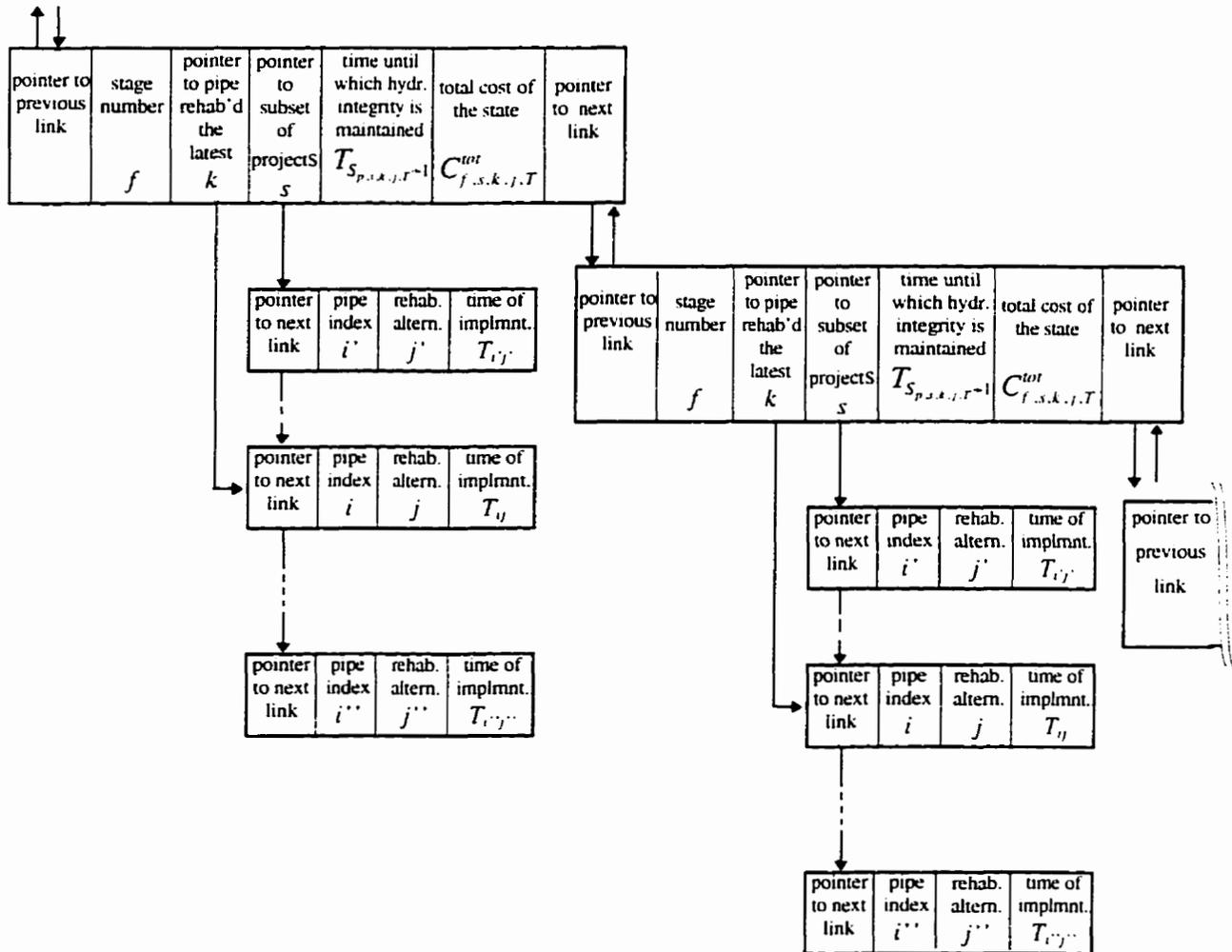


Figure 19. Graphical representation of the data structure.

At every stage the program creates a new main list which consists of all the possible states at this stage. Once a new main list is established, the old one is deleted in order to free memory. The new list is then processed to identify and delete all the inferior states.

¹ Since the ultimate number of items in the subset is known (to be equal to the number of pipes in the system), an array could be used to hold this information. However, this array would be fully utilized only at the last stage, where a link list expands with the addition of data.

8.2 Input and Output

The program expects two input files:

- (i) A file containing data that pertains to the rehabilitation alternatives and various run-time parameters, and
- (ii) The standard input file for running EPANET.

Figure 15 contains the input file with rehabilitation information for the three-pipe example that was presented in Chapter 5. The program creates four types of output files:

- (i) A continuous output file (two levels of output; full and partial).
- (ii) A file that contains the results of every stage.
- (iii) After every stage the program creates a stage file (the filename extension is assigned a number corresponding to the relevant stage (e.g., *stagefile.001*, *stagefile.002*,...,etc.)).
- (iv) A standard output file of EPANET.

The following should be noted:

- In both input files, lines that start with a semicolon are treated as comments (i.e., ignored).
- In both input files, blank lines are ignored.
- The program can be set to start running from any stage, and subsequently run any number of stages. If the program is set to start at stage n , stage file number $n-1$ is expected as an input file.
- Stage files are ASCII files, and as such can be manually modified (caution is recommended).
- The program can be set to run at different time steps. A time step of t years will check for system hydraulic integrity every n years. Furthermore, new states generated in cases (ii) and (iii), are created in increments of n years.
- Appendix 3 contains the printout of some results files for the example presented in Chapter 5.

```

[TITLE]
Sample Network (new example) - 3 Pipes

[existing pipes]
:pipe from to diam. length age ei ai Nti Ai Cbi
1 1 3 4 300 8 0.0005 0.0003 0.25 0.08 2000
2 1 2 6 400 10 0.0004 0.0003 0.3 0.07 2000
3 3 2 3 250 12 0.0005 0.0003 0.35 0.15 2000

[alternative 1]
[reline]
:pipe diam. eij aij Ntij Aij Cnj Cbij T* T**
1 4 0.0003 0.00025 0.25 0.08 40000 2000
2 6 0.00045 0.00025 0.3 0.07 55000 2000
3 3 0.00025 0.00025 0.35 0.15 30000 2000

[alternative 2]
[default replace]
:pipe diam. eij aij Ntij Aij Cnj Cbij T* T**
1 4 0.0003 0.00025 0.114 0.125 110000 2000
2 6 0.00045 0.00025 0.25 0.051 150000 2000
3 3 0.00025 0.00025 0.3 0.11 90000 2000

[alternative 3]
:pipe diam. eij aij Ntij Aij Cnj Cbij T* T**
1 6 0.00045 0.00025 0.25 0.08 150000 2000
2 8 0.0006 0.00025 0.25 0.051 180000 2000
3 4 0.0003 0.00025 0.3 0.11 110000 2000

[parameters]
a. NUMBER OF PIPES = 3
b. NUMBER OF NODES = 3
c. NUMBER OF REHABILITATION ALTERNATIVES = 3
d. MINIMUM PRESSURE HEAD ALLOWED IN ANY NODE (m) = 40
e. NUMBER OF NODES AT WHICH PRESSURE HEAD BELOW MINIMUM IS ALLOWED = 1
f. NODE NUMBERS AT WHICH PRESSURE HEAD BELOW MINIMUM IS ALLOWED:
f1. 1
f2.

g. ANALYSIS TIME HORIZON (years):40

: LIST OF FILE NAMES {full path name} (braces are required):

h. EPANET INPUT FILE {newexamp.inp}
i. EPANET OUTPUT FILE {newexamp.out}
j. EPANET EXECUTABLE FILE {epanet_q.exe}
k. OUTPUT FILE (continuous) {output.txt}
l. RESULTS FILE {results.txt}

m. INTERMEDIATE STAGE RESULTS FILES (stage)
: (the extension xxxxxx.001 , xxxxx.002 etc will be assigned by the program
: according to the stage number)

n. START DP PROCESS AT STAGE 0
: (to start at stage n, the intermediate stage results file xxxxx.00k (k=n-1)
: has to be present in the directory specified in line m above)

o. END DP PROCESS AT STAGE 3
p. DISCOUNT RATE = .05
q. TIME STEP (YEARS) = 1

[end]

```

Figure 20. Sample input file.

8.3 Program Performance

The program ran with the example presented in Chapter 5, on various computers and with various length timesteps. On a 133MHz Pentium machine, the run-time durations were 38, 17, 20, 11 and 10 seconds for 1, 2, 3, 4 and 5 years timesteps, respectively. EPANET accesses the disk drive twice every time it is run, which considerably slows the performance of the program. In order to alleviate this problem, the program and all its files are run from a virtual disk in the computer's random access memory (RAM-drive). This however is applicable only to personal computers which run under DOS or Windows operating systems and not to machines that run under the UNIX operating system. As a result, the performance of the program on UNIX machines is considerably slower than on personal computers. A modification of EPANET is required in order to enable the program to run on (much faster) UNIX machines with much better performance.

It should be noted that run-time duration, although predominantly affected by the problem dimensionality (as determined by the number of pipes, rehabilitation alternatives and number of timesteps in the analysis period) is also influenced by the condition of the distribution system in the following manner:

- The more structurally deteriorated the system is the higher the pipes breakage rate and the sooner the respective MCRTs (or T^* values) become. Consequently, the analysis will yield less cases of situation (i) and more cases of situations (ii) and (iii) (more of the latter as deterioration increases). Upon transition, rule (i) [corresponds to Situation (i)] generates from every state in the previous stage a single state in the next stage. rules (ii) and (iii) generate multiple states in the next stage for every state in the previous stage [rule (iii) generally generates more states than rule (ii)]. Most of these multiple states are likely discarded as inferior states thus there is only a minor effect on the dimensionality of the problem. Nevertheless, the generation and analysis of these multiple require additional run-time.
- The TMP of every state in the process is determined by "aging" the system one timestep at a time and monitoring the difference between the hydraulic capacity of the system and

the minimum pressure allowed. The more deteriorated the hydraulic capacity of the system, the less timesteps are required for the system to reach TMP. Consequently, less run-time is required to analyze each state.

8.4 Program Improvements and Enhancements

In order to enhance the capability and flexibility of the program to cope with realistic water distribution systems, some enhancements may be incorporated. A trend of increasing demand flow rates (due to urban development and population increase) or decreasing demand flow rates (due to implementation of water conservation programs) can often be predicted by the respective utilities and can be incorporated into the program. Moreover, different sections in a distribution system often experience different trends in demand flow rates. An option of assigning independent demand trends to different nodes could also be incorporated into the program. These enhancements could be implemented relatively easily; however, it seems that their implementation may slow down the performance of the program, hence it is suggested that they be bundled with some performance improvement that could be achieved by reducing the number of times EPANET accesses the disk during its operation as a subroutine.

9. SUMMARY AND CONCLUSIONS

Aging water supply infrastructure is a growing concern to water utility managers across the continent and indeed, worldwide. The water distribution network is the most expensive component of a water supply system, and its deterioration causes increased operation and maintenance costs, water losses, increased energy consumption, decreased level of service to the consumer and decreased level of water quality. Consequently, the rehabilitation and upgrading of pipes is an important part of effectively managing a water distribution system. Currently, the amount of effort that is applied towards the long term planning of water distribution system rehabilitation, seems to be very small relative to the size of budgets involved and to the potential savings thereof.

The long term planning of the rehabilitation and upgrading of a water distribution system involves the selection of an appropriate rehabilitation measure for each pipe in the network (e.g., reline the pipe, replace with the same diameter, replace with a larger diameter, etc.), and the implementation timing thereof, all the while maintaining adequate supply pressures in the system. The complexity of the problem lays in the fact that due to deterioration, the hydraulic capacity of the system diminishes over time and maintenance and repair costs increase over time. Any change in the hydraulic capacity of any pipe causes a redistribution of flows throughout the network. Consequently, the operational mode of any pipe in the network depends on the characteristics of all other pipes. As a result, the rehabilitation of any pipe in the network will influence the choice of pipes to be rehabilitated in the future, as well as the timing and type of future rehabilitation alternatives.

In this thesis, a multistage network rehabilitation analysis procedure (MNRAP) is developed. The objective of the MNRAP is to provide a decision support system (DSS) to aid in selecting for each pipe in an existing network, the rehabilitation alternative and its implementation timing, so as to minimize the cost of the rehabilitation investment and all maintenance costs over a pre-defined time horizon and subject to certain constraints. The decision variables are

therefore (1) the type of rehabilitation alternative to implement and (2) the time of its implementation, for every pipe in the distribution network. The constraints are (1) conservation of mass, (2) conservation of energy, and (3) minimum nodal residual supply pressure.

The MNRAP distinguishes between two types of rehabilitation measures: (1) pipe replacement - measures that improve both the structural integrity and the hydraulic capacity of the pipe (e.g. pipe replacement and certain types of sliplining) and (2) pipe relining measures that improves only the hydraulic capacity of the pipe (e.g. cement lining, calcite lining, most sliplining practices, etc.).

The MNRAP explicitly considers the deterioration over time of both the structural integrity and the hydraulic capacity of every pipe in the system in the following manner:

- For the structural integrity deterioration an exponential pipe breakage prediction model was adopted after Shamir and Howard (1979), that relates pipe breakage rate to its age. It was assumed that (as a first approximation) if a pipe replacement is selected as a rehabilitation alternative for a pipe (say replacement by the same diameter pipe or one diameter larger), than as the new pipe ages, it will eventually be rehabilitated by the same replacement alternative. This aging and replacement cycle is assumed to commence after the first replacement and continue to eternity. If a pipe is relined, then eventually it will be replaced, and the infinite cycles of aging and replacements will then commence.
- For the hydraulic capacity deterioration a logarithmic model was adopted, after Sharp and Walski (1988), that relates the Hazen-Williams coefficient of pipes to the age of the pipes.

A pipe cost function was developed that considers an infinite time stream of costs (comprising the present value of breakage repair and rehabilitation costs) that is associated with the infinite rehabilitation cycles for every pipe and every rehabilitation alternative. This function relates the total cost of each pipe to the rehabilitation measure selected for the pipe and the time of

implementing it for the first time. The cycle duration T^{**} is found such that it minimizes the infinite stream of cost cycles from the first rehabilitation to eternity. As well, the time of first rehabilitation T^* is found, such that it minimizes the total cost of a pipe from the present to eternity. Both T^{**} and T^* pertain to a single pipe (no network considerations are involved yet) and are determined for each pipe and every rehabilitation measure under consideration.

In order to evaluate the rehabilitation of single pipes in a distribution network framework, the following was assumed:

- The hydraulic capacity of a water distribution network is defined by the residual pressure at a critical demand node in the network.
- The hydraulic capacity of a water distribution network diminishes over time due to the deterioration in the Hazen-Williams coefficient of the pipes in the network.
- The rehabilitation of any pipe in the network causes an improvement (or at least does not cause degradation) in the hydraulic capacity of the network.
- If the distribution network is allowed to deteriorate to the extent that the residual pressure at the critical node is below a specified minimum, then the minimum pressure constraint is violated and the system is considered hydraulically inadequate. This point in time at which the system is about to become inadequate is therefore the latest possible time for the next rehabilitation measure to be implemented.

Next, a multistage procedure is developed based on a dynamic programming (DP) approach combined with partial and (sometimes) implicit enumeration schemes. In each stage of the procedure, every pipe combined with every rehabilitation alternative is considered for implementation. A set of rules is provided to selectively consider the time to implement each rehabilitation alternative for every pipe. These rules are derived based on the network hydraulic capacity assumptions (listed above) and the fact that the total cost function for the single pipe (described previously) is convex with a single minimum at T^* . At each stage, after all possibilities (states) have been enumerated, a criterion is provided to identify and discard all the inferior ones. At the end of every stage there are feasible states, on the basis of which the

states in the next stage are compiled. The hydraulic integrity of every state is established using a network simulation program. The number of stages is equal to the number of pipes in the network to which the procedure is applied. At the end of the procedure (after the transition to the last stage was implemented), the state that carries the least cost and at the same time provides hydraulic integrity to the system (at least) until the end of a predefined analysis period, is the one depicting the optimal sequence and timing of rehabilitation measures.

A numerical example is provided to illustrate the MNRAP. Although simple (3 pipes, 3 rehabilitation alternatives and 30 year time horizon), the solution space of this example consists of about $1.5 \cdot 10^{54}$ possibilities. The example demonstrates how this vast solution space is reduced dramatically to a manageable size by applying the rules and selection criterion of the procedure.

A computer program is developed to implement MNRAP. An elaborate memory storage scheme comprising doubly linked lists in which every link is also a gateway to another linked list, is used to store the complex data structure that MNRAP requires.

The validity of the MNRAP was established in two ways:

- (a) Some simple systems were exhaustively enumerated for all possible combinations of rehabilitation measure selection and scheduling, and their minimum cost sequences compared to those obtained by applying MNRAP. An excellent agreement was demonstrated between the two sets of results.
- (b) A study was conducted in which six water utility managers from the Greater Toronto Area and Waterloo (the participants) were presented with a sample water distribution system including all pertinent data that would typically be available when preparing a long-term rehabilitation program. The participants were required to implement their best engineering judgment and analysis tools in finding an optimal rehabilitation policy,

subject to stated constraints. These policies were then compared to that obtained by the MNRAP.

The MNRAP provides a potentially valuable decision support system for engineers and decision makers in devising long-term water distribution network rehabilitation plans. In order to increase its effectiveness, further research is recommended in the following directions:

- Dimensionality reduction is necessary to make the procedure suitable for large water distribution systems. Possibly an efficient discrete differential dynamic programming (DDDP) procedure could be developed for MNRAP (see Appendix 2, page 2-10).
- Incorporating reliability considerations into MNRAP requires further research of fast calculating surrogate reliability measures. In addition to finding such measures and adopting one suitable for MNRAP, a set of criteria has to be developed to define minimum acceptable levels of reliability (in terms of the chosen surrogate measure) for a water distribution network.
- Incorporating water quality considerations into MNRAP requires fundamental research in modeling the deterioration of water quality in pipes as a function of the condition of pipes (and subsequently as a function of the age of pipes).

Research on an ongoing basis is required to assess the suitability of the exponential breakage rate increase to pipes made with relatively new technology such as PVC pipes. It is possible (under certain conditions) to incorporate into MNRAP more than one type of deterioration function (both for structural and hydraulic deterioration) for different types of pipe.

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LIST OF NOTATIONS

p	number of pipes in the network
n	number of nodes in the network
R	number of rehabilitation alternatives to consider
H	analysis time horizon (years)
$CAP(T_{ij})$	capital cost of implementing rehabilitation alternative j in pipe i at time T_{ij}
T_{ij}	time from present that pipe i is rehabilitated with alternative j (years)
Cr_{ij}	cost of rehabilitation measure j in pipe i (\$ /km)
L_i	length of pipe i (km)
r	discount rate
t	time elapsed (years)
$N(t)_i$	number of breaks per unit length per year in existing pipe i ($\text{km}^{-1} \text{year}^{-1}$)
$N(t_0)_i$	$N(t)_i$ at the year of installation of existing pipe i (i.e., when the pipe was new)
g_i	age of existing pipe i at the present time (years)
A_i	coefficient of breakage rate growth in existing pipe i (year^{-1})
$N(t)_{ij}$	same as $N(t)_i$ except in pipe i , rehabilitated with alternative j ($\text{km}^{-1} \text{year}^{-1}$)
$N(t_0)_{ij}$	$N(t)_i$ at the year of installation of pipe i alternative j (i.e., upon replacement)
g_{ij}	age of replacement pipe i replaced by alternative j
A_{ij}	coefficient of breakage rate growth in existing pipe i (year^{-1})
$C_M(T_{ij})$	the present value of breakage repairs in pipe i for the years elapsed from the present to the year of rehabilitation with alternative j
Cb_i	cost of a single breakage repair in existing pipe i (\$)
T^f	time of first replacement of a pipe (year)
T^c	duration of a replacement cycle (years)
$C^{tot}(T^c)$	PV of total cost associated with one replacement cycle
$C_{inf}(T^c)$	PV of total cost associated with an infinite series of replacement cycles
T_{ij}^{**}	the value of T^c that minimizes C_{inf} for pipe i replaced by rehabilitation alt. j
T_{ij}^f	the time of first replacement of pipe i with replacement alternative j

T_{ij}^*	the value of T_{ij}^f that minimizes the total cost associated with the pipe from the present to infinity, also referred to as MCRT (minimum cost replacement timing)
$C_{ij}^{tot}(T_{ij})$	the PV of the total cost (present to infinity) associated with pipe i rehabilitated with alternative j at time T_{ij}
Q_i	flow rate in pipe i (m^3/s)
D_i	diameter of pipe i (m) or (ft)
$C_i^{HW}(t)$	Hazen-Williams friction coefficient pipe i at year t
e_{O_i}	initial roughness (ft) in existing pipe i when it was installed new
a_i	roughness growth rate (ft/yr) in pipe i
$e_{O_{ij}}$	initial roughness (ft) in pipe i alternative j
a_{ij}	roughness growth rate (ft/yr) in pipe i with rehabilitation alternative j
D_{ij}	diameter (m) or (ft) of pipe i rehabilitated with rehabilitation alternative j
$Q_{in,y}$	flow rate into node y
$Q_{out,y}$	flow rate out of node y
h_l	head changes in a pipe or a component that belongs to path l
$P_{v,t}$	residual supply pressure at node y in year t
$P_{min,y}$	minimum residual pressure allowed at node y in the system
s	denotes a subset of pipes that are considered to have been rehabilitated
p'	the index of the pipe that was the last to be rehabilitated in s
$\Delta t_{p',j}$	the amount of time that the system's hydraulic integrity is extended by (TMP delayed), as a result of rehabilitating pipe p' with rehabilitation alternative j
T_s	the timing of implementing the last rehabilitation project in s
T_{s+l}	the TMP (time of minimum pressure) resulting from implementing s
$CAP_{reline}(T_i)$	capital cost of relining pipe i at time T_i
$S_{f,s,k,j,T}$	a state in stage f with the five indices denoting the following: f – the stage to which S belongs s – the subset of pipes that S comprises k – the pipe that was rehabilitated last in state S j – the rehabilitation alternative implemented on pipe k T – the time at which pipe k was rehabilitated (elapsed from the present)

- $C_{f,s,k,j,T}^{tot}$ the cost of the corresponding state (the state with the same indices)
- $T_{S_{f,s,k,j,T-1}}$ the TMP of the state S
- w number of possible states that can be generated from a given state in the previous stage

Appendix 1 – Rehabilitation Technologies and Practices

Pipe rehabilitation technologies may be grouped into four major categories: cleaning, lining, pipe insertion and other methods.

- **Cleaning**

Most pipe rehabilitation projects require cleaning as a preparatory step to remove tubercles, sediments, corrosion products, biofilm, etc. On its own, cleaning is only a short term remedy. Without lining to prevent further corrosion, incrustation buildup will resume shortly after cleaning, rendering this measure less cost-effective. Several levels and techniques of cleaning are practiced:

Flushing – loose sediments can sometimes be removed by flushing problematic sections of pipe.

Scraping – heavier incrustation buildup is handled with mechanical scrapers. The scrapers can be pulled through the pipe with a winch (drag-cleaning), or propelled through the pipe by means of hydraulic pressure (pigs). For larger pipes, electric (rotating) scrapers are sometimes used. The materials of which the scrapers are constructed vary in their densities and abrasive properties to suit various conditions of pipe inner surfaces. Another type of scraper is the hydraulic-jet scraper, which uses high pressure jets of water to dislodge and remove incrustation from the pipe inner walls.

Air cleaning – air at high pressure is forced through small-diameter pipes at high speed. In order to provide added friction, small amounts of water can be injected into the air stream. Where heavy duty cleaning is required, controlled amounts of abrasive particulate material are added to the air stream making it an effective “sand blaster”.

Chemical cleaning – acid solutions (combinations of sulfuric and hydrochloric or citric acid) are used to dissolve mineral deposits in the pipe.

- **Lining**

The technique of lining pipe inner surfaces with a protective layer has been in practice for over half a century, and has been proven as one of the most effective rehabilitation methods used. Lining provides a protective coating that inhibits further corrosion and oxidation of the pipe inner surfaces. Lining does not provide the pipe with additional structural integrity. All lining practices require a thorough cleaning of the pipe as a preliminary step. Lining techniques vary according to the lining materials used:

Cement mortar lining – is oldest and most common lining technique in use. The material can be applied to a wide range of pipe diameters by either a centrifugal or a mandrel process.

The centrifugal process (more commonly used) involves a rotating head that dispenses mortar, and a series of trowels to smooth it to the pipe interior wall. The cement mortar is pumped to the unit through a hose. The centrifugal unit is pulled by a cable winch in small pipes (150 to 600 mm), or is self propelled and controlled by an operator who rides the machine, in large pipes (greater than 600 mm).

The mandrel process uses compressed air to apply the mortar and a conical mandrel to smooth it to the walls. This process is typically used for pipes 100 to 400 mm in diameter.

The thickness of the finished mortar layer is typically 2 to 10 mm, which consequently results in an inside diameter reduction of 4 to 20 mm. However, this diameter reduction may hydraulically be more than offset by a substantial

improvement in pipe friction, especially in larger diameter pipes where the liner thickness is small compared to the pipe diameter.

Some of the limitations of cement mortar lining are: (a) it is expensive for pipe with many bends; (b) service interruption (or bypass line) for 4-7 days; (c) cement mortar must be removed from service connections; and (d) high mobilization costs reduce cost-effectiveness of small relining projects.

Calcite lining – involves the circulation of a supersaturated calcite (calcium carbonate) solution through a pipe. Under controlled conditions of precise temperature, PH, and degree of saturation, a hard calcite layer will adhere to the inner pipe wall. A minimum flow velocity of 3m/s is required for the solution to deposit the hard nonporous layer. Consequently, this process becomes less cost-effective for large diameter pipes (more than 200 mm) as the required volume of calcite solution becomes very large.

Relative advantages of this technology are: (a) a very short service interruption time (a few hours); (b) service connections are not plugged and cleaning is minimal.

Although this process has been used during the last 60 years, experience with it is limited compared with other lining technologies, and it is not certain whether calcite lining is as durable as other linings.

Epoxy lining –involves the application of epoxy resin and hardener to the inner pipe walls by means of air-flow. The air-flow rate required for this application method limits the size of the pipe to be lined, to no more than 250 mm in diameter. The resulting lining is a smooth, hard, corrosion resistant film with a thickness of 1.0 mm (which makes it especially effective for small diameter pipes).

The use of epoxy lining for potable water lines has not been widespread. Some of the organic compounds used in the hardener may leach into the water, creating taste and odour problems. In addition, there have been reports of increased growth of heterotrophic bacteria in pipes after application of epoxy coating.

One last technique is the metallic phosphate lining technique, which is more a maintenance procedure than a rehabilitation project. Continuous addition of metallic phosphate (e.g. zinc orthophosphate) to potable water creates a protective film on the pipe inner wall. This film inhibits corrosion both in metallic and nonmetallic (e.g. asbestos-cement) pipes. This procedure is relatively inexpensive, but continuous monitoring and repetitions are required. Some side-effects of this procedure include possible promotion of bacterial regrowth in storage reservoirs, and a possible increase in nutrients level in wastewaters.

- **Pipe Insertion**

Inserting a new pipe within an existing pipe provides both better friction coefficients and added structural integrity. The inside diameter of the original pipe however, is reduced to that of the inserted pipe. In general, pipe insertion is most cost-effective in situations where existing structures (e.g. buildings, major highways, railroads, underground utilities, etc.) render excavation of old pipe impossible or very expensive. Also, pipe insertion is best suited for long watermains, with few sharp bends and few service connections.

Experience with pipe insertion in potable water systems is relatively limited. Most of it was gained in the natural gas industry (where there is less sensitivity to the reduction in pipe diameter). The following is a summary of existing techniques. Some of these techniques (with minor differences) may appear in the literature under different trade names.

Sliplining – involves the insertion of a rigid pipe (the liner) into an existing watermain. The typical liner materials are PVC, polyethylene, polypropylene, and polybutylene. The liner can be either pushed or pulled into place providing the

existing main is not severely crushed or collapsed. The annular space that is formed between the old pipe and the liner requires grouting or installation of spacers. Service connections must be excavated and refitted. This technique may result in a relatively high loss of hydraulic capacity due to cross-sectional area reduction. There is a technique called “pipe bursting” in which a mechanical/pneumatic device breaks and expands the old pipe, and the slipliner can then have a larger diameter than that of the original old pipe.

Swagelining – is a technique that was developed in Britain to produce a closer fitting lining. A polyethylene pipe (with outside diameter that is somewhat larger than the inside diameter of the old pipe) is inserted into the old pipe by being pulled through a diameter reduction die. The (controlled) pull tension maintains the reduced diameter of the liner throughout the insertion process. When the liner is in place, the tension is relaxed and the liner’s diameter increases towards its original size to produce a close-fit liner. Liner diameters up to 710 mm are available. The liner can be either a “thick wall” liner to provide added structural integrity to the old pipe, or “thin wall” to rehabilitate hydraulic capacity only.

Rolldown – is a technique similar to swagelining except the reduction in diameter is achieved using three pairs of compression rollers through which the insert is pushed by means of a hydraulic clamp. The deformation applied to the insert is more permanent in this method and in order for the liner to return back to its original diameter, it has to be pressurized for 24 hours after placement. Diameters are available through 470 mm.

Thermoplastic pipe – a liner is heated and deformed into a U shape for easy insertion (this can be done by the manufacturer or on site). Once in place, it is pressurized and heated in a procedure that activates the material’s “memory” of its original (pre-deformation) shape. A snug fit is thus created between the liner

and the inner walls of the old pipe. Liner material is typically polyethylene or polybutylene, but other materials may be used as well. Among the limitations of thermoplastic liners are permeability of some materials to organic contaminants in groundwater, and the need to excavate and refit all service connections.

Resin-impregnated fabric – also known as inversion lining because of the way the liner tube is inserted into the old pipe. The liner is made of polyester felt material impregnated with thermosetting resin and bonded to a plastic membrane. Liners are formed into tubes 100 to 1800 mm in diameter. The tube is inserted into the old pipe by means of water pressure. Once the tube is in place, the water inside it is heated and circulated to set the resin. Here too service connections must be excavated and refitted (fittings for this process are yet to be perfected). In addition, there is some question about the suitability of the resin material for use in potable water systems.

- **Other Methods**

Other methods of pipe rehabilitation are available for specific needs:

Chemical grouting – used to seal circumferential cracks and leaking joints, mainly in the wastewater industry and only in large pipes because a person has to enter the pipe to apply the grouting.

Reinforced shotcrete – involves placing a reinforcing mesh cage inside the pipe and then spraying it with a mixture of sand, cement and water. This method is also restricted to large diameters (800 mm and up) and is mainly used in sanitary and storm sewers and some specialized applications in water mains. No significant structural integrity is added to the pipe.

Joint rehabilitation – various methods exist to specifically rehabilitate pipeline joints to prevent leaks and faults prone to corrosion attack.

A rigorous summary of rehabilitation practices was compiled by Deb et al. (1990). An insight into the European experience is provided by Maine (1993).

Appendix 2 – An Overview of Optimization Techniques

Optimization of a system design is a formal procedure in which design variables are changed systematically within their respective feasible domains, while the system response to these changes is continuously evaluated until a “best” set of parameters is identified. The following short glossary may be useful to understand the application of optimization techniques:

objective function – a mathematical formulation of the performance criterion of a system as a function of one or more design variables termed “decision variables”. As design parameters change, the value of the objective function (system’s response) changes. The goal of the optimization procedure is to find a set of design variables that optimizes (minimizes or maximizes) the objective function.

constraints – a set of one or more mathematical expressions that describe the system that is being designed or analyzed as a function of the decision variables. Constraints can take the form of either equality or inequality constraints.

bound constraints – define the feasible domain of the decision variables.

feasible solution – a set of values of the decision variables that simultaneously satisfy all of the constraints.

feasible region – the region of feasible solutions defined by the constraints.

optimal solution – a set of values of the decision variables that satisfy the constraints and provide an optimal value of the objective function.

linear programming – optimization of a system whose objective function and constraints are all linear functions of the decision variables.

nonlinear programming – optimization of a system in which some or all of the constraints and/or the objective function are nonlinear in respect to the decision variables.

deterministic problem – defines a system whose parameters can be assigned fixed values.

probabilistic (stochastic) problem – defines a system in which the parameters are defined as random variables because of uncertainties about their true values.

static problem – defines a system in which time is not an explicit variable.

dynamic problem – defines a system in which time is an explicit variable.

continuous problem – defines a system whose decision variables are continuous variables.

discrete problem – defines a system whose decision variables are discrete variables.

integer programming – involves a system whose decision variables are all integers.

basic variables – in an indeterminate system of equations (n unknowns and m equations where $n > m$), possible solutions can be generated by letting $n - m$ unknowns equal zero and solving for the remaining m unknowns. These generated solutions are called basic solutions. The m variables are called basic variables while the $n - m$ variables are called nonbasic variables.

Edgar and Himmelblau (1988) suggested six steps to solve optimization problems:

1. Analyze the process and identify all the decision variables that may affect the outcome.

2. Determine the criterion for optimization and formulate the objective function in terms of the decision variables. The appropriate parameters and coefficients have to be determined for the objective function.
3. Formulate the problem constraints in terms of the decision variables. The constraints define the physical properties of the system, hence typically will comprise physical principles, empirical relations, external restrictions, etc. The appropriate parameters and coefficients have to be determined for the constraints.
4. If the scope of the problem is too large, breaking the formulation into manageable parts, or simplifying the model may be appropriate.
5. A suitable solution technique has to be identified or developed for the mathematical statement of the problem.
6. An examination of the results is required to determine their sensitivity to changes in coefficients and parameters of the problem.

The following is an overview of some of the more commonly used optimization techniques in water resources applications.

- **Linear Programming**

Linear programming (LP) is probably the most widely used optimization technique in resource (not necessarily water) allocation problems. The assumption of linearity (see glossary above), implies that the contribution of any decision variable (resource) to the optimization criterion (expressed in the objective function) is directly proportional to the value of this decision variable. Furthermore, it implies that at a given activity level, the overall effectiveness measure (the value of the objective function) is equal to the sum of contributions of each corresponding individual resource (i.e., the amount of one resource used has no affect on the contribution of another resource). Other required assumptions are

that activity units (resource units) can be divided into any fractional level (i.e., continuous decision variables), and all model parameters are deterministic constants.

There are several algorithms to solve an (LP) model. A graphical method is available for very small problems (2 or 3 decision variables). One widely used solution method is the *simplex method* which in essence searches for the optimum of an LP problem by examining feasible extreme points (based on the premise that an optimal solution will always contain at least one resource which is completely used). The simplex method explores the boundary of the feasible region by evaluating the objective function for a feasible set of basic variables and then (following a certain criteria) replacing old basic variables with new ones and re-evaluating until an optimal set is found. Other algorithms exist (e.g. Khatchian's ellipsoid method and Karmarkar's projective scaling method) that seek the optimal solution by moving along a search path through the interior of the feasible solution.

In linear programming problems, the feasible region and the objective function are strictly convex. Hence, if an optimum is found, it is a global optimum. This is not the case in nonlinear programming problems as is described below.

- **Nonlinear Programming**

The objective function in a nonlinear programming (NLP) problem forms a mathematical "response surface" as the decision variables are varied within their respective bound constraints. In the general case, this surface may have a feasible region that is comprised of both convex and concave segments.

An optimal solution can be viewed as a point on the response surface that is "better" than any other point (i.e. higher than all other points for a minimization problem or lower than all the others in a minimization problem). In a three-dimensional problem (three decision variables – three-dimensional response surface) this concept would be the equivalent of finding the highest mountain peak or the lowest valley.

A typical search procedure for identifying an extremum (either a minimum or a maximum) consists of two basic steps. The first step determines the search direction (a path along which the value of the objective function is improved. The second step is to search along the chosen path (line search) for the extremal point. An optimum is identified at a point where a subsequent line search yields only inferior points no matter what search direction is taken. It should be noted however, that since the response surface may have both concave and convex segments, the optimal point that was identified, may be only a local optimum (analogous to a random peak in a range of mountains, that is not necessarily the highest peak in the range). Consequently, it is likely that in one given problem, different starting points in a search sequence may yield different local optima. There is no guarantee of finding global optimum except in very specific problems in which the objective function is strictly convex.

There are many search techniques in existence. They essentially vary in their method of choosing a search direction and/or a search line algorithm. The following is a partial list of such techniques for unbound nonlinear problems:

Golden section method – is a line search which is based on splitting a line segment into two segments with a specific ratio between them.

Steepest descent method – is a search direction method in which the search direction is selected as that which results in the maximum increase (or decrease in a minimization problem) of the objective function. Essentially this is the equivalent of searching along the gradient vector at a particular point. Searching along the gradient vector may sound like the most efficient (fastest convergence) procedure, however, that is typically not the case. After an infinitesimal step was taken from a point in the direction of the gradient vector, the gradient vector at the new point will generally lie in a direction that is different from the previous one. Consequently, one of two events will transpire: (a) either the search is conducted in very small steps, in which case

the convergence will be slow (many steps will be required), or (b) the search is conducted in larger steps, in which case at the end of every step the new gradient vector will take a large change in direction, resulting in a zigzag path that is slow to converge as well.

Newton-Raphson method – modifies the gradient technique by using the second order partial derivatives of the objective function to form a weighted average of the first order partial derivatives (the gradient). This method results in a convergence in far fewer steps than the steepest descend method, but each step is computationally far more complex.

Quasi-Newton methods – (e.g. Davidson-Fletcher-Powell and Broyden-Fletcher-Goldfarb-Shanno methods) use different approximations to emulate the Newton method in order to reduce the intensity of the calculations in every step.

Conjugate direction methods – (e.g. Fletcher-Reeve and Polak-Ribiere methods) define the search direction using the gradient vector of the objective function in the current iteration as well as the one of the previous iteration. This results in a faster convergence than the steepest descend method with less complex calculations than the Newton method.

When a nonlinear objective function is bounded by constraints, the “response surface” becomes a bounded surface. Consequently, it is no longer sufficient to use one of the above techniques to identify an extremum, but the boundaries have to be explored as well for possible extremums. The following is a partial list of techniques to solve bounded nonlinear problems:

piecewise LP – is a technique that may be applied to problems with linear constraints but nonlinear objective function (e.g., an objective function that reflects economies of scale). It is possible, under certain conditions, to approximate

the objective function by a polygon (thus “linearizing” it) and then solve as a set of adjacent LP problems.

Lagrange multipliers - is a technique that uses classical differential calculus to solve optimality problems in which both the objective function and the constraints are nonlinear. This method augments the objective function and the constraints (all of which have to be expressed as equality constraints) into one function (Lagrangian) which is a new, unconstrained objective function. The augmentation is done by means of a vector of Lagrange multipliers (one such multiplier for every constraint equation). subject to certain conditions, the solution of the Lagrangian is also the solution to the original problem.

SUMT – acronym for **sequential unconstrained maximization technique**, in which some or all of the nonlinear constraints are incorporated into the objective function to form a new function termed the **barrier function**. This is done in such a manner, that upon trying to optimize the barrier function, if the search is done on (or very close to) any boundary, a large “penalty” is imposed on the barrier function. Consequently, the problem can be treated as an unconstrained problem (it is intrinsically bound, hence the name “barrier function”). If an optimum is found near a boundary, a post-optimal analysis can be performed to determine whether the true optimum is actually on the boundary itself.

GRG – acronym for **reduced gradient method**, in which the nonlinear problem is solved as a sequence of reduced problems, each of which is solved as an unconstrained minimization problem. The n decision variables are partitioned into a set of m basic variables and a set of $n-m$ nonbasic variables. The basic variables can be expressed as a function of the nonbasic variables (by iteratively solving a set of m nonlinear equations). The objective function can then be expressed in terms of only the nonbasic variables. This reformulated objective function is termed reduced objective function. The search direction is determined using

the reduced gradient method in which the gradient of the objective function is expressed in terms of only the nonbasic variables. The line search is done while optimizing the step size by solving a one dimensional optimization problem (in a given direction, the one decision variable is the step size and the objective is to advance the objective function as close as possible to the optimum). After executing a line search for a given set of nonbasic variables, if the optimum was not achieved, the process is repeated with a new set of nonbasic variables until an optimum is found.

random search techniques – while all the techniques listed above use gradient type (first derivative or first and second derivative) algorithms to search for optimal solutions, the random search techniques (e.g. Flexible Tolerance Method-FTM, Random Search with Systematic Reduction of Size of Search Region-RSSRSSR and Random Polyhedron Search-RPS) use algorithms that search optimal solutions by exploring the vertices of the polyhedrons (the equivalent of a polygon in an n-dimensional Euclidean space), that are formed by feasible solutions to the problem. The advantage of these methods is that they do not require the existence of derivatives in the objective function and the constraints of the problem. In comparison to the gradient type methods, these methods are slower to converge on simple problems, but said to be more efficient for large-dimension problems.

- **Dynamic programming**

Dynamic programming (DP) is a technique to handle problems that comprise multistage decisions, where the decision variables may be interrelated both in time and in space. DP decomposes the complex problem into a series of single-stage problems that are interrelated but are solved one at a time. It is common to associate DP with problems that involve explicit consideration of time as one of the decision variables, however, in practice DP can be effectively applied to many other problems with decision variables that can be discretized in time and/or space and in which the decisions are implemented in a spatial or temporal

The following terminology is used in DP:

- stage** – equivalent to a decision.
- state variable** – is a variable that summarizes the past history of the system up to the current stage. In every stage there is a set of input states and output states. The output states reflect the effect of the stage (decision) on the input states. The output states of one stage are the input states of the next stage.
- transition function** – also termed **transformation function**, is a set of operations that define the system's behavior upon implementing a decision (i.e., upon moving from one stage to the next).
- stage reward** – also termed **stage return**, is a function that depicts the effect of a stage (i.e., implementing a decision) on the objective function of the system to optimize.

The system evolves in stages. At any stage it starts in several feasible input states and, depending on the decision made, two events take place; (1) the system is transformed into the next stage (by implementing a decision, the transformation function is applied and output states are produced), and (2) a reward is assigned.

The DP approach stems from *Bellman's principle of optimality* as follows:

An optimal policy has the property that whatever the previous state and previous decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the previous transition.

Consequently, all transitions can be depicted in a single recursive process, in which every state after every stage contains a partial set of optimal decisions (i.e., optimal with respect to the states that comprise this partial set), while all nonoptimal states are discarded. After the final stage, the set of decisions is no longer partial (all decisions have been made), hence, the remaining state(s) constitute the optimal system. An added benefit of this process is the fact that if it is halted for some reason, the current states constitute an optimal solution to a partial

final stage, the set of decisions is no longer partial (all decisions have been made), hence, the remaining state(s) constitute the optimal system. An added benefit of this process is the fact that if it is halted for some reason, the current states constitute an optimal solution to a partial problem. Another benefit of DP is that it lends itself to solving stochastic problems somewhat better than other mathematical programming methods.

The computational time of the DP process increases approximately linearly with the number of stages, and approximately exponentially with the number of states in every stage. Consequently, the DP process can sometimes become computationally prohibitive when applied to large systems or when decision and state variables require a high degree of discretization. This phenomenon is known as the *curse of dimensionality*. In order to overcome this difficulty a procedure called **discrete differential dynamic programming (DDDP)** was suggested. In DDDP the first step is to implement a trial sequence of feasible decisions termed the *trial policy*. The sequence of states that is associated with the trial policy is called *trial trajectory*. In every stage, the states in the vicinity of the trial trajectory are selected to form a *corridor* around the trial trajectory. At this point, the traditional DP process is applied, exploring only those states that are included in the corridor, until an optimal sequence is identified within the corridor bounds. This optimal sequence is now the new trial trajectory and the process is repeated until no further improvement can be achieved. At this point, the trial trajectory defines a near optimal solution. Further accuracy can be achieved by “zooming in” on the final corridor; treating it as a new problem space, increasing the degree of state discretization within it, and repeating the DDDP process for a finer grid of states. The drawback of DDDP is that convergence time depends on the choice of the initial trial trajectory, and that the obtained optimum is not guaranteed to be a global optimum.

- **Integer and mixed integer programming**

When all or some of the decision variables in a system are strictly integers, the problem is defined as an integer or a mixed integer problem respectively. The most widely used approach for solving this type of problems utilize a method of *tree search*, also known as *backtrack algorithm*. The problem is solved by iteratively solving it as a pair of LP problems

(with relaxed integer constraints), one containing a constraint that places an upper boundary on the feasible space of one of the integer variables, and the other containing a constraint that places a lower boundary on this variable. Nonfeasible solutions are discarded, and feasible solutions are compared to previously obtained optimized solutions and the superior between them is recorded as the new optimized solution for the next comparison. In the next iteration, the problem is partitioned with respect to another integer variable and the process is repeated.

A rigorous presentation of various mathematical programming techniques can be found in Wagner (1975) and Himmelblau (1972). A rigorous presentation of the application of mathematical programming techniques to water resource problems can be found in Mays and Tung (1992).

Appendix 3 – Results Files for the Numerical Example in Chapter 5

Timestep = 1 year

Initial Stage

Pipe	Alternative	T rehab	t+delta t	cost
1	1	7	20	33862
1	3	7	40	48314
2	1	7	13	52356
2	3	7	26	66196
3	2	5	10	40917
3	3	6	13	46277

STATES FOR END OF STAGE 2:

printing state:
stage = 2
state number = 1
total cumulative cost (present value) = 73754
hydraulic integrity until (t+delta t) = 40

	pipe	alternative	year
last project:	2	2	20

list of all projects sorted by pipe number:

1	1	7
2	2	20

printing state:
stage = 2
state number = 2
total cumulative cost (present value) = 73601
hydraulic integrity until (t+delta t) = 23

	pipe	alternative	year
last project:	1	1	10

list of all projects sorted by pipe number:

1	1	10
3	2	5

printing state:
stage = 2
state number = 3
total cumulative cost (present value) = 83911
hydraulic integrity until (t+delta t) = 40

	pipe	alternative	year
last project:	1	3	10

list of all projects sorted by pipe number:

1	3	10
3	2	5

printing state:
stage = 2

state number = 4
total cumulative cost (present value) = 91113
hydraulic integrity until (t+delta t) = 17

	pipe	alternative	year
last project:	2	1	10

list of all projects sorted by pipe number:

2	1	10
3	2	5

printing state:
stage = 2
state number = 5
total cumulative cost (present value) = 93196
hydraulic integrity until (t+delta t) = 34

	pipe	alternative	year
last project:	2	2	13

list of all projects sorted by pipe number:

2	2	13
3	3	6

printing state:
stage = 2
state number = 6
total cumulative cost (present value) = 99801
hydraulic integrity until (t+delta t) = 40

	pipe	alternative	year
last project:	2	3	13

list of all projects sorted by pipe number:

2	3	13
3	3	6

STATES FOR END OF STAGE 3:

printing state:
stage = 3
state number = 1
total cumulative cost (present value) = 111821
hydraulic integrity until (t+delta t) = 40

	pipe	alternative	year
last project:	2	2	23

list of all projects sorted by pipe number:

1	1	10
2	2	23
3	2	5

Timestep = 2 years

Initial Stage

Pipe	Alternative	T rehab	t+deltat	cost
1	1	6	18	34296
1	3	6	40	50332
2	1	6	12	53150
2	3	6	26	68854
3	2	5	9	40917
3	3	6	12	46277

STATES FOR END OF STAGE 2:

printing state:

stage = 2

state number = 1

total cumulative cost (present value) = 75720

hydraulic integrity until (t+delta t) = 26

	pipe	alternative	year
last project:	2	2	18

list of all projects sorted by pipe number:

1	1	6
2	2	18

printing state:

stage = 2

state number = 2

total cumulative cost (present value) = 80864

hydraulic integrity until (t+delta t) = 40

	pipe	alternative	year
last project:	2	3	18

list of all projects sorted by pipe number:

1	1	6
2	3	18

printing state:

stage = 2

state number = 3

total cumulative cost (present value) = 99427

hydraulic integrity until (t+delta t) = 40

	pipe	alternative	year
last project:	3	3	6

list of all projects sorted by pipe number:

2	1	6
3	3	6

printing state:

stage = 2

state number = 4

total cumulative cost (present value) = 73974

hydraulic integrity until (t+delta t) = 21

	pipe	alternative	year
last project:	1	1	9

list of all projects sorted by pipe number:

1	1	9
3	2	5

printing state:

stage = 2

state number = 5

total cumulative cost (present value) = 85568

hydraulic integrity until (t+delta t) = 39

	pipe	alternative	year
last project:	1	3	9

list of all projects sorted by pipe number:

1	3	9
3	2	5

printing state:

stage = 2

state number = 6

total cumulative cost (present value) = 91797

hydraulic integrity until (t+delta t) = 17

	pipe	alternative	year
last project:	2	1	9

list of all projects sorted by pipe number:

2	1	9
3	2	5

printing state:

stage = 2

state number = 7

total cumulative cost (present value) = 78269

hydraulic integrity until (t+delta t) = 22

	pipe	alternative	year
last project:	1	1	12

list of all projects sorted by pipe number:

1	1	12
3	3	6

printing state:

stage = 2

state number = 8
 total cumulative cost (present value) = 86287
 hydraulic integrity until (t+delta t) = 40

3 3 6

	pipe	alternative	year
last project:	1	3	12

list of all projects sorted by pipe number:

1	3	12
3	3	6

printing state:
 stage = 2
 state number = 9
 total cumulative cost (present value) = 94613
 hydraulic integrity until (t+delta t) = 30

	pipe	alternative	year
last project:	2	2	12

list of all projects sorted by pipe number:

2	2	12
3	3	6

STATES FOR END OF STAGE 3:

printing state:
 stage = 3
 state number = 1
 total cumulative cost (present value) = 113228
 hydraulic integrity until (t+delta t) = 39

	pipe	alternative	year
last project:	2	2	21

list of all projects sorted by pipe number:

1	1	9
2	2	21
3	2	5

printing state:
 stage = 3
 state number = 2
 total cumulative cost (present value) = 116967
 hydraulic integrity until (t+delta t) = 40

	pipe	alternative	year
last project:	2	2	22

list of all projects sorted by pipe number:

1	1	12
2	2	22

timestep = 3 years

Initial Stage

Pipe	Alternative	T rehab	t+deltat	cost
1	1	6	18	34296
1	3	6	39	50332
2	1	6	12	53150
2	3	6	24	68854
3	2	5	8	40917
3	3	6	12	46277

STATES FOR END OF STAGE 2:

printing state:

stage = 2

state number = 1

total cumulative cost (present value) = 75720

hydraulic integrity until (t+delta t) = 39

	pipe	alternative	year
last project:	2	2	18

list of all projects sorted by pipe number:

1	1	6
2	2	18

printing state:

stage = 2

state number = 2

total cumulative cost (present value) = 99427

hydraulic integrity until (t+delta t) = 39

	pipe	alternative	year
last project:	3	3	6

list of all projects sorted by pipe number:

2	1	6
3	3	6

printing state:

stage = 2

state number = 3

total cumulative cost (present value) = 74367

hydraulic integrity until (t+delta t) = 20

	pipe	alternative	year
last project:	1	1	8

list of all projects sorted by pipe number:

1	1	8
3	2	5

printing state:

stage = 2

state number = 4

total cumulative cost (present value) = 92516

hydraulic integrity until (t+delta t) = 17

	pipe	alternative	year
last project:	2	1	8

list of all projects sorted by pipe number:

2	1	8
3	2	5

printing state:

stage = 2

state number = 5

total cumulative cost (present value) = 78269

hydraulic integrity until (t+delta t) = 21

	pipe	alternative	year
last project:	1	1	12

list of all projects sorted by pipe number:

1	1	12
3	3	6

printing state:

stage = 2

state number = 6

total cumulative cost (present value) = 86287

hydraulic integrity until (t+delta t) = 39

	pipe	alternative	year
last project:	1	3	12

list of all projects sorted by pipe number:

1	3	12
3	3	6

printing state:

stage = 2

state number = 7

total cumulative cost (present value) = 94613

hydraulic integrity until (t+delta t) = 33

	pipe	alternative	year
last project:	2	2	12

list of all projects sorted by pipe number:

2	2	12
3	3	6

STATES FOR END OF STAGE 3:

printing state:

stage = 3

state number = 1

total cumulative cost (present value) = 114259

hydraulic integrity until (t+delta t) = 38

	pipe	alternative	year
last project:	2	2	20

list of all projects sorted by pipe number:

1	1	8
2	2	20
3	2	5

printing state:

stage = 3

state number = 2

total cumulative cost (present value) = 117523

hydraulic integrity until (t+delta t) = 39

	pipe	alternative	year
last project:	2	2	21

list of all projects sorted by pipe number:

1	1	12
2	2	21
3	3	6

printing state:

stage = 3

state number = 3

total cumulative cost (present value) = 120019

hydraulic integrity until (t+delta t) = 40

	pipe	alternative	year
last project:	1	2	28

list of all projects sorted by pipe number:

1	2	28
2	2	12
3	3	6

Appendix 4 – Model Validation - Exhaustive Enumeration

Four scenarios (same network layout, pipes with different parameters) were exhaustively enumerated (enumeration scheme #2) and the ORPs obtained compared to the ORPs obtained by applying MNRAP to the same scenarios. The following are the results of these comparisons.

Scenario #1:

Enumeration scheme #2				MNRAP - 1yr timestep		
pipe	alternative	year		pipe	alternative	year
2	2	4		2	2	5
1	2	8		1	2	10
3	3	12		3	3	14
2	2	28		2	2	31 [#]
1	2	32		1	2	33 ^{##}
cost: \$256,261				\$250,828		

[#] T^{**}=26years, hence 5+26=31

^{##} T^{**}=23years, hence 10+23=33

The ORP obtained by MNRAP carries a slightly smaller cost because with a timestep of a single year the timing of the rehabilitation measures are more finely tuned.

Scenario #2:

Enumeration scheme #2				MNRAP - 1yr timestep		
pipe	alternative	year		pipe	alternative	year
1	1	4		1	1	2
2	2	12		2	2	12
3	3	16		3	3	16
1	2	24		1	2	23 [#]
cost: \$223,984				\$226,065		

[#] T^{*}=year 23

The ORP obtained by MNRAP carries a slightly higher cost because the first rehabilitation has to be implemented no later than year 2. while the exhaustive enumeration scheme (with 4-year timestep) allows year 4 (first timestep) as the earliest implementation time. The cost of relining pipe #1 is \$22,000 (\$55,000/km), at 6% discount rate, implementation at year 4 would cost about \$2,150 less than implementing at year 2. The difference almost exactly equals the cost difference between the two ORPs listed in the table above.

Scenario #3:

Enumeration scheme #2			MNRAP - 1yr timestep			MNRAP - 4yr timestep		
pipe	alternative	year	pipe	alternative	year	pipe	alternative	year
3	2	4	2	1	2	3	2	4
2	2	16	3	2	9	2	2	17
1	2	24	2	2	17	1	2	23
			1	2	23 [#]			
cost:	\$209,225			\$210,703			\$209,030	

[#] T = year 23

In this scenario it seems that there are several combinations (and sequences) of pipe rehabilitation that carry costs that are very close to the cost of the ORP (i.e., close to \$210,000). In a small network consisting of only three pipes, the sensitivity of the ORP to small changes in system conditions is much greater than larger systems, especially when several near-optimal solutions exist in close proximity to the ORP. Consequently, MNRAP with 1-year timestep obtained an ORP that is only slightly more expensive than the exhaustive enumeration (because of the timing of the first rehabilitation measure) but comprises a very different sequence and timing of rehabilitation measures. When MNRAP was applied with a 4-year timestep, the resulting ORP obtained was almost identical to the exhaustive enumeration results, at a slightly lower cost (because of a more finely tuned timing).

Scenario #4:

Enumeration scheme #2			MNRAP - 1yr timestep			MNRAP - 4yr timestep		
pipe	alternative	year	pipe	alternative	year	pipe	alternative	year
1	1	4	1	1	4	1	1	4
2	2	12	2	2	13	2	2	12
1	2	20	1	2	23 [#]	1	2	23
3	2	28	3	2	21	3	2	28
cost:	\$170,300			\$173,093			\$169,759	

* T^{*}=year 23

This scenario reveals a similar phenomenon to that described in Scenario #3, except that here a relining alternative is included in the ORPs obtained from all procedures.

Appendix 5 – Data-set Provided to Survey Participants

Dear <participant>

Re: Your participation in our study concerning the rehabilitation of water distribution networks.

Following our telephone conversation on April 22, I would like to express my sincere appreciation for taking the time to participate in what we believe is a valuable survey.

One of my Ph.D. students, Yehuda Kleiner, has developed a procedure to analyse and optimize the selection and scheduling of rehabilitation alternatives for all pipes in a water distribution system over a pre-selected analysis period. The objective and constraints of this procedure are stated as follows:

Consider a water distribution network with p pipes (links) and n nodes. Every pipe in the network may be rehabilitated by one of R rehabilitation alternatives including relining or replacement with the same or with a larger diameter pipe.

For a given time horizon of H years, minimize the present value of the total cost of maintaining (breakage repair) and rehabilitating the pipes in the network subject to the following:

- Mass conservation of flow in all nodes (continuity equations).
- Energy conservation of flow in all pipes (links).
- Residual supply pressure in every node is above a stated minimum.
- As a pipe ages, its hydraulic carrying capacity diminishes.
- As a pipe ages, its breakage rate increases.
- A pipe can be relined only once during the analysis time period.
- A pipe can be replaced more than once during the analysis time period, but if the initial replacement is alternative j , then all subsequent replacements are equal to alternative j .

The objective of the survey, in which you have agreed to participate, is to compare the proposed procedure with existing analysis practices of experienced decision makers in the water utility industry (such as yourself) and to assess its merit.

In the following pages you will find the details of a simple water distribution system. Your mission (should you decide to accept it) is to propose a policy comprising the rehabilitation alternative and its time of implementation for every pipe in the system, so as to achieve the objective stated above, subject to the constraints.

For simplicity only three rehabilitation alternatives should be considered:

- 1) clean and reline pipe,
- 2) replace with same diameter pipe, or
- 3) replace with a pipe one (nominal) diameter larger.

Pertinent data is provided in the following pages. Should you need further information please do not hesitate to call Yehuda Kleiner at (416) 978-6069 (U of T) or (905) 881-0370 (at home). We will contact you shortly after you receive the information. If you feel that some of the data provided deviates significantly from data that is prevalent in your municipality, please let us know as soon as possible so we can amend it in time for all the participants.

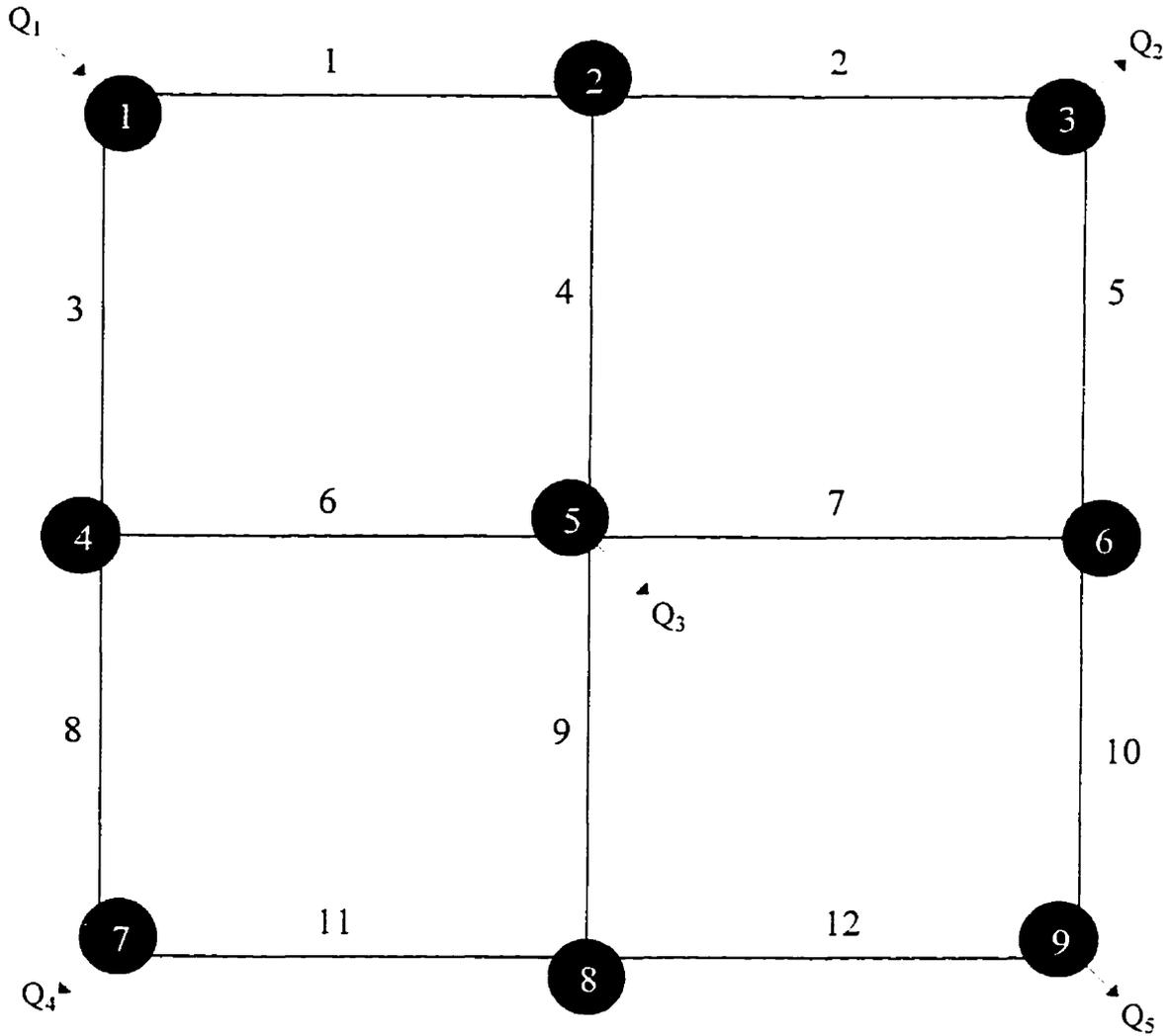
We would appreciate it if you could finish your analysis by May 26.

After the survey responses are analyzed you will be invited to participate in a concluding session in which the results will be discussed and the new methodology presented.

With appreciation,

B.J. Adams - Chair, Dept. of Civil Eng.
Yehuda Kleiner Ph.D. Candidate

University of Toronto



Node Data:

Node Number	Flow (L/s)	Elevation (m)
1	-80	45
2	0	25
3	15	23
4	0	21
5	22	26
6	0	22
7	25	19
8	0	21
9	18	23

The source node (node #1) is an elevated tank with minimum water level of 25m above grade. The demand flows are peak demands and are assumed unchanged throughout the analysis period. The minimum residual head required at each node (except the source node) is 35m.

Pipe data:

Pipe Number	From Node	To Node	Length (m)	Inside Diameter (inch)	C ^{HW} - Hazen-Williams "C" coefficient*	Installation year**
1	1	2	600	10	56	1945
2	2	3	800	6	42	1945
3	1	4	400	8	85	1945
4	2	5	500	8	62	1947
5	3	6	700	6	40	1953
6	4	5	600	6	41	1953
7	5	6	900	6	39	1953
8	4	7	500	8	55	1958
9	5	8	800	6	48	1960
10	6	9	700	6	43	1953
11	7	8	300	6	55	1963
12	8	9	600	6	56	1965

* These values were recorded in a survey conducted in 1987.

** All pipes are cast iron and assumed to have a friction coefficient of C=130 upon installation.

The table in the next page presents data pertaining to breakage (and repair) for each pipe in the system.

The following are the dates on which pipe breakage was recorded for the various pipes in the system:

Break	Pipe Number								
	1	2	3	4	5+10 *	6+7 *	8	9	11+12 *
1	03/01/65	01/26/63	07/28/58	02/08/74	12/21/65	05/11/64	11/23/64	05/25/61	11/22/68
2	08/14/69	02/28/63	01/27/60	10/11/77	02/07/71	11/17/67	12/20/66	11/24/62	05/23/70
3	12/30/71	11/16/63	07/27/61	06/20/82	02/15/71	07/30/68	03/03/69	05/24/64	11/22/71
4	07/22/75	11/17/63	05/06/62	12/12/85	05/29/71	01/05/68	11/26/70	05/15/69	02/19/77
5	03/10/78	01/04/65	01/30/64	10/17/88	12/15/73	09/17/69	12/23/70	01/17/70	03/11/79
6	11/23/78	03/07/67	05/21/66	01/06/89	12/23/73	10/29/74	01/04/73	03/20/72	02/08/81
7	01/21/79	01/21/68	07/23/63	12/02/91	12/13/74	08/24/76	02/15/77	08/17/72	11/20/82
8	09/18/79	10/05/68	01/21/65	01/08/92	01/12/76	07/30/78	12/29/77	02/16/74	07/12/84
9	01/25/80	12/28/68	07/22/66		01/17/77	06/10/80	01/06/78	08/17/75	01/13/86
10	01/07/84	11/17/69	05/06/68		01/27/77	04/26/81	10/30/78	04/18/77	05/28/87
11	03/05/85	12/05/69	12/30/68		02/06/77	03/12/82	02/28/79	03/25/78	08/20/88
12	03/28/85	01/03/70	03/19/69		01/20/79	12/29/84	12/16/81	05/01/81	09/24/89
13	02/03/87	11/26/70	08/13/81		01/14/81	01/23/86	12/26/81	08/02/80	09/09/90
14	06/25/87	12/19/71	01/30/84		06/30/81	03/19/87	02/01/82	01/19/83	07/06/91
15	01/21/88	02/03/72	02/18/86		01/17/82	06/11/88	03/16/82	02/07/85	03/12/92
16	01/05/91	02/07/72	10/11/87		02/16/82	10/04/89	02/15/83	09/30/86	09/28/92
17	01/25/93	02/29/72	01/03/89		04/12/83	03/13/90	01/28/84	12/24/87	02/25/93
18	03/29/93	10/13/72	10/30/89		01/10/85	07/31/90	01/31/85	10/19/88	06/05/93
19	02/21/94	02/08/75	03/29/90		07/19/85	11/28/90	11/25/86	03/18/89	07/25/93
20		11/04/75	03/29/90		02/21/88	03/08/91	01/15/90	01/12/90	08/29/94
21		01/20/76	11/19/91		12/20/90	05/27/91	01/16/91	11/08/90	
22		11/14/76	02/11/93		12/22/90	07/26/91	02/26/93	04/07/91	
23		02/15/77	12/08/93		09/08/91	11/03/91	03/04/93	11/27/92	
24		03/16/77			03/06/92	05/21/92	01/08/94	02/20/94	
25		12/06/77			12/30/93	10/18/92		12/17/94	
26		02/07/78			03/10/94	01/26/93		05/16/95	
27		10/14/78			07/28/94	03/17/93		10/13/95	
28		02/11/79			05/04/95	12/07/94			
29		03/10/81				12/02/95			
30		01/31/82							
31		04/13/82							
32		01/14/84							
33		01/18/84							
34		02/15/84							
35		12/16/84							
36		02/14/85							
37		03/06/86							
38		11/18/86							
39		01/06/88							
40		03/13/89							
41		12/28/91							
42		01/01/93							
43		03/11/93							
44		12/27/93							

* These pipes were treated as one pipe (for the purpose of recording their breakage events) and the data represent their combined breakage records.

Cost data:

Pipe Diameter (inch)	6	8	10	12
Breakage Repair (\$/event)	2000	2200	2500	2800
Cleaning & Relining (\$/km)	55,000	65,000	70,000	100,000
Pipe Replacement (\$/km)	150,000	200,000	220,000	300,000

Additional data and information:

- Use a discount rate of 6%
- Use an analysis period of 30 years
- Assume replacement pipes are PVC. The initial Hazen-Williams coefficient of each replacement pipe is assumed to be $C^{HW}=140$ and its deterioration rate is assumed to be one quarter¹ that of the pipe that it is replacing (i.e. it takes 4 times as long for a C^{HW} in a PVC pipe to deteriorate by the same amount as the C^{HW} in a CI pipe under the same operating conditions).
- The breakage rate and the deterioration thereof (as a function of pipe age) in a PVC replacement pipe is assumed to be one quarter¹ that of the pipe that it is replacing.
- A newly relined pipe is assumed to have a Hazen-Williams coefficient $C^{HW}=130$. The deterioration rate of the friction coefficient is assumed similar to that of the new CI pipe.
- Relining does not improve the structural integrity of the pipe, i.e., the breakage rate is not affected.
- Assume no special complaints were received from customers about water quality.

¹ Since there is no sufficient data about the deterioration with time of the hydraulic capacity and the structural integrity in PVC pipes, these values were assumed based on anecdotal information and partially related surveys.

Appendix 6 - Presentation and Analysis of the Validation Survey

The optimal rehabilitation policies (ORPs) arrived at by the six participants are presented and analyzed in this appendix along with the MNRAP results. The presentation and analysis will refer to the respective ORPs by numbers (1 through 6) rather than by the respective municipality name. In their responses, some participants elaborated more than others on the process of arriving at their ORP, but all had useful comments and highlights based on their own perspective and experience.

The hypothetical problem presented to the participants (see Appendix 5) contains some raw data and some basic assumptions and guidelines, with the intention that all analyses be implemented under similar terms. Still, as will later be seen, most participants made additional assumptions necessary for their respective analysis processes, and some deviated to a degree from the guidelines. Consequently, it was not possible to compare the ORPs to each other (and to MNRAP) on an equal basis. The ORPs will therefore be presented as obtained from the participants, followed by the author's comments. Subsequently, MNRAP's ORP is presented.

ORP #1

Participant #1 assumed breakage rate growth rate of 3% per year for all pipes in the network. Subsequently, a point rating evaluation method was applied to each pipe in the network. This method is an "in house" adoption of several rules and benchmarks suggested in professional publications like *The Journal of AWWA* and others. Following this method, points are assigned according to pipe age, breakage history, pipe type, size, depth, average operating pressure, etc. The method was applied to the system in ten year increments, i.e., year 0, 10, 20 and 30. The following is the ORP obtained by participant #1:

year	pipes	rehabilitation alt.
0	2,9,12	reline (alt. 1)
10	1,5,10	reline (alt. 1)
20	6,7,8	reline (alt. 1)
30	3,4,11	reline (alt. 1)

The total cost of this ORP was calculated by the participant to be \$661,386. This includes the present value of the investment in relining the pipes and the break repair cost during the 30 year analysis period.

Comments regarding ORP #1:

- The hydraulic capacity of the network is below the minimum even if its deterioration is not taken into account. It seems that there was a mistake in the data file of the hydraulic simulator that was used (EPANET).
- The breakage rate growth is not likely to be the same for all pipes based on the breakage history.
- The calculated cost does not account for the fact that pipe replacements will have to be implemented in the future due to the increase in breakage rate.

ORP #2

Participant #2 did not describe in detail the analysis process used, only the assumption that pipe's breakage rate throughout the analysis period was considered constant and equal to the breakage rate in the last ten years of the available break records. Based on this assumption, the participant identified that in some cases it would be more economical to replace a pipe than to reline it, in order to achieve the required hydraulic capacity. The ORP suggested by participant #2 was to replace pipes 2, 3, 8, 11, and 12 with pipes one size larger, right at the beginning of the 30 year analysis period. No reference was made to the other pipes in the network and a total cost calculation was not provided. A comment is made however that economies of scale consideration was one of the determining factors in choosing the timing of the replacements.

Comments regarding ORP #2:

- Hydraulically, ORP #2 is adequate approximately throughout the analysis period. (If the hydraulic deterioration model presented in section 3.6 is applied, the network is adequate until year 27.)
- ORP #2 does not specify rehabilitation measures for all the pipes in the network and no reference is made to costs that are expected after the time horizon.

ORP #3

Participant #3 provided very little information on the analysis process used. The participant named budget constraints and works implemented by other utilities as key factors in devising a rehabilitation plan. From the comments provided, it can be concluded that water utility #3 does not employ a rehabilitation plan and the decisions to rehabilitate are primarily reactive, based on planned road works or other sanitary works. Consequently, the ORP suggested by participant #3 is as follows:

Replace pipe 2 in the first year (or at the first opportunity, in conjunction with other infrastructure works).

Pipes 1, 3, 6, 7, 11 and 12 - reline and add cathodic protection in the first year (or at the first opportunity, in conjunction with other infrastructure works).

Pipes 4, 5, 8, 9 and 10 - reline and add cathodic protection whenever possible, in conjunction with other infrastructure works.

Comments regarding ORP #3:

- Analysis #3 was implemented without considering the hydraulic capacity of the network. Consequently, the network is hydraulically inadequate throughout the analysis period, even without considering the deterioration in the Hazen-Williams coefficients.
- Since ORP #3 is for the most part reactive, total cost cannot be calculated.
- It seems that no methodical economic evaluation was applied.

ORP #4

Participant #4 provided a detailed account of the analysis process. On the issue of pipe breakage rate, participant #4 believes that since municipal environmental conditions are dynamic, the prediction of future pipe breakage rate should rely only on recent breakage data and not on the entire break history, which for the most part may be irrelevant. Consequently, the average breakage rate of each pipe in the last ten years was calculated, and this value was taken as the (constant) breakage rate for the 30-year analysis period for each pipe respectively. The breakage rate of replacement pipes was considered negligible. On the issue of the deterioration of the Hazen-Williams coefficient, participant #4 pointed out (rightfully) that the data provided is not sufficient to compile a reliable deterioration model.

Consequently, an exponential model was adopted, based on some field data published by AWWA, as follows: $C_n = C_0 * 0.99^n$ for existing pipes, and $C_n = C_0 * 0.9975^n$ for replacement pipes, where C_n is the coefficient in year n , C_0 is the coefficient in a new pipe and n is the age of the pipe. This model was applied to all the pipes in the network. The economic/hydraulic analysis that was subsequently applied yielded the following ORP:

pipe #	rehabilitation alternative	year	subsequent rehabilitation	year
1	reline	0	replace (same)	33
2	reline	0	replace (same)	33
3	reline	0	replace (same)	33
4	reline	0	replace (same)	33
5	reline	9	replace (same)	43
6	reline	0	replace (same)	33
7	reline	0	replace (same)	33
8	reline	0	replace (same)	33
9	reline	9	replace (same)	43
10	reline	9	replace (same)	43
11	reline	9	replace (same)	43
12	reline	0	replace (same)	33

The total cost was calculated to be \$793,202 including the present value of the relining, replacement and breakage repair of all pipes from the beginning of the analysis period until the year of replacement.

Comments regarding ORP #4:

- The premise of participant #4 that the prediction of future pipe breakage rate should rely only on recent breakage data and not on the entire break history, while intuitively appealing, has a significant practical problem. The breakage rate is a net result of interaction between several mechanisms including pipe corrosion, pipe loading, operational conditions, municipal environmental conditions, etc. To date, research has not been able to model all these mechanisms in order to reliably predict the future breakage rate of a pipe. Consequently, various simplified models have been employed for that purpose, in which some inference is being made on the future breakage rate, based on historical records. By ignoring most of the breakage history of a pipe, one may lose valuable data pointing to a possible trend or pattern change in the pipe's breakage rate. Participant #4 in fact contradicted himself by deriving an average breakage rate based on a ten year period and then applying it as a constant rate for a 30-year analysis period.
- In reference to the deterioration of the Hazen-Williams coefficient, while the comment of participant #4 about too little available data is correct, this situation is typical in many water utilities, and making use of the little data that is available is likely to result in a greater accuracy than ignoring it completely. The exponential model used in ORP #4 may be as valid as any model, however, the base of the exponent should be calibrated to reflect the available data for each pipe separately. Using 0.99 as the exponent base for all the existing pipes results in an under-prediction of the hydraulic deterioration rate in all the pipes (except pipe 2).

ORP #5

The water utility represented by participant #5 has developed a standard evaluation method in which pipes are prioritized for replacement according to points they score. The criteria by

which the points are scored include breakage history, pipe and joint material, pipe age and pipe operational conditions (pressure, fire flow deficiency, proximity to a school, etc.). This method was applied to the network with the following results:

pipe	pts. for breaks	pts. for material	pts. for age	total points
1	13	30	15	58
2	10	30	15	55
3	25	30	15	70
4	8	30	15	53
5	11	30	15	56
6	17	30	15	62
7	17	30	15	62
8	20	20	10	50
9	20	20	10	50
10	11	30	15	56
11	18	20	10	48
12	18	20	10	48

With a score of 40 and over, a pipe is considered for replacement (the water utility has a policy of not relining pipes at all), which in this case means that all pipes would be considered for replacement. The scheduling of the replacements was not specified by participant #5, but it appears that it would be subject to budget constraints and in conjunction with planned road works at the pipe location. Consequently, a total cost could not be computed.

Comments regarding ORP #5:

- The point method considers costs only implicitly, through breakage history.
- Network hydraulics is not considered at all (only local hydraulic deficiencies).
- Relining can be a viable rehabilitation alternative if all that needs to be improved is the hydraulic capacity and/or the water quality in the pipe.

ORP #6

Participant #6 found the average breakage rate of every pipe in the last ten or so years, and assumed it to be the constant breakage rate of the respective pipes for the analysis period. Subsequently, an approach like Shamir and Howard (1979) was applied to find a pipe replacement time that minimizes the total cost (replacement and breakage repair) for each individual pipe (it was assumed that breakage repair cost of replacement pipe is negligible). This analysis was applied as a screening measure, to identify possible early replacements due to economical considerations. It was found by participant #6 that in the worst case, minimum cost time of replacement was year 29, which meant that no early replacements due to economical considerations were found in this particular problem.

For the deterioration of the Hazen-Williams coefficient, participant #6 assumed an exponential decay function of the type $C_{o/(1-k)^t}$, and found the appropriate parameters for each pipe, based on the data provided.

Subsequently, trial and error was used to find the ORP which included replacing pipes 2, 3, and 8 with pipes one diameter larger (alternative 3) at year 1. The total cost of ORP #6 was calculated by the participant to be \$619,000.

Comments regarding ORP #6:

- The assumption of constant breakage rate in the next 30 years is questionable (see comments regarding ORP #4) and leads to under-estimation of costs, which subsequently leads to a biased ORP (in favor of postponing the replacement of existing pipes).
- Neglecting replacement pipe costs tends to cause a bias in favor of early replacement of existing pipes.
- There seems to be a slight under-estimation of the deterioration rate of Hazen-Williams coefficients. This may be either due to the parameters selected for the exponential deterioration thereof, or the participant may have assumed a negligible deterioration rate in the replacement pipes.

ORP obtained by MNRAP

The first step in applying MNRAP was to derive the appropriate parameters from the data. Breakage rate parameters were derived for each pipe from the cumulative breakage records according to the following equations:

Breakage rate of pipe i as a function of time is

$$N(t)_i = N(t_0)_i \cdot e^{A_i(t+g_i)} \quad (\text{equation 2 – section 3.3})$$

When the breakage rate is taken from the time of installation then pipe age $g=0$ and the breakage rate becomes

$$N(t)_i = N(t_0)_i \cdot e^{A_i t} \quad (2i)$$

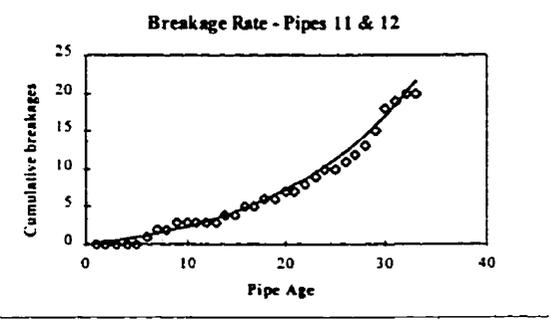
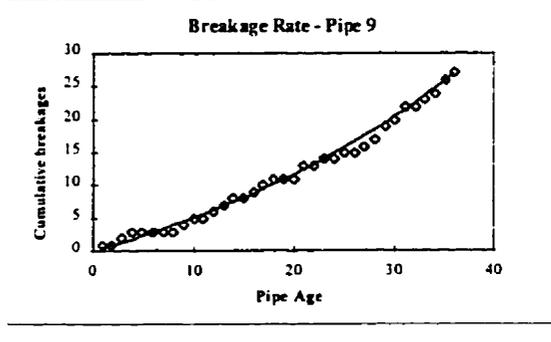
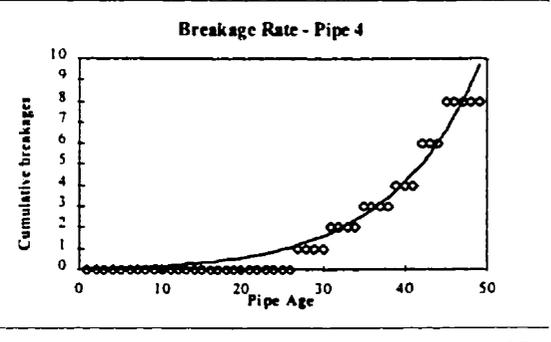
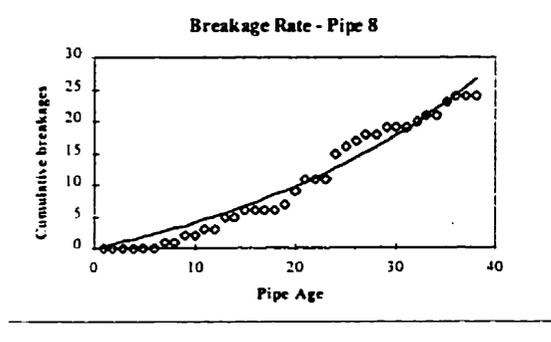
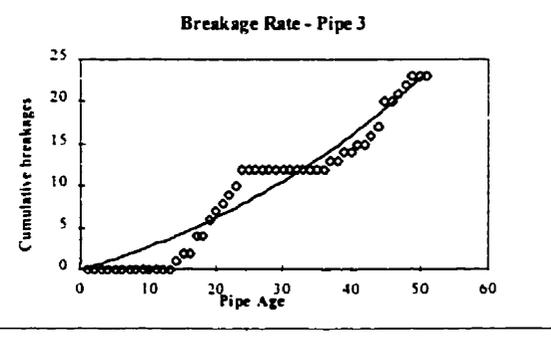
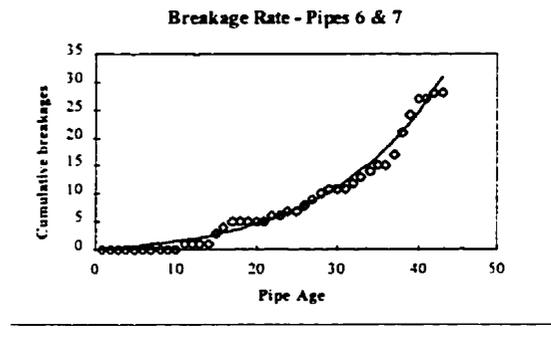
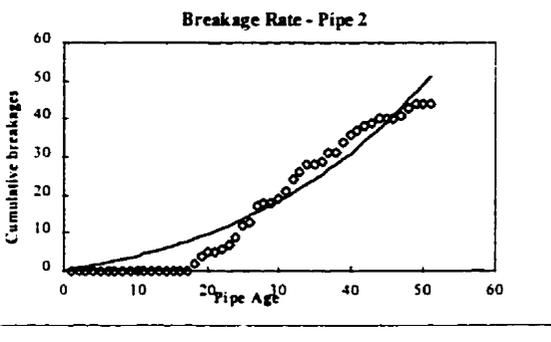
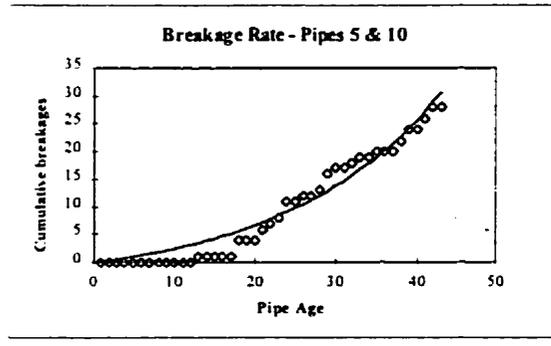
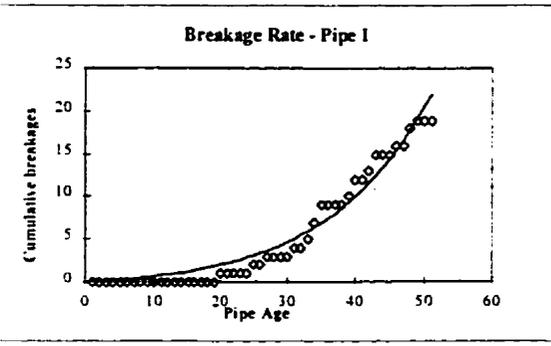
and the cumulative number of breaks from time of installation to time t becomes

$$N^c(t)_i = \int_0^t N(t_0)_i \cdot e^{A_i t} dt = \frac{N(t_0)_i}{A_i} (e^{A_i t} - 1) \quad (2ii)$$

A computer program was written for deriving the parameters $N(t_0)_i$ and A_i from the nonlinear equation (2) by the least square method. The algorithm used for the finding the parameters that result in a least square nonlinear curve is the Leveberg-Marquardt algorithm as implemented by Press et al. (1992).

After the parameters were derived, the resulting curve was plotted with the pertinent data points to allow visual inspection of the “goodness of fit” of the parameters derived for each pipe. As can be seen in the respective graphs on the next page, the exponential breakage model fits the data quite well in most case.

The $N(t_0)_i$ parameter for the replacement (PVC) pipes were assumed to be the same as the existing pipe, while the A_i parameters were assumed to be one quarter those of the respective existing pipes.



Breakage rate Parameters'

“goodness of fit”.

The parameters for the deterioration rate of the Hazen-Williams coefficient in the pipes were derived based on the equations:

$$C_i^{HW}(t) = 18.0 - 37.2 \log\left(\frac{e_{0_i} + a_i(t + g_i)}{D_i}\right) \quad (\text{equation 18a in section 3.6})$$

The C_i^{HW} was assumed equal to 130 for all existing pipes at time of installation ($t+g_i=0$).

Consequently, the parameters e_{0_i} can be directly calculated by:

$$e_{0_i} = D_i \cdot e^{\frac{130-18.0}{-37.2}} \quad (18a-i)$$

Since there is only one additional value for C_i^{HW} available for each pipe, the parameters a_i can also be directly calculated by:

$$a_i = \frac{D_i \cdot e^{\frac{C_i^{HW}(t)-18.0}{-37.2}} - e_{0_i}}{t} \quad (18a-ii)$$

The parameters e_{0_i} for the replacement (PVC) pipes were similarly calculated with an initial $C_i^{HW}=140$. The parameters a_i were taken as one quarter those of the respective existing pipes.

After all parameters were calculated, the data file for the MNRAP was compiled (see next page).

```

[TITLE]
12-pipe network for the Survey

[existing pipes]
.pipe from to diam length age e1 ai N1u A1 Cb1
1 1 2 10 600 51 0.00081 0.00187 0.0797 0.0683 2500
2 2 3 6 800 51 0.00049 0.00268 0.4139 0.0378 2000
3 1 4 8 400 51 0.00065 0.00024 0.6223 0.0222 2250
4 2 5 8 500 49 0.00065 0.00108 0.0198 0.0921 2250
5 3 6 6 700 43 0.00049 0.00375 0.1355 0.0523 2000
6 4 5 6 600 43 0.00049 0.00353 0.0686 0.0730 2000
7 5 6 6 900 43 0.00049 0.00399 0.0686 0.0730 2000
8 4 7 8 500 38 0.00065 0.00231 0.6824 0.0343 2250
9 5 8 6 800 36 0.00049 0.00287 0.5489 0.0277 2000
10 6 9 6 700 43 0.00049 0.00311 0.1355 0.0523 2000
11 7 8 6 300 33 0.00049 0.00209 0.1812 0.0711 2000
12 8 9 6 600 31 0.00049 0.00214 0.1812 0.0711 2000

[alternative 1]
[reline]
.pipe diam e1j ai1 N1u1 A1j C1j1 Cb1j T* T**
1 10 0.00081 0.00187 0.0200 0.0683 65000 2500
2 6 0.00049 0.00268 0.4139 0.0378 45000 2000
3 8 0.00065 0.00024 0.6223 0.0222 55000 2250
4 8 0.00065 0.00108 0.0198 0.0921 55000 2250
5 6 0.00049 0.00375 0.1355 0.0523 45000 2000
6 6 0.00049 0.00353 0.0686 0.0730 45000 2000
7 6 0.00049 0.00399 0.0686 0.0730 45000 2000
8 8 0.00065 0.00231 0.6824 0.0343 55000 2250
9 6 0.00049 0.00287 0.5489 0.0277 45000 2000
10 6 0.00049 0.00311 0.1355 0.0523 45000 2000
11 6 0.00049 0.00209 0.1812 0.0711 45000 2000
12 6 0.00049 0.00214 0.1812 0.0711 45000 2000

[alternative 2]
[default replace]
.pipe diam e1j ai1 N1u1 A1j C1j1 Cb1j T* T**
1 10 0.00044 0.00047 0.0200 0.0171 165000 2500
2 6 0.00026 0.00067 0.1035 0.0095 115000 2000
3 8 0.00035 0.00006 0.1556 0.0056 140000 2250
4 8 0.00035 0.00027 0.0050 0.0230 140000 2250
5 6 0.00026 0.00094 0.0339 0.0131 125000 2000
6 6 0.00026 0.00088 0.0172 0.0183 125000 2000
7 6 0.00026 0.00100 0.0172 0.0183 125000 2000
8 8 0.00035 0.00058 0.1706 0.0086 140000 2250
9 6 0.00026 0.00072 0.1372 0.0069 125000 2000
10 6 0.00026 0.00078 0.0339 0.0131 125000 2000
11 6 0.00026 0.00052 0.0453 0.0178 125000 2000
12 6 0.00026 0.00054 0.0453 0.0178 125000 2000

[alternative 3]
.pipe diam e1j ai1 N1u1 A1j C1j1 Cb1j T* T**
1 12 0.0003 0.00047 0.0200 0.0171 180000 2800
2 8 0.00016 0.00067 0.1035 0.0095 140000 2250
3 10 0.0002 0.00006 0.1556 0.0056 165000 2500
4 10 0.0002 0.00027 0.0050 0.0230 125000 2500
5 8 0.00016 0.00094 0.0339 0.0131 140000 2250
6 8 0.00016 0.00088 0.0172 0.0183 140000 2250
7 8 0.00016 0.00100 0.0172 0.0183 140000 2250
8 10 0.0002 0.00058 0.1706 0.0086 165000 2500
9 8 0.00016 0.00072 0.1372 0.0069 140000 2250
10 8 0.00016 0.00078 0.0339 0.0131 140000 2250
11 8 0.00016 0.00052 0.0453 0.0178 140000 2250
12 8 0.00016 0.00054 0.0453 0.0178 140000 2250

[parameters]
a NUMBER OF PIPES = 12
b NUMBER OF NODES = 9
c NUMBER OF REHABILITATION ALTERNATIVES = 3
d MINIMUM PRESSURE HEAD ALLOWED IN ANY NODE (m) = 35
e NUMBER OF NODES AT WHICH PRESSURE HEAD BELOW MINIMUM IS ALLOWED = 1
f NODE NUMBERS AT WHICH PRESSURE HEAD BELOW MINIMUM IS ALLOWED
f1 1
g ANALYSIS TIME HORIZON (years) 30
. LIST OF FILE NAMES {full path name} (braces are required)
h EPANET INPUT FILE {d compete.inp}
i EPANET OUTPUT FILE {compete.out}
j EPANET EXECUTABLE FILE {epanet_q.exe}
k OUTPUT FILE (continuous) {compout.txt}
l RESULTS FILE {comprsit.txt}
m INTERMEDIATE STAGE RESULTS FILES {comprstg}
. (the extension xxxxxx.001, xxxxxx.002 etc will be assigned by the program
. according to the stage number)
n START DP PROCESS AT STAGE 0
. (to start at stage n, the intermediate stage results file xxxxxx.00k (k=n-1)
. has to be present in the directory specified in line m above)
o END DP PROCESS AT STAGE 2
p DISCOUNT RATE = 06
q TIME STEP (YEARS) = 1
end]

```

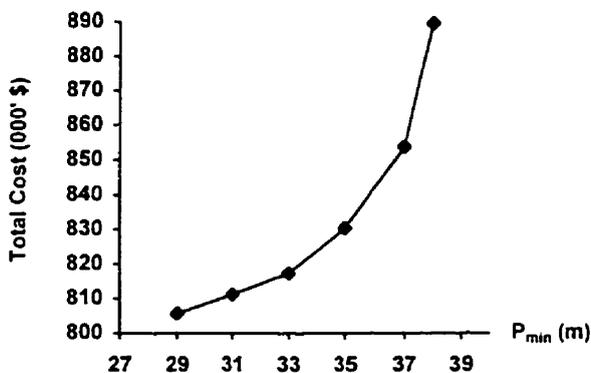
*Data file for the 12-pipe network
presented in the survey*

The ORP obtained by the MNRAP is as follows:

pipe	alternative	year
1	2 (replace same)	1
2	2	1
3	2	7
4	3 (replace larger)	1
5	2	21
6	2	12
7	2	12
8	2	1
9	2	24
10	2	18
11	2	3
12	3	1

The total cost of this ORP is \$830,521

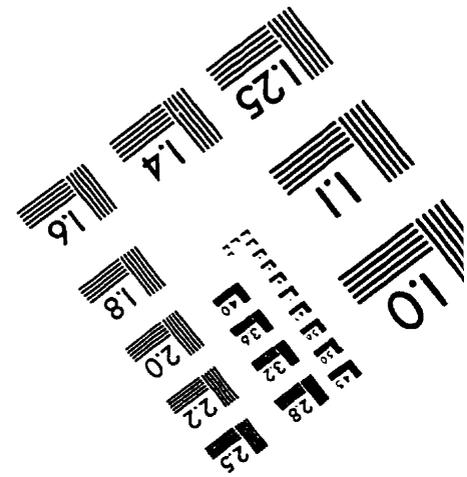
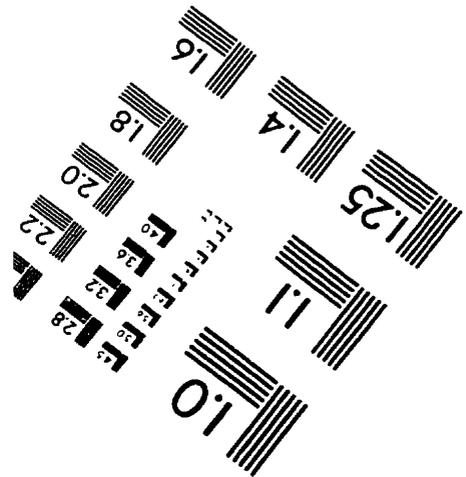
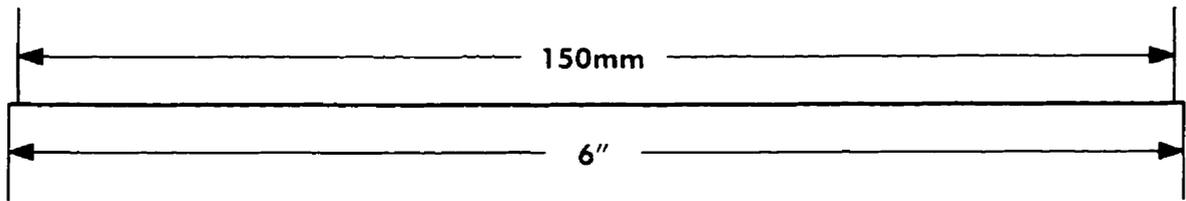
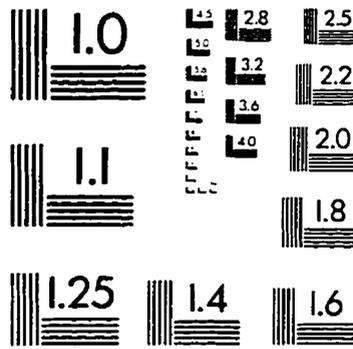
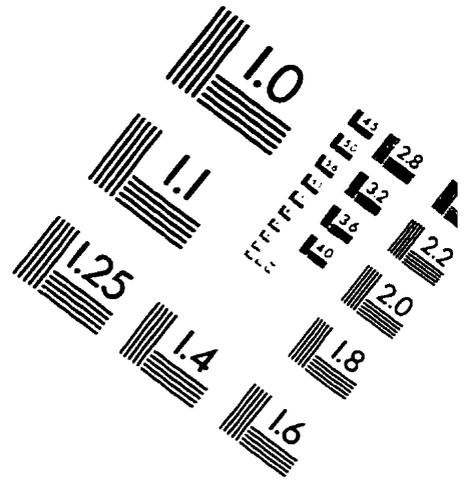
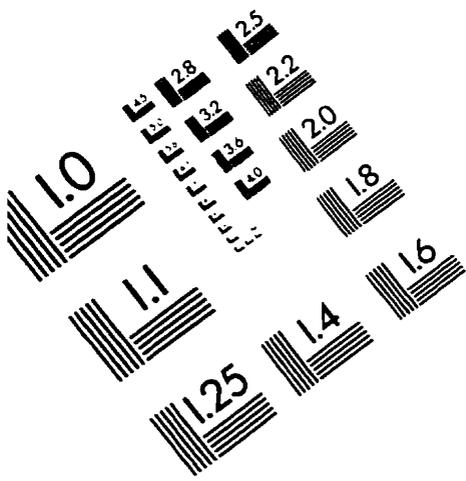
The MNRAP was subsequently applied to the network with various values for P_{min} (minimum pressure allowed). The tradeoff curve between minimum pressure and cost is:



The marginal cost of increasing P_{min} beyond 35m becomes quite high. In fact, the system cannot maintain $P_{min}=39m$ or higher for 30 years without an additional booster pump. A pump can also be considered in combination of a P_{min} that is lower than 35m.

An additional analysis revealed also that the cost of the ORP (for $P_{min}=35m$) will increase to \$834,782 (less than \$4,300) to make the system adequate for an analysis period of 40 years.

IMAGE EVALUATION TEST TARGET (QA-3)



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