An Inclusive Model of Ferromagnetic Hysteresis

by

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A Thesis

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the Degree of

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Dedication

I wish to dedicate this work to my immediate family and as well to the extended Phelps family. I am priviledged that my grandfather Charles Phelps, soon to be 90, will be here to see the completion of this thesis: the first Ph.D. thesis ever written in the Phelps family.

Abstract

A new inclusive macroscopic model of ferromagnetic hysteresis is proposed. The model is developed from a Stoner-Wohlfarth approach by adding mean field or nearest neighbour dipole-dipole interactions. Pinning of domain rotation is also postulated, and a rotational pinning extension included. The model includes the principal features of the Jiles-Atherton model in the previous Atherton-Beattie extension of the Stoner-Wohlfarth model, but still omits the domain wall energy effects included in the Globus model. The new model describes both reversible and irreversible processes, and hysteresis caused by combinations of interaction, anisotropy, and pinning. Computational approaches to both two and three dimensional calculations are detailed, and examples given. Simulations of hard magnetic materials are done, including major loops to near saturation, minor loops, and demagnetizations. The complete 2×2 magnetization tensor response is shown, including fan diagram representations. The minor loop simulations involve complicated sets of field turning points typical of the Preisach model, and the minor loops are seen to exhibit incongruence and eventual closure. The demagnetization simulations are done for both rotating and oscillating applied field cycles. Both isotropic and anisotropic polycrystalline easy axis distributions are treated.

Table of Symbols

α	mean field parameter
$\underline{\alpha}$	reduced mean field parameter
$\vec{B}$	total magnetic flux density [T]
E_A	anisotropy energy [J/m^3]
E_H	magnetostatic energy [J/m^3]
E^i	total energy of particle i [J/m^3]
$\underline{e}^i$	reduced total energy of particle i
$\underline{E}$	total energy of array
E_{mf}^i	mean field energy of particle i [J/m^3]
$\underline{e}_{mf}^i$	reduced mean field energy of particle i
E_{nn}^i	nearest neighbour energy of particle i [J/m^3]
$\underline{e}_{nn}^i$	reduced nearest neighbour energy of particle i
$\frac{\partial \underline{e}^i}{\partial \phi^i}$	derivative of $\underline{e}^i$ w.r.t. ϕ^i
$\frac{\partial \underline{e}_{mf}^i}{\partial \phi^i}$	mean field contribution to derivative
$\frac{\partial \underline{e}_{nn}^i}{\partial \phi^i}$	nearest neighbour contribution to derivative
$\hat{e}^i$	easy axis unit vector of particle i
$\{\hat{e}^i\}$	set of easy axis unit vectors for array
$f(x)$	function to be minimized by Brent's method
$\vec{H}$	total magnetic field strength [A/m]
$\vec{H}_{applied}$	external applied magnetic field strength [A/m]
$\vec{H}_{nn}$	nearest neighbour contribution to $\vec{H}$ [A/m]
$\vec{H}^j(dipole)$	magnetic dipole source from particle j [A/m]
$\vec{h}_{applied}$	reduced form of $\vec{H}_{applied}$
$h_{applied}$	magnitude of $\vec{h}_{applied}$
$\vec{h}_{nn}$	reduced nearest neighbour contribution to $\vec{h}_{applied}$
$h_{nn\ x}$	x -component of $\vec{h}_{nn}$
$h_{nn\ y}$	y -component of $\vec{h}_{nn}$
$\vec{h}^j(dipole)$	reduced form of $\vec{H}^j(dipole)$
$h_x^j(dipole)$	x -component of $\vec{h}^j(dipole)$
$h_y^j(dipole)$	y -component of $\vec{h}^j(dipole)$
k	pinning parameter
K_{u_1}	uniaxial anisotropy constant [J/m^3]

$\vec{M}$	total averaged magnetization of array $[A/m]$
$\vec{m}$	reduced form of $\vec{M}$
m_x	x -component of $\vec{m}$
m_y	y -component of $\vec{m}$
$\vec{m}(\text{old})$	old value of $\vec{m}$ in the iteration
$\vec{m}(\text{new})$	new value of $\vec{m}$ in the iteration
$\vec{M}^i$	intrinsic magnetization of particle i $[A/m]$
$\vec{m}^i$	reduced form of $\vec{M}^i$
$\hat{m}^i$	unit vector form of $\vec{m}^i$
$\{\hat{m}_0^i\}$	set of initial $\hat{m}^i$ vectors for array
m_{iJ}	magnetization tensor (2×2) i is the index of the applied field component J is the index of the magnetization component
m_s	saturation magnetization $[A/m]$
μ_0	permeability of the vacuum $[H/m]$
N	total number of particles in array
Φ	angle between $\vec{M}^i$ and $\hat{e}^i$
ϕ_h	angle of external applied magnetic field
ϕ^i	angle of intrinsic magnetization of particle i
ϕ_{ea}^i	angle of easy axis of particle i
$\{\phi_{ea}^i, \phi_0^i\}$	complete initial description of array
$\bar{\phi}_{ea}$	mean of $\{\phi_{ea}^i\}$
$\{\bar{\phi}_{ea}, \bar{\sigma}\}$	bulk description of array
$\vec{r}_{ji}$	displacement vector from source dipole j to dipole i
r_{nn}	nearest neighbour maximum interaction radius
s	rectangular array spacing parameter
$\bar{\sigma}$	standard deviation of $\{\phi_{ea}^i\}$
$\vec{\tau}^i$	torque acting on particle i
τ^i	magnitude of $\vec{\tau}^i$
$ \vec{\tau}_+ $	magnitude of magnetic torque
$ \vec{\tau}_- $	magnitude of pinning torque
(x^i, y^i)	coordinates of particle i in array

Statement of Originality

The contents of this thesis are solely the work of the author, and of the author in collaboration with his supervisor, D.L. Atherton. The work presented in chapter 2 is review. A shortened version of the work in chapters 3 and 4, as well as the results in chapter 5, and the conclusions in chapter 6 were submitted for publication in *IEEE Trans. Mag.* as [Phelps-1998a], [Phelps-1998b], and [Phelps-1999]. The work presented in appendices A and B is also review.

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*Its viewless, potent virtues men surprise;
Its strange effects, they view with wond'ring eyes
When without aid of hinges, links or springs
A pendant chain we hold of steely rings
Dropt from the stone - the stone the binding source -
Ring cleaves to ring and owns magnetic force:
Those held above, the ones below maintain,
Circle 'neath circle downward draws in vain
Whilst free in air disports the oscillating chain.*

*-Lucretius (99-55 B.C.)
De Rerum Natura, Book VI*

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Chapter 1

Introduction: Outline of Thesis

This thesis is a presentation of a new inclusive model of ferromagnetic hysteresis. It is based on the Stoner-Wohlfarth model of 1948, [Stoner-1948], which originally modeled noninteracting collections of magnetic particles. The new inclusive model, however, has several new extensions that enable modeling of interacting systems of magnetic particles. The power of current computing is harnessed to make possible computational simulations of the magnetic behaviour of macroscopic ferromagnetic materials far beyond what was originally possible in 1948.

Chapter 2 is a literature review. An historical overview of magnetism is given. The texts *Ferromagnetism* [Bozorth-1951], *Physics of Magnetism* [Chikazumi-1964], *Introduction to Magnetic Materials* [Cullity-1972], and *Introduction to Magnetism and Magnetic Materials* [Jiles-1991] provided much of the information covered. This section is followed by a review of the recent literature on modern models of ferromagnetism, including the Stoner-Wohlfarth model and various extensions to it, as well as competing models. Current applied problems in magnetics that require a model of ferromagnetism in order for a solution to be developed are also discussed, suggesting possible future applications of the inclusive model.

Chapter 3 is a detailed mathematical development of the inclusive model. The mathematics describing the original Stoner-Wohlfarth (S-W) model [Stoner-1948] is developed first. This is referred to as the classic S-W model, and contains no interactions. The various extensions to the classic S-W model, which constitute the inclusive model,

and which bring physical interactions into the picture are then developed, including the mean field interaction of the Atherton-Beattie model [Atherton-1990], a nearest neighbour dipole-dipole interaction, and a pinning interaction. It is postulated that rotational friction may possibly be present in rotation of domain magnetization, and the pinning interaction is based on this idea. The model is first formulated in terms of vector equations. A 2D geometry is then defined, and a precise 2D formulation is then extracted. Complicated sets of applied field turning points typical of the Preisach model [Preisach-1935] are considered.

Chapter 4 covers the details of constructing a numerical model based on the mathematical formalism. Minimization and calculation through iteration problems are discussed. The methods of numerically describing magnetic particle arrays with isotropic and anisotropic easy axis distributions are covered. The model parameters that describe the magnetic material properties are also covered. The different types of applied field cycles that can be used are listed. Finally, the 2×2 $m_{i,j}$ magnetization tensor is described, as in this thesis we are interested in the complete tensor magnetization response to general applied field cycles in 2D.

Chapter 5 contains the numerical results from three different computational studies. These include the study of major loops to near saturation, minor loops, and demagnetizations. The effects of isotropic versus anisotropic, as well as noninteracting versus interacting magnetic particle arrays are investigated. In the case of major loops to near saturation, the mean field, dipole-dipole, and pinning interactions are investigated. In the case of minor loops, the effect of the pinning interaction is investigated. In the case of the demagnetization simulations, both rotating and oscillating demagnetizations are performed, and the helping or hindering effects of the mean field, pinning, and combined mean field and pinning interactions are investigated. Results are displayed in the form of hysteresis loops and fan diagrams [Zhang-1995]. In some cases, the complete 2×2 magnetization tensors are shown.

Conclusions are drawn in chapter 6

Chapter 7 contains proposals for further work. It is hypothesized that one source of Magnetic Barkhausen Noise (MBN) may be discontinuous rotations of domain magnetization, in addition to the usual discontinuous displacements of domain walls. It is

therefore suggested that the inclusive model, with its rotational pinning as a mechanism of discontinuous rotation of domain magnetization, may be used to model this form of MBN. The mathematical formalism for extracting MBN energy from a hysteresis loop generated by the model is developed. A proposed 3D formulation of the model is also developed. The 3D version follows from the same fundamental vector formulation as the 2D version, but with the definition of a 3D geometry instead. The more complicated numerical considerations in 3D are discussed. Finally, it is suggested that a full-fledged 3D model could be calibrated to experimental data for a real 3D sample, using a chi-square fitting approach to find the values of the model input parameters that provide the optimum fit. The mathematical formalism is developed, and the numerical details are considered.

The appendices describe in detail the numerical techniques that were used to perform the computations in this study. Appendix A describes Brent's Method of Minimization, which was used to minimize the equation of total potential energy of a single magnetic particle, in one variable. Appendix B describes in detail the Monte Carlo techniques that were used to generate the random numbers that were sometimes needed as input to the inclusive model. Methods for generating uniform distributions and gaussian distributions are explained. The text that provided most of the information for these two appendices, as well as the actual numerical routines linked to the model code was *Numerical Recipes in C: The Art of Scientific Computing, Second Edition* [Press-1992].

Chapter 2

Literature Review

2.1 Historical Overview of Magnetism

2.1.1 The Ancient Era

The ancients of western civilization knew of electricity and magnetism as early as the classical period. To them, electricity and magnetism were separate natural phenomena.

The lodestone was known to be a natural magnet, in the classical period, several centuries before Christ. It was quarried in Asia Minor, in Magnesia, on the Aegean coast. Hence the rock was given the name 'magnetite', and its peculiar properties came to be known as 'magnetism' [Still-1946].

The study of magnetism was one pursuit for academics of that age [Roller-1959], [Still-1946]. They knew of the attractive and repulsive properties of magnets, yet it is uncertain whether or not they discovered its use as a compass. Mention of the use of a magnet as a compass has been attributed to many of the ancient writers, [Roller-1959], but the writings are very vague on that point.

By the middle ages, the study of magnetism had become steeped in superstition. Those who studied magnetism in this era tended to do so from the point of view of occultism, magic, and alchemy [Roller-1959]. The religious authorities not only

tried and put to death those suspected of practicing witch-craft, but also tended to persecute any thinkers who attempted to profess new scientific knowledge, under charges of heresy [Still-1946].

One of the earliest available written works on the navigational use of magnetite is *The Letter of Petrus Peregrinus, On the Magnet* written in 1269 A.D. [Peregrinus-1269]. It contains instructions on how to grind a magnetite sample into a sphere, how to identify and mark the magnetic poles, and how to then use it as a compass at sea by attaching it to floats and floating it in a bucket of water. The surface of the water was seen to always be horizontal, regardless of the angle the ship tilted at in the waves, and the floating sphere was seen to turn so that the magnetic poles aligned with the geographic poles of the earth.

The first scientific treatment of magnetism was by William Gilbert in the 16th century A.D. His famous treatise is entitled *De Magnete (On the Magnet)* [Gilbert-1600], published in 1600, in Latin. Among Gilbert's scientific contributions were the study of terrestrial magnetism, and the discovery that a magnet loses its magnetism at high temperatures. Gilbert's work brought the study of magnetism out of the realm of witch-craft, and into the realm of an accepted academic field of study. Gilbert was also a physician, and studied alternative methods of medical treatment using magnets.

The renaissance era saw further works on magnetism. One of these was the second paper in *Two Tracts on Electricity and Magnetism*, by Robert Boyle, in 1676 [Boyle-1675]. Electricity and magnetism remained as separate natural phenomena. The union of the two was not seen until the 19th century.

2.1.2 The Foundations of Electromagnetism

At the end of the 18th century, Coulomb discovered both the law of electrostatic interaction between two electric charges, and the law of magnetic interaction between two magnetic poles.

At the beginning of the 19th century, John Christian Oersted discovered that magnetic field were generated by an electric current in a wire [Oersted-1820]. Further work was

done by Ampere [Ampere-1827], and Biot and Savart, and resulted in the Ampere circuital law, and the Biot-Savart law, known today.

Later that century, Michael Faraday discovered the reverse effect, magnetic induction, in which an electric current was generated by a changing magnetic field [Faraday-1839]. Although Faraday expressed his discoveries in descriptive, rather than analytic mathematical language, his discovery was eventually formulated as the Faraday law of induction known today.

At the end of the 19th century, James Clerk Maxwell, in his famous *A Treatise on Electricity and Magnetism*, 1892 [Maxwell-1892], expressed the laws of electromagnetism as they are known today. The Faraday law of induction was improved by adding the displacement current term to the electric current term. Then the Gaussian laws for the electric field and the magnetic field, the Ampere circuital law, and the improved Faraday law of induction, were expressed in mathematical form as the four Maxwell equations [Lorrain-1988]:

$$\iiint \vec{D} \cdot d\vec{A} = \iiint \rho_f dV \quad (2.1)$$

$$\iiint \vec{B} \cdot d\vec{A} = 0 \quad (2.2)$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint \vec{B} \cdot d\vec{A} \quad (2.3)$$

$$\oint \vec{H} \cdot d\vec{l} = \iint (\vec{J}_f + \frac{\partial \vec{D}}{\partial t}) \cdot d\vec{A} \quad (2.4)$$

with the constitutive relations:

$$\vec{D} = \epsilon \vec{E} \quad (2.5)$$

$$\vec{B} = \mu \vec{H} \quad (2.6)$$

$$\vec{J}_f = \sigma \vec{E} \quad (2.7)$$

The natural phenomena of electricity and magnetism were at long last united into the one natural phenomenon of electromagnetism.

2.1.3 Early Theories of Ferromagnetism

The Maxwell equations as they stood described admirably the behaviour of electric and magnetic fields in homogeneous, isotropic, linear, and stationary (HILS) materials, where $\{\epsilon, \mu, \sigma\}$ were constants describing the given material. However, real materials were seen to not necessarily be HILS.

The study of magnetic materials began to address the $\vec{B} = \mu\vec{H}$ relation. We know today that a general stationary magnetic material is nonhomogeneous, nonisotropic, and nonlinear. μ must be a 3×3 tensor that takes these three complicated properties into account. We also know that the magnetic behaviour of the material is dependent on its magnetic history. Therefore μ must be history dependent as well.

While the close of the 19th century saw the completion of the theory of electromagnetism in the vacuum, the most ideal of HILS materials, the beginning of the 20th century saw the beginning of the study of real magnetic materials, and theories that would be needed to determine μ .

In describing the magnetic behaviour of magnetic materials, we prefer to use the 'magnetic susceptibility' χ , rather than the magnetic permeability μ . The magnetic susceptibility is defined as the ratio between the material magnetization $\vec{M}$ and the external applied field $\vec{H}$:

$$\vec{M} = \chi\vec{H} \tag{2.8}$$

The relation that contains $\vec{B}$, $\vec{M}$, and $\vec{H}$ together is:

$$\vec{B} = \mu_0(\vec{M} + \vec{H}) \tag{2.9}$$

where μ_0 is merely the permeability of the vacuum. The relation between the permeability and the susceptibility is therefore:

$$\mu = \mu_0(\chi + 1) \tag{2.10}$$

χ has all the same properties as μ , and is hence a 3×3 tensor, in its most general form. In the early work that is discussed next, however, χ is treated as a simple scalar that relates one component of $\vec{M}$ directly to the corresponding component of $\vec{H}$. It

is not until we discuss modern work that we will return to the tensor description.

In an early experiment in 1835, Arago attempted to magnetize a magnetic substance by using an electric current.

Warburg [Warburg-1881], in 1881, was the first to observe hysteresis, the nonlinear behaviour and history dependence described by the material permeability factor, μ , in iron. Frohlich [Frohlich-1881] and Kennelly [Kennelly-1891] made an early attempt at a purely phenomenological expression to describe anhysteretic curves, or, the magnetization curves in hysteresis-free materials. Lord Rayleigh [Rayleigh-1887], made an early attempt at a phenomenological expression to describe initial magnetization curves and small-amplitude hysteresis loops at low magnetic fields and Lamont [Lamont-1867], at high magnetic fields. An actual theory or model of ferromagnetism was still missing.

Curie investigated the magnetic properties of a number of paramagnetic solids [Curie-1895]. A paramagnetic material is a material in which neighbouring microscopic magnets (explained in the next paragraph), interact very weakly through a magnetic interaction. At finite temperatures the particles are thermally excited, and are free to take random orientations. When an external magnetic field is applied, their orientations tend to rotate slightly toward the direction of the applied field, resulting in a slight net material magnetization. For a given temperature, the magnetization $\vec{M}$ is directly proportional to the applied field $\vec{H}$. Curie formulated the Curie law of paramagnetism [Curie-1895], which states that the susceptibility is inversely proportional to the absolute temperature:

$$\chi = \frac{C}{T} \quad (2.11)$$

where C is a constant known as the Curie constant, and T is the temperature.

It was Ewing [Ewing-1900], who first systematically studied ferromagnetic hysteresis. Ewing was the first to use the term ‘hysteresis’, which means literally ‘to lag behind’ [Jiles-1991]. Ewing may also have been the first person to study magnetic phenomena from the atomistic point of view. He constructed a physical model of a ferromagnetic material, consisting of an array of magnetic compass needles, and studied its initial magnetization and hysteretic properties. He then similarly tried to explain hysteresis

in ferromagnetic materials in terms of magnetic interaction between microscopic or ‘molecular’ magnets in the magnetic material. A ferromagnetic material was hence known to be a material in which neighbouring microscopic magnets interact strongly through a magnetic interaction. At finite temperatures, they tend to align parallel with each other, even in the absence of an external applied magnetic field. When an external magnetic field is applied, their orientations tend to rotate significantly toward the direction of the applied field, resulting in a significant net material magnetization. Ewing also investigated the ‘magnetic aftereffect’, the time lag that is measured between the instant that the applied field $\vec{H}$ is changed, and the instant that the material magnetization $\vec{M}$ reaches its new equilibrium value [Ewing-1889].

Ewing was succeeded by Langevin [Langevin-1904], [Langevin-1905] and Weiss [Weiss-1906], [Weiss-1907]. Langevin gave the correct interpretation of paramagnetism, and Weiss gave the correct interpretation of ferromagnetism, from the atomistic point of view. Their theories [Chikazumi-1964] were based on statistical mechanics. In particular, Langevin formulated the relation between the magnetization of the paramagnetic material, and the temperature:

$$M = NmL(\alpha) \quad (2.12)$$

where M is the net magnetization of the material, N is the number of atomic magnetic moments in the system, m is the magnitude of the magnetic moments, (or, magnitude of the intrinsic magnetizations), and $L(\alpha)$ is the Langevin function:

$$L(\alpha) = \coth(\alpha) - \frac{1}{\alpha} \quad (2.13)$$

and α contains the temperature dependence, and is defined as:

$$\alpha = \frac{mH}{kT} \quad (2.14)$$

where m is (once more) the magnitude of the atomic magnetic moments, H is the external applied magnetic field, k is Boltzmann’s constant, and T is the temperature. The Langevin model was a classical theory, in which the atomic magnetic moments were free to orient themselves in any direction. The collection of N atomic magnetic moments was also noninteracting. From this model, the initial magnetization curve

(the plot of M vs H) of a paramagnetic material could be calculated, and the initial susceptibility χ (the initial slope of the curve) could be extracted, for a given temperature. The temperature dependent behaviour of the paramagnetic material, the Curie law, could also be extracted, namely:

$$\chi = \frac{Nm^2}{3kT} \quad (2.15)$$

Weiss extended the model to describe ferromagnetic materials. This was done by including interactions between the atomic magnetic moments. Weiss postulated a 'molecular field':

$$H_m = wM \quad (2.16)$$

where w is the proportionality factor. This meant that in the Langevin model, the substitution:

$$H \longrightarrow H + wM \quad (2.17)$$

was made, to include the molecular field interaction. The result was that the Langevin formulas stayed the same, except for the definition of α , which became:

$$\alpha = \frac{m(H + wM)}{kT} . \quad (2.18)$$

From this model, the initial magnetization curve of a ferromagnetic material could be calculated, and the initial susceptibility χ extracted. The temperature dependent behaviour of the ferromagnetic material was seen to be more involved. According to the relation between M and H , and the definition of α , as the temperature T increases, the magnetization M decreases, until it reaches zero at a critical temperature. This temperature, Θ , is called the Curie temperature, and is extracted from the model as:

$$\Theta = \frac{Nm^2w}{3k} . \quad (2.19)$$

Above the Curie temperature, the susceptibility increases from zero, and is inversely proportional to the temperature, but this time as:

$$\chi = \frac{Nm^2}{3k(T - \Theta)} . \quad (2.20)$$

This is known as the Curie-Weiss law.

These two models, in the first decade of the 20th century, were prophetic of modern models of ferromagnetism that would follow. The Jiles-Atherton model of ferromagnetism [Jiles-1983], [Jiles-1986] to be mentioned in a later section, is also a statistical mechanical model, and is based on the models of Langevin and Weiss. In it, the Langevin function is used to describe anhysteretic curves (the single-valued function relating M to H when no hysteresis occurs), and the ‘mean field’, an adaptation of the Weiss ‘molecular field’, is used to describe the magnetic interactions that result in ferromagnetic behaviour.

The next two decades saw little further development in theories of ferromagnetism in real materials. The world was occupied with the novel discovery of a whole new type of physics: quantum mechanics.

2.1.4 The Quantum Mechanical Era

In the era of quantum mechanics, a new theoretical framework for ferromagnetism was developed. The development proceeded in several stages, beginning with the theory of electrons and single atoms, and culminating in an actual quantum theory of ferromagnetism in metals.

In 1925, Uhlenbeck and Goudsmit [Uhlenbeck-1926] discovered that the electron has spin angular momentum. The spinning motion of the electrical charge could be classically thought of as the source of the observed magnetic moment of the electron, though classical theory failed to explain the quantization of electron spin into only one of two possible states: up or down, relative to an external magnetic field.

Researchers began to develop the quantum theory of the atom, to explain how the electron behaved when bound in orbit about a nucleus. The theory developed to the point that a given electron orbited a nucleus in a set pattern specified by its ‘quantum numbers’ $n = \{1, 2, \dots\}$, specifying the radial position, $l = \{0, 1, \dots, (n - 1)\}$ and $m_l = \{-l, -(l - 1), \dots, 0, \dots, (l - 1), l\}$, specifying the magnitude and orientation of the orbital angular momentum, and $s = \frac{1}{2}$ and $m_s = \{-s, +s\}$, specifying the magnitude and orientation of the spin angular momentum. They sought an explanation of the

magnetic moment of an atom in terms of its total angular momentum. Once again, according to the classical picture, the total angular momentum of the moving electron charges could be thought of as the source of the the total magnetic moment, though the classical picture failed to explain why the electrons were quantized to occupy only certain states, specified by integer and half-integer values of the quantum numbers.

In 1925, Pauli [Pauli-1925] postulated the ‘Pauli Exclusion Principle’, which dictated that no two electrons in an atom could occupy the same quantum state. This explained why all the electrons in an atom did not collapse together into the lowest possible energy state.

Also in 1925, Russell and Saunders [Russell-1925] developed the theory known as ‘Russell-Saunders Coupling’, which was a prescription for combining the orbital angular momentum numbers of all the electrons in an atom into a set of total orbital angular momentum numbers, $\{L, m_L\}$, and the spin angular momentum numbers of all the electrons into a set of total spin angular momentum numbers, $\{S, m_S\}$. These two sets of numbers could then be combined into a set of total angular momentum numbers $\{J, m_J\}$. The magnetic moment of the atom was then expressed as:

$$\vec{m} = g\sqrt{J(J+1)}\mu_B \quad (2.21)$$

where μ_B expresses the result in ‘Bohr magnetons’, and where g is the ‘gyromagnetic ratio’. The theoretical value of g was thought to be exactly 2. However, several experiments showed that g could be slightly less than or greater than 2. An overview of the gyromagnetic experiments and their results is contained in [Kittel-1986].

In 1927, Hund [Hund-1927] developed a set of rules now known as ‘Hund’s Rules’, for predicting the pattern that electrons would follow in filling the various available quantum states in an atom, in order to minimize the total energy of the atom.

The physics of the atom had by that point primarily been solved, though further work would prove to elucidate further improvements. The researchers of the time then began to apply quantum mechanics to the problem of ferromagnetism in materials.

In 1927, Brillouin [Brillouin-1927] developed a quantum correction to the Langevin and Weiss theories of paramagnetism and ferromagnetism respectively. He replaced

the Langevin function with the ‘Brillouin function’, which contained the total angular momentum J of the atomic moments being considered. The resulting expressions for the Curie temperature and the Curie-Weiss law hence contained this quantum correction related to J .

In 1928, however, Heisenberg [Heisenberg-1928] published the first full-fledged quantum theory of ferromagnetism, which was meant to replace the Weiss theory. In it, he provided a quantum mechanical explanation of the Weiss molecular field. In the original Weiss formulation, the interaction energy of a magnetic moment $\vec{m}_i$ with the ‘molecular force’ $w\vec{M}$ was:

$$E_i^{int} = -\vec{m}_i \cdot w\vec{M} \quad (2.22)$$

In the Heisenberg theory, the interaction energy of a magnetic moment $\vec{m}_i$ due to a localized electron i with the other magnetic moments due to other localized electrons is:

$$E_i^{int} = -2J\vec{m}_i \cdot \sum_{j \neq i} \vec{m}_j \quad (2.23)$$

where J is the ‘exchange integral’ which is calculated by integrating over the overlap of the wavefunctions of neighbouring electrons i and j , where it is assumed that J is identical for all electron pairs, and where the sum is carried out over all nearest neighbours j . (Or in some formulations, all the other electrons j .) The Weiss molecular field may then be considered to be:

$$w = 2J \quad (2.24)$$

The limitations of the Heisenberg model were that it did not include electron spin waves, and that it required that the electrons be localized at specific positions in space.

In 1930, Bloch [Bloch-1930] added electron spin waves to the theory, and improved the quantum mechanical explanation of the Weiss molecular field.

Stoner [Stoner-1933] and Slater [Slater-1936], in 1933 and 1936 respectively, developed the band theory of ferromagnetism, in which some electrons occupied a ‘conduction band’, and were free to move, not being localized to any one site in the material. This was an example of an itinerant electron model, as opposed to a localized electron

model.

Slater [Slater-1937] and Pauling [Pauling-1938], in 1937 and 1938 respectively, applied this theory to explain the magnetic moments of 3d metals and their alloys. The result was the ‘Slater-Pauling curve’, which gives the net magnetic moment per atom as a function of the number of 3d electrons per atom.

The difficulty of the itinerant electron model is that it is much more difficult to use for fundamental calculations than it is to use the simpler localized models of Weiss and Heisenberg.

In any event, further work was done by Zener, [Zener-1951], and extensions that explained branches from the main curve were made by Wohlfarth [Wohlfarth-1948] and Friedel [Friedel-1954].

Further developments in the quantum theory of ferromagnetism were made, and that branch of science grew into the field of solid state physics. However, the many theories that were invented and continue to be invented, are mainly suitable for describing the extremely idealized case of a single magnetic domain, in a single crystal, on the micromagnetic scale. The discoveries of magnetic domains and complicated magnetic domain patterns, as well as complicated polycrystalline structures, brought with them the need for more more complicated, and less idealized theories of ferromagnetism. The study of ferromagnetism was to move to a more macroscopic scale, and a return to the methods of classical physics was to be made.

2.1.5 Discovery of Magnetic Domains and Crystalline Structure

Some of the very early work in magnetism, before the quantum mechanical era, saw some hypotheses on the possibility of magnetic domains in magnetic materials.

In 1827, Ampere [Ampere-1827] postulated that the origin of magnetic moments was due to ‘electrical currents continually circulating within the atom’. This statement was made even before the existence of electrons was known, that negative and positive charges were distinct and separate within an atom, or even that atoms existed. This

was an important first step in the development of hypotheses on magnetic domains, as it was the first statement that elementary magnetic moments in a material had a permanent existence.

In 1852, Weber [Weber-1852] postulated that a magnetic material in a demagnetized state consisted of an array of permanent, small atomic magnetic moments randomly aligned to give a net zero vector sum. In an external applied magnetic field, the magnetic moments aligned with the field, and the material became magnetized.

In 1886, Wiedemann [Wiedemann-1886] postulated that the small atomic magnetic moments experienced a frictional resistance that opposed their tendency to align with the external applied magnetic field.

Ewing [Ewing-1900], as covered in an earlier section, was the first to systematically study ferromagnetic hysteresis. He explained the difference between an unmagnetized and a magnetized state in a fashion similar to Weber. He tried to explain the actual mechanism in terms of a microscopic dipole-dipole interaction between the magnetic moments, similar to the macroscopic dipole-dipole interactions in his array of compass needles. However, he did not realize that a magnetic material in an unmagnetized state is in fact already in a magnetically ordered state, with the atomic magnetic moments aligned in a common direction within a given magnetic 'domain', and that it was the domains themselves that could be considered randomly aligned.

In 1906 and 1907, Weiss [Weiss-1906], [Weiss-1907] made an early theoretical prediction of ferromagnetic domain structure, with his model of ferromagnetism, described in an earlier section. The Weiss molecular field was a very good approximation to the field acting on a given magnetic moment within a magnetic domain, since in a given magnetic domain, the many magnetic moments tend to be aligned parallel.

In 1919 Barkhausen [Barkhausen-1919] was the first to discover indirect experimental evidence of the existence of ferromagnetic domains. He detected the presence of small, discontinuous jumps in a magnetization curve, during the magnetization process. It was thought to correspond to the discontinuous switching and reorientation of individual microscopic magnetic domains. Today, this 'Barkhausen noise' is thought to be due to various magnetization mechanisms [Lomaev-1984]. At low fields, it is due to discontinuous jumps in the position of the walls that divide the domains, as

the domain that is closest in alignment to the applied field grows and the other, of opposite alignment, shrinks, as the wall moves in a planar fashion; At high fields, it is possibly due to discontinuous jumps in orientation of the domains themselves as they rotate to align with the applied field. The latter mechanism is in fact still only postulated, and is addressed in the final chapter of this thesis.

In 1926, Honda and Kaya [Honda-1926] measured magnetization curves for single crystals of iron. They found that the shape of the magnetization curve was dependent on the orientation of the crystallographic axes, relative to the direction of the applied field. Today, we have a theory of the crystalline anisotropy field, which, in addition to the external applied field, forms the total field which acts to reorient an individual magnetic domain parallel to it, in the modern version of the Stoner-Wohlfarth model [Stoner-1948] of ferromagnetism addressed herein.

In 1930, Frenkel and Dorfman [Frenkel-1930] calculated the critical size of a single spherical crystal, in order that it would contain only one magnetic domain. Later, more precise calculations were done by Kittel [Kittel-1946], Neel [Neel-1947], and Stoner and Wohlfarth [Stoner-1947], [Stoner-1948].

In 1931, Sixtus and Tonks [Sixtus-1931], actually succeeded in obtaining a magnetic material containing a single large magnetic domain, by stretching a thin, long Permalloy wire. They then used it to investigate the Barkhausen effect on the single, macroscopic domain.

In 1931, Bitter [Bitter-1931], and in 1932, Hamos and Thiessen [Hamos-1932] attempted the first direct observation of magnetic domain structures. The experiments were inconclusive. They ended up seeing a complicated maze-like pattern, though by this point, there were not yet any theoretical predictions as to what the pattern should look like.

In 1932, Bloch [Bloch-1932] made the first theoretical prediction of the structure not of the domains themselves, but of the domain walls that separated two adjacent domains. He showed that the boundary is not sharp, on the atomic scale, but is spread over a certain finite thickness, many atom spacings thick, with the direction of atomic spins gradually changing from the edge of one domain, through the wall, to the edge of the other domain. This structure was eventually known as a 'Bloch wall'.

Today, we know that there exist 180° and non- 180° Bloch walls, as well as Neel walls [Neel-1955].

In 1934, Kaya [Kaya-1934] made a detailed investigation of the then observed domain patterns of iron and nickel single crystals. The maze pattern was still being seen, but was explained as being due to a problem with the sample preparation technique: particularly, due to the grinding and the polishing of the crystal surface.

A milestone paper was published in 1935 by Landau and Lifshitz [Landau-1935] that made the first theoretical prediction of ferromagnetic domain structure based on minimization of the total energy, which included several energy terms, including the magnetostatic energy. They accurately predicted the alternating layer structure of domains separated by 180° walls, with the 90° walls separating the closing domains at each end of the layers. They calculated the thickness of both the domains and the domain walls. The illustration from their paper showing these predictions may be seen in figure 2.1. They also predicted the effect of an external applied field on the structure. At last, a very thorough theoretical picture of magnetic domain structure had been elucidated, and would serve to guide the experimentalists who had still not successfully photographed it in a real sample. The actual numerical results provided were for nickel. A detailed theoretical calculation was also done later, in 1944, by Neel [Neel-1944], for iron.

In 1939, Becker and Doring [Becker-1939] published *Ferromagnetismus*, a survey of the development of domain theory at the time.

In 1943, Kersten [Kersten-1943] proposed that domain wall motion was hindered by nonmagnetic inclusion sites, or, pinning sites in the crystal. The inclusion theory of domain wall pinning suggested that the domain walls moved in a planar fashion only until they encountered a pinning site. At that point, the domain wall processes would become complicated. These ideas were adopted in two of the modern models of ferromagnetism related to domain wall processes: the Globus model, and the Jiles-Atherton model, to be discussed in a later section.

Finally, in 1949, Williams, Bozorth, and Shockley, [Williams-1949], completed the long-awaited task of providing photographs of well-defined domain structures. The observations were in agreement with the predictions of Landau and Lifshitz, and of

Neel. A modern illustration of ferromagnetic domain structure is shown in figure 2.2 [Jiles-1991]. The predictions were correct.

With those long-awaited observations, knowledge of magnetic domain structure in ferromagnetic materials was basically complete. However, the basic question that Ewing had asked and had looked for in the first studies of ferromagnetic hysteresis was still only just being answered in a very new field of investigation. Namely, how does one theoretically predict the nonlinear and history-dependent path of a ferromagnetic hysteresis curve, for any possible external applied field cycle? The answer would be provided by theoretical models of ferromagnetism.

Figure 2.1: Ferromagnetic domain structure predicted by Landau and Lifshitz in 1935 [Landau-1935]. The occurrence of domain layers of alternating magnetization, as well as the flux closing domains at the ends, was predicted. The existence of the 180° and 90° was also predicted.

Figure 2.2: Ferromagnetic domain structure known at present [Jiles-1991]. The domain layers of alternating magnetization are shown, as well as the flux closing domains at the ends. The 180° and 90° domain walls are shown. The structure predicted by Landau and Lifshitz in 1935 was correct.

2.2 Modern Models of Ferromagnetism

2.2.1 The Stoner-Wohlfarth Model

The Stoner-Wohlfarth model of ferromagnetism [Stoner-1948] was published in 1948. The point of view taken was that hard magnetic materials consist of magnetic particles embedded in a nonmagnetic matrix. This assumption describes magnetic recording media quite well. Each particle was assumed to be below the critical size for which domain boundary formation is possible, and hence contained only a single domain. Each particle also possessed magnetic anisotropy. In the most fundamental form of the model, only single particle systems were considered. A particle was considered to have the geometry of an ellipsoid of revolution. It followed that the particle had shape anisotropy due to the demagnetizing field inherent in the particle's shape. In the absence of an external applied magnetic field, the natural direction of magnetization was such that the anisotropy potential energy was minimized. For the case of a prolate spheroid, this was along the long axis. This direction was termed the 'easy axis' of magnetization, and the particle magnetization vector aligned in this direction. In the presence of an external applied magnetic field, the direction of magnetization was such that the sum of the magnetostatic potential energy and the anisotropy potential energy was minimized. For a new value of applied field, the change in particle magnetization that took place was by rotation only, of the particle magnetization vector, to the new alignment. Both continuous and discontinuous rotations could occur. As the applied field increased from zero to an arbitrarily large magnitude, in a particular direction, the particle magnetization vector rotated from the easy axis direction toward the applied field direction.

Although this formed the basis for the model, the authors also explained how magnetic anisotropy could also arise from magneto-crystalline anisotropy, as well as strain anisotropy. Uniaxial magneto-crystalline anisotropy, as in the case of the hexagonal cobalt structure, could be treated with little modification of the uniaxial shape anisotropy formalism. They also simulated a magnetic material by considering a collection of noninteracting magnetic particles, with random easy axis directions. The available computation power at that time, however, was very limited, and hindered

the extent to which studies could be pursued. In particular, particle-particle magnetic interactions were left out. This original, noninteracting form of the Stoner-Wohlfarth (S-W) model, will be referred to as the 'classic' form of the model.

Since 1948, extensions to and applications of the S-W model have been investigated. We now cover some of these studies, including some very recent ones.

Watson, [Watson-1972], modeled the effects of a combination of cubic magneto-crystalline anisotropy and mechanical stress. Magnetic interaction effects were also modeled through energy averaging techniques.

Wuori and Judy, [Wuori-1984], modeled ferromagnetic films by considering a 2D array of magnetic particles, interacting through sheet demagnetizing fields.

Furlani and Baker, [Furlani-1986], included magnetic interactions in terms of a 'mean field' interaction, similar to the Weiss mean field. This was the mean field interaction that appeared in the Jiles-Atherton model [Jiles-1986] (described in a later section), in which the mean magnetization is taken over a collection of domain magnetic moments, as opposed to a collection of atomic magnetic moments within one single domain, as the original Weiss formulation amounted to. They also included temperature effects by using the Maxwell-Boltzmann distribution of total energy, where the energy terms, including both interactive and anisotropic terms, were functions of the easy axis angles.

Beardsley, [Beardsley-1986], developed a model of digital magnetic recording. The classic S-W model was used to generate the magnetization history of the magnetic recording material, at each stage of the recording process.

McCurrie and Viccary, [McCurrie-1986], compared S-W model results with actual experimental results, for an Mn-Al-C permanent magnet material. They used the S-W theory to predict the magnetic torque that would occur in the magnetization process, and compared it to the measured experimental torque. They hoped to conclude whether or not rotational hysteresis processes were occurring in the material. They reached a negative conclusion, though according to other researchers of the time, the results amounted to a positive conclusion.

Koehler, [Koehler-1987], developed an approximation to the S-W model, which re-

sulted in faster computation times.

Atherton and Beattie, [Atherton-1990], formulated a mean field extension to the S-W model, using the mean field formulation from the Jiles-Atherton model [Jiles-1986] (to be described in a later section). The Atherton-Beattie model was essentially a one-dimensional treatment.

Fredkin, Chen, and Koehler, [Fredkin-1992], studied the demagnetizing effect of the dipole-dipole interaction. They modeled a collection of 750 spherical magnetic particles, with no magnetic anisotropy, but with every particle pair interacting through the dipole-dipole interaction. The collection of particles was randomly distributed over a finite square, for the 2D case, and over a finite cube, for the 3D case. Without the dipole-dipole interaction, the resulting hysteresis curves would have been square loops with vertical sides. They found, however, that in 2D the inclusion of the dipole-dipole interaction had a demagnetizing effect that resulted in the usual sigmoid-shaped hysteresis curves. Their results in 3D were inconclusive.

Arrot, Templeton, and Yoshida, [Arrott-1993] considered magnetic particle arrays with differing interior and surface anisotropy. The surface anisotropy was set to be stronger than the interior anisotropy. They also included an exchange interaction between one magnetic moment and its nearest neighbours in the array.

Lu, Huang, and Klik, [Lu-1994], developed an extension to study thermally excited, noninteracting S-W particles under time-dependent oscillating applied fields at a range of frequencies. They found that the remanence decreased as the frequency was decreased, because the magnetization of each particle had more time, at low frequencies, to make the rotation to its new orientation at a lower energy state, under thermal excitation, even though the applied field had not overcome the energy barrier to the irreversible transition. The classic S-W loops corresponded to the case of infinite frequency, where the magnetization vector had no time to make the transition before the applied field overcame the energy barrier, while thermal equilibrium loops corresponded to the case of zero frequency, where there was always sufficient time to make the transition.

Zhang and Atherton, [Zhang-1995], published a thorough study of demagnetized states in the S-W model. The magnetization vectors of the collection of magnetic

particles were graphically displayed in the form of ‘fan diagrams’ [Chikazumi-1964]. They also showed that the particular demagnetized state influenced the shape of the subsequent initial magnetization curve, in the classic S-W model.

Surig and Hempel, [Surig-1996], used the S-W model with a mean field extension to predict microwave absorption loss along minor loops, in particulate recording media. Experimental work was also done, and the model parameters for their particular mean field extension were extracted.

Callegaro and Puppini, [Callegaro-1997], modeled the effects of applied uniaxial stress in ferromagnetic films. Magnetic interaction effects were included using a one dimensional mean field formulation, similar to the Atherton-Beattie model [Atherton-1990]. Stress effects were included using a stress anisotropy energy term in addition to the crystalline anisotropy and the magnetostatic energy terms. The model was a one-dimensional formulation, as the external applied field, the interaction mean field, and the uniaxial applied stress were all taken to be along a common axis.

Yu and Atherton, [Yu-1997a], [Yu-1997b], formulated a 3D version of the S-W model, and included the one dimensional treatment of the mean field from the Atherton-Beattie model, [Atherton-1990]. They chose the set of model parameters to describe a ferrite permanent magnet cube with an overall bulk easy axis, and they calculated the diagonal and off-diagonal magnetization tensor loop response to an applied field cycle along each of the three principal axes. They also performed actual hysteresis experiments with a real ferrite permanent magnet cube. A good comparison of theory with experiment was obtained.

Chen *et. al.*, [Chen-1997], used the classic S-W model to study the angular dependence of remanent magnetic properties on the in-plane easy axis alignment of barium ferrite films. These model results were called the ‘coherent’ rotation model results. They also performed actual experiments with real barium ferrite films, and found the experimental results to differ from the model results. They proposed that ‘incoherent’ rotation of magnetization was occurring, and that it was dependent on grain size and intergranular interactions.

Gunal and Visscher, [Gunal-1997], performed a molecular dynamics type of simulation of a magnetic ink, a colloidal suspension of acicular particles. The magnetization

behaviour of the particles was determined using the classic S-W model. The resulting magnetization vector in each particle was used to assign magnetic free poles at the ends and at the sides of the particles. The magnetic free poles were then used to determine the physical rotational motion of each particle through interaction with the others. Magnetic hysteresis loops were generated based on both the magnetic and physical dynamic processes.

Orozco *et. al.*, [Orozco-1998], extended the classic S-W model by including more general crystalline anisotropy terms. Cubic and quartic terms were included, in addition to the usual quadratic term. The new model predicted new magnetic behaviour. Real experiments were done with monocrystal nanoparticles, and the measured magnetic behaviour matched the predicted magnetic behaviour, providing evidence of the more general crystalline anisotropy in the monocrystals.

Thamm and Hesse, [Thamm-1998], used the classic S-W model to calculate the various forms of remanent magnetization for differing orientation of the applied magnetic field, to study the dependence of remanence on the processes of demagnetization used to initially prepare the sample.

2.2.2 Competing Models

In addition to the S-W model, there are three other principal models for describing ferromagnetism in macroscopic materials. These are the Preisach model, the Globus model, and the Jiles-Atherton model. The S-W model has been used as part of various recent forms of the Preisach model. In addition to the macroscopic models, there are also several approaches to describing micromagnetics. The S-W formalism has been used as part of some of these models as well. These competing macroscopic and microscopic models are now described.

Micromagnetics

In the classic Landau and Lifshitz paper, [Landau-1935], the time dependent dynamic behaviour of an atomic magnetic moment in an external applied field is described by

the Landau-Lifshitz equation, or, the gyromagnetic equation of motion with damping:

$$\frac{d\vec{M}}{dt} = \gamma \vec{M} \times \vec{H} - \frac{\lambda}{M} \vec{M} \times (\vec{M} \times \vec{H}) \quad (2.25)$$

The first term describes the gyromagnetic precessional motion of the magnetic moment. The second term describes the additional motion due to damping. Taken together, the equation describes a very complicated rotational motion of the magnetic moment in both rotational degrees of freedom. Many studies have been done which treat ferromagnetism as the motion, according to the Landau-Lifshitz equation, of collections of magnetic moments, in the external applied field.

Kikuchi, [Kikuchi-1956], derived analytic solutions to the Landau-Lifshitz equation for the cases of a single domain sphere and a single domain sheet. The time-dependent behaviour of the magnetic moments was solved, and the actual numeric minimum magnetization reversal times were computed.

In an exceptional paper, Zhu and Bertram, [Zhu-1988], performed supercomputer numerical simulations of a 2D array of magnetic moments described by the Landau-Lifshitz equation. They took the field $\vec{H}$ in the equation to be the sum of a crystalline anisotropy field, a granular self-interaction field, an intergranular nearest neighbour exchange interaction field, and the external applied field. The crystalline anisotropy field was derived from the crystalline anisotropy energy term from S-W theory. Bertram and Seberino have continued this line of study in a recent work [Bertram-1999].

Brown [Brown-1959], and Aharoni [Aharoni-1959] developed a completely different micromagnetic theory, in which the basic concept is a spontaneous magnetization whose direction varies continuously with position. Domain and wall concepts emerge mathematically from the theory, without needing to be postulated. The fundamental equations of this theory, though, constitute a system of coupled, nonlinear partial differential equations, and solution is difficult.

Finally, there are micromagnetics models that are similar to the S-W model, in that they involve an array of magnetic moments that take the orientation that minimizes a total potential energy sum. Hartman and Potter, [Hartman-1978], developed a 2D

model where the total field in the magnetostatic energy term was the sum of the external applied field, an interaction field, and a particle demagnetizing field. The interaction field for a given magnetic moment was the sum of the long-range dipole fields of the other moments. The dipole-dipole interaction formalism that was used was the elaborate solid state physics formalism of Born and Huang, [Born-1968].

Yan and Della Torre, [Yan-1988], developed a 3D model and performed a detailed study of the occurring numerical error. Williams and Dunlop, [Williams-1989], also developed a 3D model to study zero field solutions of a 3D array of magnetic moments. The total potential energy was taken as the sum of a crystalline anisotropy energy term, a nearest-neighbour exchange energy term, and a long-range dipole-dipole interaction energy term.

Preisach Model

The model of ferromagnetism that is currently used most commonly in magnetic recording applications is the Preisach model. This model was originally published in 1935 by Preisach, in Germany [Preisach-1935]. The Preisach model is of a mathematical, rather than a physical, basis. It is essentially a complicated curve-fitting method. However, if simple hysteresis curves for a magnetic recording material are already known, from experiment, then the Preisach model may be effectively fit to the experimental curves, such that the model may be used to predict the more general magnetization histories that occur in magnetic recording.

The mathematical basis of the Preisach model is that a hysteresis loop of a material is the sum of many elementary loops. The elementary loops are square loops, with differing positive and negative switching fields, H_+ and H_- . A single domain that behaves according to one of these elementary square loops can only have one of two possible magnetization states: magnetization parallel, or anti-parallel to the direction being considered. Therefore only irreversible magnetization processes may be modeled.

A plane with axes H_+ vs. H_- is defined. Each elementary square loop is located in this plane according to the set of coordinates (H_+, H_-) defined by its own H_+ and H_- values. A complicated set of external applied field turning points corresponds to

a complicated set of areas swept through the plane by a vertical line sweeping to the right (or a horizontal line sweeping down), corresponding to an increase of the applied field H in the positive direction (or in the negative direction). If a coordinate point (H_+, H_-) is swept, then a certain number of domains with the elementary square loop with those switching field values, switches its magnetization from $+$ to $-$, or vice versa.

The actual number of domains that switches is specified by a 2D distribution function, called the Preisach function, which is defined over the H_+ vs. H_- plane. The goal in Preisach modeling is to find the Preisach function such that the resulting model hysteresis loops match the known experimental hysteresis loops. The Preisach function hence contains, in an abstract sense, all of the information that specifies the physical interactions in the magnetic material. If the Preisach model is used strictly in the curve-fitting sense, then the physical interactions in the material are never known.

Del Vecchio, [Del Vecchio-1978], [Del Vecchio-1980], has written some very good introductory papers describing the Preisach model. Mayergoyz, [Mayergoyz-1991], has done considerable work to extend the Preisach model.

A recent trend in Preisach modeling has been to use the physically based S-W model to study proposed physical interactions in the magnetic material, to derive a corresponding Preisach function, to be fed to the mathematically based Preisach model, for predicting complicated magnetization histories. Stancu has done much work in this area [Stancu-1994], [Stancu-1997], [Stancu-1998], [Stancu-1999].

Della Torre *et. al.*, have also written recent papers on the Preisach model [Lou-1998], [Reimers-1998]. In addition, some very recent books on the Preisach model have appeared, by Bertotti [Bertotti-1998], and by Della Torre [Della Torre-1998].

Hodgdon [Hodgdon-1988], has developed her own model of ferromagnetism. It is similar in spirit to the Preisach model, in that it is of primarily a mathematical basis.

Globus Model

In soft magnetic materials, magnetization at low fields is due mainly to domain wall motion, as opposed to domain rotation in hard magnetic materials at high fields. The

types of domain wall motion that occur are wall bowing and wall displacement. The domain wall bowing may be thought of as elastic deformation of a membrane. In this analogy, the membrane edge is fixed, or 'pinned', at the crystal grain boundary, and the membrane sheet is bowed in one direction or the other, under the action of the external applied field. This constitutes the reversible wall motion. At higher fields, the pinning at the membrane edges is overcome, and planar domain wall displacement occurs. This constitutes irreversible motion, as energy is lost in overcoming the pinning sites. Many of these ideas were originally suggested by Kersten [Kersten-1943].

The Globus model of ferromagnetism describes magnetization in this fashion. The original formulation, [Globus-1962], considered the reversible domain wall bowing only. Experimental work was also done [Globus-1966]. The model was then used to calculate initial magnetization curves [Globus-1969], [Globus-1970], [Globus-1971]. The extended Globus-Guyot model, [Globus-1972], included both domain wall bowing and domain wall displacement. This model was used to calculate coercivity and remanence, as well as the critical field beyond which the irreversible processes begin. Further work attempted to calculate the hysteresis loss due to the creation and annihilation of domain wall surface area, and the energy loss due to continual pinning and unpinning of domain walls [Guyot-1973], [Guyot-1977]. A final paper summarized earlier work, and gave a general formula for the initial magnetization and hysteresis loss for a single crystal grain [Globus-1977]. Polycrystalline materials may be modeled by considering a collection of such grains.

Jiles-Atherton Model

The Jiles-Atherton model [Jiles-1983], [Jiles-1986], [Jiles-1991], is a statistical mechanical approach to describing ferromagnetism in soft magnetic materials at low fields, where the magnetization processes are domain wall bowing and domain wall displacement. The Jiles-Atherton model is particular about the distinction between irreversible magnetization due to domain wall displacement and reversible magnetization due to domain wall bowing. The irreversible changes involve the overcoming of 'pinning sites', impurities in the crystal that resist a change in magnetization, and require the expenditure of energy to change. As the domain wall sweeps in a planar fashion through a volume, it encounters several of these pinning sites along the way.

The pinning constant k of the Jiles-Atherton model, is defined as:

$$k = \mu_0 \frac{dE_{loss}}{d\vec{M}} \quad (2.26)$$

where dE_{loss} is the energy lost in overcoming the pinning site, and $d\vec{M}$ is the resulting change in magnetization. The total effective field at work is:

$$\vec{H}_e = \vec{H} + \alpha \vec{M} \quad (2.27)$$

where $\vec{H}$ is the external applied field, and $\alpha \vec{M}$ is a ‘mean field’ similar to the Weiss molecular field in the Weiss theory of ferromagnetism discussed in an earlier section. The differential equation describing the irreversible changes is:

$$\frac{d\vec{M}_{irr}}{d\vec{H}} = \frac{\vec{M}_{an} - \vec{M}_{irr}}{k - \alpha[\vec{M}_{an} - \vec{M}_{irr}]} \quad (2.28)$$

and the differential equation describing the reversible changes is:

$$\frac{d\vec{M}_{rev}}{d\vec{H}} = c \left(\frac{d\vec{M}_{an}}{d\vec{H}} - \frac{d\vec{M}_{irr}}{d\vec{H}} \right) \quad (2.29)$$

where c is a constant. These two contributions are summed, to yield a single differential equation:

$$\frac{d\vec{M}_{tot}}{d\vec{H}} = \frac{d\vec{M}_{irr}}{d\vec{H}} + \frac{d\vec{M}_{rev}}{d\vec{H}} \quad (2.30)$$

$\vec{M}_{an}$ is the anhysteretic value of magnetization, and is given by the Langevin function, as in the Weiss theory of ferromagnetism.

The Jiles-Atherton model is a statistical mechanical model, such that only the one equation is needed to describe the entire macroscopic material.

An important step in the quest to model a specific material is to find the values of the model parameters that give the best fit to experimental results. Jiles, Thoenke, and Devine, [Jiles-1992] derived an analytic method that accomplishes this. Recently, Jiles and Sablik, [Sablik-1987], [Sablik-1988], [Sablik-1993a], [Sablik-1993b], [Sablik-1999] have extended the model to include stress effects.

2.3 Current Problems in Magnetism that Need a Model of Ferromagnetism

Magnetic hysteresis is a nonlinear, history-dependent, and in general, a nonisotropic phenomenon. There are many current problems in magnetism that require a model of ferromagnetism in order to include hysteretic effects with the finite element analysis that is done. Magnetic recording models is one application. Often the magnetic recording media is considered a 2D film consisting of magnetic particles embedded in a nonmagnetic binder material. A 2D model based on S-W theory is useful for this application. Electrical machines is another possible application. Finite element analysis is often performed over 2D machine cross-sections, or even over the complete 3D geometry of the machine components. The components are built from electrical steel, often laminated, to reduce eddy current losses. Perhaps a 2D or 3D model based on S-W theory might be a useful attempt at including hysteretic effects into the electromagnetic simulations of these machines, in the design stage.

Our research group, however, is concerned with the study of Magnetic Barkhausen Noise (MBN) in the field of nondestructive testing. In 1919, Barkhausen, [Barkhausen-1919] discovered that a magnetization curve actually consisted of very many small, discontinuous steps. Our research group has published several papers on experimental study of Barkhausen noise [Krause-1994a], [Krause-1994b], [Krause-1996], [Krause-1997]. Of particular interest is the effect of external applied stress on the Barkhausen signal. If a correspondence between external applied stress and the Barkhausen noise signal can be well established, then measurement of the Barkhausen noise in a magnetic material such as pipeline steel or construction steel will be a good indication of the mechanical stresses that it is being subjected to, and could alert of imminent failure.

Therefore it is of value to have a model of ferromagnetic hysteresis that may be used to predict a magnetic Barkhausen signal. The discontinuous jumps in magnetization that comprise the signal are thought to be due to discontinuous domain wall displacements that occur as pinning sites are overcome, in the low field regime. It is proposed that another source of the magnetic Barkhausen signal could be discontinuous domain rotations that would occur as a proposed pinning torque is overcome,

in the high field regime. One of the extensions to the S-W model that is proposed in this thesis is the inclusion of pinning torque. It will be seen, in some of the results that are presented, that inclusion of pinning torque results in discontinuous steps in the hysteresis loops generated by the model. It is therefore possible that future versions of this model that would include stress anisotropy, possibly as in the Callegaro-Puppin model, [Callegaro-1997], may potentially be useful for predicting a magnetic Barkhausen signal in a stressed material. A proposal for how magnetic Barkhausen signals may be extracted from hysteresis loops generated by this model, is presented in the final chapter of this thesis.

Chapter 3

The Extended Stoner-Wohlfarth Model

3.1 Development of the Classic Non-Interacting S-W Model

3.1.1 Introduction

The Stoner-Wohlfarth (S-W) model [Stoner-1948] of ferromagnetic hysteresis describes magnetization processes in terms of domain magnetization rotation in an anisotropic crystal. The ferromagnetic materials whose behaviour is particularly well modelled are the ‘hard’ magnetic materials, such as ferrite permanent magnets. Bulk or macroscopic anisotropy in a polycrystalline ferromagnet results from a preferred orientation of crystals which themselves exhibit magnetocrystalline anisotropy.

The classic S-W model, in its original form, modelled a hard anisotropic ferromagnetic material as a set of mutually independent ‘magnetic particles’ interacting with an applied magnetic field. The necessary calculations for a model of even this simplicity were tedious without the use of electronic computers.

We have extended the original model to include different physical interactions, specifically a mean field interaction, as introduced in the Atherton-Beattie extension of the

S-W model [Atherton-1990], and a nearest neighbour dipole-dipole interaction. We have also included pinning, which was treated in the Jiles-Atherton model [Jiles-1986]. These new developments have resulted in a more realistic model. The model continues to be most relevant to hard magnetic materials at high fields. The specific material that we modeled in our study was a hard barium ferrite, of chemical composition $BaO \cdot 6Fe_2O_3$. The ultimate goal is to develop a three dimensional (3D) model and calibrate it to our existing experimental data. Most of the developments can be described in terms of the simpler case of 2D effects.

We begin with the most fundamental description of the model: the energy equation in vector form. Then we specialize to a 2D model. We demonstrate the development to the point of the original classic S-W model, and then go beyond to the extensions that we have made, to include the mean field and nearest neighbour interactions, and pinning.

3.1.2 The Energy Equation: Vector Formulation

We consider first the total energy of a single magnetic particle in an array of particles. The total energy of the array itself will then be simply the sum of the energy terms of all particles.

Consider first a single magnetic particle. The particle, with an intrinsic magnetization $\vec{M}^i$, will be situated in an applied magnetic field $\vec{H}$, which will act to rotate it in such a way that the magnetization vector will tend to align parallel to the applied field vector. The magnetostatic energy term is therefore:

$$E_H = -\mu_0 \vec{M}^i \cdot \vec{H} . \quad (3.1)$$

In addition, the magnetic material will have a given crystal structure. Within a crystal structure, there exist preferred directions of magnetization, the ‘easy axes’. This is termed ‘crystalline anisotropy’. In the absence of an applied field, the particle magnetization tends to align parallel or antiparallel to the one of the easy axes.

We consider the simplest case, uniaxial crystalline anisotropy, in which there is only one easy axis in the particle. We define the angle Φ as that between the particle’s

intrinsic magnetization $\vec{M}^i$, and its easy axis direction $\vec{e}^i$. The anisotropy energy term, E_A , is a minimum for a Φ of 0° and 180° , and a maximum at 90° . It is therefore, in general, expressed as a power series in $\sin^2 \Phi$ of the form:

$$E_A = K_{u_1} \sin^2 \Phi + K_{u_2} \sin^4 \Phi + \dots \quad (3.2)$$

Usually, just the first term is taken. In vector form,

$$\sin \Phi = \frac{|\vec{e}^i \times \vec{M}^i|}{|\vec{M}^i|} \quad (3.3)$$

or:

$$\sin \Phi = |\vec{e}^i \times \hat{M}^i|, \quad (3.4)$$

where $\hat{M}^i$ is the unit vector form of $\vec{M}^i$. Therefore the anisotropy energy term, in vector form, becomes:

$$E_A = K_{u_1} |\vec{e}^i \times \hat{M}^i|^2. \quad (3.5)$$

The total energy of a particle then becomes:

$$E^i = -\mu_0 \vec{M}^i \cdot \vec{H}_{\text{applied}} + K_{u_1} |\vec{e}^i \times \hat{M}^i|^2. \quad (3.6)$$

Normalizing in terms of the saturation magnetization m_s such that:

$$\vec{M}^i = m_s \vec{m}^i, \quad (3.7)$$

we obtain:

$$\left(\frac{E^i}{K_{u_1}} \right) = - \left(\frac{\mu_0 m_s}{K_{u_1}} \right) (\vec{m}^i \cdot \vec{H}_{\text{applied}}) + |\vec{e}^i \times \hat{m}^i|^2. \quad (3.8)$$

Using reduced units:

$$\vec{h}_{\text{applied}} = \left(\frac{\mu_0 m_s}{K_{u_1}} \right) \vec{H}_{\text{applied}} \quad (3.9)$$

$$\underline{e}^i = \left(\frac{E^i}{K_{u_1}} \right), \quad (3.10)$$

the energy equation for one particle becomes:

$$\underline{e}^i = -\vec{m}^i \cdot \vec{h}_{\text{applied}} + |\vec{e}^i \times \hat{m}^i|^2. \quad (3.11)$$

As a further simplification, all reduced magnetizations $\vec{m}^i$ are normalized to unit magnitude. The energy equation then becomes:

$$\underline{e}^i = -\vec{m}^i \cdot \vec{h}_{\text{applied}} + |\vec{e}^i \times \vec{m}^i|^2 . \quad (3.12)$$

Note that the energy of the array is the sum of the energies of all particles in the array:

$$\underline{E} = \sum_i \underline{e}^i . \quad (3.13)$$

3.1.3 The Energy Equation: 2D Model Formulation

We now treat the geometry for a 2D model. The vectors and angles are defined in Figure 3.1.

The reduced energy becomes:

$$\underline{e}^i = -\underline{h}_{\text{applied}} \cos(\phi_h - \phi^i) + \sin^2(\phi^i - \phi_{ea}^i) . \quad (3.14)$$

This formulation may be used to treat an applied field that varies in both magnitude and direction. For the case of an applied field with fixed direction, varying in magnitude only, we may align the applied magnetic field along the x -axis, so that:

$$\vec{h}_{\text{applied}} = \underline{h}_{\text{applied}} \hat{x} , \quad (3.15)$$

and the energy takes the simple form:

$$\underline{e}^i = -\underline{h}_{\text{applied}} \cos \phi^i + \sin^2(\phi^i - \phi_{ea}^i) . \quad (3.16)$$

The derivative w.r.t. ϕ^i gives the magnitude of torque, $\vec{\tau}^i$, acting on the particle to change its magnetization orientation:

$$|\vec{\tau}^i| = \left| \frac{\partial \underline{e}^i}{\partial \phi^i} \right| = \left| \underline{h}_{\text{applied}} \sin \phi^i + \sin 2(\phi^i - \phi_{ea}^i) \right| . \quad (3.17)$$

This is the expression for the energy of a single non-interacting magnetic particle

in the classic S-W model. It forms the basis of the ferromagnetic hysteresis model developed here. The additional effects that are introduced are a mean field interaction, a nearest-neighbour dipole-dipole interaction, and magnetic domain pinning. The first two effects are included by adding terms to the basic energy equation above. The third effect is included by relating the torque acting to change the particle magnetization, with a pinning torque acting to keep it fixed. We next add the first two effects.

Note that the model is inherently three dimensional if the vectors and appropriate angles are defined in a 3D geometry. This is treated in chapter 7.

Figure 3.1: Geometry of the 2D model showing the easy axis direction vector $\hat{e}^i$, the intrinsic magnetization vector $\hat{m}^i$, the applied field vector $\vec{H}_{applied}$, and their corresponding angles ϕ_{ea}^i , ϕ^i , and ϕ_h^i for magnetic particle i .

3.1.4 Hysteresis Curves

The results generated by our inclusive model are hysteresis curves. These are the nonlinear and history-dependent relations between the applied magnetic field intensity $\vec{H}$ and the resulting magnetic flux density $\vec{B}$. The alternate relation which is of more interest here is that between $\vec{H}$ and the resulting magnetization vector $\vec{M}$.

For a given history of applied field $\vec{H}$, the corresponding hysteresis curve is extracted by minimizing the energy equation. Consider the case of a single magnetic particle in an external applied field, described by the energy equation given in the previous section. For a given value of applied field, h_{applied} , there exists a corresponding value of intrinsic magnetization angle, ϕ^i , for which the energy of the particle $\underline{e}^i$ is a minimum. This is evident from curves of $\underline{e}^i$ vs ϕ^i , for varying values of h_{applied} , for a fixed easy-axis angle ϕ_{ea} , as shown in Figure 3.2. For purposes of illustration, we have chosen $\phi_{ea} = 30^\circ$.

Note, however, that there may be two minima. At low applied field, there are 2 minima: one local, and one global minimum. As the applied field is increased, it reaches a critical value at which the local minimum becomes an inflection point. As the applied field is increased past this critical value, the inflection point vanishes, and there remains just one global minimum.

This is the fundamental mechanism of hysteresis in the model. To illustrate, consider a particle in a zero applied field, with an initial magnetization orientation of 30° , in line with the particle easy-axis. As the applied field is increased in the positive direction, the minimum in the series of energy plots shifts from 30° to the left. At an infinite value, it would shift all the way to 0° . As the applied field is reduced back to zero, the minimum shifts back to 30° . This is an example of a reversible magnetization process, a continuous change in magnetization, and involves small continuous rotations of ϕ^i .

Suppose then that the applied field is increased in the negative direction. Initially the local minimum shifts to the right. At some critical value of applied field, though, that local minimum disappears. ϕ^i then jumps to the one remaining minimum: the global minimum. This is an example of an irreversible magnetization process, a discontinuous change in magnetization known as hysteretic jump, and involves a large

discontinuous rotation of ϕ^i . As the applied field is increased further in the negative direction, the new minimum shifts to the left. At an infinite value, it would shift all the way to 180° . However, as the applied field is decreased back to zero, the minimum shifts to a value of -150° , opposite to its original orientation in zero applied field.

The cycle is completed by then increasing the applied field in the positive direction again, to infinity. The local minimum shifts from 150° to the right, and then makes a second irreversible jump. The new minimum then shifts to the left, approaching 0° .

The hysteresis cycle is more easily visualized with the aid of a ‘fan diagram’ [Chikazumi-1964], a diagram that shows the orientations of the applied field, easy axis, and magnetization vectors. (See Figure 3.3.) The portions of the hysteresis cycle that have been explained so far are diagrams *a* to *m* in the series, with the magnetization vector rotating to the correct minimum on the energy diagram.

Finally, the hysteresis curve is extracted from the fan diagram. The conventional hysteresis curve is simply the $\hat{x}$ -projection of the magnetization vector, m_X , vs the applied field defined to be along the $\hat{x}$ -direction, h_x . A plot of this curve is shown in Figure 3.4. More generally, this is the m_{xX} -component of the magnetization hysteresis tensor, as explained subsequently.

This constitutes the fundamental principles upon which the classic S-W model of magnetism is based. We next discuss the extensions made in the new inclusive model.

Figure 3.2: Total energy of a single S-W particle in an applied field. e^i is the total energy, ϕ^i the intrinsic magnetization angle, and ϕ_{ea}^i the easy axis angle (here $\phi_{ea}^i = 30^\circ$). The series of h_{applied} quantities are specific reduced applied field values.

Figure 3.3: Fan diagram series for a single S-W particle in an applied field. $\hat{e}$ is the easy axis direction vector (the fine line), and $\hat{m}$ is the intrinsic magnetization vector (the heavy line). $\vec{H}_{\text{applied}}$ is the applied field direction.

Figure 3.4: m_{xX} and m_{xY} major hysteresis loops to near saturation for a single S-W particle, with an easy axis angle of $\phi_{ea} = 30^\circ$

3.2 Extensions

3.2.1 Mean Field Interaction

The mean field interaction was treated in the Jiles-Atherton model [Jiles-1986]. It was later included as an extension to the S-W model in the Atherton-Beattie model [Atherton-1990]. Here, we include it using the latter formulation. If in the magnetostatic energy term we replace the applied field $\vec{H}_{applied}$ with:

$$\vec{H} = \vec{H}_{applied} + \alpha \vec{M} \quad (3.18)$$

where:

$$\vec{M} = \sum_j \vec{M}^j \quad (3.19)$$

is the total averaged magnetization of the array and α is the mean field parameter, then in addition to the magnetostatic energy term from before, we then obtain the mean field energy term:

$$E_{mf}^i = -\alpha \mu_0 \vec{M}^i \cdot \vec{M} = -\alpha \mu_0 \sum_{j \neq i} \vec{M}^i \cdot \vec{M}^j . \quad (3.20)$$

As before, we then write:

$$\frac{E_{mf}^i}{K_{u1}} = -\frac{\alpha(\mu_0 m_s^2)}{K_{u1}} \hat{m}^i \cdot \vec{m} , \quad (3.21)$$

and using reduced units:

$$\underline{\alpha} = \left(\frac{\mu_0 m_s^2}{K_{u1}} \right) \alpha , \quad (3.22)$$

obtain:

$$\underline{e}_{mf}^i = -\underline{\alpha} \hat{m}^i \cdot \vec{m} . \quad (3.23)$$

In terms of the 2D geometry defined earlier, this becomes:

$$\underline{e}_{mf}^i = -\underline{\alpha} [m_x \cos \phi^i + m_y \sin \phi^i] , \quad (3.24)$$

where the net averaged magnetization components are the sums:

$$m_x = \frac{1}{N} \sum_j^N \cos \phi^j \quad (3.25)$$

$$m_y = \frac{1}{N} \sum_j^N \sin \phi^j . \quad (3.26)$$

And finally, the contribution to the first derivative is:

$$\frac{\partial e_{mf}^i}{\partial \phi^i} = +\alpha [m_x \sin \phi^i - m_y \cos \phi^i] . \quad (3.27)$$

3.2.2 Dipole-Dipole Interaction

If, in the magnetostatic energy term, we replace the applied field $\vec{H}_{applied}$ with:

$$\vec{H} = \vec{H}_{applied} + \alpha \vec{M} + \vec{H}_{nn} \quad (3.28)$$

where:

$$\vec{H}_{nn} = \sum_{j \neq i}^{within r_{nn}} \vec{H}^j(dipole) \quad (3.29)$$

is the total nearest neighbour field from all neighbouring dipoles within an interaction radius of r_{nn} , where

$$\vec{H}^j(dipole) = \frac{1}{4\pi} \frac{3\hat{r}_{ji}(\hat{r}_{ji} \cdot \vec{M}^j) - \vec{M}^j}{r_{ji}^3} V \quad (3.30)$$

is the dipole field [Jackson-1975], where $\vec{r}_{ji}$ is the displacement vector from source dipole j to dipole i , where V is the volume occupied by one dipole, which for simplicity is taken as:

$$V = s^3 \quad (3.31)$$

and where s is one rectangular array spacing, then in addition to the magnetostatic energy term and the mean field energy term from before, we obtain the nearest

neighbour energy term:

$$E_{nn}^i = -\mu_0 \vec{M}^i \cdot \vec{H}_{nn} . \quad (3.32)$$

As before, we may write:

$$\frac{E_{nn}^i}{K_{u_1}} = -\frac{(\mu_0 m_s)}{K_{u_1}} \hat{m}^i \cdot \vec{H}_{nn} , \quad (3.33)$$

and using reduced units:

$$\vec{h}_{nn} = \left(\frac{\mu_0 m_s}{K_{u_1}} \right) \vec{H}_{nn} , \quad (3.34)$$

obtain:

$$\underline{e}_{nn}^i = -\hat{m}^i \cdot \vec{h}_{nn} . \quad (3.35)$$

In terms of the 2D geometry defined earlier, this becomes:

$$\underline{e}_{nn}^i = -[h_{nnx} \cos \phi^i + h_{nny} \sin \phi^i] \quad (3.36)$$

where the nearest neighbour field components are the sums:

$$h_{nnx} = \sum_{j \neq i}^{\text{within } r_{nn}} h_x^j(\text{dipole}) \quad (3.37)$$

$$h_{nny} = \sum_{j \neq i}^{\text{within } r_{nn}} h_y^j(\text{dipole}) . \quad (3.38)$$

where

$$\underline{h}^j(\text{dipole}) = \left(\frac{\mu_0 m_s^2}{K_{u_1}} \right) \frac{1}{4\pi} \frac{3\hat{r}_{ji}(\hat{r}_{ji} \cdot \hat{m}^j) - \hat{m}^j}{(\frac{r_{ji}}{s})^3} \quad (3.39)$$

Finally, the contribution to the first derivative is:

$$\frac{\partial \underline{e}_{nn}^i}{\partial \phi^i} = +[h_{nnx} \sin \phi^i - h_{nny} \cos \phi^i] . \quad (3.40)$$

In this extension, a domain is treated as one dipole. Therefore the volume $V = s^3$ of each dipole is the volume of a single domain, and the length scale s of the interaction is the width of a single domain.

3.2.3 Pinning

Pinning was introduced in the Jiles-Atherton model [Jiles-1986] as an effect acting on domain wall motion. We instead formulate pinning as a nonconservative frictional torque within an array particle that impedes the tendency of the particle magnetization vector to rotate to its new orientation. In a dynamic model, a model in which the rotation is modelled by a second-order differential equation of motion in time, the frictional torque would appear as a nonconservative term in the equation.

A nonconservative force term, however, cannot usually be simply included as a term in an energy minimization formulation. Therefore in order for us to include pinning, we must choose a different approach.

The torque acting on a particle magnetization was expressed in chapter 1 as:

$$|\vec{\tau}_+^i| = \left| \frac{\partial \underline{e}^i}{\partial \phi^i} \right| \quad (3.41)$$

where we have included the subscripted $+$ to indicate that the torque is acting to change the particle magnetization. We postulate simply that there is also a pinning torque $\vec{\tau}_-$ acting to impede the change in the particle magnetization. If $|\vec{\tau}_+| > |\vec{\tau}_-|$ then we allow the particle magnetization to rotate to its new orientation, where the net torque is zero: $|\vec{\tau}_+| - |\vec{\tau}_-| = 0$. But if $|\vec{\tau}_+| \leq |\vec{\tau}_-|$ then we say that the magnetization is ‘pinned’, and we keep it fixed at its old orientation.

As for the value of $\vec{\tau}_-$, we have used:

$$|\vec{\tau}_-| = k|\vec{m}| \quad (3.42)$$

where k is a fixed constant (one of the model input parameters), and $\vec{m}$ is the array magnetization. A future extension to this model may be to consider distributions of k . This formulation was chosen because it is both simple and descriptive, and also describes quite well the pinning of domain wall motion at low magnetization. We, however, have postulated that it may also serve to model pinning of domain rotation at high magnetization.

3.2.4 Field Turning Points

Our model also handles the complicated sets of field turning points that are typical of the Preisach model [Del Vecchio-1978], [Del Vecchio-1980], [Mayergoyz-1991], [Bertotti-1998], [Della Torre-1998]. It is therefore possible to model sets of minor loops, which are generated by small field reversals at various points in a major loop. Since our model is a proper 2D treatment, it is also possible to model the magnetization response of general 2D vector applied field cycles. Examples of more general applied field cycles are discussed in the next chapter.

Chapter 4

Numerical Modeling

We now describe the details necessary to construct our model of magnetic hysteresis. The numerical method of minimizing the energy equation is discussed, followed by the method of iteration that is needed to solve for the magnetization components. Next, we describe the model input parameters, which include the numbers necessary to provide a description of the magnetic particle array, as well as material parameters. The various external applied field cycles that may be specified in 2D, as the input signal to the model, are listed. Finally, we discuss the magnetization loop tensor that we expect to achieve with this model.

4.1 Minimization of Energy

The principal numerical method used in this model is minimization. As explained previously, the magnetization angle ϕ^i is that which minimizes the particle's total energy $\underline{e}^i$. The problem is hence a minimization in 1 variable, and is conveniently solved using Brent's method of minimization [Press-1992]. A detailed explanation of Brent's method is given in appendix A.

This method is applied to each particle energy $\underline{e}^i$. This constitutes one iteration step for the array.

4.2 Iteration

The magnetization vector, $\vec{m}$, is obtained by summing particle magnetization vectors $\hat{m}^i$ over the array. The orientations of the particle magnetization vectors are obtained by minimizing the particle energies $\underline{e}^i$. However, when we include the mean field interaction, the particle energy contains the implicit term:

$$\underline{e}_{mf}^i = -\underline{\alpha}\hat{m}^i \cdot \vec{m} \quad (4.1)$$

which contains the magnetization vector $\vec{m}$, and must therefore be solved through iteration. Within one iteration, we are attempting to obtain a new value of magnetization $\vec{m}(new)$ by summing over the array. To achieve this, we are using an old value of magnetization $\vec{m}(old)$ in the energy term $\underline{e}_{mf}^i$. This is repeated until the two values of magnetization differ by less than a pre-defined tolerance. The value $\vec{m}(new)$ is then the solution to the implicit problem in $\vec{m}$.

Note that once the nearest neighbour dipole-dipole interaction is included as well, the degree of iteration required to solve for $\vec{m}$ increases.

4.3 Description of Array

The magnetic particle array is completely described by the set of particle easy axis direction vectors, $\{\hat{e}^i\}$, and the set of particle initial magnetization direction vectors, $\{\hat{m}_0^i\}$. In this 2D model, the directions are described simply by the vector angles. Therefore the set of angles $\{\phi_{ea}^i, \phi_0^i\}$ constitutes a complete description.

Different types of materials may be described by different types of distributions of ϕ_{ea}^i . A perfectly isotropic material may be described by a set of N angles spaced uniformly around the unit circle. An isotropic, yet perhaps more realistic material may be described by a set of N angles of random distribution around the unit circle. A nonisotropic material with an overall bulk easy axis may be described by a set of N angles of gaussian distribution around the unit circle, with a mean angle of $\bar{\phi}_{ea}$, and a standard deviation of $\bar{\sigma}$. In this sense, the set of angles $\{\bar{\phi}_{ea}, \bar{\sigma}\}$ provides a bulk description of the magnetic material.

The numbers of gaussian distribution were taken using the Monte Carlo technique known as the ‘transformation method’ [Press-1992]. At the core of all random number generation was a typical pseudo-random number generator. A detailed treatment of these Monte Carlo techniques is given in appendix B.

4.4 Extensions of Array Description

Inclusion of the nearest neighbour dipole-dipole extension requires that the magnetic particles in the array have well-defined array coordinates. In 2D, particle i has coordinates (x^i, y^i) . Therefore it is necessary to extend the array description to include the set of coordinates $\{x^i, y^i\}$ for all the particles. It is in fact sufficient to use as coordinates the integer values of particle position in the array, as the array spacing parameter s cancels out of the formulation. This is fortunate, as it eliminates the need to make s a model parameter. In any case, it would be difficult to decide on a numerical value for the dimension s .

4.5 Material Parameters

When defining the reduced units in the energy equation, there were some material parameters that needed to be defined. These are the uniaxial anisotropy constant K_{u1} and the saturation magnetization m_s . When the mean field interaction is included, there is also the mean field parameter α . In order to include the nearest neighbour dipole-dipole interaction, there will be the nearest neighbour maximum interaction radius r_{nn} . Finally, there is the pinning parameter k .

All these may be regarded as input parameters for the model, independent of the particle array description. Essentially, once an array is generated according to the array description, the same array may be used for many different material property descriptions, and vice-versa.

The ultimate goal of the modeling process would be to regard all the model input parameters as fitting parameters, and to then find the best possible fit with known

experimental results. For now, a trial-and-error approach is taken. Eventually, one would hope for an automated fitting algorithm. However, this would constitute a multidimensional minimization problem, with the number of dimensions equaling the number of fitting parameters. Even in principal, this is a difficult problem, since in this case it is the global minimum that is desired, while minimization algorithms typically return the closest local minimum. The problem is discussed in chapter 7.

4.6 Applied Field Cycles

Hysteresis curves are generated by incrementing the applied field $\vec{h}_{applied}$ slightly from its previous value, calculating the new particle magnetization vector orientations, then summing over the array to obtain the bulk magnetization vector $\vec{m}$. The various components of $\vec{m}$ are then plotted against the various components of $\vec{h}_{applied}$, resulting in the (for this model) 2×2 magnetization loop tensor.

The applied field, $\vec{h}_{applied}$, is set to vary in various patterns in order that we may obtain the different types of hysteresis curves. The following is a list of the principal patterns that are of interest:

1. Major cycle to near saturation
2. Minor cycle
3. Rotating field of constant magnitude
4. Rotating field of diminishing magnitude (An AC demagnetizing field)
5. Oscillating field of diminishing magnitude (An AC demagnetizing field)
6. Superposition of a DC field and an AC demagnetizing field

The major cycle to near saturation is the common pattern that results in the sigmoid-shaped major hysteresis loop. The applied field starts at 0, increases to a maximum $+|h_{max}|$, decreases to a minimum $-|h_{max}|$, and then increases back to the maximum $+|h_{max}|$. The magnitude $|h_{max}|$ is chosen such that the bulk magnetization approaches the saturation value.

The minor cycle has as its base the major loop cycle, but contains applied field reversal cycles at various points within the major cycle. These minor cycles result in minor hysteresis loops: closed loops contained within the sigmoid-shaped major hysteresis loop, with widths Δh and Δm in the applied field and magnetization axes respectively.

The rotating field of constant magnitude is simply that. It is expected to result in a rotating magnetization vector, which may perhaps either lag or be synchronous with the applied field vector. The magnetization magnitude may perhaps be either constant or varying.

The rotating field of diminishing magnitude is an attempt to demagnetize the array; i.e., to result in a final magnetization state that is identical to its initial non-magnetized state. The oscillating field of diminishing magnitude is a similar attempt, except that the applied field is fixed to oscillate along a given axis, as opposed to rotating about a given axis.

The final applied field type is used to calculate the anhysteretic curve. This is achieved by super-imposing a fixed DC field over one of the two possible AC demagnetizing field cycles. Once the demagnetizing field has diminished to zero, all that remains is the DC field, and the final state of the array constitutes one point on the anhysteretic curve. The DC field is then incremented to the next value, and the demagnetization is repeated. Such a calculation is quite costly in computation, and is not attempted in this thesis.

4.7 The 2×2 m_{iJ} Tensor

As the applied field, $\vec{h}_{\text{applied}}$, is changed and the magnetization, $\vec{m}$, is calculated, the various components of the magnetization vector (m_X, m_Y) may be plotted versus the various components of the applied field (h_x, h_y) to yield the 2×2 magnetization loop tensor:

$$m_{iJ} = \begin{pmatrix} m_{xX} & m_{xY} \\ m_{yX} & m_{yY} \end{pmatrix}. \quad (4.2)$$

m_{iJ} does not fit the usual definition of a tensor, as each element is a highly nonlinear and history-dependent curve. The diagonal components (m_{xX}, m_{yY}) are relatively well-known. The off-diagonal components, though, are not so well-known. In a perfectly isotropic array, the off-diagonal components should be zero. However, we have discovered that the off-diagonal components exhibit quite interesting behaviour in near-isotropic and non-isotropic particle arrays.

Chapter 5

Results

5.1 Major Loops

5.1.1 Various Array Types

Single Particle Arrays

The basic set of material parameters for all studies in this thesis are listed in table 5.1. In cases where we have taken non-zero interaction parameters α , τ_{nn} , and k , we explicitly state the values.

We start by presenting elementary results for a single particle, with the easy axis angle, ϕ_{ea}^1 , set to from 0° to 90° , in increments of 10° . For each value, of ϕ_{ea}^1 , the magnetization loop tensor m_{ij} is generated by our numerical model. The complete 2×2 tensors are shown in figure 5.1. The corresponding fan diagrams would be similar to those in figure 3.4. The single particle loop tensors in a sense form the basis for multiparticle array studies, as the multiparticle arrays, setting aside interaction considerations, are collections of single particles of varying easy axes.

Table 5.1: Model parameters of the magnetic material

K_{u1}	$1.5 \times 10^5 J/m^3$
m_s	$3.8 \times 10^5 A/m$
α	0.0
r_{nn}	0
k	0.0

The first two parameters are the values given in [Yu-1997a] for the barium ferrite cube, with chemical composition $BaO \cdot 6Fe_2O_3$.

Figure 5.1: Hysteresis tensor major loops to near saturation for a single S-W particle with varying easy axis. The easy axis angle, ϕ_{ea} , varies from 0° to 90° , in increments of 10° .

Isotropic Arrays

We next present results for a 100-particle, uniform, isotropic array, for the case of isolated, non-interacting particles. The set of figures 5.2 to 5.6 shows an initial magnetization curve and a major loop to near saturation, with accompanying fan diagrams, for interaction and pinning parameters set to zero. An important point is to distinguish between reversible and irreversible magnetization processes, or more simply, reversible and irreversible rotational motion of the magnetic moments. These two different processes occur in this simple major loop. We use the hysteresis curve to see the macroscopic effect, and the fan diagrams to see the underlying motion of the individual magnetic moments.

In the return from saturation, the material starts off undergoing reversible motion. In the hysteresis curve this is seen as a smooth curve, with slope close to zero at saturation, and with gradually increasing slope as the return from saturation is made. In the fan diagrams the reversible motion is seen as a gradual motion of each magnetic moment in the fan toward the direction of applied field. The magnetic moments stay evenly distributed about the portion of the unit circle that the fan occupies.

At a well defined value of applied field, some irreversible motion occurs. In the hysteresis curve this is seen as a sudden vertical step. In the fan diagrams, it is seen as a set of magnetic moments in quick succession with increasing applied field making an irreversible jump from the old location in the fan to a far removed new location. This results in a banding pattern in the fan. A band is a distribution of angles about a central angle. This term may apply to the particle magnetic moments. The fan of magnetic moments forms various banding patterns, bands of magnetic moments separated by empty space, at various points in the hysteresis curve. We stress that the banding pattern is not caused by interactions between magnetic moments, as no interactions have been included here. Rather, a step of a certain size in increasing applied field results in a corresponding interval of magnetic moments in the fan making the irreversible jump. This motion is termed irreversible for the reason that if the applied field were reversed at this point, the hysteresis curve would not retrace back along its history. Instead, it would take a new path, and a minor loop would result. This is demonstrated in the section on minor loops.

On the continuing approach to saturation, a combination of reversible and irreversible motion occurs. As the applied field increases, an irreversible jump is made by all the magnetic moments that have not previously jumped. Meanwhile, the magnetic moments that have previously jumped, continue to undergo reversible motion in the approach to saturation. An applied field reversal anywhere here, in the approach to saturation, would also result in an interior minor loop. Eventually all the irreversible motion is finished, as all magnetic moments have made the irreversible jump, and the remainder of the approach to saturation consists only of reversible motion.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.2: Major loop to near saturation for a 100-particle isotropic, uniform array, of noninteracting particles. The material parameters are as listed in table 5.1. The circles on the curve indicate the points for which the fan diagrams are shown in the following series of figures (5.3 to 5.6).

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.3: The remanence state of the major loop in figure 5.2. Only reversible motion has occurred to this point in the major loop. This series of figures (5.3 to 5.6) corresponds to the points indicated in the major loop in figure 5.2. The particle fan is shown on the right. The short lines radiating from the edge of the unit circle are the particle easy axes. The long lines radiating from the center of the circle are the particle intrinsic magnetizations. The iJ -components of the (2×2) hysteresis tensor are shown on the left. The x -axes are the i -components of the applied field, and the y -axes are the J -components of the magnetization. The open dot on each curve indicates the current state.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.4: The last point of reversible motion in the major loop in figure 5.2. Only reversible motion has occurred to this point in the major loop.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.5: The coercive force state of the major loop in figure 5.2. This is the first point in the major loop at which some irreversible motion has occurred. Note that banding has occurred in the fan of magnetic moments. This is due to the irreversible jumping of an interval of magnetic moments in the fan, creating new occupied bands, and leaving behind unoccupied bands. The magnetic moments that did not make an irreversible jump underwent the usual reversible motion.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.6: Further irreversible and reversible motion in the major loop in figure 5.2.

Banded Arrays

In the case of an anisotropic multi-particle array, the bandedness provides an important description of the particle easy axis distribution. As stated earlier, a band is a distribution of angles about a central angle. This term may also apply to the particle easy axes. Various banding patterns may exist in the easy axis distribution of the array. In general, an array containing n bulk easy axes distributed around the unit circle is $2n$ -banded. The reason for this duplicity is that with each particle, the easy axis extends in both the positive and negative directions. Therefore each bulk easy axis is the center of two corresponding bands: one centered on the positive and the other centered on the negative directions of the axis, with each band being a distribution of particle easy axes. A 0-banded array is the isotropic case, in which the particle easy axes are distributed around the unit circle with no overall bulk easy axis. A 2-banded array consists of two distributions 180° apart. The type of distribution may vary. In this study, a uniform distribution was chosen. The other important descriptive parameter is the offset angle of the first bulk easy axis, $\bar{\phi}_{ea}$, specifying the center of the first band.

In the case of the 100-particle array of non-interacting particles, a 2-banded pattern is chosen, with the offset angle of the bulk easy axis, $\bar{\phi}_{ea}$, taking the same series of values as the single easy axis did in the single particle case. For each value of $\bar{\phi}_{ea}$, the magnetization loop tensor is generated. The complete tensors are shown in Figure 5.7. A corresponding series of fan diagrams is shown for the case of $\bar{\phi}_{ea}$ equal to 30° , in figures 5.8 to 5.14. The specific fan diagrams that are shown are those for the initial unmagnetized state, and the saturation, remanence, and coercive force states for both the positive and negative magnetization.

In the figure for the initial unmagnetized state (Figure 5.8), we see that there are two magnetization bands, which match the two easy axis bands. The magnetization vectors are unpaired in this initial state. In the figures for the remanence states (Figures 5.10 & 5.13) we see that there is only one magnetization band, which matches one of the two easy axis bands. The magnetization vectors have become paired. They cannot be made unpaired again by field reversal alone. Of particular interest are the figures for the coercive force states. (Figures 5.11 & 5.14.) Three magnetization

bands are evident. The band closest to the applied field direction consists of particles that have made an irreversible jump. The band opposite to the applied field direction consists of particles that have undergone reversible motion only. The band in between these consists of particles that have at that point undergone reversible motion only, but are about to make an irreversible jump.

100-particle, anisotropic, uniform array, with varying bulk easy axis

$$\alpha = 0.0$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.7: Hysteresis tensor major loops to near saturation for a 100-particle, anisotropic 2-banded array with uniform bands, for varying bulk easy axis, and with noninteracting particles. The bulk easy axis angle, $\bar{\phi}_{ea}$, varies from 0° to 90° , in increments of 10° . The material parameters are as listed in table 5.1. The following series of figures (5.8 to 5.14) show the fan diagrams for various points in the $\bar{\phi}_{ea} = 30^\circ$ loops.

100-particle, anisotropic, uniform array, with 30° bulk easy axis

$$\alpha = 0.0$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.8: Initial unmagnetized state of array, in the major loop in figure 5.7. This series of figures (5.8 to 5.14) is a set of fan diagrams corresponding to the loops in figure 5.7 for the 2-banded array of 100 S-W particles with bulk easy axis of $\bar{\phi}_{ea} = 30^\circ$. The short lines radiating from the edge of the unit circle are the particle easy axes. The long lines radiating from the center of the circle are the particle intrinsic magnetizations. The iJ -components of the (2×2) hysteresis tensor are shown on the left. The x -axes are the i -components of the applied field, and the y -axes are the J -components of the magnetization. The open dot on each curve indicates the current state.

100-particle, anisotropic, uniform array, with 30° bulk easy axis

$$\alpha = 0.0$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.9: Near saturation state in the positive direction of magnetization, in the major loop in figure 5.7.

100-particle, anisotropic, uniform array, with 30° bulk easy axis

$$\alpha = 0.0$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.10: Remanence state in the positive direction of magnetization, in the major loop in figure 5.7.

100-particle, anisotropic, uniform array, with 30° bulk easy axis

$$\alpha = 0.0$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.11: Coercive force state after near saturation in the positive direction of magnetization, in the major loop in figure 5.7.

100-particle, anisotropic, uniform array, with 30° bulk easy axis

$$\alpha = 0.0$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.12: Near saturation state in the negative direction of magnetization, in the major loop in figure 5.7.

100-particle, anisotropic, uniform array, with 30° bulk easy axis

$$\alpha = 0.0$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.13: Remanence state in the negative direction of magnetization, in the major loop in figure 5.7.

100-particle, anisotropic, uniform array, with 30° bulk easy axis

$$\alpha = 0.0$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.14: Coercive force state after near saturation in the negative direction of magnetization, in the major loop in figure 5.7.

5.1.2 Interactions

Non-Interacting Arrays

The results that have been presented to this point are for noninteracting arrays. The classic S-W model was hence sufficient to generate these results.

Next, we present results for interacting arrays, which are made possible by the three extensions of the inclusive model: a mean field interaction, a nearest neighbour dipole-dipole interaction, and pinning.

Mean Field Interaction

We discuss results for the mean field interaction. The array chosen for analysis of the mean field interaction is the 100—particle, uniform, isotropic array. The coordinates of the particles are not important, and hence the array is considered as a simple set of particles. No pinning is included. The major loops to near saturation for varying values of the mean field parameter α are shown in figure 5.15. As α is increased, the loops become more square. The irreversible jump becomes more abrupt as the strength of the interaction increases. This behaviour is well known from the Atherton-Beattie extension to the Stoner-Wohlfarth model [Atherton-1990]. We extend the analysis further by considering the corresponding fan diagrams.

Consider the series of figures 5.16 to 5.21, for the case of $\alpha = 0.5$. Figure 5.16 is merely the initial un-magnetized state of the array, and figure 5.17 is merely the near saturation state in the positive magnetization direction. Figure 5.18 is the remanence state, and in it we see an effect of the mean field interaction. Due to the interaction, the fan of particles is clustered on the right-hand side, about the direction of positive applied field, and has not returned to the semi-circle distribution typical for the non-interacting case. This is indicative of the particles acting on one another to keep them clustered, and is a form of memory of the previously positive applied field.

Figure 5.19 is the state just before the irreversible jump. It resembles the fan diagram corresponding to the non-interacting case. Figure 5.20 is the state just after the irreversible jump, and is quite different from that corresponding to the non-interacting

case. Almost the entire fan has made the irreversible jump, due to the interaction field. The banding structure seen in the non-interacting case has not occurred; the array has jumped almost as a whole. As the applied field is further increased in the negative direction, the particles complete the reversible approach to saturation, shown in figure 5.21.

100-particle, isotropic, uniform array

α varying

$k = 0.0$

Major loop to near saturation

Figure 5.15: Major loops to near saturation for a 100-particle, uniform, isotropic array, with varying values of the mean field parameter α . The other material parameters are as listed in table 5.1. As α is increased, the loops become more square at the sides as the irreversible jump becomes more abrupt. The following series of figures (5.16 to 5.21) show the fan diagrams for various points in the $\alpha = 0.5$ loop in this figure.

100-particle, isotropic, uniform array

$$\alpha = 0.5$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.16: Initial un-magnetized state of array for the $\alpha = 0.5$ loop in figure 5.15. This series of figures (5.16 to 5.21) corresponds to various points in the $\alpha = 0.5$ loop in figure 5.15. The particle fan is shown on the right. The short lines radiating from the edge of the unit circle are the particle easy axes. The long lines radiating from the center of the circle are the particle intrinsic magnetizations. The iJ -components of the (2×2) hysteresis tensor are shown on the left. The pinning parameter is set to $k = 0.0$.

100-particle, isotropic, uniform array

$$\alpha = 0.5$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.17: Near saturation state in the positive direction of magnetization for the $\alpha = 0.5$ loop in figure 5.15.

100-particle, isotropic, uniform array

$$\alpha = 0.5$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.18: Remanence state for the $\alpha = 0.5$ loop in figure 5.15. Note that the fan of particles is clustered on the right-hand side, about the direction of positive applied field. This form of memory of the previously positive applied field, even though the applied field has returned to zero, is enhanced by the presence of the mean field.

100-particle, isotropic, uniform array

$$\alpha = 0.5$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.19: The last state just before the irreversible jump for the $\alpha = 0.5$ loop in figure 5.15. The horizontal line in the fan diagram at the 180° position is the applied field direction vector.

100-particle, isotropic, uniform array

$$\alpha = 0.5$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.20: The first state just after the irreversible jump for the $\alpha = 0.5$ loop in figure 5.15. The banding structure that occurs in the non-interacting case has not occurred here; the array has irreversibly jumped, almost as a whole. The one particle that has not jumped is that at the 0° position. It rests in a position of unstable equilibrium.

100-particle, isotropic, uniform array

$$\alpha = 0.5$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.21: Near saturation state in the direction of negative magnetization for the $\alpha = 0.5$ loop in figure 5.15.

Dipole-Dipole Interaction

Next we discuss a dipole-dipole interaction extension to the S-W model. To begin with, we present a mathematically precise treatment of the dipole-dipole interaction on the microscopic scale, for an array of interacting dipoles, with periodic boundary conditions, with no crystalline anisotropy present. A series of array diagrams corresponding to various points in a major loop to saturation is shown in figures 5.22 to 5.27, for a 10×10 array of isolated magnetic dipoles, interacting through the dipole-dipole interaction. The interaction radius is set to $r_{nn} = 4$, which is the maximum value that can be used for an array of this size, before some dipole would be considered more than once as a dipole field source. It is interesting to note that the dipole-dipole interaction alone is sufficient to cause hysteresis. The initial state in figure 5.22 is the zero field state. It corresponds to the state of the local minimum in the array total energy that was closest to an initial random guess, and hence is dependent on the initial guess. In this particular example, the zero field state exhibits antiferromagnetic alignment in the vertical direction.

As the applied field is increased in the positive direction, to the right, the array configuration changes. Transition from the initial antiferromagnetic alignment to ferromagnetic alignment in the direction of the applied field occurs. This transition is seen in figures 5.22 to 5.25. Perfect ferromagnetic alignment is seen in the saturation state, in the positive direction of magnetization, in figure 5.25. A magnetic memory effect is seen in the remanence state in figure 5.26. The saturation state persists, even as the applied field is reduced to zero and then reversed. The irreversible jump occurs in figure 5.27, where the ferromagnetic alignment of the array flips from right to left. The array exhibits square hysteresis loops, with the flat upper and lower branches representing complete saturation in the positive and negative directions of magnetization respectively, and with the vertical sides representing instantaneous switching in magnetization direction. The sigmoid shaped hysteresis loops would only occur with the presence of the edge demagnetization effects that would accompany nonperiodic boundary conditions. Temperature effects have not been considered here. If temperature effects were included, then sigmoid shaped hysteresis loops would also occur at finite temperatures.

Our treatment of this interaction on the macroscopic scale of magnetic materials is more phenomenological than we have shown for the microscopic scale of isolated dipoles. We have attempted to use it to model arrays of single-domain crystals, with each domain represented as a single magnetic moment. On this scale the crystalline anisotropy term must be present. Our findings indicate that a direct application of a dipole-dipole field would have little observable effect, as it would tend to be overpowered by the crystalline anisotropy field. Therefore we have introduced the phenomenological factor α_{nn} , by which we multiply the dipole-dipole field, resulting in the modified dipole-dipole energy term:

$$\underline{e}_{nn}^i = -\alpha_{nn} \hat{m}^i \cdot \vec{h}_{nn} . \quad (5.1)$$

We have determined that a value of $\alpha_{nn} = 4.0$ results in a noticeable effect. The magnitude of the magnetic field of a single source dipole at a distance r is proportional to r^{-3} . However, in 2D, there would occur a number of source dipoles at radius r , proportional to r , with the result that the total dipole field experienced from all source dipoles, at radius r , is proportional to r^{-2} . This type of dependence falls to zero quickly, and hence we may choose interaction radius values r_{nn} accordingly. In 3D, the dependence would be proportional to r^{-1} . Therefore in 3D the dipole-dipole interaction would have a stronger effect, and an amplification factor of α_{nn} would probably not be necessary. In 3D, for the same reason, it might also be necessary to use much larger interaction radius values r_{nn} .

The array chosen for study of the dipole-dipole effect was a 10×10 particle, random array, with an overall bulk easy-axis. The coordinates of the individual particles were important, and were for simplicity taken as those of a regular square array, with periodic boundary conditions to simulate a region of magnetic material deep inside a macroscopic sample. The coordinates may be taken as simply the integer number of array spacings in the $\hat{x}$ - and $\hat{y}$ - directions, as the array spacing parameter, s , cancels out of the formulation. The dipole-dipole effect seemed to be most noticeable for arrays with the overall inherent order of a bulk easy axis. The particular array chosen was one with a bulk easy axis of $\bar{\phi}_{ea} = 30^\circ$ and a distribution deviation of $\sigma = 10^\circ$. No pinning was included. Figures 5.28 to 5.32 show the dipole-dipole effect for increasing values of the interaction radius r_{nn} .

The difference between $r_{nn} = 0$ and $r_{nn} = 1$ is dramatic. The dipole-dipole interaction results in a butterfly shaped hysteresis loop, with the approach to saturation consisting of large steps. Increasing the maximum interaction radius to $r_{nn} = 2$ seems to result in earlier saturation. It is increased up to $r_{nn} = 4$, the maximum possible value for an array of this size.

We attempt to explain the regions of large steps that result in a butterfly shaped curve. Consider the set of array diagrams shown in figures 5.33 to 5.39, corresponding to the $r_{nn} = 4$ loop. Figure 5.33 is the initial un-magnetized state, and figure 5.34 is the near saturation state in the direction of positive magnetization. Figure 5.35 is the remanence state, and is very similar to the way it would appear for the non-interacting case. This indicates that the return from saturation in the applied field segment $h_{\text{applied}} = +|h_{\text{max}}|$ to 0 is affected very little by the dipole-dipole interaction.

Figure 5.36 is the coercive state, and is the beginning of a horizontal step segment. It is the beginning of the portion of the curve from there to saturation in the direction of negative magnetization, which is affected by the dipole-dipole interaction. Antiferromagnetic alignment has occurred. Figure 5.37 is the end of the horizontal step segment. There has been a small increase in overall order, as is evident from the slight change in magnetization value toward negative saturation. The rows of particles that were aligned in the direction of the applied field have become more strongly aligned in that direction. Many rows remain aligned opposite the applied field direction. Some particles in these rows are oriented at odd angles, as they are close to undergoing an irreversible jump in orientation. The dipole-dipole interaction, for this type of array in which the particle easy axes are distributed about an overall bulk easy axis, has effectively hindered the approach to saturation, and the increase of overall magnetic order, resulting in the flat segment of the step. Figure 5.38 is the bottom of a sudden vertical step segment. A sudden increase in overall magnetic order has occurred. Some rows of the array have completely flipped orientation to align with the applied field. Some rows remain aligned opposite the applied field direction.

The above stepping process is repeated a few times between the coercive field point and the near saturation point. The dipole-dipole field has resulted in a stepped approach to overall magnetic order, in this region of the hysteresis curve, which gives the butterfly shaped loop.

10 × 10-particle array of magnetic dipoles with $K_{u_1} = 0.0$

$$\alpha = 0.0$$

$$k = 0.0$$

$$r_{nn} = 4$$

$$\alpha_{nn} = 1.0$$

Major loop to saturation

Figure 5.22: Zero field state of array, showing antiferromagnetic alignment in the vertical direction, in a major loop to saturation, for a 10 × 10-particle, random array of interacting magnetic dipoles, with $r_{nn} = 4$. The array is a 10 × 10 arrangement of magnetic dipoles interacting through the dipole-dipole interaction alone, with periodic boundary conditions, and with no crystalline anisotropy. The nearest neighbour maximum interaction radius is $r_{nn} = 4$. This is the maximum interaction radius that can be used for an array of this size, before some dipole would be considered more than once as a dipole field source. The other material parameters are as listed in table 5.1, except for the crystalline anisotropy, which by definition is set to $K_{u_1} = 0.0$. This is the state corresponding to the local minimum in total energy of the array that was closest to an initial random guess, and is hence dependent on the initial guess. This series of array diagrams (figures 5.22 to 5.27) corresponds to various stages in a major loop to saturation. The particle array is shown on the right, with the long lines within the unit circles representing the current particle magnetic moment alignments. The iJ -components of the (2 × 2) hysteresis tensor are shown on the left.

10 × 10-particle array of magnetic dipoles $K_{u1} = 0.0$

$$\alpha = 0.0$$

$$k = 0.0$$

$$r_{nn} = 4$$

$$\alpha_{nn} = 1.0$$

Major loop to saturation

Figure 5.23: Transition to ferromagnetic alignment in the applied field direction, in a major loop to saturation, for the array in figure 5.22. The applied field direction is to the right.

10 × 10-particle array of magnetic dipoles $K_{u1} = 0.0$

$$\alpha = 0.0$$

$$k = 0.0$$

$$r_{nn} = 4$$

$$\alpha_{nn} = 1.0$$

Major loop to saturation

Figure 5.24: Continuing transition to ferromagnetic alignment in the applied field direction, in a major loop to saturation, for the array in figure 5.22.

10 × 10-particle array of magnetic dipoles $K_{u1} = 0.0$

$$\alpha = 0.0$$

$$k = 0.0$$

$$\tau_{nn} = 4$$

$$\alpha_{nn} = 1.0$$

Major loop to saturation

Figure 5.25: Saturation state in the positive direction of magnetization, in a major loop to saturation, for the array in figure 5.22.

10×10 -particle array of magnetic dipoles $K_{u_1} = 0.0$

$$\alpha = 0.0$$

$$k = 0.0$$

$$r_{nn} = 4$$

$$\alpha_{nn} = 1.0$$

Major loop to saturation

Figure 5.26: Remanence state in a major loop to saturation, for the array in figure 5.22. Saturation persists due to the dipole-dipole interaction.

10 × 10-particle array of magnetic dipoles $K_{u_i} = 0.0$

$$\alpha = 0.0$$

$$k = 0.0$$

$$r_{nn} = 4$$

$$\alpha_{nn} = 1.0$$

Major loop to saturation

Figure 5.27: Switching in magnetization, for the interacting dipole array in figure 5.22. The applied field direction is now to the left. The array of magnetic dipoles interacting through the dipole-dipole interaction, with periodic boundary conditions, exhibits square hysteresis loops, with the array magnetization at saturation in each direction of magnetization. The sigmoid shaped hysteresis loops would only occur with the presence of the edge demagnetization effects that would accompany nonperiodic boundary conditions.

10 × 10-particle anisotropic, random array, with 30° bulk easy axis and 10°
distribution deviation

$$\alpha = 0.0$$

$$k = 0.0$$

$$r_{nn} = 0$$

$$\alpha_{nn} = 4.0$$

Major loop to near saturation

Figure 5.28: Major loop to near saturation for a 10 × 10-particle, anisotropic, random array with a bulk easy axis of $\bar{\phi}_{ea} = 30^\circ$ and a distribution deviation of $\sigma = 10^\circ$, for $r_{nn} = 0$, $\alpha_{nn} = 4.0$. Periodic boundary conditions are in place. This series of figures (5.28 to 5.31) shows major loops to near saturation for a 10 × 10-particle, random array with a bulk easy axis of $\bar{\phi}_{ea} = 30^\circ$ and a distribution deviation of $\sigma = 10^\circ$, for varying values of the nearest neighbour maximum interaction radius r_{nn} . The multiplication factor is set to $\alpha_{nn} = 4.0$. The other material parameters are as listed in table 5.1. Both the m_{xX} and m_{xY} components are shown.

10 × 10-particle anisotropic, random array, with 30° bulk easy axis and 10°
distribution deviation

$$\alpha = 0.0$$

$$k = 0.0$$

$$r_{nn} = 1$$

$$\alpha_{nn} = 4.0$$

Major loop to near saturation

Figure 5.29: Major loop to near saturation for a 10 × 10-particle, anisotropic, random array with a bulk easy axis of $\bar{\phi}_{ea} = 30^\circ$ and a distribution deviation of $\sigma = 10^\circ$, for $r_{nn} = 1$, $\alpha_{nn} = 4.0$. Periodic boundary conditions are in place. Comparison of this figure with the previous one indicates that the difference between no interaction and a dipole-dipole interaction with $r_{nn} = 1$ is quite dramatic. The dipole-dipole interaction results in a butterfly shaped hysteresis loop. The approach to saturation is seen to consist of large steps of alternating horizontal segments where the approach to magnetic order is delayed, and vertical segments where a sudden jump in magnetic order is made.

10 × 10-particle anisotropic, random array, with 30° bulk easy axis and 10°
distribution deviation

$$\alpha = 0.0$$

$$k = 0.0$$

$$\tau_{nn} = 2$$

$$\alpha_{nn} = 4.0$$

Major loop to near saturation

Figure 5.30: Major loop to near saturation for a 10 × 10-particle, anisotropic, random array with a bulk easy axis of $\bar{\phi}_{ea} = 30^\circ$ and a distribution deviation of $\sigma = 10^\circ$, for $\tau_{nn} = 2$, $\alpha_{nn} = 4.0$. Periodic boundary conditions are in place.

10 × 10-particle anisotropic, random array, with 30° bulk easy axis and 10°
distribution deviation

$$\alpha = 0.0$$

$$k = 0.0$$

$$r_{nn} = 3$$

$$\alpha_{nn} = 4.0$$

Major loop to near saturation

Figure 5.31: Major loop to near saturation for a 10 × 10-particle, anisotropic, random array with a bulk easy axis of $\bar{\phi}_{ea} = 30^\circ$ and a distribution deviation of $\sigma = 10^\circ$, for $r_{nn} = 3$, $\alpha_{nn} = 4.0$. Periodic boundary conditions are in place.

10 × 10-particle anisotropic, random array, with 30° bulk easy axis and 10°
distribution deviation

$$\alpha = 0.0$$

$$k = 0.0$$

$$r_{nn} = 4$$

$$\alpha_{nn} = 4.0$$

Major loop to near saturation

Figure 5.32: Major loop to near saturation for a 10 × 10-particle, anisotropic, random array with a bulk easy axis of $\bar{\phi}_{ea} = 30^\circ$ and a distribution deviation of $\sigma = 10^\circ$, for $r_{nn} = 4$, $\alpha_{nn} = 4.0$. Periodic boundary conditions are in place. This is the maximum interaction radius that can be used for an array of this size, before some dipole would be considered more than once as a dipole field source.

10 × 10-particle anisotropic, random array, with 30° bulk easy axis and 10°
distribution deviation

$$\alpha = 0.0$$

$$k = 0.0$$

$$r_{nn} = 4$$

$$\alpha_{nn} = 4.0$$

Major loop to near saturation

Figure 5.33: Initial un-magnetized state of the 10 × 10-particle, anisotropic, random array in the major loop in figure 5.32 This series of figures (5.33 to 5.39) is a set of array diagrams corresponding to the loop for the 10 × 10-particle, anisotropic, random array with a bulk easy axis of $\bar{\phi}_{ea} = 30^\circ$ and a distribution deviation of $\sigma = 10^\circ$, for $r_{nn} = 4$, $\alpha_{nn} = 4.0$ in figure 5.32. The particle array is shown on the right, with the tick marks within the unit circles representing the particle easy axes, and the long lines representing the current particle intrinsic magnetizations. The iJ -components of the (2×2) hysteresis tensor are shown on the left. The pinning parameter is set to $k = 0.0$.

10 × 10-particle anisotropic, random array, with 30° bulk easy axis and 10°
distribution deviation

$$\alpha = 0.0$$

$$k = 0.0$$

$$r_{nn} = 4$$

$$\alpha_{nn} = 4.0$$

Major loop to near saturation

Figure 5.34: Near saturation state in the direction of positive magnetization for the 10 × 10-particle, anisotropic, random array in the major loop in figure 5.32. The applied field direction is to the right.

10 × 10-particle anisotropic, random array, with 30° bulk easy axis and 10°
distribution deviation

$$\alpha = 0.0$$

$$k = 0.0$$

$$\tau_{nn} = 4$$

$$\alpha_{nn} = 4.0$$

Major loop to near saturation

Figure 5.35: Remanence state of the 10 × 10-particle, anisotropic, random array in the major loop in figure 5.32. The array configuration is very similar to the way it would be for the non-interacting case, as the magnetic moments do not deviate much from the closest easy axis direction. This is an indication that the return from saturation in the applied field segment $\underline{h}_{applied} = +|\underline{h}_{max}|$ to 0 is affected very little by the dipole-dipole interaction.

10 × 10-particle anisotropic, random array, with 30° bulk easy axis and 10°
distribution deviation

$$\alpha = 0.0$$

$$k = 0.0$$

$$r_{nn} = 4$$

$$\alpha_{nn} = 4.0$$

Major loop to near saturation

Figure 5.36: The coercive force state of the 10 × 10-particle, anisotropic, random array in the major loop in figure 5.32. The applied field direction is now to the left. The hysteresis loop segment from this point to saturation in the direction of negative magnetization is strongly affected by the dipole-dipole interaction; the segment will be seen to consist of large steps of alternating horizontal and vertical segments. This figure is the first state of a horizontal segment. By definition, there is no overall magnetic order in this state. However, there is much local order. Antiferromagnetic alignment has occurred, with some rows of particles oriented in the direction of the applied field, and some oriented in the reverse direction.

10 × 10-particle anisotropic, random array, with 30° bulk easy axis and 10°
distribution deviation

$$\alpha = 0.0$$

$$k = 0.0$$

$$r_{nn} = 4$$

$$\alpha_{nn} = 4.0$$

Major loop to near saturation

Figure 5.37: The final state of the current horizontal segment, just before an irreversible jump, for the 10 × 10-particle, anisotropic, random array in the major loop in figure 5.32. There has been a small increase in overall magnetic order, as the horizontal segment was not completely level, but dropped slightly toward negative saturation. The rows of particles that were aligned in the applied field direction (to the left) are now more strongly aligned in that direction. A few of the particles from the rows that were aligned opposite the applied field direction (to the right) are now at odd angles, as they are about to undergo a flip in orientation.

10 × 10-particle anisotropic, random array, with 30° bulk easy axis and 10°
distribution deviation

$$\alpha = 0.0$$

$$k = 0.0$$

$$r_{nn} = 4$$

$$\alpha_{nn} = 4.0$$

Major loop to near saturation

Figure 5.38: The first state just after an irreversible jump, and the beginning of a new horizontal segment, for the 10 × 10-particle, anisotropic, random array in the major loop in figure 5.32. A discontinuous increase in overall magnetic order has occurred. Two rows have flipped orientation to align with the applied field. The rows of particles that were aligned opposite the applied field direction (to the right) are now more strongly aligned in that direction.

10 × 10-particle anisotropic, random array, with 30° bulk easy axis and 10°
distribution deviation

$$\alpha = 0.0$$

$$k = 0.0$$

$$r_{nn} = 4$$

$$\alpha_{nn} = 4.0$$

Major loop to near saturation

Figure 5.39: Near saturation in the direction of negative magnetization, after the final irreversible jump, for the 10 × 10-particle, anisotropic, random array in the major loop in figure 5.32. All rows are now aligned in the applied field direction (to the left).

Pinning Interaction

We have conducted several studies with the 100-particle, uniform, isotropic array to illustrate the pinning interaction. Here, we present results on pinning in major loops. Minor loops and pinning will be covered in the section on minor loops.

Figure 5.40 shows a set of initial magnetization curves and major loops to near saturation, for varying values of the pinning parameter k . In comparing the loops, it is seen that the magnitude of magnetization at the maximum applied field point of 1.0 MA/m , decreases for increasing value of k . This is easily explained using the set of accompanying fan diagrams in figures 5.41 to 5.43. As the pinning constant is increased and the magnitude of pinning torque increases, the motion of the magnetic moments becomes increasingly inhibited in the approach to saturation. In fact, the higher the pinning, the less dense the distribution of magnetic moments is in the fan, when the applied field has reached the maximum field point. In addition, some of the magnetic moments have not jumped to the applied field direction by that point. Both effects result in a smaller value of magnetization magnitude at the maximum field point.

A principal feature appearing in the loops with pinning is a 'flattening' effect. Essentially, it is a zero slope region of the curve, where there has been a zero change in the magnetization, for a corresponding finite change in the applied field. It is most dramatic at the return from saturation. In the unpinned loop, the return from saturation is a curve of continuously increasing slope. In the pinned loops, it is a flat region. The reason the flat region occurs is that the magnetic torque from the applied field has not overcome the pinning torque acting on the magnetic moments, and they have remain fixed, resulting in the magnetization remaining fixed. This effect also occurs in minor loops, as will be seen shortly. Since in our model the pinning torque magnitude is proportional to the magnetization magnitude, pinning occurs to a greater degree closer to saturation. This is easily seen in the greater degree of flattening closer to saturation.

The flattening effect, in which the magnetization remains fixed, often occurs in succession with discontinuous changes in magnetization, resulting in a 'stepping' effect. This is seen in the approach to saturation in the pinned loops. The reason it occurs is

that the loops alternately go through regions where the pinning is not overcome and the magnetic moments are held pinned, and regions where the pinning is overcome, and the magnetic moments make a discontinuous, irreversible jump. This stepping effect is physically expected with the inclusion of the pinning interaction. It may be thought of as a form of Magnetic Barkhausen Noise (MBN), with the underlying mechanism being pinning of domain rotation, as opposed to the usual pinning of domain wall motion that occurs in the soft magnetic materials.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

k varying

Major loop to near saturation

Figure 5.40: Major loop to near saturation for a 100-particle, isotropic, uniform array, with varying pinning parameter k . The other material parameters are as listed in table 5.1.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Major loop to near saturation

Figure 5.41: The maximum applied field point for the $k = 0.0$ major loop in figure 5.40. All the magnetic moments are contributing to the fan in the applied field direction.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.5$$

Major loop to near saturation

Figure 5.42: The maximum applied field point for the $k = 0.5$ major loop in figure 5.40. Due to pinning, not all the magnetic moments are contributing to the fan in the applied field direction. The fan in the applied field direction is also less densely distributed than in figure 5.41.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 1.0$$

Major loop to near saturation

Figure 5.43: The maximum applied field point for the $k = 1.0$ major loop in figure 5.40. Due to pinning, not all the magnetic moments are contributing to the fan in the applied field direction: less, in fact, than in figure 5.42. The fan in the applied field direction is also less densely distributed than in both figures 5.41 and 5.42.

5.2 Pinning and Minor Loops

Major and minor loop studies have been made using a 100-particle array to illustrate the pinning interaction. The array that was chosen was a uniform, isotropic array. The basic material parameters are as listed in table 5.1. No mean-field or nearest neighbour interactions were included here. The pinning parameter k was varied.

The primary topic of this section is minor loops, and the effect of pinning on minor loops. We begin with a series of minor loops with no pinning. Figure 5.44 shows a set of small minor loops originating from a major loop perimeter, in both the upper branch and the left near vertical segment. The width of the minor loops, along the applied field axis, is $0.2 MA/m$. Such a small change in applied field ensures that only small rotations in the magnetic moments occur over the minor loops. The absence of pinning furthermore ensures that the small rotations are entirely reversible. Therefore all the minor loops are closed, that is, contain no area. For this reason, some of the minor loops here are not even visible. The minor loops originating from a purely reversible portion of the major loop, in this case the upper branch, are not visible, as they follow exactly the path of the major loop. The minor loops originating from an irreversible portion, in this case the left near vertical segment, are visible, as they deviate from the path of the major loop. This serves as a clear illustration of the irreversible character of the near vertical segments, where the irreversible hysteretic jumps occur. We have included a set of minor loops in the 2nd quadrant. These loops could be of interest to design engineers in the design of permanent magnet devices [Parker-1990], as they would serve as the recoil lines that are needed to design a permanent magnet of maximum possible recoil energy. Note that these minor loops are curved, as opposed to the straight lines that are often used as approximations. Their curvature is concave downward due to the presence of reversible motion only, and is close to but slightly differing from that of the major loop upper branch. Curvature concave upward would occur only if some irreversible motion was present.

We next show a further series of minor loops with no pinning. Figure 5.45 is a set of large minor loops originating from a major loop perimeter. The left endpoints are taken along the left near vertical segment, and the loops involve a complete return to

the maximum field point to the right. As a result, the minor loops rejoin the major loop along the upper branch on the approach to saturation. These minor loops involve a change in applied field that is large enough to cause irreversible motion, resulting in the minor loops being open, that is, containing area. Of particular interest is the way in which these large minor loops compare to those of soft magnetic materials in the Jiles-Atherton model [Jiles-1986]. In particular, some are seen to cross the initial magnetization curve. The reversible segments of these minor loops extend quite far to the right side of the diagram before the irreversible segment occurs. The transition interval is the applied field domain interval in which the slope is discontinuous and becomes near vertical.

In figure 5.46 we show just one of the large minor loops from figure 5.45. The circles indicate the various interesting points along the loop for which we have included the corresponding fan diagrams in the series of figures 5.47 to 5.50. Of particular note here is the way in which the minor loop closely follows the initial magnetization curve from the initial unmagnetized point to the elbow, and then abruptly deviates. This may be considered a clear indication that the model is highly history-dependent.

Figure 5.47 is the fan diagram for the left endpoint of the large minor loop. The magnetic moment banding structure that is evident is due to the irreversible motion that has occurred in the preceding segment of the major loop. The reason why irreversible motion results in a banding structure has been investigated was discussed in the section on major loops. Figure 5.48 is the last point of purely reversible motion in the minor loop. As is characteristic of this type of motion, the banding structure of the previous figure has been preserved, though the moments have rotated reversibly to the right. Figure 5.49 is the first point of irreversible motion in the minor loop. The banding structure has changed, as there have been irreversible jumps by some bands of magnetic moments. Finally, figure 5.50 is the closing right endpoint of the minor loop. All the irreversible motion in the lower branch of the minor loop is complete. The banding structure is gone, leaving the magnetic moments uniformly distributed over the fan. A field reversal to the left would initially result in purely reversible motion once again.

Next we consider the effects of pinning on minor loops. Figure 5.51 shows a set of small minor loops originating from a major loop perimeter, in both the upper

branch and the left near vertical segment. The width of the minor loops, along the applied field axis, is $0.2 MA/m$. This small change in applied field ensures that only small rotations in the magnetic moments occur over the minor loops. The presence of pinning, however, means that even these small rotations constitute irreversible motion. Therefore all the minor loops are open, that is, they contain area. For this reason they are all visible. The minor loops originating from the upper branch of the major loop have the same general alignment as the major loop. Though this time they are open, and hence visible. The minor loops originating in the left near vertical segment, the hysteretic jump region, continue to deviate from the major loop. The shapes of the minor loops in this figure vary significantly. The shape of a particular minor loop depends on the mechanisms of flattening and small rotations. The greater the magnitude of pinning torque that is present, the greater the degree of flattening there occurs at the start of both the lower and upper segments of the minor loop, as the segments are traced from lower to upper in the counter-clockwise direction. The flattening occurs at the beginning of each segment, until the applied field has increased enough in the current direction, to overcome the pinning. From that point until the end of the segment, small rotations occur, and the segment curves. These two mechanisms result in the particular shape of the minor loop. The pinning mechanism depends on both the value of k and the proximity of the minor loop to saturation.

The circles in figure 5.51 indicate the points for which the corresponding fan diagrams are shown, in figures 5.52 to 5.55. Each pair of figures shows the left and right endpoints of a minor loop, and demonstrates that in the small minor loops, the magnetic moments have made small rotations, with no jumps, toward the right, the direction of increase in applied field. They also indicate that the fan structure is preserved within the small minor loop. The rectangle in figure 5.51 indicates an individual minor loop which is shown in greater detail in figures 5.59 and 5.60, and which is discussed at the end of this section.

Of particularly important note are the ways in which this model compares and contrasts with the Preisach model [Preisach-1935]. The properties that are compared are the number of magnetic domains that undergo a change in a minor loop, the degree of change, closure of minor loops, and incongruence of minor loops with common field turning points.

In the Preisach model, a minor loop results from a switching in polarity of a small number of magnetic domains, and hence only irreversible magnetization processes may be modeled. In this model, however, a small minor loop results from a small rotation in all the magnetic moments. This is seen in the series of fan diagrams in figures 5.52 to 5.55. Note the fan diagrams corresponding to the left endpoint of a loop and the right endpoint of the loop. The difference between the left and right endpoints is that all the magnetic moments have rotated by a small amount. In short, minor loops in the Preisach model involve a small number of domains making big changes, while in contrast, minor loops in this model involve a large number of domains making small changes.

In the Preisach model, minor loops with common field turning points are the same in shape. (Though there is a nonlinear version of the Preisach model for which this is not necessarily the case [Mayergoyz-1991]). This is the property of congruence, and is due to the fact that the same region of magnetic domains switches in the same way for the same set of minor loop turning points, irrespective of the magnetization coordinate of the minor loop in the hysteresis diagram. In our model, though, minor loops with common field turning points are incongruent in shape. As was explained earlier, the shape of a minor loop is dependent on its proximity to saturation. Therefore in a set of minor loops with common minor loop turning points, the shape of a particular loop is dependent on its magnetization coordinate. This is the property of incongruence of minor loops with common field turning points, and is another property in contrast to the Preisach model. Figure 5.56 demonstrates this property with a set of interior minor loops with common field turning points of 0.0 and 0.2 MA/m , and varying magnetization coordinates in the hysteresis graph. The circles in the figure indicate the turning points of one of the minor loops, for which the corresponding fan diagrams are shown in figures 5.57 and 5.58. It is a small minor loop, and the fan diagrams demonstrate again that the magnetic moments have made small rotations with no jumps toward the direction of increase in applied field, while preserving the banding structure. There may exist a class of magnetic materials for which these predictions are accurate.

The final property of the Preisach model that we consider here is the property of closure of minor loops. In our model, minor loops do not necessarily exhibit closure

after just one cycle, but do eventually close after several cycles. This behaviour is expected from experiment [Ewing-1900]. Close inspection of the minor loops in figure 5.51 reveals that the minor loops originating in the left near vertical segment, where the irreversible hysteretic jumps occur, are not closed at the left endpoints. The minor loop indicated with the surrounding rectangle is shown in greater detail in figure 5.59. On this scale, it is clear that the minor loop is not closed at the left endpoint. However, when the minor loop is cycled through several times to allow it to stabilize, the left endpoint eventually becomes closed, as is seen in figure 5.60. Therefore in this model, minor loops do exhibit eventual closure.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Perimeter minor loops

Figure 5.44: Small minor loops originating from a major loop perimeter for a 100-particle, isotropic, uniform array, with zero pinning constant, $k = 0.0$. All minor loops here involve a small change in applied field from one endpoint to the other (0.2 MA/m), and hence small rotations only occur there. The absence of pinning ensures that the small rotations are entirely reversible. Therefore all the minor loops are closed. The minor loops originating from the reversible portion of the major loop, the upper branch segment, are not visible, as they follow exactly the path of the major loop. The minor loops originating from the irreversible portion of the major loop, the hysteresis jump segment, are visible, as they deviate from the path of the major loop. The minor loops in the 2nd quadrant could easily serve as recoil lines for design engineers, in the design of permanent magnet devices. Note that these minor loops are curved concave downward, as opposed to the straight line approximations that are often used, due to the presence of reversible motion only. There is some asymmetry in the stepping patterns in the approach to saturation in the positive and negative directions of magnetization, due to differing applied field increments.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Large minor loops

Figure 5.45: Large minor loops originating from a major loop perimeter for a 100-particle, isotropic, uniform array, with zero pinning constant, $k = 0.0$. All minor loops here involve a complete return to the maximum field point, a large change in applied field, and hence irreversible motion occurs. Therefore the minor loops are open. Note that pinning is still absent. The properties of these loops are quite different than those of soft magnetic materials in the Jiles-Atherton model. In particular, some are seen to cross the initial magnetization curve. The reversible component of the minor loops originating on the left side of the diagram extends quite far to the right side of the diagram before the irreversible motion occurs. The transition interval is the applied field domain interval in which the slope is discontinuous and becomes near vertical. One of these large minor loops is shown singly in figure 5.46, and some accompanying fan diagrams are shown in figures 5.47 to 5.50.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Single large minor loop

Figure 5.46: A single large minor loop from figure 5.45. The circles indicate the points in the minor loop for which the corresponding fan diagram is shown, in figures 5.47 to 5.50. The minor loop closely follows the initial magnetization curve closely, from the initial unmagnetized point to the elbow, at which point it deviates. This is a clear indication that the model is highly history-dependent.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Single large minor loop

Figure 5.47: Left endpoint of the large minor loop in figure 5.46. The magnetic moment banding structure that is evident is due to the irreversible motion that has occurred in the major loop [Phelps-1998a]. This series of figures (5.47 to 5.50) corresponds to the points indicated in the major loop in figure 5.46. The particle fan is shown on the right. The short lines radiating from the edge of the unit circle are the particle easy axes. The long lines radiating from the center of the circle are the particle intrinsic magnetizations. The iJ -components of the (2×2) hysteresis tensor are shown on the left. The x -axes are the i -components of the applied field, and the y -axes are the J -components of the magnetization. The open dot on each curve indicates the current state. In this study, only the m_{xX} component is of interest.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Single large minor loop

Figure 5.48: Last point of reversible motion in the large minor loop in figure 5.46. The banding structure seen in the previous figure has been preserved, though the bands have rotated reversibly toward the applied field direction, to the right.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Single large minor loop

Figure 5.49: First point of irreversible motion in the large minor loop in figure 5.46. The banding structure has changed, as there have been irreversible jumps in some bands of magnetic particles.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Single large minor loop

Figure 5.50: Right endpoint of the large minor loop in figure 5.46. All irreversible motion in the lower branch of the minor loop is complete. The banding structure is gone, leaving the magnetic moments uniformly distributed over the fan.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.1$$

Perimeter minor loops

Figure 5.51: Minor loops originating from a major loop perimeter for a 100-particle, isotropic, uniform array, with pinning constant $k = 0.1$. The presence of pinning has introduced irreversibility into all motion of the magnetic moments, even the small rotations. Therefore all the minor loops are open and hence are visible. The circles mark the minor loop turning points for which the corresponding fan diagrams are displayed in figures 5.52 to 5.55. The rectangle marks an individual minor loop which is studied in greater detail in figures 5.59 and 5.60, regarding the property of closure.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.1$$

Perimeter minor loops

Figure 5.52: Left endpoint of minor loop in figure 5.51, with endpoints 0.0 and 0.2 MA/m . This minor loop originates from a region in the major loop in which no jumps have occurred. This series of figures (5.52 to 5.55) corresponds to the points indicated in the major loop in figure 5.51.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.1$$

Perimeter minor loops

Figure 5.53: Right endpoint of minor loop in figure 5.51, with endpoints 0.0 and 0.2 MA/m . The magnetic moments have made small rotations, with no jumps, toward the right, the direction of increase in applied field. The fan structure has been preserved in the small minor loop. All the magnetic moments have moved a small angle, in contrast to the Preisach model, in which only a few magnetic domains would change by switching polarity.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.1$$

Perimeter minor loops

Figure 5.54: Left endpoint of minor loop in figure 5.51, with endpoints -0.35 and -0.15 MA/m . This minor loop originates from a region in the major loop in which irreversible jumps have occurred, as is evident from the banding pattern in the magnetic moment distribution [Phelps-1998a].

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.1$$

Perimeter minor loops

Figure 5.55: Right endpoint of minor loop in figure 5.51, with endpoints -0.35 and -0.15 MA/m . The magnetic moments have made small rotations, with no jumps, toward the right, the direction of increase in applied field. The banding structure has been preserved between the minor loop endpoints, though it has rotated.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.2$$

Interior minor loops

Figure 5.56: Interior minor loops, for a 100-particle, isotropic, uniform array, with pinning constant $k = 0.2$. This series of interior minor loops, with common field turning points, but varying magnetization coordinates in the hysteresis plot, indicates the property of incongruence in this model. The circles mark the minor loop turning points for which the corresponding fan diagrams are displayed in figures 5.57 and 5.58.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.2$$

Interior minor loops

Figure 5.57: Left endpoint of the interior minor loop in figure 5.56 with endpoints 0.0 and 0.2 MA/m . This set of figures (5.57 and 5.58) corresponds to the points indicated in the major loop in figure 5.56.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.2$$

Interior minor loops

Figure 5.58: Right endpoint of the interior minor loop in figure 5.56 with endpoints 0.0 and 0.2 MA/m . The magnetic moments have made small rotations, with no jumps, toward the right, the direction of increase in applied field. The banding structure has been preserved in the minor loop, though it has rotated.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.1$$

Multiple cycles through single perimeter minor loop

Figure 5.59: An individual minor loop from figure 5.51. On this scale, it is evident that the minor loop is not closed at the left endpoint. However, it is expected that the property of closure will hold if the minor loop is cycled several times, as in the following figure. The arrows indicate the direction followed through the minor loop.

100-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.1$$

Multiple cycles through single perimeter minor loop

Figure 5.60: Multiple cycles through the minor loop in figure 5.59. The minor loop has actually been cycled 3 times. This has successfully closed the loop at the left endpoint. We therefore maintain that the property of closure holds in this model. The arrows indicate the direction followed through the minor loop. The numbers 1, 2 & 3 refer to the cycle number.

5.3 Demagnetization

In this section, we use our inclusive model to conduct computational studies in demagnetization of hard magnetic materials. We investigate the demagnetizing response of both rotating and oscillating demagnetizing fields on both isotropic and anisotropic idealized polycrystalline materials described by the various particle easy axis distributions discussed earlier. We also investigate the additional effects introduced by the inclusion of the mean field interaction, as well as the inclusion of low, medium, and high pinning. Real demagnetization of hard magnetic materials in the laboratory through purely magnetic means is very difficult. We hope that our computational results will help to determine the most effective methods that can be used for the classes of hard magnetic materials considered.

In the laboratory, a soft magnetic material sample is demagnetized with a series on the order of 100 cycles, of amplitude diminishing from near saturation to zero. One demagnetization takes on the order of a couple of minutes [Jiles-1984]. The more cycles used, the better the demagnetization achieved. Hard magnetic materials are inherently much more difficult to demagnetize than soft magnetic materials.

In our numerical simulations, we specified a demagnetizing field that was either rotating or oscillating, while diminishing in magnitude. For the rotating demagnetization, the applied field rotation was performed in the xy -plane of the array, about the origin, in the counter-clockwise direction. The applied field magnitude was reduced in a set of 600 cycles from 1.0 to 0.4 (MA/m), then reduced in very fine steps in a set of 1000 cycles from 0.4 to 0.3 (MA/m), and finally in a set of 300 cycles from 0.3 to 0.0 (MA/m). Not every cycle is shown in the figures. In the middle set of 1000 cycles, only every 100th cycle is shown. In the first and last sets, only the first and last cycles are shown. The tip of the reduced array magnetization vector, $\vec{m}$, is plotted in polar form as the Y -component vs. the X -component, where $\vec{M} = m_s \vec{m}$. For the oscillating demagnetization, the applied field oscillation was performed along the x -axis of the array. The applied field in this case is essentially the x -component of the applied field used in the rotating case. The field magnitude limits are the same, and the cycle numbers that are shown in the figures are the same as well.

The basic material parameters used are listed in table 5.1. α and k were given various

nonzero values in the various interacting cases.

The simulations were costly in computation time. A single demagnetization took approximately 3 hours for a noninteracting case, and approximately 2 days for an interacting case, on a Pentium II, 300 MHz machine.

The first case considered is demagnetization with continuously decreasing magnitude, for a 99-particle, isotropic, uniform array, with no pinning or interactions, shown in figure 5.61. For the rotating demagnetizing field, demagnetization was nearly achieved with this method, as the final points in the hysteresis tensor elements are close to the respective origins. Banding in the magnetic moments has occurred, resulting in an anisotropic near-demagnetized state, with uniform bands. For the oscillating demagnetizing field with amplitude decreased every $1/2$ cycle, demagnetization was not achieved by this method. In addition, for this type of demagnetizing cycle and for this type of array, no magnetic hysteresis occurs perpendicular to the applied field, as the array is magnetically symmetric in that direction.

A closely related case is demagnetization with magnitude decreased every $3/2$ cycles, for a 99-particle, isotropic, uniform array, with no pinning or interactions, shown in figure 5.62. For the rotating field, demagnetization was not achieved by this method. We may conclude that rotating demagnetization cycles with continuously decreasing magnitude are effective, while rotating demagnetization cycles with magnitude decreased every $3/2$ cycles are not, perhaps because the latter demagnetization cycle lacks the rotational symmetry of the former. For the oscillating field, demagnetization was nearly achieved. We may conclude that oscillating demagnetization cycles with magnitude decreased every $3/2$ cycles are effective, while those with magnitude decreased every $1/2$ cycle are not. Fine banding has occurred in the particle magnetic moments, resulting in a near-uniform and near-isotropic near-demagnetized state. In comparing this near-isotropic near-demagnetized state, figure 5.62(c), with the clearly anisotropic near-demagnetized state of figure 5.61(b), we may tentatively conclude that oscillating demagnetization cycles with magnitude decreased every $3/2$ cycles are superior to rotating demagnetization cycles with continuously decreasing magnitude, for this class of uniform isotropic magnetic materials.

The next noninteracting case we tried was demagnetization with continuously de-

creasing magnitude for the rotating case (and with magnitude decreased every $3/2$ cycles for the oscillating case), for a 400-particle, isotropic, random array, with no pinning or interactions, shown in figure 5.63. For the rotating field, demagnetization was nearly achieved with this method, as the final points in the hysteresis tensor elements are close to the respective origins. The particle magnetic moments lie in a near-isotropic, random distribution, resulting in a near-demagnetized state. It is not fully demagnetized, as the distribution appears slightly more dense in the top-left quadrant. For the oscillating field, complete demagnetization was achieved with this method, as the final points in the hysteresis tensor elements are at the respective origins. The particle magnetic moments lie in a near-isotropic, random distribution. As for the uniform isotropic materials, we may tentatively conclude that oscillating demagnetization cycles with magnitude decreased every $3/2$ cycles are superior to rotating demagnetization cycles with continuously decreasing magnitude, for this class of random isotropic materials.

For our final noninteracting case, we departed from the isotropic materials, and performed a demagnetization with continuously decreasing magnitude for the rotating case (and with magnitude decreased every $3/2$ cycles for the oscillating case), for a 99-particle, anisotropic array with uniform bands, with bulk easy axis at 0° , 45° , or 90° to the x -axis, with no pinning or interactions, shown in figure 5.64. For the rotating field, demagnetization along the bulk easy axis direction was unsuccessful. For the oscillating field, three different relative bulk easy axis orientations were tried. In the first case, the applied field axis was parallel to the bulk easy axis direction. Demagnetization was unsuccessful. Note also that no magnetic hysteresis occurs perpendicular to the applied field, as the array is magnetically symmetric in that direction. In the second case, the applied field axis was at 45° to the bulk easy axis. Demagnetization was successful. In the third case, the applied field axis was at a right angle to the bulk easy axis. Demagnetization along the array x -axis, was unsuccessful. From the rotating field case and the three oscillating field cases, it appears that the best method for demagnetizing a magnetic material with a single bulk easy axis is to apply an oscillating demagnetizing field at some angle to the bulk easy axis, between 0° and 90° , as shown in the 45° case, in (d).

We investigated the effects of mean field and pinning interactions on demagnetization.

The effects of a mean field interaction alone were investigated with a demagnetization with continuously decreasing magnitude for the rotating case (and with magnitude decreased every $3/2$ cycles for the oscillating case), for a 99-particle, isotropic, uniform array, with a mean field parameter of $\alpha = 0.01$, and no pinning, shown in figure 5.65. For both the rotating and oscillating fields, demagnetization was unsuccessful, due to the magnetizing effect of the mean field interaction.

The effects of a pinning interaction alone were investigated for three different pinning strengths. The low pinning case involved demagnetization with continuously decreasing magnitude for the rotating case (and with magnitude decreased every $3/2$ cycles for the oscillating case), for a 99-particle, isotropic, uniform array, with no interactions, and with a pinning parameter of $k = 0.01$, shown in figure 5.66. For the rotating field, demagnetization was successful, and was aided by the pinning. Comparison of this figure with that for the non-interacting case, figure 5.61(b), makes this evident. The 5-fold banding structure in the magnetic moments that was seen there has been preserved, though the degree of uniformity in the bands has been improved, resulting in a better demagnetized final state. For the oscillating field, demagnetization was also successful, and was aided by the pinning. Comparison of this figure with that for the non-interacting case, figure 5.62(c), makes this evident. The degree of isotropy and uniformity in the magnetic moment distribution has been improved, resulting in a better demagnetized final state.

The medium pinning case involved demagnetization with continuously decreasing magnitude for the rotating case (and with magnitude decreased every $3/2$ cycles for the oscillating case), for a 99-particle, isotropic, uniform array, with no interactions, and with a pinning parameter of $k = 0.05$, shown in figure 5.67. For the rotating field, demagnetization was aided by the pinning, though it was less successful than in the $k = 0.01$ case shown in figure 5.66(b). It appears that the banding structure for $k = 0.05$ is somewhere between the 5-fold structure for $k = 0.01$ in figure 5.66(b), and the 3-fold structure to be seen for $k = 0.5$ in figure 5.68(b). For the oscillating field, demagnetization was also successful, and was aided by the pinning. A highly uniform and isotropic magnetic moment distribution has occurred for this value of pinning. The degree of uniformity and isotropy in the magnetic moment distribution is the best in all our results for oscillatory demagnetization.

The high pinning case involved demagnetization with continuously decreasing magnitude for the rotating case (and with magnitude decreased every $3/2$ cycles for the oscillating case), for a 99-particle, isotropic, uniform array, with no interactions, and with a pinning parameter of $k = 0.5$, shown in figure 5.68. For the rotating field, demagnetization was successful, and was aided by the pinning. A 3-fold banding structure in the magnetic moments has occurred for this value of pinning. The degree of uniformity in the bands is the best in our results for rotational demagnetization. We conclude that the best final demagnetized states, for rotational demagnetization, occur at low and high values of pinning, where 5- or 3-fold magnetic moment banding structures occur. For the oscillating field, demagnetization was not quite achieved for this value of pinning. We conclude that the best final demagnetized states, for oscillatory demagnetization, occur at mid-level values of pinning, where highly uniform and isotropic magnetic moment distributions occur.

Finally, the effects of a both a mean field and pinning interaction were investigated with a demagnetization with continuously decreasing magnitude for the rotating case (and with magnitude decreased every $3/2$ cycles for the oscillating case), for a 99-particle, isotropic, uniform array, with a mean field parameter of $\alpha = 0.01$, and a pinning parameter of $k = 0.05$, shown in figure 5.69. For both the rotating and oscillating fields, demagnetization has occurred, with the final magnetic moment distribution being quite similar to that of the $\alpha = 0.0$, $k = 0.05$ case in figure 5.67, indicating that the pinning has overpowered the magnetizing effect of the mean field interaction. The magnitude of the pinning parameter must be several times the magnitude of the mean field parameter in order for this to occur.

Figure 5.61:

(a)

99-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Rotating applied field with continuously decreasing magnitude

(b)

99-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Rotating applied field with continuously decreasing magnitude

(c)

99-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Oscillating applied field with magnitude decreased every 1/2 cycle

Figure 5.61, Continued.

Demagnetization with continuously decreasing magnitude for the rotating case (and with magnitude decreased every 1/2 cycle for the oscillating case), for a 99-particle, isotropic, uniform array, with no pinning or interactions.

(a) Rotating demagnetization. The applied field rotation was performed in the xy -plane of the array, about the origin, in the counter-clockwise direction. The applied field magnitude was reduced in a set of 600 cycles from 1.0 to 0.4 (MA/m), then reduced in very fine steps in a set of 1000 cycles from 0.4 to 0.3 (MA/m), and finally in a set of 300 cycles from 0.3 to 0.0 (MA/m). Not every cycle is shown. In the middle set of 1000 cycles, only every 100th cycle is shown. In the first and last sets, only the first and last cycles are shown. The tip of the reduced array magnetization vector, $\vec{m}$, is plotted in polar form as the Y -component vs. the X -component, where $\vec{M} = m_s \vec{m}$. The fan diagram for the final point is shown in (b).

(b) Fan diagram [Chikazumi-1964] for the final point in the demagnetization shown in (a). The particle fan is shown on the right. The short lines radiating from the edge of the unit circle are the particle easy axes. The long lines radiating from the center of the circle are the particle magnetic moments. The iJ -components of the (2×2) hysteresis tensor are shown on the left. The x -axes are the i -components of the applied field, and the y -axes are the J -components of the magnetization. The open dot on each curve indicates the current state. Demagnetization was nearly achieved with this method, as the final points in the hysteresis tensor elements are close to the respective origins. Banding in the magnetic moments has occurred, resulting in a near-uniform but anisotropic near-demagnetized state.

(c) Oscillating demagnetization with magnitude decreased every 1/2 cycle. The applied field oscillation was performed along the x -axis of the array, and is essentially the x -component of the rotating applied field used in (a), (b). The demagnetization history is shown in the xX -element of the magnetization tensor. The xY -element is a flat line. For this array, no magnetic hysteresis occurs perpendicular to the applied field, as the array is magnetically symmetric in that direction. The fan diagram for the final point is also shown. Demagnetization was not achieved by this method.

Figure 5.62:

(a)

99-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Rotating applied field with magnitude decreased every 3/2 cycles

(b)

99-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Rotating applied field with magnitude decreased every 3/2 cycles

(c)

99-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.0$$

Oscillating applied field with magnitude decreased every 3/2 cycles

Figure 5.62, Continued.

Demagnetization with magnitude decreased every $3/2$ cycles, for a 99-particle, isotropic, uniform array, with no pinning or interactions.

(a) Rotating demagnetization. The fan diagram for the final point is shown in (b).

(b) Fan diagram for the final point in the demagnetization shown in (a). Demagnetization was not achieved by this method. Rotating demagnetization cycles with continuously decreasing magnitude are superior to rotating demagnetization cycles with magnitude decreased every $3/2$ cycles.

(c) Oscillating demagnetization. Demagnetization was nearly achieved with this method, as the final points in the hysteresis tensor elements are close to the respective origins. Oscillating demagnetization cycles with magnitude decreased every $3/2$ cycles are superior to those with magnitude decreased every $1/2$ cycle. Fine banding has occurred in the particle magnetic moments, resulting in a near-uniform and near-isotropic near-demagnetized state.

Figure 5.63:

(a)

400-particle, isotropic, random array

$$\alpha = 0.0$$

$$k = 0.0$$

Rotating applied field with continuously decreasing magnitude

(b)

400-particle, isotropic, random array

$$\alpha = 0.0$$

$$k = 0.0$$

Rotating applied field with continuously decreasing magnitude

(c)

400-particle, isotropic, random array

$$\alpha = 0.0$$

$$k = 0.0$$

Oscillating applied field with magnitude decreased every 3/2 cycles

Figure 5.63, Continued.

Demagnetization with continuously decreasing magnitude for the rotating case (and with magnitude decreased every $3/2$ cycles for the oscillating case), for a 400-particle, isotropic, random array, with no pinning or interactions.

(a) Rotating demagnetization. The fan diagram for the final point is shown in (b).

(b) Fan diagram for the final point in the demagnetization shown in (a). Demagnetization was nearly achieved with this method, as the final points in the hysteresis tensor elements are close to the respective origins. The particle magnetic moments lie in a near-isotropic, random distribution, resulting in a near-demagnetized state. The distribution appears slightly more dense in the top-left quadrant.

(c) Oscillating demagnetization. Complete demagnetization was achieved with this method, as the final points in the hysteresis tensor elements are at the respective origins. The particle magnetic moments lie in a near-isotropic, random distribution.

Figure 5.64:

(a) 99-particle, anisotropic, uniform array, with 0° bulk easy axis

$$\alpha = 0.0$$

$$k = 0.0$$

Rotating applied field with continuously decreasing magnitude

- (b) 99-particle, anisotropic, uniform array, with 0° bulk easy axis

$$\alpha = 0.0$$

$$k = 0.0$$

Rotating applied field with continuously decreasing magnitude

- (c) 99-particle, anisotropic, uniform array, with 0° bulk easy axis

$$\alpha = 0.0$$

$$k = 0.0$$

Oscillating applied field with magnitude decreased every 3/2 cycles

- (d) 99-particle, anisotropic, uniform array, with 45° bulk easy axis

$$\alpha = 0.0$$

$$k = 0.0$$

Rotating applied field with continuously decreasing magnitude

- (e) 99-particle, anisotropic, uniform array, with 90° bulk easy axis

$$\alpha = 0.0$$

$$k = 0.0$$

Oscillating applied field with magnitude decreased every 3/2 cycles

Figure 5.64, Continued.

Demagnetization with continuously decreasing magnitude for the rotating case (and with magnitude decreased every $3/2$ cycles for the oscillating case), for a 99-particle, anisotropic array with uniform bands, with bulk easy axis at 0° , 45° , or 90° to the x -axis, with no pinning or interactions.

(a) Rotating demagnetization. The bulk easy axis is at 0° to the x -axis. The fan diagram for the final point is shown in (b).

(b) Fan diagram for the final point in the demagnetization shown in (a). The bulk easy axis is at 0° to the x -axis. Demagnetization along the bulk easy axis direction was unsuccessful.

(c) Oscillating demagnetization. The bulk easy axis is at 0° to the x -axis. Hence the applied field oscillation was performed along the bulk easy axis direction. No magnetic hysteresis occurs perpendicular to the applied field, as the array is magnetically symmetric in that direction. Demagnetization was unsuccessful.

(d) Oscillating demagnetization. The bulk easy axis is at 45° to the x -axis. Hence the applied field oscillation was performed at 45° to the bulk easy axis direction. Demagnetization was successful.

(e) Oscillating demagnetization. The bulk easy axis is at 90° to the x -axis. Hence the applied field oscillation was performed at 90° to the bulk easy axis direction. Demagnetization along the x -axis, perpendicular to the bulk easy axis direction, was unsuccessful.

Figure 5.65:

(a)

99-particle, isotropic, uniform array

$$\alpha = 0.01$$

$$k = 0.0$$

Rotating applied field with continuously decreasing magnitude

(b)

99-particle, isotropic, uniform array

$$\alpha = 0.01$$

$$k = 0.0$$

Rotating applied field with continuously decreasing magnitude

(c)

99-particle, isotropic, uniform array

$$\alpha = 0.01$$

$$k = 0.0$$

Oscillating applied field with magnitude decreased every 3/2 cycles

Figure 5.65, Continued.

Demagnetization with continuously decreasing magnitude for the rotating case (and with magnitude decreased every $3/2$ cycles for the oscillating case), for a 99-particle, isotropic, uniform array, with a mean field parameter of $\alpha = 0.01$, and no pinning.

(a) Rotating demagnetization. The fan diagram for the final point is shown in (b).

(b) Fan diagram for the final point in the demagnetization shown in (a). Demagnetization was unsuccessful, due to the magnetizing effect of the mean field interaction. Comparison of this figure with that for the non-interacting case, figure 5.61(b), makes this evident.

(c) Oscillating demagnetization. Demagnetization was unsuccessful, due to the magnetizing effect of the mean field interaction. Comparison of this figure with that for the non-interacting case, figure 5.62(c), makes this evident.

Figure 5.66:

- (a) 99-particle, isotropic, uniform array
 $\alpha = 0.0$
 $k = 0.01$
Rotating applied field with continuously decreasing magnitude

(b)

99-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.01$$

Rotating applied field with continuously decreasing magnitude

(c)

99-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.01$$

Oscillating applied field with magnitude decreased every $3/2$ cycles

Figure 5.66, Continued.

Demagnetization with continuously decreasing magnitude for the rotating case (and with magnitude decreased every $3/2$ cycles for the oscillating case), for a 99-particle, isotropic, uniform array, with no interactions, and with a pinning parameter of $k = 0.01$.

(a) Rotating demagnetization. The fan diagram for the final point is shown in (b).

(b) Fan diagram for the final point in the demagnetization shown in (a). Demagnetization was successful, and was aided by the pinning. Comparison of this figure with that for the non-interacting case, figure 5.61(b), makes this evident. The 5-fold banding in the magnetic moments that was seen there has been preserved, though the degree of uniformity has been improved, resulting in a better demagnetized final state.

(c) Oscillating demagnetization. Demagnetization was successful, and was aided by the pinning. Comparison of this figure with that for the non-interacting case, figure 5.62(c), makes this evident. The degree of isotropy and uniformity has been improved, resulting in a better demagnetized final state.

Figure 5.67:

(a)

99-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.05$$

Rotating applied field with continuously decreasing magnitude

(b)

99-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.05$$

Rotating applied field with continuously decreasing magnitude

(c)

99-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.05$$

Oscillating applied field with magnitude decreased every $3/2$ cycles

Figure 5.67, Continued.

Demagnetization with continuously decreasing magnitude for the rotating case (and with magnitude decreased every $3/2$ cycles for the oscillating case), for a 99-particle, isotropic, uniform array, with no interactions, and with a pinning parameter of $k = 0.05$.

(a) Rotating demagnetization. The fan diagram for the final point is shown in (b).

(b) Fan diagram for the final point in the demagnetization shown in (a). Demagnetization was aided by pinning, though it was less successful than in figure 5.66(b). It appears that the banding structure for this value of pinning is somewhere between the 5-fold structure of figure 5.66(b), and the 3-fold structure to be seen in figure 5.68(b).

(c) Oscillating demagnetization. Demagnetization was successful, and was aided by the pinning. A highly uniform and isotropic magnetic moment distribution has occurred for this value of pinning. The degree of uniformity and isotropy is the best in our results for oscillatory demagnetization.

Figure 5.68:

(a)

99-particle, isotropic, uniform array

$$\alpha = 0.0$$

$$k = 0.5$$

Rotating applied field with continuously decreasing magnitude

- (b) 99-particle, isotropic, uniform array
 $\alpha = 0.0$
 $k = 0.5$
 Rotating applied field with continuously decreasing magnitude

- (c) 99-particle, isotropic, uniform array
 $\alpha = 0.0$
 $k = 0.5$
 Oscillating applied field with magnitude decreased every $3/2$ cycles

Figure 5.68, Continued.

Demagnetization with continuously decreasing magnitude for the rotating case (and with magnitude decreased every $3/2$ cycles for the oscillating case), for a 99-particle, isotropic, uniform array, with no interactions, and with a pinning parameter of $k = 0.5$.

(a) Rotating demagnetization. The fan diagram for the final point is shown in (b).

(b) Fan diagram for the final point in the demagnetization shown in (a). Demagnetization was successful, and was aided by pinning. A 3-fold banding structure in the magnetic moments has occurred for this value of pinning. The degree of uniformity is the best in our results for rotational demagnetization.

(c) Oscillating demagnetization. Demagnetization was not quite achieved for this value of pinning.

Figure 5.69:

(a)

99-particle, isotropic, uniform array

$$\alpha = 0.01$$

$$k = 0.05$$

Rotating applied field with continuously decreasing magnitude

(b)

99-particle, isotropic, uniform array

$$\alpha = 0.01$$

$$k = 0.05$$

Rotating applied field with continuously decreasing magnitude

(c)

99-particle, isotropic, uniform array

$$\alpha = 0.01$$

$$k = 0.05$$

Oscillating applied field with magnitude decreased every 3/2 cycles

Figure 5.69, Continued.

Demagnetization with continuously decreasing magnitude for the rotating case (and with magnitude decreased every $3/2$ cycles for the oscillating case), for a 99-particle, isotropic, uniform array, with a mean field parameter of $\alpha = 0.01$, and a pinning parameter of $k = 0.05$.

(a) Rotating demagnetization. The fan diagram for the final point is shown in (b).

(b) Fan diagram for the final point in the demagnetization shown in (a). Demagnetization has occurred, and in fact, the magnetic moment distribution is quite similar to that of figure 5.67(b), indicating that the pinning has overpowered the magnetizing effect of the mean field interaction.

(c) Oscillating demagnetization. Demagnetization has occurred, and in fact, the magnetic moment distribution is quite similar to that of figure 5.67(c), indicating that the pinning has overpowered the magnetizing effect of the mean field interaction.

Chapter 6

Conclusions

We have presented a new macroscopic model of ferromagnetic hysteresis, based on the Stoner-Wohlfarth approach, but also including the mean field or dipole-dipole interaction, and pinning. The mean field interaction is included using an approach based on the previous Atherton-Beattie extension to the S-W model. The pinning effect is included using a new formulation based on the presence of domain rotational friction, in contrast to domain wall pinning in the Jiles-Atherton model. The new model describes both reversible and irreversible processes, and hysteresis caused by combinations of interaction, anisotropy, and pinning. We have outlined the computational approaches to both two and three dimensional calculations, and have given examples for the two dimensional geometry.

In models of magnetism based primarily on the classic S-W model, such as ours, both reversible and irreversible rotational motion of the magnetic moments can occur. Reversible motion is characterized by small rotations of many or all of the magnetic moments in the fan distribution. Irreversible motion is characterized by the sudden jumping of a magnetic moment to a new angular position which is far from the original one. A given change in applied field may result in a band of magnetic moments in the original fan making the irreversible jump, resulting in an occupied band at the new location, and leaving behind an unoccupied space at the old location. This banding effect is not necessarily related to any interactions between magnetic moments.

The inclusion of the mean field interaction results in a major hysteresis loop with

square sides as the irreversible jump becomes more abrupt. The presence of the mean field causes the array to jump as a whole, and the banding structure that occurs for the partial irreversible motion in the non-interacting case does not occur. The remainder of the approach to saturation is reversible. Increasing the mean field parameter α increases the strength of the mean field, with the effect that the remanence and the squareness of the loop increase as the abruptness of the irreversible jump increases.

The dipole-dipole interaction alone is sufficient to cause hysteresis on the microscopic scale, with no anisotropy energy terms present. Periodic boundary conditions result in square hysteresis loops for an array of interacting magnetic dipoles. Array patterns that quite distinctly demonstrate minimization of the array energy may be achieved with a microscopic model.

The inclusion of the dipole-dipole interaction on the macroscopic scale, with anisotropy energy terms present, results in butterfly shaped major loops with regions of large steps on the approach to saturation, between the coercive field point and the maximum field point. The dipole-dipole field results in a stepped approach to overall magnetic order in this region of the loop. The difference between no interaction and a dipole-dipole interaction with a maximum nearest neighbour interaction radius of 1 is dramatic. Increasing the interaction radius results in some changes in the shape of the loop. In 2D, the total dipole field from the many source dipoles in the array at distance r from a given dipole, falls off as r^{-2} . Therefore small interaction radius values may be used. In 3D, it would fall off as r^{-1} , and much larger interaction radius values would have to be used.

The dipole-dipole interaction must be amplified by a multiplication factor in order for it to have a noticeable effect in a macroscopic model. Without the amplification factor, the dipole-dipole interaction is insignificant. Inclusion of the dipole-dipole interaction as an extension to the S-W model is done on a phenomenological basis, rather than the more mathematically precise basis of a microscopic model. In 3D, the multiplication factor would probably not be necessary.

The inclusion of pinning in the model results in several noticeable effects, including stepping in the curve on the approach to saturation, flattening in the curve on the return from saturation, and a decrease in the magnetization amplitude of the curve

for a given maximum field point. The stepping effect may be physically interpreted as a form of Magnetic Barkhausen Noise which occurs here due to pinning of domain rotation, as well as due to the irreversible hysteretic jumps that already occur in the classic S-W model, as opposed to the usual pinning of domain wall motion.

Small minor loops in this model are characterized by small rotational motion only, between the minor loop endpoints. Without pinning, the small rotational motion is entirely reversible, and the minor loops remain closed. With the inclusion of pinning, even the small rotational motion becomes irreversible, and the minor loops tend to open. Large minor loops may contain irreversible jumps between the minor loop endpoints, and hence may be open. The actual shape of the minor loop depends on the proximity of the loop to saturation and/or the magnetization coordinate of the minor loop in the hysteresis diagram, and on the magnitude of the pinning constant. Of primary note are the ways in which this model contrasts with the Preisach model. In this model, small minor loops result from small changes in many or all of the magnetic moments, whereas in the Preisach model they result from the switching of a small number of domains. This model demonstrates the property of incongruence in minor loops, while in the Preisach model, congruence holds, as minor loops with common field turning points result from switching of the same set of domains, irrespective of the magnetization coordinate in the m, h plane.

In this model minor loops do exhibit eventual closure, though it is sometimes necessary, in regions where irreversible hysteretic jumps have previously occurred, to cycle the minor loop several times.

We have used our model of ferromagnetism to model the magnetic tensor response of hard magnetic materials in demagnetization simulations. The two general types of demagnetization cycles used were the rotating and the oscillating cycles, with amplitude diminishing from near saturation to zero. Two different variations of each were tested. In the rotating case, the magnitude was either decreased continuously, or every $3/2$ cycles; in the oscillating case, either every $1/2$ cycle or $3/2$ cycles.

In the rotating demagnetizations, it was seen that demagnetizations with continuously decreasing magnitude are superior to demagnetizations with magnitude decreased every $3/2$ cycles. In the oscillating demagnetizations, it was seen that demagnetizations

with magnitude decreased every $3/2$ cycles are superior to those with magnitude decreased every $1/2$ cycle. Overall, it was seen that the method of oscillating demagnetization is superior to the method of rotating demagnetization, for hard magnetic materials. For isotropic uniform arrays, the rotating demagnetizations resulted in final near or fully demagnetized states, depending of the interaction parameters, but with anisotropic banding patterns in the magnetic moment distribution. For the same arrays, the oscillating demagnetizations, however, resulted in final near or fully demagnetized states, depending on the interaction parameters, with isotropic, uniform distributions of magnetic moments about the unit circle. These final states more closely resembled the initial unmagnetized state of the array.

For oscillating demagnetizations, and for arrays that are symmetric about the axis of the applied field, no magnetic hysteresis occurs perpendicular to the applied field, as the array is magnetically symmetric in that direction.

For anisotropic arrays with a single bulk easy axis, it appears that the best method for demagnetization is an oscillating demagnetizing field at some angle to the bulk easy axis, between 0° and 90° .

For cases with a mean field interaction but no pinning interaction, demagnetization by any of the methods tried was unsuccessful, due to the magnetizing effect of the mean field. Conversely, for cases with a pinning interaction but no mean field interaction, demagnetization by any of the methods tried was aided by the pinning.

More specifically, for the cases with the pinning interaction, for rotational demagnetization, the best final demagnetized states occurred at low and high values of pinning, where 5- or 3-fold banding patterns in the magnetic moment distributions occurred. For oscillatory demagnetization, the best final demagnetized states occurred at mid-level values of pinning, where isotropic, uniform magnetic moment distributions occurred.

For cases in which both a mean field interaction and a pinning interaction were included, the magnitude of the pinning parameter had to be set to several times the magnitude of the mean field parameter, in order for the pinning to overpower the magnetizing effect of the mean field interaction. In successful demagnetizations, the magnetic moment distribution was quite similar to that for cases with the same value

for the pinning parameter and a zero value for the mean field parameter.

Chapter 7

Proposals for Further Work

7.1 2D Model

7.1.1 Modeling Magnetic Barkhausen Noise

In experimental Magnetic Barkhausen Noise (MBN) studies, a time varying potential, $V_{input}(t)$ is applied to a coil wrapped around a soft magnetic U-core in contact with a magnetic material. Another time varying potential, $V_{output}(t)$ is detected in a pick-up coil also in contact with the magnetic material, and located between the arms of the U-core. Although a smooth signal may be applied as input $V_{input}(t)$, invariably the output signal $V_{output}(t)$ contains noise. This is termed the MBN signal, to which is ascribed an energy termed the MBN energy. It is defined as:

$$MBN_{Energy} = A \sum_{i=1}^N \int V_{output}(t)^2 dt \quad (7.1)$$

where A is an amplification constant, and the summation is over N events, where an event is defined as a voltage peak that exceeds a cutoff level, $V \geq V_{cut-off}$, which effectively filters out background noise.

The output voltage in the pick-up coil is related to the change in flux in the magnetic material by:

$$V_{output}(t) = -cN \frac{d\Phi(t)}{dt} \quad (7.2)$$

where $\Phi(t)$ is the magnetic flux through the magnetic material, N is the number of turns in the pick-up coil, and c is a coupling constant that relates the magnetic flux through the pick-up coil to the magnetic flux that is actually in the sample. $V_{input}(t)$ is typically a low frequency input signal. The resulting output signal $V_{output}(t)$, however, is a high frequency noise signal, and therefore the magnetic flux $\Phi(t)$ only penetrates the sample close to the surface, according to the skin depth equation for a semi-infinite slab, and its magnitude falls off exponentially with increasing depth. The pick-up coil can either be a circulating coil wrapped around the sample, in which case it measures the resulting magnetic flux signal parallel to the applied field in the U-core, or it can be a coil that lies flat on the surface of the sample, in which case it measures the resulting magnetic flux signal perpendicular to the applied field in the U-core. In either case, the detected magnetic flux represents the magnetic response of the material close to the surface of the sample. The magnetic flux, of course, is the sum over area of the magnetic flux density:

$$\Phi = \int \vec{B} \cdot d\vec{A} \quad (7.3)$$

The MBN energy can therefore be written in terms of the magnetic flux Φ through the magnetic material. Substituting the expression for output voltage (7.2) into the definition of MBN energy (7.1), we obtain:

$$MBN_{Energy} = k_1 \sum_{i=1}^N \int \left(\frac{d\Phi(t)}{dt} \right)^2 dt \quad (7.4)$$

where k_1 is some constant. If we consider a very small cross-sectional area of the magnetic material, such that the flux density may be considered constant across it, then we may write:

$$\Phi = BA. \quad (7.5)$$

Equation (7.5) is a good approximation in the case of the pick-up coil lying flat on the surface of the sample, detecting the perpendicular magnetic flux signal, as the flux is expected to be basically constant across the coil area. Equation (7.3) would be necessary in the case of the circulating pick-up coil, detecting the parallel magnetic flux signal, as the flux magnitude is expected to fall off exponentially with depth into the sample. In the latter case, the MBN energy may be written in terms of the

magnetic flux density:

$$MBN_{Energy} = k_2 \sum_{i=1}^N \int \left(\frac{dB(t)}{dt} \right)^2 dt \quad (7.6)$$

where k_2 is some constant. Finally, if we take the external applied field H as a function of time:

$$H = H(t) \quad (7.7)$$

so that:

$$dH = \left(\frac{dH}{dt} \right) dt \quad (7.8)$$

then we may write the MBN energy in purely magnetic terms:

$$MBN_{Energy} = k \sum_{i=1}^N \int \left(\frac{dB(H)}{dH} \right)^2 \left(\frac{dH}{dt} \right) dH \quad (7.9)$$

where k is some constant, where the summation is over N events, and where an event is defined as a differential permeability that exceeds a certain cut-off permeability, and filters out background noise:

$$\frac{dB(H)}{dH} = \mu' \geq \mu_{min} \quad (7.10)$$

It is postulated that one possible source of Magnetic Barkhausen Noise may be discontinuous domain rotations, in addition to the usual source of discontinuous domain wall displacements. This was first suggested by Barkhausen [Barkhausen-1919], and later by Lomaev [Lomaev-1984]. MBN may then be modelled by our rotational 2D model of ferromagnetism, in which the pinning extension provides a mechanism for discontinuous domain rotations. In following the 2D formalism, we may calculate a 2×2 MBN energy tensor corresponding to the 2×2 magnetization response tensor:

$$(MBN_{Energy})_{iJ} = \begin{pmatrix} (MBN_{Energy})_{xX} & (MBN_{Energy})_{xY} \\ (MBN_{Energy})_{yX} & (MBN_{Energy})_{yY} \end{pmatrix} \quad (7.11)$$

where :

$$(MBN_{Energy})_{iJ} = k \sum_{i=1}^N \int \left(\frac{dB_{iJ}(H_i)}{dH_i} \right)^2 \left(\frac{dH_i}{dt} \right) dH_i \quad (7.12)$$

where:

$$\frac{dB_{iJ}(H_i)}{dH_i} = \mu'_{iJ} \geq \mu_{min} \quad (7.13)$$

A study such as this should prove to be highly interesting. The various methods we have developed, such as descriptions of isotropic and nonisotropic magnetic crystalline arrays, descriptions of various applied field cycles such as major loops, minor loops, and rotating fields, and descriptions of magnetic particle interactions and pinning, could all be studied using the MBN analysis techniques developed above. If a stress anisotropy energy term was added to the energy equation, such as in the Callegaro-Puppini model [Callegaro-1997], then the effect of stress on the MBN signal could be modeled. Some work has been done in this area by Mitra and Jiles [Mitra-1997] from the point of view of the Jiles-Atherton model.

Presently, these studies are left as proposals for future research.

7.2 3D Model

7.2.1 The Formulation in 3D Geometry

The vector form of the energy equation, in reduced variables, was:

$$\underline{e}^i = |\hat{e}^i \times \hat{m}^i|^2 - \hat{m}^i \cdot \vec{h}_{applied} - \underline{\alpha} \hat{m}^i \cdot \vec{m} - \alpha_{nn} \hat{m}^i \cdot \vec{h}^{nn} . \quad (7.14)$$

We are free to define a geometry similar to the 2D geometry shown in figure 3.1, but in 3D, with vector direction angles of $\{(\theta^i, \phi^i)\}$ for the magnetic moments, $\{(\theta_{ea}^i, \phi_{ea}^i)\}$ for the easy axes, and (θ_h, ϕ_h) for the applied field.

The various vector products in the vector energy equation may be written in rectangular form:

$$|\hat{e}^i \times \hat{m}^i|^2 = [e_y^i m_z^i - e_z^i m_y^i]^2 + [e_z^i m_x^i - e_x^i m_z^i]^2 + [e_x^i m_y^i - e_y^i m_x^i]^2 \quad (7.15)$$

$$\hat{m}^i \cdot \vec{h}_{applied} = m_x^i h_x + m_y^i h_y + m_z^i h_z \quad (7.16)$$

$$\hat{m}^i \cdot \vec{m} = m_x^i m_x + m_y^i m_y + m_z^i m_z \quad (7.17)$$

$$\hat{m}^i \cdot \vec{h}^{nn} = m_x^i h_x^{nn} + m_y^i h_y^{nn} + m_z^i h_z^{nn} \quad (7.18)$$

where the various rectangular components, in terms of the above vector direction angles, are written:

$$m_x^i = \sin \theta^i \cos \phi^i \quad (7.19)$$

$$m_y^i = \sin \theta^i \sin \phi^i \quad (7.20)$$

$$m_z^i = \cos \theta^i \quad (7.21)$$

$$m_x = \sum_i m_x^i \quad (7.22)$$

$$m_y = \sum_i m_y^i \quad (7.23)$$

$$m_z = \sum_i m_z^i \quad (7.24)$$

$$e_x^i = \sin \theta_{ea}^i \cos \phi_{ea}^i \quad (7.25)$$

$$e_y^i = \sin \theta_{ea}^i \sin \phi_{ea}^i \quad (7.26)$$

$$e_z^i = \cos \theta_{ea}^i \quad (7.27)$$

$$\underline{h}_x = \underline{h}_{applied} \sin \theta_h \cos \phi_h \quad (7.28)$$

$$\underline{h}_y = \underline{h}_{applied} \sin \theta_h \sin \phi_h \quad (7.29)$$

$$\underline{h}_z = \underline{h}_{applied} \cos \theta_h \quad (7.30)$$

$$\underline{h}_x^{nn} = \underline{h}^{nn}(\text{dipole})_x \quad (7.31)$$

$$\underline{h}_y^{nn} = \underline{h}^{nn}(\text{dipole})_y \quad (7.32)$$

$$\underline{h}_z^{nn} = \underline{h}^{nn}(\text{dipole})_z \quad (7.33)$$

Thus the solution to the 3D model involves finding the closest local minimum of the energy function in 2 variables (θ^i, ϕ^i) , at each step of the applied field in 3 components $(\underline{h}_x, \underline{h}_y, \underline{h}_z)$ (or in spherical coordinates $(\underline{h}_{applied}, \theta_h, \phi_h)$). This may be accomplished with a multidimensional minimization algorithm, such as the conjugate gradient method [Press-1992]. This method requires gradient information of the multidimensional function to be minimized.

The gradient of the energy function is written:

$$\vec{\nabla} \underline{e}^i = \frac{\partial \underline{e}^i}{\partial \theta^i} \hat{\theta}^i + \frac{1}{\sin \theta^i} \frac{\partial \underline{e}^i}{\partial \phi^i} \hat{\phi}^i \quad (7.34)$$

in the $\theta^i - \phi^i$ plane, where the partial derivatives are:

$$\frac{\partial \underline{e}^i}{\partial \theta^i} = \frac{\partial \underline{e}^i}{\partial m_x^i} \frac{\partial m_x^i}{\partial \theta^i} + \frac{\partial \underline{e}^i}{\partial m_y^i} \frac{\partial m_y^i}{\partial \theta^i} + \frac{\partial \underline{e}^i}{\partial m_z^i} \frac{\partial m_z^i}{\partial \theta^i} \quad (7.35)$$

$$\frac{\partial \underline{e}^i}{\partial \phi^i} = \frac{\partial \underline{e}^i}{\partial m_x^i} \frac{\partial m_x^i}{\partial \phi^i} + \frac{\partial \underline{e}^i}{\partial m_y^i} \frac{\partial m_y^i}{\partial \phi^i} + \frac{\partial \underline{e}^i}{\partial m_z^i} \frac{\partial m_z^i}{\partial \phi^i} \quad (7.36)$$

where the nontrivial factors are:

$$\frac{\partial \underline{e}^i}{\partial m_x^i} = -\underline{h}_x - \underline{\alpha} m_x - \alpha_{nn} \underline{h}_x^{nn} + 2[e_z m_x^i - e_x m_z^i] e_z - 2[e_x m_y^i - e_y m_x^i] e_y \quad (7.37)$$

$$\frac{\partial \underline{e}^i}{\partial m_y^i} = -\underline{h}_y - \underline{\alpha} m_y - \alpha_{nn} \underline{h}_y^{nn} + 2[e_x m_y^i - e_y m_x^i] e_x - 2[e_y m_z^i - e_z m_y^i] e_z \quad (7.38)$$

$$\frac{\partial \underline{e}^i}{\partial m_z^i} = -\underline{h}_z - \underline{\alpha} m_z - \alpha_{nn} \underline{h}_z^{nn} + 2[e_y m_z^i - e_z m_y^i] e_y - 2[e_z m_x^i - e_x m_z^i] e_x \quad (7.39)$$

where the above algebra may be carried out simply by using the identity:

$$\frac{\partial}{\partial \hat{m}^i} (\hat{e} \times \hat{m}^i) = 2(\hat{e} \times \hat{m}^i) \times \hat{e} . \quad (7.40)$$

Since the torque acting on the magnetic moment with this energy function is

$$\vec{\tau}_+^i = -\vec{\nabla} \underline{e}^i \quad (7.41)$$

then the magnitude of the torque acting to change the magnetic moment orientation is:

$$|\vec{\tau}_+^i| = [\vec{\nabla} \underline{e}^i \cdot \vec{\nabla} \underline{e}^i]^{\frac{1}{2}} = \left[\left(\frac{\partial \underline{e}^i}{\partial \theta^i} \right)^2 + \frac{1}{\sin^2 \theta^i} \left(\frac{\partial \underline{e}^i}{\partial \phi^i} \right)^2 \right]^{\frac{1}{2}} \quad (7.42)$$

while the magnitude of the torque acting to keep the magnetic moment pinned is, as formulated in the 2D model,

$$|\vec{\tau}_-| = k |\vec{m}| . \quad (7.43)$$

This constitutes the complete 3D formulation of the model that we have presented in detail in the 2D formulation. Both formulations follow directly from the vector form

of the reduced energy equation.

7.2.2 Model Calibration to a Real Sample

The ultimate goal of a 3D model of ferromagnetic hysteresis is to find the optimum values of the model input parameters that result in the best possible fit with experimental data. This may be treated as a multivariable minimization problem as follows.

A set of experimental data for a given sample is a 3×3 magnetization tensor of major loops, m_{ij}^e :

$$\begin{pmatrix} m_{xX}^e & m_{xY}^e & m_{xZ}^e \\ m_{yX}^e & m_{yY}^e & m_{yZ}^e \\ m_{zX}^e & m_{zY}^e & m_{zZ}^e \end{pmatrix}. \quad (7.44)$$

A curve-fitting routine is then performed, resulting in a 3×3 tensor of functions, $f_{ij}^e(h_i)$, that are fit, say, to the upper branch of the major loops, m_{ij}^e :

$$\begin{pmatrix} f_{xX}^e(h_x) & f_{xY}^e(h_x) & f_{xZ}^e(h_x) \\ f_{yX}^e(h_y) & f_{yY}^e(h_y) & f_{yZ}^e(h_y) \\ f_{zX}^e(h_z) & f_{zY}^e(h_z) & f_{zZ}^e(h_z) \end{pmatrix}. \quad (7.45)$$

where $f_{ij}^e(h_i)$ may be a polynomial in h_i . A single polynomial generally supplies a poor fit to the sigmoid pattern of experimental points. Therefore a piecewise fit should be performed, involving a set of polynomials, with a given polynomial fitted only to the closest few points.

The corresponding set of model data, in the form of the 3×3 magnetization tensor of major loops is:

$$\begin{pmatrix} m_{xX} & m_{xY} & m_{xZ} \\ m_{yX} & m_{yY} & m_{yZ} \\ m_{zX} & m_{zY} & m_{zZ} \end{pmatrix} \quad (7.46)$$

A chi-square fit is performed on, say, the major loop upper branch data. For a given

tensor element, we generate:

$$\chi_{iJ}^2 = \sum_k \left[\frac{(m_{iJ})_k - (f_{iJ}(h_i))_k}{\sigma_k} \right]^2 \quad (7.47)$$

where the sum is over all points in the model data for the major loop upper branch, and where we have simply defined all the standard deviations as:

$$\sigma_k \equiv 1 \quad (7.48)$$

All nine χ_{iJ}^2 values are summed to yield the total chi-square value, which expresses goodness of fit:

$$\chi^2 = \sum_{\substack{i = x, y, z \\ J = X, Y, Z}} \chi_{iJ}^2 \quad (7.49)$$

However, the χ^2 value may be considered a function of the model input parameters:

$$\chi^2 = \chi^2(x_1, x_2, \dots, x_n) \quad (7.50)$$

where $x_1, x_2, \dots, x_n$ are the n model input paramters. Therefore, given an initial guess of the model input parameters,

$$\{x_1^0, x_2^0, \dots, x_n^0\}, \quad (7.51)$$

an nD minimization of χ^2 is performed to find the optimal set of model input parameters:

$$\{x_1, x_2, \dots, x_n\}. \quad (7.52)$$

Then we may say that the model has been successfully calibrated to the experimental data.

A possible numerical algorithm for performing the nD minimization is Powell's Method [Press-1992]. This method does not require gradient information of the function being minimized, which is an important feature when the function is very complicated and strictly numerical, as in this case.

Note that the principal difficulty in obtaining the solution is that Powell's Method, like most minimization algorithms, returns only the closest local minimum. We would really like to find the global minimum. Therefore the best approach would be to choose several different initial guess sets. The minimization would then be performed for each set, resulting in the corresponding best fit sets. The set corresponding to the smallest value of χ^2 would then be taken as the solution.

Given experimental data for a 3×3 magnetization tensor for a real material, the 3D version of the inclusive model of hysteresis presented here could be calibrated to the experimental data using the above procedure. The optimum values of the various model input parameters could be computed. Once calibrated to the real sample, the model could be used to predict the magnetization response for very complicated applied field cycles. This would be of important use in 3D finite element modeling of electromagnetic devices.

Appendix A

Brent's Method of Minimization

Brent's Method of minimization is used to find the abscissa of the minimum point of function of one variable. The algorithm requires as input a 'bracketing triplet' of points:

$$\{(a, f(a)), (b, f(b)), (c, f(c))\}$$

where

$$a < b < c$$

and where the function value at the middle point is less than at the two end points:

$$f(b) < f(a) \text{ and } f(b) < f(c) . \tag{A.1}$$

Therefore the (local) minimum is guaranteed to be somewhere in the interval (a, c) . Brent's Method is a quick way of finding it, to within a given tolerance. Brent's Method uses a combination of 'parabolic interpolation' and the 'golden step' methods. The parabolic interpolation method gives superlinear convergence. In cases where this method fails, the algorithm resorts to the golden step method, which gives linear convergence to the final answer.

The next two sections detail the parabolic interpolation and the golden step methods. A third section outlines the problem of finding the bracketing triplet in the first place, from a given starting point on the x -axis. Taken together, all the methods of this chapter may be combined to find the closest local minimum to a given starting point

on the x -axis, of a function in one variable.

Note that finding a global minimum is a more difficult problem. One would need to find all the local minima, possibly by using Brent's Method, and then taking the least of the local minima as the global minimum. This generally requires much more knowledge of the function behaviour than is needed for finding the closest local minimum to a given starting point.

A.1 The Parabolic Interpolation Method

Given the set of three points:

$$\{(a, f(a)), (b, f(b)), (c, f(c))\},$$

the parabola that intersects them all has an equation of the form:

$$y = Ax^2 + Bx + C. \quad (\text{A.2})$$

Substituting each set of points into the above equation yields the system:

$$f(a) = Aa^2 + Ba + C \quad (\text{A.3})$$

$$f(b) = Ab^2 + Bb + C \quad (\text{A.4})$$

$$f(c) = Ac^2 + Bc + C \quad (\text{A.5})$$

or in matrix form:

$$\begin{bmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} f(a) \\ f(b) \\ f(c) \end{bmatrix} \quad (\text{A.6})$$

Solving for (A, B, C) gives us the equation of the intersecting parabola. Taking the first derivative, setting it to zero, and solving for x yields the abscissa of the minimum point:

$$x = b - \frac{1}{2} \frac{(b-a)^2[f(b) - f(c)] - (b-c)^2[f(b) - f(a)]}{(b-a)[f(b) - f(c)] - (b-c)[f(b) - f(a)]} \quad (\text{A.7})$$

The function f is then evaluated at this point. In the case that $b < x$, the new bracketing triplet is then either:

$$\{(b, f(b)), (x, f(x)), (c, f(c))\} \text{ if } f(x) < f(b)$$

or

$$\{(a, f(a)), (b, f(b)), (x, f(x))\} \text{ if } f(b) < f(x)$$

with a similar choice for the other case, $x < b$. This method is repeated until the bracketing interval is of size less than the specified tolerance, at which instance the answer is taken as the middle point. An illustration of this method is shown in figure A.1 [Press-1992].

Note that the method of parabolic interpolation fails if either the three points are collinear, or if they specify a parabola with a maximum instead of a minimum point. In these cases, Brent's Method resorts to the Golden Step Method.

Figure A.1: [Press-1992] Convergence to a minimum by inverse parabolic interpolation. A parabola (dashed line) is drawn through the three original points 1,2,3 on the given function (solid line). The function is evaluated at the parabola's minimum, 4, which replaces point 3. A new parabola (dotted line) is drawn through points 1,4,2. The minimum of this parabola is at 5, which is close to the minimum of the function.

A.2 The Golden Step Method

As before, we start with a 'bracketing triplet' of points:

$$\{(a, f(a)), (b, f(b)), (c, f(c))\}$$

where

$$a < b < c$$

and where the function value at the middle point is less than at the two end points:

$$f(b) < f(a) \text{ and } f(b) < f(c) . \quad (\text{A.8})$$

We choose a new point, x , either in (a, b) or (b, c) . As an example, let us choose x to be in (b, c) . We evaluate $f(x)$. The new bracketing triplet is then either:

$$\{(b, f(b)), (x, f(x)), (c, f(c))\} \text{ if } f(x) < f(b)$$

or

$$\{(a, f(a)), (b, f(b)), (x, f(x))\} \text{ if } f(b) < f(x)$$

This process is repeated until the interval is tolerably small. To explain what we mean by tolerably small, we write the function $f(x)$ near b , by Taylor's theorem:

$$f(x) \approx f(b) + \frac{1}{2}f''(b)(x - b)^2 \quad (\text{A.9})$$

The second term will be negligible compared to the first term when it is a factor ϵ smaller:

$$\frac{\frac{1}{2}f''(b)(x - b)^2}{f(b)} < \epsilon \quad (\text{A.10})$$

or

$$|x - b| < \sqrt{\epsilon}|b| \sqrt{\frac{2|f(b)|}{b^2 f''(b)}} \quad (\text{A.11})$$

Since for most functions the radical factor is ≈ 1 , then this simplifies to approximately:

$$|x - b| < \sqrt{\epsilon}|b| \quad (\text{A.12})$$

Since for double precision computing the lowest value we can have for ϵ is:

$$\epsilon \approx 10^{-15}$$

then:

$$|x - b| \leq 3 \times 10^{-8} |b| \quad (\text{A.13})$$

Therefore the fractional tolerance for the golden step method should be not less than (3×10^{-8}) , relative to the central abscissa.

The actual strategy for finding the new point x is as follows. Suppose that b is a fraction w of the way between a and c :

$$\frac{b - a}{c - a} = w; \quad \frac{c - b}{c - a} = 1 - w \quad (\text{A.14})$$

Suppose also that the next trial point x is an additional fraction z beyond b :

$$\frac{x - b}{c - a} = z \quad (\text{A.15})$$

Then the next bracketing interval will be of length either $w + z$ or $1 - w$. An illustration of the various intervals used here appears in figure A.2.

If we want to minimize the worst possible case, then we set these two intervals to be equal, yielding:

$$z = 1 - 2w \quad (\text{A.16})$$

Note that z is positive only if $w < \frac{1}{2}$. Therefore in our example, x must lie in the larger segment.

However, the value w itself must have come from a previous iteration. Therefore, supposing that x lies between b and c , we have the scale similarity:

$$\frac{|x - b|}{|c - b|} = \frac{|b - a|}{|c - a|} \quad (\text{A.17})$$

yielding:

$$\frac{z}{1 - w} = w \quad (\text{A.18})$$

Solving equations (A.16) and (A.18) for w gives:

$$w^2 - 3w + 1 = 0 \quad (\text{A.19})$$

or

$$w = \frac{3 \pm \sqrt{5}}{2} \quad (\text{A.20})$$

with numerical values:

$$w = 2.618\ 033, 0.381\ 966 \quad (\text{A.21})$$

The first solution makes no physical sense in this problem, and is discarded. This leaves us with the second solution as our answer for w .

Since we assumed that the new point x lay in the larger of the two intervals, each new interval is of fractional size 0.618 034 compared to the previous. This is the ‘golden ratio’, known since ancient times, and is the meaning behind the name of this method. Convergence is linear, meaning that additional significant figures in the minimizing abscissa are yielded linearly with additional iterations of the golden step.

The C version of Brent’s Method in *Numerical Recipes in C: Second Edition* is termed `brent`, and returns the minimizing abscissa, given an appropriate bracketing triplet.

Figure A.2: Convergence to a minimum by the golden search method. We start with a bracketing triplet of points $\{a, b, c\}$ where $a < b < c$ and where the function value at the middle point is less than at the two end points. We choose a new point, x , either in (a, b) or (b, c) . In this illustration, we have chosen x to be in (b, c) . We evaluate $f(x)$. If $f(b) < f(x)$ then the new bracketing triplet is $\{a, b, x\}$ (case (a)). If $f(x) < f(b)$ then the new bracketing triplet is $\{b, x, c\}$ (case (b)). b is a fraction w of the way between a and c . The next trial point x is an additional fraction z beyond b . Then the next bracketing interval is of length either $w + z$ or $1 - w$. Optimal convergence occurs for $w = 0.381\ 966$.

A.3 Bracketing the Minimum

In order to begin Brent's Method, we need the bracketing triplet of points. Usually, though, all we have initially is a single point x_0 , and we desire to find the closest local minimum.

Therefore, we initiate a bracketing algorithm. We start with the two points:

$$(x_0 - \frac{\delta}{2}, x_0 + \frac{\delta}{2})$$

where δ is a very small number, and where we ensure that:

$$f(x_0 - \frac{\delta}{2}) \neq f(x_0 + \frac{\delta}{2})$$

We then derive a third point by taking a golden step in the downhill direction; this time we choose the new point such that the new interval is of size 1.618 034 times the old interval above.

Therefore we have a set of three points. We continue iterating by moving one of the outside points (the one on the downhill side), in the downhill direction, by one of two methods: either by the method of parabolic extrapolation, or by the modified method of the golden step we just described. The method of parabolic extrapolation is performed using the same formula as for the method of parabolic interpolation; the difference is that the new abscissa is outside the original interval specified by the three points used.

At each iteration the bracketing algorithm checks to see if the three points form a suitable bracketing triplet, and iteration is continued until they do. The result is the required bracketing triplet of points:

$$\{(a, f(a)), (b, f(b)), (c, f(c))\}$$

where

$$a < b < c$$

and where the function value at the middle point is less than at the two end points:

$$f(b) < f(a) \text{ and } f(b) < f(c) . \quad (\text{A.22})$$

This forms the required input for Brent's Method.

The C version of this bracketing algorithm in *Numerical Recipes in C: Second Edition* is termed `mnbrak`, and returns a bracketing triplet, given a pair of endpoints for an initial interval.

Appendix B

Monte Carlo Techniques

B.1 Uniform Distributions on (0,1)

Park and Miller set down a pseudo-random number generator which they termed the 'Minimal Standard'. It is a simple multiplicative congruential algorithm:

$$I_{j+1} = aI_j \pmod{m} \quad (\text{B.1})$$

where

$$\begin{aligned} a &= 7^5 &= 16\,807 &\approx 1.7 \times 10^4 \\ m &= 2^{31} - 1 &= 2\,147\,483\,647 &\approx 2.1 \times 10^9 \end{aligned}$$

The algorithm is given a 'seed' integer, I_0 , and returns a new integer, I_{j+1} , with each call. Divided by m , the numbers become deviates with a uniform distribution over the interval $(0, 1)$.

It is not possible to implement the Park and Miller algorithm directly in a high-level language, because the product $a(m-1)$ exceeds the maximum integer value for 32-bit arithmetic.

However, this issue is overcome by Schrage's algorithm, in which an approximate factorization of m is performed:

$$m = aq + r \quad (\text{B.2})$$

where

$$\begin{aligned} q &= \left\lfloor \frac{m}{a} \right\rfloor = 127\,773 \\ \tau &= m(\bmod a) = 2\,836 \end{aligned} \tag{B.3}$$

where the square brackets denote the integer truncation of the enclosed number.

Using this factorization, for $I_j \in (0, m-1)$,

$$I_{j+1} = aI_j(\bmod m) = \begin{cases} a[I_j(\bmod q)] - \tau\left[\frac{I_j}{q}\right] & , \text{if it is } \geq 0 \\ a[I_j(\bmod q)] - \tau\left[\frac{I_j}{q}\right] + m & , \text{otherwise} \end{cases} \tag{B.4}$$

where it may be shown that both $a[I_j(\bmod q)]$ and $\tau\left[\frac{I_j}{q}\right]$ are in the interval $(0, m-1)$.

This random number generator must be seeded with $I_0 \in (0, m-1)$, and has a period of $2^{31} - 2 \approx 2.1 \times 10^9$.

The Park and Miller algorithm plus the Schrage algorithm constitute a portable version of the Minimal Standard which may be coded in any high-level language on any platform. The C version of this algorithm in *Numerical Recipes in C: Second Edition* is termed `ran0`.

However, this algorithm suffers from a form of sequential correlation which subtracts from the degree of randomness. Successive random integers I_j and I_{j+1} differ by only a multiple of 16 807 out of a modulus of 2.1×10^9 . Therefore, once the random numbers have been normalized to the interval $(0, 1)$, a small random number will tend to be followed by another small random number. For example, a number that is less than 10^{-6} will always be followed by a number that is less than 0.0168. This form of sequential correlation makes this algorithm ill suited for many applications.

The sequential correlation may, however, be removed by instituting a shuffling algorithm. Figure B.1 is an illustration of the following explanation. A table of 32 values $\{Iv_1, \dots, Iv_{32}\}$ plus a single value Iy are initially filled with random numbers generated by the Minimal Standard. On a given random number call, Iy is used to select a number Iv_j from the table. Iv_j is returned as the output, and also becomes the next Iy . The place in the table that was occupied by Iv_j is replaced with a new number generated by the Minimal Standard.

The C version of this algorithm in *Numerical Recipes in C: Second Edition* is termed

ran1, and is the fundamental random number generator used to generate random deviates with a uniform distribution on the interval $(0,1)$. This basic distribution may then be operated on to yield more complicated distributions, as is detailed in the next section. This algorithm is suitable for applications that call for a relatively small set of random numbers, that is, a set of less than $m \approx 2.1 \times 10^9$ numbers.

Figure B.1: [Press-1992] Shuffling procedure used in **ran1** to break up sequential correlations in the Minimal Standard generator. Circled numbers indicate the sequence of events: On each call, the random number in **iy** is used to choose a random element in the array **iv**. That element becomes the output random number, and also is the next **iy**. Its spot in **iv** is refilled from the Minimal Standard routine.

B.2 The Transformation Method: Gaussian Distributions

Suppose we have a uniform probability distribution:

$$p(x)dx = \begin{cases} dx & , \quad 0 < x < 1 \\ 0 & , \quad \text{otherwise} \end{cases} \quad (\text{B.5})$$

normalized so that:

$$\int_{-\infty}^{\infty} p(x)dx = 1 \quad (\text{B.6})$$

Taking some prescribed function of the uniform deviates x ,

$$y = y(x) \quad (\text{B.7})$$

then the probability distribution of y , $p(y)dy$, is, according to the fundamental transformation law of probabilities,

$$|p(y)dy| = |p(x)dx| \quad (\text{B.8})$$

or

$$p(y) = p(x) \left| \frac{dx}{dy} \right|. \quad (\text{B.9})$$

So:

$$p(y)dy = p(x) \left| \frac{dx}{dy} \right| dy. \quad (\text{B.10})$$

This generalizes to multi-dimensions. If $x_1, x_2, \dots$ are random deviates with a joint probability distribution $p(x_1, x_2, \dots)$, and if $y_1, y_2, \dots$ are functions of the x 's:

$$p(y_1, y_2, \dots)dy_1dy_2 \cdots = p(x_1, x_2, \dots) \left| \frac{\partial(x_1, x_2, \dots)}{\partial(y_1, y_2, \dots)} \right| dy_1dy_2 \cdots. \quad (\text{B.11})$$

Suppose we choose a transformation $(x_1, x_2) \longrightarrow (y_1, y_2)$, where x_1, x_2 are two uniform deviates on $(0, 1)$.

$$y_1 = \sqrt{-2 \ln x_1} \cos 2\pi x_2 \quad (\text{B.12})$$

$$y_2 = \sqrt{-2 \ln x_1} \sin 2\pi x_2 \quad (\text{B.13})$$

The inverse transformation is then $(y_1, y_2) \longrightarrow (x_1, x_2)$

$$x_1 = \exp\left(-\frac{1}{2}(y_1^2 + y_2^2)\right) \quad (\text{B.14})$$

$$x_2 = \frac{1}{2\pi} \tan^{-1}\left(\frac{y_2}{y_1}\right) \quad (\text{B.15})$$

The Jacobian determinant is then:

$$\frac{\partial(x_1, x_2)}{\partial(y_1, y_2)} = \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} \end{vmatrix} = - \left[\frac{1}{\sqrt{2\pi}} e^{-\frac{y_1^2}{2}} \right] \left[\frac{1}{\sqrt{2\pi}} e^{-\frac{y_2^2}{2}} \right] \quad (\text{B.16})$$

Therefore our distribution function is:

$$p(y_1, y_2) dy_1 dy_2 = - \left[\frac{1}{\sqrt{2\pi}} e^{-\frac{y_1^2}{2}} \right] \left[\frac{1}{\sqrt{2\pi}} e^{-\frac{y_2^2}{2}} \right] dy_1 dy_2 \quad (\text{B.17})$$

In other words, the new distribution function $p(y_1, y_2)$ is a product of two independent normal distributions in y_1 and y_2 , with a mean of 0 and a standard deviation of 1.

Therefore, taking two uniform deviates (x_1, x_2) , and performing the transformation $(x_1, x_2) \longrightarrow (y_1, y_2)$ results in two normal deviates (y_1, y_2) .

This is the transformation method of the normal distribution.

The C version of this algorithm in *Numerical Recipes in C: Second Edition* is termed `gasdev`, and is used to generate random deviates with a gaussian distribution, with a mean of 0.0 and a standard deviation of 1.0. This distribution may then be operated on in a trivial way to yield a gaussian distribution with any mean and any standard deviation.

Bibliography

- [Ampere-1827] Ampere, A.M. *Theorie Mathematique des Phenomenes Electrodynamiques Uniquement Deduite de l'Experience*. 1827. Reprinted by Blanchard, Paris, 1958.
- [Aharoni-1959] A. Aharoni *J. Appl. Phys.* **30** 70S-78S (1959)
“Some recent developments in micromagnetics at the Weizmann institute of science”
- [Arrott-1993] A.S. Arrott, T.L. Templeton, and Y. Yoshida *IEEE Trans. Mag.* **29** 2622-2624 (1993)
“Model for nonuniform magnetization processes in particles with enhanced surface anisotropy”
- [Atherton-1990] David L. Atherton and J.R. Beattie *IEEE Trans. Mag.* **26** 3059-3063 (1990)
“A mean field Stoner-Wohlfarth hysteresis model”
- [Barkhausen-1919] H. Barkhausen *Phys. Z.* **20** 401-403 (1919)
“Zwei mit hilfe der neuen verstärker entdeckte erschweinungen”
- [Beardsley-1986] I.A. Beardsley *IEEE Trans. Mag.* **22** 454-459 (1986)
“Modeling the record process”
- [Becker-1939] Becker, R. and Doring, W. *Ferromagnetismus* Springer, Berlin (1939)
- [Bertotti-1998] Bertotti, Giorgio *Hysteresis in Magnetism*. Academic Press, 1998.

- [Bertram-1999] H. Neal Bertram and Christian Seberino *J. Magn. Magn. Mater.* **193** 388-394 (1999)
 “Numerical simulations of hysteresis in longitudinal magnetic tape (invited)”
- [Bitter-1931] F. Bitter *Phys. Rev.* **38** 1903-1905 (1931)
 “On inhomogeneities in the magnetization of ferromagnetic materials”;
Phys. Rev. **41** 507-515 (1932)
 “Experiments on the nature of ferromagnetism”
- [Bloch-1930] F. Bloch *Z. Physik.* **61** 206-219 (1930)
 “Theory of ferromagnetism”
- [Bloch-1932] F. Bloch *Z. Physik.* **74** 295-335 (1932)
 “Zur theorie des austauschproblems und der remanenzerscheinung der ferromagnetika”
 “Theory of the exchange problem and of residual ferromagnetism”
- [Born-1968] M. Born and K. Huang *Dynamical Theory of Crystal Lattices* Oxford University Press, London, 248-255 (1968)
- [Boyle-1675] Hon. Robert Boyle *Two Tracts on Electricity and Magnetism*. Reprinted from the rare editions of 1675 and 1676. London: Bedford Press, 1898.
- [Bozorth-1951] Bozorth, Richard M. *Ferromagnetism* IEEE Press, Piscataway, NJ (1951)
- [Brillouin-1927] L. Brillouin *J. de Phys. Radium* **8** 74-84 (1927)
 “Moments of rotation and undulatory mechanics”
- [Brown-1959] William Fuller Brown, Jr. *J. Appl. Phys.* **30** 62S-69S (1959)
 “Micromagnetics, domains, and resonance”

- [Callegaro-1997] Luca Callegaro and Ezio Puppini *IEEE Trans. Mag.* **33** 1007-1011 (1997)
"Rotational hysteresis model for stressed ferromagnetic films"
- [Chen-1997] Yingjian Chen, Wei Yang, David N. Lambeth, and Mark H. Kryder *IEEE Trans. Mag.* **33** 2998-3000 (1997)
"Incoherent rotation in barium ferrite thin film recording media"
- [Chikazumi-1964] Chikazumi, S. *Physics of Magnetism*. John Wiley & Sons, Inc., 1964.
- [Cullity-1972] Cullity, B.D. *Introduction to Magnetic Materials*. Addison-Wesley Publishing Co., 1972.
- [Curie-1895] P. Curie *Ann. Chem. Phys.* **5** 289-405 (1895)
"Magnetic properties of bodies at various temperatures"
- [Del Vecchio-1978] R.M. Del Vecchio *Westinghouse Internal Report* (1978)
"Analytical representation of arbitrary paths in magnetic materials"
- [Del Vecchio-1980] R.M. Del Vecchio *IEEE Trans. Mag.* **MAG-16** 809-811 (1980)
"An efficient procedure for modeling complex hysteresis processes in ferromagnetic materials"
- [Della Torre-1998] Della Torre, Edward *Magnetic Hysteresis*. Piscataway, NJ: IEEE Press, 1998.
- [Ewing-1889] J.A. Ewing *Proc. Roy. Soc. (London)* **46** 269-286 (1889)
"On time-lag in the magnetisation of iron"
- [Ewing-1900] Ewing, J.A. *Magnetic Induction in Iron and Other Metals, Third Edition, Revised*. London: "The Electrician" Printing and Publishing Company, Ltd., 1900.
- [Faraday-1839] Michael Faraday *Experimental Researches in Electricity* Dover Publications, Inc., New York (1965)

- [Fredkin-1992] D.R. Fredkin, W. Chen, and T.R. Koehler *IEEE Trans. Mag.* **28** 2380-2381 (1992)
 "Hysteresis in random collections of isotropic interacting particles"
- [Frenkel-1930] J. Frenkel and J. Dorfman *Nature* **126** 274-275 (1930)
 "Spontaneous and induced magnetisation in ferromagnetic bodies"
- [Friedel-1954] J. Friedel *Adv. Phys.* **3** 446-507 (1954)
 "Electronic structure of primary solid solutions in metals"
- [Frohlich-1881] O. Frohlich *Electrotech Z.* **2** 134-141 (1881)
 "Investigations of dynamoelectric machines and electric power transmission and theoretical conclusions therefrom"
- [Furlani-1986] Edward P. Furlani and Alan G. Baker *J. Appl. Phys.* **59** 3815-3819 (1986)
 "Mathematical model for the calculation of magnetization in anisotropic materials"
- [Gilbert-1600] Gilbert, William *De Magnete*. Dover Publications, Inc., 1958.
- [Globus-1962] Anatol Globus *Comptes Rendus Acad. Seances* **255** 1709-1711 (1962)
 "Influence des dimensions des parois sur la permeabilite initiale"
- [Globus-1966] A. Globus and P. Duplex *IEEE Trans. Mag.* **MAG-2** 441-445 (1966)
 "Separation of susceptibility mechanisms for ferrites of low anisotropy"
- [Globus-1969] A. Globus and P. Duplex *Phys. Stat. Sol.* **31** 765-774 (1969)
 "Initial susceptibility of ferrimagnetic materials and topography of domain walls"

- [Globus-1970] A. Globus and P. Duplex *Phys. Stat. Sol. (a)* **3** 53-61 (1970)
 "Size of Bloch wall and parameters of the magnetic susceptibility in ferrimagnetic spinels and garnets"
- [Globus-1971] Anatol Globus, Paul Duplex, and Marcel Guyot *IEEE Trans. Mag.*, Intermag Conference, 617-622 (1971)
 "Determination of initial magnetization curve from crystallites size and effective anisotropy field"
- [Globus-1972] A. Globus and M. Guyot *Phys. Stat. Sol. (b)* **52** 427-431 (1972)
 "Wall displacement and bulging in magnetization mechanisms of the hysteresis loop"
- [Globus-1977] A. Globus *Physica* **86-88B** 943-944 (1977)
 "Universal hysteresis loop for soft ferrimagnetic polycrystals"
- [Gunal-1997] Y. Gunal and P.B. Visscher *IEEE Trans. Mag.* **33** 3070-3072 (1997)
 "Computer simulation of switching in Stoner-Wohlfarth colloids"
- [Guyot-1973] M. Guyot and A. Globus *Phys. Stat. Sol. (b)* **59** 447-454 (1973)
 "Determination of the domain wall energy from hysteresis loops in YIG"
- [Guyot-1977] M. Guyot and A. Globus *J. de. Phys.* **38** C1-157 - C1-162 (1977)
 "Determination of the domain wall energy and the exchange constant from hysteresis in ferromagnetic polycrystals"
- [Hamos-1932] L.V. Hamos and P.A. Thiessen *Z. Physik.* **71** 442-444 (1932)
 "Über die sichtbarmachung von bezirken verschiedenem ferromagnetischen zustandes fester korper"
 "Variation of ferromagnetic quality made visible in solids"
- [Hartman-1978] Keith Hartman, Robert I. Potter, and Irene B. Ortenburger *IEEE Trans. Mag.* **MAG-14** 223-227 (1978)
 "A vector model for magnetic hysteresis based on interacting dipoles"

- [Heisenberg-1928] W. Heisenberg *Z. f. Phys.* **49** 619-636 (1928)
 “Zur theorie des ferromagnetismus”
 “Theory of ferromagnetism”
- [Hodgdon-1988] Marion L. Hodgdon *IEEE Trans. Mag.* **24** 3120-3122 (1988)
 “Mathematical theory and calculations of magnetic hysteresis curves”
- [Honda-1926] K. Honda and S. Kaya *Sci. Rept. Tohoku Imp. Univ.* **15** 721-753 (1926)
 “On the magnetization of single crystals of iron”
- [Hund-1927] F. Hund *Linienpektren und Periodisches System der Elemente* Julius Springer, Berlin (1927)
- [Jackson-1975] Jackson, John David *Classical Electrodynamics, Second Edition.* John Wiley & Sons, Inc., 1975.
- [Jiles-1983] D.C. Jiles and D.L. Atherton *IEEE Trans. Mag.* **MAG-19** 2183-2185 (1983)
 “Ferromagnetic hysteresis”
- [Jiles-1984] D.C. Jiles, D.L. Atherton, H.E. Lassen, D. Noble, J. deVette, and T. Astle *Rev. Sci. Instrum.* **55** 1843-1848 (1984)
 “Microcomputer-based system for control of applied uniaxial stress and magnetic field”
- [Jiles-1986] D.C. Jiles and D.L. Atherton *J. Magn. Magn. Mater.* **61** 48-60 (1986)
 “Theory of ferromagnetic hysteresis”
- [Jiles-1991] Jiles, David *Introduction to Magnetism and Magnetic Materials.* Chapman & Hall, 1991.
- [Jiles-1992] D.C. Jiles, J.B. Thoeke, and M.K. Devine *IEEE Trans. Mag.* **28** 27-35 (1992)

“Numerical determination of hysteresis parameters for the modeling of magnetic properties using the theory of ferromagnetic hysteresis”

- [Kaya-1934] S. Kaya *Z. Physik.* **89** 796-805 (1934)
“Powder figures on a magnetised iron crystal”;
Z. Physik. **90** 551-558 (1934)
“Powder figures on magnetised iron crystals”;
S. Kaya and J. Sekiya *Z. Physik.* **96** 53-61 (1935)
“Mapping of magnetic field of nickel crystals”
- [Kennelly-1891] A.E. Kennelly *Trans. Am. IEE.* **8** 485-517 (1891)
“Magnetic reluctance”
- [Kersten-1943] Kersten, M. *Underlying Theory of Ferromagnetic Hysteresis and Coercivity* Hirzel, Leipzig (1943)
- [Kikuchi-1956] Ryoichi Kikuchi *Journ. Appl. Phys.* **27** 1352-1357 (1956)
“On the minimum of magnetization reversal time”
- [Kittel-1946] C. Kittel *Phys. Rev.* **70** 965-971 (1946)
“Theory of the structure of ferromagnetic domains in films and small particles”
- [Kittel-1986] Kittel, C. *Introduction to Solid State Physics, 6th Edition* Wiley, New York (1986)
- [Koehler-1987] T.R. Koehler *J. Appl. Phys.* **61** 1568-1575 (1987)
“A computationally fast, two-dimensional vector hysteresis model”
- [Krause-1994a] Thomas W. Krause, J.M. Makar, and D.L. Atherton *J. Magn. Magn. Mater.* **137** 25-34 (1994)
“Investigation of the magnetic field and stress dependence of 180° domain wall motion in pipeline steel using magnetic Barkhausen noise”

- [Krause-1994b] Thomas W. Krause, L. Clapham, and David L. Atherton *J. Appl. Phys.* **75** 7983-7988 (1994)
 "Characterization of the magnetic easy axis in pipeline steel using magnetic Barkhausen noise"
- [Krause-1996] T.W. Krause, L. Clapham, Andras Pattantyus, and D.L. Atherton *J. Appl. Phys.* **79** 4242-4252 (1996)
 "Investigation of the stress-dependent magnetic easy axis in steel using magnetic Barkhausen noise"
- [Krause-1997] T.W. Krause, K. Mandal, C. Hauge, P. Weyman, B. Sijgers, and D.L. Atherton *J. Magn. Magn. Mater.* **169** 207-219 (1997)
 "Correlation between magnetic flux leakage and magnetic Barkhausen noise: stress dependence in pipeline steel"
- [Lamont-1867] Lamont, J. *Handbuch des Magnetismus* Voss, Leipzig (1867)
- [Landau-1935] L. Landau and E. Lifshitz *Phys. Zeits. d. Sowjetunion* **8** 153-169 (1935)
 "Theory of the dispersion of magnetic permeability in ferromagnetic bodies"
 E. Lifshitz *J. Phys. USSR* **8** 337 (1944)
- [Langevin-1904] P. Langevin *Comptes Rendus* **139** 1204-1207 (1904)
 "Electronic theory of magnetism"
- [Langevin-1905] P. Langevin *Annal. Chim. Phys.* **5** 70-127 (1905)
 "Electronic theory of magnetism"
- [Lomaev-1984] G.V. Lomaev, V.S. Malyshev, and D.P. Degterev *Sov. J. NDT.* **20** 189 (1984)
- [Lorrain-1988] Paul Lorrain, Dale P. Corson, and Francois Lorrain *Electromagnetic Fields and Waves* W.H. Freeman and Co., New York (1988)

- [Lou-1998] Jie Lou, Lawrence H. Bennett, and Edward Della Torre *IEEE Trans. Mag.* **34** 1279-1281 (1998)
 "Identification of the Preisach parameters of a three quadrant medium"
- [Lu-1994] Jing Ju Lu, Huei Li Huang, and I. Klik *J. Appl. Phys.* **76** 1726-1732 (1994)
 "Field orientations and sweep rate effects on magnetic switching of Stoner-Wohlfarth particles"
- [Maxwell-1892] Maxwell, James Clerk *A Treatise on Electricity and Magnetism*. London: Oxford University Press, 1892.
- [Mayergoyz-1991] Mayergoyz, M.I. *Mathematical Models of Hysteresis*. New York: Springer-Verlag, 1991.
- [McCurrie-1986] R.A. McCurrie and M.W. Viccary *IEEE Trans. Mag.* **MAG-22** 1849-1858 (1986)
 "Rotational Hysteresis in an anisotropic Mn-Al-C permanent magnet"
- [Mitra-1997] A. Mitra and D.C. Jiles *J. Magn. Magn. Mater.* **168** 169-176 (1997)
 "Magnetic Barkhausen emissions in as-quenched Fe-Si-B amorphous alloy"
- [Neel-1944] L. Neel *J. Phys. Radium* **5** 241-276 (1944)
 "Laws of magnetization and subdivision of elementary domains of iron"
- [Neel-1947] L. Neel *Compt. Rend.* **224** 1488-1489 (1947)
 "Properties of a finely-powdered ferromagnetic of cubic structure"
- [Neel-1955] L. Neel *Advances in Phys.* **4** 191-243 (1955)
 "Some theoretical aspects of rock-magnetism"

- [Oersted-1820] John Christian Oersted *Ann. Phil.* **16** (1820)
 “Experiments on the effect of a current of electricity on the magnetic needle”
- [Orozco-1998] E. Bonet Orozco, W. Wernsdorfer, B. Barbara, K. Hasselbach, R. Benoit, and D. Mailly *IEEE Trans. Mag.* **34** 979-981 (1999)
 “Single particle measurement showing agreement with the model of uniform rotation”
- [Parker-1990] Parker, Rollin J. *Advances in Permanent Magnetism*. New York: Wiley, 1990
- [Pauli-1925] W. Pauli *Z. Physik.* **31** 765-783 (1925)
 “Relation between the closing in of electron-groups in the atom and the structure of complexes in the spectrum”
- [Pauling-1938] Linus Pauling *Phys. Rev.* **54** 899-904 (1938)
 “The nature of the interatomic forces in metals”
- [Peregrinus-1269] Brother Arnold (Translator) *The Letter of Petrus Peregrinus, On the Magnet, A.D. 1269*. New York: McGraw Publishing Company, 1904.
- [Phelps-1998a] B.F. Phelps, F. Liorzou, and D.L. Atherton *IEEE Trans. Mag.* Submitted (1998)
 “An inclusive model of ferromagnetic hysteresis”
- [Phelps-1998b] B.F. Phelps and D.L. Atherton *IEEE Trans. Mag.* Submitted (1998)
 “Pinning and minor loops in an inclusive model of ferromagnetic hysteresis”
- [Phelps-1999] B.F. Phelps and D.L. Atherton *IEEE Trans. Mag.* Submitted (1999)
 “Demagnetization in an inclusive model of ferromagnetic hysteresis”

- [Preisach-1935] F. Preisach *Zeits. f. Physik* **94** 277-302 (1935)
 "Über die magnetische nachwirkung"
 "Magnetic after-effect"
- [Press-1992] Press, W.H., Teukolsky, S.A., Vetterling, W.T. and Flannery, B.P. *Numerical Recipes in C: The Art of Scientific Computing, Second Edition*. Cambridge University Press, 1992.
- [Rayleigh-1887] Lord Rayleigh *Phil. Mag.* **23** 225-248 (1887)
 "On the behaviour of iron and steel under the operation of feeble magnetic forces"
- [Reimers-1998] Ann Reimers and Edward Della Torre *IEEE Trans. Mag.* **34** 3857-3866 (1998)
 "Fast Preisach-based magnetization model and fast inverse hysteresis model"
- [Roller-1959] Roller, Duane H.D. *The De Magnete of William Gilbert* Menno Hertzberger, Amsterdam (1959)
- [Russell-1925] H.N. Russell and F.A. Saunders *Astrophys. J.* **61** 38-69 (1925)
 "New regularities in the spectra of the alkaline earths"
- [Sablik-1987] M.J. Sablik, K. Kwun, G.L. Burkhardt, and D.C. Jiles *J. Appl. Phys.* **61** 3799-3801 (1987)
 "Model for the effect of tensile and compressive stress on ferromagnetic hysteresis"
- [Sablik-1988] M.J. Sablik and D.C. Jiles *J. Appl. Phys.* **64** 5402-5404 (1988)
 "A model for hysteresis in magnetostriction"
- [Sablik-1993a] Martin J. Sablik and David C. Jiles *IEEE Tran. Mag.* **29** 2113-2123 (1993)
 "Coupled magnetoelastic theory of magnetic and magnetostrictive hysteresis"
- [Sablik-1993b] M.J. Sablik, S.W. Rubin, L.A. Riley, D.C. Jiles, D.A. Kaminski, and S.B. Biner *J. Appl. Phys.* **74** 480-488 (1993)

- "A model for hysteretic magnetic properties under the application of noncoaxial stress and fields"
- [Sablik-1999] Martin J. Sablik and David C. Jiles *IEEE Trans. Mag.* **35** 498-504 (1999)
 "Modeling the effects of torsional stress on hysteretic magnetization"
- [Sixtus-1931] K.J. Sixtus and L. Tonks *Phys. Rev.* **37** 930-958 (1931)
 "Propagation of large barkhausen discontinuities";
Phys. Rev. **39** 357-358 (1932)
 "Propagation of large barkhausen discontinuities";
Phys. Rev. **42** 419-435 (1932)
 "Propagation of large barkhausen discontinuities. Part II."; *Phys. Rev.* **43** 70-80, (1933)
 "Propagation of large barkhausen discontinuities. Part III. Effect of a circular field with torsion";
Phys. Rev. **43** 931-940 (1933)
 "Propagation of large barkhausen discontinuities. Part IV. Regions of Reversed Magnetisation"
- [Slater-1936] J.C. Slater *Phys. Rev.* **49** 537-545, (1936)
 "Ferromagnetism of nickel";
Phys. Rev. **49** 931-937 (1936)
 "Ferromagnetism of nickel, Part II";
Phys. Rev. **52** 198-214 (1937)
 "Theory of ferromagnetism, Lowest energy levels"
- [Slater-1937] J.C. Slater *J. Appl. Phys.* **8** 385-390, (1937)
 "Electronic structure of alloys"
- [Stancu-1994] Alexandru Stancu and Constantin Papusoi *IEEE Trans. Mag.* **30** 4308-4310 (1994)
 "Hysteresis modeling of recording media with an interacting Stoner-Wohlfarth particles system"

- [Stancu-1997] Alexandru Stancu and Irinel Chiorescu *IEEE Trans. Mag.* **33** 2573-2579 (1997)
 “Crossover condition in the coherent rotation model and the Preisach-type models”
- [Stancu-1998] Alexandru Stancu and Leonard Spinu *IEEE Trans. Mag.* **34** 3867-3875 (1998)
 “Temperature- and time-dependent Preisach model for a Stoner-Wohlfarth particle system”
- [Stancu-1999] Alexandru Stancu, Philip R. Bissell, and Roy W. Chantrell *J. Magn. Magn. Mater.* **193** 395-398 (1999)
 “Interactions characterisation in particulate magnetic recording media. Experiments and Preisach model”
- [Still-1946] Still, Alfred *Soul of Lodestone: The Background of Magnetical Science* Murray Hill Books, Inc., New York (1946)
- [Stoner-1933] E.C. Stoner *Phil. Mag.* **15** 1018-1034 (1933)
 “Atomic moments in ferromagnetic metals and alloys with non-ferromagnetic elements”;
Repts. Progr. Phys. **11** 43 (1947)
- [Stoner-1947] E.C. Stoner, and E.P. Wohlfarth *Nature* **160** 650-651 (1947)
 “Interpretation of high coercivity in ferromagnetic materials”
- [Stoner-1948] E.C. Stoner, and E.P. Wohlfarth *Phil. Trans. Roy. Soc.* **240A** 599-642 (1948)
 “A mechanism of magnetic hysteresis in heterogeneous alloys”
IEEE Trans. Mag. **27** 3475-3518 (1991)
 (Reprint)
- [Surig-1996] C. Surig and K.A. Hempel *J. Appl. Phys.* **80** 3426-3429 (1996)
 “Interaction effects in particulate recording media studied by ferromagnetic resonance”
- [Thamm-1998] Stefan Thamm and Jurgen Hesse *J. Magn. Magn. Mater.* **184** 245-255 (1998)

- “The remanence of a Stoner-Wohlfarth particle ensemble as a function of the demagnetisation process”
- [Uhlenbeck-1926] G.E. Uhlenbeck and S. Goudsmit *Nature* **117** 264-265 (1926)
“Spinning electrons and the structure of spectra”
- [Warburg-1881] E. Warburg *IEEE Ann. Physik* **13** 141-164 (1881)
“Magnetic investigations”
- [Watson-1972] John N. Watson *IEEE Trans. Mag.* **MAG-8** 201-204 (1972)
“The static M-H loops of stressed magnetic materials with intergranular interaction”
- [Weber-1852] W. Weber *Pogg. Ann.* **LXXXVII** 167 (1852)
- [Weiss-1906] P. Weiss *Comptes Rendus* **143** 1136-1139 (1906)
“Temperature-variation of ferromagnetism”
- [Weiss-1907] P. Weiss *Journ. de Physique* **6** 661-690 (1907)
“Hypothesis of the molecular field and ferromagnetism”
- [Wiedemann-1886] G. Wiedemann *Phil. Mag.* **52** (1886)
- [Williams-1949] H.J. Williams, R.M. Bozorth, and W. Shockley *Phys. Rev.* **75** 155-178 (1949)
“Magnetic domain patterns on single crystals of silicon iron”
- [Williams-1989] Wyn Williams and David Dunlop *Nature* **337** 634-637 (1989)
“Three-dimensional micromagnetic modelling of ferromagnetic domain structure”
- [Wohlfarth-1948] E.P. Wohlfarth *Proc. Roy. Soc.* **195A** 434-462 (1948)
“Collective electron ferromagnetism. III. Nickel and nickel-copper alloys”
- [Wuori-1984] Edward R. Wuori and J.H. Judy *IEEE Trans. Mag.* **MAG-20** 1867-1869 (1984)
“Demagnetizing influence on a Stoner-Wohlfarth particle array”

- [Yan-1988] Ying Dong Yan and Edward Della Torre *IEEE Trans. Mag.* **24** 2368-2370 (1988)
 “Discretization errors in numerical micromagnetic models”
- [Yu-1997a] Y. Yu and D.L. Atherton *J. Magn. Magn. Mater.* **169** 199-206 (1997)
 “Tensor magnetic hysteresis loops for a ferrite permanent magnet cube”
- [Yu-1997b] Y. Yu and D.L. Atherton *IEEE Trans. Mag.* **33** 3390-3392 (1997)
 “Study of magnetization vector rotation process using tensor magnetic hysteresis loops”
- [Zener-1951] C. Zener *Phys. Rev.* **81** 440-444 (1951)
 “Interaction between the *d* shells in the transition metals”;
Phys. Rev. **82** 403-405 (1951)
 “Interaction between the *d*-shells in the transition metals. II. Ferromagnetic compounds of manganese with perovskite structure”;
Phys. Rev. **83** 299-301 (1951)
 “Interaction between the *d*-shells in the transition metals. III. Calculation of the Weiss factors in Fe, Co, and Ni”;
Phys. Rev. **85** 324 (1951)
- [Zhang-1995] Yanjing Zhang and David L. Atherton *IEEE Trans. Mag.* **31** 2233-2240 (1995)
 “Levels of demagnetized states”
- [Zhu-1988] Jian-Gang Zhu and Neal H. Bertram *J. Appl. Phys.* **63** 3248-3253 (1988)
 “Micromagnetic studies of thin metallic films (invited)”