QUANTIFICATION OF GAMMA-RAY COMPTON-SCATTER NONDESTRUCTIVE TESTING

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Abstract

A method for quantifying the information obtained from the indications of the gamma-ray Compton scattering technique in nondestructive testing was developed. The method enables the detection of the presence of anomalies, and the determination of the location, size and the density of occupying material. A measurement model was formulated, based on once-scattered photons, to provide a numerical estimate of the detector response for a given size and density of anomaly. This model was successfully verified against Monte Carlo simulations. An inversion process of the measurement model was developed to determine the density of a discontinuity in an object by inverting the detector response within the energy range of interest, after compensating for photon attenuation in the surroundings.

The quantification process was applied to independently-obtained experimental indications, with considerable success. A normalization process relating the experimental measurements to the measurement model was developed, and the inversion algorithm was slightly modified to accommodate the background subtraction process involved in obtaining the experimental indications.

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Chapter 1

Introduction

1.1 Introduction

Electromagnetic radiation, in the form of microwaves, x-rays and gamma-rays, is used in nondestructive testing because of its ability to penetrate deep into matter. Microwaves are good in penetrating dielectric materials (like ceramics) but inspection of metals is limited by the inability of microwaves to penetrate more than skin depth. Moreover, positioning of microwave test equipment and test specimen is often critical, requiring complex arrangements and calibrations in the alignment of waveguides and detectors (Bray and McBride, 1992). X-rays are not as penetrating as gamma-radiation since x-rays typically

have lower photon energies than gamma rays. However, increased penetrating depth can be achieved by using a high current x-ray machine to increase the source intensity. X-rays produce a continuous multi-energetic spectrum due to the deflection of electrons by the strong electromagnetic fields surrounding the nuclei of the atoms of the material emitting the radiation (Lamarsh, 1975). The multi-chromatic nature of x-ray photons makes it more difficult to directly obtain quantitative information. On the other hand, gamma-rays produce an energy spectrum that has distinct emission energy peaks, thus providing well-defined photon energies that enable easier analysis of the measured signal. Moreover, gamma-ray isotopic sources are readily portable, self-contained, self-energizing and useable in hostile environments.

In nondestructive testing (NDT) with radiation, the transmission modality is usually employed, where a source and a detector (e.g., a radiographic film) are placed on opposite sides of the object, and the attenuation of the source photons through the object is measured. However, transmission measurements may not always be possible due to surrounding space constraints, and/or if the object is too bulky to produce sufficient radiation penetration. Transmission provides line-integrated information along the path of radiation from the source to the detector, which masks the position of an anomaly present along the transmission line. Therefore, it is difficult to determine the position of an anomaly directly from transmission measurements. Scattering provides, therefore, an alternative method.

In scattering, the detector is positioned so that it is located outside the direct field of view of the source, and thus it measures radiation deflected from the object. Moreover, in scattering, point-wise information can be obtained by focusing the field of view of the source

and detector so that they intersect around a point. In the scattering technique, the information obtained is strongly related to the material density, thus allowing changes in the material uniformity to be monitored (Hussein, 1989). Scattering also eliminates the need of accessing two opposing sides of the target object. The gamma-ray scatter technique is discussed in more details in the following section, together with a review of recent progress in Compton-Scatter Nondestructive Testing (CS-NDT).

1.2 Compton-Scatter NDT

Compton scattering is the interaction of photons with the free electrons of the atom. Therefore, it is a viable tool for inspecting material since it is an interaction which is strongly

Figure 1.1 Collimated-detector Compton-scatter inspection system

dependent on the electron density of the scattering medium, and in turn, its mass density. A basic inspection system, used by many workers (Berodias and Peix, 1988; Whynot and Hussein, 1989; Tuzi and Sato, 1990; among others), is depicted in Figure 1.1. The intersection of a collimated beam of photons with a collimated field of view of a detector defines the inspection (scattering) volume. The intensity of the scattered radiation then depends on the electron density of the material in the inspection-volume. In such a system, the detector's field of view is limited to a small narrow beam by the use of a collimator made of a heavy metal (such as lead). In such arrangement, only one inspection-volume can be monitored at a time, without moving the source and/or detector.

The Compton scattering kinematics provide a unique relationship between the energy and angle of scattering, such that (Evans, 1955):

$$E(\theta) = \frac{E_o}{1 + \frac{E_o}{m_o c^2} (1 - \cos \theta)} \quad (1.1)$$

where E_o is the incident photon energy, $E(\theta)$ is the energy of photons scattered by angle θ and $m_o c^2$ is the rest mass energy of the electron (0.511 MeV). This relationship shows that, for single-scattered photons, the scattering direction can be determined from the scattering angle for a mono-energetic source of known energy, E_o . Therefore, many angles of scattering can be monitored simultaneously using a single detector capable of measuring the photon energy. This allows simultaneous inspections of many points along the incident beam. This concept has been used in NDT applications by a number of workers, in the configuration depicted in Figure 1.2.

Figure 1.2 Wide-angle Compton-scatter inspection system

Anghaie *et al.* (1990a, 1990b) employed the above concept for anomaly detection, sizing and location. However, Anghaie *et al.* were able, using a wide angle detector, to detect an anomaly, determine its location and estimate its size from the energy spectrum of scattered photons. This method works well for single isolated anomalies. If multiple anomalies are present within the field of view of a wide angle detector, their indication will overlap within the wide spectrum and will not be detectable by the method of Anghaie *et al.* In order to be able to use Eq. (1.1) to relate the energy to angle of scattering, a point-to-point correspondence must be established between the radiation source, inspection volume and the detector. In practice, for a physical detector with a given size, radiation can scatter from different points on the path of the incident radiation with the same angle, and consequently same scattering energy, to different points on the detector surface. This makes it difficult to determine a unique angle of scattering from the detector to a position on the incident beam,

by directly applying Eq. (1.1).

Mullin and Hussein (1994) attempted to solve the detector-size problem by converting the detector to equivalent three point-detectors (one at the centre and two at the edges). They developed a scheme to relate the energy-to-angle of scatter to the equivalent detectors. However, difficulties were encountered in relating the measured energy spectrum to the energy of the photons incident on the detector, due to the spread in the measured energy spectrum that occurs even for incident mono-energetic radiation.

Figure 1.3 Confined wide-angle Compton-scatter inspection system

Jama (1997) confined the detector to almost a point. This was achieved by equipping the detector with a wedge collimator that has a narrow opening, as shown in Figure 1.3. This enabled the establishment of a point-to-point correspondence between the scattering point and the detector. More definitive information about anomaly detection, location, and sizing were, therefore, obtained.

Given the above work, as well as the work of many others as evident by the 65 abstracts of work published between 1988 and 1994 (NDT Abstracts, 1995), the Compton scattering technique is sufficiently mature as a qualitative method to locate an anomaly, determine its location and provide an indication of its size. However, no quantitative method for nondestructive testing application is available, although imaging methods have been developed (Prettyman *et. al.*, 1993; Arendtsz and Hussein, 1995a, 1995b). Imaging requires many projections and is a tedious process that is not suited, or needed, in many NDT application. In NDT, the objective is to find and locate anomalies, that are typically limited in volume and size. This is in contrast to imaging, which provides a portrait of the entire object. Moreover, in NDT, the nature of the inspected object is usually known and the purpose is to determine any disturbance in the measured signal that can indicate the presence of an anomaly. The purpose of this work is to take the Compton Scattering NDT method a step further by developing a quantitative method of analysis. Such a method will enable one to obtain indications that are more definitive and easier to report and analyze.

1.3 Objectives

The objective of this work is to develop a method for quantifying the information obtained from the indications of the Compton scattering technique in NDT. This is achieved by formulating a model which provides a numerical estimate of the detector response for a

given anomaly size. Such a model is called the “measurement model”. Using the measured response function, the anomaly location, size and density can then be obtained with the aid of the measurement model. The development of the measurement model is discussed in the next Chapter. The model is then verified against Monte Carlo simulations which better depicts the physical process. Once the model is verified, and corrected if needed, it is applied to quantify indications obtained from experimental results.

Chapter 2

Measurement Model

As indicated in Chapter 1, there is a need to develop a method to quantify the information obtained from the indications of Compton scattering techniques for nondestructive testing. An essential step for this quantification process is to formulate a measurement model which provides a numerical estimate of the detector response for a given anomaly size at a given location in some target. This Chapter introduces such a model and verifies it against Monte Carlo simulations.

2.1 Formulation

In order to formulate a model, the scattering of a photon at a point is considered. Along that point, an infinitesimal volume is constructed, as shown in Figure 2.1. It is assumed that only single-scatter events contribute to the detector, since the indication signal is monitored at an energy corresponding to that of single scattering at a given angle of scattering. Multi-scattered photons suffer more energy loss than single-scattered photons, and as thus are not expected to appear within the monitored energy range.

Figure 2.1 Scattering of Photon at a Point

The number of photon collisions, N_s , that takes place within ΔV can be expressed as:

$$N_s = \Sigma_s \psi \Delta V \quad (2.1)$$

where Σ_s is the scattering cross section (probability per unit length) and ψ is the photon fluence within ΔV , since the fluence can be defined as the number of photons tracks per unit volume, refer to Appendix I. The fluence, ψ , is related to the incident fluence, ψ_o , at the surface of the object within ΔV by:

$$\psi = \psi_o \exp\left(-\int_0^{x_m} \Sigma_{in} dx\right) \quad (2.2)$$

where Σ_{in} is total (absorption plus scatter) cross section of the material present along the path of the incident radiation and x_m is the distance from the surface of the object to the point of scattering. The exponential term in Eq. (2.2) accounts for the attenuation of photons along the path of travel to the point of scattering. The photons scattered in ΔV can scatter in all directions. The number of photons scattered towards the detector within a small cone define by $d\Omega$ can be expressed as:

$$N_\Omega d\Omega = N_s p(\Omega) d\Omega \quad (2.3)$$

where $p(\Omega)d\Omega$ is the probability of scattering within a small cone $d\Omega$. In terms of the polar angle θ and the azimuthal angle ϕ , $d\Omega = d\mu d\phi$, then:

$$p(\Omega)d\Omega = p(\phi)p(\mu)d\mu d\phi = p(\phi)p(\mu)d\Omega \quad (2.4)$$

where $\mu = \cos \theta$ and $p(\mu)d\mu$ is the probability of scattering about μ within $\mu \pm d\mu$, and $p(\phi)d\phi$ is the probability of scattering about the azimuthal angle, ϕ , within $\phi \pm d\phi$. The value of $p(\mu)$ is given, to a good approximation, by the Klein-Nishina relationship (Los Alamos National Laboratory, 1993):

$$p(\mu) = \frac{\eta^3 + d_1 \eta^2 + d_2 \eta + d_3}{c_1 \eta^2 + c_2 \eta + c_3} \left(\frac{\alpha'}{\alpha} \right)^2 \left(\frac{\alpha}{\alpha'} + \frac{\alpha'}{\alpha} + \mu^2 - 1 \right) \quad (2.4a)$$

where $\alpha = E_o/mc^2$ (where E_o is the incident photon energy, m is the mass of the electron and c is the speed of light), $\mu = \cos \theta$ for the angle θ of deflection from the line of flight of the incident photons, and $\alpha' = \alpha/[1 + \alpha(1 - \mu)]$. The variables, $\eta = 1 + 0.222037\alpha$, $c_1 = 1.651035$, $c_2 = 9.340220$, $c_3 = -8.325004$, $d_1 = 12.501332$, $d_2 = -14.200407$, and $d_3 = 1.699075$ as shown in Appendix II. The scattering in the azimuthal direction is isotropic since it does not affect the kinematics of the scattering process, therefore, $p(\phi) = 1/2\pi$. Consequently:

$$N_\Omega d\Omega = \frac{N_s}{2\pi} p(\mu) d\Omega \quad (2.5)$$

The photon fluence reaching a detector of a small area dA at distance R from the scattering point can be expressed as:

$$\psi_d = \frac{N_\Omega d\Omega}{dA} \exp\left(-\int_0^{y_{out}} \Sigma_{out} dy\right) \quad (2.6)$$

where Σ_{out} is the total cross section of the material present along the path of the scattered radiation and y_{out} is the distance from the scattering volume to the surface of the object, as shown in Figure 2.1. Substituting $N_\Omega d\Omega$ from Eq. (2.5) into Eq. (2.6), and using the fact that $d\Omega = dA/R^2$, one obtains:

$$\psi_d = N_s \frac{p(\mu)}{2\pi R^2} \exp\left(-\int_0^{y_{out}} \Sigma_{out} dy\right) \quad (2.7)$$

Substituting N_s from Eq. (2.1) into Eq. (2.7), gives:

$$\psi_d = \Sigma_s \psi \Delta V \frac{p(\mu)}{2\pi R^2} \exp\left(-\int_0^{y_{out}} \Sigma_{out} dy\right) \quad (2.8)$$

The fluence ψ at the scattering volume can be related to that incurred on the object using Eq. (2.8), resulting in:

$$\psi_d = \Sigma_s \Delta V \psi_o \exp\left(-\int_0^{x_{in}} \Sigma_{in} dx\right) \frac{p(\mu)}{2\pi R^2} \exp\left(-\int_0^{y_{out}} \Sigma_{out} dy\right) \quad (2.9)$$

The scattering volume, ΔV , can be expressed as $\Delta A_s \ell$ where ΔA_s is the surface area of ΔV from the incident beam and ℓ is the width of ΔV . Then, the source intensity directed towards the scattering volume, ΔV , is $\psi_o \Delta A_s$. Therefore, the intensity of the photons scattered to the

detector per unit source (the detector response, S_{model}) can be expressed as:

$$S_{model} = \frac{\psi_d}{\psi_o \Delta A_s} = \frac{\psi_d}{\psi_o} \frac{\ell}{\Delta V} = \Sigma_s \ell \exp\left(-\int_0^{x_{in}} \Sigma_{in} dx\right) \frac{p(\mu)}{2\pi R^2} \exp\left(-\int_0^{y_{out}} \Sigma_{out} dy\right) \quad (2.10)$$

Since for most materials, the Compton scattering cross section, Σ_s , is proportional to the mass density of the material (Lamarsh, 1975) then one can write:

$$\Sigma_s = \Sigma_{ref} \bar{\rho} \quad (2.11)$$

where Σ_{ref} is the Compton scattering cross section with respect to a reference material and $\bar{\rho}$ is the density of the scattering material relative to the reference material. In nondestructive testing, the bulk material of the inspected volume can be taken as the reference material; thus enabling the detection of any variation from the reference density.

Finally, combining Eq. (2.10) and Eq. (2.11) gives:

$$S_{model} = \Sigma_{ref} \ell \bar{\rho} \exp\left(-\int_0^{x_{in}} \Sigma_{in} \bar{\rho} dx\right) \frac{p(\mu)}{2\pi R^2} \exp\left(-\int_0^{y_{out}} \Sigma_{out} \bar{\rho} dy\right) \quad (2.12)$$

The above equation represents the measurement model for the detector response. It should be noted that the model does not take into consideration the detector efficiency, since it can be accounted for in practice by calibration with a source of known density.

Having formulated the measurement model, a method is needed to verify that it faithfully represents the encountered physical process. For this purpose, the Monte Carlo method is employed, as explained in the next section.

2.1.1 Monte Carlo Simulations

The Monte Carlo method is a stochastic numerical simulation method that takes into account all the physical and geometrical aspects of the problem. This method is used in radiation transport studies for simulating the transport of neutrons, photons and electrons, particularly in irregular geometries. In the Monte Carlo approach, individual particle transport histories are generated by randomly sampling particles as they travel through the geometry. These transport histories are then analyzed to provide information on quantities of interest, such as particle fluence or detector response. The Monte Carlo method is most effective in solving problems that are difficult to simulate with common deterministic methods. This method also provides an answer with a statistical uncertainty associated with it, so that a confidence level in the results can be established (Hussein, 1997).

The MCNP (Monte Carlo Neutron Photon) transport code is a widely used and recognized code for neutron and photon transport (Los Alamos National Laboratory, 1990). This code was therefore adopted for use in this work. MCNP is a general purpose, continuous energy, generalized geometry, time dependent, coupled neutron-photon Monte Carlo transport code. The energy of the simulated photons can range from 0.001 to 100 MeV. Photon cross-section (probability of interaction) information in MCNP is point-wise form, derived from widely-used libraries (Storm and Israel plus ENDF/B evaluations, Los Alamos National Laboratory, 1990).

The MCNP code requires a user input file to define the problem's geometry, source and detectors. MCNP defines geometry cells in a Cartesian coordinate system which is user

defined and gives the user the flexibility to define geometrical cells from surfaces through a Boolean combination process. The material content of each cell is then defined by the model's element composition.

The source is specified in MCNP by listing the geometric location, direction of flight, energy and particle weight. The source can be isotropic or biased and can emanate from a point, a surface or a volume. A particle is tracked until a pre-assigned parameter is reached, or until the particle escapes the system geometry.

The detector is specified in MCNP through a tallying function, specifying how the detector response is established (e.g., over a surface, a volume or at a point). The detector response is normalized per source particle.

For the purpose of verifying the measurement model, Monte Carlo simulations were performed with sufficient number of particles, so that a fractional standard deviation (i.e., standard deviation with respect to the mean of the estimated quantity) of less than 5% is obtained. In the simulations, 5 million particles were used to achieve a fractional standard deviation (or relative error) of less than 0.005.

2.2 Verification of Measurement Model

In this section, the measurement model is verified against Monte Carlo simulations using the MCNP code, first for isolated small objects then for single anomalies, i.e., isolated objects in the form of solid sphere scatterers were used, as they present the simplest form of geometry and enable direct verification of the model. Spheres of various diameter and density are used in the simulations. A flaw, represented by a cylindrical hole in an aluminum block, was then simulated. Different diameters of flaws were considered. Based on the simulation results, the measurement model's capabilities and limitations are assessed and aspects requiring further development are identified.

2.2.1 Spherical Scatterers

Small spheres of different densities and diameters were used in the Monte Carlo simulations. These are the same diameters used in the experiments conducted by Jama (1997). The setup of the MCNP simulation is shown in Figure 2.2, where a source beam is made incident directly on the scatterer and a point detector is positioned at an angle of 77.37° with respect to the incident beam to monitor the scattered photons. A Cesium-137 source (662 keV photon peak) was utilized the form of a collimated (pencil) beam of width 2 mm. A collimated beam was used as the effects of source beam divergence was not considered at this time. A point detector was used in the simulations, rather than a volume detector, to

Figure 2.2 Geometry used in the Monte Carlo simulations for a single scatterer

enable direct comparison with the model.

Figure 2.3 shows the results of the simulation in the form of energy spectra for target spheres of 4.76 mm, 9.52 mm and 19.04 mm in diameter. The presence of a scatterer can be distinguished from the spectrum by the peak corresponding to the angle of single-scattering from the source beam to the detector, in this case, at 329 keV or 77.36° in accordance with Eq. (1.1). The width of the peak indicates the range of angles and energies over which the photons would have scattered from the spheres. This range of energies corresponds, therefore, to the width of the scatterers. For example, the results for the Monte Carlo simulation for the 4.72 mm sphere, show that the range of angles for the sphere spans from 324 to 334 keV, with corresponding scattering angles from 78.76° to 75.99° . This angle/energy range indicates, from geometry, that the width of the scatterer is 4.57 mm, which is close to the actual diameter of the sphere of 4.76 mm.

Figure 2.3 Energy spectrum of three target spheres (4.76, 9.52, 19.02 mm in diameter)

The area under the peak is indicative of the amount of photons scattered, which is proportional to the amount of material present in the target spheres. As the scatterer size increases, the size of the corresponding peak area also increases. This increase in peak size does not, however, appear to be linearly proportional to the amount of material in the scatterer. This may be due to the attenuation of photons within the sphere (so-called self-attenuation). Therefore, to quantify the size of the scatterer from the peak area, one must rely on the measurement model as it includes the effect of self-attenuation. We begin first by comparing the strength of this peak to the detector response obtained from the measurement model. For comparison with the measurement model, the target spectrum (denoted by S) was determined by integrating the detector response over the energy range indicated by the peak

in the energy spectrum.

Different scatterer sizes and densities were simulated to provide a suitable set of data for comparison. The density of aluminum was artificially altered for comparative purposes. Table 2.1 summarizes the results obtained from the simulations, S , as well as those calculated by the measurement model (S_{model}), Eq. (2.12). The model detector response was obtained assuming scattering to occur at the centre of the spherical scatterer (i.e., in effect, the scattering within the volume of the sphere is lumped into a point).

Table 2.1 Monte Carlo and Measurement Model Results for a Spherical Scatterer

Scatterer Dia. (mm)	Scatterer Density (kg/m ³)	Target (S) (photons/cm ²)	Model (S_{model}) (photons/cm ²)	S_{model}/S
4.76	675	1.554×10^{-5}	1.630×10^{-5}	1.05
9.52	675	3.140×10^{-5}	3.171×10^{-5}	1.01
19.04	675	6.007×10^{-5}	5.998×10^{-5}	0.999
4.76	1350	3.032×10^{-5}	3.171×10^{-5}	1.05
9.52	1350	5.966×10^{-5}	5.998×10^{-5}	1.01
19.04	1350	1.087×10^{-4}	1.073×10^{-4}	0.990
4.76	2700	5.774×10^{-5}	5.942×10^{-5}	1.03
9.52	2700	1.082×10^{-4}	1.073×10^{-4}	0.992
19.04	2700	1.786×10^{-4}	1.717×10^{-4}	0.961
4.76	4050	8.253×10^{-5}	8.509×10^{-5}	0.976
9.52	4050	1.473×10^{-4}	1.440×10^{-4}	0.978
19.04	4050	2.201×10^{-4}	2.060×10^{-4}	0.936
4.76	5400	1.057×10^{-4}	1.073×10^{-4}	1.02
9.52	5400	1.781×10^{-4}	1.717×10^{-4}	0.964
19.04	5400	2.419×10^{-4}	2.197×10^{-4}	0.908

Relative error in each result is very small. (Typically less than 1%)

The detector response calculated by the measurement model increases with the scatterer's size and density as Table 2.1 shows. This was anticipated since the measurement model estimates the amount of scattering provided by the present material, which tends to increase with density and scatter size. However, the measurement model tends to underestimate the detector response in comparison with the Monte Carlo results by 1% to 9%, as the scatterer diameter and density increases. On the other hand, for small scatterers, the measurement model overestimates the simulated detector response by 2% to 5%. Therefore, the measurement model was carefully examined to explain the cause of this under/overestimation. To facilitate this process, the model represented by Eq. (2.12) is considered, along with the schematic diagram of Figure 2.4. Each portion of the model is scrutinized separately.

Figure 2.4 Schematic showing incident beam path and path of scattered photons from three scattering points

The schematic in Figure 2.4 shows the extremities of the incident and scattered photon paths, as these extremities are likely to cause the largest deviation from the lumped (into a point) nature of the model. These extreme values have the largest difference when the scatterer size is large. Therefore, the results of the largest scatterer diameter used in the simulation, 19.04 mm, are examined below.

The effect of averaging among the extreme points is examined in comparison with the value given by the model which represents the central path of the radiation (UO and OP in Figure 2.4) Referring to the measurement model, the term $p(\mu)$, accounts for the probability density for scattering towards the detector. The largest possible value of the probability density for scattering is $p(\mu)$ at θ_1 where θ_1 is the scattering angle from point U to point P shown in the Figure, while the smallest value is $p(\mu)$ at θ_3 (scattering angle from point L to point P). Numerically, these values are 0.4133 and 0.3440, respectively. The average of these two extreme values is 0.3787, while the model uses the value of 0.3742 for $p(\mu)$ at angle θ_2 (scattering from O to P). Comparing the two values, the results suggests that the model value is smaller than the average value by a factor of 0.988. This is not sufficiently significant to explain the deviation between the model and the simulation results.

The $1/R^2$ term which accounts for the post-scatter divergence of the photons is now examined. The largest possible value of the divergence term is $1/R_U^2$, where R_U is the distance from U to P shown in the Figure 2.4, and in this case is equal to 0.01114. The smallest value is $1/R_L^2$, where R_L is the distance from L to P , and has the value of 0.01218. The average value of the $1/R_U^2$ and $1/R_L^2$ is 0.01166. The computed model value is 0.01176, which corresponds to $1/R^2$ where R is the distance from O to P . The model value is greater

than the average value by a factor of 1.009. This factor is also not significant enough to explain the deviation between the model and the simulation results.

For the pre-scatter term in the model equation, $\exp(-\sum_{in}\rho|UO|)$, the largest possible value is $\exp(-\sum_{in}\rho|UL|)$, where UL is the distance from U to L shown in Figure 2.4. There is no photon attenuation for the other extreme point (point U in Figure 2.4), since the incident photons are directly scattered to the detector from this point. The average value for the attenuation term from the extreme point is 0.8409 and the computed model value is 0.8258. The model value is smaller than the average value by a factor of 0.982 which is not very significant.

The next term in the model equation, $\exp(-\sum_{out}\rho|OQ|)$, accounts for the post-scatter attenuation. The attenuation term for the extreme point is $\exp(-\sum_{in}\rho|UX|)$, where distance UX is shown in Figure 2.4. There is no attenuation for the other extreme point (point L in Figure 2.4), since photons are scattered directly to the detector from this point. Numerically, the average value for the attenuation term from the extreme points is 0.9282. The computed model value is 0.7749. The model value is smaller than the average value by a factor of 0.835, giving an upper limit of deviation. This factor is very significant to the deviation between the computed model and the averaged results. Therefore, the post-scatter attenuation factor is the most significant parameter responsible for underestimation of the model results.

Combining all the above factors together results in an overall upper deviation limit of 0.817. Therefore, one would expect the model to always underestimate the Monte Carlo results, due to its lumping effect. However, Table 2.1 shows that, for the small scatterer, the model overestimates the Monte Carlo results. Therefore, one must look more carefully at the

Figure 2.5 Schematic of the interaction volume in a spherical scatter

last main parameter in the model, i.e., the interaction rate factor, $\Sigma_s \ell$.

The interaction rate per unit volume is equal to $\Sigma_s \psi$, where Σ_s is the scattering cross section (probability) per unit length and ψ is the photon fluence at the surface of the target object. The fluence, ψ , for a collimated beam of radius b and unit intensity is equal to $1/\pi b^2$. For an object of volume V , the total number of scatters will be equal to $\Sigma_s \psi V$. The space in which the source photons interact within the target matter represents a cylinder with a volume V , equal to $\pi b^2 \ell$, where ℓ is the length of the interaction volume created by the incident photons within the target. Therefore, the interaction term, $\Sigma_s \psi V$, becomes equal to $\Sigma_s \ell$.

For a spherical scatterer, representing the interaction volume by a cylinder is an approximation, as can be seen in Figure 2.5. The Figure indicates that the assumption of an cylindrical interaction volume leads to the creation of an additional volume outside the

sphere, which does not include scattering material. The model assumes, in effect, that the additional volume outside the spherical scatterer, but inside the interaction cylinder, contains a scattering material. This additional material will lead to overestimation of the amount of scattering, as modeled by Monte Carlo simulations. To further confirm this effect, a cylindrical scatterer was simulated to imitate the interaction volume of the model. The width of the cylindrical scatterer was taken equal to the diameter of the spherical scatterers. The larger the size of this additional volume, the more overestimation one would expect to obtain. However, the size of the additional volume decreases as the beam width decreases, while the relative size of this additional volume, with respect to the scatterer size, decreases with the increased diameter of the scatterer. Therefore, the magnitude of overestimation due to the additional volume should decrease as the beam width decreases and the scatterer's size increases. This explains the overestimation observed for the smallest scatterer size (4.76 mm diameter) observed in Table 2.1. The detector response for a cylindrical scatterer obtained from the Monte Carlo simulations was 5.978×10^{-5} photons/cm² and the detector response for the spherical scatterer obtained was 5.774×10^{-5} photons/cm². Comparing the two responses with the model response of 5.942×10^{-5} photons/cm², the response of the cylindrical scatterer becomes closer to that of the model. This certifies that the additional volume missed in modeling in the spherical scatterer is responsible for the observed model's overestimation.

In summary, the Monte Carlo simulations revealed that as the diameter of the scatterer and the size of the beam (relative to the scatterer) increases, the model tends to overestimate the Monte Carlo results. This is due to the additional volume, as explained

above, which causes the overestimation. On the other hand, as the size of the scatterer increases with no increase of beam size, the model tends to underestimate the Monte Carlo results. This, in part, is due to the post-attenuation term, $\exp(-\sum_{out}\rho|OQ|)$, that cause a significant deviation between the model value and the average value of the extreme points on the scatterer. To reduce this discrepancy, one should decrease the width of the beam, limit the size of the scatterer, and/or find a way to improve the measurement model. Model improvement can be achieved by averaging the response over many scatterer points (i.e., consider more than one incident/scatter rays).

2.2.2 Single Flaw

Having investigated the physical model for single scatterers, the model is examined for a single flaw. Target objects in the form of 60 x 50 x 50 mm³ aluminum blocks having a hollow cylinder to represent a flaw void were stimulated with the MCNP code. The setup is shown in Figure 2.6. The same target sizes used in the experiments conducted by Jama (1997) were employed. These involve aluminum blocks with flaw size of 9.52 mm, 4.76 mm, 2.38 mm and 1.19 mm in diameter. A solid block, of the same size as the test object, was also simulated as a reference.

An indicator of the presence of a flaw was obtained by subtracting the detector response of the reference block from that of the test target. This process provides a “negative” indication of the absent material. This subtraction process also results in a scattering signal originating from the region in the object containing the flaw. Since the flaw

Figure 2.6 Geometry used in the Monte Carlo simulations for a single flaw

size is very small, mainly single scatter events are expected to contribute to the detector. Therefore, simulations were carried out initially considering only single scattering. Subsequent simulations were performed to examine the effect of multiple scattering.

The indication signal can be seen as being equivalent to that represented by an isolated scatter occupying the void space of the flaw. This is because the detector response for the target can be considered as being equal to that of the reference block minus the response of material that would have been present if the flaw was absent. That is:

$$\begin{aligned}
 \text{Indication} &= S_i = S_{ref} - S_{target} \\
 &= S_{ref} - (S_{ref} - S_{material}) \\
 &= S_{material}
 \end{aligned} \tag{2.13}$$

where S_{ref} designates the detector response of the reference object, S_{target} is that for target with the flaw and $S_{material}$ is the response for the material that would fill in the flaw. Therefore, in accordance with the above expression, $S_{material}$ is equivalent to the response of a single

scatterer in the place of the flaw. This enables direct comparison with the measurement model by comparing S_i to that calculated for a equivalent material filling entirely the flaw space. The energy spectra provided by the MCNP code for a target with a 9.52 mm diameter flaw are shown in Figure 2.7 and the indication for this target is given in Figure 2.8. The indication spectra distinguishes the presence of a flaw by a peak at an energy corresponding to the angle of the single-scattering from the source beam to the detector (329 keV and 77.36° in this case). The width of the peak reflects the range of angles over which the photons would have scattered from the flaw region if it was filled with material.

The area of the peak is indicative of the amount of material missing from the target block which in this case ranges from 319 to 339 keV with corresponding scattering angles from 80.20° to 74.65°. This energy/angle range indicates, from geometry, that the width of

**Figure 2.7 Energy spectrum of target block
(single 9.52 mm diameter flaw)**

**Figure 2.8 Indication spectrum of target block
(single 9.52 mm diameter flaw)**

the flaw is 9.16 mm, which is close to the actual diameter of the flaw of 9.52 mm. Therefore, the strength of this peak can be compared to the detector response obtained from the measurement model for a single scatterer equivalent to that caused by the missing material.

Consequently, for comparison with the model, the indication spectra should be accumulated within the energy range covering the width of the spectrum. Therefore, S_{ref} and S_{tar} in Eq. (2.13) were obtained by integrating the detector response within the energy range obtained from the indication spectrum.

In order to compare the measurement model with Monte Carlo results, different flaw sizes were simulated using Monte Carlo methods. Table 2.2 summarizes the results obtained from the indication, S_i , obtained from the Monte Carlo simulations, as well as those calculated by the measurement model (S_{model}), Eq (2.12).

Table 2.2 Monte Carlo Simulations and Measurement Model Results for Single Flaws

Flaw Diameter (mm)	Indication (S_i) (photons/cm ²)	Model (S_{model}) (photons/cm ²)	S_{model}/S_i
9.52	3.834×10^{-5}	8.622×10^{-5}	2.25
4.76	2.589×10^{-5}	4.804×10^{-5}	1.86
2.38	1.163×10^{-5}	2.564×10^{-5}	2.20
1.19	3.698×10^{-6}	1.319×10^{-5}	3.57

Relative error in each result is very small. (Typically less than 1%)

Table 2.2 shows that the detector response calculated by the measurement model increases with flaw size. This is anticipated as the model estimates the amount of scattering that would have been provided by the absent material; the amount of which increases with the flaw size. The measurement model tends to overestimate the indication values often by a factor of 2 or more. This trend continued even though the beam size was reduced so that it matched the flaw width, so as to eliminate any possible effects of beam size. It was concluded that this overestimation is caused by some physical effect. Therefore, the field of view of the detector was carefully examined to determine where the photons reaching the detector are emerging.

Figure 2.9 shows a schematic of the target block with a single void anomaly, together with the source beam path and the detector angle of view. The detector view angle is defined by the energy range within the portion of the angle of the indication spectrum corresponding to the flaw. Within this range of detection, four regions are recognized and marked by "A", "B", "C" and "D". These regions are formed as the detector's angle of view and the source

Figure 2.9 Schematic showing the source beam path and detector angle of view

beam overlap each other.

The subtraction process for regions “A”, “B”, “C” and “D” would appear to give identical indication in both the target and reference (i.e., in the absence of flaw). Careful examination, however, showed that this is not necessarily true, as explained in the ensuing paragraphs.

Considering region “A”, photons scattered in this region within the target will have to cross the flaw region in order to reach the detector. Since there is no material in the flaw region, no photon removal (attenuation) takes place. On the other hand, photons scattered from region “A” within the reference block are attenuated by the material present as they travel towards the detector. In the subtraction process of the indication spectrum, the effect of region “A” is not canceled out. In fact the subtraction process results in a negative value

due to the lack of attenuation in the anomaly. This tends to result in an underestimation in the indication response.

On the other hand, incident source photons traveling to region “B” in the target must cross the anomaly region. Since there is no attenuation in the anomaly region, there will be more photons available to scatter to the detector. But photons scattered from the same region (“B”) in the reference are attenuated by the material present as they travel through it. Therefore, in the subtraction process of the indication spectrum, the effect of region “B” is not cancelled out. This also tends to produce an underestimation in the indication response.

In region “C”, the situation is more complicated as it involves a change in both the incident and the scattered photons. Incident photons traveling to region “C” in the target have to cross the flaw region. Unlike photons scattering from region “B” the scattered photons have to travel through the anomaly region again before reaching the detector. Once again, since there is no attenuation in the flaw region this will further increase the detector response. As before, photons traveling through the same region in the reference are attenuated by the present material. The subtraction process results in a negative value for the indication spectrum. Therefore, this tends also to further affect the underestimation of the indication response.

Incident source photons pass through the same material before reaching region “D” in both the target and the reference block. Similarly, scattered photons leaving region “D” towards the detector encounter the same material. Therefore, region “D” has a null effect on the subtraction of the indication spectrum.

To summarize, region “A”, “B” and “C” of Figure 2.9, result in an overall

underestimation of the indication spectrum due to the lack of photon attenuation within the area occupied by the anomaly. This demonstrates that the subtraction process (reference minus target spectra) used in the indication process does not reflect only the flaw region, but it also includes the effect of the surroundings. Indeed, the subtraction process gives an indication of the inspection region defined by the intersection of the incident beam and the view field of the detector. The flaw may not necessarily occupy the entire inspection region. However, by narrowing the beam width and/or the field of view of the detector, one should be able to minimize the effect of the surroundings. Alternatively, one can attempt to model scattering from the entire inspection region, including the assumed anomaly geometry, and the additional area seen by the detector.

2.3 Conclusions

In this chapter, a measurement model for Compton Scattering was formulated and verified against Monte Carlo simulations. The Monte Carlo simulations revealed that the measurement model tends to underestimate the Monte Carlo detector response for isolated scatterers and tends to overestimate it for single flaws. In both cases, careful examination of the interaction of the incident and scattered photons showed the region inside the interaction volume, but outside the scatterer/flaw volume was the cause of the underestimation and overestimation. Therefore, to reduce the underestimation and overestimation in modeling the

scatterers and flaws, one could

- limit the field of view of the detector,
- decrease the width of the source beam and
- improve the measurement model.

Model improvement can be achieved by averaging the response over more scatterer points, as shown in the next Chapter.

Chapter 3

Inversion of Measurement Model

3.1 Introduction

The measurement model, developed in Chapter 2, relates the sought parameters (e.g., size and density of flaws) to the measured indication spectrum. In turn, inversion of the measurement model enables the determination of the sought parameters from the measurements. However, one must first deal with the limitations revealed by Monte Carlo verification of the model, discussed in Chapter 2. These limitations included;

1. The missing-volume effect, i.e., the portion of the space in the inspection volume not accounted for by the model.

2. The lumping effect over a large volume, i.e., the application of a point-wise model to a large volume.

To overcome these limitations, a narrow incident beam combined with a narrow detector field-of-view is needed. The former can be achieved physically by collimating the source, while the latter can be achieved by considering one small detector-energy-interval at a time, and applying the model to the corresponding inspection volume. This should decrease the missing-volume effect.

The above approach is first tested on single scatters, as described in the next section. After establishing the proper procedure for these relatively simple cases, the approach is applied to multiple flaws, single and multiple inclusions. Subsequently, the effect of ignoring multiple scattering (as indicated in section 3.7) is readdressed. Finally, the proposed procedure is summarized.

3.2 Inversion Over Energy Intervals

In order to account for the missing volume effect, it is desirable to reduce the detector field of view. This, in turn, should reduce the missing-volume effect by considering a small inspection volume, and hence reducing the amount of material unaccounted for in the lumping process of the model. This can be done by physically collimating the detector, i.e., using a narrow collimator made of a photon-absorbing material. This, however, takes away the major advantage of using an energy-sensitive detector, i.e., the ability to inspect a large

Figure 3.1 Narrow detection energy intervals (ΔE is energy interval and $\Delta\theta$ is corresponding scatter-angle range)

portion of the object. One can, however, apply the model to one small energy-interval at a time. For each energy-interval, the field of view coincides with a small range of angles, thus defining a small area within the inspection volume, within which the model can be applied. By considering the other energy intervals as well, one can cover a wide inspection area, as shown in Figure 3.1. Using this approach, the detector response from Monte Carlo simulations for small spheres of different diameters was inverted using the measurement model, i.e., the density is obtained from the measurements. Unlike lumping the detector response over the entire energy range in the part of the energy spectrum indicating the presence of the flaw, as performed in Chapter 2, the detector response is analyzed within a small energy interval. Within the small energy interval, the detector response is inverted using the measurement model, a relative density, $\bar{\rho}$, is calculated by applying Eq. (2.12) to every energy interval. Referring to Figure 3.1, the relative density of the first energy interval

is calculated from:

$$S_{\Delta E(1)} = \sum_{ref} \ell_1 \bar{\rho}_{(1)} \exp(-\sum_{in} \bar{\rho}_{(1)} \frac{\ell_1}{2}) \frac{p_1(\mu)}{2\pi R_1^2} \exp(-\sum_{out} \bar{\rho}_{(1)} \gamma_{out(1)}) \quad (3.1)$$

where $S_{\Delta E(1)}$ is the detector response within a small energy interval, ΔE , ℓ_1 is the width of the small inspection volume corresponding to ΔE , obtained from geometry, and the rest of the parameters are as defined following Eq. (2.12). Eq. (3.1) is solved iteratively to obtain $\bar{\rho}_{(1)}$; a computer program for this iterative solution is given in Appendix III. Note that the inversion of Eq. (3.1) to obtain $\bar{\rho}$ can result in two different solutions, due to the presence of the exponential terms in the left-hand-side of the equation. However, one of the solutions is usually physically unacceptable, i.e., too high in value, as shown in Appendix III. The second interval, $S_{\Delta E(2)}$ is adjusted for the attenuation caused by interval 1. Therefore, $S_{\Delta E(2)}$ is calculated iteratively from:

$$\Delta E(2) \{ \exp(\sum_{ref} \ell_1 \bar{\rho}_{(1)}) \} = \sum_{ref} \ell_2 \bar{\rho}_{(2)} \exp(-\sum_{in} \bar{\rho}_{(2)} \frac{\ell_2}{2}) \frac{p_2(\mu)}{2\pi R_2^2} \exp(-\sum_{out} \bar{\rho}_{(2)} \gamma_{out(2)}) \quad (3.2)$$

where $\exp(\sum_{ref} \ell_1 \bar{\rho}_{(1)})$ accounts for the photon attenuation within the area corresponding to the first energy interval, and all other terms are analogous to those of Eq. (3.1). The process is repeated sequentially for the other bins. Therefore, for bin k , $\bar{\rho}_{(k)}$ is obtained from:

$$E(k) \{ \exp(\sum_{i=1}^{k-1} \sum_{ref} \bar{\rho}_{(i)} \ell_k) \} = \sum_{ref} \ell_k \bar{\rho}_{(k)} \exp(-\sum_{in} \bar{\rho}_{(k)} \frac{\ell_k}{2}) \frac{p_k(\mu)}{2\pi R_k^2} \exp(-\sum_{out} \bar{\rho}_{(k)} \gamma_{out(k)}) \quad (3.3)$$

where the exponential term in the left-hand-side accounts for photon attenuation within the preceding $k-1$ intervals. The above procedure is applied to single scatterers in the following sections.

3.3 Application to Scatterers

The above inversion procedure is applied to scatterers for different sizes. Figures 3.2 to 3.4 show the relative density calculated from Monte Carlo results for three scatterers with diameters of 4.76, 9.52 and 19.04 mm, for a beam size 2 mm wide. In the following graphs, the bar-chart represents the relative densities obtained from the small energy intervals, while the dash-line represents the relative density calculated by lumping the entire detected energy interval into one segment, and the error-bars represent uncertainties in the indication corresponding to the uncertainties in the MCNP results.

For the smallest scatter size, Figure 3.2, where the missing volume effect is smallest, using narrow energy intervals does not improve the model's predictions. However, applying the model over small energy intervals shows where the missing-volume effect is most prominent, in this case at the edges of the indication spectrum. This in turn indicates that the inspected object is not uniform in geometry, rather it has reduced density at the edges. In other words, the narrow-energy interval indication provides some geometrical shape indication. The situation is similar for the medium size scatterer, Figure 3.3.

For the largest scatterer size, Figure 3.4, the application of the model over narrow

Figure 3.2 Relative densities obtained from a 1 keV energy-detection intervals (bar chart) and the entire energies range (dashed line) for a scatterer 4.76 mm with a beam width of 2 mm.

Figure 3.3 Relative densities obtained from a 1 keV energy-detection intervals (bar chart) and the entire energies range (dashed line) for a scatterer 9.52 mm with a beam width of 2 mm.

Figure 3.4 Relative densities obtained from a 1 keV energy-detection intervals (bar chart) and the entire energies range (dashed line) for a scatterer 19.04 mm with a beam width of 2 mm.

energies is clearly an improvement over the wide energy model. The estimated relative density is closer to the actual density of unity at all intervals except at the edge closer to the beam (higher-energy range). The latter effect is due to the missing-volume carried by the spherical shape of the scatterer. Note, however, that at the edge far away from the beam, no missing-volume effect was observed. This is due to the inclination of the detector with respect to the object, as can be seen in Figure 3.1. Note that the far-edge effect was noticeable for the small scatterer sizes due to the incident beam (2 mm wide) covering relatively a large volume of the scatterer, thus resulting in a large missing-volume effect.

In the above section, a 1 keV energy-detection interval was arbitrarily chosen. The width of the energy intervals determines the volume upon which the measurement model is lumped. The smaller the energy interval, the better is the spatial resolution of the obtained

indication. However, with a narrow energy interval, a smaller number of counts will be registered, hence resulting in a poorer statistical uncertainty. For practical reasons, it is desirable to increase the energy interval to obtain better statistics with a reduced source strength (and subsequent shielding) and/or counting time. The effect of increasing the width of the energy interval was examined for one scatterer's size (4.76 mm) for demonstration purposes. Figures 3.5 to 3.7 show the result of increasing the size of the energy intervals to 2 keV. It can be seen that with 2 keV energy intervals, a reasonable indication can still be observed. However, for large energy interval, the relative density at the edges of the scatterer is underestimated. Therefore, at least for the considered source and detector energies, an interval as large as 2 keV can be used. It is wise, however, to set a narrow energy window at the edges to reduce the missing-volume effect.

Figure 3.5 Relative densities obtained from a 2 keV energy-detection intervals (bar chart) and the entire energies range (dashed line) for a scatterer 4.76 mm with a beam width of 2 mm.

Figure 3.6 Relative densities obtained from a 4 keV energy-detection intervals (bar chart) and the entire energies range (dashed line) for a scatterer 4.76 mm with a beam width of 2 mm.

Figure 3.7 Relative densities obtained from a 8 keV energy-detection intervals (bar chart) and the entire energies range (dashed line) for a scatterer 4.76 mm with a beam width of 2 mm.

3.4 Application to Single Flaws

Using narrow-energy intervals revealed that, for single scatterers, the inversion process produced results that are closer to the actual relative densities of the material. The same procedure is used in this section for single cylindrical flaws. From the MCNP simulations, indications of zero scattering within the indication spectrum indicates the presence of a void (zero density), as shown in Figure 3.8 for a flaw diameter of 9.52 mm. However, in order to further test the inversion process, the density of the absent material was

Figure 3.8 Indication spectrum for a single flaw of size 9.52 mm with a beam size of 2 mm, indicating a void in the energy interval 318 keV to 340 keV.

obtained. For this purpose, the detector response of the target block was subtracted from the detector response of the reference block. The net response within the indicated energy spectrum, $S_{\Delta E}$, is inverted using Eq. (3.3) to obtain the density of the absent material.

Three target blocks with flaws of diameters 4.76 mm, 9.52 mm and 19.04 mm were simulated. Figures 3.9 to 3.11 show the relative densities of the missing material within the flaw region. From the Figures, it can be clearly seen that the narrow-energy-interval inversion process provide a relative-density of the absent material closer to that of the reference material than when lumping over the entire indication into a single energy band. This further shows that the inversion process is successfully applied.

Figure 3.9 Relative densities obtained from narrow detection intervals (bar chart) and the entire energy range (dashed line) for a flaw size 4.76 mm with a beam size of 2 mm, shown as negative value to indicate an absent material.

Figure 3.10 Relative densities obtained from narrow detection intervals (bar chart) and the entire energy range (dashed line) for a flaw size 9.52 mm with a beam size of 2 mm, shown as negative value to indicate an absent material.

Figure 3.11 Relative densities obtained from narrow detection intervals (bar chart) and the entire energy range (dashed line) for a flaw size 19.04 mm with a beam size of 2 mm, shown as negative value to indicate an absent material.

3.5 Application to Multiple Flaws

The inversion process for a single scatterer and a single flaw was tested for three extreme sizes (test sizes of 4.76 mm, 9.52 mm and 19.04 mm). However, all subsequent inversion processes are conducted only for a medium-size flaw or inclusion (9.52 mm) for demonstrative purposes only. In this section, the inversion process is applied to multiple flaws.

Two flaws (spherical flaws of 9.52 mm diameter in size) along the path of the incident beam within a target block were used to test the inversion process. The target spectrum is differenced with the reference spectrum to indicate the location of the energy bins which correspond to the flaw regions. As before, the same procedure is applied to the first flaw facing the beam. However, to account for the lack of photon-attenuation through the first void region, correction for the second and subsequent flaws is needed. Therefore, the indication for the second flaw was adjusted as follows,

$$\begin{aligned}
 & [\exp(\sum_{ref} d_1)] (S_{\Delta E \chi(k)}) \{ \exp(\sum_{i=1}^{k-1} \sum_{ref} \ell_{2(k)} \bar{\rho}_{2(k)}) \} \\
 & = \sum_{ref} \ell_{2(k)} \bar{\rho}_{2(k)} \exp(-\sum_{in} \bar{\rho}_{2(k)} \frac{\ell_{2(k)}}{2}) \frac{p_{2(k)}(\mu)}{2 \pi R_{2(k)}^2} \exp(-\sum_{out} \bar{\rho}_{2(k)} \gamma_{out 2(k)})
 \end{aligned} \tag{3.4}$$

where the term inside the square brackets in the left-hand-side of the equation compensates for the photon attenuation through the first flaw region with d_1 being the predicted width of the first flaw. This modified procedure also needs to be applied as well for subsequent flaws, if they exist. However, only results for a second flaw are demonstrated. Figure 3.12 shows

Figure 3.12 Relative densities obtained from narrow detection intervals (bar chart) and the entire energy range (dashed line) for multiple flaw of size 9.52mm with a beam size of 2 mm, shown as negative value to indicate an absent material.

the relative densities obtained for two subsequent flaws along the path of the incident beam.

The relative densities for the two flaws are close to the actual relative density of unity except at the edges of the flaws. This is due to the spherical shape of the flaw. Although the relative density of the absent material in the two flaws was correctly predicted, the first flaw appears to be wider than the second. This, however, is caused by the conversion between energy and angle according to Eq. (1.1). Note that the detector sees the first flaw at an angle of 103° and the second flaw at an angle of 90° , which according to Eq.(1.1) gives a large angular interval for the first flaw.

3.6 Application to Inclusions

The inversion process is applied in this section to a single high-density inclusion, multiple high-density inclusions and multiple low-density inclusions. Unlike imbedded flaws, inclusions provide a non-zero indication, enabling direct inversion of the measurement without referring to the reference indication spectrum. The procedure is first applied to a single high-density inclusion.

3.6.1 Single High-Density Inclusion

The presence of a high-density inclusion is recognized in the indication spectrum by an increase in the count rate within a particular energy range, as shown in Figure 3.13

Figure 3.13 Indication Spectrum for a single high-density inclusion of size 9.52 mm with a beam size of 2 mm, within energy-interval of 318 keV and 340 keV.

between energies 318 keV and 340 keV for a spherical iron inclusion of 9.52 mm in diameter. The inversion process is applied in this energy range to determine the relative density of the inclusion. Photons are attenuated before reaching the inclusion and after scattering off the inclusion by the surrounding material as shown in Figure 3.14. It is,

Figure 3.14 Distance for the attenuation coefficients used in the inversion process for single inclusion of size 9.52 mm and a beam size of 2 mm.

therefore, necessary to compensate for this attenuation process. This is achieved by adjusting $S_{\Delta E}$ of Eq. (3.3) for a single scatterer by the factors, e_i and e_j leading to:

$$\begin{aligned}
 & [e_i \ e_j] S_{\Delta E(k)} \{ \exp(\sum_{i=1}^{k-1} \Sigma_{ref} \ell_k \bar{\rho}_{(k)}) \} \\
 & = \Sigma_{ref} \ell_k \bar{\rho}_{(k)} \exp(-\Sigma_{in} \bar{\rho}_{(k)} \frac{\ell_k}{2}) \frac{p_k(\mu)}{2 \pi R_k^2} \exp(-\Sigma_{out} \bar{\rho}_{(k)} y_{out(k)})
 \end{aligned} \tag{3.5}$$

where $e_i = \exp(\Sigma_{in} \rho_{ref} x_i)$ and $e_j = \exp(\Sigma_{in} \rho_{ref} x_j)$, compensate respectively for photon

where $e_1 = \exp(\sum_{in} \rho_{ref} x_1)$ and $e_3 = \exp(\sum_{in} \rho_{ref} x_3)$, compensate respectively for photon attenuation before and after leaving the inclusion zone.

Eq. (3.15) is inverted for a single iron inclusion of size 9.52 mm with a density of 7900 kg/m³ which is equivalent to $\bar{\rho} = 2.92$, relative to the density of the aluminum reference-material (2700 kg/m³). The results of the inversion process are shown in Figure 3.15 for the relative densities within the inclusion region. From the Figure, it can be seen that the attenuation-adjusted inversion process provides a relative density of the inclusion material that is not very different from the actual relative density of 2.92. However, the relative densities are overestimated at higher energies and are underestimated at the subsequent lower energies. In order to investigate the cause for this discrepancy, the

Figure 3.15 Relative densities obtained from 1 keV detection intervals (bar chart) and the entire energy range (dashed line) for single inclusion of size 9.52 mm with a beam size of 2 mm.

Figure 3.16 Relative densities obtained from 1 keV detection intervals (bar chart) and the entire energy range (dashed line) for reference block within the indications from an iron inclusion of size 9.52 mm with a beam size of 2 mm.

for this process which indicates that the model is successful in predicting $\bar{\rho}$ for aluminum, which is equal to unity. The discrepancy in the case of the iron inclusion is therefore attributed to other causes than the application of the measurement model. This can be due to several factors that the measurement model did not take into account; namely, the photoelectric-effect, the effect of bounded-electrons and multiple scattering. The latter factor is investigated in Section 3.7, the other two factors are considered below.

The measurement model assumes the dominance of Compton scattering, ignoring absorption by the photoelectric-effect (photon absorption by electrons). This effect was neglected because it is not as prominent as Compton scattering in the energy range

considered and most materials (Evans, 1955). However, for a material with high density and a high atomic-number (Z) (e.g., iron), the effect of photoelectric-absorption increases. By neglecting photoelectric-absorption, the model overestimates the detector response. Consequently, when the detector response is inverted, $\bar{\rho}$ is overestimated. This effect is seen in the first few energy intervals of Figure 3.15. Since $\bar{\rho}$ in the first few energy intervals is overestimated, photon attenuation (before-scattering attenuation) is overestimated for the subsequent energy intervals. This, in turn, underestimates $\bar{\rho}$ for the subsequent energies. However, at the low energies, y_{out} (the distance from the scattering point to the exit surface of the inclusion) is shorter than at the center of the inclusion, which reduces the effect of neglecting the photoelectric-effect. This is seen by the upward trend at the low energies shown in Figure 3.15. In order to accommodate the photoelectric-effect, the measurement model is adjusted by adding the photoelectric cross section to the Compton cross- section in the absorption (exponential) terms as follows:

$$\begin{aligned}
 [e_1 \ e_3] S_{\Delta E(k)} \{ \exp(\sum_{i=1}^{k-1} \Sigma_{ref} \ell_k \bar{\rho}_{(k)}) \} \\
 = \Sigma_{ref} \ell_k \bar{\rho}_{(k)} \exp [- (\Sigma_{in} \bar{\rho}_{(k)} + \Sigma_{in(pe)}) \frac{\ell_k}{2}] \frac{p_k(\mu)}{2 \pi R_k^2} \exp [- (\Sigma_{out} \bar{\rho}_{(k)} + \Sigma_{out(pe)}) y_{out(k)}]
 \end{aligned} \tag{3.6}$$

where $\Sigma_{in(pe)}$ and $\Sigma_{out(pe)}$ are the photoelectric-absorption cross section at the incident photon and scattered energies respectively. The values of these cross section are estimated by looking them up from a cross-section library, such as XCOM (Berger and Hubbell, 1968), for an element of density comparable to the previously estimated $\bar{\rho}$. The model is then inverted to obtain a new estimate for $\bar{\rho}$. When this was done for the problem at hand, no

significant change in the results was observed. One, then, has to consider the effect of the other parameters ignored in the model rather than the photoelectric-effect. This is due to the fact that although Σ_{pe} for iron (0.0069 cm^{-1} at 662 keV) is larger than that for aluminum (0.00018 cm^{-1} at 662 keV), it is still much smaller than the Compton cross-section for iron (0.56564 cm^{-1} at 662 keV). This small increase in attenuation is further suppressed by the exponential nature of the attenuation process.

The essence of Compton scattering is the interaction of photons with free electrons, i.e., the binding energy of the atomic electrons is neglected. However this is not true for materials with high Z numbers, where the binding energy of the inner electron is high. For example, the binding energy for the inner most electron for aluminum is 5.86 eV and 7.5 eV for iron (Krane, 1983). Therefore, using the Compton cross-section for aluminum multiplied by the relative density $\bar{\rho}$ to approximate the cross section for a higher density material is not quite correct. For example, using XCOM to obtain data for cross sections at the incident energy of 662 keV, the cross section (Σ_{ref}) for aluminum is 0.2011 photons/cm, $\Sigma_{ref}\bar{\rho}$ for the higher density material = 0.587212 photons/cm while for iron $\Sigma_{Fe} = 0.57275$ photons/cm. One can see that the Σ_{Fe} for iron is less than the value used in the model, $\Sigma_{ref}\bar{\rho}$. To overcome this hurdle, the Compton cross section for an element with density closest to $\bar{\rho}\rho_{ref}$ is corrected as follows:

$$\begin{aligned}
 & [e_1 \ e_3] S_{\Delta E(k)} \left\{ \exp\left(\sum_{i=1}^{k-1} \Sigma_{ref} \ell_k \bar{\rho}_{(k)}\right) \right\} \\
 & = \Sigma_{in} \ell_k \bar{\rho}_{(k)} \exp\left(-\Sigma_{in(mat)} \frac{\ell_k}{2}\right) \frac{p_k(\mu)}{2\pi R_k^2} \exp\left(-\Sigma_{out(mat)} y_{out(k)}\right)
 \end{aligned} \tag{3.7}$$

where $\Sigma_{in(mat)}$ and $\Sigma_{out(mat)}$ are the cross sections at the incident and scattered energies respectively obtained from tables for a material close in density to $\bar{\rho}_{ref}$.

For the problem at hand $\bar{\rho}_{ref} = 7.884$, which is close to that of iron. The cross-section for iron was incorporated in Eq. (3.7) and a new $\bar{\rho}$ value was obtained and shown in Figure 3.17. The Figure shows some improvement, in comparison to the uncorrected results of Figure 3.15. This procedure should be repeated a few times until a convergence is obtained. However, even with a single iteration, improvement is observed. Therefore, the new procedure is used for multiple inclusions, as discussed in the section below.

Figure 3.17 Re-inverted relative densities obtained from 1 keV detection intervals (bar chart) and the entire energy range (dashed line) for a single inclusion of size 9.52 mm with a beam size of 2 mm.

3.6.2 Multiple High-Density Inclusions

The inversion process is applied in this section to multiple high-density inclusions. Multiple inclusions (spherical flaws of 9.52 mm diameter in size) within a target block were used to test the inversion process. The inversion procedure is applied to the first inclusion and the relative density for each energy interval is obtained. If the obtained $\bar{\rho}$ is high, it indicates that the density of the first inclusion is high and the model is adjusted for the bounded-electron effect using Eq. (3.7), as shown in the previous section. For the second inclusion, photons are attenuated before reaching the inclusion and after scattering off the inclusion by the surrounding material as shown in Figure 3.18. It is, therefore, necessary to

Figure 3.18 Distance for the attenuation coefficients used in the inversion process for multiple inclusions of size 9.52 mm and a beam size of 2 mm.

compensate for this attenuation process by adjusting Eq. (3.5) for a single inclusion by the factors e_1 , e_2 , e_4 and e_5 , which compensate respectively for photon attenuation along the distance x_1, x_2, x_3, x_4 and x_5 , as shown in Figure 3.18, leading to:

$$\begin{aligned} & [e_1 \ e_2 \ e_4 \ e_5] S_{\Delta E(k)} \{ \exp(\sum_{i=1}^{k-1} \Sigma_{ref} \ell_k \bar{\rho}_{(k)}) \} \\ & = \Sigma_{ref} \ell_k \bar{\rho}_{(k)} \exp(-\Sigma_{in} \bar{\rho}_{(k)} \frac{\ell_k}{2}) \frac{p_k(\mu)}{2 \pi R_k^2} \exp(-\Sigma_{out} \bar{\rho}_{(k)} y_{out(k)}) \end{aligned} \quad (3.8)$$

$e_1 = \exp(\Sigma_{in} \rho_{ref} x_1)$, $e_2 = \exp(\Sigma_{in} \rho_{ref} x_2)$, $e_4 = \exp(\Sigma_{in} \rho_{ref} x_4)$ and $e_5 = \exp(\Sigma_{in} \bar{\rho} x_5)$ and the “x” variables are designated in Figure 3.18. However, if $\bar{\rho}$ obtained for the second inclusion is high, the model is adjusted for the bound-electron effect as follows:

$$\begin{aligned} & [e_1 \ e_2 \ e_4 \ e_5] S_{\Delta E(k)} \{ \exp(\sum_{i=1}^{k-1} \Sigma_{ref} \ell_k \bar{\rho}_{(k)}) \} \\ & = \Sigma_{ref} \ell_k \bar{\rho}_{(k)} \exp(-\Sigma_{in(mat)} \bar{\rho}_{(k)} \frac{\ell_k}{2}) \frac{p_k(\mu)}{2 \pi R_k^2} \exp(-\Sigma_{out(mat)} \bar{\rho}_{(k)} y_{out(k)}) \end{aligned} \quad (3.9)$$

The above equation is inverted for detector response for two inclusions and the results are shown in Figure 3.19. The Figure shows that the inversion process, adjusted for the bounded-electron effect, is reasonably successful in determining the density and size of the two inclusions. It should be noted that the second inclusion appears to be wider than the first. This is due to the conversion between energy and angle according to Eq. (1.1), which affects the projection of width, i.e., angular sections into energy range.

Figure 3.19 Relative densities obtained from 1 keV detection intervals (bar chart) and the entire energy range (dashed line) for multiple inclusion of size 9.52 mm with a beam size of 2 mm.

3.6.3 Multiple Low-Density Inclusions

The presence of a low-density inclusion is recognized in the indication spectrum by a decrease in the count rate within a particular energy range, as shown in Figure 3.20. The low-density inclusions (spherical inclusion of diameter 9.52 mm in size) are simulated in this test as having a density equivalent to half that of aluminum (i.e., $\bar{\rho} = 0.5$). As before, the relative density for the first inclusion is obtained by inverting Eq. (3.5) for a single inclusion. Since the obtained $\bar{\rho}$ value is low, as shown in Figure 3.20, no further adjustment for the bounded-electrons effect is needed. The relative density for the subsequent inclusion is

Figure 3.20 Indication Spectrum for multiple low-density inclusions of size 9.52 mm with a beam size of 2 mm, at energy-intervals of 281 keV to 297 keV and 318 keV to 340 keV.

obtained by inverting Eq. (3.7).

As Figure 3.21 shows, the relative density of the low-density inclusion in the two regions is correctly predicted. It should be noted again that although the two inclusions should have the same size, the conversion of energy and angle according Eq. (1.1) gives a larger angular interval for the second inclusion than the first. At the edges of the inclusions, there is an overestimation of the relative densities which indicates that at the edges there is a combination of the original reference density and the inclusion density as a result of the circular shape of the inclusion. Therefore, such high-indication at the edges indicates that the inclusion is not uniform in shape.

Figure 3.21 Relative densities obtained from 1 keV detection intervals (bar chart) and the entire energy range (dashed line) for multiple low-density inclusions of size 9.52 mm with a beam size of 2 mm.

3.7 Effect of Multiple Scattering

The effect of multiple scattering is addressed in this section to investigate how it affects the inversion process. Several simulations using the aluminum reference block were conducted, since multiple scattering is more likely to occur within the entire block than in the usually small flaw region. Simulations restricted to once-scattered, twice-scattered and thrice-scattered were conducted and compared to the unrestricted case (all collisions

Figure 3.22 Indication spectrums for a reference block with once-scattered, twice-scattered and multiple scattered photons using a beam of 2 mm in size, within an energy-interval of 245 keV and 352 keV.

allowed). The indication spectra obtained from the simulations are shown in Figure 3.22. The

indications for once-scattered photons, Eq. (1.1), are used to determine the edges of the block by converting the energy group into corresponding angular sectors. All indications outside the energy group of the block (245 keV to 352 keV) are mainly caused by multiple scattering and since the energy group of interest is outside the detector monitored range, it does not play any significant role in the inversion process. Within the energy range corresponding to the aluminum block, it is noted that the count rate (indication) increases as the number of allowed scatters increases. From the Figure, it is seen that twice-scattered photons increases the count rate significantly from those of once-scattered photons within the energy group of

beyond that of twice-scattered is not as significant.

In order to explain how double scattering occurs within the energy-range of once-scattered photons corresponding to the energy range of the reference block, let us consider the two possible scenarios shown in Figure 3.23. In the first case, a photon forward scatters by angle θ_1 , then forward scatter again by angle θ_2 to reach the detector. For this to occur, θ_1 must be a reasonably small angle, otherwise a large energy loss will occur and subsequently the second collision will put the photon beyond the energy range of interest. Since θ_1 is allowed to be small, then the energy according to Eq. (1.1), will be also small and the second-scattered may lie within the once-scattered energy range of the detector. In the second case, a photon back scatters by angle θ_3 as shown in Figure 3.23. This will result, according to Eq. (1.1), in a large energy loss which will put the photon out of the energy range of interest as will subsequent collisions. Therefore, the most likely twice-scattered

Figure 3.23 Schematic showing double scattering caused by two possible scenarios.

event that can contribute to the detector's response within the energy range of interest is due to a small-angle forward-scatter at the first scatter, followed by a second scatter within an angle that can reach the detector. Such an event has to occur upstream of the detector location with respect to the incident beam. Fortunately, however, one can protect the detector against such an event by blocking it with a high-density material such as lead so that these twice-scattered photons will not hit the detector, and consequently be absorbed before reaching the detector, as shown in Figure 3.24. This was done in the experiments of Jama (1997), discussed in the next chapter.

Figure 3.24 Schematic showing detector shielding to protect against twice-scattered photons

3.8 Conclusions

In this chapter, an inversion process is presented to determine the width and density of a discontinuity in an object using the Compton scattering indication. The inversion process is used to invert the detector response within the energy range of interest to obtain a relative density ($\bar{\rho}$) of the discontinuity. This is achieved by applying the measurement model to one small energy-interval at a time. Within the small-energy interval, the detector response is inverted and solved iteratively to obtain $\bar{\rho}$. A computer program is used to obtain $\bar{\rho}$ for this iterative process. Indications for subsequent energy interval are adjusted for attenuation caused by the previous energy bins and the corrected indications are inverted as before. Moreover, it is necessary to compensate the detector response for photon attenuation before reaching and after scattering off a discontinuity by the surrounding material as shown by Eq. (3.5). For multiple discontinuities, the procedure for adjusting the detector response is similar for the first discontinuity. However, adjustments for photon attenuation before (including attenuation through the first discontinuity) and after scattering off the discontinuity is necessary as shown by Eq. (3.3). With this process, direct inversion of the measurements to obtain a relative density can be achieved without referring to a reference indication spectrum.

The inversion process was applied to single scatterers of small spheres of different diameters (4.76 mm, 9.52 mm and 19.04 mm). The calculated relative densities from the inversion process over small energy-intervals showed improvement, in comparison to

applying the model over the wide-energy model as described in Chapter 2. For the purpose of demonstration, all subsequent inversion processes were conducted for a medium-size discontinuity (9.52 mm). The inversion was then applied to a single flaw, multiple flaws, as well as single and multiple inclusions. In the case where a high $\bar{\rho}$ for the discontinuity was obtained, photon attenuation compensation for the bounded-electron effect was shown to be necessary. The inversion process needs to be first applied to the discontinuity using Eq. (3.5) to obtain an initial $\bar{\rho}$. Thereafter the Compton cross section for an element with density closest to $\bar{\rho}\rho_{\text{ref}}$ is obtained from tables. Using Eq. (3.7), which incorporates the bounded-electron effect, a new $\bar{\rho}$ is calculated. The procedure must be repeated a few times until convergence is obtained.

The effect of multiple scattering was reexamined to investigate its influence on the inversion process. Simulations using an aluminum reference block were conducted for once-scattered, twice-scattered, thrice-scattered and unrestricted cases (all collisions allowed). Within the energy range of the block, it was found that twice-scattered photons are the most dominant form of multiple scattering. Double scattering within the energy-range of once-scattered is mainly caused by the forward-forward scattering of photons upstream from the incident beam source. Double scattering can be minimized by shielding the detector with a high-density material capable of absorbing photons.

Chapter 4

Inversion of Experiment Data

4.1 Introduction

In NDT, the first step is to determine the presence of an anomaly which is achieved in Compton scattering by the presence of a peak or a dip in the energy spectrum of scattered photons. The second step in NDT is to determine the location of the anomaly. This is easily obtained by using the Compton scattering relationship of Eq. (1.1), with the scattering energy corresponding to the peak/dip energies. Thirdly, one needs to determine the size of the anomaly which is related to the width of the peak/dip. Finally, it is desirable to know the

density of the material occupying the anomaly which can be determined by the height of the peak/dip. In the previous chapter, an inversion process was presented to determine the width and density of a discontinuity in an object from Compton scatter indications. In this chapter, the process is applied to experimental results. In order to accommodate source parameters (strength and collimation) and detector parameters (efficiency, area and collimation), one must introduce a normalization constant to the experimental results. In addition, since background radiation (radiation scattered from the surrounding shielding and collimators) always exists, a net signal must be obtained by subtracting the measurements obtained from those of the background. The net count rate obtained for small scatterers are used to determine the normalization parameters. The ratio between the indication spectrum for a scatterer and that obtained using the measurement model of Chapter 3, determines the normalization constant, which incorporates the source and detector parameters.

This Chapter shows how the above procedure can be applied to the experimental results obtained by Jama (1997), who used a collimated ^{137}Cs source and a high purity germanium detector. In the next section, the use of single scatters to determine the normalization parameter is explained. Section 4.3 introduces a method for determining the width of the anomaly from the indication spectrum. The method introduced in these two sections are then applied to the indication spectra obtained from scatterers, flaws and inclusions, in Sections 4.4 and 4.5, respectively. Based on the results obtained in this Chapter, recommendations are provided for future experiments to facilitate the quantification process of Compton scatter NDT.

4.2 Normalization of Experimental Indications

The indication obtained from a single scatter facilitates the determination of the experimental source and detector parameters that were not incorporated in the measurement model developed in Chapter 2, which assumed a unit source area, unit detection area and 100 % detection efficiency. This is due to the fact that a small scatterer resembles a scattering point, thus eliminating test-section related parameters, such as self-attenuation and missing volume effects, and provides a link directly between the source and the detector effects. However, a very small scatterer (in size) provides a very small scattering count rate (a weak indication that may be difficult to distinguish from background), making the normalization process difficult and unreliable. A large size scatterer, on the other hand, gives a stronger indication but over a wider angle and energy interval, hence introducing the self-attenuation and missing volume effects discussed earlier in this work. One, therefore, has to find a scatterer with a reasonable size (volume) that is sufficiently small to be considered as a point, but large enough to provide a reliable indication. Jama (1997) recorded the indication spectra from four aluminum scatter rods of diameter 1.58 mm, 3.23 mm, 6.45 mm and 9.47 mm.

A normalization parameter is calculated from the net (over one energy interval) bulk indication of each of those scatterers. The count rate for the experimental indication is taken as the integral of the signal within an energy interval defined by the full-width-half-maximum (FWHM) of the peak as demonstrated in Appendix III. This is the process conventionally used in dealing with peaks. (Knoll, 1989).

The normalization factor, K , for a scatterer is defined from the relationship:

$$C = K S_m \quad (4.1)$$

where C is the net count rate and S_m is the scatterer's response from MCNP simulations. As shown in Section 2.2, the measurement model matches well with MCNP results. This simple normalization factor can then be used to adjust the measured indication spectrum before applying the inversion process described in Section 3.3. It should also be noted that the factor, K , is based on the count rate, i.e., the number of counts per unit time. However, the choice of unit time is arbitrary and K remains the same, for a fixed counting period. The factor K is obtained from three different scatter sizes, as described below.

Using relationship (4.1), the normalization factors were calculated for the three scatterers used in Jama's (1997) work and shown in Table 4.1. It should be noted that the smallest scatterer was not used in obtaining the normalization factor due to the weakness of the indication. (relative high statistical uncertainty)

Table 4.1 The ratio between the count rate and MCNP results for three scatterers

Scatterer Diameter (mm)	$K = C/S_{mc}$	
	Single Scatter ($\times 10^6$)	Multiple Scatter ($\times 10^6$)
3.23	5.007	5.000
6.45	9.648	9.602
9.47	13.539	13.493
Average K	7.049	7.023

As the Table shows, multiple scattering has a very small effect on the value of K . However, K decreases with the scatterer's diameter. Factors causing this unexpected trend were examined and were attributed, as explained below, to the use of a well-collimated beam in Monte Carlo simulations, while experimentally the source photons diverged with distance. A collimated beam was used in the simulation for efficiency of calculations, since it is much simpler to simulate such a source than to simulate a diverging source. Figure 4.1 shows a schematic of both the experimental setup and the Monte Carlo setup. In the experimental setup, a small size source, Figure 4.1(a), which can be represented by a point, is placed inside

Figure 4.1 Schematic showing the geometry of the experimental and Monte Carlo setups

a lead shielding (200 mm x 200 x 100 mm). A small slit cylinder (2 mm width and 10 mm in height) was milled within the lead shield to function as a collimator. The test object was centered around a point 280 mm away from the edge of the collimator. From the MCNP simulations, Figure 4.1(b), a collimated beam with a width of 2 mm was used, for efficient tracking of photons. Therefore, in the MCNP simulations all scatterers are exposed to the same number of source photons (normalized to unity 1). However, in the experiment, a larger scatterer is exposed to more source photons than a smaller one, as it occupies a larger area within the view field of the diverging source. This means, in effect, that not all source photons exposure area in the experiment is utilized by the object, leading to a photon occupancy factor, defined as

$$\text{Photon Occupancy Factor} = \frac{38}{d_i(\text{mm})} \quad (4.2)$$

which represents the rate of area (per unit length) present by the source that occupies by the scatterer, where 38 mm is the width of the diverging beam at the center of the object.

With beam divergence, one also must take into account the fact that the intensity of radiation decreases with the inverse square of the distance. Therefore, the incident fluence over a larger scatterer should be higher than that for a smaller scatterer centered at the same point, due to the proximity of the latter to the source. When the K factor of Eq.(4.1) is adjusted by the photon occupancy factor of Eq. (4.2) and the reduction in intensity due to divergence, one obtains:

$$k_{eff} = K \frac{38}{d_i(mm)} \frac{D_o^2}{D_i^2} \quad (4.3)$$

where D_o and D_i as defined in Figure 4.1 (a). The values of the adjusted normalization factors are shown in Table 4.2.

Table 4.2 Normalization factor for three scatterers

Scatterer Diameter (mm)	k_{eff} ($\times 10^8$)
3.23	5.933
6.45	5.754
9.47	5.552
Ave k_{eff}	5.746

Table 4.2 still shows a slight difference between the normalization factor for different scatterer sizes. This slight difference may be attributed to the difference in the average track length between MCNP simulations and experiments due to the type of beams used, as schematically shown in Figure 4.1, indicating a larger mean track length of photons within the same object for the source used in the simulations. This deviation increases with the size of the scatterers causes more photon interaction in the MCNP setup than in the experiment, which explains the slight decrease in the normalization factor with scatterer diameter observed in Table 4.2.

4.3 Size Determination

The size of an anomaly affects the width of the peak/dip in the energy spectrum of the indication; the larger the flaw, the wider is the dip in the spectrum. However, the width of the peak/dip in the spectrum is also affected by the incomplete energy deposition of photons in the detector (i.e. photons escaping the detector before being fully absorbed within its volume). For the inversion process described in Section 3.3, the relative density determined for a certain energy interval affects the values obtained in the subsequent intervals, it is important to properly determine the width of the anomaly (factor ℓ in Eq. (2.12)). When dealing with peaks, the concept of full-width-at-half-maximum (FWHM) is usually used in radiation spectroscopy. The FWHM value corresponds to about two standard deviations around the most probable value in the Gaussian distribution (see Appendix 4), and therefore covers most of the relevant counts of a peak indication.

The obvious approach is to use the FWHM to mark the energy range under the peaks which defines two energies called here E_1 and E_2 . From Eq. (1.1), the corresponding angles of scatter to E_1 and E_2 , called here θ_1 and θ_2 , can be calculated. As schematically shown in Figure 4.2, the angles θ_1 and θ_2 , determine the width of the anomaly, i.e., ℓ . For the 90° scattering setup used in Jama's (1997) work, one can determine ℓ as:

$$\ell = R \left(\frac{1}{\tan(\pi - \theta_1)} + \frac{1}{\tan \theta_2} \right) = R \left(\frac{1}{\tan \theta_2} - \frac{1}{\tan \theta_1} \right) \quad (4.4)$$

Figure 4.2 Schematic showing the width of anomaly bounded by angles θ_1 and θ_2

where R is the distance normal to the indicated beam from the center of the anomaly to the detector. For small flaws, θ_1 and θ_2 will be very close to 90° , the nominal scattering angle, which makes Eq. (4.4) very susceptible to errors in determining θ_1 and θ_2 . Therefore, Eq. (4.4) is obviously not suitable for small anomalies.

An alternative approach suitable for small flaws is to consider angle ϕ_1 and ϕ_2 in Figure 4.2. The angles represent the spread in the peak caused by the anomaly and are, therefore, small for small anomalies. Consequently,

$$\begin{aligned} \ell &= R (\tan \phi_1 + \tan \phi_2) \\ &\approx R (\phi_1 + \phi_2) \end{aligned} \quad (4.5)$$

since $\theta_1 - \phi_1 = \pi/2$ and $\theta_2 + \phi_2 = \pi/2$. Then $|\frac{\partial\theta}{\partial E}| = |\frac{\partial\phi}{\partial E}|$ where the subscripts 1 and 2 are dropped for convenience. From Eq. (1.1),

$$|\frac{\partial E}{\partial\theta}| = \frac{\frac{E_o^2}{m_o c^2} \sin \theta}{[1 + \frac{E_o}{m_o c^2} (1 - \cos \theta)]^2} \quad (4.6)$$

At $\theta = \pi/2$,

$$|\frac{\partial E}{\partial\theta}| = |\frac{\partial E}{\partial\phi}| = \frac{\frac{E_o^2}{m_o c^2}}{(1 + \frac{E_o}{m_o c^2})^2} \quad (4.7)$$

For $E_o = 662$ keV, (Cesium-137) used in Jama's (1997) work and $m_o c^2 = 511$ keV, then

$$|\frac{\partial E}{\partial\theta}| = |\frac{\partial E}{\partial\phi}| = \frac{\frac{662^2}{511}}{(1 + \frac{662}{511})^2} = \frac{662}{4.067} = \frac{\Delta E}{\Delta\phi} \quad (4.8)$$

$$\therefore |\Delta\phi| = \frac{4.067}{662} |\Delta E|$$

where $\Delta\phi$ is in radians and ΔE is in keV. For small flaws, ϕ_1 and ϕ_2 in Figure 4.2 will correspond to $\Delta\phi_1$ and $\Delta\phi_2$, respectively. Since $\phi = 0$ if $\theta = \pi/2$. Therefore, for Eq. (4.5),

$$\ell = R \left(\frac{4.067}{662} \right) (E_2 - E_1) \quad (4.9)$$

where E_1 and E_2 correspond to the upper and lower energy limits as determined from the FWHM under the peak.

In summary, one can calculate the width of the anomaly using Eq. (4.4) for large anomalies and Eq. (4.9) for small anomalies. In order to find out what represents a small anomaly and a large one, the above two approaches are applied to the indication from the scatterers discussion section. Table 4.3 shows the results obtained using the above two approaches. It is clear that both approaches tend to overestimate the size of the anomaly, with the second approach of Eq. (4.9) giving a lower degree of overestimation. One can, therefore, conclude that the approach of Eq. (4.9) is slightly more suited, given the susceptibility of the approach of Eq. (4.4) to small changes in the scattering angle. This FWHM method is demonstrated in the example shown in the ensuring section.

Table 4.3 Size determination using Approach 1 and Approach 2

Actual Width (mm)	Approach 1 Width (mm) Using Eq. (4.4)	Approach 2 Width (mm) Using Eq.(4.9)
3.23	6.11	5.83
6.45	9.25	8.75
9.47	10.70	10.20

4.4 Application to Measurements

In the above sections, a value for the normalization constant that relates the measurements to the calculations was obtained, and methods were developed for determining the size of an anomaly from the indication spectrum. With the aid of these parameters, one can now proceed to employing the inversion model of Chapter 2 to the experimental results obtained by Jama (1997). We start first with the case of scatterers, followed by flaws and inclusions in an aluminum block. The inversion process is applied only to single anomalies since the inversion process for multiple anomalies is equivalent, in effect, to that for a single anomaly once the presence of the first anomaly is accounted for, as explained in Chapter 3. The inversion process is also applied only to bulk indication, rather than to small energy intervals, for simplicity and because the latter approach applies the same procedure but in a repetitive manner.

4.4.1 Scatterers

Jama (1997) obtained the Compton spectrum for four scatterer sizes (9.47 mm, 6.45 mm, 3.23mm and 1.58 mm) and the net measured indications are shown in Figure 4.4. From the Figure, one can determine:

1. An indication for the presence and absence of an anomaly,
2. The location of the anomaly,

3. The size of the anomaly, and
4. The density for the anomaly.

Figure 4.3 Measured indication obtained by Jama (1997) for four scatterers size 1.58 mm, 3.23 mm, 6.45 mm and 9.47 mm

The above four key parameters (indication, location, size and density) are found for the measurements, plotted in Figure 4.3 as follows:

Indication

The presence of a peak indicates a material of a density higher than that of the background. Since the background here was simply due to the surroundings, the peaks indicate the presence of scatterers.

Location

From Figure 4.3, the energy corresponding to the top of the four peaks is 282.4, 281.1, 282.4 and 282.4 keV, from the largest to the smallest indication respectively. This indicates a scattering angle of 90.14° , which is very close to the experimental angle of 90° used by Jama (1997). The location of the scatterer can be determined by the scatterer angle.

Size

The size of the scatterer can be deduced from the energy of the peak and its relationship to the angle using Eq.(1.1). In this case, an energy of all peaks appear at about 282 keV. This is equivalent to 90° angle of scatter; for a ^{137}Cs (662 keV) beam source. The width of the flaws along the incident beam can be derived using the procedure outlined in Section 4.3. The results are summarized in Table 4.4, in accordance with the height of the peaks (with the Top being the highest count).

Table 4.4 Estimated width of scatterers from results of Figure 4.3

Indication	Approach 1 (mm) Eq. (4.4)	Approach 2 (mm) Eq. (4.9)	Actual Width (mm)
Top	10.70	10.20	9.47
2 nd Top	9.25	8.75	6.45
2 nd Bot	6.11	5.87	3.23
Bottom	1.53	1.46	1.58

As Table 4.4 shows, reasonable estimates for the width of the largest scatterer were obtained using both approaches. For the middle size scatterers, both approaches overestimated the size, with the 2nd Approach being closer to the actual value. For the smallest scatterer size, both approaches surprisingly gave good estimate of the scatterer size, due to the narrow spread of the indication.

Density

Using the value of the normalization constant of section 4.2, and the scatterer width value reported in Table 4.4, one can now use the inversion process of Chapter 3 to determine the density of the scatterers. The results are shown in Table 4.5 (recalled that the density is a ratio determined with respect to aluminum) and Approach 2 were used for the scatterer size (Table 4.4).

Table 4.5 Estimated relative density of scatterers for result of Figure 4.3

Indication	Estimated Relative Density	Actual Relative Density
Top	0.97	1.0
2nd Top	0.75	1.0
2nd Bot	0.51	1.0
Bottom	0.48	1.0

As Table 4.5 shows, its density was slightly underestimated because of the overestimation of the scatterer size for the largest scatterer (refer to Table 4.4). For the smallest scatterer, the

density was significantly underestimated due to errors introduced in the normalization process (i.e., the use of average normalization parameters). Although the density estimations are slightly off the actual value, they are offset to some extent by the overestimation of the flaw size.

In summary, from the experimental data for the scatterers, one can quantify both the size and density of the scatterers, with some uncertainties. However, given the many experimental uncertainties involved, i.e., statistical uncertainties, positioning effects, experimental discretization of energy into intervals, averaging of the normalization parameters, etc., the obtained results were still indicative of changes in the scatterers attributes.

4.4.2 Application to Flaws

Figure 4.4 shows the experimental indications obtained by Jama (1997) for four holes through an aluminum block ($60 \times 50 \times 50 \text{ mm}^3$), simulating voided flaws. The diameters of the holes were 1.58 mm, 3.23 mm, 6.35 mm and 9.52 mm. The experimental indications were obtained by subtracting the measured scattering spectrum for a target containing a simulated flaw from that of the flawless reference block. These scattering data were collected at a scattering angle of 77.5° . The following is a quantitative analysis of the indications obtained.

Figure 4.4 Measured indication obtained by Jama (1997) for four flaws in an aluminum block of size 1.58 mm, 3.23mm 6.45mm and 9.52 mm

Figure 4.5 Geometry used in Jama's experiments (1997) for a single flaw

Indication

The negative indications shown in Figure 4.4 signify the presence of a voided anomaly, i.e., reduced scattering in comparison to the reference block. It should be noted that, as explained by Jama (1997), the peak to the left of the dip (due to voiding), is caused by the fact that the absence of material in the flaw area results in a high photon fluence downstream of the flaw due to the lack of attenuation in comparison to the reference block. The smoothness of the dip indicates a regular shape of the anomaly since a rough flaw, such as a crack, would have resulted in jagged indications.

Location

The energy corresponding to the bottom of the four dips of Figure 4.4, is about 326.7, 325.4, 326.7 and 324.1 keV, largest to smallest indication respectively. Using Eq. (1.1), this indicates a scattering angle of about 78.3° , which is close to the experimental angle of 77.5° used by Jama (1997). This angle of scattering enables the determination of the location of the flaw.

Size

Applying Approach 1 for size determination, as explained in section 4.3, to the dips of Figure 4.5, resulted in the width estimates shown in Table 4.6. It should be noted that Approach 2 for size determination was not used as it is only suitable for a scattering angle near 90° , which is not the case here.

Table 4.6 Estimated Width of Flaws from Results of Figure 4.4

Indication	Approach 1 (mm) Eq. (4.4)	Actual Width (mm)	Appr. 1/Act. Width
Top	10.27	9.52	1.08
2nd Top	8.42	6.35	1.33
2nd Bottom	5.43	3.23	1.68
Bottom	2.44	1.58	1.54

As Table 4.6 shows, the size of the flaws is overestimated. This is likely attributed to the skewness of the dips corresponding to the flaws. This skewness is caused by the uneven attenuation of the incident within the area corresponding to the flaw in the reference block; which is used for obtaining the net indication. In the future, attention should be given to compensating for the skewness effect. Nevertheless, the predicted flaw size is still indicative of the actual flaw size.

Density

Obviously the density of a voided flaw is zero. However, the indications of Figure 4.5 do not necessarily indicate a voided flaw, since a reduced density flaw (e.g., due to corrosion) may also result in an indication similar to that of a voided flaw but not as deep. Therefore, it is useful to either determine the density of the absent material as was done in Section 3.4 for Monte Carlo results, or directly estimating the density of the material occupying the flaw region, which should be equal to zero, if the flaw is actually voided. Both approaches are used here.

In order to determine the density of the absent material correction for the attenuation before and after the anomaly must be performed, as explained in Section 3.4. After utilizing this correction process, and normalizing the count rate using the normalization factor of Section 4.2, the densities for the four anomalies of Figure 4.5 were estimated using the inversion procedures of Section 3.4. The results are given in Table 4.7.

Table 4.7 Estimated relative density of flaw for results of Figure 4.4, using FWHM approach

Indication	Estimated Relative Density of Flaw Material	Actual Relative Density of Flaw Material
Top	0.57	1
2nd Top	0.54	1
2nd Bottom	0.21	1
Bottom	0.13	1

Table 4.7 shows an underestimation of the density of the anomaly that greater than can be explained by the overestimation of the width (Table 4.6). This again is attributed to the abrupt drop to zero count rate for the flaw. This abrupt change tends to spread over a number of counting channels, corresponding to the region around the edges of the flaw due to statistical fluctuations. Therefore, a more realistic approach is to include the negative counts surrounding the dip, and out of the region covered by the FWHM region. After performing the above correction using the negative-count indication, Table 4.8 summarizes the results obtained.

Table 4.8 Estimated relative density of flaw for results of Figure 4.4, using FWHM approach and using the negative-counts indication

Indication	Estimated Relative Density of Flaw Material	Actual Relative Density of Flaw Material
Top	0.83	1
2nd Top	0.70	1
2nd Bottom	0.20	1
Bottom	0.14	1

Table 4.8 still shows some underestimation of the density. However, for the large flaws, this underestimation is less severe and can be attributed to the overestimation of the flaw size, which should lead to underestimation of density. For the smaller flaws, the density is significantly underestimated. The effect of the error in overestimating the flaw size, is that one has a large volume than would need to be filled by a less dense material to produce the same amount of scattering effect as the original material. In addition, error in the flaw size significantly affects the inversion algorithm (Section 3.4) since the flaw width appears in the exponential terms. Therefore, when the flaw size is estimated to be about twice as large as the original one, as is the case for the two smallest flaws (see Table 4.6), one would expect a severe underestimation of the density of the absent material as shown in Table 4.8.

The other approach to density determination is to directly determine the density of the material with the anomaly. This was not done for Monte Carlo simulation results, as it would have resulted in a trivial zero density due to the fact that the Monte Carlo estimation of the indication matched well with the modeled value. The relative density of the material

in the anomaly can be estimated from the following relationship:

$$\frac{S_{ref} - S_{target}}{k_{eff}} \quad (4.10)$$

$$= e_1^{-1} e_3^{-1} \sum_{ref} \ell \frac{p(\mu)}{2 \pi R^2} \cdot$$

$$[\exp(-\sum_{in} x)(1) \exp(-\sum_{out} y) - \exp(-\sum_{in} \bar{\rho} x)(\bar{\rho}) \exp(-\sum_{out} \bar{\rho} y)]$$

$$\frac{(S_{ref} - S_{target}) e_1 e_3}{k_{eff}} \quad (4.11)$$

$$= \exp(-\sum_{in} x) \sum_{ref} \ell \frac{p(\mu)}{2 \pi R^2} \cdot$$

$$\exp(-\sum_{out} y) [1 - \exp(-\sum_{in} (\bar{\rho} - 1) x) (\bar{\rho}) \exp(-\sum_{out} (\bar{\rho} - 1) y)]$$

where $e_1 = \exp(-\sum_{in} \rho_{ref} x_1)$ and $e_3 = \exp(-\sum_{in} \rho_{ref} x_3)$ and all other notations are defined in Chapter 3. The above equation can be solved iteratively for $\bar{\rho}$ leading to the results shown in Table 4.9, using the integral of negative-counts approach discussed earlier.

Table 4.9 Relative density of flaw material using Eq. (4.11)

Indication	Estimated Relative Density of Flaw Material	Actual Relative Density of Flaw Material
Top	0.103	0
2 nd Top	0.229	0
2 nd Bottom	0.25	0
Bottom	0.843	0

As Table 4.9 shows once again the density of the material within the anomaly is quite small for the two largest flaws, but is not zero due to the error in overestimating the flaw size. The error in the two smallest flaws is due to the significant overestimation of the size as explained earlier. As one would expect, the estimated density almost complements the estimated density of the absent material. This may be seen by comparing Table 4.8 to Table 4.9.

The above results show that the error in the flaw size has a strong influence on the value of the estimated density. In order to demonstrate the above, the density of the anomaly was recalculated using the correct value for the flaw size and the results are shown in Table 4.10.

Table 4.10 Relative density of flaw material using Eq. (4.11) and the actual flaw size

Indication	Estimated Relative Density of Flaw Material	Actual Relative Density of Flaw Material
Top	0.06	0
2nd Top	0.06	0
2nd Bottom	0.60	0
Bottom	0.77	0

As Table 4.10 shows, the two largest flaws resulted in values very close to the expected zero density, but for the two smallest flaws, the values are still unrealistic. One can attribute this problem to the fact that in the experiment the same count period was used for the flaws (Jama, 1997). This put the smallest flaws at a disadvantage due to the associated large

uncertainty and subsequent fluctuation in the indication. In addition, the indication for the flaw is obtained by subtracting the count rate for the reference block, which compounds the statistical uncertainty.

In summary, for flaws of the size of 6 mm or larger, the inversion process of anomaly size and density worked reasonably well. This was not the case for smaller flaws due to the weakness of the signal. This can be remedied by increasing the count period. Nevertheless, one can still detect the presences of a small flaw, determine its location and provide a conservative estimate of its size.

4.4.3 Application to Inclusions

The experimental indications obtained by Jama (1997) for four steel inclusions through an aluminum block (60 x 50 x 50 mm³), are shown in Figure 4.6, for inclusion diameters of 1.58 mm, 3.23 mm, 6.35 mm and 9.52 mm. The experimental indications were obtained by subtracting the measured scattering spectrum for the flawless reference block from that obtained with a target containing a simulated inclusion. As for the flaws, these scattering data were collected at scattering angle of 77.5° as schematically shown in Figure 4.7. A quantitative analysis of the experimental indications is given in the following pages.

Figure 4.6 Measured indication obtained by Jama (1997) for four inclusions in an aluminum block 1.58 mm, 3.23 mm, 6.45 mm and 9.52 mm

Figure 4.7 Geometry used in Jama's experiments (1997) for a single inclusion

Indication

The indications shown in Figure 4.6 imply the presence of inclusions due to the increased scattering in comparison to the reference block. As explained by Jama (1997), the dip to the left of the peak is caused by the increased attenuation downstream of the anomaly due to the higher density of the inclusion.

Location

The four peaks of Figure 4.6 correspond to 329.3, 332.0, 328.0 and 324.1 keV, which using Eq. (1.1) corresponds to a scattering angle of about 77.6° . This is very close to the experimental angle of 77.5° used by Jama (1997). This angle of scatter enables the determination of the location of the flaw.

Size

When Approach 1 for size determination (given in Section 4.3) is used, the peaks of Figure 4.6, produced the width estimates shown in Table 4.11.

Table 4.11 Estimated width of inclusions from results of Figure 4.6

Indication	Approach 1 (mm) Eq. (4.4)	Actual Width (mm)	Appr. 1/ Act. Width
Top	12.3	9.52	1.29
2nd Top	8.57	6.35	1.35
2nd Bottom	4.23	3.23	1.31
Bottom	2.45	1.58	1.55

The overestimation of the size of the inclusions, shown in Table 4.11, is again attributed to the skewness of the peaks corresponding to the inclusions. The skewness is caused by the difference in the degree of attenuation between the inclusion and reference materials. However, the estimated inclusion size is still indicative of the size of the inclusion.

Density

Since the experimental indications involved subtracting the count rate for a reference block, the inversion algorithm of Chapter 3, see Eq. (3.5), is slightly altered. Similar to Eq. (4.11), the relative density of the inclusion anomaly can be estimated from the relationship:

$$\begin{aligned} & \frac{(S_{target} - S_{ref}) e_1 e_3}{k_{eff}} \tag{4.12} \\ &= \exp(-\Sigma_{in} x) \Sigma_{ref} \ell \frac{p(\mu)}{2 \pi R^2} \cdot \\ & \exp(-\Sigma_{out} y) [\exp(-\Sigma_{in} (\bar{\rho} - 1) x) (\bar{\rho}) \exp(-\Sigma_{out} (\bar{\rho} - 1) y) - 1] \end{aligned}$$

where the notations are identical to those used in Chapter 3. Solving Eq. (4.12) iteratively for $\bar{\rho}$ leads to the results shown in Table 4.12.

Table 4.12 Relative density of inclusion material using Eq. (4.12)

Indication	Estimated Relative Density of Inclusion Material	Actual Relative Density of Inclusion Material
Top	1.90	2.9
2nd Top	1.62	2.9
2nd Bottom	1.17	2.9
Bottom	1.08	2.9

The resulting relative densities shown in Table 4.12 are lower than the actual values, mainly due to the overestimation of the flaw size, attributed to the use of the FWHM approach explained in Section 4.3. This approach is appropriate for a symmetric peak, which is not the case for the skewed peaks shown in the indications of Figure 4.6. Therefore, in the future effort should be made to find a better method for size determination, as indicated in the above discussion of flaw-size estimation. It should be noted here, unlike the case of flaws, the indication counts integrated within the energy range identified by the FWHM criterion was used. That is, there was no need to include additional counts near the tail ends of the indication, since the change in counts between the inclusion material and the reference material is not as severe as that between a voided flaw and the reference material.

In summary, the quantification process for inclusion size and density determination worked well, with the overestimation of the former compensated for by a corresponding decrease in the latter. For example, the overestimation of the largest inclusion size from the actual value of 9.52 mm to 12.3 mm, which results in an increase in the equivalent area by a factor of about 1.67. This is matched by a decrease in relative density from 2.9 to 1.90,

which have a ratio of 1.52. Therefore, the spread of the inclusion density over a larger apparent area resulted, as one would expect, in a proportional decrease in the equivalent density. Consequently, to obtain a better estimate of density, one needs to develop a more accurate method for size determination that takes into account the observed skewness of the indications.

4.5 Conclusions

The quantification process of determining the location, size and density of an anomaly, developed in Chapter 3, was applied to the experimental indications, independently obtained by Jama (1997). First, a process was applied to normalize the experimental count rates to the inversion model, using the results obtained for single scatterers. This enabled using the application to flaws and inclusions. However, the inversion model for determining the density of an anomaly was slightly modified to take into account the subtraction of the count rates for a reference object used in obtaining the experimental indications. Although the inversion process was not applied to multiple anomalies, nor within small energy bins, as was done in Chapter 3 for the Monte Carlo simulated results, there is no fundamental reason that the developed approach not be applicable. This is due to the fact that the two main aspects of normalizing the experimental count rate and taking into account the effects of background radiation (i.e., the subtraction of the count rate of a reference object) have been dealt with in this Chapter. However, since accounting for the skewness of the obtained

indications was shown to significantly affect both the estimated size of anomaly and its density, effort should be devoted to overcoming this problem, before applying the process to the more complex cases of multiple anomalies and finer energy intervals, which involve an error-propagation process.

Chapter 5

Summary and Conclusions

5.1 Summary

In this work, a method for quantifying the information obtained from the indications of the Compton scattering technique in NDT was developed. While the location of an anomaly can be directly determined from the location of its indication (dip or peak) on the measured energy spectrum, its size can be estimated from the width of the corresponding indication. In order to determine the density of the anomaly, a measurement model was formulated to provide a numerical estimate of the detector response for a given size and

density of anomaly. This model was successfully verified against Monte Carlo simulations, in Chapter 2.

An inversion process of the measurement model was developed in Chapter 3 to determine the density of a discontinuity in an object by inverting the detector response within the energy range of interest to obtain a relative density ($\bar{\rho}$) of the discontinuity. This was better achieved by applying the measurement model to one small energy-interval at a time. It was, however, necessary to compensate the detector response for photon attenuation before reaching and after scattering off a discontinuity by the surrounding material. For multiple discontinuities, the procedure for adjusting the detector response is similar for the first discontinuity, with adjustment performed to account for the effect of preceding discontinuities. The inversion process was successfully applied to single scatterers, multiple scatterers, single flaws, multiple flaws, single inclusions and multiple inclusions, using Monte Carlo simulated results, as was shown in Chapter 3. Monte Carlo simulations also enabled the examination of the effect of multiple scattering on the inversion process. For the geometries and materials considered, it was shown that, within the energy range of interest, twice-scattered photons were the most dominant cause of multiple scattering. However, double scattering occurs mainly in the forward direction and can be minimized by shielding the detector with a high-density material capable of absorbing photons.

The quantification process of determining the location, size and density of an anomaly was applied to the experimental indications, independently obtained by Jama (1997), with considerable success. A normalization process relating the experimental measurements to the measurement model was developed with the aid of single scatterers

(rods). Moreover, the inversion model for determining the density of an anomaly was slightly modified to take into account the subtraction of the count rates for a reference object used in obtaining the experimental indications.

5.2 Conclusions

For the Compton-scattering NDT system studied, which employed a collimated ^{137}Cs gamma-ray source and a high-purity germanium detector to examine voided-flaws and higher-density inclusions in a $60 \times 50 \times 50 \text{ mm}^3$ aluminum block, a measurement model based on tracking once-scattered photons was shown to adequately simulate the physical process. Although, Monte Carlo simulations showed that twice-scattered photons may affect the measurements, their influence can be minimized by careful shielding of the detector. Applying the measurement model directly to large anomalies resulted in some discrepancies due to the “lumping to a point” nature of the model. This can, however, be overcome by applying the model sequentially to small regions within the anomalies, corresponding to the fine energy intervals measured by the high energy-resolution detector used in the experiments.

In inverting the measurement model to obtain the density of an anomaly, care must be taken to correct for the attenuation effect of material or other anomalies present along the beam path of radiation. Due to the nonlinear nature of the measurement model, its inversion may result in more than one estimate of density, one of which is usually physically

unacceptable. The effect of measurement error can be taken into account in the inversion process by utilizing both the upper and lower estimates of a measurement to provide bounds for the estimated density.

In quantifying experimental indications, normalizing the results to match those of the theoretical model is important. Although a normalization process was presented in this work, it should be kept in mind that the proposed process is based on averaging the results for four different scatterers. The effect of measurement background should also be incorporated into the inverting algorithm, as shown in Chapter 4. The method based on the concept of full-width at half-maximum (FWHM) approach usually used in analyzing peaks in radiation counting was used in this work both to determine the size of an anomaly and to define the energy region along which the count rate corresponding to the anomaly should be integrated. However, due to the skewness observed in the experimental indications, (caused by the subtraction of a reference signal), the FWHM approach tended to result in an overestimation of the size of the anomaly. Upon performing the inversion process, this overestimation resulted in an underestimation in the density of the anomaly.

In spite of the above mentioned difficulties, the quantification process of the Compton scatter NDT technique was reasonably successful. While the position of anomalies can be adequately determined, its size and density can be approximately determined. With some improvement in the processes of determining the flaw size, the quantification process can be further improved. Therefore, future work should focus on replacing the FWHM method with a more adequate approach suitable for the skewed signals observed experimentally. This perhaps may be done by multiplying the indication values by the value

of the reciprocal of the attenuation factor, after obtaining a first-estimate of the density, to rectify the skewness. Application of peak fitting and background stripping technique used in radiation spectroscopy may also prove to be useful. When performing laboratory experiments for use in the quantification process, care should be taken to obtaining results with good counting statistics, within fine energy intervals.

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Appendix I

Scattering Cross Section

The scattering cross section, Σ_{ref} , can be obtained by multiplying the atomic cross section, σ_a , by the number of atoms per unit volume, N :

$$\Sigma_{ref} = \sigma N \quad (I.1)$$

The atomic cross section, σ , can be expressed in terms of electron cross section, σ_e , as:

$$\sigma = Z \sigma_e \quad (I.2)$$

where

$$N = \frac{N_A}{A} \rho \quad (I.3)$$

where Z is the atomic number of the material, A is the material mass number, N_A is Avogadro's Number (6.023×10^{23} atoms/mole), and ρ is the material mass density. Hence, the scattering cross section is:

$$\begin{aligned}
\Sigma_s &= \sigma_e \rho \frac{Z}{A} N_A = \sigma_e \rho_{ref} \frac{Z}{A} N_A \frac{\rho}{\rho_{ref}} \\
&= \sigma_e \rho_{ref} \frac{Z}{A} N_A \rho_{rel} \\
&= \Sigma_{s, ref} \rho_{rel}
\end{aligned} \tag{I.4}$$

From most materials, with the exception of hydrogen, the ratio of Z/A is approximately equal to $1/2$. Moreover, σ_e and N_A are both constants. Therefore, the probability of the Compton scattering per atom, Σ_s , of the detector depends on the number of electrons available as scattering targets in the object, and therefore increases linearly with the atomic number, Z (Knoll, 1979). The number of photons scattered is then a measure of the electron density of the material that scattered it. This can in turn help to identify the presence of a flaw or an inclusion.

Appendix II

Klein-Nishina Formula

The differential scattering cross-section is given by the well known Klein-Nishina formula, which determines the differential probability of scattering of photons, σ , into a small solid angle Ω at a given scattering angle and incident photon energy:

$$K(\alpha, \mu) d\mu = \pi r_o^2 \left(\frac{\alpha'}{\alpha} \right)^2 \left[\frac{\alpha'}{\alpha} + \frac{\alpha}{\alpha'} + \mu^2 - 1 \right] d\mu \quad (\text{II.1})$$

where r_o is the classical electron radius 2.817938×10^{-13} cm, $\alpha = E_o/mc^2$ (where m is the mass of the electron and c is the speed of light), $\mu = \cos \theta$ for the angle θ of deflection from the line of flight of the incident photons, and $\alpha' = \alpha/[1+\alpha(1-\mu)]$. For estimators such as detectors used in Monte Carlo simulations, the probability density for scattering toward the detector point must be calculated:

$$p(\mu) = \frac{1}{\sigma_t^K(Z, \alpha)} K(\alpha, \mu) \quad (\text{II.2})$$

where $\sigma_t^k(Z, \alpha)$ is the total Klein-Nishina cross section obtained by integration $K(\alpha, \mu)$ over all angles for angles for energy α . Since this is a difficult integration, the empirical formula of Hastings, given in Lux and Koblinger (1991) is used:

$$\sigma_i^K = \pi r_o^2 \frac{c_1 \eta^2 + c_2 \eta + c_3}{\eta^3 + d_1 \eta^2 + d_2 \eta + d_3} \quad (\text{II.3})$$

where $\eta = 1 + 0.222037\alpha$, $c_1 = 1.651035$, $c_2 = 9.340220$, $c_3 = -8.325004$, $d_1 = 12.501332$, $d_2 = -14.200407$, and $d_3 = 1.699075$. Thus,

$$p(\mu) = \frac{\eta^3 + d_1 \eta^2 + d_2 \eta + d_3}{c_1 \eta^2 + c_2 \eta + c_3} \left(\frac{\alpha'}{\alpha} \right)^2 \left(\frac{\alpha}{\alpha'} + \frac{\alpha'}{\alpha} + \mu^2 - 1 \right) \quad (\text{II.4})$$

Appendix III Algorithm and Code for Inversion of Measurement Model

Eq. (3.1) is solved iteratively to obtain the relative density at an energy-group. A FORTRAN program based on the $x = g(x)$ method of Hilderbrand (1987). The solution can also be readily obtained using a hand-held calculator, such as HP48SX. However, it should be noted that Eq. (3.1) can result in two different solutions due to the presence of the exponential terms. Figure III.1 shows a typical behavior of the function of Eq. (3.1) for a relative density range from 0 to 10. Nevertheless, the peak relative density is quite high; in the case presented it is equal to $1.34\text{E-}03 \text{ 1/cm}^2$, which corresponds to $10,800 \text{ kg/m}^3$. Therefore, almost always the relevant solution is below the maximum relative density. It is, therefore, necessary to start with an initial guess below the maximum value, in order for the solution to converge to a physically acceptable value.

Figure III.1 The measurement model, S_{model} , as a function of the relative density, $\bar{\rho}$, for a scatterer of 9.52 mm in diameter and 90° scattering angle.

PROGRAM PXGXIT

```
C-----
C
C
C PROGRAM DESCRIPTION
C-----
C THIS PROGRAM CALLS SUBROUTINE XGXIT AND COMPUTES THE
RESULTS OF
C A FUNCTION:
C
C SI =
EXP(-SIGIN*X*XIN)*SIGIN*X*L*PMU/(2*PI*R**2)*EXP(-SIGOUT*X*YOUT)
C
C BY THE ITERATION METHOD (X=G(X)).
C
C SI      : DETECTOR RESPONSE
C SIGIN   : ATTENUATION COEFFICIENT IN
C X       : RELATIVE DENSITY
C XIN     : DISTANCE TRAVELLED FROM SURFACE OF OBJ TO
SCATTERING      POINT
C L       : LENGTH OF THE INTERACTION VOLUME
C PMU     : KLIEN NISHAN
C PI      : 3.141592654
C R       : DISTANCE FROM SCATTERING POINT TO DETECTOR
C SIGOUT  : ATTENUATION COEFFICIENT OUT
C YOUT    : DISTANCE TRAVELLED FROM SCATTERING POINT TO EXIT
SURFACE
C E5      : ATTENUATION COEFFICIENT WITHIN THE FLAW/INCLUSION
C-----
C
C
C      REAL GFCN,X,SI,SIGOUT,PMU,R,L,YOUT,XIN,SIGIN,PI,E5
C      CHARACTER*15 IND, CON, OUT
C      INTEGER I, NLIM, SFLAG, BINS
C      COMMON SI,SIGOUT,PMU,R,L,YOUT,XIN,SIGIN,PI
C      EXTERNAL GFCN
C
C      STOL = 1E-5
C      NLIM = 100
C      SIGIN = 0.20061
C      PI = 3.141592654
```



```

E5 = 1.0
C
PRINT *, 'NAME OF INDICATION FILE?(INCLUDE PATH IF NEEDED)'
READ *, IND
PRINT *, ''
PRINT *, 'NAME OF CONSTANT FILE?(INCLUDE PATH IF NEEDED)'
READ *, CON
PRINT *, ''
PRINT *, 'NAME OF OUTPUT FILE?(INCLUDE PATH IF NEEDED)'
READ *, OUT
PRINT *, ''
PRINT *, 'NUMBER OF ENERGY BINS'
READ *, BINS
OPEN(9,FILE=IND)
OPEN(8,FILE=CON)
OPEN(5,FILE=OUT)
C
DO K = 1, bins
  X = 0.0
  I = 0
  READ(9,*) SI
  SI = SI * E5
  READ(8,*) SIGOUT,PMU,R,L,YOUT
  XIN = L/2
C  PRINT *, SI,SIGOUT,PMU,R,L,YOUT,XIN
  PRINT *, 'SI = ',SI
  CALL XGXIT(GFCN,X,STOL,NLIM,I)
c  E5 = EXP(SIGIN * L * X) * E5
ENDDO
C
C IF I=1, THE ROOT IS EQUAL TO X.
C
  STOP
  END
C -----
-----
C
  SUBROUTINE XGXIT (GFCN,X,XTOL,NLIM,I)
C
C -----
-----
C
C  SUBROUTINE XGXIT :

```

```

C   THIS SUBROUTINE FINDS THE ROOT OF  $F(X) = 0$  BY ITERATION WITH
C    $X = G(X)$ .
C
C -----
C
C   PARAMETERS ARE :
C
C   GFCN  -FUNCTION TO COMPUTE  $G(X)$ , MUST BE DECLARED
EXTERNAL IN
C           CALLING PROGRAM. IT HAS ONE ARGUMENT,X.
C   X      -INITIAL VALUE TO BEGIN ITERATIONS. X ALSO RETURNS
THE
C           VALUE OF THE ROOT TO THE CALLER.
C   XTOL  -TOLERANCE VALUE FOR CHANGE IN X TO TERMINATE
ITERATIONS.
C   NLIM  -LIMIT TO NUMBER OF ITERATIONS.
C   I      -A SIGNAL FOR HOW TERMINATED.
C   I=1   -MEETS TOLERANCE FOR X VALUES.
C   I=-1  -NLIM EXCEEDED.
C   I=-2  -ITERATIONS APPEAR TO DIVERGE IN INITIAL CALCULATIONS.
C
C   WHEN THE SUBROUTINE IS CALLED, THE VALUE OF I INDICATES
WHETHER TO
C   PRINT EACH VALUE OR NOT. I = 0 MEANS PRINT, .NE. 0 MEANS DON'T.
C
C -----
C
C   REAL GFCN,X,XTOL,SIGIN,L
C   INTEGER NLIM,I,J
C   REAL SAVEX,DEL1,DEL2
C   COMMON SIGIN,L
C
C -----
C
C CHECK INITIAL VALUES
C
C   J = 1
C   SAVEX = X
C   X = GFCN(X)
C   DEL1 = ABS(X - SAVEX)

```

```

      IF (DEL1 .LE. XTOL) THEN
        I = 1
          X = -ALOG(1-SIGIN*X*L)/(SIGIN*L)
          PRINT *,X
          PRINT 202, J, SAVEX, X
          WRITE (5,300) X
          RETURN
        END IF
      IF (I .EQ. 0) THEN
        PRINT 199, J, SAVEX, X
      END IF

C
C -----
C
C GENERATE SEQUENCE OF POINTS
C
  DO 20 J=2,NLIM
    SAVEX = X
    X = GFCN(X)
    DEL2 = ABS(X - SAVEX)
  C
  C -----
  C
  C CHECK FOR XTOL CONDITION
  C
    IF (DEL2 .LE. XTOL) THEN
      I = 2
        X = -ALOG(1-SIGIN*X*L)/(SIGIN*L)
        PRINT *, X
        PRINT 202, J, SAVEX, X
        WRITE(5,300) X
        RETURN
    END IF

C
C -----
C
C CHECK FOR DIVERGENCE AT START
C
  IF (J .EQ. 2) THEN
    IF (DEL1 .LE. DEL2) THEN

```

```

        I = -2
        X = -ALOG(1-SIGIN*X*L)/(SIGIN*L)
        PRINT 201
        WRITE (5,201)
        RETURN
    END IF
END IF
IF (I .EQ. 0) PRINT 199, J, SAVEX, X
20  CONTINUE
C
C -----
C
C WHEN LOOP IS NORMALLY TERMINATED, NLIM IS EXCEEDED.
C
    I = -1
    PRINT 200, NLIM, SAVEX, X
    WRITE (5,1000) X
    RETURN
C
C -----
C
199  FORMAT ('AT ITERATION',I3,4X,'X =',F9.4,4X,'G(X) =',F9.4)
200  FORMAT ('METHOD DID NOT CONVERGE IN',I4,'ITERATIONS. FINAL '
+      'VALUES ARE X =',F12.5,'G(X) =',F12.5)
201  FORMAT (' FIRST THREE VALUES INDICATE DIVERGENCE.')
202  FORMAT (' XTOL MET IN ',I4,' ITERATIONS. X =',F12.5,
+      ' G(X) =',F12.5)
300  FORMAT (F7.5,/)
1000 FORMAT ('DID NOT CONVERGE, FINAL X =',F12.5)
    END
C
C -----
C
C
    REAL FUNCTION GFCN(X)
C
C -----
C
    REAL X, SI, SIGOUT, PMU, R, L, YOUT, XIN, SIGIN, PI
    COMMON SI, SIGOUT, PMU, R, L, YOUT, XIN, SIGIN, PI

```

C

```
      GFCN =  
SI*2*PI*R*R/(SIGIN*L*PMU)*EXP(SIGIN*X*XIN+SIGOUT*X*YOUT)  
      RETURN  
      END
```

C

Appendix IV The Full-Width-at-Half-Maximum (FWHM) Approach

The FWHM for a typical peak is shown in Figure IV.1, for a function , $f(x)$. The integral,

$$\int_{x_p - \frac{FWHM}{2}}^{x_p + \frac{FWHM}{2}} f(x) dx \quad (IV.1)$$

Figure IV.1 Schematic representation of the FWHM approach

represent the strength of the signal used in the analysis of Chapter 4.

In order to demonstrate the significant of the FWHM, assume $f(x)$ represents a

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x - \bar{x})^2}{2\sigma^2}\right) \quad (\text{IV.2})$$

normal distribution, which is a common statistical distribution, as shown in Eq. (IV.2),

where $\bar{x}$ is the mean value of x and σ^2 is the variance of the distribution. At $x = \bar{x}$, $f(x)$

is maximum and is equal to $\frac{1}{\sqrt{2\pi}\sigma}$. Therefore, one can obtain the value of x at half the

maximum value using; i.e., $x_{1/2}$,

$$\frac{1}{2\sqrt{2\pi}\sigma} = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x_{1/2} - \bar{x})^2}{2\sigma^2}\right) \quad (\text{IV.3})$$

which leads to,

$$\ln(0.5) = -\frac{(x_{1/2} - \bar{x})^2}{2\sigma^2} \quad (\text{IV.4})$$

or

$$x_{1/2} = \bar{x} \pm 0.980 \sigma$$

Therefore, integrating between the two values of x covers about 68 % of the strength of peak, which corresponds to a one-sigma confidence interval.