

INVESTIGATING
MATHEMATICS ANXIETY
THROUGH THE MEDIUM OF A WORKSHOP

By

Sean Etches ©

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Abstract

This study probed mathematics anxiety among pre-service teachers through the medium of a workshop. In particular, this study attempted to answer the following question: What impact would a workshop have in altering the attitudes and conceptions of mathematics that would in turn reduce mathematics anxiety? The workshop was administered by the researcher to seven Education students at Lakehead University.

Data were collected by means of an anecdotal survey as well as concept maps, poems, and images of mathematicians created by the participants. Further data were gathered through transcription of tape recorded discussions. To augment these qualitative materials, an attitude survey was administered to the participants at the start of the first session, and at the end of the last session.

The results of the study indicate that in order to reduce anxiety, one's learning of mathematics needs to relate to one's everyday life. Further stress would be reduced if the ambiguity of word problems could be minimized, and if there was a balance between independent and cooperative learning tasks in the mathematics classroom. The perceived invasive nature of mathematics and a love/hate relationship with the discipline were two ideas, emerging from this study, which are not visible in the literature on mathematics anxiety. This study reaffirmed the importance of a workshop in reducing feelings of isolation experienced by sufferers of this type of anxiety.

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Chapter 1

Introduction

Description of Research Study

Mathematics anxiety is a phenomenon that has been extensively researched and has been shown by researchers such as Tobias (1980) to be potentially debilitating. The purpose of this study was to investigate this phenomenon through the medium of a workshop. In particular, this study attempted to answer the following question: What impact would a workshop have in altering the attitudes and conceptions of mathematics that would in turn reduce mathematics anxiety? Some of the existing literature on this subject suggests that carefully designed workshops might be of some benefit to sufferers of this type of anxiety. Donady and Auslander (1980) and Hadfield (1992) have pointed out that the implementation of well-structured workshops¹ has, in the past, met with success in the alleviation or reduction of math anxiety.

The workshop was made available to students in the Bachelor of Education program at Lakehead University in the second half of January, 1997. The workshop was divided into two sections to accommodate for scheduling conflicts, and each section ran for a total of seven and a half hours. It involved discussion of the thoughts, feelings, and experiences associated with mathematics anxiety as well as the various ideas related to mathematics. The

¹ For the purpose of this study, a well-structured workshop is one that consists of a series of components similar to those suggested by Donady and Auslander (1980), which may include relaxation techniques, role playing activities, and sentence completion exercises.

ideas demonstrated perceptions of the discipline that are somewhat different from the perceptions as shared by mathematicians. This was a concern addressed by researchers such as Frankenstein (1984). Also, the workshop included hands-on activities in which the participants would use manipulatives as a way of discovering mathematical relationships and building self-confidence, such as those suggested by Sobel and Maletsky (1975). The workshop concluded with each of the participants filling out an anecdotal evaluation form that allowed them to comment on the strengths and weaknesses of the workshop as well as suggest ways in which the workshop might be modified in order to be more effective.

Also, the extent, if any, to which the participants' sense of the discipline of mathematics is changed over the course of the workshop was investigated. This has been shown to be of significance in relation to mathematics anxiety by Frankenstein (1984). This was accomplished by administering portions of Quilter and Harper's (1988) survey on attitudes pertaining to mathematics, both at the start of the first session and the end of the last session, then analyzing the results. The use of the Quilter and Harper survey, as well as the anecdotal evaluation form and field notes taken by the researcher while listening to the tape recorded sessions of the workshop, was indicative of the researcher's intention to use both quantitative and qualitative research methods in this study. However, due to the small sample size ($N=7$), a descriptive, as opposed to inferential, analysis of the survey was done. The results of this study would thus be of interest to educational researchers who wish to probe reasons for students' success in mathematics by illuminating the causes of mathematics anxiety and possible strategies for reducing the anxiety.

Rationale

Tobias (1980), Williams (1988) and Folk (1995) consider that mathematics anxiety is a serious problem, particularly because mathematics has practical applications in several fields of study, including the physical and biological science, the social sciences, computer science, finance, and others. Many occupations require a solid grounding in the fundamental concepts of mathematics. The Ontario Ministry of Education's 1985 Curriculum Guidelines for mathematics in the Intermediate Senior Division refers to a link between mathematics anxiety and mathematics avoidance. This may lead to serious consequences. If there are individuals who suffer from such a high level of math anxiety that they do not pursue a study of mathematics beyond the minimum required number of courses to graduate from secondary school, then they could end up missing out on several career opportunities. Math-anxious teachers who are faced with having to teach mathematics may unconsciously transmit their anxiety to their students. Folk (1995) pointed out that math-anxious teachers allow their anxiety to get the better of them and cause them to teach the subject in an inflexible manner, in the sense that such teachers are non-reflective practitioners who do not recognize the cumulative nature of mathematics. Therefore, it would seem that if math-anxious Bachelor of Education student teachers have their math anxiety examined, and at least partially alleviated, before they begin their teaching careers, then that would ultimately be of benefit to their future students. An exponential effect would be created whereby each graduate of the Bachelor of Education program, leaving the program with confidence in teaching mathematics, would transmit that confidence to hundreds of students throughout his or her career. In turn, some of these students would become mathematics students who would

transmit their confidence to hundreds more students, and so forth. Thus, a well-structured workshop could be of great societal value, if it works. It is the focus of this study to determine the value of such a workshop.

Researchers have pointed to a societal need for numeracy. According to Joram, Resnick and Gabrielle (1995), "Numbers are pervasive in our daily lives. They are used to acquaint us with facts and to persuade us of a writer's point of view." (p.346). Joram et al went on to argue that, "The ability to understand and interpret arguments that hinge on numbers is essential for evaluating many of the claims that people encounter daily in advertisements, forecasts, and public policies and thus for becoming intelligent consumers who can make informed decisions." (p.346). Paulos (1988) examined innumeracy in a number of contexts, including a possible link between innumeracy and a gullibility regarding stock market scams and fraudulent medical treatments. One is less likely to be victimized by such illegal and unethical activities if one has a basic understanding of the laws of probability. Paulos linked innumeracy with mathematics anxiety and pointed out that feelings of terror and intimidation when confronted with mathematics, accompanied by the notion "that there are mathematical minds and non-mathematical and that the former always come up with answers instantaneously whereas the latter are helpless and hopeless" (p.88), all have the potential of leading to a major stumbling block for numeracy. Such comments suggest an urgent need for educators to address the problem of mathematics anxiety. One must realize, however, that the meaning of numeracy and related terms like quantitative literacy change over time and from place to place. Joram et al, quoting Cockroft (1982), defined numeracy as "those mathematical skills that enable an individual to cope with the practical demands of everyday

life.” (p.347). Galbraith, Carss, Grice, Endean and Warry (1992) quoted Australia’s national definition for quantitative literacy - “the ability to use mathematical operations, such as addition and multiplication, to solve problems included in written material” (p.581) - but suggest that advancement in calculators and computer software could change what mathematical skills are regarded as important.

Mathematics anxiety has been tied to difficulties that people experience when doing mathematics. Levine (1995) stated, “Mathematics anxiety is often perceived as debilitating and is linked with impaired performance, both in traditional academic settings (e.g. school examinations) and in daily life, where it can interfere with successful completion of a task or achievement of a goal.” (p.42). She indicated that males consistently score higher on SAT mathematics scores and suggested that mathematics anxiety for females might be an underlying cause for this. It is the researcher’s position that the presence of too many math-anxious teachers could ultimately impair the mathematical performance of society as a whole.

Definition of Terms

The following terms were central to the workshop and for the purpose of this study:

- (1) *Mathematics Anxiety*: “(a) fear of mathematics or an intense, negative emotional reaction to the subject” (Kennedy and Tipps, 1994, p.13)
- (2) *Innumeracy*: “the inability to deal comfortably with the fundamental notions of number and chance” (Paulos, 1988, p.3)

- (3) *Mathematician*: “someone who conducts research in, or educates others in, mathematics” (Researcher’s definition derived from Webster’s Third New International Dictionary, 1971, p.1303)
- (4) *Polyomino*: “a set of squares connected along their edges” (Sobel and Maletsky, 1975, p.72).

Research Problem and Questions

Through the process of investigating mathematics anxiety using the medium of a workshop, it is important to ascertain whether the workshop is of any value to those who choose to participate in it. Thus the researcher has chosen to examine the problem.

Research Problem:

What impact would a workshop have in altering attitudes and conceptions of mathematics that would in turn reduce mathematics anxiety?

Research Questions:

- (1) *What are the overwhelming memories that triggered the mathematics anxiety in the participants?*

Probing such memories would help the workshop participants isolate the sources of their mathematics anxiety and allow them to reflect on possible means of coming to terms with these sources and, in turn, diminishing the effect these sources have on their confidence of their own mathematical ability.

(2) *What prior teaching styles have contributed to mathematics anxiety?*²

Researchers such as Haines (1982) have traced mathematics anxiety to the manner in which mathematics is taught.

(3) *How might the views of the participants regarding their own abilities evolve over the course of the workshop?*

Since researchers like Frankenstein (1984) have linked mathematics anxiety to views on mathematics, determining how the views of the participants evolved would help to determine the overall effectiveness of the workshop.

Overview

In Chapter 1, the researcher briefly describes the mathematics anxiety workshop and provides a rationale for the study by addressing the problems of mathematics avoidance and innumeracy. Also, the central terms are defined and the research problem and questions are stated. In Chapter 2, the literature is reviewed to address the problem of mathematics anxiety, ways to alleviate mathematics anxiety, views on mathematics, the mathematics anxiety workshop, and the various data collection instruments used in this study. The third chapter deals with the design of the workshop, the activities conducted during the workshop, and the data collection process. The fourth chapter presents information gathered from the various instruments and discusses the emerging themes. In the final chapter, the research

² Examples of teaching styles would include straight lecture, interactive lecture, and seminar.

problem and questions are revisited, and recommendations for the alleviation of mathematics anxiety and for further research are suggested.

Chapter 2

Literature Review

The Problem of Mathematics Anxiety

There are a number of definitions of mathematics anxiety to be found in the literature. To give two examples, Kennedy and Tipps (1994) defined mathematics anxiety as “a fear of mathematics or an intense, negative emotional reaction to the subject” (p.13), and Kvarnes (1980) defined the phenomenon as “a basic situation in which a person experiences a loss of self-esteem if and when he confronts a situation involving math.” (p.3). Different researchers use different words and phrases when creating a definition, but all the various definitions are essentially saying the same thing - namely, that mathematics anxiety is a condition in which people become scared when confronted with a task that is mathematical in nature.

An underlying question is: “Why do some people suffer from mathematics anxiety?” In other words, what are the origins of mathematics anxiety? Following the work of Kennedy and Tipps (1994), one could hypothesize that the techniques by which teachers teach mathematics is a significant contributing factor. Kennedy and Tipps have identified several problems in connection with teaching that could lead to the development of mathematics anxiety in learners. Such problems include a lack of variety in the teaching and learning process, emphasis on memorization, speed, and doing one’s own work, and authoritarian teaching practices. Perceptions of what mathematics is and what people who teach mathematics or study mathematics in depth are like play another role. Donady and Auslander (1980) noted that “many math-anxious people see mathematics as strict, unemotional and

specific at all stages.” (p.13). Through their implementation of a mathematics anxiety workshop, Donady and Auslander (1980) also learned that many sufferers of mathematics anxiety have their own image of a mathematician, perceiving a mathematician to be “a cold, rigid, rational, uncreative, insensitive man who works only with facts and figures” (p.13), thus regarding mathematics as something separate from the rest of humanity.

In contemplation of the reasons behind the occurrence of mathematics anxiety, one would wonder, as suggested by Levine (1995), whether or not there was a connection between this phenomenon and gender. Tobias (1980), who has done extensive research into this type of anxiety, has stated her belief in a gender link in several of her published works, including her 1980 book Overcoming Math Anxiety. Tobias has perceived mathematics anxiety to be an issue predominantly associated with females and has stated that “feminists trace math anxiety to political and social forces that oppress women.” (p.92). Tobias further stated that, in American society, “Men are not free to avoid math, women are.” (p.70).

Tobias’ (1980) assertions regarding the gender link are controversial. Haines (1982), in his doctoral dissertation Brief Counselling Interventions in College, pointed out that “women have been considered to constitute a disproportionate share of this mathematically disabled group.” (p.1). This is consistent with Tobias’ (1980) assertion that math anxiety is predominantly a female phenomenon.

Tracy and Davis (1989) expressed the idea that math anxiety in pre-service elementary teachers may be partially due to a lack of available information on female mathematicians and alluded to the work of Sadker and Sadker (1985) on sexism in classrooms in the 1980s. Tracy and Davis suggested a number of tasks that could be given to pre-service and in-service

teachers to familiarize them with a number of female mathematicians. One such task would be to interview a female mathematics major and prepare a written report based on that interview. Another would be to construct a display, mixing pictures with words, that highlights a female mathematician. Yet another task would be to access a book featuring the lives of women mathematicians, and from that book paraphrase one of the biographical sketches. Tracy and Davis had their students in the pre-service program engage in these tasks. After completing the tasks, these student teachers expressed their belief that they would be able to reduce mathematics anxiety and sexism as related to mathematical learning in their future students. Fraser (1995), in the context of a review of the literature on math anxiety, stated his belief that there is a higher incidence of math anxiety among females than among males. However, Aksu and Saygi (1988), writing in relation to Turkish society, stated that "with the changing role of women in society, math anxiety is slowly moving in the direction of becoming an equal opportunity disability" (p.391), suggesting that career changes among women have forced them to resolve their math anxiety and thus, women no longer constitute a disproportionate share of the math anxious group with both men and women suffering equally from this problem. Hembree (1990) stated that females report higher levels of math anxiety but, "this does not translate into depressed performances for females" (p.45) as it does for males. Hembree's study was the result of combining 151 studies into a meta-analysis for the purpose of integrating "the findings of the research on mathematics anxiety regarding its nature, effects, and relief." (p.35). The studies for Hembree's meta-analysis were derived from three data bases - dissertation abstracts, psychological abstracts, the Educational Resources Information Centre (ERIC), and from "tracking citations from study

to study.” (p.35). Flessati and Jamieson (1991) noted that, although females tend to be more self-critical about their math anxiety, “males may under-report their levels of math anxiety because of cultural pressures against males having math anxiety” (p.310), and further stated that “math anxiety may be a weakness acceptable for females but which may be less acceptable for males.” (p.310). A researcher would therefore be careful about jumping to conclusions about the possible gender link until further research has been done.

Ways to Alleviate Mathematics Anxiety

How could a teacher help prevent or alleviate math anxiety in his or her students? Clearly, the phenomenon of math anxiety is so extraordinarily complex that there are no easy answers to this question. However, many researchers have offered a number of suggestions. Kennedy and Tipps (1994) cited several ways to alleviate math anxiety which include the following:

- (1) Provide a relaxed atmosphere for learning.
- (2) Give credit to process and thinking as well as the answer.
- (3) Be sensitive to children’s feelings.
- (4) Spend time explaining concepts thoroughly.
- (5) Look at all aspects of mathematics.
- (6) Do not use mathematics as punishment.
- (7) Use various concrete materials.
- (8) Work in a variety of settings. (p.17).

Implementing these suggestions would help the learner to understand how mathematics can be applied to everyday events. They can also help demonstrate to learners that mathematics is a way of thinking and not just right or wrong answers. The Ontario Ministry of Education's Curriculum Guideline for mathematics in the Intermediate Senior Division acknowledges the existence of mathematics anxiety and suggests ways of alleviating it, including the following:

The provision of many opportunities for students to experience success and positive reinforcement; patience, receptivity and understanding; the avoidance of unnecessary tension and pressure in the classroom; a matching of the reading level of resource to that of the students; the use of a variety of assessment techniques (e.g. frequent short tests, take-home tests, projects); provisions for re-testing if necessary; the acceptance of alternative solutions; the use of diagnostic interviews to probe beyond a superficial identification of learning problems associated with mathematics; the relating of mathematical concepts and generalization to application familiar to the learner. (p.13).

These statements might help teachers in increasing their awareness of the problem of mathematics anxiety.

Fairbanks (1992) investigated the use of an optional contract as a means of treating math anxiety. The contract which Fairbanks shared with 111 of his college students (a group consisting of 74 females and 37 males, of which 51 females and 12 males signed and returned the contract) was designed to assure students of a passing mark in the mathematics course provided they met certain non-evaluative requirements such as regular class attendance, bringing textbooks, notebooks and calculators to every class, and correcting the errors made on examinations. Although some of his colleagues were concerned that some students who did not know the course material would pass the course, Fairbanks discovered that those students who signed the contract scored better on final examinations than those who did not.

Commenting on this, Fairbanks stated that he “sensed that the contract relieved anxiety” and further noted that “several students stated that knowing they would pass the course relaxed them.” (p.430).

A teacher might wish to determine if opportunities to engage in collaborative problem solving would help students to cope with their anxiety. Matz and Leier (1992) suggested that a series of word problems could be presented in the form of a script that could be role-played by a group of students. During such activity, a narrator would shout “freeze” whenever a computation would need to be performed. The audience in turn would perform the computation. After practising with pre-existing scripts, students could work in groups to design their own scripts. Matz and Leier pointed out that, in such situations, students learn how to work cooperatively toward a common goal, delegate responsibilities, and develop their communication skills. Matz and Leier alluded to the non-competitive nature of such activities and stated that “Regardless of ability, every student can be a successful participant and an essential contributor.” (p.17). If a math-anxious learner feels that he or she has an integral part of the group process, might that help to lessen the anxiety?

Views on Mathematics

Frankenstein (1984) identified a number of “misconceptions” that some math-anxious people have regarding the discipline. Examples of such misconceptions include:

- (1) People who are good in math do it fast, in their heads, and in one sitting.
- (2) It is unacceptable to make mistakes or ask questions in class.

- (3) The teacher knows the one correct answer and the one correct way of finding the answer. (p.172-175).

If math-anxious students cannot generate correct answers quickly, in their heads, and in one sitting, then that in turn increases their anxiety. Frankenstein (1984) suggested techniques to assist students in breaking down those misconceptions, such as small group discussions, analyzing student mistakes, showing a number of methods to solving problems, and carefully explaining the nature of mathematical symbolism. Haines (1982) attempted to rectify certain misconceptions and irrational ideas that sufferers of mathematics anxiety have as part of his cognitive intervention scheme for his doctoral dissertation study. Two particular mathematics classes that Haines taught were given a treatment consisting of 14 experimental handouts dealing with such topics as student and instructor responsibilities, the "Math Anxiety Bill of Rights", math myths, and games to play both alone and with others, while two mathematics classes which did not receive this treatment served as the control groups. Both experimental and control groups were taught in a similar lecture manner, but the experimental groups were exposed to three-to-five-minute presentations that related to each of these handouts. However, through an analysis of variance, Haines reported that he found "no significant main-effect difference between the experimental and control groups" (p.149), which led him to suggest that further research was needed in this area. Commenting on this conclusion, Haines stated that "whether this is a function of inappropriate treatment, population characteristics (e.g. age, intact classes), and/or other factors, remain speculation and an issue for further study" (p.149), which led this researcher to suggest that this study has the potential of making an important contribution to the literature of mathematics anxiety.

The significance of this study is to increase the available knowledge regarding the effectiveness of a workshop in the reduction of mathematics anxiety.

The Math Anxiety Workshop

How useful would a math anxiety workshop be in the reduction of this anxiety? What would such a workshop involve? Donady and Auslander (1980) described a number of techniques in which instructors of typical math anxiety workshops would use a resource manual that was part of a 1980 project on math anxiety, directed by Tobias. These techniques would include sentence completion exercises, role playing activities, relaxation techniques to control breathing, keeping a math diary in which participants would note successful and unsuccessful experiences in mathematics, and psychological homework involving the use of all of the above techniques. Donady and Auslander felt that such a workshop could be of considerable value, and previous successes in the implementation of workshops led them to discover that “members no longer want to hide their fear, and as they build self-confidence and self-esteem they no longer have to.” (p.8).

Recent Research

Recent research into mathematics anxiety has offered some illuminating insights. Levine (1995) supported the notion of gender differences in attitudes toward mathematics and ways of approaching mathematical tasks, which has implications for the study of mathematics anxiety. Her research has suggested that girls tend to have less confidence in their mathematical ability, which could adversely affect performance. She also suggested that,

compared to boys, “Math-anxious girls take fewer chances, relying instead on the logic and the lessons of the classroom. This reliance on algorithmic solutions is at odds with the expectations of timed examinations like the SATs, which reward creative invention and shortcuts with higher scores.” (p.45).

Bandura (1968) defined self-efficacy beliefs as “people’s judgments of their capabilities to organize and execute courses of action required to attain designated types of performances.” (p.391). Pajares and Miller (1995) dealt with the contention that “self-efficacy beliefs strongly influence the choices people make, the effort they expend, the strength of their perseverance in the face of adversity, and the degree of anxiety they experience.” (p.190). Pajares and Miller’s study centred around 391 undergraduate university students (247 women and 144 men) from three large public universities in the southern United States who were asked to perform three types of mathematics self-efficacy judgments - “confidence to solve mathematical problems, confidence to succeed in math-related courses, and confidence to succeed in math-related tasks” (p.190), using a descriptive survey, the Mathematics Self-Efficacy Scale-Revised (Betz & Hackett, 1983), and a problem solving test. It was discovered that “students’ reported confidence to solve the problems they were later asked to solve was a more powerful indicator of that performance than was either their confidence to perform math-related tasks or to succeed in math-related courses” (p.190), and confidence to succeed in math-related courses more strongly predicted choice of math-related majors than the other two types of self-efficacy. The researcher believes that the multi-dimensional nature of mathematical self-efficacy might suggest a multi-dimensional nature for mathematics anxiety.

This multi-dimensional nature became apparent in Bessant's (1995) study in which a factor analysis of the results of the administration of the Mathematics Anxiety Rating Scale, devised by Richardson and Suinn (1972), to 173 university students led to the identification of six factors. These are as follows:

- (1) *General Evaluation Anxiety*: "apprehension over completion of mathematics-related tasks"
 - (2) *Everyday Numerical Anxiety*: "mathematics anxiety induced while using basic arithmetic to carry out budgetary or financial affairs"
 - (3) *Passive Observation Anxiety*: anxiety associated with such activities as "watching someone work with a calculator, sitting in a math class and waiting for the professor to begin, and walking into a math class"
 - (4) *Performance Anxiety*: anxiety related to "being observed by others while carrying out basic arithmetic tasks"
 - (5) *Mathematics Test Anxiety*: anxiety related to "thinking about an upcoming math test"
 - (6) *Problem-Solving Anxiety*: anxiety related to solving mathematical problems.
- (p.6-7).

These factors emphasize the multi-dimensional nature of mathematics anxiety. Bessant also related mathematics anxiety to approaches to learning, and concluded that intrinsically motivated learners who engage in syllabus-free inquiry tend to suffer less anxiety. On the other hand, learners who rely on rote memorization when studying mathematics tend to suffer from comparatively high levels of general evaluation mathematics anxiety.

Instruments

A number of instruments were used in this study. These need to be described in some detail.

Quilter and Harper Survey

Quilter and Harper (1988) noted the problem of mathematics anxiety and its adverse effect on mathematics achievement but remarked on a lack of clarity as to how mathematics anxiety and other variables contribute to an inability to do mathematics. Quilter and Harper designed a study “to identify the reasons offered by adults (who succeed to some degree in their own specialist subjects) for their difficulties, anxieties, fears and inability to cope with mathematics at more than a rudimentary level.” (p.122). The researchers devised a survey that would be administered to “a professional group, the name of which it withheld...to preserve confidentiality” (p. 122), from which a target group was selected who were judged to have a “negative attitude” regarding mathematics on the basis of an assessment of the surveys made by the researchers and a mathematician. From that target group, a random sample was selected for an interview. There were 147 respondents to the survey and 15 of those respondents were selected for an interview.

The original draft of the survey contained 57 statements “derived from two sources: verbatim reports in Buxton’s Do You Panic about Mathematics (1981), and from tape recordings of an extended free group discussion on mathematics education attended by eight self-confessed poor mathematicians following the Master of Education degree course at the University of Bath.” (p.123). Two pilot studies of this survey were conducted on students

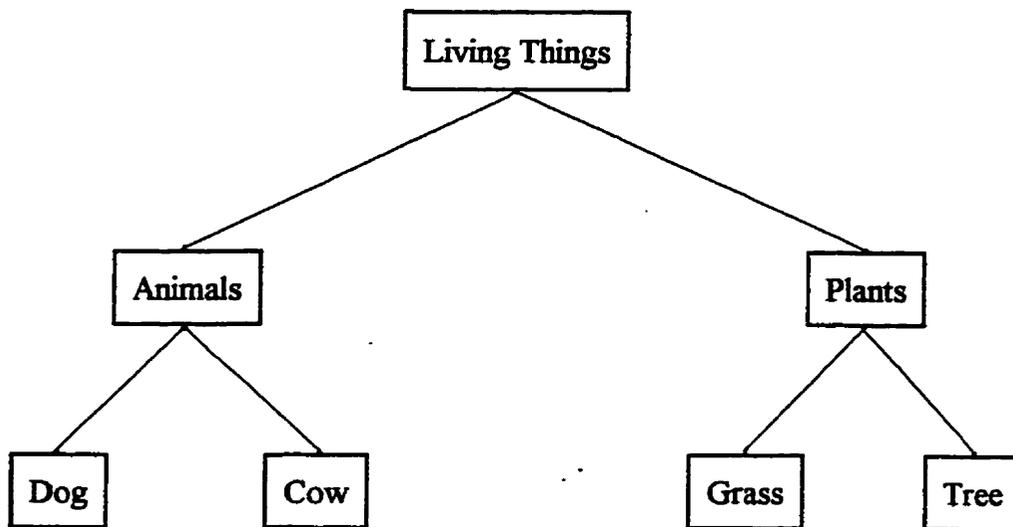
at the University of Bath and tests of internal reliability were completed after each study, thus reducing the original 57 statements to 24.

Quilter and Harper (1988) concluded from the interviews conducted during their study that mathematics anxiety arises from a dislike of teachers who are perceived in the following manner: "They are "arrogant" and "assume knowledge of others" or they "belittle inability" and are "dogmatic" and "lack flair - both social and professional". (p.125). The interviewees associated dislike of teachers with "other variables, in particular, teaching methodology, the lack of relevance in mathematics, and the perceived rigidity of the subject." (p.125). Further insight about the learning of mathematics was gathered through these interviews. Some interviewees suggested "the existence of a dyscalculic syndrome akin to dyslexia." (p.124). Such a syndrome, if it does exist, should be investigated further so that techniques could be developed to help learners cope with this problem. The interviewees also pointed to conceptual difficulties when faced with algebra for the first time, their perception that the language of mathematics is alien, and their "need for open-ended or multi-factored conclusions" (p. 127), which they do not find in mathematics.

Concept Maps

Concept mapping can be a meaningful part of the learning process. Novak and Gowin (1989) stated that "Concept maps are intended to represent meaningful relationships in the form of propositions. Propositions are two or more concept labels linked by words in a semantic unit." (p.15). Novak and Gowin commented that such maps are visual road maps showing pathways taken by learners to connect meanings of concepts. White and Gunstone

(1992) remarked that concept maps show structure and linkages in a manner that is quicker, more direct and less verbal than essays. Novak and Gowin also pointed to the valuable social functions of concept maps, if they are constructed in small groups, by triggering classroom discussions and allowing for meanings to be “shared, discussed, negotiated and agreed upon.” (p.20). An example of a concept map taken from White and Gunstone is as follows (p.18):



Novak (1995) commented on the conditions of meaningful learning to suggest that “The material to be learned must be conceptually clear and presented with language and examples relatable to the learner’s prior knowledge.” (p.230). Novak further stated that “Concept maps can be helpful to meet this condition, both by identifying large general concepts prior to instruction in more specific concepts, and by assisting in the sequencing of learning tasks though (sic) progressively more explicit knowledge that can be anchored into developing conceptual frameworks.” (p.230). Moving beyond the scope of learning, Novak has also

determined the value of concept maps as a tool for evaluation. Valid and invalid ideas can be identified by an instructor analyzing a learner's map. The effectiveness of such maps as a tool for evaluation can in turn, as Novak pointed out, encourage students to engage in meaningful learning (as opposed to rote learning). Through the construction of these concept maps the complexity of mathematics anxiety is made clearer.

Poems

The researcher was inspired to use mathematical poetry as a means of investigating mathematics anxiety during the workshop, by Bartley, the researcher's thesis supervisor, who acquired information via the Internet from Panitz (1997), an instructor at Cape Cod Community College. Panitz has given students in his mathematics classes an assignment in which they were asked to reflect on their experiences, feelings and problems with mathematics and translate these reflections into any form of poetry they choose. Panitz was impressed with the effort and sincerity that most of his students put into the poems, an example of which is given here:

Math Doesn't Come Easy to Me

Math doesn't come easy to me
It's not like my A, B, C's
 $1 + 2 + 3$ equals 6
But what does $2A + 6AB$ mean?
In life we use adding, subtracting,
Multiplying and dividing
To answer everyday problems.
But I've never come across a day,
When algebra and geometry meant anything.
The two seem to have no meaning in life.
So why do professors waste their time teaching?
But I will accept, and try not to fret
Because math doesn't come easy to me!

This poem eloquently captures the confusion regarding mathematics and detachment from forms of mathematics other than arithmetic, as experienced by a student.

Recreational Mathematics

One source for examining recreational mathematics is that of Azzolino, Silvey and Hughes (1986) who attempted to demonstrate a possible integration of mathematics and humour. This was accomplished through a compilation of cartoons, anecdotes, jokes, and humorous poetry. If use of such material makes a student laugh, the laughter might help relieve the feelings of tension.

Sobel and Maletsky (1975) compiled a number of activities that have potential recreational value. Among such activities are ones involving magic squares which, according to Sobel and Maletsky, “date back to ancient times when people believed they held strange mystic powers because of their special properties.” (p.89). Others include the construction of polyominoes (see Definition of Terms) and a card game (see Appendix C). A teacher or workshop instructor could experiment with a number of such activities, determine which are enjoyable and suitable for a particular group of learners and, through observation and questioning, determine if the learners feel relaxed when engaging in those tasks.

The Nature of Mathematics and Mathematics Education **(Moritz & Shaw)**

How might math-anxious learners respond to ideas about what mathematics is and how it is taught? Such a question might be answered by having learners respond to certain

quotes. Moritz (1942) compiled hundreds of such quotes into a single volume. Moritz expressed his hope that “the present volume will prove indispensable to every teacher of mathematics, to every writer of mathematics, and that the student of mathematics and the related sciences will find its perusal not only a source of pleasure but of encouragement and inspiration as well.” (p.2). Moritz later described the development of a better appreciation of mathematics as his central aim. Despite the fact that this source predates this study by 55 years, the ideas contained within it have the potential of sparking discussion by comparing them to more recent ideas.

An interesting experiment might be to test a math-anxious learner’s response to the idea of mathematics as an art. This is an idea that has found support from a number of mathematicians, including Shaw (1966). Shaw viewed mathematics as “on the artistic side, a creation of new rhythms, orders, designs and harmonies, and on the knowledge side, a systematic study of the various rhythms, orders, designs and harmonies.” (p.259). Consolidating these ideas into a definition of mathematics, Shaw explained that “mathematics is, on one side, the qualitative study of the structure of beauty, and on the other side is the creator of new artistic forms of beauty.” (p.259). Part of Shaw’s love for the discipline stemmed from his belief that mathematics is the one subject which “has steadily added to its riches and never has thrown away anything.” (p.239). Perhaps an examination of material by Shaw, or someone with a similar perspective, might encourage learners to reflect on their thoughts as to what mathematics is and possibly broaden their interest in the subject. This heightened interest might make them feel less anxious when faced with mathematical tasks.

Chapter 3

Methodology and Design

Design

This workshop was aimed at students who were in a Bachelor of Education program. Enrollment consisted of a self-identified sample of seven participants who responded to either a written advertisement of the workshop or a brief oral description of the workshop given by the researcher to three Primary/Junior Environmental Studies classes. The workshop was held in a classroom. The workshop was divided into two sections to accommodate for differences in schedules among the participants. The workshops were held between January 21 and January 31, 1997, and the total time of each workshop was seven and a half hours (five ninety-minute sessions for Workshop A and two ninety-minute sessions plus one two-hour session and one two-and-a-half-hour session for Workshop B).

This study combines the ethnographic techniques of observation and transcription with a pre-test and post-test administration of the Quilter and Harper (1988) objective style survey. The survey consisted of 24 items, in which the participants were asked to indicate strong agreement, agreement, strong disagreement, disagreement, or undecidedness about each item. There was also a brief section for comments. A copy of the survey (Figure 1) is to be found on the next page.

Methodology

The first session of the workshop began with the administration of sections B, C and D of Quilter and Harper's (1988) attitude survey. See this page and the following page. No time limit was given for its completion.

Figure 1: Quilter and Harper (1988) Survey

SURVEY QUESTIONS	STRONGLY AGREE	AGREE	UNDECIDED	DISAGREE	STRONGLY DISAGREE
1. Mathematics is creative.					
2. I find math threatening.					
3. Math is bewildering.					
4. I think guesswork has a part to play in solving mathematical problems.					
5. Math would be alright if it was expressed in words instead of symbols.					
6. You can get a math question half right.					
7. Math problems can make your brain seize up.					
8. Mathematics teachers appear to be high priests of some domain of secret knowledge.					
9. The trouble with math is that it is too abstract.					
10. Minus numbers are daft.					
11. I don't panic about math.					
12. The rightness or wrongness of my mathematics work is often obvious to me.					
13. I become embarrassed if I'm stuck on a math problem.					
14. Much of mathematics is an affront to common sense.					
15. Juggling around in math lessons with symbols and numbers have no meaning.					
16. I find mathematics symbols (e.g. π) can be frightening.					
17. I find it shameful if I can't do a piece of mathematics.					
18. 'x', the unknown, is full of mystery.					
19. People who can do math are clever.					

20. Math is not an experimental activity.					
21. I don't find math frightening.					
22. Math is irritating.					
23. High anxiety prevents me from learning math.					
24. As much can be learned from a wrong answer in mathematics as from a right one.					

Do you feel there were any particular significant factors that led you to your particular attitudes?

Please add any other comments you may wish to make.

The researcher gave the participants a copy of the survey which they were asked to complete. Then the researcher gave a detailed explanation of the workshop's purpose and format. He informed the participants that he is a Master of Education student who is in the process of writing a thesis on mathematics anxiety, and the data collected from this workshop would be used in this thesis. The researcher assured the participants that their names would not be used in the thesis and any references to what they said during the workshop would be made pseudonymously. He also assured the participants that they did not have to say anything about themselves or their experiences that they were uncomfortable talking about.

The researcher then instructed the participants to work in groups and construct a concept web for mathematics anxiety. He then gave a presentation using chart paper that involved defining mathematics anxiety, tracing the possible origins of mathematics anxiety, explaining possible techniques for reducing mathematics anxiety, and the controversy surrounding the possible link between mathematics anxiety and gender. The researcher welcomed questions from the participants both during and after the presentation.

After the presentation, the researcher informed the participants that, although he has deep respect for mathematics and has studied mathematics at the post-secondary level, he has not always been utterly immune to mathematics anxiety. He told them of his own math-anxiety-inducing experiences including long division in grade four, doing work involving straight edge and compass constructions in grade nine, and taking a statistics course in university. Afterward he encouraged the participants to talk about any negative experiences that may have triggered the mathematics anxiety within themselves, as well as the thoughts, feelings and physical symptoms associated with the math-anxious state. The participants were

also given the task of creating their own image of a mathematician using either pictures, words, or both.

Subsequent sessions of the workshops varied slightly between Workshop A and Workshop B due to the fact that the number of sessions and the time period for each session differed for each workshop. The participants from both workshops wrote poems that captured their attitudes, conceptions and emotions associated with mathematics, and commented on a series of cartoon strips designed with the intent of combining mathematics with humour and a series of brief quotes related to the nature of mathematics and the teaching of mathematics taken from Moritz (1942) and Shaw (1966). These exercises were given in order to determine how the participants' attitudes toward mathematics and their understanding of the nature of the discipline might contribute to their mathematics anxiety.

The participants were also given the opportunity to engage in a number of activities related to mathematics which were selected from Sobel and Maletsky's (1975) book Teaching Mathematics: A Sourcebook of Aids, Activities and Strategies, and were chosen with the intent that they might have a greater appeal for the participants than what more traditional mathematical exercises would have. The activities chosen for Workshops A and B were slightly different from one another to take into account the possibility that due to scheduling conflicts, a participant from Workshop A might attend a session for Workshop B or vice versa. Both workshops had activities involving the construction of magic squares, simple probability experiments through coin tossing, and puzzles of a geometric nature. Workshop A also included an activity involving the creation of polyominoes (see Definition of Terms) whereas Workshop B included a card game in which the object of the game was to generate

the sum of 22 using the cards. Both workshops also engaged in collaborative problem making and collaborative problem solving. The source for such exercises was Tobias' (1980) Resource Manual for Counsellors and Instructors. Specific detail regarding the content of the workshops is given here:

<u>Session</u>	<u>Activities</u>	<u>Rationale</u>
1	<ol style="list-style-type: none"> 1. Administration of the Quilter and Harper (1988) survey 2. Construction of a concept map for mathematics anxiety (group activity with one participant volunteering to be spokesperson to present the map) 3. Instructor's presentation on mathematics anxiety (using a concept map as a visual aid) 4. General discussion about mathematics anxiety 5. Construction of an image of a mathematician (using words, pictures, or both) 	<ol style="list-style-type: none"> 1. Initial assessment of attitudes related to mathematics 2. Participants reflect and discuss what they believe to be the essential elements underlying the phenomenon of mathematics anxiety 3. Participants become familiarized with the problem as described in the literature (e.g. definitions, causes, method of reduction) 4. Instructor determines the negative experiences that the participants had in connection with the study of mathematics 5. Instructor determines what qualities the participants believe to be part of the personal and professional nature of mathematicians
2	<ol style="list-style-type: none"> 1. Creation of math poems (Any style of poetry is acceptable. The instructor reads examples of math poems by students of Panitz (1997), then asks the participants to create their own poems. Afterwards, they read their poems out loud.) 2. Discussion of cartoons by Azzolino, Silvey and Hughes (1986) 3. Discussion of selected quotes from Moritz (1942) 	<ol style="list-style-type: none"> 1. Participants capture their thoughts and feelings about mathematics in a creative manner 2. Instructor determines if such cartoons could evoke a humorous response from the participants 3. Instructor probes further for their thoughts and attitudes about mathematics

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|---|--|---|
| 3 | <ol style="list-style-type: none"> 1. Discussion of Shaw (1966) quote 2. Participation in the "Geometric Tidbits" activity from Sobel and Maletsky (1975) 3. Collaborative problem making (Participants work as a group to create problems based on information contained in "A Glorious Day", a short story by Donady and Auslander (1980).) 4. Collaborative problem solving (Participants work as a group to select a problem from a list of problems, then attempt to solve it.) 5. Construction of magic squares (activities of Sobel and Maletsky (1975)) | <ol style="list-style-type: none"> 1. Instructor determines if mathematics holds any aesthetic appeal for the participants 2. Instructor determines if such exercises are fun and relaxing for the participants 3. Instructor determines if problem making is more relaxing than problem solving 4. Instructor determines if working as a group to solve a math problem is less anxiety inducing than working independently 5. Instructor determines if constructing magic squares is a relatively anxiety free activity |
|---|--|---|
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|---|--|---|
| 4 | <ol style="list-style-type: none"> 1. Completion of magic squares activities 2. Coin tossing probability experiment by Sobel and Maletsky (1975) (Participants repeatedly toss coins to determine the probability of tossing two consecutive heads.) | <ol style="list-style-type: none"> 1. See above 2. Instructor determines if the coin tossing probability experiment is a relatively anxiety free activity |
|---|--|---|
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|---|--|---|
| 5 | <ol style="list-style-type: none"> 1. Additional activities by Sobel and Maletsky (1975) (e.g. polynomials and card games) 2. Discussion of workshop activities 3. Administration of anecdotal evaluation form 4. Administration of the Quilter and Harper (1988) survey | <ol style="list-style-type: none"> 1. Instructor determines if such activities are relatively anxiety free 2. Participants express their opinion about the strengths and limitations of the workshop 3. Final assessment of attitudes related to mathematics |
|---|--|---|

Research Process

The research process was documented in the form of field notes that the researcher took while the participants were engaged in activities. He also transcribed notes from tape recordings of each of the workshops. What follows is description of the various types of data used in the study.

Concept Maps

In each of Workshops A and B, the researcher had the participants work together to design a concept map for mathematics anxiety using coloured markers and chart paper. After the concept maps were completed, one participant volunteered to be spokesperson and gave a brief oral presentation of the material within the maps. This provided the researcher with insight into the participants' understanding and experience of mathematics anxiety which could then be augmented with the researcher's insight into the phenomenon as revealed through the literature and consolidated into the researcher's own concept map. The researcher could then compare what the literature had to say about the nature, origins, and possible means of reduction of mathematics anxiety with the participants' beliefs regarding these issues. Working as a group on these concept maps also allowed the participants to socialize and share ideas.

Poems

The researcher was fascinated with the idea of studying mathematics anxiety through the medium of poetry and decided to implement mathematical poetry into the workshop. Six of the participants composed a math poem during the workshop then read that poem out loud to the group. (There was one absentee during the poetry session.) Afterward, the researcher collected the poems from the participants and these poems became part of the researcher's data base.

Images of Mathematicians

Donady and Auslander (1980), as discussed in the review of the literature, noted that many people have an unflattering image of a mathematician. Such an image emphasizes strictness and a lack of creativity and sensitivity.

Donady and Auslander linked the perceived unemotional nature of mathematicians which in turn was linked to mathematics anxiety. These linkages inspired the researcher to ask his participants to create their own image of a mathematician. The participants worked independently on this activity and were allowed to use pictures, words, or a combination of the two. The participants completed their image of mathematicians during the workshop, then submitted their images to the researcher.

Anecdotal and Objective Surveys

The Quilter and Harper (1988) survey was administered to the participants at the beginning and at the end of the workshops for the purpose of determining whether the

experience of participating in a mathematics anxiety workshop led to any changes in their thoughts and feelings regarding mathematics. The survey consisted of 24 objective style items, along with a section for comments. Due to the small sample (N=7), it was determined that the use of inferential statistics on the results would yield little value. However, the results could still be analyzed on an item by item basis to determine if there were any notable changes from the initial administration to the final administration of the survey. (ie. If six participants strongly disagreed with the item "Mathematics is creative" on the initial survey, but six participants strongly agreed with the same item in the final survey, that might be meaningful.)

It was determined by the researcher in consultation with the supervisor that to give proper consideration to the research problem (ie. What impact would a workshop have in altering the attitudes and conceptions of mathematics that would in turn reduce mathematics anxiety?), an anecdotal evaluation should also be given during the last session of the workshops. The anecdotal evaluation survey was designed by the researcher and the participants were asked to provide written comments relating to the strengths and limitations of the workshop as well as suggestions for modification.

Field Notes

The sessions of the workshop were tape recorded. During the two weeks following the data collection period, the researcher transcribed the tape recorded discussions by hand for the purpose of searching for emerging themes. The researcher also made notes based on observations of the actions of the participants as they engaged in the various workshop activities.

Field notes were described by Stainback and Stainback (1988) as “a written record of what the researcher has seen and heard in the field and his or her feelings, reactions and thoughts.” (p.57). The purpose of field notes according to Stainback and Stainback is “to keep track of the development of an investigation, visualize how the research plan has been affected by the data collected, and to remain self-conscious of how he or she has influenced or has been influenced by the data.” (p.57).

Data Analysis

The field notes, transcribed notes and anecdotal evaluation forms were studied and emerging themes were extracted and categorized. Each theme is reported in depth. While extracting themes, the researcher kept in mind the literature and personal experiences that have influenced his thinking. Researchers like Kennedy and Tipps (1994) have, for example, pointed to the problem of authoritarian teaching practices and the need to make mathematics concrete. Reflection on personal experiences reinforced such points. The notes, poems, anecdotal surveys, concept maps and images of mathematicians were read through carefully. Items within these sources of data (e.g. a statement made by a participant during a discussion, transcribed from a tape recording) that corresponded to a particular theme were identified by placing a symbol representing that theme beside the item. The symbols used can be found on page 69.

In section B of Quilter and Harper’s (1988) survey are 31 items related to feelings and perceptions regarding mathematics. Participants must indicate whether they strongly agree, agree, strongly disagree, disagree, or are undecided regarding each item. Quilter and Harper

(1988) identified seven statements that “were not included in the summated scores on the basis of relatively poor construct or criterion related validity arising from the main survey.” (p.129). Thus, the researcher eliminated those items from his administration of the survey. The items were ranked on a scale ranging from +2 to -2 in which +2 represents a response with the potential of inducing much anxiety and -2 represents a response with the potential of inducing little or no anxiety. The entries in tables 1 and 2 (see Chapter 4) were multiplied by its rank number and the results were added to give a cumulative ranking. The differences of the cumulative ranking of each item between the initial and final administration of the survey were also noted (ie. Final-Initial). Thus, alterations in the participants attitudes and conceptions regarding mathematics could be determined.

Chapter 4 Findings

Introduction

A total of seven people participated in the workshops - one male, six females. Six of the participants were Bachelor of Education student teachers in the Primary/Junior Division and the other participant was a third-year Concurrent Education student. Attendance at the workshop was not consistent due to conflicting commitments on the part of the participants. This problem with attendance was noted as one of the limitations of the workshop when the participants completed the anecdotal evaluation forms. However, the instructor attempted to rectify this by allowing a participant from Workshop A to come to a session of Workshop B, and vice versa, if scheduling problems emerged. Thus, five of the participants received seven and a half hours of "treatment", another received six hours, and the other received four and a half hours. The concept maps, surveys, poems, notes and images of mathematicians all yielded a considerable amount of data. Examples and interpretations of each type, as well as the themes that emerged from this data, are discussed in this chapter.

What follows is a table indicating the pseudonym of each participant and their total time of attendance:

Table 1: Workshop Attendants (Total Time in Hours)

Participant	Yvonne	Nancy	Terri	Joanne	Rhonda	Greg	Cyndi-Lynn
Total Time	7.5	7.5	7.5	7.5	7.5	6	4.5

The Participants

The following descriptions contain information told to the researcher by the participants as well as the researcher's general impressions.

Cyndi-Lynn

Cyndi-Lynn was a third year Concurrent Education student majoring in mathematics. She joined the workshop mid way through its run. She learned about the workshop from a posted advertisement and contacted the researcher by telephone.

Cyndi-Lynn had two reasons for participating. First, she hoped to come to terms with the frustration that the study of mathematics had caused her. This frustration had been a problem for her despite the fact that she had sufficient interest in mathematics to pursue it at the university level, with the ambition of becoming a mathematics teacher. Secondly, she was seeking the pleasures of social interaction which was limited for her given her responsibilities as a mother and as a student.

The researcher's general impression of Cyndi-Lynn was of someone who enjoys working in a group setting. She was consistently eager to share her ideas about mathematics anxiety and her insights into mathematics with the other members.

Greg

Greg, as was the case with all the other participants yet to be mentioned, was in the one-year Bachelor of Education program, in the Primary/Junior Division, who responded to the researcher's oral promotion of the workshop.

Before attending the first session of the workshop, Greg told the researcher that he had not experienced any significant amount of mathematics anxiety until taking a current university level mathematics course. The anxiety that this course triggered, however, was of a sufficient degree that he was willing to participate in this workshop with the hope of gaining some insight into this phenomenon.

The researcher's general impression of him is that he is serious minded and assertive, yet always willing to listen to the ideas of others. It should be noted that Greg was the only male participant. Is this indicative of the contention of some researchers that mathematics anxiety is a problem more associated with females than with males? Perhaps, but seven is too small a sample size from which to make generalizations.

Rhonda

Rhonda, who has struggled for much of her student career with mathematics, has a son who has proven to be gifted in mathematics. She believes that because of this, mathematics may become an important part of her life in the near future. Her reason for participating was to have the opportunity to discuss possible ways of reducing her mathematics anxiety. Then in turn she would be of more help to her son in nurturing his mathematical talent.

The researcher's general impression of her was of someone who is practical minded and goal oriented. Mathematical work that she did not perceive as practical was of little interest to her.

Joanne

Joanne participated in the workshop because she wanted to find out more about the underlying causes of mathematics anxiety. If she knew what caused mathematics anxiety she might be able to prevent her own students from developing it.

Joanne opened herself enormously during the workshop, speaking at length and in depth about her ideas and experiences. She mentioned that she was married to someone who had a Masters in Business Administration and had considerable mathematical ability. He had helped her solve computational problems like what mark was needed on a particular examination to pass a particular course.

Terri

Terri had found mathematics difficult. When she had had to take mathematics courses, she had invested a tremendous amount of time and energy into them, to the point of "living off coffee and cigarettes." She had become frustrated when confronted with mathematical problems that she had difficulty solving. Perhaps her motivation for participating in the workshop stemmed from a desire to find a coping mechanism for that frustration.

Terri, according to the researcher's general impression, was a highly self-motivated individual with a strong desire to confront and successfully overcome challenges. Her participation in the workshop might represent one stage in her confrontation with mathematics anxiety.

Nancy

Nancy's mathematics anxiety was so severe that she described her reaction to mathematics as "visceral." Perhaps her awareness of the extreme nature of her problem with mathematics sparked her interest in the workshop where she would share ideas with others who would understand what she has been through.

Nancy presented herself as an outgoing individual who smiled and laughed a lot and enjoyed social interaction. She was open to a wide range of ideas and was careful not to be too judgmental regarding people and things of which she had a limited understanding.

Yvonne

Yvonne had traced her anxiety to the early years of secondary school, and negative experiences in mathematics classes during those years had left their mark on her. Yvonne wanted insight into all aspects of mathematics anxiety in order to effectively help any future students of hers who might have this problem.

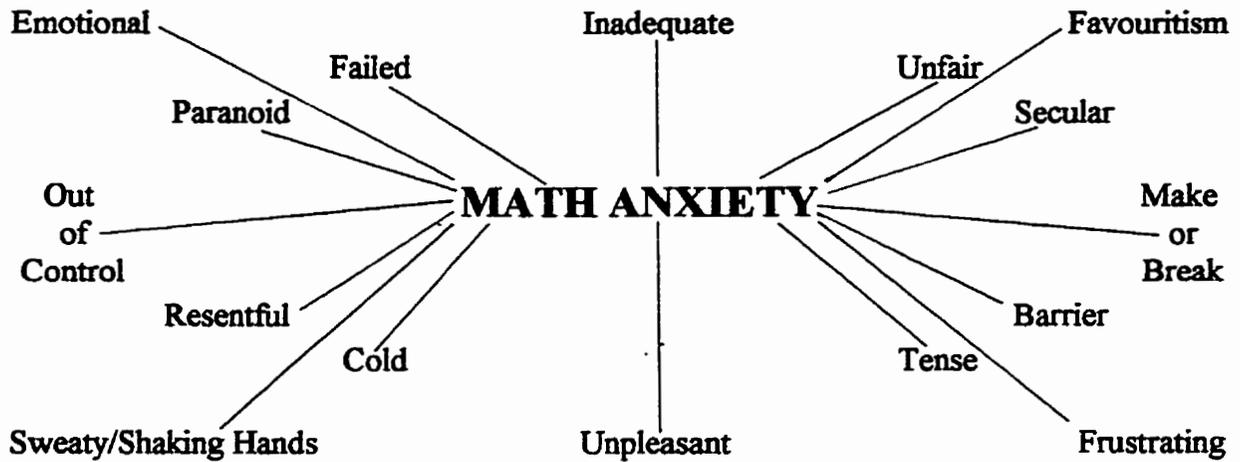
Yvonne, in the mind of the researcher, was a very humane individual who was sensitive about the feelings of others. She evoked an impression of being passionate about her ambition to become a teacher.

Concept Maps

Small group work led to the construction of two concept maps related to mathematics anxiety. These maps provided insight into how the participants interpret what mathematics anxiety is and what concepts or ideas they associate with the phenomenon. The maps were

somewhat different from the concept maps shown by Novak and Gowin (1989) in the sense that the participants drew arrows leading from the term “math anxiety” to other words, phrases or statements without joining prepositions which were normally placed beside the arrows in examples provided by Novak and Gowin. An example of one of the concept webs constructed during one session of the workshop is given on this page.

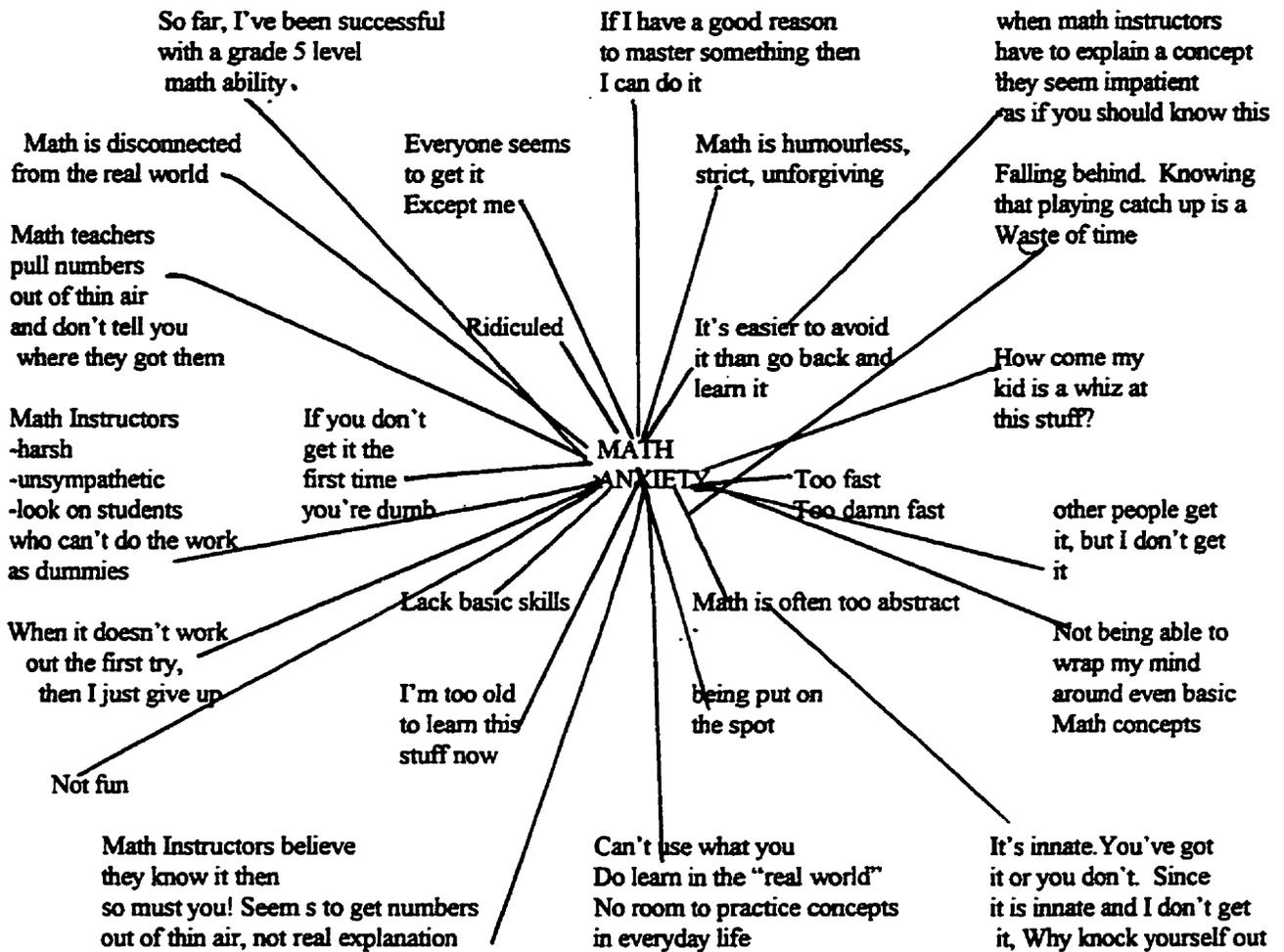
Figure A



This concept map provided information about the physical and psychological reactions associated with mathematics anxiety as well as the effect this problem has on their outlook on mathematics, mathematics teachers, the events that transpire in a mathematics classroom, and themselves. One limitation within the concept webs is a lack of suggestion as to how their anxiety might be reduced.

The second concept map is to be found on the next page. This map is particularly rich in thoughts, feelings and ideas. It illustrates an extremely bleak view of mathematics teachers, who seem like magicians who are unwilling to reveal their secrets to their audience. A feeling of embarrassment at not being able to do mathematics well is glaringly evident in this map. Other revelations offered by this map are the perception that mathematics lacks a recreational dimension, the idea that people either do or do not have a mathematical mind, and a lack of a perceived need to learn any mathematics beyond a minimal level. Much of the information contained in the map fits well with the literature on mathematics anxiety, but there were some additional insights, including the perception that mathematics becomes too difficult to learn when one is beyond a certain age, and puzzlement over not being mathematically inclined, yet having a child who is.

Figure B



Poems

Six of the participants wrote poems that captured their thoughts and feelings about mathematics. The participants seemed eager to express themselves through poetry and seemed pleased with the results of their creative endeavours. Three of them asked if they could place their math poems in their Language Arts folder. The researcher agreed to this and he photocopied those poems, gave the originals to the participants, and kept the copies to add to his data base. An example of one of the poems is the following:

[*I Don't Get Math*]³

Why is it I don't get math?
It's not that I'm stupid
Not that I don't try
However the fact that I can't do
Up to grade 8 math
Makes me so embarrassed, I could
Just die!
What am I lacking?
What is it that I should do?
I just don't get math
Why is it so much easier for you?

The question in the first line of the poem is an important one. What do people who do well in math possess that others do not? This poem subtly suggests that some individuals possess an innate mathematical talent that allows them to solve mathematical problems effortlessly. On the other hand, those who do not have this talent must struggle in vain.

The other poems also proved to be extremely illuminating.

³ Titles of poems that appear in square brackets are titles invented by the researcher.

[*Drowning*]

Fatal flaws with fractions
hinder my actions
and make me feel divided

Gigantic guesses in geometry
that I still can't see
make me ask "y"

Swirling shapes
make me ache for sense and reason
but still make me feel square

I want to feel big
like it used to be
before

Help I'm drowning in a sea
of x's and o's and tic tac toes,
that make it difficult for me
to look at foes
like calculus and geometry
without some time to woe
about the answer I can't see

Here, the writer expressed the belief that mathematics is essentially meaningless. Solutions to problems in calculus and geometry lack sense and the underlying rationale behind pursuing such disciplines is elusive. Mathematics apparently was not always a problem for the writer, but faced with growing complexity and abstraction, it began to attack his/her self-esteem.

Endless Lessons in Humility

I
sit
shamed

certain
that everyone can
see into my
empty head

I
don't hate Math

I
hate myself
because

I
don't understand

It would seem from this poem that the study of mathematics has led to several humiliating experiences for this individual. However, such experiences have not caused him/her to undervalue the discipline but to undervalue himself/herself. To what extent were his/her teachers responsible for this suffering? More is said about this poem later in this chapter when the perceived abusive nature of mathematics is discussed.

[*Twisted Path*]

I rather think math
is a twisted path
that sidewinds along
and doesn't really belong
it's not part of my life
it's like an unwieldy knife
that cuts
my world
in two

The writer of this poem perceives extremely little practicality to mathematics. Despite this separation of the discipline from the mundane aspects of daily life, he/she has had to deal with mathematics as a subject in school. Mathematics, in this respect, is like a warrior invading the writer's life, and the writer lacks the necessary tools for defending himself/herself against this attack. Further insight into this poem is revealed when the perceived abusive nature of mathematics is explored.

[*Meaninglessness*]

Around and around
in a sea of symbols
Numbers and letters
like vegetable soup
in my mind

A sludge of equations
and odd sounding words
Making no sense at all
No worthwhile meaning
of any kind

Moving and tickering
Marks on a page
Formulas with not a trace
of life in them
Feeling resigned

As the game goes on
around me in the fog
Numbers, more numbers
Problems and lessons
No hope I find

The complexity and abstraction of mathematics is clearly conveyed in this poem. To this writer, mathematics is a game in which the object and the rules are lost to him/her. The search for meaning has been almost given up as beyond his/her ability. Despite this lack of comprehension, mathematics continues to haunt this individual. He or she is forced to continue playing this “game” without any hope of ever winning. Perhaps this poem is a declaration by the writer that he or she has given up trying to make sense out of complex mathematical ideas.

[Mathematics: An Acrostic Poem]

Mumbled words
Anxious feelings
Trembling hands
Horrid fear
Exasperating
Mixed up
Anger building
Terrified beyond belief
Intimidating air
Confusing thoughts
Scared stiff

This acrostic poem captures a number of physical, mental and emotional reactions to dealing with mathematics. The fear and anger work together to interfere with the thought process. The resulting confusion undoubtedly leads to increased fear and anger, thus creating a vicious cycle of negativity. Since every letter in mathematics is associated with an unpleasant feeling or sensation, one might infer that for this person, mathematics is a discipline lacking in any positive qualities.

These poems captured frustration, embarrassment, puzzlement, fear, helplessness, hopelessness, self-hatred, resignation, despair, anger and terror. What the poems did not capture were ways of confronting and overcoming these feelings. In the researcher's opinion, the poems demonstrated considerable writing skills on the part of the participants, which is understandable given the fact that they are university students in the Bachelor of Education program. If the participants had poor writing skills, then the poems might have had limited value. However in this study, the poems seem to capture what having mathematics anxiety means to the participants. In their brutal honesty their poems make visible their emotional scars and the debilitating repercussions of these scars.

Images of Mathematicians

Some participants drew pictures to convey their images of mathematicians. Others used words. Here is an example of one participant's written verbatim description of a mathematician:

- old
- male
- boring voice
- involved in numbers, not interested in people
- no humour
- smarter than average in some things, ignorant in others
- works for the government or big business or the mob
- wears a polyester plaid suit
- doesn't get enough exercise
- lacks creativity
- squeaky shoes
- life has no meaning, contributes nothing to the community
- graduate of higher education
- not a good listener
- can't convey knowledge in everyday language
- limited.

Such an image as this portrays an individual detached from the social mainstream and is somewhat reminiscent of the cold, insensitive man alluded to by Donady and Auslander (1980). It is a harsh, though possibly brutally honest description, yet one might wonder whether the participants created their images on the basis of a large sample of mathematicians, or on one emotionally scarring example. These images allowed the participants to examine critically their own views on mathematicians, and allowed the researcher to compare these images with the one referred to by Donady and Auslander (1980) (See p.9).

The other participants' images need to be examined. Here is another verbatim written description:

- strict
- rigid
- unforgiving
- self-important
- mathematically brilliant
- socially inept
- bad "bedside" manner
- self-absorbed
- lazy
- willing to share only with the gifted students
- ignores the students who are slow
- tends to make assumptions
- lack of humour/fun

- lack of creativity
- everything is black or white, right or wrong
- power hungry - enjoys putting people on the spot
- likes to show off his/her brilliance
- no patience for slow learners
- tends to openly favour the bright students.

In this image, the mathematician does not seem to possess many important social and practical skills. In fact, mathematical ability would seem to be the only favourable quality. Both written images of a mathematician seem to portray such an individual as a social outcast, yet approach this portrayal from different perspectives. The first description describes a perceived physical appearance in some detail, whereas the second description focuses more on mental and emotional characteristics (e.g. self-absorbed, self important).

The other images of mathematicians were portrayed using pictures. These pictures appear on pages 55 to 57, and for the purpose of interpretation, they have been labelled as Picture 1, Picture 2, and Picture 3.

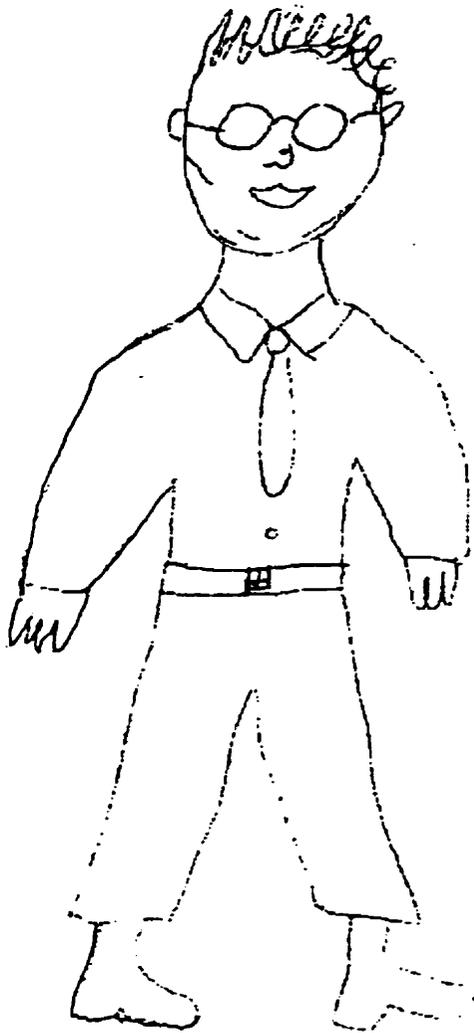
Picture 1 shows an individual that has a somewhat masculine appearance (as interpreted by the researcher), although there is no positive identification as to gender. He/she is dressed in a shirt, necktie, belt, pants and shoes. The presence of a necktie suggests a business-minded professional, but the pants are too short, suggesting carelessness or a lack of concern for one's public image. The spectacles and unruly hair convey the stereotypical image of a "nerd", which relates to the "outcast" theme that seems to be present in the written description.

Likewise with Picture 2, one cannot be certain if the mathematician is male or female, although this one is also wearing a necktie, suggestive of masculinity. The bizarre squiggles surrounding the person might represent an unpleasant odour. Like Picture 1, the hair is unruly, but this person is not wearing spectacles. He/she seems to have a very troubled facial expression (e.g. the wavy mouth and the lines underneath the eyes), which contrasts with the smile in Picture 1. Nancy commented during one of the workshop discussions that she believed mathematicians were essentially unhappy due to a lack of meaningful interaction with others. This picture might also reflect that belief.

Picture 3 actually depicts an entire classroom with the students sitting at their desks, arranged in rows. The mathematics instructor is standing in front of the class, using a metre ruler or stick (or some long, narrow instrument) to point at multiplication questions on the blackboard. Again, the appearance of a necktie suggests a male, though certain identification cannot be made. The short hair and spectacles bear some similarity to Picture 1. This individual is straight faced, suggesting a business-like demeanor. Might such a demeanor be intimidating for some learners?

All of these images suggest that if a mathematics teacher presents to students certain characteristics, like strictness, or untidiness, and fails to present other characteristics like a sense of humour, then that teacher might be unwittingly contributing to a student's mathematics anxiety. This anxiety in turn might create a perception of the mathematics teacher in which his or her flaws are overemphasized and his or her virtues are ignored.

Picture 1

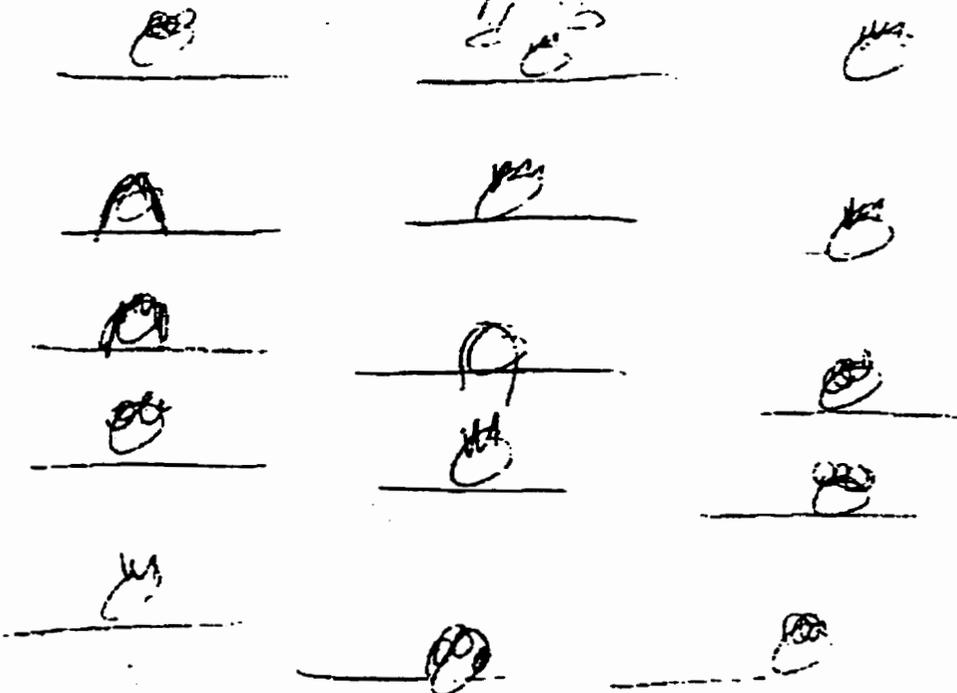


Picture 2



Picture 3

$\begin{array}{r} 0.23 \\ \times .46 \\ \hline \end{array}$	$\begin{array}{r} 4.36 \\ \times 2.12 \\ \hline \end{array}$
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The Quilter and Harper Survey

The results of the Quilter and Harper (1988) survey have been consolidated so that the number of individuals who strongly agreed, agreed, strongly disagreed, disagreed or were undecided with each particular item on the initial and final administration of the survey was determined. These results appear on the following page. Items that were left blank or in which two check marks were placed in two different columns for the same item were ignored. The consolidation of the results of this survey appear on the next two pages.

Although there was relatively little change in most of the items on the survey between its initial and final administration, three items deserve some consideration. For the first item (ie. "Mathematics is creative"), there was a significant change (a difference of -8 in the cumulative ranking). Thus, the participants at the end of the workshop were more inclined to give credence to the idea that mathematics has room for creativity and less inclined to view mathematics as merely a rigid collection of rules and symbols.

The tenth item (ie. "Minus numbers are daft") also exhibited a significant change (a difference of -7 in the cumulative ranking). Although minus (negative) numbers were not given special attention during the workshop, there was one probability activity used on the workshop that involved the use of a number line ranging from -5 to +5 (see Appendix C). Perhaps after engaging in this activity, the participants perceived a use for negative in their connection to hands-on activity.

The sixth item (ie. "You can get a math question half right") also changed significantly but in a way that the researcher did not expect (the difference in the cumulative ranking was +7). The researcher ranked this item in such a way that agreement with this item would

induce little anxiety because in his or her prior experience a half right solution to a problem was usually entitled to some part marks. However, it emerged through discussion that not all teachers accept partial solutions. Yvonne recalled secondary school teachers that marked solutions as either right or wrong without awarding any part marks. This discussion may have lead to the unanticipated change.

The comments on the survey provided some background. One participant recalled being slapped by mathematics teachers in elementary school in the 1960's for using concrete objects. Another recalled lagging behind in mathematics in elementary and secondary school. Another alluded to exposure to mathematics teachers he or she deemed to be incompetent. Problems involving teaching were reinforced in the workshop discussions.

Table 1
Results of the Initial Administration of the Survey

Ranking System ⁴	Item Number	Strongly Agree	Agree	Undecided	Disagree	Strongly Disagree	Blank	Cumulative Ranking
-	1	0	2	0	4	1	-	+4
+	2	3	3	1	0	0	-	+9
+	3	2	3	1	0	0	1	+7
-	4	1	2	2	2	0	0	-2
+	5	1	1	4	1	0	1	+2
-	6	3	3	0	1	0	-	-8
+	7	5	1	0	0	0	1	+11
+	8	3	4	0	0	0	-	+10
+	9	2	3	1	1	0	1	+6
+	10	2	0	1	3	0	1	+1
-	11	0	0	1	4	2	-	+8
+	12	1	2	0	3	1	-	-1
+	13	2	3	0	1	0	1	+6
+	14	0	1	3	2	1	-	-3
+	15	1	1	1	2	2	-	-3
+	16	3	2	0	2	0	-	+6
+	17	2	2	0	3	0	-	+3
+	18	2	3	0	1	1	-	+4
+	19	3	3	0	1	0	-	+8
+	20	2	2	1	2	0	-	+4
-	21	0	0	1	2	4	-	+10
+	22	3	3	1	0	0	-	+9
+	23	2	3	2	0	0	-	+7

⁴ + means a ranking of +2, +1, 0, -1, -2 consecutively; - means a ranking of -2, -1, 0, +1, +2.

Table 2
Results of the Final Administration of the Survey

Ranking System	Item No.	Strongly Agree	Agree	Undecided	Disagree	Strongly Disagree	Blank	Cumulative Ranking	Change
-	1	1	4	0	2	0	-	-4	-8
+	2	1	4	2	0	0	-	+6	-3
+	3	0	4	1	2	0	-	+2	-5
-	4	0	1	2	4	0	-	+3	+5
+	5	0	3	1	3	0	-	0	-2
-	6	3	2	0	1	1	-	-1	+7
+	7	1	5	0	1	0	-	+6	-5
+	8	1	4	0	2	0	-	+4	-6
+	9	1	4	1	1	0	-	+5	-1
+	10	0	2	0	2	3	-	-6	-7
-	11	0	0	1	4	2	-	+8	0
+	12	1	2	2	2	0	-	+2	+3
+	13	1	4	0	1	0	-	+5	-1
+	14	0	3	0	2	1	-	-1	+2
+	15	0	4	1	1	1	0	+1	+4
+	16	1	5	0	1	0	-	+6	0
+	17	1	3	2	1	0	-	+4	+1
+	18	1	2	1	3	0	-	+1	-3
+	19	1	5	0	1	0	-	+6	-2
+	20	0	3	2	0	2	-	-1	-5
-	21	0	0	0	6	1	-	+8	-1
+	22	2	4	0	1	0	-	+7	-2
+	23	2	3	0	2	0	-	+5	-2

Anecdotal Evaluation Surveys

All the participants filled out the anecdotal evaluation forms. (See Appendix A for an example of the form.) Some of the positive comments were as follows, along with possible interpretations:

- (1) I had a good time challenging myself and my memory and I had fun doing the questions. *(This suggests that effective workshops should offer some, but not too much, intellectual stimulation. It might also suggest that mathematics courses would be fun if the activities were perceived as being creative.)*
- (2) Everyone had the chance to express themselves. *(All participants of a workshop should be encouraged to express their ideas so that none of them is left feeling isolated.)*
- (3) Meeting other people who are experiencing difficulty in math, I don't feel isolated with this problem. *(In a mathematics anxiety workshop, each participant needs to know that he or she is not the first person to suffer from this problem. Talking with other people also helps to generate ideas that might help reduce their anxiety.)*

Other comments were as follows:

- (1) Discuss in more detail how to eliminate math anxiety and help children get over their anxiety. *(Eliminating mathematics anxiety might be impossible. However, a workshop instructor might work with the participants to develop a list of ideas of how to reduce this anxiety. The participants may also discover that certain types of mathematical activities are less stressful than others.)*

(2) More word problems - to learn to eliminate the unnecessary. (*If more word problems were used, they should be of a practical nature, in keeping with the need as experienced by the participants, to relate mathematics to everyday life.*)

(3) It would have been better for me to attend a workshop like this when I'm not in school full time and coping with a workload of 9 courses. (*This is an inevitable problem. The best any instructor can do is set up days and times for the workshop that are of greatest convenience to the greatest number of people interested in participating in the workshop.*)

Overall the results of the anecdotal evaluation indicate that most of the participants found the workshops to be enjoyable and worthwhile, but the workshops could be modified to address a wider range of difficulties.

Field Notes

What follows is a sample from the transcribed notes (Workshop A, Session 2):

INSTRUCTOR: Are you influenced by the media in your image of a mathematician?

YVONNE: No, I don't watch television.

TERRI: My idea of what you would call a nerd is very stereotypical.

NANCY: I think so too. Mind you, I have seen some neat stuff too, like Stephen Hawking's Brief History of Time. I really liked that. I had to read some paragraphs twice over to get it, and once you get it, it's like, wow, this is really interesting and fascinating stuff; I really liked it. Mine is more of a comical take on it; I guess that's what Steve

Urkel⁵ is, kind of a commentator but somebody who's so oblivious to everything going on around him because that's all. That's it. That's math. Food down your tie, down your shirt, never noticed it. Never has the time to get a haircut because you are so wrapped up in quantum physics that you actually leave your house and you're locked. I don't think they're happy.

YVONNE: I did very well in math until Grade 9. Looking back on it, I did well in math right up to Grade 8. Then in Grade 9 when I had this teacher, I was so busy fighting his rude remarks, crude conversations in class, I went back to my Grade 7 and 8 math teachers for help. Maybe I just lost the basis that you needed in grade 9 to continue on, I don't know.

The remarks about mathematics educators align themselves with the harsh images of mathematicians created by the participants, thus showing how one type of data could be used as a reinforcement for another type of data.

Mathematics and Humour

When the instructor asked if there was any connection between mathematics and humour, Terri responded with an emphatic "No such thing!" Nevertheless, the instructor attempted to demonstrate a humorous side to mathematics by handing out to the participants

⁵ Steve Urkel is a character on the television series *Family Matters*. He is a teenage boy and presents himself in the stereotypical image of a nerd. This image is conveyed through spectacles, a squeaky voice, and social awkwardness. Urkel also has tremendous ability and interest in mathematics and science. This may contribute to a perception that people who do well in mathematics are nerds.

cartoons by Azzolino, Silvey and Hughes (1986). (See Appendix D.) Reaction to these cartoons was minimal. No laughter was heard. However, Joanne found the cartoon showing the father and the teenage girl to be “cute.” The instructor felt that this cartoon represents the misunderstandings that can occur when communication involving numbers takes place, and perhaps frequent misunderstandings of this nature could intensify feelings of anxiety.

The lack of desired effect (ie. laughter) on the participants that the cartoons had may be due to a perception of the humour contained in them as being droll and unsophisticated. The instructor asked the participants if this was the case and some of them nodded in agreement. However, a more significant reason is simply that the cartoons contained mathematical knowledge and symbolism with which the participants were unfamiliar. For example, the cartoon involving the Fibonacci series was ineffective because they didn't know what this series is. Also obscure was the man who said, “ $H/\sqrt{-1}$.” (ie. “Hi!”). Complex numbers are sums of real numbers and imaginary numbers in the form $a + bi$ where a and b are real and i equals $\sqrt{-1}$. (Bendick, Levin, and Simon, 1965).

Rhonda recalled a previous encounter with an attempt at mathematical humour. She said, “I was looking for a Halloween card for my kids and one of the cards said, “What do you get when you divide the circumference - pumpkin pie?”” (Note: she must have meant “divide the circumference of the pumpkin by its diameter.”) She smiled when she said this, showing some mild amusement.

Although the cartoons did not amuse the participants, this does not necessarily suggest that mathematics and humour are incompatible. A teacher might use different cartoons on his or her students, or try a different medium like jokes or humorous poetry. If

a mathematics teacher is able to get the students to laugh, that laughter might help them to relax. For instance, cartoons that emphasized feelings of helplessness in mathematics, or cartoons that satirized mathematics might have been more appropriate.

Discussion of Quotes

The instructor read to his participants the final paragraph of the essay Mathematics - The Subtle, Fine Art by Shaw (1966). The paragraph is quoted here:

Many mathematicians have been artists in other ways. Some wrote poetry, some composed music. The inquiry conducted several years ago into mathematicians' activities found that most of them were seriously interested in some phase of art. And most of them reported that their discoveries or creations came just as do the inspirations of artists in other lines. The mathematician dreams, and in his dream an elusive spirit goes in and out; floats in the mist, and vanishes; glides back at unexpected moments, but slips away from the hand that would grasp her; reappears in intricate dance, and phantasmal play of color; disappears; and one day steps out to clasp the hand that has awaited her, with Kummer's ideal numbers for a gift. The mathematician dreams and in the spinning chaos fairy flowers in fantastic forms bloom and vanish; mists wind through them with birds flashing now and then; strains like Debussy's nocturnes are faintly heard; a seething, bewildering multitude of forms are created out of the void, they drop back into the void; and then in one rapturous moment a new form appears, superbly beautiful, and Prosperous wand is held stationary to bid the cloud-castle, the flowers, the wild birds, the haunting music, the spirits of light and beauty stay, and a new branch of mathematics is born, the linear associative algebra of Benjamin Peirce. This is an enchanted land, and the city, like Hugh Ferriss' Metropolis of Tomorrow, is, in Tennyson's words, "built to music, therefore never built at all, and therefore built forever." It is a world that knows no second law of thermodynamics, a world that guarantees to man his creative nature, his eternity of time, his imperishability. Here grows the ash-tree Yggdrasill, supporting the universe, its roots in nature, its trunk of the fibers of logic, its foliage in clearest ether of intuition, its inflorescence the living imagination. In this land of enchantment the queen is beauty, who turns men into gods. (p.260-261)

The instructor found this sample of Shaw's writing to be aesthetically appealing, and

he wondered if the participants would feel likewise. Unfortunately they didn't. Terri's reaction was as follows: "I'm sorry, I didn't get that. My mind is blank." Joanne's comment was as follows: "It hasn't been my experience. I can understand it being someone's experience." In an effort to explore the possible aesthetic dimension of mathematics further, the instructor asked the participants if they could perceive any connection between mathematics and art. Most of the participants had a difficult time seeing mathematics being connected in any way with art, although Joanne did allude to the golden rectangle and Nancy recalled watching a television program about Martin Gardner that mentioned how Gardner's ability to draw geometric patterns influenced women to pursue studies in mathematics.

The instructor also read quotes found in a collection of quotes compiled by Moritz (1942). Here is one of them: "The essence of mathematics lies in its freedom." (George Cantor). Joanne responded by saying, "I don't think of it that way at all. There's always some rule or regulation or formula." Terri wondered if the author of that quote was being sarcastic and added, "Maybe he's trying to say the opposite because it is so obvious that it is not true." Rhonda recalled, "One time when I did find the use of math freeing was when I was working in real estate. Getting a mortgage for \$100,000 at 12%, but maybe a relative could lend them \$60,000 at 10% and \$40,000 at 15%. Which is the better deal? I could help to negotiate, and because I'm such a strong-minded language person, I could educate my clients, and they appreciated that they had choices." The instructor speculated whether the quote was meant to convey the idea that work in pure mathematics is unbounded by ethical constraints. It was the instructor's thought that a mathematician has limitless freedom to manipulate mathematical symbols without causing harm to others.

Another quote discussed during the workshop was as follows:

The ends to be attained [in the teaching of mathematics in the secondary schools] are the knowledge of a body of geometrical truths, the power to draw correct inferences from given premises, the power to use algebraic processes as a means of finding results in practical problems, and the awakening of interest in the science of mathematics. (1912 American Report - International Commission on the teaching of Mathematics)

The instructor wondered if the ideas expressed in this passage might be viewed today as rather archaic. Terri was in partial agreement of his view. “Awakening in science and mathematics should start earlier. That’s archaic; but as far as a body of geometric truth and stuff - as far as I know they are still teaching that.” Joanne described a shift in emphasis from then and now. “The way they worry about problem solving. You no longer have to add, subtract, or do long division - more emphasis on solving the problem.” Is this shift in emphasis toward problem solving one that would be beneficial in reducing mathematics anxiety? This would depend on whether the students are trained in the necessary skills to solve problems effectively and if the content of the problems relate to students’ everyday lives.

The Emerging Themes

Analysis of the notes, concept maps, poems, evaluation forms, surveys, and other data have yielded seven emerging themes. Such analysis was conducted through a lens shaped by personal experiences and reviews of the literature, all of which connect mathematics anxiety to the dry, abstract, impractical manner in which mathematics is sometimes taught. Each of these themes were examined in their relationship to mathematics anxiety. The themes (not

in rank order) are as follows:

	Theme	Code
(1)	The perceived practicality of the discipline	P
(2)	The perceived complexity of the discipline	C
(3)	Collaborative versus independent mathematics work	C/I
(4)	Ambiguity of word problems	A
(5)	Connection between mathematics and puzzles and riddles	M/P/R
(6)	The perceived invasive nature of mathematics	INV
(7)	A love/hate relationship with mathematics	L/H

The Perceived Practicality of the Discipline

The concept maps of mathematics anxiety designed by the participants indicated the stressful feelings that emerge when mathematics seems disconnected from everyday life. This suggests a need on the part of teachers to demonstrate the practical applications of the mathematical concepts that they are presenting to their students. However, one question needs to be asked: To what extent is the discipline of mathematics itself perceived as being of practical value by math-anxious students?

During the workshop discussions, varying attitudes emerged. For example, one participant, Rhonda, described her career in real estate which involved much financial mathematics that was useful, practical, concrete, and made sense to her. Mathematics also played a role in her hobby as a quilt maker when she needed to measure particular lengths of material. Joanne also used math in particular situations like completing income tax forms,

balancing her checkbook, and baking. In these familiar situations, the mathematics involved do not generate anxiety for her. She has experienced anxiety, however, when doing work in algebra (e.g. finding “t”) or solving obscure word problems. In such cases, mathematics was seen as separate from real life. Cyndi Lynn also emphasized that stress originates from a perceived lack of possible application. Mathematical thought does take on an added dimension when set in a real life context. Fensham (1992) alluded to Lave’s (1988) study of women shoppers in a grocery supermarket. It was discovered through this study that the women were able to solve far more complex arithmetic problems while shopping than on a pencil and paper test despite the added complexity of the “real life” situation.

However, the degree to which mathematics itself is applicable in a practical sense was unclear to some of the participants. Greg viewed mathematics as being about solving riddles which might imply that he perceives of the discipline as having minimal practical value. Rhonda alluded to a personal dilemma in which she was not sure if mathematics itself was “dumb” and as such was unsuccessful at it, or if she was “dumb” and was merely trying to convince herself that the mathematics was “dumb” through a kind of rationalization. This dilemma was also expressed in the final seven lines on the poem Endless Lessons in Humility. Here, the participant has decided that the discipline of mathematics itself is not to blame for his/her anxiety. Rather, the problem might lie within the self. This leads to a provocative question: Does mathematics anxiety arise from an innate disability to cope with the complexity of the discipline? This lack of understanding would undoubtedly make one anxious and might contribute to a perception that certain mathematical concepts are more complex than he or she is able to grasp.

The Perceived Complexity of the Discipline

The idea of complexity was evoked through descriptions and metaphors related to mathematics that were used in the workshop discussions and through the medium of math poetry. Greg referred to mathematics as a tree without a root structure, which might suggest a perceived complexity without a corresponding foundation of simplicity or elegance. Feelings of drowning were alluded to by a number of the participants. Cyndi Lynn has memories of compulsory mathematics courses that she described as being over her head.

In the poetry, one participant described mathematics as “a twisted path that sidewinds along and doesn’t really belong.” Another participant expressed the complexity in the last stanza of the poem Drowning. The idea of “drowning in a sea of x’s and o’s and tic tac toes” suggests that the participant is faced with far too many symbols when doing mathematics. In this complexity of symbolism, it is difficult for the participant to find whatever it is he or she is supposed to find within this perceived chaos.

Another participant made use of similar imagery in the first stanza of the poem Meaninglessness. There is some similarity between this stanza and the previously alluded-to stanza from Drowning. Once again, there is the image of immersion in a “sea of symbols.” The lines in this stanza suggest that too much complexity might lead to the perception that mathematics is essentially meaningless.

Such expressions strongly suggest the need for teachers not only to provide students with a practical base for studying mathematics, but to provide students with a proper, thorough grounding in the basics (e.g. computational skills) before too much complexity is introduced. Might the concern about complexity be effectively addressed if teachers expand

their vision of mathematical learning to incorporate more than just independent work? Hamm and Adams (1992) hypothesized that “co-operative learning improves academic performance among high and low achieving students.” (p.8). Might this improved performance be partially attributable to improved ability at handling complexity?

Collaborative versus Independent Math Work

The participants in the workshops had the opportunities to collaborate on a number of activities. In some cases, such as exercises in problem making and problem solving, the participants were specifically asked to work as a group, while in other instances they were given an option of either working collaboratively or independently. It was observed that even during the exercises that were intended to be collaborative in nature, the participants would spend at least part of the time working quietly on their own. For example, they might read a problem quietly to themselves, work out their own solutions, then compare their solutions to those of the other participants, and from that comparison of notes would work together, asking questions and exchanging ideas, toward a commonly agreed-upon solution. This tendency to spend at least part of the time working on their own probably might stem from their personal backgrounds in mathematics where the vast majority of their work would probably have been done independently.

Certain participants seemed to feel that the opportunity to do collaborative work helped to reduce their anxiety. Terri realized that she was not the only one who could not solve a problem right away. The sharing of ideas seemed to be quite beneficial. One cannot comment further on this topic from this study, therefore more research would undoubtedly

have to be done to determine a suitable balance between collaborative and independent work in the context of mathematics. Further study would also help to determine to what extent the sharing of ideas about word problems could reduce the anxiety felt by some when they confront this problematic aspect of mathematics.

The Perceived Ambiguity of Word Problems

Difficulty in solving word problems was noted by several of the participants as a major source of anxiety. In the comment section of the Quilter and Harper (1988) survey, one participant stated that “reading problems always causes me to freeze - I have no idea how to eliminate unnecessary data.” Joanne definitely found some word problems quite awkward, alluding somewhat vaguely to one problem she has encountered involving a plane taking off and landing at various times, but unable to remember the exact details.

What does the stress in connection with word problems stem from? Does the inability in dealing effectively with word problems originate from a lack of essential skills on the part of those attempting to solve the problems, or does it originate from a degree of ambiguity inherent within a large number of word problems? Matz and Leier (1992), while acknowledging problem solving as an essential skill, recognized that many learners find it difficult. When confronted with word problems, “the student must read the problem and decide what operations could apply. These decisions are made difficult when a problem requires several steps.” (Matz & Leier, 1992, p.14). Clearly, a considerable amount of research would have to be done before a definitive answer could be reached as to why some

students find word problems so difficult, but the results of the workshop has offered some clues.

While engaged in problem-solving activities during a session of a workshop, it was noted that there were complications inherent in the interpretation of some of the problems. One that created particular confusion was a problem taken out of Sobel and Maletsky's (1975) book which asked the reader to determine how many pennies could be arranged such that every penny touched every other penny. After spending some time manipulating pennies, some of the participants wondered whether or not the problem could be interpreted in such a way that pennies could be arranged in a straight line, in which case the answer to the problem would be an infinite number and not the published answer of five.

The participants were engaged in both problem making and problem solving. They tended to find the act of problem making less stressful than problem solving. An example of a problem-making activity is one devised by Donady and Auslander (1980) in which one is asked to create mathematical word problems based on a short story entitled A Glorious Day. After completing this exercise, Greg noted, when asked to compare problem making with problem solving, that in the problem-making exercise he could see all the information, which might suggest that when asked to solve a given problem, all of the relevant information may not be visible. Also noteworthy was the comment, also made by Greg, that problem making is "math with a million rights," suggesting that being able to create a word problem without having to find a solution to it can be an intensely liberating experience. This leads one to wonder whether mathematics anxiety could be considerably reduced if students had a greater

awareness of the enormous breadth of the discipline, including what might normally be thought of as puzzles and riddles.

Connections Between Mathematics and Puzzles and Riddles

Greg remarked that mathematics is about solving riddles. Opinions varied, however, as to the relationship between mathematics and riddles and puzzles. Joanne commented that certain activities administered during the workshop sessions (e.g. counting the number of rectangles in a rectangular design) were brain teasers that might do in her spare time but she would not think of such activities as mathematics. She later noted that such activities were good tricks but were not very useful. Rhonda agreed with this, stating that there was little to the activities beyond entertainment value. When asked to define mathematics, Rhonda stated that to her mathematics is numbers, and Joanne said that mathematics involves numbers and a logical way of thinking. To both of them, the activities involving magic squares represented the best of both worlds in the sense that it involves both mathematics and the element of a puzzle. If more teachers made use of activities of this nature, it is conceivable that a more positive attitude toward mathematics might be nurtured, leading in turn to less anxiety.

The Perceived Invasive Nature of Mathematics

The imagery that math-anxious people use to describe the process of teaching and learning mathematics can be quite powerful and suggestive of an abusive or invasive nature of the discipline. This was most evident in the math poetry that the participants wrote.

Consider the first seven lines of the poem Endless Lessons in Humility. In those lines there is a description of the embarrassment the participant feels when she perceives that “everyone can see into my head.” To the researcher, the idea that people are looking into one’s “head,” which is probably synonymous with “mind” and should be a private place, might be invasive. Such an act might even be interpreted as “the act of a mental voyeur.” Although no such “intrusion” would be intentionally perpetrated on the part of the teacher, the feelings experienced by the participant are no less real.

Consider the final five lines in the poem Twisted Path. Here, mathematics is identified as something that is “not part of my life.” Thus, mathematics might constitute an unwelcome intrusion. The metaphor of the “unwieldy knife” suggests that mathematics might be viewed as a potentially dangerous weapon. The knife cuts the participant’s world in two, suggesting a feeling of victimization.

Consider the first stanza of the poem Drowning. The participant is describing error-plagued work with fractions. The idea that such work makes the participant “feel divided,” although it could be interpreted as a “tongue in cheek” remark, might be viewed in a different way. Perhaps by “divided,” the participant is attempting to convey the sensation of feeling torn open and exposed when doing mathematics.

Other pieces of evidence offer subtle clues to this invasive quality. On one of the concept webs, the word “paranoid” is used, which would be an appropriate word choice for someone feeling threatened by an invasive, external force. Also suggestive of this is the result on the Quilter and Harper (1988) survey in which six people out of seven either strongly agreed or agreed with the statement “I find math questions threatening” on the pre-treatment

administration of the survey and five people agreed or strongly agreed with the same item on the post-treatment administration on the survey. Since an invasive situation can be threatening for the victim, one might wonder whether some severely math-anxious individuals might feel “invaded” when forced to do mathematics in, for instance, a classroom setting.

A Love/Hate Relationship with Mathematics

Greg said in connection with mathematics that “It’s amazing if you get it; if you don’t get it, it’s hell.” This statement hinted at a love-hate relationship with the discipline. Although it is doubtful that all of the participants would describe a love/hate relationship in the context of their mathematical experiences, this idea was evoked by Cyndi Lynn, who is the third year concurrent education student, also majoring in mathematics. The other participants were initially quite surprised that a mathematics major would suffer from mathematics anxiety. She recalled times when she would be crying because “Math gets to me,” yet these feelings have not stopped her from deciding to pursue a career as a mathematics teacher and expressing a love for the discipline. She agreed with the researcher’s assertion that mathematics was frightening and fascinating at the same time, and the act of doing mathematics actually involved a whole range of emotions. It would be interesting to know if the feelings Cyndi Lynn expressed were common to other math-anxious mathematics majors. Although it was noted that this love/hate tension regarding mathematics was demonstrated by only two of the participants, this is an area of personal interest for the researcher, and one that he might study further in the future.

Chapter 5

Discussion

Introduction

It is clear from that mathematics anxiety has a number of causes. Information provided by the participants in this study suggested such causes as transmissional modes of instruction, impatient and uncaring instructors, a lack of positive reinforcement, a lack of cooperative learning strategies, over-reliance on textbooks to guide instruction, over-reliance on testing as a means of evaluation, and a difficulty in adequately grasping word problems.

Given the considerable number of potential causes of mathematics anxiety, finding ways of reducing mathematics anxiety becomes a complex task. Teaching styles for mathematics, as well as mathematics curricula, need to be revised and modified if society's goal is to bring about a major, lasting reduction in the mathematics anxiety experienced by learners. Workshops, however, might have a small yet valuable role, particularly in their potential for increasing both teacher and student awareness of this problem.

Now that the themes emerging from the research have been discussed, as well as the research problem, recommendations regarding mathematics anxiety, its alleviation, and avenues for further research need to be addressed.

Revisiting the Questions and Problem

Questions:

- (1) *What are the overwhelming memories that triggered the mathematics anxiety in certain individuals?*

Many of the initial memories that have led to this problem seems to involve particular teachers and teaching styles. For instance, in the comment section of the Quilter and Harper (1988) survey, one participant alluded to unpleasant memories dating back to the second half of the 1960's when there were instances of "being slapped for attempting to use concrete objects to solve problems," and in subsequent grades "having teachers refuse to answer my questions because "that was supposed to be learned in Grade 8".” Rowan and Bourne (1994) referred to one of the goals in the NCTM Standard: "Children must be actively involved in doing mathematics," (p.11) and stated that children "must interact with the physical world and with other children and supporting adults. Concrete materials are essential to provide the foundations for children just developing the ability to think abstractly" (p.11). Rowan and Bourne stressed that "Hands on activity is critical to "doing" mathematics, and the basic language components - speaking, listening, reading and writing - serve as necessary links between action and mathematical understanding." (p.12). Thus, his or her use of concrete materials for mathematical use in the 1960's was consistent with curriculum reforms in mathematics in later decades. It also demonstrated the extremely rigid and autocratic nature of the teachers. Another respondent to the survey alluded to teachers' negative and complicated instruction. Yet another respondent described experiences related to high school

mathematics - specifically Grade 9 - without specifying the precise nature of those experiences.

The participants in this study provided evidence that mathematics anxiety doesn't necessarily begin in early childhood, and can in fact develop within someone at any age. Cyndi Lynn experienced little anxiety until she began taking mathematics at the OAC level. Greg did not experience any considerable amount of mathematics anxiety until he had exposure to a current university-level mathematics course. Mathematics might have a much greater potential for inducing anxiety when it becomes more advanced and abstract rather than the computational style of mathematics that tends to be emphasized in the elementary grades. However, everyone has his or her own limit as far as mathematical ability is concerned. Learners could expect to experience some difficulties as their mathematical studies become more complex, and some anxiety is bound to occur as they confront such difficulties. Levels of anxiety are also dependent on the stakes involved. For example, a required mathematics course for a professional program can be extremely stressful, if successful completion of the program requires successful completion of the course.

(2) *What prior teaching styles have contributed to mathematics anxiety?*

When asked to construct an image of a mathematician, one student drew a classroom setting in which all of the students are sitting in rows, and the teacher is standing in front of the students, behind a blackboard, and is pointing at multiplication questions with some sort of stick. This is suggestive of the authoritarian teaching practices alluded to by Kennedy and Tipps (1994) as one of the origins of mathematics anxiety. Teaching styles that play a contributing role in this type of anxiety seem to relate somewhat to a math-anxious learner's

image of a mathematician and many of the participants understandably designed their image based on prior exposure to mathematics educators. Teaching styles involving strictness, putting students on the spot, displays of teachers' own brilliance, and a lack of humour or fun have been problematic for one participant. Another participant alluded to teaching styles that reflect a lack of creativity and poor voice modulation.

At least one participant made a conscious effort to avoid using teaching styles while student teaching that she was uncomfortable with as a student. As a student teacher, Terri avoided the lecture approach to teaching mathematics, spending as little time as possible in front of the classroom explaining something to the class. She preferred to have them work with concrete, manipulative objects like geoboards and gave the students the opportunity to position themselves anywhere in the classroom while working, so an observer in her classroom would see some students sitting down on the floor, or perhaps lying down on the floor while engaged in mathematical activities.

It also emerged from the workshop discussions that having the opportunity to work collaboratively was less stressful than working independently all the time. This would suggest that teachers who structure their lessons in such a way that all the mathematical learning experiences are in an independent role might be contributing to the problem.

(3) *How might the views of the participants evolve over the course of the workshop?*

Seven and a half hours of treatment was reasonably close to the ten hour time period suggested by Donady and Auslander (1980). It is the hope of the researcher that the discussions and activities will give the participants material for reflection. Through a

descriptive analysis of the Quilter and Harper (1988) survey, it was observed that fewer students reported finding that math questions were threatening in the post-treatment administration of the survey. Also during this post-treatment survey, fewer participants disagreed with the idea that mathematics is creative. Some differences in the items of the survey between pre-treatment and post-treatment seemed to the researcher to be minimal. Near the end of the workshop the participants seemed slightly more inclined to accept the idea that mathematical activities could effectively contain the element of a puzzle. When questioned about the effectiveness of magic squares, the participants agreed that they serve the best of both worlds in the sense of being a presentation of a fundamental aspect of mathematics, namely addition, but the nature of this presentation is that of a puzzle.

On the anecdotal evaluation form, one participant commented that he or she no longer felt isolated with the problem of mathematics anxiety. Others noted the value in having the opportunity to discuss their feelings amongst each other, which might suggest that they no longer feel as though they are the only ones who suffer from this problem. Perhaps the most significant evolution for the participants could be defined as a heightened awareness of the commonality of mathematics anxiety.

Problem:

What impact would a workshop have in altering attitudes and conceptions of mathematics that would in turn reduce mathematics anxiety?

Reflection occurred during the course of the workshops, and the researcher is hopeful that the experience of participating in the workshop will lead to further reflection on the part of the participants. Perhaps the greatest function that a workshop of this nature could

provide would be to increase people's awareness of the problem and provide a situation whereby the causes of mathematics anxiety could be explored and possible solutions to the problem could be discussed.

The impact that a workshop has on its participants and the instructor is dependent on its content and structure. Based on the anecdotal evaluation forms, the workshop might be modified for the better if there were to be a greater emphasis on techniques for solving problems. One favourable aspect of the workshop was the incorporation of hands-on mathematics activities. This would suggest that such activities should be an integral part of any mathematics anxiety workshop, thus providing an opportunity for participants possibly to equate the act of doing mathematics with having fun. Impact could be maximized if the participants could leave a workshop with experience of alternative approaches to doing a variety of mathematical problems. The value of the workshop stems from the potential it has in triggering reflection within the participants to examine their own attitudes about mathematics and how such attitudes might be contributing to their own anxiety. A workshop administered to Bachelor of Education students, allows for an understanding their own anxiety which is a crucial step in helping their future students deal with their anxiety.

Strengths and Limitations of the Workshop

On the basis of the anecdotal evaluation forms, the strengths of the workshop were:

- (1) The opportunities of the participants to learn from others (*The researcher was pleased to discover that the contributions of the participants made for a positive learning experience.*)
- (2) The comfortable small group setting (*The small group size allowed all participants the opportunity to express themselves and get to know each other.*)
- (3) The discussion of the phenomenon of mathematics anxiety (*The workshop discussions involved examining what mathematics anxiety is and from where it originates.*)
- (4) The opportunity to participate in hands-on activities (*The hands-on activities were undoubtedly a welcome change from a merely abstract manipulation of symbols.*)
- (5) The opportunity to socialize (*Hard working people like the participants may not find much time to socialize, but if an opportunity arises that combines practical and social benefits they may wish to find the time to participate.*)

The limitations as identified by the participants on the anecdotal evaluation forms were:

- (1) An insufficient number of word problems and breadth of methods presented to deal with word problems (*This comment suggests that word problems are a*

major contributing factor to mathematics anxiety and that strategies for analyzing word problems may not have been fully addressed by the instructor of this workshop.)

- (2) *The problem of attendance (This was inevitable given the participants' busy schedules. This limitation might be linked to a group desire for the regular contributions of certain group members. The heavy workload of Bachelor of Education students also meant that the workshop had only a small number of participants.)*
- (3) *The absence of a test for mathematics anxiety (The participant might be referring to a survey in which his or her anxiety could be rated on some kind of scale. The researcher believed that administering a mathematics anxiety scale would have taken valuable time away from discussion and mathematical activities.)*

Suggestions for modification of the workshops included:

- (1) *Spending more time examining ways to eliminate mathematics anxiety (It is doubtful that mathematics anxiety can be eliminated but some of the ideas raised on the workshop, like the use of hands-on activities, might be implemented with the effect of reducing this anxiety.)*
- (2) *Alternative approaches to problem solving (Students might feel more comfortable if they realize that there is often more than one way to solve a given problem.)*

- (3) Alternatives to testing (*When evaluating in mathematics, it might be beneficial to examine ways of doing this that are more creative and less stressful than tests. For example, students could compose short essays on the connections between mathematics and art, give oral presentations on the practical application of particular concepts, or investigate the history of mathematics.*)

One major limitation of the workshop as conducted was the small sample size (N=7). Due to the small sample size, the results cannot be generalized, in the same way that a predominantly quantitative study might be generalized. Merriam, (1988), raised the question: "Is generalization from a single case possible?" (p.174). Merriam referred to Stake's (1978) idea of naturalistic generalization, in which one can take a detailed knowledge of the particular and look for similarities in a new context. Merriam also alluded to Wilson's (1979) idea of reader or user generalizability which "involves leaving the extent to which a study's findings apply to other situations up to the people in those situations." (p.177). However, the small sample size allowed the instructor to examine the mathematics anxiety of the participants in considerable depth, thus creating a rich supply of data from this case study. The data gave rise to themes that have not been adequately addressed in the literature. Thus, this study represents a significant contribution to the understanding of mathematics anxiety.

Recommendations

Two types of recommendations need to be discussed. One concerns the alleviation of mathematics anxiety. The other concerns possible avenues for further research.

Mathematics Anxiety and Its Alleviation

The goal of alleviating mathematics anxiety mandates a responsibility on the part of both the teacher and the learner. The mathematics teacher needs to be observant and make an effort to look for signs of mathematics anxiety in his or her students. An important question needs to be raised. Do any of the students appear tense when writing a test or examination? If there are no non-verbal cues, such as fidgeting or a worried facial expression, indicating anxiety during such a situation, a teacher might ask the students how they are feeling. Then, an elementary school teacher may wish to probe the students' feelings if he/she gives his/her students a mathematics test and, if the student were feeling tense or scared, he/she would take the test away from them and re-teach the material. This may not be appropriate for secondary school mathematics teachers, who have a large amount of material to cover within a limited time frame. However, such teachers also need to have some awareness of their students' feelings, and if necessary alter the style of teaching. All mathematics teachers need to find ways to tie mathematical concepts to practical situations. Such practical aspects could be reinforced by providing students with opportunities to engage in a variety of hands-on activities, some of which could be done in small groups. Furthermore, some time needs to be spent helping students learn metacognitive strategies for solving word problems. Beyond the instructional aspect of teaching, teachers might reduce the anxiety associated with mathematics assessment if they implement a contract system. Such contracts would guarantee a passing grade if certain requirements are met.

One of the goals of mathematics teaching should be to have students become aware of their own styles of learning. This means encouraging students to engage in ongoing

reflection. For example, students could keep a mathematics journal in which they record their thoughts and feelings while engaged in mathematical tasks. This might help students to answer the following question: Why am I experiencing this anxiety? If the learner could isolate the negative experiences that triggered the anxiety, he or she might attempt to separate the experiences from his or her perceptions of his or her mathematical ability. The learner might also reflect on whether or not his or her thoughts and attitudes regarding mathematics are really justifiable.

In order to alleviate or reduce mathematics anxiety, mathematics curricula must be reviewed and possibly revised. Certain questions should be asked. Is the content of a given mathematics course suitable for the needs and interests of students? Does it take into account the practical aspects of mathematics? Are the objectives or outcomes reasonable given the time frame of the course? Have the assessment instruments used to determine the extent to which those objectives or outcomes have been reached chosen wisely? Such questions might be used as a guide for any curriculum reform effort that would take into account mathematics anxiety. The 1997 Ontario Curriculum, Grades 1-8, Mathematics addresses the importance of using a wide variety of theoretically sound teaching strategies, including the use of concrete manipulatives. Some practical applications, like connections between number sense and grocery shopping are recognized, but further suggestions in this regard might be helpful. Teachers could reflect on the questions addressed here and attempt to fit concepts to the lived experiences of their students. Stallings and Tascione (1996) studied student self-assessment and self-evaluation which, according to the researchers, find support from NCTM's 1995 Assessment Standards. Stallings and Tascione stated that, "According to the Assessment

Standards, student self-assessment can be used to improve student's confidence in their ability to do mathematics and allow them to become more independent in their learning of mathematics." (p.548). Recognition of such alternative forms of assessment would suggest that NCTM (National Council of Teachers and Mathematics) is conscious of the need to deal with the problem of mathematics anxiety.

A workshop should be a medium for the discussion of mathematics anxiety and the instructor should search for themes that emerge from that discussion. These themes could then be probed for ways of dealing with this problem. As an outcome of this workshop, the researcher recommends that mathematics courses be aligned with practical concerns, suggest strategies for analyzing word problems, and mix independent and cooperative learning tasks.

Recommendations for Further Research

The workshops that were central to this study, while providing some insight into the phenomenon of mathematics anxiety, have also generated a number of questions that could serve as avenues for further research. One such question would be as follows: How might a mathematics anxiety workshop be created, taking into consideration time frames, content, organization, and management in order to maximize its value? Such concerns were addressed through the anecdotal evaluation surveys and the researcher's reflections. Clearly, such a workshop could be designed in a number of different ways, and this type of workshop should have sufficient internal flexibility to allow for different needs and desires on the part of the participants. An emphasis on word problems and a more compact schedule (perhaps sessions of two and a half hours over four days) would have been helpful to the researcher's

participants. One might pay particular attention to the nature of the workshop activities. This particular set of workshops focused on activities that were mathematical in nature, but this is not the only possible approach. Other approaches are more psychological in nature. For example, Walker (1981) conducted a mathematics anxiety workshop in Austin, Texas for high school students in which the emphasis for the entire five sessions was on discussions of fearful mathematics experiences, and the physical and emotional effects of the resulting anxiety. Perhaps a workshop of this nature would better suit some individuals than a workshop that is more mathematically intensive. The researcher proposes that two streams of workshops could be offered. One would focus on discussion, the other would focus on mathematical activities. Participants could be identified for a particular stream by administering a survey and analyzing the results.

The mathematics poems that the participants wrote during this study served as a remarkably rich source of information. To what extent could math poetry be useful in probing the nature of mathematics anxiety? Age and writing ability would certainly be major factors for consideration. In this particular situation, the participants were all students in the Faculty of Education, which strongly suggests a considerable amount of ability in articulating thoughts and feelings in written form. Other groups of participants might struggle with written expression and might become anxious while dealing with such a struggle. However, such participants might be able and comfortable expressing themselves through other creative modes of expression like drama, music, or visual art. Poetry and other creative modes of expression might also be implemented into the mathematics classroom, whereby a teacher would encourage his or her students to articulate not only mathematical concepts and ideas,

but thoughts and emotions related to the act of doing mathematics, in creative fashion, and in so doing may help students not only to retain mathematical knowledge, but also to reduce any mathematics anxiety that they might be experiencing.

Cyndi Lynn's contributions to the workshops have inspired a question of some significance. To what extent do university students majoring in mathematics have to cope with mathematics anxiety? While such students have chosen mathematics as their academic focus, there may be a significant number of such students who might be reacting to the opposing forces of attraction and frustration and in turn confirming and doubting their own mathematical ability. A comparative analysis might be undertaken whereby self-identified math-anxious non-mathematics majors and math-anxious mathematics majors would be studied on the basis of their level of mathematics anxiety, the underlying causes of this anxiety and their coping mechanisms for dealing with this anxiety. Communication of the results of such a study might heighten an awareness among members of both groups of a common experience, which in turn might lead to reflection on the perceived dichotomy, composed of people who can do mathematics and people who cannot do mathematics, which Tobias (1987) claimed to be a false belief.

The Significance of this Study

The significance of this study is its potential to increase awareness of possible effectiveness of a workshop as a means of investigating the problem of mathematics anxiety. Given time limitations inherent in the design of such workshops, a workshop obviously does not constitute a cure to mathematics anxiety by itself, but if careful thought is given to its

content and structure, it might serve as a vehicle for reflection, and this reflection could in turn lead to increased self-confidence.

The problem of mathematics anxiety should not be underestimated. The comments made by all of the participants strongly suggest that the problem can have some severe side effects. Mathematics anxiety has manifested itself not only in terms of serious emotional effects but also physical effects. Yvonne reported feeling her hands tremble and sweat in situations involving mathematics, and Nancy described her reaction as visceral, meaning that severe anxiety the night before a mathematics test would cause her to vomit. A workshop could serve the needs of math-anxious people by offering a time and a place for sufferers of mathematics anxiety to meet and discuss these debilitating effects. Such a workshop might be of particular value to math-anxious teachers or student teachers, for if their problem is not solved, they may find themselves unconsciously becoming part of the problem.

Conclusion

What was there to learn from the experience of participating in and conducting the mathematics anxiety workshop? The participants learned that they are not alone in their suffering of mathematics anxiety. They also seemed more inclined to accept the idea that doing mathematical work can be a creative, experimental endeavour, and that mathematics is more than just a collection of rules and formulas.

The instructor has concluded that the information acquired during the workshop not only reinforces the literature but to some extent augments the literature. The need to make mathematics practical, referred to in the literature, was a central concern for the participants.

The perceived invasive nature of mathematics is a theme that offers a unique perspective in the problem of mathematics anxiety. Also, from a purely personal point of view, the researcher discovered he is not the only mathematics major who has had to cope with mathematics anxiety. The workshop reinforced the need for mathematics teachers to be cognizant not only of the anxiety felt by their students but also how the way in which they present themselves and their material contributes to that anxiety.

The value of the workshop is through the generation of an increased awareness of the complexity of this problem. Through this increased awareness, the instructor has learned that he needs to look for signs of mathematics anxiety in his future students and be careful that he structures his lessons and evaluations in such a way as not to increase the anxiety felt by the students. The participants, after learning more about their own mathematics anxiety, will likewise be sensitive to the needs and feelings of their future anxious students.

Appendix A: Anecdotal Evaluation

1. Are there any aspects of mathematics anxiety that were not covered in this workshop? If so, what are they?
2. What do you consider to be the strengths of this workshop?
3. What do you consider to be the weaknesses of this workshop?
4. How might the activities, presentations, discussions or format of this workshop be modified in order to make it more effective?

Appendix B: Quilter and Harper (1988) Survey

	Strongly agree	Agree	Undecided	Disagree	Strongly disagree
1. Mathematics is creative.					
2. I find math questions threatening.					
3. Maths is bewildering.					
4. I think guesswork has a part to play in solving mathematical problems.					
5. Maths would be alright if it was expressed in words instead of symbols.					
6. You can get a maths question half right.					
7. Maths problems can make your brain seize up.					
8. Mathematics teachers appear to be the high priests of some domain of secret knowledge.					
9. The trouble with maths is that it is too abstract.					
10. Minus numbers are daft.					
11. I don't panic about maths.					
12. The rightness or wrongness of my mathematical work is often obvious to me.					
13. I become embarrassed if I'm 'stuck' on a maths problem.					
14. Much of mathematics is an affront to common sense.					
15. Juggling around in maths lessons with symbols and numbers have no meaning.					
16. I find mathematical symbols (e.g. π , $\int e^{-\lambda x^2} dx$) can be frightening.					
17. I find it shameful if I can't do a piece of mathematics.					
18. 'x', the unknown, is full of mystery.					
19. People who can do maths are clever.					
20. Maths is not an experimental activity.					
21. I don't find maths frightening.					
22. Maths is irritating.					
23. High anxiety prevents me learning maths.					
24. As much can be learned from a wrong answer in mathematics as right one.					

Do you feel there were any particularly significant factors that led to your particular attitudes?

.....

.....

.....

Please add any other comments you may wish to make.

.....

.....

.....

PLEASE RETURN THIS FORM IN THE ENVELOPE PROVIDED

Appendix C Mathematical Activities

What follows are samples of problems and activities used in the workshop and selected from Sobel and Maletsky (1975).

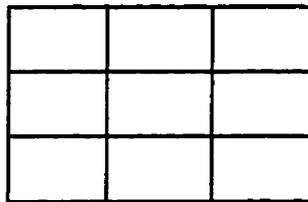
(A) From "Geometric Tidbits" (p. 39-40)

(i) How many pennies can you arrange such that each penny touches every other penny?

(ii) The digits 0 to 8 have been classified with the letters A, B, and C. How would you classify the digit 9?

A	1	4	7	
B	2	5		
C	0	3	6	8

(iii) How many squares are in this figure?



(B) From "Magic Square Activities" (p. 88-94)

Study these arrays. Then complete them so that they become magic squares.

16	2	12
	18	

8		14	3
15	2	9	4
	11		13

$1\frac{1}{6}$		
	$\frac{11}{12}$	
		$\frac{2}{3}$

Complete these magic squares⁷:

	1	
3		
		2

		8		
				10
			9	

			1			

See if you can complete these magic squares using the same set of numbers, 1 through 9. Remember, the numbers in each row, column, and diagonal still must add to 15

6		
2		4

	3	
6		2

		8
		1
	7	

		2
8	3	

		3
	1	8

		2
	5	
	1	

What happens when you add a constant to each entry in a magic square? Will you get another magic square?

5	6	1
0	4	8
7	2	3

⁷ Magic Squares are “numbers arranged in a square so that each row, column, and diagonal add up to the same total..” (Bendick, Levin, & Simon, 1965, p. 124.)

Start with the magic square above. In each case add the constant given to each entry to form a new array. Then check to see if it, too, is a magic square.

Add 1

Add 5

Add 25

Add $1/2$

Add $1 \frac{1}{4}$

Add 1.5

Does it appear that, when you add the same number to each entry in a magic square, another magic square is formed?

Now multiply each entry in the original magic square by these numbers.

Multiply by 9

Multiply by $3/4$

Multiply by 1.3

Does it appear that, when you multiply each entry in a magic square by the same number, another magic square is formed?

The entries in this magic square are integers. Show that the numbers in each row, column, and diagonal add to 0.

-3	2	1
4	0	-4
-1	-2	3

Start with the magic square given above and perform the operations indicated. In each case, see if the resulting array is also a magic square.

Add -3

Subtract 4

Add -1.5

Subtract -2

Multiply by -3

Divide by -2

**Multiply by $-\frac{1}{3}$ then
add 2**

**Divide by $\frac{3}{4}$
then subtract 1**

**Add $-\frac{3}{4}$ then
multiply by $-\frac{1}{2}$**

(C) From "Probability Experiments" (p. 127-129)
Each student draws a number line from -5 through 5. Then starting at 0 he moves one unit at a time to the right or left depending upon whether a tossed coin lands heads or tails. The first one to reach -5 or 5 tossing his own coin wins.

Reaching a 5 requires 5 heads; reaching -5 requires 5 tails. In 5 tosses, the probability of all heads is $\frac{1}{32}$, as it is for all tails. So if you allow just 5 tosses, the chances are 1 in 16 that a student will reach -5 or 5. In an average class, then, at least one student should be expected to win in just 5 tosses.

An interesting follow-up is to find how many in class landed at each number and compare these results with those from a list of all 32 ways in which the 5 tosses could fall.

		Probabilities
1	way to stop at -5 TTTTT	$1/32 = .03125$
5	ways to stop at -3 TTTTH TTTHT TTHTT THTTT HTTTT	$5/32 = .15625$
10	ways to stop at -1 TTTHH TTHTH THTTH HTTTH TTHHT THTHT HTTHT THHTT HTHTT HHTTT	$10/32 = .31250$
10	ways to stop at 1 HHHTT HHTHT HTHHT THHHT HHTTH HTHTH THHTH HTTHH THTHH TTHHH	$10/32 = .31250$
5	ways to stop at 3 HHHHT HHHTH HHTHH HTHHH THHHH	$5/32 = .15625$
1	way to stop at 5 HHHHH	$1/32 = .03125$
		1.00000

(D) The Game of "22" (p. 50)

A game is played by using a set of 16 cards consisting of the four aces, four 2's, four 3's, and four 4's.

Players alternate selecting one card at a time from the pile of 16 cards, without replacements. As before, cumulative sums are kept. The winner is the first person to select a card that brings the total to exactly 22, or forces his opponent to go over 22. The set of numbers here is {2, 7, 12, 17, 22}. The strategy for winning is to go first and begin with 2. There after, select the complement of your opponent's number relative to 5. That is, if he picks 4, you chooses 3. This would force you to repeatedly choose 2 and you would run out of 2's prior to reaching the objective number of 22. Can you consider alternative strategies for such a situation?

(E) From "Polyominoes" (p. 72-74)

A polyomino is merely a set of squares connected along their edges. The simplest form is a single square, called a monomino. Two connected squares are called domino, three squares are called a triomino, and four connected squares are called a tetromino. In this enrichment topic we are concerned with the total number of possible arrangements of such figures that are not congruent to one another.

Supply students with graph paper since this is the most convenient way of drawing and studying polyominoes. Together with the class, demonstrate the following figures and arrangements.

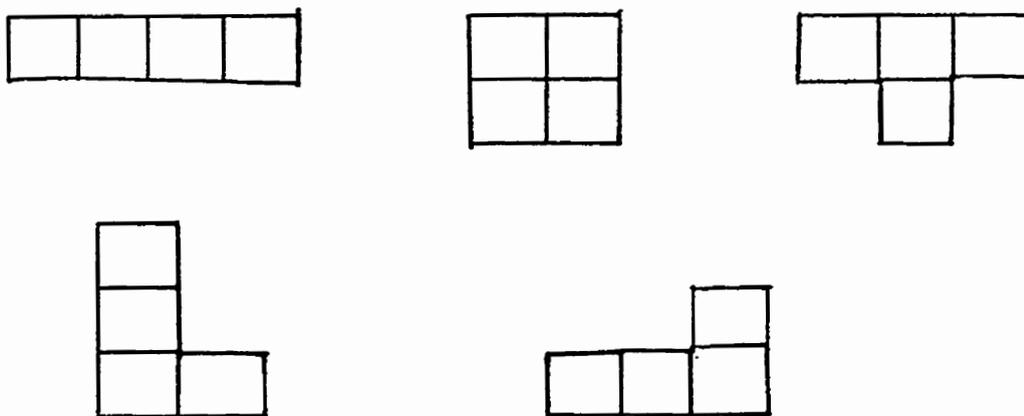
There is only one type of monomino and one domino.



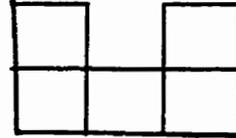
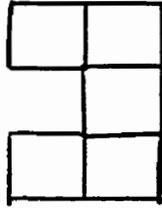
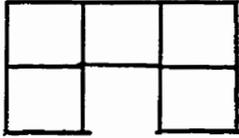
There are two possible arrangements using three squares.



There are five possible tetrominoes.



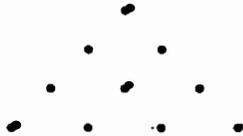
Having offered a class the preceding exposition, ask them to find all the possible pentominoes, figures formed by five connected squares. Caution them not to include any that are congruent to one another. For example, each of the following consists of just a single arrangement.



Participants experimented with the construction of polynomials, using graph paper, made up of four, five, and six squares.

(F) From Geometric Tidbits (pp 39-40)

(a) Move just three dots to form an arrow pointing down instead of up.



(b) Form four equilateral triangles with just six toothpicks.

(c) How many pennies can you arrange such that each penny touches every other penny?

(d) To mount a picture two thumbtacks are needed in any two corners. What is the least number of tacks needed to mount four pictures?

(e) Rearrange three toothpicks to form a figure that consists of three squares of the same size.

(f) Rearrange three toothpicks to form a figure that consists of five squares of the same size.

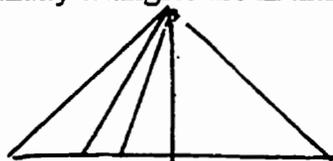
(g) Here the letters A through H have been classified with the digits 1, 3, and 5.

1		A E F H
3		C
5		B D G

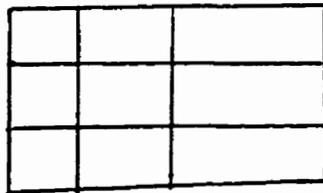
(h) Without lifting your pencil from the paper, try to draw four connected lines that pass through all nine points. Remember, the lines must be straight.



(i) How many triangles are in this figure?

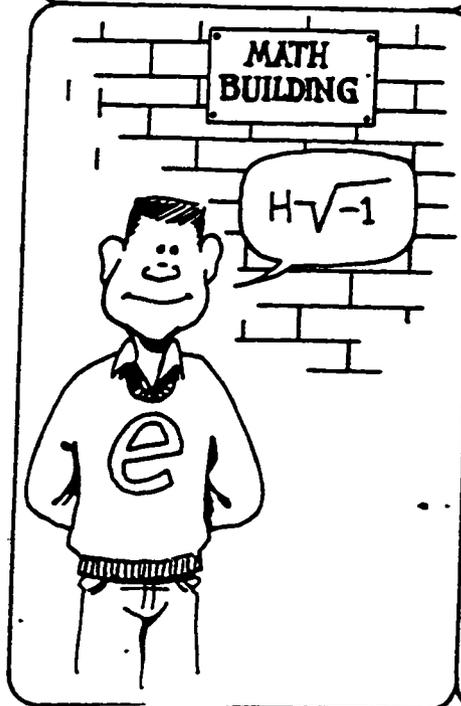
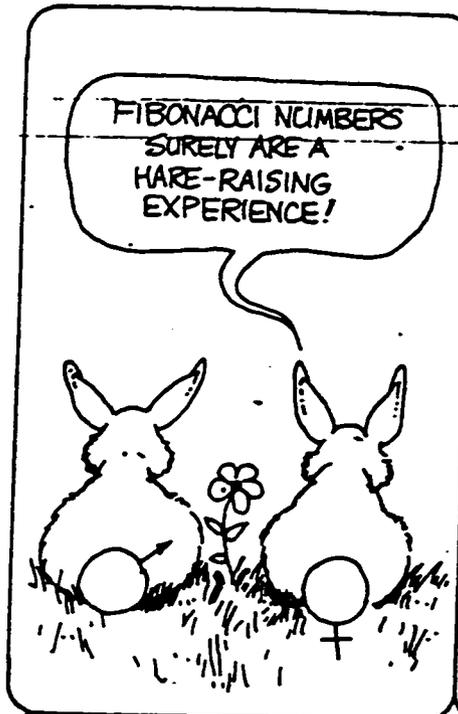


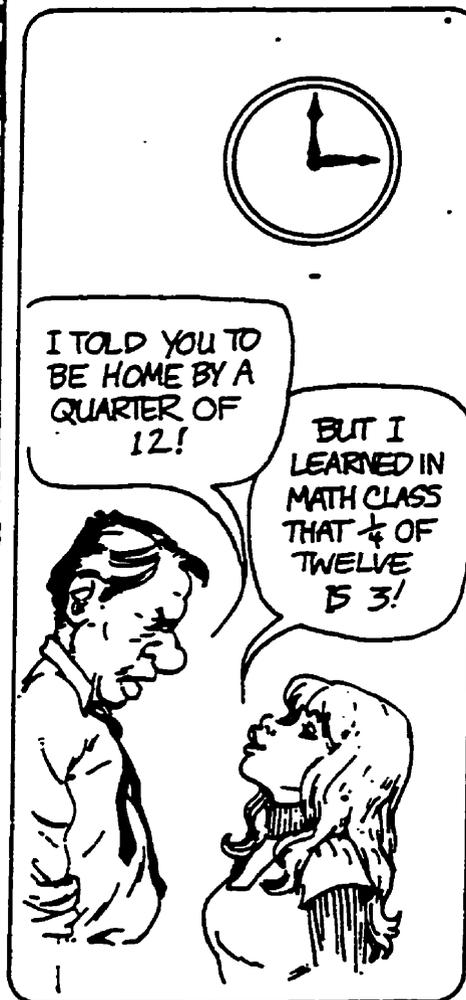
(j) How many rectangles are in this figure?

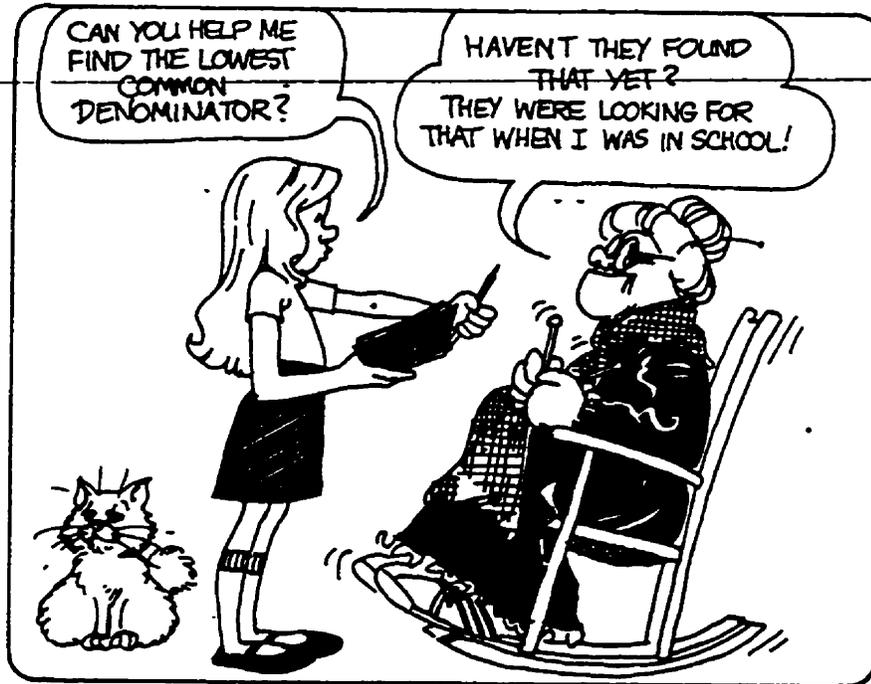


Appendix D

Cartoons (Azzolino, Silvey & Hughes, 1986)







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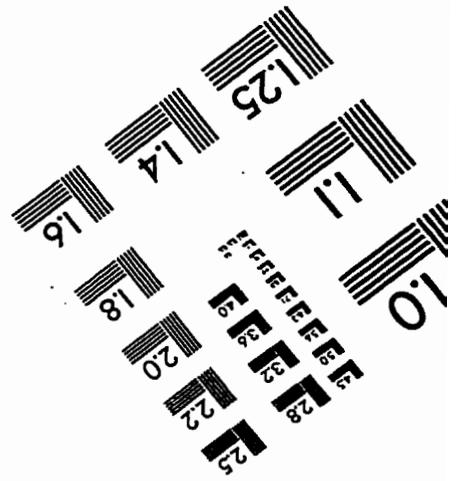
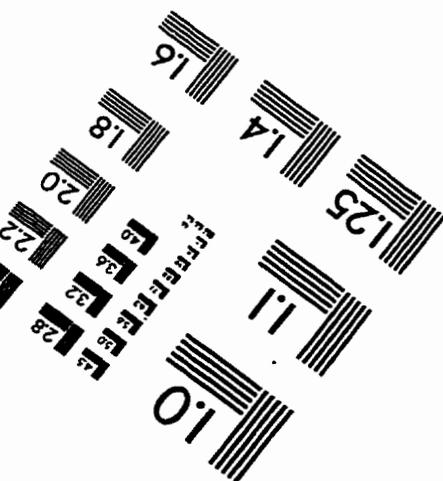
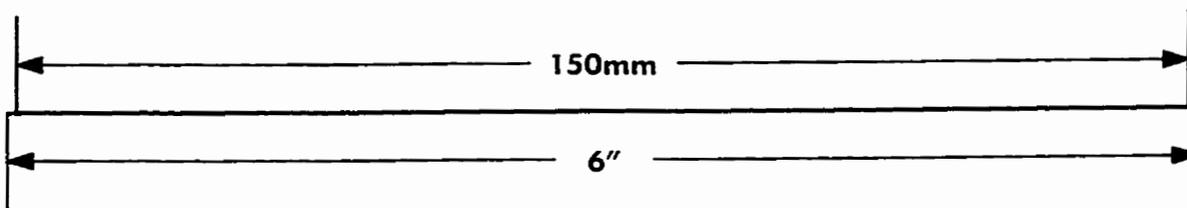
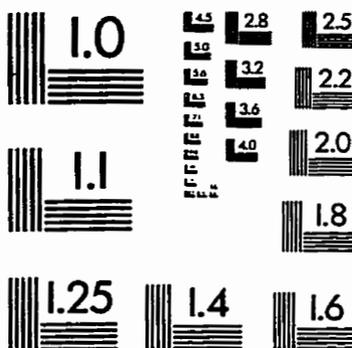
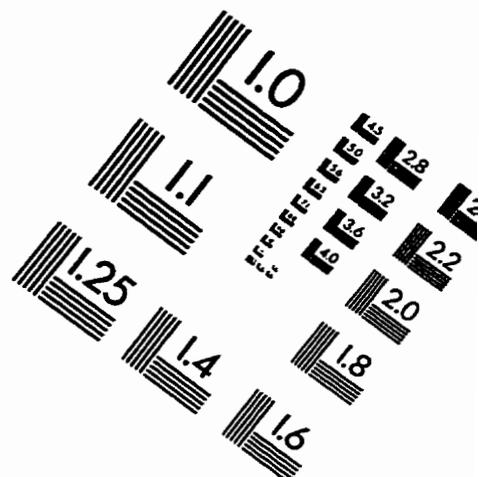
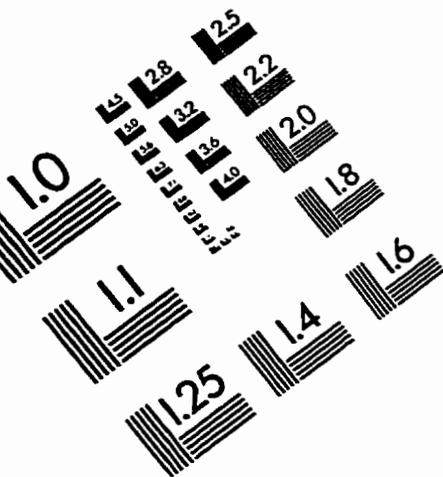
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