JOURNAL WRITING IN AN ELEMENTARY MATH CLASSROOM AND ITS EFFECT ON STUDENTS' UNDERSTANDING OF DECIMALS

by

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Abstract

This is an exploratory, action research study of the use of journal writing, focused on four students in a regular grade five math class and their growing understanding of decimals. They were given journal assignments as well as oral and manipulative-based activities during a series of ten, forty minute lessons. Data collection methods included field notes, audio taped student-student and teacher-student interaction, and written journal entries.

The findings of the study indicate that students' use of journal writing, in conjunction with other constructivist tools such as manipulative-based activities, significantly improved: their conceptual understanding of decimals; their ability to work through to solutions not previously understood; their ability to consolidate learning initiated through other methods; their awareness of their own understanding; their confidence; and their ownership of their learning. Students' journal entries allowed me, the researcher, to programme and assess students' needs quickly and consistently.

Acknowledgments

I would like to thank the students involved in this study for responding honestly, diligently and enthusiastically to my many queries.

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Chapter 1 — Introduction

Background and rationale

Over the past couple of decades. there has been a growing concern about math education in North America. Among other difficulties. North American students have not done well on non-numeracy sections of international standardized tests such as the Third International Mathematics and Science Study (NCES. 1998). Overall. students' basic numeracy skills were good. but students appeared to have greater difficulties with other strands of mathematics. including problem solving. Another significant concern in math education has been the alarming number of North American students that drop math at the Secondary school levels (Noddings. 1994). Evidence shows that one of the reasons for both of these problems is that students have not received the proper conceptual math education that should have occurred at every level of their formative years. Students were so inundated with math facts. rules and memorized procedures rather than a solid conceptual understanding of mathematics. that they did poorly on international tests. As well. at a local level. when the curriculum became more challenging. these same students felt overwhelmed and opted out of math class.

In response to this. and other similarly related problems. "The National Council for Teachers of Mathematics" (NCTM) mathematical standards for education has become a stronger focus in educational circles. These standards have included elements of constructivism (Confrey. 1990) whereby. with the use of constructivist tools. students could gain a greater understanding of mathematical concepts. This in turn. would allow them to build a solid conceptual. rather than a merely procedural. foundation which would then support an increasingly difficult set of mathematical ideas at more advanced levels (Skemp. 1974; Hiebert. 1990).

Even though this critically important mathematics reform has been in existence for more than a decade. it has been slow to spread for the simple reason that teaching mathematics from a conceptual perspective is more difficult than teaching mathematics from a procedural perspective. However. with our ever changing society. a teacher can no longer ask a child merely to memorize math facts and formulas. S/he must develop solid problem solvers and mathematicians who understand the reasons behind the mathematical algorithms. Teaching for conceptual understanding is a considerably greater educational challenge than merely encouraging rote memorization.

Having been a graduate student for the past several years. I have learned a great deal about constructivism and situated learning (Confrey. 1990; Brown. 1989). I have worked on implementing these educational philosophies in my teaching of junior-aged math classes. Because of this knowledge and its implementation. I felt that I have become a better educator and facilitator. Having children discover math concepts and seeing them truly understand those previously mindless algorithms has been extremely rewarding for them and for me as the teacher.

However. as much as I agreed with and practiced the concept of constructivism. I still had some reservations about a few of the constructivist-based tools. I had heard many comments about the benefits of math journal writing. but I was a skeptic. I believed that discovering what a child knew about a subject could be done more easily through an oral interview during class. I felt as though journal writing would be a waste of good learning math time. On the other hand. I did feel that journals could serve. potentially. as an assessment tool. It is for this reason that I decided to find out more about the process of writing and reading math journals. I could hardly judge what I didn't know about in the first place.

Goals and Objectives

The main goal of this enquiry was to study the effects of journal writing in conjunction with the use of manipulatives on grade five students as they developed an awareness of decimals. The specific cognitive math goals to be achieved by the students were the conceptual understanding of a decimal being a part of a whole. tenths being ten parts of one whole, hundredths being a hundred parts of one whole and thousandths being a thousand parts of one whole. As well. the students were to grasp the concept of what happened to the parts or decimals of a whole when they were used in situations where one needed to add and subtract them.

The following were objectives for this study:

i) to teach a unit of ten lessons involving the concept of decimals using concrete materials within a constructivist framework;

ii) to explore the effectiveness of journal writing as an assessment tool through the comparison of in-class manipulative work and student's corresponding journal entries;

iii) to explore the effectiveness of journal writing as a way in which to determine a student's views and understanding of a math topic before the topic's actual introduction — in this case decimals;

iv) to explore the effectiveness of open ended journal writing on math-related experiences;

v) to identify/explore other ways in which math journal writing can assist in a child's growing understanding of mathematics.

Research Questions

The following research questions were investigated:

i) How effective is math journal writing as an assessment tool in determining the level of conceptual understanding gained from in-class manipulative work?

ii) How effective is math journal writing as an accurate assessment tool in which to determine a student's base line knowledge of a math topic?

iii) How effective are open-ended math journal entries for student or educator?

iv) In what other ways can math journal writing assist a child's growing understanding of mathematics?

Plan of the Thesis

The first chapter of this thesis gives a very brief overview of the background of current mathematics reform in North America. The second chapter reviews literature that provides both a theoretical and practical framework for the rest of the study. The third chapter explains the methodology used. The fourth chapter describes each of the ten math lessons as well as detailed results from each lesson. The final chapter summarizes and discusses the results as they relate to the initial research questions and to the literature review.

Chapter 2 — Literature Review

This literature review looks at the existing body of written work in three different areas. The first set of reviewed articles looks at the widely varied approaches to math journal writing itself and how it can be used. Different methods and their implementation at the elementary, secondary and overall grades 4-12 levels are discussed in these articles.

The second set of reviews deals with student/teacher discourse and oral communication. The purpose of math journals is to help students communicate and articulate their math ideas. However, communication is not strictly written. Communication includes elements of listening and speaking, or in National Council for Teaching Mathematics' (NCTM) Standard terminology — teacher/student discourse.

The third set of reviews includes different educational theories found in the overall theoretical framework of this study. Articles dealing with situated learning, constructivism, Piagetian educational philosophy, and procedural versus conceptual understanding will be reviewed at the end of this literature review.

Journal Writing Methods and Implementation

Norwood and Carter (1994) describe a simple, concise and practical approach to math journal writing in an elementary classroom. The authors explain that the purposes of a math journal can be somewhat diverse. Math journals can focus students on a review of a concept. They can serve as a knowledge indicator in order to gain insight into a student's views on a topic before it's introduction. They can also be used as an assessment tool for the teacher to find out how well concepts covered in class have been understood. This latter kind of journal writing allows the teacher to have insight into the student's basic constructs that, in a test situation, would be overlooked completely. The example given is that of a child that has memorized the multiplication algorithm. The child can get a perfect mark on a test, but has little, to almost no knowledge of the basic concept behind the algorithm. Cognitive journal writing would allow a teacher to assess a child's true understanding of a math concept.

The authors describe practical ways and methods in which math teachers can incorporate journal writing in their daily/weekly math classes. Preparing a special booklet just for math journal writing allows a student to take ownership and pride not only of the actual journal, but of their math ideas as well. Setting time limits on journal writing periods will provide a more structured environment. Responding to children's math journal entries allows the children to see that you value their ideas. When a teacher introduces the concept of journal writing to the students, it's important to have the children warm up to the concept by answering more generic and broad questions as opposed to specific cognitive type math questions. Many of these ideas are not new and innovative, but they serve as gentle reminders of what should be done.

As excellent as many of these suggestions are, I have a few difficulties with some of their other suggestions. The authors of the article state that a teacher should choose a particularly clear and concise journal entry from the class and put it on display. Since one of the purposes of constructivism is to value every child's set of constructs to the point of ensuring that all children feel they have an equal voice in the room, this setting up of one child's work as an example for the rest of the class tends to eat away at this concept. Suddenly, there are bright children and not so bright children. There certainly are differences in students' ability levels, but as an educator, I feel that it's important that all children feel bright. Wrong answers should be corrected, but the correction should take place in such a way that the child with the wrong answer doesn't feel stupid.

Stix (1994) suggests that the best way for elementary students to learn is through a multimodal approach. If fractions are being taught, in order for the concept to be fully understood the student would need the concrete or manipulative stage to be presented as well as the pictorial or picture representation of the concrete, and finally representation of the abstract or symbolic stage. Stix firmly believes that the same holds true for journal writing. Her concept of *pic-jour math* is journal writing which includes pictures, numbers, symbols and manipulatives.

She describes a current trend with regards to math learning. She states that, "...most schools still have a distinct tendency to reward only verbal proficiency" (page 85). This focus on verbal proficiency has allowed students to listen to the words given verbally by the instructor, memorize them and then regurgitate the words at will, yet not really understand the concept behind them. She goes on to state that a multimodal approach allows students to mix visual clues, verbal knowledge and personal experience in a way that allows them to come up with a more complete understanding of a math concept. Visualizing math ideas helps to bridge the gap for students between the concrete and the symbolic, while verbal knowledge allows students to get a deeper understanding of the concept covered.

Stix applies these theoretical concepts to journal writing. A typical pic-jour entry should include pictures, writing and numbers. In order to not be distracted from the mathematical task, s/he is told that the grammar and writing style are not going to be marked. A pic-jour entry would be particularly pertinent immediately after a lesson that has involved extensive manipulatives. This will allow a child to "...link his or her manipulative experience to the numbers . . ." (page 87). As well, the actual writing of the journal entry will assist a child in better understanding the verbal and visual processes s/he has just experienced.

Further arguments in favour of multimodal journal entries include the fact that they allow children more tools to draw on to make their point. Since children have different learning styles and come from different backgrounds, this kind of multimodal learning is more likely to address a variety of needs such as the desire to convey one's ideas through pictures. A student no longer needs to be confined to the strict parameters of the written word.

Overall, this article not only touched on the key theoretical points of a child's math learning processes, but it applied this theory clearly and concisely to the activity of journal writing. From inception to demonstration to assessment to conclusion, Stix has outlined a very workable and effective way to implement math journal writing, or pic-jour math in an elementary classroom.

Frances Curcio (1990) makes an excellent point relating to whether or not elementary aged students should concern themselves with specific math terminology. In order for elementary school aged children to understand concepts. Curcio believes it is better to build on the students' informal math language first and then, after extensive discussion, writing, reading and working with concrete manipulatives, slowly introduce the appropriate math terminology. One way to build that understanding is through journal writing.

Stewart and Chance (1995) report on a study of the use of math journals in four secondary first-year algebra classes. They explain the various purposes behind their three ways of journal writing, and then they connect these different types of journal entries to the NCTM standards. To illustrate each of these connections, they provide student and teacher journal samples which provide an excellent insight into the success (or failures) of each of the methods of journal writing.

The first kind of journal writing deals with mathematics concepts and procedures. The following type of prompt was given to the students,"subtracting is the same as adding the opposite because ...." (page 92). The purpose behind this kind of inquiry was to find out the student's understanding of concepts presently being taught in class. The teacher marked these entries for the level of mathematical understanding demonstrated, as well as the child's attention to detail.

The second kind of journal entry is broader in nature. It deals with curriculum issues. A sample journal starter might be,"one mathematics activity I really enjoy is...because..." (page 92). These types of entries were marked by the teacher with the nature of the content in mind. Over a period of time, the teacher would be able to recognize patterns emerging in their students' thinking.

The third kind of journal writing is a free-write. Students were allowed to write about whatever they wanted to do. The only stipulation was that, "...the entries involve the writers as students of mathematics" (page 92). Similar to the math journal written with curriculum issues in mind, these free-style entries were also marked for the nature of the entry.

Stewart and Chance (1995) tie the last two types of math journal writing to the fifth NCTM standard. I was particularly interested by the fifth standard which is entitled, "The Learning Environment" (page 94). The authors speak of the importance of adapting students' ideas and suggestions into the daily math class. They report on a study where the students were given the opportunity to state their general views of math to the teacher in the form of a journal. The teacher was able to adapt her programme accordingly based on the use of journals. One student wrote,

> I like journal writing. It is beneficial to both student and teacher. The teacher can use the writing of the students to help them and maybe plan fun things to do. The teacher can get ideas and learn from the journals and use them in class. (page 95)

Aside from meeting the NCTM guidelines, the authors also address the issue of math anxiety. They draw upon the "Mathematics Anxiety Rating Scale for adolescents" (page 94) as a resource. During one of the free-style journal writing sessions, a child struggling with this type of anxiety wrote,

> Writing in journals lets you get rid of some stress because instead of keeping your feelings bunched up you can express them to your teacher on paper . . .I am not having as much trouble with algebra as I used to. I like it and feel good about what I am learning. (page 94)

This article provides an excellent argument as to why math journals are effective learning tools for students. It states clearly, and gives empirical evidence, that math journals not only assist in the cognitive development of students, but in the social-emotional needs of the children in a math class. This approach looks at the whole child. From an elementary teacher's perspective, there would have to be some significant modifications to this particular math journal writing programme in order for an elementary-aged child to benefit, but the basic concepts are sound.

Margaret E. McIntosh's (1991) approach to math journal writing is similar to that of Stewart and Chance (1995), but she adds an additional few components to the basic premise that children should have input. Along with the concept of math journals (she writes with the secondary student in mind), she suggests a learning log as an alternative to math journals. Learning logs include responses as they happen. A child might write out an entry in their learning log just before asking a question in math class. The entry would describe the question in detail. Similarly, when a child has just experienced a mathematical break-through, the child is encouraged to write out, in detail, what the break-through was.

McIntosh suggests that since math writing is being used to enhance the math programme, by the very nature of mathematics, there should be a precision in the general writing style. On the other hand, she also states that since the concept of writing is more open ended, writing about math can bring about a less threatening view of math. There should be a happy medium between concise mathematical writing and open ended writing. McIntosh quotes a math teacher as saying,

> Mathematics is, after all, communication, but communication in math involves a compact, unambiguous symbolism that to many students is cold and rigid. Writing, on the other hand, is a less structured way of expressing ideas. (page 430)

McIntosh's study is at the secondary level. Students, therefore, have a higher level of language development allowing math definitions to come more easily. However, this concept could be applied, to a limited degree, to students at the elementary level (this was touched upon briefly in the article by Curcio reviewed earlier).

Tobias (1989) focuses on the therapy element of journal writing. She states that math journals can assist a math-phobic individual in several ways. Firstly, as someone who feels dumb in math writes about one's feelings and math constructs (or mis-constructs), this will allow the individual the opportunity to release their stress and anxiety. Secondly, by allowing the instructor to view the journal entry and any math misinformation, it will help to guide the instructor's programming accordingly.

She suggests that to optimize this therapy concept, a divided-page approach would be helpful. The right side of the page is to be used for the figuring out of the actual problem. This could include calculations, diagrams, etc. The left side of the page is the therapy side. The student is encouraged to right down her feelings about answering the question as she goes along.

She, like others, also suggests the use of journals for assessment purposes, but the implementation is somewhat unique. She calls this type of journal writing "Minute papers about muddiest points" (page 53). Four or five times a term, a teacher will ask the following two questions: What is the most significant thing you learned today? What question is uppermost in your mind at the conclusion of this class session? The purpose of these questions is evaluative. They allow the teacher to find out how much a student has learned and they allow the teacher to find out if there are any unanswered questions left on the minds of her/his students.

Similarly to Tobias' "muddy minutes", L. Diane Miller (1991) speaks about secondary students writing quick journal entries in about three to four minutes' time. She even promotes the use of a stop watch to enforce this! However, the purpose of these particular entries is not evaluative in nature. Instead, Miller sees this kind of math journal writing as a way of improving student's intellectual skills by forcing them to think sharply and more concisely about math related issues. She promotes the use of journal writing as an assessment tool only on an infrequent basis.

An area in which I disagree with Miller is her views on math journals and grades. In spite of the fact that Miller believes that journal writing can be used as a teaching tool to sharpen students' mathematical understanding, she introduces the idea that journal writing should not be used for grading purposes. A student should feel comfortable in writing, or not writing out a journal entry. From an elementary teacher's perspective, I feel that if a fair amount of time is going to be spent on the process of math journal entries, then the grades should reflect this especially when a math journal entry is being used as a less formal, more comfortable and flexible assessment tool.

Miller provides a more humanitarian view of a teacher in a classroom with a potential student size of 30+ children. She states that reading and responding to every journal entry, every day is unnecessary. She feels that a teacher can respond to students' math journal entries in a variety of ways. Personal responses are good, but a teacher can also respond to the class's entries as a whole by stating, at the beginning of the class following the one in which an entry was written before that, as a result of reading the math entries, programming has been altered to better meet the needs of the students in the group.

Nahrgang and Petersen (1986) give excellent suggestions on how a math journal can be a learning tool, not just for clarification or assessment purposes, but for the actual learning process as well. Although the authors write about the secondary experience, many of the ideas can be adopted for use at the elementary level. The main example given is a secondary school math class focusing on factoring and finding products of polynomials. The students were given the question, "think of a non-mathematical relationship that is analogous to the process of finding a product and factoring" (page 464). One particularly remarkable answer from a student was as follows:

> One relationship similar to finding a product vs. factoring is that of taking a carburetor apart and then putting it back together. When the carburetor is together it is difficult to clean and repair. Therefore it is disassembled to make it easier to work with. This is analogous to factoring.
>
> When the parts of the carburetor are in working condition, they must be put back together before they will work as desired. This is analogous to finding a product. (page 464)

In this example, as a result of the information learned in a math unit and, instigated by the teacher's succinct question, the student was able to use a variety of skills such as "...synthesis, interpretation, translation, analysis, and evaluation" (page 465).

Student/Teacher Discourse

As mentioned earlier, a discussion on journal writing should also include what the NCTM standards refer to as "The Teachers' Role in Discourse" (Standard 2), "The Student's role in Discourse" (Standard 3), and "Tools for enhancing Discourse" (Standard 4). Deborah Loewenberg Ball (1991) gives this definition of discourse.

> An unfamiliar term to many, 'discourse' is used to highlight the ways in which knowledge is constructed and exchanged in classrooms. Who talks? About what? In what ways? What do people write down and why? What questions are important? Whose ideas and ways of knowing are accepted and whose are not? What makes an answer right or an idea true? What kinds of evidence are encouraged or accepted? (page 44)

Ball emphasizes the influential and crucial role a teacher plays in the way the discourse in a classroom occurs. Looking at two extremes, a teacher responding in a less intrusive way to a student's question can further the students' exploration resulting in, eventually, the working out of the right answer. A teacher responding to a question simply by giving the right answer would merely promote the child's dependency on the teacher and what the child perceives to be the right answer, thus robbing the child of the opportunity of learning through the process of exploration. Teachers' interactions have a great deal of influence on a student's way of learning. On a larger scale, this, in turn, effects the atmosphere of the classroom.

Ball doesn't suggest that there is a right way of teaching. Instead, Ball includes a transcript of a grade three class which she taught. Immediately beside the transcript itself, she jots down her personal feelings with regards to each situation as they arose. However, instead of providing us with an interpretation of her transcript, she allows the reader to come up with our own questions and answers about teacher/student discourse through our perusal of the transcript. Having said that, she still seems to favour the viewpoint that a teacher should be more of an unbiased, but clearly directed questioner, rather than the omniscient source of all knowledge.

Since journal entries are reflective of what happens during math class, the atmosphere of the class, including children's desire to risk-take and learn from their own processes is critically important. Ball's article raised some interesting questions about the kinds of questions that should be asked of the children as well as how the instructor should be responding to these questions and comments in a way that would encourage the children to further their investigations.

Theoretical Framework

Having reviewed the current literature on the topic of journal writing, it is also important to mention some of the inherent theory included in this study. The critical need for children to actively use manipulatives is a recurring theme in mathematics research. Piaget, according to Labinowicz (1980), states that in order for children to have any real understanding of a mathematical concept at the abstract level, they must first be able to work successfully at the concrete stage using manipulative materials. When children work fairly consistently at this level (on any given concept), according to Piaget they are in the concrete operational stage of development. Once they have a solid grasp of the concrete they move into the pictorial level of understanding, then eventually into the abstract level of understanding. Piaget calls this abstract level of development, the formal operational stage. For example, if children are going to fully understand the concept of decimals, they must be exposed to a variety of activities involving physical manipulatives first. Once the concept is understood at this level, they will then be able to work towards a more abstract understanding of the concept.

The difference between relational and instrumental understanding (Skemp, 1976) is important to the understanding of the use of journals. Skemp is a firm believer in the concept that meaningless algorithmic memorization, or instrumental learning, does not help a child truly understand any mathematical concept. If a child is taught not only the how, but the why, s/he will have a solid and deeper relational understanding of math concepts covered. He states,

> [by the term relational understanding] is meant what I have always meant by understanding, and probably most readers of this article: knowing both what to do and why. Instrumental understanding I would until recently not have regarded as understanding at all. It is what I have in the past described as 'rules without reasons'... (page 21)

Recently math educators and researchers have changed the terms of 'relational' and 'instrumental' to the terms 'conceptual' and 'procedural' respectively. They call for more emphasis on developing conceptual understanding (Hiebert. 1990).

The theory of Constructivism supports the use of journals. Constructivism advocates the idea that children construct their own knowledge through interaction with their environment (Confrey. 1990). A teacher is required to find out as much as possible about their students' individual mental constructs regarding mathematics and then programme accordingly. Confrey (1990) advocates the use of individual case studies for each student. This would allow a teacher to be thoroughly familiar with each child's specific mental construct and plan accordingly.

Clements (1997) describes constructivism as an approach that should include. "...time for 'experience'; for 'intuitive learning'; for learning by listening; for practice; and for conscious reflective thinking" (page 198). The reflective thinking portion is very much what journal writing is all about. In fact math journals themselves. allow a teacher to better see and understand the child's own personal mental constructs.

Another theory derived in part. from social constructivism. which appears frequently throughout the course of this paper. is the more global educational concept of situated learning (Brown. 1989). This theory states that children have difficulty understanding abstract concepts taught in an isolated. decontextualized environment such as the school room. If students of today are going to have more than just a cursory. procedural understanding of math. the curriculum must be presented to them within a socially understandable. practical context and within a culture of mathematical practice. As Roth (1992) states.

> These [mathematical] tasks should be situated in conceptual, practical. and social contexts that reflect realistic simulation of practice. Both the NCTM and AAAS documents highlight the importance of integrating real-world problems with school subject matter... (page 307)

The educational concept of situated learning should apply to all strands of mathematics. Even the more abstract mathematical concepts such as decimals and fractions should be taught in a socially applicable context which a child can understand. otherwise the child may not see a purpose for this knowledge and might promptly forget whatever she may have learned.

Chapter 3 — Methodology

This classroom based-inquiry (Hubbard. 1993) was an exploratory study on the use of journal writing in an elementary classroom. It followed a qualitative research paradigm (Glesne. Corrine & Peshkin. 1992). This action research based inquiry included four child case studies involving grade 5 students and their learning processes during a unit of ten lessons on decimals.

Classroom Setting

At the time of this study. I had been an elementary school teacher for approximately eight years. I had taught in the same school throughout these eight years. This made me a known entity as a teacher within the school's parent community. During the three years immediately previous to this study. I had taught grade five in some form either as a split or straight grade.

My teaching assignment for this study was a split grade 5/6 class. The school in which I taught was located in an economically and ethnically mixed community. By and large the parents in this particular community were supportive of their children. They took an interest in their children's learning and general well being.

There were eight children in the grade 5 portion of the class and eighteen children in the grade 6 portion of the class. This study analyzed the learning processes of four of the eight students in the grade 5 portion of the class.

In order to meet the needs of both grades properly. I taught the two grades math at different times of the day. During one period of the day. while the grade five students were working on a Language Arts assignment. the grade 6 children were being taught math at the carpet. During a second period of the day. the situation was reversed. Even though I was required to programme for and supervise all twenty-six students during the two forty minute periods of time in question. I was able to work on math with my eight. grade 5 students for forty minutes every day.

Participants

The selection of the four participants was based on a convenience model. As the eight grade 5 students in the room were girls. I selected four female participants. In choosing which four girls should be included, several considerations were taken into account. Four of the eight children were taught by me in the previous year and had been exposed to a more constructivist math programme. As a result of working with the other four children who were not in my class last year, I discovered that they had experienced math teaching of a more traditional nature. With this in mind, I chose two students whom I had taught from the previous year and two students whom I had not taught. Within these parameters, I chose one student with an average to below average math ability level, two students with average to strong math abilities and one student with a very high math ability level.

Alice

Alice was one of the two students in this study whom I had taught the previous year. She had above average math skills. Her overall academic skills were above average. She had an excellent long term memory. She would consistently wrestle with a problem until she figured it out. She wanted life or in this case mathematics, to fit nicely into a given set of rules and if they did not fit, she often became frustrated. She did not take risks easily. She was a born leader, but she was not bossy (rather remarkable for a child).

Carly

Carly was the second student whom I had taught in my homeroom the previous year. Her math skills were slightly above average to, at times, well above average. She was extremely conscientious and hard working. Although she was capable of outstanding work all the time, there were times in which she would make do with just enough to get by. She could be rather hard on herself at times. She was very fair, kind and thoughtful. Carly was a socially, academically and emotionally well adjusted child. She knew how to work well and she knew how to play well.

Lynn

Lynn was one of the students that I had not taught in the previous year. Her math skills were well above average, if not excellent at times. She had a natural aptitude to math. Overall, her academic skills were very good to excellent. However, her actual writing, ie., editing, spelling and grammar skills were not on the same level as her conceptualization of math ideas. Even though her past math experience had been procedural, because she was a fast learner, this did not deter her math learning in the least. She was a confident, strong, independent thinker not easily swayed by her peers' points of view. She rarely said anything that she did not emphatically feel.

Tamar

Tamar was the second of the two students that I had not taught the previous year. Her math skills were average to, at times, below average depending on general motivation. Tamar was a conscientious and hard working student with overall above average academic skills except for math. Since her language skills were higher than her math skills, she was included in this study in the hope that the process of journal writing might help her to improve her confidence and allow her to pursue difficult math concepts more diligently until they were solved.

She was an extremely sensitive, caring and empathetic individual. She was always socially sensitive to circumstances around her, occasionally, to the detriment of her own learning.

Since she had not been exposed to the constructivist approach to math last year, and had been exposed to the more algorithmic, memorize-the-rule-and-you've-got-the-concept approach, not only was it necessary to pull her out of the algorithmic thinking mode, but she needed to adjust to the concept that math can be creative and concretely understandable.

Data Collection

Three types of data were collected. The first type of data included the teacher's/ investigator's observations of participants' involvement during in-class manipulative-based lessons. Field notes were made by the investigator during the 10 lesson plans. While initiating and moderating group discussions, observations were occasionally written down during the lesson, but were primarily recorded at the end of the lesson. As there was a great deal of discussion and interaction with the students, it was more feasible for notes to be taken at the end of the lesson. Field notes were also written extensively during the period of time that the students wrote up their journal entries.

The second type of data collected included students' discussions of math concepts during teacher/student instruction and peer conferencing. Audio tapes recorded student/student and teacher/student interaction when the students worked with concrete manipulatives.

The third type of data collected included different types of journal entries such as mini-journal writing entries, brainstorming entries, base-line determination entries and general cognitive-type of entries. They were usually written by students after using manipulatives or after large or small group discussions on decimals. Journal entries were collected at the end of most of the lessons.

Data were collected during ten, forty minute lessons on decimals. The lesson plans were initially designed taking into account each student's general base line and rate of learning. The data were collected over a four week period. The lesson plans generally included some or all of the following: an initial full group instruction session; a manipulatives session in which I would conference with smaller groups of children while they were working in partners with manipulatives; a final full group wrap-up discussion; and a journal writing session. Small groups of children usually consisted of two children chosen by me. The children were paired up differently each day thus enabling the students the opportunity to work with students that had varying ranges of mathematical abilities.

The specific outline for each of the following lessons was set up once the children's knowledge gained from the previous lesson was ascertained. The main source of understanding for this knowledge came through review of the children's journal entries, the audiotapes and the field notes. The motivation and intent of each lesson was as follows:

Lesson #1: Journal writing session in which the children were to brainstorm what they knew about decimals.

Lesson #2: As the majority of the children had some understanding of decimals, the second lesson delved into the more concrete concept of adding and subtracting decimals, specifically using paper money.

Lesson #3: As students demonstrated a fairly solid understanding of exchanging pennies for nickels, nickels for dimes, etc. in the previous lesson, they totaled up grocery receipts using the base ten blocks instead of photocopied money.

Lesson #4: As the students demonstrated a solid understanding of adding money with base ten blocks in lesson #3, they were given the more abstract task of adding decimals using paper and pencil.

Lesson #5: As a result of a difficult and frustrating concept being covered the day before, the children were asked to do a free-write journal entry.

Lesson #6: It was apparent from previous lessons that the students needed to have a greater situational understanding of decimals. The children were introduced to the idea of linear measurement being a whole and a part of a whole depending on the specific measurement being used, and peripherally, the idea of subtraction of decimals.

Lesson #7: From the previous lesson, it was apparent that they were feeling comfortable with the basic concept of decimals and, to a certain extent they felt comfortable with the pertinent scientific notation. However, it was evident that they needed to find objects to measure with non-standard units in order for the children to understand the idea that tenths can be 10 parts of ANY whole and similarly, hundredths can be one hundred parts of any whole. The children measured fruit roll-ups for the majority of this lesson. Based on the student's needs, a different type of journal writing, or a mini-write, was given to the children in the midst of the manipulative portion of the lesson itself. (The children were allowed to eat the manipulatives at the end of the lesson.)

Lesson #8: As a result of the fruit roll-up and measurement lessons, I decided to consolidate and assess the student's understanding of decimals. The concept of a changing whole was covered using measuring tapes.

Lesson #9: With a more solid grasp of decimals as they relate to measurement, the children were asked to make comparisons between decimals as they related to money and decimals as they related to measurement. They wrote their findings in a journal entry.

Lesson #10: The previous lesson was supposed to have been the final lesson of this study. However, I was surprised by the limited understanding demonstrated in their journal entries. I felt that they knew and understood much more than they had demonstrated. In order to find the best way to uncover this knowledge, I set up a basic oral review in this lesson through a question and answer approach.

To facilitate the reading of the research findings related to each of these lessons, a detailed description of each of these lessons is included in the "Results" chapter.

Data Analysis

In order to increase the trustworthiness and validity of the results, methodological triangulation (Glesne, Corrine & Peshkin, 1992) was used in the data-collection for this study. All the audio tapes, field notes and journal entries were initially analyzed separately. In preparing for this study, there were a number of predetermined cognitive and theoretical criteria from which all three types of data were to be analyzed. As the study got under way, the data were analyzed for potentially emerging themes as well.

The data were analyzed with the following cognitive themes in mind: tenths as being ten parts of one whole; hundredths being a hundred parts of a whole; thousandths being a thousand parts of the whole; ten hundredths equaling one tenth; ten thousandths equaling one hundredth; place value as it relates to decimals; and the concept of a changing whole as it related to a decimal and the addition and subtraction of decimals.

The data were analyzed with the following theoretical themes: the Piagetian perspective that a student's understanding of a concept depends on movement between the concrete, symbolic and abstract levels; Skemp's relational versus instrumental understanding (translated into current education philosophy, the concrete versus the procedural); constructivism and the identification of a child's developing constructs in different settings, comparing evidence of different levels of understanding in one child as she worked in different mediums — partner work, full group discussions and individual journal entries.

Since many of the field notes were so closely connected with the transcribed audiotapes the two areas were reported on under one heading in all but one lesson. The different forms and varieties of journal entries have been placed at the end of each lesson under a separate sub-title.

Ethical Considerations

It's important to note that ethical reviews were completed and approved by both the Ontario Institute for Studies in Education of the University of Toronto, and the Board of

Education where the inquiry took place. As well, before the inquiry began, written permission was obtained from each of the four student participants and their parents. The names of the participants were changed for confidentiality purposes.

Chapter 4 -- Results

Lesson #1

Purpose

This was the first lesson or introduction that referred to the concept of decimals this year. In order to programme further. I needed to find out what the children learned in previous years with other teachers. or in the case of the students that had me as a teacher. what they remembered from last year's decimal work.

Strategies/Procedures

With no previous discussion the students were asked to write out a journal entry answering the following question: What is a decimal? They were asked to brainstorm individually and think about any and everything that related to decimals. They were told that they would not be marked on the entry. but all entries would be read and. based on the content. programming would be set up accordingly.

Even though a math class usually lasted for about forty minutes. since this was an introduction. the lesson only lasted about 20 minutes.

Observation Notes

After I assembled the students at the carpeted area. I explained that they were to write out a journal entry answering the question "what is a decimal?" I asked the children if they had any questions. Immediately. Alice asked if she could talk about something else like money — specifically. change. I answered affirmatively.

Carly expressed concern about the fact she didn't know what decimals were at all. Lynn wondered how she could write about something she didn't know very much about. I asked her. if that was the case. to write out her feelings about decimals. or the fact that she didn't know anything about them. I also suggested that if they were really stuck for ideas. that they could ask questions regarding decimals.

During the journal writing portion of the lesson. Carly wrote for the full 20 minutes. She wrote with confidence. Alice and Lynn wrote intermittently. Tamar wrote almost non-stop. Towards the end. she started to play with her kleenex box and her Halloween toys. Overall. once the students started. they felt rather comfortable with the assignment.

Journal entries

Alice

Alice's entry displayed a primarily procedural understanding. She recited the basic place value vocabulary as it related to decimals but was unable to. or at least did not describe what a tenth. or a hundredth actually was:

> A decimal sep[a]rat[e]s the hundreds. tens and ones column from the tenths. hundre[d]ths and thousa[nd]ths column. . . . Decimals have to do with place value . . . Place value is important because it tells you where to put hundreds. tens and ones.

Although it was primarily procedural. her description of a place value chart was indicative of her excellent observational skills. There was a large place value chart that sat up by my desk at the front of the room.

Mid-way through her entry. she moved to a conceptual and situational explanation when she talked about money.

> When you go shopping a decimal sep[a]rates the dollars and the change. Decimals are very important because without the[m] you couldn't divide some columns from other columns and cash from change. . . Without decimals you wouldn't have any change.

Although Alice had a good initial understanding of where one might find decimals, this journal entry indicated that her conceptual understanding was still limited.

Carly

She started off demonstrating an initial. situationally based. very limited understanding of decimals through the use of money. However. she confused the concepts of cash and change. She did have a general idea that a decimal separated one kind of thing from another. She stated:

> A decimal is an important dot that sep[a]rates change from dollars. . . With a decimal it is e[as]y to tell how much change you have and how much cash you have[. W]ithout a decimal it would all be put together.

In the process of trying to remember something, or rather anything from the past about decimals. she then reverted to a more procedural understanding by stating. "...a decimal is used in place value[. A]nything before a decimal is millions. thousands. hundreds. tens or ones. Anything after a decimal is tenths. hundredths or thousandths."

Like Alice. Carly sat near the place value chart by my desk. She too. was able to recite place value vocabulary without being able to provide much of a conceptual explanation. At one point. Carly. in her explanation of money. attempted to describe her procedural understanding and. because it was merely procedural. became confused. She thought that. in scientific notation. the decimal or part of the whole numbers were represented by the numbers to the left of the decimal. and the whole numbers were represented by the numbers to the right of the decimal. She stated.

> If you just wan[t] to read change. you just look on the left side of the decimal. If you wanted to say 50 it could be .50 or .5. they both stand for 50. If you wanted to say 5 you would put a zero after the decimal then put 5.

Carly showed caution and wrote only what she felt certain about. She knew that decimals related to money. This had been confirmed for her during the earlier discussion at the carpet. when Alice had asked the leading question. whether or not she could talk about money. Secondly. she spoke about a procedural understanding of place value. Again, this was a safe discussion for Carly since the place value chart in the room had the word 'decimal' on it and an arrow pointing to the decimal which sat between the ones and the tenths column. In essence, Carly's entry was not so much a brainstorming as a safe recounting of facts that she knew for certain were correct.

Lynn

Lynn saw the question from an almost completely situationally-based premise. She wrote about decimals as they related to percentage, marks and calculators:

> I think decimals are if you are saying a percentage of something and it is below 1 than you put zero, point what the numbers [are], or if [you] were being marked on a test and we got half a mark than it would be your mark then point 5. [I]f you['re] measuring som[e]thing you may write 2m.02 cm, and if you want to find the square root of something on a calculator and it [is] a number that can't go in[to] 2 than on a calculator it will show a number and then a decimal and then you will see other numbers and if you need to find what percentage of [something] you [would] calculate and you [would] get som[e]thing and a half. [Y]ou [would have a] percentage [which would] have a number decimal and then more numbers.

Her understanding of square roots, albeit procedural in nature, demonstrated a limited awareness of yet another kind of situation in which a decimal could be used.

Even though she became confused in the actual process of writing out a whole number and decimal together, she demonstrated a more conceptually based understanding of a decimal being similar to a fraction when she spoke about a decimal being a number/percentage that is below 1. Her understanding of a half a mark being the same as .5 demonstrated a correct conceptual understanding of decimals being similar to fractions as well.

Lynn took my suggestion and saw this entry more as a brainstorming exercise. She wrote down anything and everything she suspected might be related to decimals.

Tamar

Tamar tried to remember where in life she had seen decimals and described these experiences accordingly:

> A de[c]imal is a dot that comes after three numbers or in an amount of money like $10.39, or if it[']s like 100.000.0. If there weren't any decimals you wouldn't know how to say 100.000.0 one billion or something like that or w[h]ether you say one t[h]ousand and thirty nine doll[a]rs or ten doll[a]rs thirty nine cents. So decimals are extre[me]ly important. They also tell you sort of what [time it is] because 2:30 could be 23 o[']clock. So they put two decimals like this :. A decimal tells you how to. read time[.] read money amounts[,] how to read high numbers like. 336.443.22.246. What ever number that would be! So decimals are really important.

Aside from her comment with regards to money and seeing a decimal between $10.39. she demonstrated an extremely limited understanding of decimals. She mistook commas for decimals in place value (she'd never lived in Europe to be confused with their way of expressing numbers). She mistook decimals for the two dots in a colon used in the notation for time.

Similarly to Lynn. she felt comfortable in this journal entry to brainstorm and share with me anything and everything she thought might be connected to decimals anywhere in life.

Summary and reflections

The four children in the study were particularly eager and enthusiastic about starting this first lesson. I'd contacted each child's parents just prior to this first lesson and both parents and students were eager to help out their teacher. or in their eyes .a fellow learner. This eagerness and enthusiasm spilled over into the rest of the grade five students as well.

The different levels of decimal understanding amongst the non-participants mirrored those of the participants. The range spread from a child who demonstrated a fairly good conceptual understanding which lead to a solid abstract understanding similar to Lynn. to a very limited conceptual understanding similar to Tamar.

Since the abilities were so similar between the participants and the non-participants, programming separately for the other non-participants was unnecessary.

Lesson #2

Purpose

Since most of the students' initial knowledge base included the concept of decimals as it related to money. I decided to start the unit on decimals with the addition and subtraction of money. Another purpose for this lesson was to bring up. indirectly. an opportunity to correct some of the misconceptions expressed in their journal entries through the next several lessons. Since lessons two and three were similar in nature. it's not until the conclusion of lesson #3. that the students were asked to answer the open ended journal question. "what is a decimal?"

This question was used again to see if they were able to demonstrate a greater concrete understanding of the relationship between decimals and money because of their work with the manipulatives.

Strategies/Procedures

Using photocopied Canadian coins and bills as counters. the students were asked to work in groups of two. They were to add up the first 10 items on the grocery receipts that they had brought from home. Paper and pencil were not allowed to be used in the computation.

Observation notes and transcribed audiotapes

Sitting at the carpet. I put the children into groups of two. Alice and Carly worked together in one group and Tamar and Lynn worked together in another group.

All the children approached the task with enthusiasm and confidence. Each of the grocery tapes indicated something slightly different. One tape included a cost reduction of an item because it was on sale. One tape included a cost reduction for a returned item. The children initially discussed these differences before they started to use their paper money to add up each of the items.

Alice and Carly had no problem exchanging different coin and bill values to add up the items on their grocery list. They worked together well and came to their total quickly and effectively.

Carly	The first thing is 59 cents.
Alice	Here's a quarter and another. Here's a nickel and cents — 6, 7, 8,9 and the other one is 59 cents isn't it?
Carly	Wait.
Alice	And the other one is 69 cents, but it was on sale for 59 cents.
Carly	Here's 2 more quarters
	(*Alice then put another nickel down and adds it, with 4 cents to the first 59 cents*)
Alice	We need to add it all together
Carly	No, we don't add it all together yet. You just leave it like this *(2 separate piles)*
Alice	No, now we have to add them together.
MEE	What are you going to do now?
Alice	We are going to put them all together.
Carly	That's a dollar *(taking the 4 quarters and putting them together)*
MEE	How did you get the dollar?
Alice	'Cause 4 quarters equal a dollar
	(*Turning back to her photocopied money*)
Alice	Then that's . . .
MEE	What's that? *(Thus prompting them to verbalize what they were physically doing with the money)*
Alice	Two five cents equal a dime
	(Alice pushed away 2 nickels that were in her 'together' pile and pulled 1 dime towards her)
	Then there is 1,2,3,4,5, 6, 7, 8 pennies which equals 8 pennies. There.
MEE	Great.

From their initial work at the carpet. Alice and Carly had a fairly solid understanding of the concept of regrouping and carrying pennies. nickels. dimes. etc.

Tamar and Lynn had a few more difficulties. General cooperation was a problem from time to time. Initially the girls wanted to do the work in their head. It's interesting to note that the way in which each child did her calculations was different but still ended up with the correct answer. However. neither child was willing to give in to the other child in terms of the process. However. once they were redirected by me to not do the work in their head and use only the concrete manipulative or paper money to find the answer. they started over again and worked together to arrange the money accordingly. However. when Lynn suspected that they had gotten the wrong total. she reverted back to mentally calculating the answer on her own to see if the two of them had figured out the correct answer using the paper money.

Tamar	2.99 plus .99 is. 99 and 99 is 2 dollars. so that's 4 dollars
Lynn	Here. it's like 3 dollars
Tamar	No. It's like 2 dollars plus 2 dollars
Lynn	No. that's 99 cents.
Tamar	No. 1 more cent and that's another dollar.
Lynn	So that would be 4 dollars. no 3.98 would be these two.
MEE	How are you doing?
Lynn	We are adding these 2 *(pointing to 2.99 and .99 on the receipt slip)* and then we're going to add these 2 to it. *(Pointing to the 3.84 and 1.29 on the receipt slip).* I rounded this up to 3 and then I plussed a dollar. which is 4 dollars and then minus two.
MEE	Before you go on. could you show me 2.99 with the money and then the .99 cents on top of that? Go ahead.
Tamar	2.99 — here's 2 dollars. 10.20,30.40 um. 50. 60. 70. 80,90. Now we need 9 cents
Lynn	Here's 5
Tamar	Now we need four cents — 1.2.3.4. So that's 2.99

Lynn	Why don't we use quarters?
Tamar	Why don't we change this *(the dimes and nickels for quarters)* — that's three quarters. 75. 85. 95 and I need 1.2.3.4 cents. So that's 2.99 plus .99. We need quarters for .99 cents. Quarters — 1. 2. 3, that's 75 cents. 85. 95 plus 4 cents which makes 99 cents.
Lynn	Why don't I do the next one. 3.84 — 3 dollars (*pulling out dollars*). I need another quarter.
MEE	(seeing that they hadn't finished adding up the first two. I asked them to do so now.) Now that you have the two separate piles. can you show me how you are going to put the 2.99 plus .99 piles together?
Tamar	We did 2 dollars plus 75 cents then we did 2 dimes which would be 95 plus 4 cents is .99. That's 2.99. Then we did another 25. 50. 75 (*counting quarters*). 85. 95 (*counting dimes*) and then four cents.
MEE	How are you going to merge them?
Lynn	2.99 plus .99 is . . .
MEE	Don't do it in your head. Normally that's excellent. however for this. add it together using the money
Tamar	Like that? *(She put the two piles together)*
MEE	That's a heck of a lot of change. Can we do anything about it?
Tamar	4 quarters is a dollar. so we can have that *(taking 4 quarters from the pile).* 50.60.70.80.90.95. 96.97.98..99 — another dollar.
Lynn	Why don't you start with 3 dollars (*they got rid of the pile of change Tamar just counted out and exchanged it for a dollar*).
Tamar	So the total is 4 dollars and two cents.
Lynn	Something's wrong. That can't be.
Tamar	This is a dollar and one cent and this is a dollar and one cent.
Lynn	Hear. listen to me Tamar *(said in a gentle tone).* you round this *(pointing to 2.99)* up to 3 dollars and you round this *(pointing to the receipt where it says .99 cents)* up to a dollar so that makes 4 dollars. but then you have to minus 2, so that's 3.98.

Tamar	3.98 oh . . . *(At that point. she counted out 3.98 in paper money).*
Lynn	I don't think we have the money we had in the first place. You're putting it all over here.
Tamar	3.98 there. Shall we start again?
Lynn	OK here's 2.00. here's 25. 50. 75 (*counting out quarters*)
Tamar	OK. so we have 75. 85. 95. 1. 2. 3. 4. That's 99 cents?
Lynn	This is 2.99
Tamar	Now we need another 99 cents
Lynn	Here's 50 cents. 75. 85. 95. now 1.2.3.4. Now we add this.
Tamar	Let's put all the rest of the money away. That pile is 99 cents and that pile is 99 cents.
Lynn	That is 2 dollars and 25 cents and 2 dollars and 50. 75 and that's 3 dollars and 25 cents. 50. 60. 70. 80. 90. 1.2.3.4.5.6.7.8 which equals 3.98.
MEE	Is that right? Can you change this so you don't have so many coins?
Tamar	Yup
MEE	How?
Lynn	You put them altogether.
Tamar	*(Counting out 4 quarters)* That's a dollar. so you put these back and get a loonie. You can get 25 cents from the pennies and nickels.
Lynn	You can't make another dollar with 98 cents. Check again. (*After reworking the coins and loonies and toonies.*)
Lynn	Now we have 3.98
MEE	Great. That's your 'smallest amount of change' pile.

It was apparent. for the most part. that Lynn and Tamar were able to regroup and carry as well. They understood the basic concept of adding up enough pennies until you could exchange them for nickels or dimes and so on.

At this point, since the majority of the students understood the basic carrying of money concept, at the end of the lesson on adding receipt entries, we discussed the difficulties as well as overall observations as a whole class. The full group discussion was fascinating on several counts. Tamar and Lynn were able to articulate their problems clearly.

MEE	What did you find hard with this exercise?
Lynn	We were getting the money confused with the money over here and stuff. We were flicking it out of their piles.
Tamar	We kept having to change dollars for two dollars for less coins.
MEE	So you were doing a lot of exchanging back and forth?
Tamar	Yes
Alice	That kept happening to us too. We probably mixed up some of the money, like two quarters from maybe a dollar.

Since the children had been working with the concrete manipulatives, or in this case the photocopied money for the last 30 minutes, I attempted to move their frame of reference from the manipulative, situated learning experience of exchanging change for bills, etc., to the more abstract level of thinking by asking questions which would have led them to look at the role of decimals within the context of money. It was a difficult transition because the students had to overcome the concept that, change was always the portion of money that was merely a part of a dollar. Loonies and toonies could also be considered change. However, once this problem was eradicated, it was interesting to note the direction they took with the questioning.

MEE	What does this have to do with decimals? (*Silence*) What did you have to exchange?
Tamar	Dollars for 25 cents and all that other stuff for dollars and toonies, like cents and five cents and ten cents.
MEE	What are all those dimes and stuff?
Alice	Change

MEE	What do we call change?
Lynn	Cash
MEE	What about change versus what else?
Carly	Bills
MEE	It's unfortunate for this lesson, but what have we created recently, that we used to call bills?
Alice	Loonies?
MEE	Great. What else?
Carly	Toonies?
MEE	Great. If we were going to make a distinction between loonies and toonies. Let me start again, first of all what is a penny, a nickel, a dime and quarter?
Tamar	A penny is 1 cent, a nickel is five cents, a dime is 10 cents, a quarter is 25 cents.
MEE	What do they all have in common?
Lynn	They're solid. They're not like paper money. They're not paper like a 5 dollar bill.
MEE	What else do they have in common? How is that money different from other money?
Carly	They're change
MEE	What's change? Describe change?
Carly	It's like, if you gave someone at a cash register a 20 dollar bill, and something only cost you 11.58, then you'd get the change back.
MEE	O.K. What do they have in common? Don't think physical appearance so much.
Carly	They're all hard except for the bills.
Tamar	They are all cents. They are all change.
MEE	What do you mean by all cents?
Tamar	Cents, pennies, quarters and dimes.

In order to change the direction of the questioning to encourage the children to make a natural division between whole dollars and change' or parts of a dollar, I changed my line of questioning. With the new line of questioning, Lynn quickly picked up on the concept by explaining that 'cents are decimals'.

MEE	Can a dime be turned into a loonie? (*Silence . . .*)
	How many pennies in a dime?
Tamar	10
MEE	How many pennies in a quarter Lynn?
Lynn	25, and they're the decimal part.
MEE	What does that mean 'the decimal part'?
Lynn	Well, the cents are decimals. Because there are bills and then there are the cents and the cents are decimals.
MEE	That sounds good to me.

I was hoping that the conversation would head towards the children gaining a greater understanding of 10 dimes in a dollar which in turn meant that 0.1 represents one dime out of a dollar. Instead, the conversation changed drastically, but came out with an even better conclusion than I'd hoped for! A child, separate from the children in the study group, answered my initial question by referring to the general notation of decimals. Through the course of the discussion, the children touched on the correct understanding of tenths, hundredths and thousandths! Also, as can be seen in the following dialogue, as a result of the turn in discussion, Carly's initial lack of understanding of decimal notation demonstrated in her initial journal entry was corrected in such a way that she was not put on the spot, but still had her faulty construct corrected!

MEE	There is the dollar part, and then you have the decimal part. The decimal part is what? Can we figure out from here what a decimal is?
	[Another child: "The part after the decimal."]
MEE	That's right, it's the part after the decimal — everything that is on the what side of the decimal?

Carly	The left
Alice	(*Raised hand almost out of her arm socket obviously feeling that Carly's answer was wrong.*)The right?
MEE	On the right side of the decimal. What else. they are the pennies and they are all what? You've almost got it. They are all pennies. They always make up . . . . .
Tamar	If you didn't have nickels. quarters or dimes. if something was 1.25, you'd have to put down a dollar and. like 25 pennies.
MEE	OK Having different denominations of coins helps us not have to carry so many pennies around.

At this point. I was hoping the children would refer to the change as being a part of the whole. They went in another direction instead.

MEE	However. look at the change. Forget the loonies and toonies, forget that they are change. each of the change items are what? They are the decimal and they are the . . .?
Alice	The tenths. hundredths and thousandths?
MEE	Oh. she's been listening to my teaching the 6's!! Well done!
Alice	No. I've been looking at the chart (*she pointed to the place value chart on the wall*). I remember looking at it last year as well.
MEE	Excellent. There are the tenths. the hundredths and thousandths. How many pennies are in a dollar?
Lynn	100 pennies
MEE	What is a part of something? What is a penny a part of?
Carly	A quarter
MEE	What's a quarter a part of?
Tamar	A quarter is a part of a dollar?
MEE	Could a penny be a part of a dollar?
Alice	Yes

MEE	What else could be a part of a dollar?
Carly	A dime
MEE	What else?
Lynn	a nickel
MEE	Could two quarters be part of a dollar?
Lynn	Yup.

Even though most of the children briefly touched on the specific decimal notation, they seemed to have a fairly solid understanding of change, ie., pennies, nickels, dimes and quarters being a part of a whole. The conceptual idea behind a tenth being ten parts of a whole, etc. had not yet been realized.

Summary and reflections

Alice and Carly worked well together and demonstrated a conceptual understanding of the addition of money while using concrete manipulatives. Sarah and Lynn had more difficulty perhaps as a result of the initial differences in understanding. Where Lynn had already conceptually understood the addition of decimals and wanted to figure out the answer on a more abstract level. Tamar felt more comfortable staying and working at the concrete, manipulative based level.

The other two sets of non-participant pairs worked well together, but experienced similar difficulties. The non-participants experienced difficulties in exchanging change for change and change for bills, or the whole.

The final discussion was unexpected. Where I had thought that the students might gain a grasp of a tenth being one tenth of a whole part, the abstract concept of decimal notation was discussed instead. Even though there was a limited understanding behind the notation introduction, at least they'd been introduced to it within the context of decimals.

Lesson #3

Purpose

The students demonstrated a fairly solid understanding of exchanging pennies for nickels, nickels for dimes. etc. Some of the children hinted at the idea that pennies, nickels, dimes, quarters were a part of a whole. I decided to start the next lesson with the addition of the last 10 items on their receipts. This time. they had to use the base ten blocks instead of photocopied money. This would eliminate the concept of nickels and quarters and build up the children's base ten understanding. Before the lesson started. an initial review of the previous day's work was covered.

Strategies/Procedures

Using base ten blocks. the students were asked to work in groups of two to add up the last 10 items on the grocery receipts that they brought from home. Similar to yesterday's lesson, the use of paper and pencil during the computational portion of the lesson was not allowed. They had to use the base ten blocks exclusively.

Observation notes and transcribed audiotapes

After the initial review and introduction of the lesson, the children were paired up. The students worked on this task for about 25 minutes. The groupings remained the same as yesterday. Alice worked with Carly while Tamar and Lynn worked together. Changing from the concept of adding decimals using money to the more abstract concept of using base ten blocks was a slight challenge. although not a serious impediment to the students.

Carly and Alice remained on task throughout the lesson. Both students worked hard at describing each and every movement they made. Tamar and Lynn had difficulties determining a way to do the actual work. Listening to the tapes. it was apparent that Tamar and Lynn overheard what Alice had said about the conversion of money to base ten blocks and they started to add up each of their items. As a result. each of the two groups figured out the conversion in very similar ways. Alice and Carly summarized the situation most succinctly. Their initial question was $1.99 + $2.99 +$1.99. The conversation was as follows:

Alice	How will this work? I guess that a flat would be the same as a dime, no, as a dollar a flat would be a dollar. A dime would be a ten stick and a centicube would be . . .
Carly	We need nine ten sticks. 1, 2, 3, 4, 5, 6, 7, 8, 9 centicubes and one flat.
Alice	Here is the one flat
Carly	We need 2 flats — 9 ten sticks. 1, 2, 3, 4, 5, 6, 7, 8, 9 and nine centicubes
Alice	1, 2, 3, 4, 5, 6, 7, 8, 9
Carly	We add them together
Alice	We take 10 ten sticks and exchange it for a flat.
Carly	Take 10 centicubes and exchange it for a ten stick. Then take 10 ten sticks and exchange it for a flat. (*For the third item from the bottom*) Take a hundreds flat and 9 ten sticks and 9 centicubes.
Alice	Our total is 6 flats, 9 ten sticks and 7 centicubes or $6.97.

The full group discussion following the activity was particularly helpful in furthering the children's understanding of decimals. The children were asked what materials they found easiest to work with. The answer I had been expecting was the base ten blocks because there were less 'denominations' to work with than the money. Lynn came up with this answer, but the other answers I got were insightful as well, and allowed me once again, to better understand all the participants' mental constructs.

MEE	Can someone tell me what you found easier, adding with the paper money or adding with centicubes?
Alice	I think that adding with the money is easier because there is one cent, and five cents and ten cents and twenty-five. So you didn't have to wait as long to get the

other thing. Like, with the cubes, you had to wait until you got 10 cents to get a ten stick, but with the money, you only had to wait to get to five cents.

Tamar I thought the base ten blocks were easier, because all the money kept getting mixed up, but the blocks weren't getting mixed up. They wouldn't blow away.

Lynn I thought the base ten blocks were easier, because it was easier to count with only three of them, not with the 1, 5, 10, and 25. So if you had 10 of the ten sticks then you would get one flat, instead of 4-25 cents.

MEE So you found it easier to work with fewer items in the base ten blocks than with the four items in money.

Lynn Yeh.

Carly I think the money was easier cause it's like, if you had 9 centicubes, you couldn't exchange it for a ten stick, cause a ten stick has ten in it, but if it was 9 pennies, you could exchange 5 pennies of it, 5 pennies for a nickel, and you wouldn't have as much single stuff left over.

MEE At the beginning of today's lesson, what did you ask yourself? (*Silence*)... When I asked you to work with the base ten blocks, instead of the money?

Carly How am I going to do this?

MEE OK, what was the problem?

Carly I don't know.

[Another child: "Which kinds of blocks should I use for the dollars and the cents?"]

MEE O.K. What block am I going to use for dollars and cents. What else?

Alice What am I going to use for all those cents?

Lynn What am I going to use for the decimals?

MEE What do you mean by that?

Lynn Well, the cents are a decimal. What am I going to use for a decimal.

MEE Can you explain something to me — there are two columns after the decimal. What do the columns mean? Give me the answer in money, not according to the place value chart. What column means what?

Lynn	The column to the right [of the decimal] is the ten cent column and that one [pointing to the hundredths column] is the one cents column. The column to the left of the decimal is the one dollar column.
MEE	Suppose you had no cents and ten cents, what would that be?
Carly	dot zero one?
MEE	I have a question. What is bigger, dot zero one or zero dot one?
Tamar	dot one zero?
MEE	Why?
Tamar	If it's dot zero one, that would be one cent and dot one zero would be ten cents.
MEE	How many dot ones would you need to make a whole?
Alice	Ten?
MEE	Show me.
Alice	10 ten sticks equals a flat and 10 centicubes equals a ten stick
MEE	So 10 centicubes equals a ten stick. What's the whole in this set up?
Alice	These 10 flats are a whole loonie.

Lynn's comments at the end were excellent. She didn't know how money related to decimals until she realized that change was considered to be decimals. Tamar wasn't sure how to write out a decimal. Earlier in the discussion, she thought that $1.80 would be written 100.80. When I asked the group how you could write one dollar and eighty cents, they had difficulties. Another child in the group understood that 100.80 would in fact be 100 dollars and not one dollar.

Some other questions relating to the way in which the scientific notation was to be recorded were answered for Carly when the issue came up and her initial concept of the decimal being to the left of the decimal point instead of to the right of the decimal point was corrected by Alice during the group discussion.

However. based on the actual lesson. it seems as though the students were still having problems understanding the concept of change being part of a dollar.

Journal entries

The following journal entries answered. once again. the question "What is a decimal?" The children were allowed to draw on their work with partners. information gained from class discussions and work with manipulatives. Since the children had been exposed to adding with base ten blocks as well as the centicubes. the intent of these entries was to allow me to see how well the children had solidified not only the concept of the addition of decimals. but ultimately. change as being *part* of a whole and. hopefully. the concept that pennies are hundredths and dimes are tenths.

During the actual journal writing process. Lynn felt somewhat frustrated because she wasn't sure what to write about. Tamar. Alice and Carly wrote fairly continuously for the full 20 minutes.

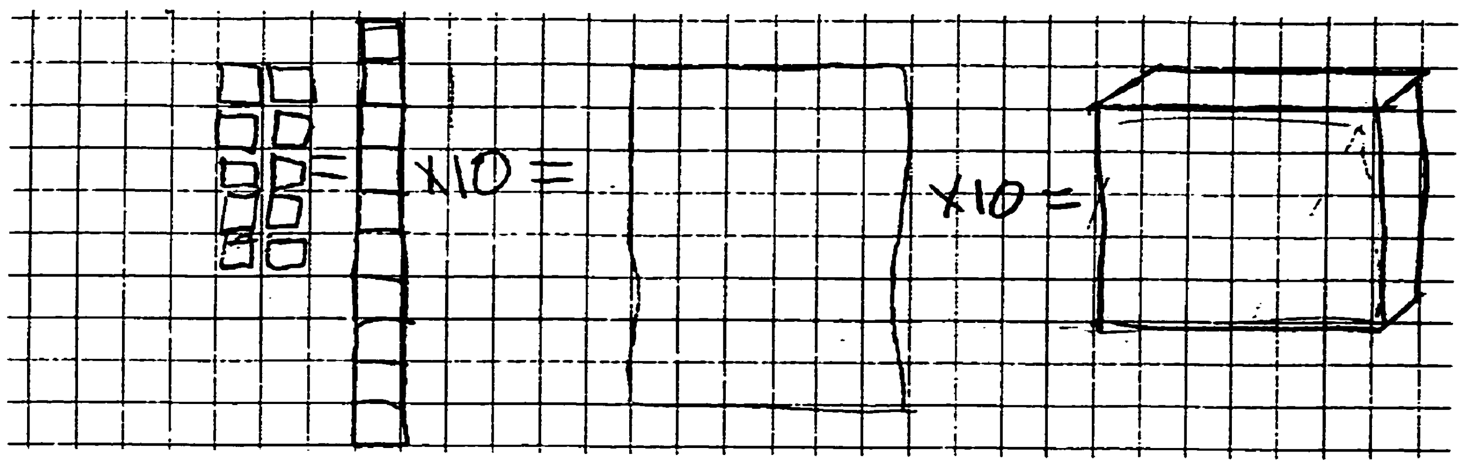
Alice

Alice wrote her entry from a procedural perspective when speaking of place value and addition within the context of decimals rather than actually answering the question with regards to what a decimal really was — ie.. decimals being a part of a whole. She stated that. "when you have 10 centicubes it equals 1 ten stick. when you have 10 ten sticks. it equals 1 flat and when you have 10 flats it equals a thousands cube."

Her picture. supporting her understanding of equivalency was fairly straightforward. After her to-scale picture of the centicubes and ten stick. she decided to make the flat and thousands cube symbolic in size. but inserted the numbers for clarification. (See figure #1).

Her conceptual understanding of the addition of decimals within the framework of place value was fairly good. She stated that:

Figure 1 **Alice, Lesson 2**



> When you're using decimals [.] the first column to the right is the column where you put dimes or tens sticks and the column beside that is the column that you put pennies or centicubes. The columns to the left of the decimal is where you put the hundreds flat which is like a dollar.

Compared to her first entry, this entry demonstrated a greater conceptual awareness. Where she initially spoke in a procedural manner about place value, in this entry she demonstrated a brief description of how money, particularly dimes and base ten blocks, works in relation to decimals. As well, in this second entry, she was able to make the distinction between where the dollar amount and the change goes in relation to the decimal point.

Carly

Similar to her oral input given during this lesson, Carly understood the concept that change was a part of a whole. However, she tried to apply procedural rules to her concrete understanding of what a decimal was and became procedurally mixed up. She understood that 10 dimes made a dollar and that 100 pennies made a dollar, but she confused the tenths column for the tens column and the hundredths column for the ones column. She stated:

> A decimal is change. On the right there is 2 columns the tens colum[n] and a ones colum[n]. .1 is bigger than .01 because .1 is in the tens colum[n]. .01 is in the ones colum[n]. To have a dollar before the decimal you will have to [have] ten dimes and to get the dimes you will need 100 pennies. Ten pennies = a dime. 10 dimes = a dollar. It is just like 10cm cube = a ten stick and 10 ten sticks = a flat.

Compared to her last "what is a decimal?" entry, she showed a greater conceptual understanding when she stated that a penny was of less value than a dime. This gave her a hint that the further one goes to the right of a decimal, the lower in value the number becomes. She also indicated a fairly solid understanding of base ten, place value and addition. However, her understanding of what a decimal is, in terms of change being part of a whole loonie, was still weak. She hinted at this idea by stating that "a decimal is change", but she didn't elaborate further.

As a result of the manipulative work in lesson #2 and #3, this journal entry demonstrated the correct concept that decimals, or the parts of the whole, were to be recorded to the right of the decimal instead of to the left as she had indicated in her initial journal entry.

Compared to her first journal entry in which she merely referred to the concept of place value as it related to a picture on the board, she started to demonstrate an initial awareness of a base ten structure and decimal notation within this journal entry.

Lynn

Compared to her previous "what is a decimal?" entry, Lynn demonstrated a more specific conceptual understanding of what a decimal is as it related to money in this entry. She stated:

> A decimal is the part after the whole, like in money the doll[a]r is the whole and the cent is the decimal... The decimal says how much you have leftover...A decimal is always after the dot.
>
> With base ten blocks the cent[i]cubes, ten of them equal a ten stick[,] a[nd] ten ten sticks equals a flat, a flat is like a dollar and a ten stick is like ten cents and cent[i]cubes are like cents.

She then started a lengthy explanation of the scientific notation for decimals.

> You can tell what the number is after the dot and before. Before the dot if it is a one, than you put it with a zero before the number and if its a ten it comes before the zero and same with the hundreds only you have to put more zeros after the number same with the thousands.

Overall, this journal entry confirmed that her oral understanding, demonstrated during lessons #2 and #3, was soundly based. Compared to the first journal entry in which she answered the same question, she further refined her understanding of decimals by being able to describe the concept of decimals being something that is "left over".

Tamar

After lessons two and three, it was difficult to know exactly where Tamar's conceptual understanding of decimals stood. She volunteered one or two answers during the course of the group discussion regarding the procedural, scientific notation of decimals, but she volunteered little else. During her work with her partner, much of the work was done by her partner.

The following journal entry confirmed my suspicions that she didn't fully understand the material covered. Also, as a result of this journal entry, it became apparent how she was trying to figure out this concept. She tried to find the answer through the procedural process rather than truly understanding the concept first and then being able to make the natural transition to the abstract. She took the activity of adding change and dollars together, as well as centicubes and ten sticks together and used the addition process to describe what decimals were. She stated:

> A decimal is a dot like this . That is used mostly in money amounts or like 10 centicubes is 1 ten stick and 10 ten sticks are 1 flat and 10 flats are a 1 thousands cube.
>
> You would read one dollar and eighty cents like this 1.80 or 1.8.

The activities in lessons two and three expanded her concept of decimals from almost nothing to the understanding that decimals were found in money. However, she saw the addition of decimals with base ten blocks and money as a place value exercise. This journal entry confirmed the fact, previously hinted at in the lessons, that she was not yet able to view decimals as being a part of a whole.

Summary and reflections

Even though the students worked through the addition of their receipts with a greater sense of ease compared to their work in Lesson #2, and even though each child demonstrated a greater oral understanding than in Lesson #1, this did not necessarily translate into a greater conceptual understanding being demonstrated in their journal entries.

Alice was the only participant in the study group whose journal entry exceeded her oral expression of understanding during group work. One other non-participant child also exceeded her oral expression of understanding in her journal entry as well.

Carly confirmed her oral understanding of place value in her journal entry, but got confused on the scientific notation portion of the process. Lynn's journal entry clearly matched her oral understanding expressed in class, demonstrating a solid conceptual understanding. Tamar's oral understanding of decimals matched her journal entry, but she was still not able to identify decimals as being part of a whole.

The non-participant grade 5's, for the most part, had journal entries that matched their oral expression of understanding of decimals in the group discussion.

Analyzing the relationships between students' oral and written expressions of understanding was helpful. Whereas I would have thought that their oral expression would have displayed greater understanding than their written expression, the level of understanding expressed in both mediums was much the same.

Lesson #4

Purpose

After having had the children add decimals for the last two days using money and base ten blocks, I thought that I would introduce the more abstract concept of adding decimals on paper. An initial review of the previous day's work was covered.

Strategies/Procedures

Three addition questions with decimals were written horizontally, one at a time, on the board. The three questions were: 4.3 +3.06, 12.34 +11.49 and .93 +.99. The children were asked to discuss with their partner strategies to come up with the answer while using the base ten blocks as tools. The children worked on this for 35 minutes. At the end of the lesson, the children were asked to explain, in a journal entry, how to add decimals. This took the children about 15 minutes.

Observation notes and transcribed audiotapes

Throughout this lesson. it was apparent that the kids knew how to mechanically add the different questions.

MEE	We've done adding of grocery receipts. We used money and base ten blocks. I'm going to write two numbers on the board and I want you to pull out the blocks and discuss with your partner how you are going to add the two items up. We 're going to come back together in a few minutes and you are going to tell me what you did. There's not just one right way of doing it . Please keep in mind that there's different ways of doing this.
	[4.3 ÷ 3.06 was the first question put on the board.]
	[*Alice worked with Lynn and Carly worked with Tamar . . .*]
Lynn	Let's start with 4 + 3
Alice	No. not yet. you take 4 hundreds blocks. no flats and 3 ten sticks. 3 flats and 6 one centicubes
Lynn	So that's 6 and then plus them.
Alice	Add them together so we take 4 flats and 3 flats which is 7 flats and then we take 6 ones
Lynn	And we put it on to that. umm ...
	[*At this point. she became concerned because the pile of blocks in front of her was disorganized.*]
Alice	We're supposed to do it in columns. If you had the ones column and then you had the tens column and you add it together.
Lynn	Yes. are we ever smart.
MEE	Can you describe what you did?
Lynn	There's nothing to describe because you're not carrying. You don't have to get 10 of these (*centicubes*) and get one of these (*1 ten stick*). or 10 of these (*ten sticks*) and get one of these (*a flat*).
	[As a group . . .]

Tamar	We took 4 flats for 4.3 and 3 ten sticks.
MEE	Why 3 ten sticks?
Carly	Because .3 stands for 30 but it just doesn't have a zero at the end.
Tamar	and then for 3.06 we took 3 flats and 6 centicubes because it's not 6.0, it's .06 and then we added it together.
MEE	How did you add it together?
Tamar	We put the 3 flats onto the 4 flats.
Carly	Then you have 7 [*flats*] thirty-six.
MEE	Why was it easy?
Tamar	Because you didn't have to exchange stuff.
Carly	There wasn't enough to get a thousands cube and there wasn't enough of these (*10 sticks*) to get a flat.

Even though the basic addition was clear, what wasn't always clear was that the decimals, or the numbers to the right of the decimal point, were parts of a whole and the numbers to the left of the decimal point represented the whole number. They could add by carrying, but they couldn't make the connection between what the whole number was and what or if, there was a portion of a whole number.

Related to this point is the fact that the students had difficulties understanding that the hundredths column was not the ones column and the tenths column was not the tens column. Similarly, the tens column was not the thousandths column. Lynn and Alice discussed the situation as follows:

MEE	Explain how you did it.
Lynn	First you have to add 12 and 11.
MEE	What's the question?
Lynn	12.34 plus 11.49.

Alice	We got 12. I mean a thousands cube and 2 flats and 3 ten sticks and 4 centicubes. We took a flat and a thousandths cube and we took 4 ten sticks and 9 centicubes then we added them together.
Lynn	Then we got another 10 stick.
Alice	We exchanged 10 centicubes for a ten stick.
MEE	How many centicubes do you have in total?
Lynn	We had 13.
MEE	Then you counted out how many?
Lynn	10
MEE	O.k. Go ahead.
Alice	You then take the 10 sticks. there are 7 and you put them in the tenths column.

By this section of the transcript. Alice was able to understand the concept of the tenths column and not the tens' column. It will be interesting to note if this beginning use of vocabulary. not prompted by me. shows up in her journal entry thus demonstrating the beginning of a more conceptual understanding.

MEE	How many ten sticks did you have left over?
Alice	7
Lynn	8
Alice	Oh yeh — 8
Lynn	Then we had 3 centicubes because we took away 10 from 13 so we had 3 centicubes left over.
Alice	Our answer is 23.83

On the other hand. during the class discussion. Alice suddenly became stuck on the place value names for decimals. She thought that the centicubes represented the ones column which was true in money. but certainly not in measurement or time. It's apparent too. that almost all of the participants had the same difficulty. When I tried to ask the group more conceptually-based questions like. "what does the 3 represent?" (meaning 3 tenths as a fraction) the group couldn't completely understand this. They had a beginning understanding of the concept without understanding the whole picture. They seemed to understand that. being base ten, you were supposed to continue to add from one column to the next. They hadn't yet discovered that the tenths and hundredths columns were differing parts of the whole. The conversation was as follows:

MEE	Once you've got the piles. what do you find the easiest thing to do. What order do you add them?
Alice	You just add the columns together.
MEE	Which columns?
Alice	Ones. tens and hundreds...
MEE	What's the ones column?
Alice	The centicubes
MEE	Do we call it the ones column?
Alice	Yeh
MEE	Does anybody have a different name for it?
Carly	It's the column that has something below ten in it.

It was evident that the students were having difficulty in understanding the conceptual basis behind what each column in the scientific notation stood for. In an attempt to have them discover that their tens and ones columns were really the hundredths and tenths columns, I asked the students.

MEE	Suppose I gave you this number — 12.34. What's the 2 column? [Another child: "The ones column."]
MEE	What column is the 4?
Tamar	The ones column.
MEE	So the 2 and the 4 are in the ones column.

Lynn	The 2's the hundreds column, no it's the thousands column cause it's part of the 12.
	[Another child: "The 2 is in the bigger ones column."]
MEE	So you think there is a bigger ones column and a smaller ones column.

In spite of the fact that Alice was the first participant to verbalize the misconception, she was also the first student to understand the correct concept. As can be seen by the discussion below, Carly started to demonstrate a greater understanding of the material covered.

Alice	I think it's wrong. I think the 4 is in the hundredths column and the 3 is in the tenths column, the 2 is in the ones column and the one is in the tens column.
Lynn	That's what I was going to say.
MEE	Do we agree or disagree with that?
Carly	Yeh, because the decimal is there and the 3 and 4 are BEHIND the decimal.
MEE	What does any number in the tenths column mean?
Lynn	It's a decimal and it's after the whole.
MEE	How many parts of a whole is this? [*pointing to the .3*]
Tamar	3?
MEE	How many parts of a whole is it?
Lynn	An extra 7 would fit into one whole.
MEE	You mean if you took the 3 and added 7 more it would be one whole?
Lynn	Yeh.
MEE	What kind are we talking about?
Lynn	3 tenths.
MEE	If this is 3 tenths, what is the 4?
Alice	The hundredths column.
MEE	How many of these would you need to fit into a whole [*pointing to the hundredths column*]?
	How many hundredths would you need to fit into a whole?

[Another child: "100."]

MEE If you're not sure, think money. What's that? [pointing to the hundredths column]

Lynn Pennies column.

MEE How many pennies are in a dollar?

Carly 100

MEE This column [pointing to the tenths column] will always represent the . . .

Alice dimes?

MEE How many dimes are in a dollar?

Tamar Ten?

Tamar A nickel can fit into it too.

MEE That's true. How much is a nickel?

Tamar 5 cents

MEE When does something no longer fit just into the hundredths column?

Tamar When there's 2 nickels.

MEE Exactly.

I was becoming concerned that Tamar had become less verbal over the past few days. At first I thought that she was feeling intimated by the other children who were becoming more verbal, but when I watched her working with the cubes, she seemed to get more out of the learning process when she had physical manipulatives to work with as opposed to being directed by me or her classmates. I was discovering however, that for this kind of a child, journal writing provided me with an excellent assessment tool that allowed me to better understand her constantly modulating mental mathematical constructs.

Journal entries

Alice

By the conclusion of the lesson the idea of place value and decimals had been covered. It's obvious that Alice took what she learned during the lesson and applied it directly to an addition question using a place value chart. (Even though the terms listed on the place value chart were mentioned during class, at no point were the words 'place value' discussed during the lesson.) Her explanation before the diagram explains the diagram perfectly. (See figure #2). She stated:

> For the question 4.34 + 2.96 you start with 4.34 and you put the number 4 farthest to the right and put it in the hundredths column[. T]hen you take the 3 and put it in the tenths column[. T]hen you take the other 4 and put it in the ones column with the 2.96[. Y]ou take the 6 and put it in the hundredths column then you take the 9 and put it in the tenths column and put the 2 in the ones column[. T]hen you add them together by adding up the hundredths column first then so on . . . and you[r] answer is 7.30.

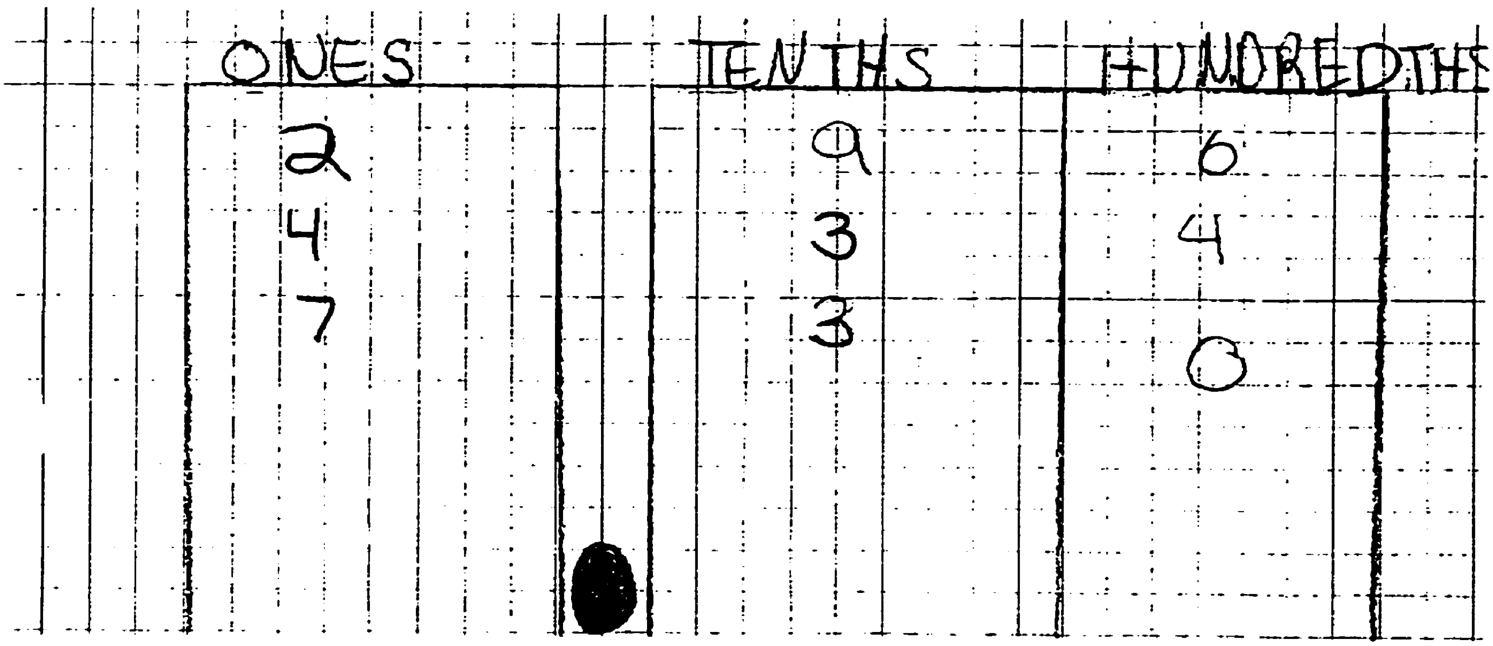
Even though Alice had some difficulties during the lesson understanding the difference between the hundredths versus the ones column and the tenths versus the tens columns, she eventually figured it out. This journal entry confirmed her more procedural knowledge of place value to a point where she could clearly and precisely verbalize her findings/ understanding. However, she was still unable to demonstrate an understanding of what tenths or hundredths column conceptually represented in her journal entry.

Carly

Whereas she had experienced earlier difficulties understanding that a) the decimal portion of the number was to the right of the decimal and not to the left and b) that the hundredths column was not the ones column, through this journal entry, she clearly understood the name and purpose of each of the columns. The actual process of writing the entry seemed to solidify her correct, mathematical construct of decimals. She stated (through the scribble):

> To add a decimal you have to add up colum[ns]. For example 4.34 + 2.96 first you add up the hundre[d]ths colum[n]. 4 + 6 = 10. Then you add up the tenths colum[n] 3 + 9 = 12. Then you add up the ones colum[n] 4+2=6. Then you add it all together.

Figure 2 **Alice, Lesson 4**



At this point, she decided to abandon her explanation and draw the question in order to further explain the carrying portion. Midway through the drawing, she explained the carrying process:

> You exchange 10 cm cubes for a 10 stick. Then you exchange 10 ten sticks for a flat. Then you have your answer. But before your answer you have to add the blocks together. Your answer is 7.3.

She then proceeded to draw a picture of the answer. (See figure #3).

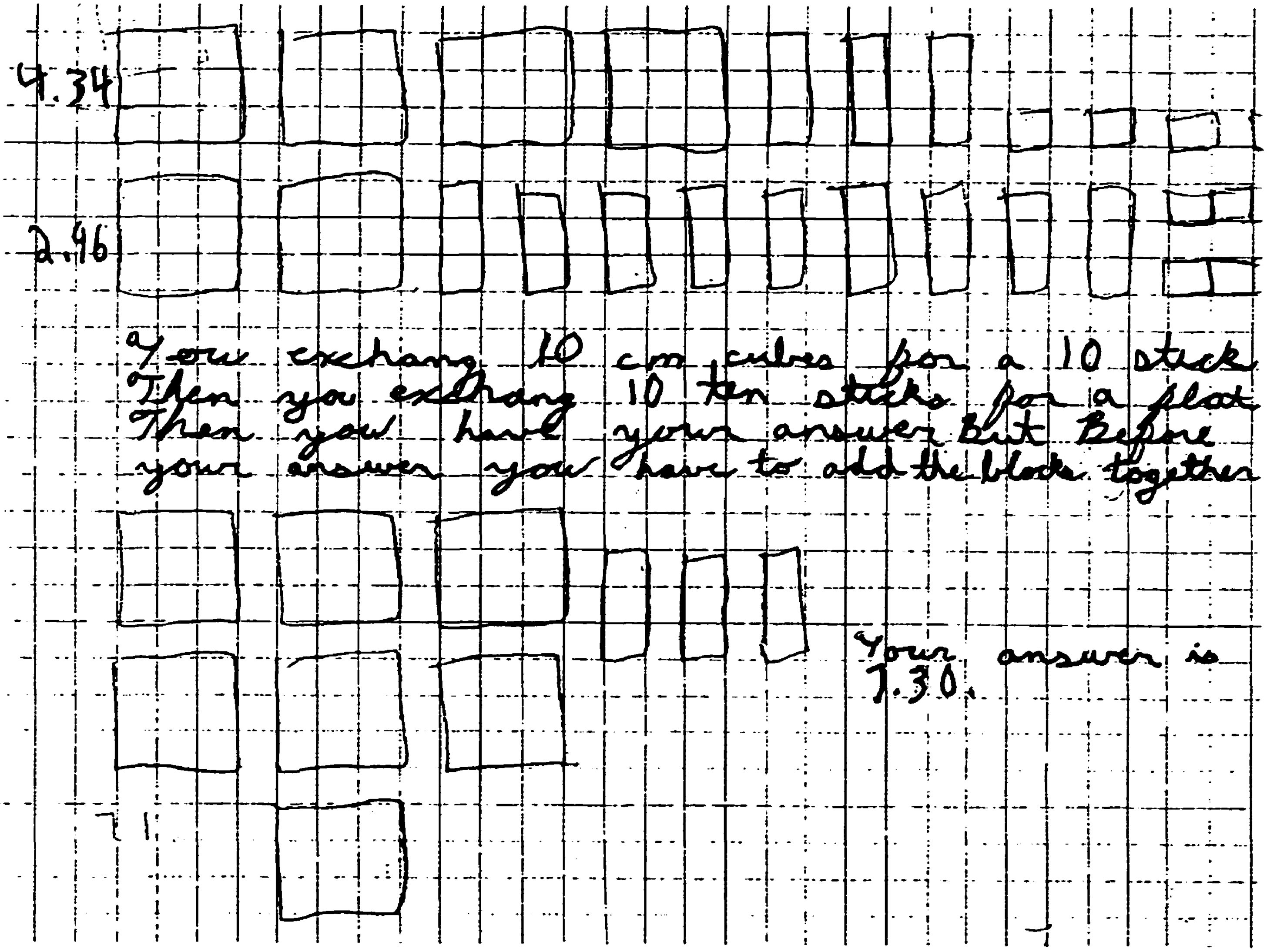
Lynn

Throughout this lesson, Lynn orally demonstrated a solid understanding of the concept of decimals. She volunteered the information that if you had .3 or 3 tenths, you would need .7 or 7 more tenths to make one whole. She also understood that the hundredths column meant that you needed a hundred of something (her example was pennies) to make up a whole (dollar). She solidified this conceptual understanding in her journal entry. She explained the concept of exchanging parts of a whole if they were numbers to the right of the decimal, or the wholes themselves if they were to the left of the decimal. She started off by saying,

> If you were to do 4.34 + 2.96 you would add up the hundreds (4.6) w[hi]ch is ten and you can make a tens stick with that because a tens stick is made out of ten cent[i]cubes. [T]hen you add up the tens colum[n], (9+3) w[hi]ch equals 12, plus the ten stick that was carried over from the ones, so you will have 13 tens sticks and you take ten of the tens sticks and make a flat, w[hi]ch is made out of ten tens sticks and a hundred cent[i]cubes, so you have 3 tens sticks and no cent[i]cubes. Then you add the ones up w[hi]ch is 4 and 2 plus the flat you carried so it is 4 flats plus 2 flats plus 1 flat and that is 7 flats and the 3 tens sticks from before so the answer is 7.30. You put the de[c]imal always before the tens colum[n] because you can't [put] hundreds into 1 hundred. The cent[i]cubes are called the hundreds colum[n] because you fit a hundred ones in a hundred.

One difficulty, evident in this entry, is spelling. She understood what tenths were but then proceeded to misspell it when she stated that "...you put the decimal always before the tens column." Similarly, she demonstrated an understanding of hundredths when she stated that "...centicubes are called the hundreds column because you fit a hundred ones in a hundred [cube]". She demonstrated a conceptual understanding of the whole being the cube and a

Figure 3 **Carly, Lesson 4**



hundredth being one, one-hundredth of the cube, but when it came to the scientific notation, she spelled the place value terms incorrectly.

Tamar

Tamar was not very verbal during the manipulative portion of the math class. She demonstrated a manipulative and visual understanding rather than an oral one, as was evident by her intense concentration on the manipulatives during class time. Her journal entry supported this idea as well.

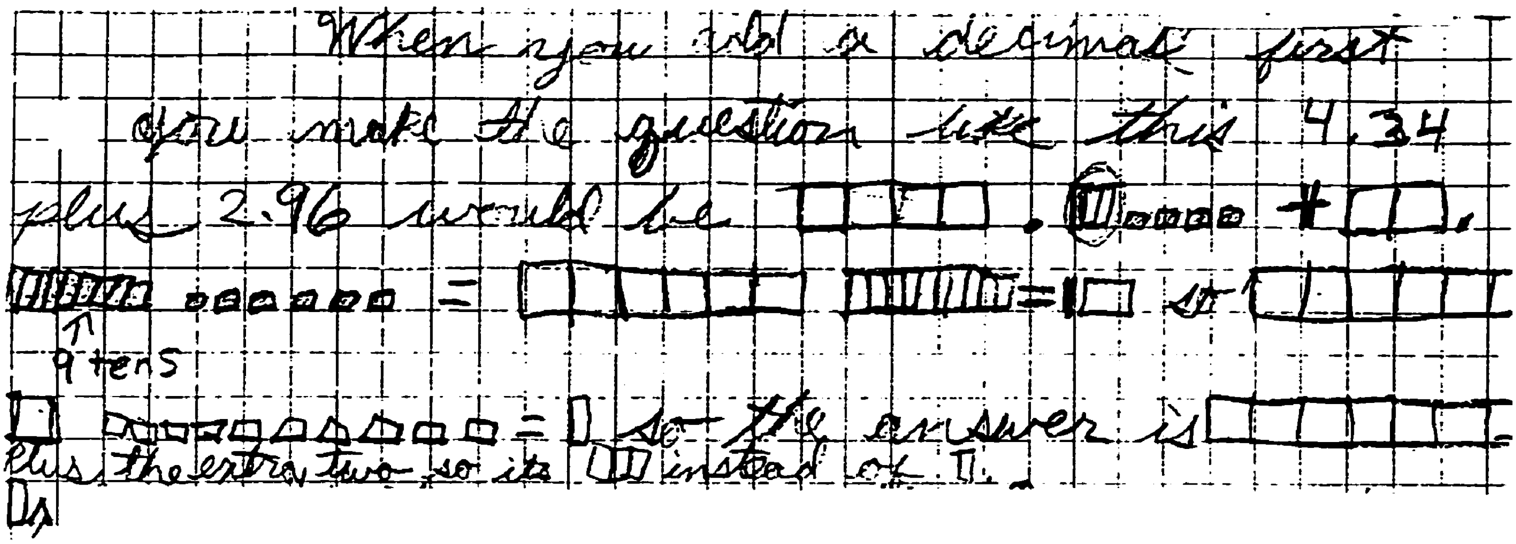
In this journal entry, Tamar showed her understanding of decimals primarily through the use of diagrams. She described every step of the way from the original question, through the carrying portion and finally to a picture of the final answer. Tamar demonstrated a solid understanding of the process of carrying in addition when she drew pictures of the numbers. For 4.34, she drew 4 large boxes for the 4 ones, she drew 3 smaller sized boxes for the tenths and she drew 4 considerably smaller boxes for the hundredths. In her first answer, she showed the answer in such a way that carrying had not occurred and then drew another equal sign and showed the boxes where carrying had occurred. (See figure #4).

At the end, she wanted to make certain that I knew that she understood the concept when she told me "in words", i.e. by also showing the algorithm, what she did.

> In words, seven point three or 7.30. So if you're right you do:
>
> $$\begin{array}{r} 4.34 \\ \underline{+2.96} \\ 7.30 \end{array}$$

Her legend at the bottom of the page explaining the different size boxes was interesting as well. She wanted to make sure that she clearly defined the difference between centicubes, ten sticks and hundred flats.

Figure 4 **Tamar, Lesson 4**



Throughout the entry though, she did not use the words hundredths, tenths or whole or parts of a whole. Even though she understood the concept of carrying, the terminology was still problematic. From her journal entry, it was difficult to tell just how much conceptual understanding she had.

Summary and reflections

This lesson cleared up a particular problem. Base ten blocks have traditionally been the best concrete material to teach decimals. However, I discovered that once I had taught the children the names and values for each of the items — centicubes equal one whole, a ten stick equals ten wholes, a flat equals one hundred wholes and one thousandths cube equals one thousand wholes — the children stuck with these names and values. When I tried to use base ten blocks to teach the concept of decimals, the students could not make the transition from understanding that all the base ten blocks were multiples of a whole to the understanding that these same base ten blocks now represented, nothing but, parts of a whole. The names of the materials, as they related to the four operations and whole numbers, did not relate to their purpose in decimals!

Once I was able to understand this problem, during the ensuing group discussion, I was able to have them discover the problem and consequently discover the correct understanding of the scientific notation. Returning to the concept of money helped the students make this discovery.

Both the participants and the non-participants expanded their view and conceptual understanding of place value and decimals throughout this lesson, as was evident by their journal entries. However, even though they were able to give the names of the different place values, they were still not using vocabulary that would lead me to believe that they understood that you would need one hundred .01's to make up one whole number, dollar, metre, etc.

Lesson #5

Purpose

The students had already written a number of different kinds of journal entries for me. However, they had not written a free-write up to this point. As a result of the previous day's lesson (not recorded here), I decided to have them do a free-write.

During the previous day's math class, I introduced the concept of decimals as it related to time and discovered, as I did in last year's study, that the concept was too difficult and not terribly relevant to the students' frame of reference. I introduced the concepts of time and of parts of a second being decimal portions of a second. An additional problem arose when the textbook also required the students to include minutes as well as seconds. For example, the children were asked to find the difference between times such as 3:45.45 and 4:51.45. Not only were the children required to understand the concept of base ten decimals with regards to the parts of a second, but they were simultaneously required to understand the concept of base 60 as it related to the addition and subtraction of minutes within the same question. The results of the lesson, from all perspectives, produced frustration and anxiety. In an effort to relieve some of this anxiety, I decided that an open free-write might help the situation.

Strategies/Procedures

At the beginning of the period, I asked the students to tell me, in the form of a journal entry, about their likes, dislikes, what frustrated them, what they would like to have more or less of in a math class. It was completely open ended as long as it related to math in some way. They were given fifteen minutes to complete their entries.

Observation Notes

After explaining to the students what this journal entry was to include, I asked the students if there were any questions. Alice wanted to know how critical she could become. She wanted to know if she could talk about the frustrations of the new math programme. I told her that she could most certainly do this, as long as she also told me why she felt the way she did. She felt comfortable with doing this.

During the writing portion of the class, Tamar was purposeful in her endeavours. She wrote for about 10 minutes and then got stuck. Alice felt comfortable writing for about 15 minutes. Lynn stopped and started throughout the time period. Carly wrote fairly consistently for about 15 minutes.

The results of the free-write were surprising. They all had very specific ideas that they wanted to share with me. When I have done this type of writing in the past, the students have usually gone blank. This was not the case for these students. This further demonstrated their enthusiasm and energy for any and everything that they do in class.

Journal entries

Alice

Alice expressed herself in a clear and articulate manner. She stated:

> Sometimes I get a little frust[ra]ted about the [new text] book because some of the questions are hard. I would like to use the other math books a little. I like doing a unit on decimals but could we do a unit on fractions later in the year[?] Math is fun. I like it when we get to work with partners. I like it better this year because there[']s less people.
>
> I get frust[ra]ted [with the new text] because some of the questions are hard and nobody understands them.
>
> Could we use the paper money more because we cut all of it up?

Her comments about the new math programme were thought provoking. She recognized the fact that there was a radical difference between the old textbook and the new textbook. The largest problem she had with the new programme was that it was significantly more difficult than the old one.

Her request for fractions was particularly insightful. In spite of the fact she did not have a solid, conceptual understanding of decimals at that point, she was able to conclude that there was a correlation between the two and she, potentially, could be helped by understanding fractions better. This knowledge in itself, pointed to an understanding, by inference, that both decimals and fractions were parts of a whole! This conclusion was also supported by comments made earlier on the some of the non-reported transcripts for this study.

Carly

Carly expressed herself in a direct and articulate manner. She stated, "...I enjoy doing math on the carpet. One thing I like about it is working with base ten blocks and money. I would like to work more with the money. I like working in partners or as a group."

Similarly to Alice, she felt fairly strongly about the new math programme. She stated, "...one of the things I don't like is the [new math] book. Sometimes it is o.k but I personal[l]y like numbers."

This reference to "lik[ing] numbers" related to the fact that when the children first looked at the new math programme, their initial comments were that there were hardly any numbers in the new textbook. She continued on,

> I get frust[rated] when we do math in our heads and I forget so I enjoy doing math on paper or with base ten blocks or with money. I like to add and subtract and multiply small numbers. I am not good at dividing.

In spite of the fact she was not an avid fan of the new math programme, she still enjoyed working with manipulatives which, ironically enough, were a direct by-product of the new math programme. Her enjoyment of manipulatives also pointed to the fact that she was a strong tactile and visual learner. She understood best when she was allowed to physically manipulate the materials involved in the teaching of a concept.

It is amazing how this free-write journal entry capsulized Carly's learning style as it related to math! From an educator's perspective, she was clearly a strong tactile and visual learner. It was interesting to note how she naturally gravitated towards the educational manipulatives that best helped her.

Lynn

Lynn continued to be enthusiastic about everything she did. This enthusiasm showed up in her journal entry. She gave a clear response describing what she did and did not enjoy. She stated:

> I feel about math that it is one of my favourite subjects because I liked doing multiplying and decimals. I didn't like investigating your school as much because [we were doing a] different thing ... every day. In decimals I thought it was helpful us[ing] the base ten blocks because it makes sense and is fun.

Lynn demonstrated her excellent long term memory skills. She itemized each of the units covered in math to date which is impressive in itself because I had not reviewed the different units before asking the students to start writing their entries. As well, I separated all the grade 5's from one another so there was no opportunity for unofficial peer conferencing.

Her assessment of what she did not like about the new math programme was specific and relevant. She stated:

> I don't think it was helpful when in multipl[i]cation when we were doing pointless math, because for instance I will not use a multipl[i]cation chart when I get older because I would do multiplication the normal way.

She articulated what she enjoyed about the programme. For the same reasons that other students disliked the new math programme, Lynn saw it to be a wonderful challenge and thoroughly enjoyed it.

> I like using the [new math programme] because the other book of math is all practically question and answers not problem solving answers. I didn't like math last year or the year before or the

> year before or the year before. I like writing in my journal because I think it helps me study for tests and it refreshes my memory.

Lynn's reference to "my journal" related to an ongoing type of journal that I had the students add to throughout the year. Whenever a new concept had been covered in class and the children had worked with the idea for a few days, I asked the students to come together as a group and describe to me what they had learned. I asked them to tell me the concept, the tips, tricks and rules in their own words. I, in turn, wrote out their comments on the board, as they dictated them to me, correcting information when necessary. After they had finished telling me their ideas, each child copied the information from the board. At different times during the year — test times, reviews, or when a concept previously covered came up again, they were able to refer to these notebooks. Lynn referred to these journals.

Lynn's reference to the old textbook being worse than the new because "...the other book of math is all practically question and answers not problem solving answers..." was fascinating. Like Carly, she expressed a preference for what she enjoyed, whether she stated this specifically in these terms or not. Previously, she had been bored. This programme offered her the opportunity to use her intellectual muscle and thus she appreciated the different approach.

Tamar

Tamar's entry was insightful. She felt the need to verbalize her frustrations with the difficult concepts covered during the previous day's lesson. She felt extremely frustrated and decided to figure the problem out on paper in her free-write. At first I found this to be rather odd and then I realized that I've often obsessed in my own life when it came to things that I found a challenge. I sometimes have not been able to get things out of my mind and I have needed to explore/air my ideas until the issues were resolved. Often this exploration was in the form of writing. Tamar's entry was just such an entry. She stated:

> The part where we use the blocks in a question like 3:26 minus 3:38, I get the part like you do 8 minus 6 is 2 and then you do 2 minus 3 but you can't do that so you carry the 3 and make it a 2 and you put a 1 beside the 2 and it's 12 so it's 12 minus 3 which is 9 and then you do 2 minus 3

> which is impossible and you don't have anything to carry so how do you do that? I'm so confused just because of that. It's confusing because there is nothing else to carry and you can't add another colum[n] or take from the last colum[n] or second last colum[n].

She became confused when she started off by trying to take away a larger number from a smaller number. When, in the second clause it looked as though she had switched the numbers around and in fact is subtracting the smaller number from a larger one, she then reverted to the process of trying to take a larger number from a smaller number.

Summary and Reflection

The journal entry for this lesson was difficult to assess because of the initial format. I had set the assignment up with only a few lines for each answer (as opposed to a full blank page). Both the participants and the non-participants of the group viewed it as a fill-in-the-blank exercise rather than a journal mini-write. Assessment was difficult as a result.

However, even with this limited amount of information, it was interesting to see each student's level of conceptual versus procedural understanding of decimals. Since both Alice and Carly avoided using decimals in their decimal number sentences, it was evident that they weren't totally understanding the concept of adding decimals with decimals and whole numbers with whole numbers. Lynn on the other hand, succinctly described her answer as 2 dm, 3 cm and .5 thus indicating some understanding of .5 being a part of a whole. Tamar wrote out the correct answers, but it was uncertain as to whether or not she had merely memorized the procedure or rule which states that when you add decimals, you "line up the decimals and add".

As a result of this lesson, I discovered that it was much easier to show the concept of decimals in linear measurement than in time, or even money. Linear measurement demonstrated thousandths where money only demonstrated hundredths. As well, linear measurement, unlike money and time, lent itself to the discussion of the concept of a decimal as a changing whole.

Lesson #6

Purpose

It was apparent that the students needed to have a greater situational understanding of decimals. They had a fairly clear understanding of decimals and addition as they related to money, but not of concept that decimals were a part of the whole (with the exception of Lynn). Before embarking on this lesson relating to measurement, a study of time had been looked at (as was mentioned previously). Even though this particular topic of time and decimals was abandoned, I felt that it was important to expose the students to another situation in life in which decimals were used. The children were introduced to the specific idea of the whole and a part of a whole as it related to linear measurement. As well the concept of linear measurement being introduced, the concept of the subtraction of decimals was also touched upon. Children were encouraged to discover their own scientific notation for the subtraction of decimals.

A worksheet was chosen as the follow-up activity. It was necessary that the children be able to write out decimals as scientific notation.

Strategies/Procedures

After a review of the basic premise of linear measurement, the students were given measuring tapes and asked individually to measure a number of bodily parts. They were asked to compare the differences between bodily parts using linear measurement. Basic information was recorded on a work sheet.

Observation notes and transcribed audiotapes

The beginning of this lesson served as an assessment of the children's understanding of linear measurement. Initial findings showed that Carly forgot that there was something smaller than a centimetre. Lynn remembered that 10 centimetres equalled a decimetre. Alice and Carly could remember centimetres, and little else. When they first measured themselves, all the children used centimetres only.

MEE	(*After the introduction*) The first thing I want you to measure is the bottom of your foot. . . When you've got the answer, raise your hand and tell me.
Carly	24.5 cm.
Alice	25.5 cm.
Lynn	23.5 cm.
Tamar	24 cm.

Although they had forgotten a fair bit, they had obviously covered the basic concepts before since they came back to them fairly quickly, as was apparent by the following discussion:

MEE	Take a look at your tape. Put your index and thumb and put it on zero. Take the index and thumb of your other hand, and put it on the spot you measured to. Now, hold that strip in front of you and tell me what you see on the tape? (*Silence...*)
MEE	Ok, I want you tell me a different way of saying the same thing. (*Silence... the students had quizzical looks on their faces.*) Is there a different way of saying the same thing?
Lynn	2 dm. and 3 cm. and .5
MEE	What would the .5 be?
Lynn	The decimal, or mm.
MEE	Tamar, what's yours?
Tamar	2 dm. and 4 cm.

After the children had finished completing the work sheets, I decided to discuss and consolidate the ideas covered in them. This discussion was interesting from Carly's perspective. She reaffirmed that she was a more visual and tactile learner. In this exchange, she saw the subtraction process on the measuring tape. She asked if she had to write anything down since she knew the answer just like that. Lynn, on the other hand, knew the mental algorithm and did it in her head.

MEE Tell me how you figured out the distance between your nose and your thumb?

Carly I measured my nose and then I figured out the difference between them.

MEE And how did you figure out the difference?

Carly I figured out the difference somehow. I don't know. I just looked at the numbers.

MEE What number did you look at?

Carly I looked at, if they had any mm. on the tape, and if one did and the other didn't, that would count as a difference. And then I looked at the cm. and if one had more cm., I figured out how many more cm. by.

MEE Terrific!

Lynn I minussed the number that was smaller from the number that was bigger.

MEE Tamar, what did you do?

Tamar I did the same as Lynn.

In an attempt to apply decimals (and possibly changing decimals) to the situation, I moved the questioning in a slightly different direction. After they finished measuring themselves and they worked on the worksheet, it was surprising how Alice and Carly came to the conclusion that one needed to compare like things. In the following example, Carly stated that one couldn't compare decimetres to centimetres or centimetres to decimetres without first changing one to the other. She stated:

MEE Another question. If my foot was 2.6 dm. long and Lynn's foot was 23 cm. long, whose foot is longer and by how much. Use your tape measures. *(After a few moments...)* What's the first problem that you need to figure out?

Alice One is dm. and one is cm.

MEE Ok, so what's the problem with that?

Alice Nothing

MEE Nothing?

Alice I figured out that the decimal separated the dm. from the cm.

MEE The decimal in 2.6 separated the . . .

Alice	The dm. from the cm.
MEE	Is there any other problem?
Lynn	Well. the dm. is the same as the cm.. sort of. if you take the decimal away, and then you can take 2.6 dm. and take away the decimal. it's cm. So if you minus now. it's ok.
MEE	So why was it not ok before?
Lynn	It wasn't ok before because dm. aren't cm. and if you took away the two. the answer would be in between.
MEE	What do you mean in between?
Lynn	Like. you wouldn't know what to put at the dm. You have to change the dm. to cm.
MEE	Going back to the question.

At this point in the discussion Lynn and Carly primarily. and Alice secondarily. were understanding the concept of a decimal being a part of a whole. or centimetres being part of decimetres. This concept was solidified in the following exchange with a student who was not part of the study. The other child in the group raised her hand stating that it would be easier to write out the question and the way in which you should write out the question is that 2.6 - 23 — the 2 of the 23 was supposed to go under the 6 — and the answer would be 24.3. Two non-group members supported this theory for about 5 minutes. Part way through this. both Carly and Alice spoke up and made a statements that demonstrated an excellent conceptual understanding.

Carly	I disagree. with the 2.6 dm.. it could very well be 26 cm., and 23 cm. could be 2.3 dm. because in 23 there's 2 groups of 10 and in 20 and there's 2 groups of ten and 10 cm. makes a dm. The difference between those is 3cm.
Alice	How did she get .43? There's no 4 or 3 anywhere.
	(*At this point. the non-participant child said that she was getting mixed up and her answer was wrong*)
MEE	Back to the question . . .

Alice	But you don't have to change it because when it's 23 cm., it already has 2 dm. making it 20.
MEE	How would you suggest we do it?
Alice	2.6 minus 2.3 and then the answer.
MEE	Ok, how would I do this?
	[Another child stated that "the answer would be 3".]
MEE	Ok, are we done?
	[Another child stated that "you have to put the decimal in the middle".]
MEE	Why would you put the decimal in the middle?

Tamar had been quiet for sometime. I gave her a knowing look at different times hoping that with my visual cue she might participate, but she had declined to offer an answer. However, at this point in the discussion, she volunteered the following procedural information.

Tamar	Because you wouldn't have 0.3 decimal. If you didn't have the decimal, you wouldn't have any decimal numbers.
MEE	What are the decimal numbers?
Tamar	3
MEE	What does the 3 represent? What does the 3 mean?
	[Another child answered, "3 mm. , no 3 cm."]
MEE	We have 2 answers. Agree or disagree?
Carly	Which answer are we saying agree or disagree?
MEE	You tell me.
Carly	I think that 3 cm. is right.
MEE	Why?
Carly	On there (*pointing to the question written on the board*) the 3 and the 6 are both cm.
MEE	You're right!
Carly	So it's just like taking 3 cm. away from 6 cm. which equals 3 cm.

MEE	Exactly! So you're comparing all the cm. right? And then you're comparing all the dm. right? Take a look at your tapes. In money, what is a dime?
	[Another child answered, "10 cents".]
MEE	How do you write a dime?
Alice	.1
MEE	Great. If I said to you that my foot was 2 dm. and 8 cm. what is that as a decimal?
Tamar	2.8?
MEE	2.8 dm. or cm.?
Lynn	If it's dm., then the cm. are the decimal. If it's cm., than the mm. are the decimal.
MEE	What do you mean by 'it's'?
	[Another child answered, "The bigger part that you are measuring".]
MEE	Ok, what else? What did we call it in money?
Carly	The whole
MEE	The whole. If our whole is a dm., what will the decimal be?
Alice	cm.
MEE	What if your whole is a cm., what will your decimal be?
Tamar	mm.
MEE	If your whole is a meter, what would your decimal be?
Alice	A dm.
MEE	A dm. Good. Suppose though that you had your whole as a meter and your decimal would be a decimeter. Suppose though you had something that measured 1m, 6 dm. and 5 cm. What would the 5cm. be?
Lynn	Part of the decimal
MEE	What part of the decimal would it be? What column would it be?
Lynn	The hundredths column.

It was clear by the end of the lesson that Lynn, and to lesser degrees Alice, Carly and Tamar, demonstrated a fairly solid conceptual understanding of decimals and measurement.

A final and critically important conclusion reached as a result of this lesson was the change in understanding that the students were going through. They were moving from the concrete to the pictorial to the abstract level of understanding. Most of the children in the class had a solid grasp of the concrete concept and they were slowly and somewhat voluntarily working towards a better abstract concept by the end of the period. Without taking their journal entries into consideration, the majority of students demonstrated a fairly solid verbal understanding of decimals being part of a potentially changing whole.

Work Sheet:

Alice

Alice recorded most answers by describing full sets of measurements rather than just the decimals. For example she stated that the length from her right hand thumb from the tip to the second knuckle was, "5 cm. 5 mil".

The only time in which she switched over to a full fledged decimal was when she needed to subtract the numbers. She set up the question by writing it vertically on the page. After that, she wrote out in full, how much bigger her index finger was than her thumb and why. In her explanation, though, she confused the portion after the decimal as being a part of a millimetre instead of being part of a decimetre. She stated, "Thumb 5.50. Index 6.00. My index is bigger because its .5 of a mil bigger."

This showed the beginnings of a conceptual reasoning behind the abstract notation. However, it was evident that she still needed assistance in the concrete to help make a smoother transition to the abstract.

Carly

In spite of the fact that she was the first to recognize that the 2 in 2.8 decimetres and a dollar in money are the whole, Carly recorded her answers as full measurement sentences without making the decimal connection. In answer to the question, "*Using the information above, how* *much longer is your index finger than your thumb and by how much?*" She stated, that "...the difference is 5 mm. I got the answer by finding out the difference between 5 cm. 5mm and 6 cm."

Since her answer did not include a decimal or any other written explanation, I asked her how she figured out the answer on paper. She stated that she'd preferred to use just her tape and her head.

Lynn

She immediately saw the connection between the whole and then all other left overs being part of a decimal. Every question on the sheet was answered either as a whole number, or as a decimal with the appropriate type of measurement beside it. She understood the difference between her longest finger and her shortest finger to be, "6.5cm - 5 cm. = 1.5cm".

She understood that the whole remains the same in the question and in the answer. In answer to the question, "Using the information above, how much longer is your index finger than your thumb and by how much?", she also felt a decimal number sentence was enough and wrote, "5.5cm - 5 cm. = .5mm".

She was a little confused about the decimal being part of a millimetre rather than .5 being a part, or half of a centimetre. This was the exception rather than the rule. However, since the uncertainty had been expressed, it allowed me to review the concept quickly the next day.

It was apparent from the rest of the sheet that she had a generally solid understanding of the concept. For example, in answer to her question "*How much different is your longest finger and your nose?*" she wrote, "...the difference between my longest finger and my nose is: 6.5cm - 3.5cm = 3cm."

Tamar

Tamar demonstrated a good understanding of decimals as they relate to scientific notation. Again relating to the question "*Using the information above, how much longer is your index finger than your thumb and by how much*?" She wrote out the following number sentence, "6.1 - 5 = 1.1 cm."

She understood the measurement concept and the abstract notation. Because she did not write out more of a journal type answer, it was difficult to know just how thoroughly she understood the concept conceptually as opposed to her merely parodying the procedural steps.

Summary and reflections

The children completed a worksheet designed by me as a way in which to attempt to bridge the student's concrete knowledge with the abstract concept of the subtraction of decimals. However, analysis was difficult for this work sheet. Because the majority of the sheet was a fill-in-the-blank, the children didn't have to write out a journal entry if they weren't so inclined. Since most of the children only filled in the blanks with the numerical answer and they did not write out a full journal entry, it was difficult to assess the degree of their conceptual knowledge! It became obvious that the process of journal writing had to be included in the actual assessment of scientific notation.

Lesson #7

Purpose

The participants' understanding in the area of decimals has grown considerably since the beginning of this study. From the previous lesson, it was apparent that they were feeling comfortable with the basic concept of decimals and, to a certain extent, the pertinent scientific notation. However, having gone over all their work including the previous journal entries and lessons, I decided it was important to find objects to measure with non-standard units in order for the children to understand that tenths could be 10 parts of ANY whole and similarly, that hundredths could be one hundred parts of any whole. I chose an item of food that could, fairly easily, be divided into 10 equal parts and subsequently, that those tenths could then be cut up into hundredths or 10 equal parts again. Finally, I wanted to reinforce their understanding of the pertinent scientific notation.

By asking six addition and subtraction questions based on the decimal amount of fruit-roll ups that I held in my hands, they were to write out the addition/subtraction questions in their own way. They were allowed to draw pictures. They could give me a written description. They could write out an algorithmic sentence. Each question from the time that I showed them an addition or subtraction question to the time that they finished their mini-write took about 10 minutes.

The purpose for this was to encourage all the children to think on their own. Partner and larger group work was certainly productive, but I discovered in previous tapes that there was occasionally a domineering partner between any two children who would consistently provide the answer. Full journal entries written after the lesson, only allowed me to understand the weaker child's construct after she had been influenced by the stronger child in the group. In order to counteract this, I asked the children a question orally and each child was required to write out their OWN answer on a separate sheet of paper. These mini-entries occurred immediately after having worked with the fruit roll-up thus making the leap from the concrete to the abstract a little less foreboding. After each entry, I was able to quickly check their work and programme for the next ten minutes accordingly.

Having learned from the previous worksheet entry not to provide brief fill-in-the-blank type questions if I wanted a fuller, more descriptive answers, the children were given smaller pieces of *blank* paper.

Strategies/Procedures

I unrolled a fruit roll-up and asked the students to find out what .1 of the roll-up was. Once this was accomplished, the children were asked to write out a series of addition and subtraction questions based on the portions of the roll-up that I had cut up. They were required to write out the answer after each physical array.

Observation notes and transcribed audiotapes

The students were extremely excited about the fruit roll-ups. They were, for the first few minutes, more interested in the roll-ups than in the lesson. However, once they got beyond that, their views of the project were fascinating. Alice was quieter this day and perhaps a little more uncertain. Tamar was more forthcoming in her answers. Carly forged ahead and each time she got stuck on a problem she felt comfortable enough to just state that she didn't know the answer. Lynn was in perfect form. She understood the purpose of the lesson immediately by stating that in order to find out what .1 was of the roll-up, you just had to measure the fruit roll-up with a tape measure and divide it by 10. This would give .1.

I initially asked the students to tell me what .1 of the roll-up was. A series of folding explanations was given. When they had exhausted this possibility, the students brought up the idea of measuring the roll-up with a tape measure. As hesitant as I was with this, I went ahead with it. I quickly discovered the difficulties with this. The children got stuck on the fact that .1 of the fruit roll-up was actually .8 centimetre. This created a problem for each of the subsequent questions. The answer could be expressed as a fraction relating to the specific metric linear measurement, but the answer could also be expressed as a generic part of a whole. After a few minutes, since the students were becoming confused, I put the measuring tapes away.

The children returned to the folding process again. It was suggested that the tape be folded and folded again until you hit 10 equal parts. I was told to fold it in half which I did. I then asked the children what the folded portion represented.

Alice	.5?
MEE	Agree? Disagree?
Lynn	Disagree.

MEE Why?

Lynn Well because that's 2 halves of a whole.

MEE Ok, 2 halves of a whole.

Lynn No, it's like a half of a whole. Like, you folded it in half so it's not .5 because .5 would be just a little bit of it.

MEE How much would .5 be?

Lynn If you had it all into tens, you could split one of the tens in half and you would get point 5. No, it wouldn't because .5 is more. No, .5 is right.

MEE You think this (*pointing to half of the fruit roll-up*) is right?

Lynn Because .5 is just like half.

MEE That's great! You just talked yourself out of the wrong answer and into the right one!

A folding lesson occurred in which the kids felt that if I folded the roll-up in half and then in half and then in half again, I'd come up with 10 equal parts. I did what they told me, but they discovered that you could only come up with 2, 4, 8, or 16 equal parts. It by-passed 10. At that point another child suggested the guess and check method. She said that I should take a little piece of roll-up and keep rolling it until I had 10 equal parts. If I had too much at the end, I should make the first part bigger and vice versa.

Carly This would be so much easier if the fruit roll-up was 100 cm.

MEE Why?

Carly Cause you would just be able to use the 10cm mark.

MEE So what did you do to 100 to come up with 10?

Carly There's 10.. If you think of money, there's 10 dimes in a dollar

MEE Exactly!

Alice There's 10 sets of 10 cm. in a hundred cm.

MEE Wow! That's a great explanation.

At this point, I was positively giddy with excitement! Carly had made an excellent comparison to money. In order to nudge the children into discovering that .1 was a tenth of something and all you had to do was divide the whole by 10 to arrive at .1, I asked the students,

MEE	What's .1 of $5.00?
Lynn	25
MEE	Think so?
Lynn	50
MEE	If .1 of $5.00 is 50 cents, what's .1 of 83?
Carly	I don't know, but the question could be like $8.30.
Lynn	I think it's 50, it's 500 it's 50, it's 35.5, it's 8.3.
	(Transcriber's note: It's clear why Lynn said "50, 500, 50" however, I have NO idea how she came to 35.5 and then 8.3. Even though I can't figure out the construct that arrived at this conclusion, as a result of her talking things through for herself, she came up with the correct answer.)
MEE	Are you sure?
Lynn	It's 8.3.
Alice	It's 8 cm. and 3 mm.

What was of particular interest through this lesson was the fact that even though Carly, Lynn and Alice couldn't explain why, they still came up with the correct answers!

Journal entries

I asked, or showed, very specific questions and asked each child to write the answer on a small 3 cm. x 3 cm. piece of paper.

First Question:

After the children had discovered how to make .1 of the roll-up, I asked the children to tell me how to make .3 of the roll-up. At that point, they had evolved to the point where they told me to cut up 2 more .1 strips for 3 — .1 strips in total. My first question to them was, "if I hold these [I held the three .1 strips in my hand], how much do I have left over?"

Alice

Alice wrote that there would be .57 cm. left over. Even though I was aiming for .7, she went farther and gave me the exact measurement based on her understanding of decimals! When I asked her what she did, she told me,

> If .1 of the fruit roll-up was 8 point and a little bit left over of a cm., and there were 7— .1's left then 7 times 8 equals 56. Since there was a little bit left over, you add a little bit more onto it. and that makes about .57 cm.

This demonstrated an excellent conceptual understanding which was applied to a tougher question!

Carly

Carly stated that there was 32 centimetres left over. When I asked Carly what her rationale was she said, "...if you take the 83 cm. in the first place, and fold it in half, of the part that was more than a half, this equalled about 32cm."

It's clear that she had difficulty understanding the purpose of the question.

Lynn

She wrote that there was .7 left over. She said that "...since I took away 3—.1's, since there was 10 of the them to start with, then there would be .7 left over."

It was interesting during the discussion taking up this question, upon hearing other's answers, she asked in an anxious tone, "I answered the question in decimals . . . is that ok?"

It was clear that Lynn had a fairly solid grasp of what a decimal was.

Tamar

Tamar stated that there would be 64 centimetres left over. Her oral rationale when we took up the questions was that "...if .1 was approximately 8, then 8 times 7 was 64 cm."

In spite of the fact the actual answer was wrong because she had multiplied incorrectly, she understood the basic premise.

Second Question:

I cut up a few more .1's so there were five, .1's on the table and one, .5. After having done this, I put the five, .1's on the table along with the .5 strip left over. I asked the children to come up with an addition question in which everything on the table was the answer.

Alice

Alice quickly and quietly wrote the following, " .5 + .1 + .1 + .1 + .1 + .1 = 1.0."

She clearly understood the concept that you need 10 — .1' s to come up with a whole.

Carly

She came up with the idea that .5 + .5 = 80. She set up the question in such a way that, to look at the fruit roll-up, she saw .5 and .5, however, she then decided that the answer should be an exact measurement and instead she wrote out 80 instead. [The fruit roll-up, by this point, was 80 cm. long].

She could visually see .5 and .5. She put the addition sentence together in generic decimals, but then she became confused and gave an answer in linear measurement! As confused as she was, she demonstrated a good, basic understanding of generic and situationally real decimals.

Lynn

In the previous question she clearly went for the procedural approach, however for this question, she returned to the conceptual by coming up with this question, "8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + = 80".

Since she had been given a blank piece of paper, she wrote out the answer in her own way. She showed me that she understood the concept that .1 as 8 centimetres, and the concept that there must be ten tenths, or in this case "8's" to make a whole. However, she didn't have enough of a solid understanding of notation to turn the numbers into decimals. However, she did understand that the uncut section of roll-up was .5 of the whole and then she added it all up as if the uncut .5 section of the roll-up was cut up into 5 equal pieces!

Tamar

Where there had been a series of gaps in her decimal understanding, Tamar demonstrated an understanding of the concept that each strip of the fruit roll-up, put back together, equaled one whole. She stated, ".5 + .1 + .2 + .2 = 1.wh".

The notation of "wh" showed an understanding of the fact that the sum of all the parts was one whole. It also showed an extension of the conventional notation with the use of the "wh" abbreviation.

Although she was grasping at the concept of wholes and parts of the whole, she still misinterpreted the question and did a little additional adding of .2 +.2 instead of .1 +.1+.1+.1.

Third question:

I asked the students to tell me what to do in order to make a piece of fruit roll-up that was .01 big.

Alice

She demonstrated a solid grasp of the basic concept. She stated, "...to get .01 you divide .1 into 10 pieces."

Carly

As she stated earlier in the transcribed audiotapes, Carly felt comfortable with this question because she realized it related to money. She was able to easily transpose the situation and come up with the following written suggestion. "...you could show point .01 by cutting the .1 into ten equal pieces and take one of those pieces and that's point .01."

Lynn

Lynn's confusion with regards to the actual size of the .1 portion of the fruit roll-up continued. As well as the fundamental confusion that .1 of the fruit roll-up was 8 centimetres., she further confused herself by working on the principal of a base 8 structure. She stated, "You would cut each .1 up into 8 equal parts."

Tamar

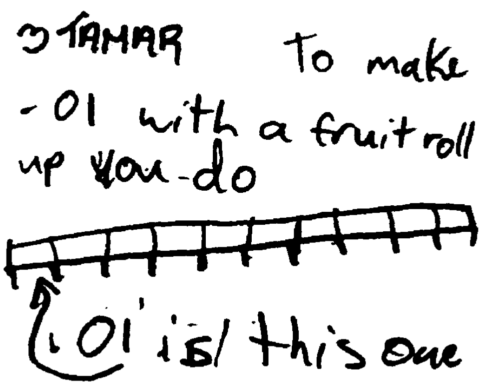
Tamar's confusion arose from the fact that she felt that .1 is the same as .01. (See figure #5).

Fourth Question:

The fourth question was a basic subtraction question. I held out 5— .1 strips of fruit roll-up. I took 2 — .1's away. I asked the children to describe what I had just done. I asked them to write out a sentence. I immediately heard groans until Alice piped up and asked if it could be a number sentence to which I replied affirmatively.

Figure 5

Tamar, Lesson 7



Alice

Alice understood the concept. It's interesting to note though, that in each of the following number sentences, she always wrote out the question vertically and she always lined up the decimals. She wrote out the correct question and answer.

$$\begin{array}{r} ".5 \\ \underline{-\ .2} \\ .3\ " \end{array}$$

Carly

She decided to be more verbally descriptive. She understood clearly the basic concept of the subtraction of decimals. She stated, "...first it started off as .5 but then she [the teacher] took away .2 so there was .3 left."

Lynn

Lynn as well, understood clearly the concept of the subtraction of decimals. She stated, ".5 - .2 = .3 "

Tamar

Similarly Tamar stated that, ".5 - .2 = .3 "

Fifth Question:

The fifth question was a little more difficult. I cut up one of the .1 pieces into two equal pieces. I then held up 2 — .1 pieces with the left hand and then brought 2 — .1 pieces and 1— .05 piece with my right hand to the left hand. I asked the children to write a sentence for me.

Alice

Alice had difficulty counting up the .1's. Instead of .2 plus .25, she felt that I had shown .2 plus .35. Her conclusion was:

".2
+.35
.55 "

Carly

Carly became completely confused and assumed that I was asking for a multiplication question. Of the six questions, this is the only one where she wrote out a number sentence. All the other questions, she wrote out the question and answer in words. As is clear from this question, she is way off track. Since she didn't conceptually understand the process, she reverted to a procedural explanation.

"24
x20
00
480
480 "

Lynn

Lynn got over her confusion between looking at each piece of the fruit roll-up as a decimal versus looking at it in terms of specific measurement amounts. She did, however, demonstrate a problem when she gave her answer. Her understanding of tenths was clear, but she felt it necessary to make a distinction between tenths and hundredths in her answer by adding an additional decimal, an understandable although incorrect assumption. She stated, ".2 + .25 = .4.5 ".

Tamar

Instead of understanding that the question was an addition question, she thought it was a subtraction question. Her question and answer were correct for a subtraction question. It is interesting to note that she wrote the question horizontally as opposed to vertically, thus hinting at a more conceptual understanding of the subtraction concept as opposed to merely following the subtraction algorithm, i.e. to line up the decimals and subtract. She wrote, ".25 - .2 = .05".

Sixth Question:

The sixth question was a basic subtraction question. I held out 2 — .1 and 2 — .05 strips in my left hand. I then took away 1 — .1 strip and 1 — .05 strip with my right hand. In essence, I showed them the subtraction question: .3 - .15 =

Alice

Alice, again lined up her decimals, but didn't line up her numbers. It would appear that she felt more comfortable with writing out number sentences vertically, but did not necessarily follow through on the mindless algorithm of lining everything up and then subtracting because she did not go through the borrowing process. She genuinely understood the subtraction process as it related to decimals. She wrote:

".3
<u>-.15</u>
.15"

Carly

Carly understood the question well enough, but forgot to include the answer. She stated, "She [the teacher] started of[f] with point .3 but took away .15."

Lynn

Lynn's confusing second decimal disappeared and she showed a solid understanding of the subtraction of decimals. She wrote, ".3 - .15 = .15".

Tamar

Tamar understood the question well enough to write it out, however, she became confused with the final answer. She wrote, ".3 - .15 = .25".

Summary and reflections

The concept of having the children write out answers on a small piece of paper, even explanation type questions was helpful to me since I was able to figure out what each of the 8 children were thinking and not just a) the children in my study group and b) the more vocal ones in the group. As well, because I was able to read the small sheets quickly, I could fix misconceptions as they arose by spontaneously altering programming to help correct the misconceptions.

Throughout the exercise all the grade 5's responded extremely well to this kind of stop-and-break-to-write approach. At one point when one of my questions looked as though the answer wouldn't fit on the small sheet of paper, I asked the students if they would like a larger one. They immediately turned this down stating that it was fun to write on such small pieces of paper.

Alice stated that writing on a smaller piece of paper meant that they didn't have to write as much. The students felt less intimidated with the smaller piece of paper, so that many of them actually wrote more per question on the smaller piece of paper than they would have on a larger piece of paper had they been given one!

Lesson #8

Purpose

As a result of the student's measurement experience, it was logical to set up a lesson that consolidated and assessed the student's understanding of decimals and a changing whole.

Strategies/Procedures

With the measuring tapes in front of them, I asked the students a number of questions based on the concept of a changing whole. I gave the children one linear measurement and then requested that they come up with a number of different ways to describe the same whole. At the end of this group discussion. I asked the students to answer the following question as a journal entry, "What is a changing decimal as it relates to measurement?"

Observation notes and transcribed audiotapes

I started the lesson with the children using measuring tapes. I asked them to measure various parts of their bodies and then asked them to give me the answer in different ways. We analyzed the way in which centimetres could be expressed as decimetres by merely changing the whole. Here are three different examples of ways in which the children discovered what a changing decimal was.

Example #1:

MEE	Can you measure from the tip of your thumb down to the second knuckle? Raise your hand and tell me how far it is when you've got it.
Alice	5.5 cm.
Carly	5.5 cm.
Tamar	6 cm.
MEE	Lynn, what did you get?
Lynn	5
MEE	5 what?
Lynn	5
MEE	5 what?
Lynn	5 cm.
MEE	Take your finger and put it on whatever total you got. If your total was 5.5 cm., you're going to pinch the 5.5 cm. mark on your tape. Take a look at that distance. Is there some way in which you can give me that information, but in a different way, but still as a decimal? *(Silence . . .)*
Lynn	50 mm.
MEE	O.k., any other way?

Carly	55 mm.
Tamar	60 mm.
	[Another child: "4.9 mm."]
MEE	*(In an effort to have the children self-discover what was wrong...)*
	Ok., can everyone in the class pinch the number that you measured your thumb to be and then with your other hand, can you all show me 4.9 mm.
	(Kids looked on the tape . . .)
Lynn	Ms. Edwards, the tape has a metal part on it so you can't see 4.9 mm.
MEE	So how big is 4.9 mm.? *(to the other child in the class),* do you want to revise what you said?
	[Another child: "Yes, 49 mm."].
MEE	Everyone, show me 49 mm. While you're at it, how many mm. are in a cm.?
Lynn	10
MEE	If there are 10 mm. in a cm. and you have 49 mm., that means . . .
Tamar	You have 4.9
MEE	4.9 what?
Tamar	4.9 cm.
MEE	If I said 4.9 cm., what's the whole?
Lynn	4.9 cm. No 4.
MEE	What's the part of the whole?
Tamar	9
MEE	So 4.9 cm. really means what?
Alice	4 cm. and 9 mm.

Example #2:

MEE	Can each of you measure the length of your foot. Once you have the answer, raise your hand.
Tamar	I got new shoes and they're 2 sizes too big *(at this point, she was trying to kneel on her shoes to hide them.)*

MEE	Oh, don't worry about size honey. I can guarantee that my foot is much bigger than yours and many of the grade 6's have larger feet than your feet. [Another child in the group: "I got 27.4 cm."]
Lynn	I got 24
MEE	24 what?
Lynn	24 cm.
MEE	Carly what did you get?
Carly	24.5 cm.
MEE	Alice, what did you get?
Alice	25 cm.
Tamar	I got 26 cm.
MEE	Can you all give me your measurements again as a different decimal?
Tamar	2 dm. and 6 cm.
MEE	Give it to me again as a single decimal.
Tamar	2.6 dm.
MEE	So what's the whole?
Tamar	2
MEE	2 what?
Tamar	2 dm.
MEE	...and what's the decimal?
Tamar	6
MEE	6 what?
Tamar	6 cm.
MEE	Excellent. Ok, Lynn,
Lynn	.24 mm
MEE	OK, can everyone on their tape show me .24 mm.?
Lynn	No, I want to change my answer. That's wrong. Because that would be way less. It's 240 mm.
MEE	Can everyone show me 240 mm. on your tapes? Try pinching 240 mm.

	(*The kids all pinch in the correct spot*)
MEE	I have a question for you. Is 240 mm. the same as 2.4 dm.? Look at your tape if you're stuck. Carly, what do you think?
Carly	No, because dm. are bigger?
MEE	How many dm. are in 2.4 dm.?
Carly	2?
MEE	Can you show me?
	(*She shows me 2 dm. on the tape*)
MEE	Excellent. Show me the point 4 (*which she does correctly*). Excellent. Where are 10 mm.? (*Which she does correctly*). Where's 240 mm.? (*Which she does and her two sets of index fingers and thumbs are overlapping each other.*) Are 240 mm. the same as 2.4 dm.?
Carly	Yes
MEE	Good. What are 3 ways to describe 240 mm.? 240 mm. is one way, what's another way.
	[Another child: "2.4 dm."]
MEE	And what's the last one?
Alice	24 cm.

Example #3:

MEE	We'll do one more and then I'll ask you to write out a journal entry. I want you to put your finger on 110 cm. Stretch your hand out so that one hand is on the zero and the other hand is holding the tape at the 110 m spot. How can you describe that as a quantity and describe it for me? Describe it in a new way that we haven't talked about just yet.
Alice	1.10 dm.
MEE	Agree or disagree?
Tamar	I agree. Because there's 100 cm. and then there's another 10.
MEE	Alice said dm. How many dm. are in 110 cm.?

Tamar	Oh, I disagree. It's 11.0.
MEE	11.0 what?
Tamar	11.0 dm.
Lynn	It's a metre.
MEE	You're absolutely right, but let's just finish off Tamar's thought first. Lynn, hold onto your thought though and we'll talk about it in a minute. Is 11 dm. right? Agree or disagree? (*the group nodded in agreement*) Lynn, you had a point to make.
Lynn	100 cm. is a metre and 1 dm. would be .1
MEE	So give it to me in a different way.
Lynn	1.1
MEE	1.1 what? What's the whole? (*Not answering my intermediary question, she quickly jumped to the answer.*)
Lynn	1.1 m.
MEE	Agree or disagree? (*Lot's of heads nod in agreement*)
MEE	What's the whole? In 1.1 m.
Lynn	A metre
MEE	What's the decimal?
Carly	The 1?
MEE	One what?
Carly	Dm.?
MEE	Every time I ask you to come up with a new name for something, what changes?
Alice	The decimal changes
Lynn	The whole?
MEE	Absolutely. What's the 3rd thing that changes? (*Silence . . .*) You change the decimal, the whole and the what? What did I change from 25 cm. to 2.5 dm. to 250 mm.? I'm changing the whole, decimal and the . . .
Lynn	The measurement?

MEE Excellent.

MEE I'd like you to take the tape with you back to your desk and I want you to describe how you could have the same distance and still call it by different names.

Lynn Can we have an example?

MEE You can create one. You need to describe for me the what a changing decimal is as a journal entry.

It's hard to know when to give the children the answers, and when not to. On the one hand, Alice felt frustrated not knowing the answer and she started to cry. However, when I gave her the answer, she felt much better. On the other hand, even though Alice floundered a little bit this day, Tamar started to feel more comfortable about thinking out loud and thinking through her answers. However, when she saw Alice feeling uncomfortable and crying over the disagreement of her answer, Tamar backed off. Tamar is an extremely sensitive child, but I'm concerned that her sensitivity may come in the way of her learning and answering/asking questions.

Carly was feeling a little frustrated. Similar to other situations, when Carly concentrated on the manipulative and visual, she in turn understood the concepts (as is evident from the audio tapes). However, not being as much of a strong oral learner, today's lesson was a little more of a challenge for her.

Lynn was very much in her element throughout this lesson.

Journal entries

Alice

Alice's entry showed a solidification of the procedural concept covered during class time. She stated:

> A changing decimal is when every time you change the name to a decimal, a millimetre or a centimetre you change the name to the right of the decimal. For example when you measure

> from your 3[rd] finger to the palm of your hand it's 16 centimetres [or] 1 decimetre and 6 centimetres which [is] 1.6 decimetres.

> You change it by, say you have 16 centimetres to get it to decimetres you just have to put the decimal between the 1 and the 6.

She concentrated on the algorithmic approach as to how a changing decimal worked on paper as opposed to describing the idea from a more conceptually-based approach which should have included the fact that because 10 cm. equal 1 dm., the decimal could change from 16 cm. to 1.6 dm. This entry, for the most part, supported my suspicions which arose during class that she had more of a procedural rather than conceptual understanding of a changing decimal. There is a comment from her earlier in the transcripts to the effect that when one changed the way a measurement numerically looked, one changed the decimal. However, similarly to the journal entry, during the lesson she felt unable — out of frustration — to explain the principle that decimals were base ten oriented.

Carly

In the few volunteered comments during class that day she expressed a hesitancy and a lack of confidence. She answered questions based on specific questions, but she was unclear or hesitant in her answers. It was difficult to confirm just how thoroughly she understood the basic concept.

Her journal entry, however, showed a definite conceptual as well as abstract understanding of the concept.

> From my third finger to my wrist is 15 cm. you could also say 1 dm. and 5 cm. because in 15 cm. there is at least ten cm. which makes a dm. Then there is 5 cm. left. If you say it in dm. it will be 1.5. The whole is the dm. and decimal is cm. Both ways you do this you have the same amount of measurement.

She gave the more procedural approach first stating that for 15 centimetres "you could also say 1 dm. and 5 cm." but then she explained the statement by stating that "in 15 cm. there is at least ten cOm. which makes a dm.". She explained the base ten concept in decimals and then further stated how to relate the concept of a changing decimal to this concept.

In order to verify what she stated earlier, she further explained her thought processes by applying the concept of a changing decimal to mm. She stated, "...another way to say it is 150 mm. When I do this I look for the amount of cm. or mm or dm. that could fit into the whole."

Whereas in the main portion of the lesson, Carly demonstrated less than a solid understanding of a changing decimal, on paper — in the form of a journal entry, she accurately described in detail both the abstract and conceptual ideas involved in the process. In this case, oral expression was not the final assessor of understanding.

Lynn

Lynn showed a good conceptual understanding of a changing whole throughout the course of the lesson. She was able to visualize the concept .24 mm being "way too small" when she tried to figure out if 240 mm. was another way of describing 24 cm. She also was able to immediately jump to the understanding of how a metre could be a changing whole when expressed by varying units of measurement. It's interesting to note that she visualized the differences between decimals without verbalizing the idea that the decimals were tenths or hundredths of the whole.

Her journal entry demonstrated a fairly good conceptual as well as abstract understanding of the material covered. However, she started off at a procedural level by regurgitating what was stated in class.

> A changing decimal can change in three different ways. [It] changes the whole, it changes the decimal, and [it] changes the measurement.

She then volunteered an example of this procedural process demonstrating a limited conceptual understanding.

> If you were to take a measuring tape and measure the length from the tip of your finger to w[h]ere you[r] wrist begins (mine is 14cm) you would [have] 14 cm. as centimeters and 140mm as millimetres and 1.4 as decimeters ...

She then decided to give a second example of how this theory worked. However, mid-way through her explanation, she included a full conceptual understanding of the question by including the base ten idea as well.

> ...and if it was something even longer like from one tip of the finger to the other tip of the finger with your arms spread out (mine is 137 cm. you have 4 different types of measurement, the millimetres the centimetres and the decimeteres and the meters w[hi]ch a whole meter is made up of a thousand millimetre a hundred cm. and ten decimeters. If you were to right down how long it is from one tip of the finger to the other using meters you would wri[te] 1.37m.

By the end of her entry, she clearly expressed her views both conceptually and abstractly further substantiating the point that the requirement to set out the idea on paper may be instrumental in reviewing a working understanding.

Tamar

The transcribed tapes of the previous lesson showed increased understanding developing in Tamar's constructs. Her progress slowed down a little bit when Alice's frustrations started to surface as a result of Tamar's recognition that Alice's answer was incorrect. However, this was a significant breakthrough for Tamar who, to this point in the study, had not felt strong or comfortable enough to agree or disagree strongly about an answer given and to be able to correctly and accurately explain why she felt that way. Often she would offer an answer after someone else had given a correct answer along the same lines, therefore feeling safer to give an answer.

It's interesting to note that this attitude change resulted in a greater understanding of the topic. She demonstrated both a conceptual and abstract understanding. She started off merely regurgitating what was stated in class:

> A changing decimal is when you have from your middle finger to the bottom of your hand which is 16cm.[. Y]ou can also say 1.6 decimeters which is 1 decimeter and 6 cent[i]meters. Or you could say 160 mil[l]imetres

However, she demonstrated a more conceptual understanding when she volunteered the brief base ten concept:

> I changed the centimeters to decimeters by looking [at] how many centimeters I had (16) and thought 10 centimeters is 1 dec. And 6 centimeters is the same. The way I changed the 1.6 dec's to mili's is I well, if I have 1 dec that would be 100 milis and 6 centi's would be 60 mili's so that's how I got 160 milis.

Her visual reference to the actual distance demonstrated greater conceptual awareness as well:

> Like in 110 [m] you can say 11 dec's or 110 centi's or 1010 milis. That's a lot! So that's how I change a decimal.

Up to this point, Tamar had been more of a procedural thinker, a rule memorizer. She tried to do the rule in her head as opposed to working things out with the manipulatives. This last reference to the concept of 1010 mm. being "a lot" showed a greater conceptual understanding (even though the actual amount given in millimetre's was incorrect and should have read 1100 mm.).

Summary and reflections

The journal entry for this lesson worked well. I was able to assess each child's true understanding of the concepts covered in class. By allowing the children to write on a fairly specific, yet open ended question, I was able to discern if the knowledge demonstrated in class verbally was truly understood based on the information they put down on paper. As a result of this journal entry, by having the students write out their thoughts, I discovered that Carly actually reached a greater understanding through the process of writing out her ideas. I discovered that Tamar also understood more than she had demonstrated in class, but because she felt momentarily uncomfortable expressing herself in class because of a difficult social moment, the math journal gave her the opportunity for her to prove what she knew!

The journal results from the other members of the class also served as excellent assessment tools allowing me to discern if their understanding was merely skin deep.

Perhaps because of the previous day's free-write, one of the other children included a side-bar suggestion of additional programming relating to today's lesson! Because I'd allowed children input into their learning environment, this one particular child in turn, felt comfortable putting forth a few more suggestions as well.

Lesson #9

Purpose

Each of the participants in the study had at least begun to demonstrate an understanding of: decimals working on a base 10 principle, decimals as changing wholes and, the addition and subtraction of decimals. However, I felt that further review to consolidate their understanding seemed to be in order. I designed the journal entry question for this lesson to be open-ended and broad enough in scope allowing the students to draw on more of their newly acquired skills. I asked the students to explore the similarities and differences between money and measurement when it came to decimals.

Strategies/Procedures

At the beginning of the math period, the students were asked to write a journal entry in which they were to describe the differences and similarities between decimals in money and measurement. They were given 20 minutes to write out their entries. I let them know that they could have had a longer period of time if they needed the time.

Observation Notes

When I first asked the students to answer this question, they were completely confused. At first there was complete silence, which is rather unusual. Lynn asked for clarification. I responded by writing the journal assignment on the board.

During the actual journal writing portion of the lesson, there were a lot of blank looks. Lynn called me over and stated that she didn't know what I was looking for. I told her that it's not what I was looking for that was important, but what she was thinking. This made her feel a little more at ease.

Journal entries

Alice

As represented in this journal entry, Alice's understanding of the differences between measurement and money were limited and confused. She started off by saying that,

> The difference between decimals, money and measurement are that you call them different things like in money there's the tens column and ones column and in decimals the[y're] called the tenths and hundredths column and in measurement the[y']re called the decimeter column and the centimetre column.

The tenths and hundredths columns were all called different things whether it be tens (dimes) and ones (pennies) for money, tenths and hundredths in decimals and decimetres and centimetres in measurement. The concept of a changing whole seemed to have eluded her a little bit. She stated, somewhat assertively, that the tenths and hundredths columns were called the decimeter and centimetre column respectively without explaining any reasons why this would not be completely accurate.

Her similarities were not terribly specific, but a little more accurate. She explained:

> The similarities are that once you get one [w]hole it goes in the ones column and all of them use the numbers in the same way like if [it's] in the hundredths column and it reaches ten it goes into the next column.

Here, Alice described the concept of carrying with a very basic base ten understanding. She stated that if you had more than 10 hundredths, you'd carry and the tenths column would change by increasing by one (in this case).

Even though this entry demonstrated a greater conceptual understanding of a changing whole, the generic concept of tenths (such as dimes or decimetres) being ten parts of a whole, or hundredths (such as centimetres or pennies) being a hundred parts of a whole had not been truly solidified in her thought yet.

Carly

Carly started off her entry by referring to an obvious difference, "A difference of decimals in money and in measurement is in measurement you use the words meters and decimeters, cm. and mm. In money you use the [words] that have to do with money."

She then moved on to a similarity that showed a fairly good understanding of the base ten concept behind decimals.

> A sim[i]larity is that in measurement and money you can count by tens. There are 10 mm in a cm. and 10cm in a dm. and 10dm. in a meter. For money there is 10 penni[e]s in a dime and 10 dim[e]s in a dollar.

She reiterated what she said earlier with a slightly different spin by saying, "...a difference is that for measurement there are four words that you use and for money there is three words you could use to get up to a dollar."

Her reference pointed to the fact that there were four words — millimetres, centimetres, decimetres, and millimetres in measurement and there were only three words — pennies, dimes and dollars — in money. Her observation was an excellent one, but she failed to follow through with the fact that this meant that you could divide the whole in measurement into thousandths, whereas in money, you could only split the whole up into hundredths. By understanding that there was one more word in measurement though, the mental landscape was fertile for understanding the concept of the thousandths column.

Lynn

Lynn started her entry with an extremely rudimentary but accurate comparison. Her explanation was that measurement and money could be mathematically expressed in decimals.

> I think they are the same because the[y] have a whole and the[y] have decimals. They are different because in money the[y] use different names for stuff like penn[ies], dimes, lo[onies], quarters [as] [op]posed to millimeters, centimeters, decimeters and meters but they are the same because the way you wr[ite] the decimal like centimeteres are in the same spot [as] a cent would be the same as dimes and decimeters, meters and loonies.

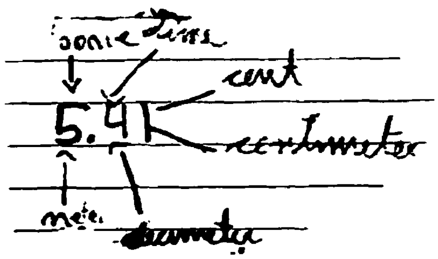
Her difference was a surface description. Her similarity could have been explained further. If she had fully understood the concept of a changing decimal as it related to measurement, she would have realized that pennies or the hundredths column could be defined as millimetres, centimetres, decimetres, etc. in linear measurement.

Unlike anything Lynn had written before, she decided to explain herself through a diagram. She gave an outstanding example of the difference between money and measurement. She spoke about the idea that a dollar could only be split up into a hundred equal parts, whereas in measurement, you could go smaller than a centimetre into millimetres, or thousandths. She explained, "Like they are different because money doesn't have anything smaller than a cent (in decimals I don't think you use quarters)." (See figure #6).

Throughout her entry though, Lynn did not come to a conclusion that money, measurement, or any other math concept that could be divided into decimals could be split because it's based on the base ten concept or, tenths are 10 parts of a whole, etc.

Figure 6

Lynn, Lesson 9



Tamar

Tamar's similarity was superficial and somewhat meaningless. She described the idea that the numbers 2.6 could be written to describe decimetres as in 2.6 decimetres, or dollars — 2.6 dollars. However, her digression mid-way through her explanation demonstrated a greater conceptual understanding of a changing whole. She correctly explained one conversion of 26 centimetres = 260 millimetres (even though she gave an incorrect example of 26 centimetres=2.6 centimetres). If she had gone just a little farther, she would have recognized the difference between the two (we don't really use changing wholes in money, but we do in measurement) She stated:

> One similarit[y] for measurement and money using decimals is in measurement you [can say] for example: My foot leng[th] is 26 centimetres you could say 2.6 centimetres. Or 260 milis but that's not with the decimal. You could say a lot of stuff and also for money you could say for example; 2.6 dollars. Dollars are like dec's, and cents are like mili's.

Summary and reflections

Not only were the children in the study group stymied, but all the children in grade five were confused by the extremely open ended question given to them to answer. Although the idea of children comparing mathematical concepts was a good one, because they had not been used to this kind of mathematical mode, they did not do well. Even Lynn, usually the most successful child of the group had difficulties with the exercise. Her entry was good, but not as nearly as coherent as some of her other entries.

Another unfortunate result of this exercise occurred when I started to see the frantic math-phobic look in some of the non-grade 5 participants. Two of the non- participants looked positively uncomfortable. Their final entries however demonstrated the fact that they overcame this fear to a certain extent.

This type of an open ended question was an excellent idea because it forced the students to look at their newly acquired knowledge and analyze it from a slightly different perspective. However, because of the initial confused reaction and the lack of familiarity of the format from the students' perspective, I realized that the students needed to be exposed to this type of questioning on a more regular basis before a conclusion as to its ultimate usefulness could be determined.

Lesson #10

Purpose

Originally, the previous lesson was to have been the final lesson in this study. However, I was surprised by their final journal entries. Whether it was because of the overly open-ended nature of the question, or overall fatigue, I felt that they knew and understood more than they had demonstrated. In order to find the best way to determine this, I decided to try a basic oral review a few days later. I decided to change the atmosphere of the class. I didn't pull any math manipulatives out. I sat on the floor with the students. Up until now, I had been sitting on a short chair while the rest of the eight students sat in a circle on the carpet.

Strategies/Procedures

I asked the students a series of questions on decimals about objects completely separate from measurement or dollars. The children, as a group (all eight children) discussed and explored the various questions throughout the period.

Observation notes and transcribed audio tapes

I held up a book from a nearby shelf. The following conversation occurred:

MEE	If I asked you to give me .07 of this cover, what and how would you do it?
Lynn	You'd cut the cover into 100 equal pieces and then I'd give you 7 of the pieces.
MEE	Another question (*At this point, I was so bowled over that I didn't want to express ecstatic joy in front of the children and make it look as though I was shocked that they should come up with the right answer.*) Suppose I asked you to give me .4 of this book cover, what would you do?

Alice	I'd cut up the book cover into 10 equal parts and I'd give you 4 pieces of the cover.
MEE	Agree or disagree?
	[Another child: "I disagree. I'd cut the book up into quarters and give you 4 pieces."]
MEE	Agree or disagree and tell me why you agree or disagree?
Tamar	I agree because you want to take out 4 pieces and so you . . . no, I disagree because you want to cut up the book cover into 10 pieces and then . . .
MEE	What are you going to do with the ten pieces?
Tamar	I'm going to cut up the cover into 10 pieces and then I'm going to give you 40 pieces.
MEE	Where will the 40 pieces come from?
Tamar	No, I mean I will give you 4 pieces.
MEE	I have another one for you. Suppose you give me .09 of this cover, how will you do this?
Carly	I'll cut the book cover into 100 equal pieces and I'll give you 9 pieces.
MEE	One last question. Suppose I want you to give me .837 of this book cover, what and how would you do it?
Lynn	I'd cut the cover into 1000 equal pieces and I would give you 837 pieces. No, I'd give you .837 . . . No, I'd give you 837 of the thousand pieces.

I was dumbfounded! I'd been striving to have the students understand the basic premise, through the medium of measurement, money and time (to a certain extent) the concept that tenths were 10 parts of a whole, hundredths were one hundred parts of a whole, etc. from the beginning of this study. Based on their past journal entries, transcribed tapes, oral exercises and work with manipulatives in class, even though each student had touched upon a certain aspect of decimals, with the exception of Lynn, they hadn't demonstrated in a clear and concise fashion, the solid conceptual understanding that a decimal was part of a whole! Suddenly, in a quick moment of desperation, or perhaps inspiration, I decided to ask them what the scientific notation meant as it related to a book cover. In answer to the question, they demonstrated perfectly, a conceptual understanding of decimals as it related to a specific, situationally relevant circumstance.

Summary and reflections

The students demonstrated a fairly solid conceptual understanding when using concrete and pictorial depictions. However when I shifted to the abstract, this seemed to be a moving, elusive target for these children based on the results of the last journal entry.

After a few days of being away from the study, even though they had NOT demonstrated the concept of decimals as being part of a whole at the abstract level, suddenly, as a result of a simple question that I asked during this final lesson, they finally understood that a) a decimal is part of a whole and, more specifically that b) tenths are 10 parts of a whole c) hundredths are a 100 parts of a whole c) thousandths are 1000 parts of a whole. (This was highly unusual since only a few references had been made to the thousandths column!). The final icing on the cake was that they understood that the parts must be equal!

In this study, it was at first difficult to determine how the students managed to come to this conclusion. I had to ask myself, what happened from the lesson merely three days earlier and this lesson? They hadn't had homework on the topic whereby their parents helped them out. Because of various interruptions, they hadn't even had one math lesson in between!

I rediscovered two elements about teaching and learning. As a learner, I have often found the need to reflect, ponder and think about new concepts before truly understanding them. Similarly I rediscovered the need of children to be able to take time to digest information or process their learning. When this gestation period has been completed, new ideas emerge or are clarified.

As a teacher, I discovered that there is often more than one way to proverbially "skin a cat". Whereas I had been heavily focusing upon math manipulatives and math journals to further the student's learning, once I changed my focus to include a laterally different situated learning environment, as simple a concept as determining what .032 of a book cover might be, allowed the children to view the concept from a fresh perspective and thus allow me to see that they clearly understood the conceptual concept.

Chapter 5 — Summary, discussion and conclusions

Summary of Case Studies (by student)

Before discussing the differences of learning styles and the effects of journal writing on the students, a few commonalities should be included at this point. One overwhelmingly obvious effect of journal writing was that it served as an excellent indicator of the degree to which each child had an understanding of a given math concept. It also allowed me to figure out where the children stood in each of the concrete, pictorial/symbolic and abstract levels of understanding. Journal writing also allowed me to better understand each child's constructs and learning styles, and programme accordingly. The purpose of journal writing at the beginning of a unit served as an excellent way of getting baseline data. This was as far as the commonalities went.

A somewhat surprising conclusion from this case study was the fact that each child reaped different benefits from the different types of journal writing styles. Some children related to and utilized the free-write journal writing style more than the descriptive math journal entry. Some students got more out of the descriptive math journal entry than the shorter mini-writes. Lynn and Carly used journal writing primarily as a consolidation of their conceptual understanding of decimals. For Alice, many entries allowed her to remain in the safe arena of the procedural. This in itself was not beneficial, but it allowed me to assess her procedural versus slowly emerging conceptual understanding throughout the study. Different types of journal writing met different types of students' needs.

Alice

Alice's journal entries started out being variations of a previously-mentioned rule. In essence, her initial entries were procedural rather than conceptual in nature. However, during group and partner work she demonstrated an improved conceptual understanding when manipulating concrete materials and was able to correctly answer a given question on that basis. As the study progressed though, the entries started to change from being merely procedural to explaining more conceptual aspects of the unit. Once she made this transition, by the third lesson, I was able to clearly see what her conceptual constructs were!

At the beginning of the study when her entries were more procedural than conceptual in nature, I was able to better assess Alice's learning style by analyzing both her journal entries and audio taped work in class. To start with, Alice felt more comfortable telling me previously memorized algorithms and processes than trying to explain the conceptual reasoning behind a mathematical idea in her journal entries. With her excellent long term memory, she was able to stay out of the conceptual why's and regurgitate memorized rules for a longer period of time than most children!

Evidence of greater conceptual understanding started to appear in lesson three. Her conceptual understanding of the addition of decimals within the framework of place value was fairly good. Through the in-class manipulative session combined with the journal entry, she started to break down the procedural, surface level answer with one that demonstrated a greater understanding of decimals.

By lesson seven, she primarily wrote at the conceptual level of understanding with a strong abstract level of understanding. She understood not only the abstract idea that .1 was one tenth of one whole, but that if you had to figure out specifically what .7 was of something that was 80 centimetres long (or the whole was 80 centimetres), all you'd have to do is find out what one tenth of 80 was and then multiply it by seven!

In terms of analyzing Alice's conceptual versus procedural levels of understanding, another challenge occurred by lesson nine. There was no easy answer to the open-ended question given for that lesson. She was forced to put her own constructs together. She could not use the procedural level of thinking at all. As a result, she came up with an answer that allowed me to clearly assess her conceptual level of understanding and provided her an opportunity to solidify her knowledge on paper. Ultimately, more open ended journal writing forced Alice to look at the conceptual level of math and not merely spout the procedural rule.

Overall, Alice's entries certainly helped me to programme better for her needs, even though it was an ongoing challenge to try to find a question that would crack her procedural facade!

As well, journal writing allowed me to recognize not only Alice's learning style, particularly allowing me to recognize greater verbal than written acuity, but her conceptual versus procedural levels of understanding. For a child like Alice, journal writing eventually allowed me to see and understand the gaps between the concrete, symbolic and pictorial levels of learning. The free-write, allowed a venting process that was most beneficial to her and me in terms of daily programming.

Carly

Journal writing, for such an open and conscientious child like Carly, was extremely beneficial for a number of different reasons. It helped me best to identify her as a visual, tactile and manipulative/concrete based learner. When given a choice between money and base ten blocks, she preferred to work with money. She clearly preferred the manipulative, relevant situated learning approach to decimals. This theme was supported by several of her other journal entries including her free-write.

Journal entries helped me to assess, as we went along from lesson to lesson whether or not her basic constructs were correct or not. Since Carly was so open and articulate in her journal entries, I was able to recognize errors and correct them either immediately or in subsequent lessons. Her written journal explanation in lesson 1 that decimals went to the left of the decimal was quickly and easily corrected during the following day's larger group discussion.

Journal writing allowed me to identify and correct mistakes made during the transition between her concrete and the abstract level of understanding. During class discussion she was able to explain subtraction of measurement items by stating that,

> ...[I] looked at, if they had any mm. on the tape, and if one did and the other didn't, that would count as a difference. An then I looked at the centimetres and if one had more centimetres., I figured out how many more centimetres by.

She naturally understood the concept of subtracting apples from apples and oranges from oranges. However, her mini-write for this lesson didn't quite make the leap to the abstract. She gave her answers as full measurements, ie., 5 millimetres and 6 centimetres as opposed to 6.5 centimetres. She understood the process of subtracting centimetres for centimetres and decimetres from decimetres., but couldn't put the whole, abstract concept together just yet.

Similarly, during the fruit roll-up lesson, she was able to orally make connections with money, understanding that 10 equal parts of the whole fruit roll-up were similar to 10 dimes in a dollar. However, when she tried to work more at the abstract level in her mini-write, she became a little confused with the actual measurement of the roll-up being 80 centimetres as opposed to 100 centimetres. However, once she went back to working at the conceptual level of understanding she was fine, "...you could show point .01 by cutting the .1 into ten equal pieces and take one of those pieces and that's point .01."

The journal entries served as a consolidation of her in-class work as well. During later lessons in the unit, she became quieter during small group or class discussions. Since she had previously talked her way through difficult questions, I often became concerned that she was completely lost. However, when I read her journal entries they served as a way for her to talk or rather write her way through the problem and express herself accordingly. The example in lesson 4 of her writing out a procedural explanation and then reverting to a more comfortable explanation from a concretely/pictorially manipulative perspective was an indication of this.

Journal writing for a child like Carly allowed me to best identify her as a visual, tactile and manipulative/concrete based learner. From lesson to lesson, journals helped me assess whether or not her basic constructs were correct. They allowed me to identify and correct mistakes between her concrete and abstract level of understanding. The process of actually writing journal entries often served as a consolidation of her in-class work as well.

Lynn

Journal writing for Lynn served several purposes. It served as a solidifying process for her general knowledge. Not having had a great deal of exposure to decimals the year before, she demonstrated an excellent initial understanding of where you would find decimals and a general abstract understanding with a little bit of a conceptually based understanding. I was able to programme accordingly. What was also interesting about this entry was that I had asked the children to brainstorm on their own and come up with as many possible answers as they could. Lynn did precisely this. In fact she used the journal writing process, in and of itself, to help her clarify and articulate this brainstorming process.

The second journal entry allowed Lynn to translate her excellent oral skills to paper. She was the first to understand that pennies were "the decimal part". In lesson three, during the recap, she was able to bridge the concrete to pictorial level quickly by realizing that adding with base ten blocks was easier than money because there were fewer variables — there weren't nickels, quarters, etc. Her journal entry in lesson three served as a way for her to refine her excellent oral understanding. It also allowed me to see the growth in her understanding of the concept by comparing her first entry with her second one to see where the clarifications and deeper perceptions were occurring in her constructs.

Similarly in some of the lessons (four, six and eight), her journal entry allowed me to assess if her written work was as strong as her oral knowledge. In lesson four, she demonstrated an equivalence in her understanding, but the journal entry was helpful in allowing me to identify her difficulties with her math vocabulary in general. She misspelled hundredths and tenths thus putting into question just how solidly she understood the actual concept. Generally, she didn't pay a great deal of attention to spelling, but I still needed to know if the spelling she used was intentional or merely misspelling. I was able to address this in class the following day. In lesson eight's entry, she gave a couple of excellent examples and explanations of a changing whole which showed a solid conceptual and ultimately abstract understanding of the concept.

Lynn's free-write was helpful for her as it served as an outlet for her feelings. It allowed her to tell me that she loved math! She had felt free in the past to tell me what she didn't like, but because of this journal entry, it allowed her the opportunity to tell me what she did enjoy!

Lynn's free-write was also helpful to me for programming purposes. Her assessment of different elements of the new math programme, particularly the section in which she spoke about the multiplication chart, was not only valuable to me for programming purposes, but it allowed me to share my thoughts with the group and let her and the rest of the group understand that I appreciated their input to the point where I could alter my programming for future grade 5 classes. As mentioned earlier, her journal entry also betrayed her personal interest in problem solving.

The quick mini-write of lesson seven was an excellent help to Lynn. Initially, she became rather confused between trying to make a decimal out of the actual measurement of the fruit roll-up and understanding, abstractly what half of the roll-up was as a fraction. Because I had been able to see each scientific notation or mini-write as we went along, I quickly discovered Lynn's errors and, through question and answer, corrected the mistaken construct. By the end of the series of mini-writes she was able to write out that ".5-.2=.3".

I was interested to see what a child like Lynn would do with a more open-ended question such as the one given in lesson nine. This journal entry question was so open ended that she was able to brainstorm and show her knowledge from a number of different perspectives. The entry not only allowed her to go where she wanted to go, solidifying knowledge gained from the unit, but it allowed me to see a wider view of her mental constructs as expressed through journal writing. With this wider view, I was able to recognize and identify what needed correcting.

Tamar

There were many purposes, functions and benefits of journal writing for Tamar. She was not always a strong oral contributor during class or partnered discussions, however, she did feel comfortable taking a risk on paper and showing me her constructs, concerns and ideas in her journal entries. In lessons two and three she had not spoken up during class and when I asked her a question when I thought she knew the answer, she still did not want to volunteer an answer for fear of being wrong. (Although the portions of tape in which this happened were not included in this study, it happened a fair number of times.)

However, her next journal entry gave me greater insight into how the manipulative work in the second and third lessons had affected her math constructs. In her journal entry, she was beginning to identify the different components of decimals in terms of place value. Her understanding was still at a procedural level, but the information was important for me since it allowed me to recognize and alter programming accordingly for the following day.

Similar to the other children, journal writing allowed me to discover a base-line of knowledge for Tamar. Reading her initial journal entry served solely as a fact-finding mission for me. It allowed me to look inside her mental construct called decimals and view what her previous constructs on decimals were. This exercise didn't really serve as a brainstorming session for her since she did not have any previous decimal based constructs, but the exercise served as a way for me to determine her base level of decimal knowledge.

Because of the journal entries, I was able to discover that she was a visual and tactile learner. As well, I was able to discover where she stood on the concrete, pictorial/symbolic, abstract continuum. Throughout the manipulative portion of lesson five, she was able to explain literally what she did and how she moved each of the manipulative pieces without understanding why we were doing it. Any questions that she answered were purely at the basic, concrete level. Her journal entry allowed me to assess how far she had progressed from merely pushing cubes around to understanding why and what she did. Of particular importance was the fact that she chose to draw her explanation of how to add decimals thus showing the beginning steps of a pictorial/symbolic understanding. This picture also allowed me to see that she, similarly to Carly, was more of a visual and tactile learner.

The free-write allowed Tamar a venue to vent and clarify her thoughts. Even though she did not approach the journal entry in the way that I had requested, her response was more telling and helpful to her and to me than if she had answered my question directly. Through the process of journal writing, she found an outlet for her math frustrations. The mere process of venting or working through concepts on paper brought about a resolution to the problem, or helped to reduce the overall frustration levels that had built up over a period of time. Tamar used the journal entry for just a therapeutic purpose.

The mini-journal write served as an exceptionally helpful tool for Tamar since I was able, minute-by-minute, to detect her level of understanding of decimals and correct or support her work accordingly. Even though she got a little muddled up trying to understand the difference between looking at the fruit roll-ups' actual lengths as decimals and looking at the fruit roll-ups' lengths as an abstract decimal, ie., .1 is a tenth of the fruit roll-up, the process whereby I was able to monitor closely her thoughts and steer her in the right direction where necessary, worked well. By the end of the lesson, she was able to understand that, ".5+.1+.2+.2=1.wh".

By allowing Tamar to answer the question in her own way, she was able to demonstrate a conceptual understanding that the sum of the parts equaled one whole! As well, by allowing Tamar the opportunity to answer the question in her own way, she felt comfortable returning to a pictorial/symbolic level. It was as a result of this lesson that a break-through in conceptual understanding occurred. Looking at lesson eight, it was interesting to note that as a result of feeling more successful through this write-and-check approach, she gained more confidence to speak up in class.

In lesson eight, since she had felt greater confidence in the previous lesson, Tamar was willing to volunteer more abstract concepts of decimals during the oral portion of the lesson. She did well until the agree/disagree portion of the lesson. Even though she gently disagreed with another child in the group and kindly explained why, the child whose answer was being discussed broke into tears. Tamar, being a sensitive and sweet child, thought the tears were brought on by her comment, and said virtually nothing else for the rest of the lesson. This made me concerned that she had switched from educational to social mode thus losing the import of the lesson. However, after reading her journal entry, I was sufficiently satisfied that, even though she did not make an oral contribution for the rest of the class, she was still working her way mentally through all of my questions. In her entry, she not only described the physical manipulative description, but she demonstrated a level of abstract thought with a full explanation of her conceptual thinking processes! At one point she stated, "...I changed the centimetres to decimetres by looking [at] how many centimetres I had (16) and thought 10 centimetres is 1 dec."

She was clearly understanding the base ten concept and, if it hadn't been for the journal entry, I wouldn't have known this about her!

It's obvious from the first journal entry to her entry in lesson nine that she went from knowing literally nothing about decimals, to understanding the concept of base ten, adding and subtracting decimals as well as understanding the concept that decimals can have a changing whole!

Overall Summary and Discussion

This study confirmed what was reported in the literature review that journal writing in a math classroom is a critically important tool. Depending on the type of journal entry used, it can serve as an excellent tool for an educator to:

1) determine base line data.
2) identify a student's constructs,
3) assess children individually,
4) determine a child's procedural versus concrete understanding,
5) identify a child's learning style.

Depending on the type of journal entry used, it can serve as an excellent tool for the student to:

1) consolidate her/his understanding of a math concept,
2) work through math frustrations and anxieties.
3) communicate with the teacher privately.
4) to have input into her/his own programming,
5) think about a math concept from a broader perspective.

Journal Writing as a tool for educators

Journal writing was an essential activity for determining a base line. Even though the teacher/student ratio for this study was outstanding, each child in the classroom came with his or her own constructs. Before a teacher can programme for their students, they have to determine what each of the students' starting constructs were on a given topic and then programme accordingly. At the beginning of this study, within the selected group of four students, the range of knowledge was considerable. It extended from Tamar who thought that decimals were the same thing as the colons separating the hours and minutes in time to Lynn who had a reasonably good situated knowledge foundation and understood decimals and where they could be found — for example, part marks in tests or percentages. It is difficult to identify all the different children's constructs merely from small or larger group discussions. Whether this type of journal writing is described as a base line indicator or a knowledge indicator as explained by Norwood and Carter (1994), it nevertheless allowed me to identify each child's learning levels and then programme accordingly.

Similarly, construct identification was something that was important not just at the beginning of the unit but throughout the unit in order to keep abreast of students' developing constructs. Sometimes, as a result of the flow of a discussion, the children understood parts of the discussion and thus felt comfortable to participate, but s/he may not have had a full understanding of the concept being taught. Journal writing, when it was done immediately after a given lesson, allowed the children to fully explain her current knowledge on the topic. This in turn allowed me to have a more accurate description through which to identify the child's constructs. It also allowed me to recognize the misconstructs or gaps and programme accordingly.

Journal writing helped me assess the children's conceptual understanding of scientific notation. In lesson six, when the students were asked to fill in a worksheet (with an option for very short journal writing), the majority of the children opted for the short cut. It met with some success, however, there was some difficulty in determining with a child like Tamar, if she really understood the scientific notation that she'd jotted down, or was she merely parroting a previously-learned procedural rule. As a result of this lack of clarity, I had to ask the students to explain their answers orally. If more space had been provided and a need for written entries had been stressed, a better understanding of their constructs might have emerged from their written work, and more importantly, the students would have further solidified their understanding of scientific notation by writing out a full explanation!

Mini-journal writing sessions, such as the fruit roll-up lesson, were beneficial because there was no time delay between the concept taught and the recording of the new concept in the child's own words! It fostered a greater sense of individuality. There was no influence from others since each child had to write out the answers completely on their own. Subsequently, because they were so short, the children's concisely written constructs could be assessed quickly and mistaken beliefs corrected immediately! They did not have to wait until the next math class to find out if they held a mistaken understanding of a concept. As well, no one child could dominate the discussion since EVERY child was required to give their input in the form of a very quick, mini-write.

Tobias (1989) and Miller (1991) both propounded the virtue of mini-writes. Tobias called her mini-writes, "minute papers about muddiest points". Miller called her mini-writes, "quick entries". Tobias valued these entries purely for assessment purposes. Miller viewed these entries as ways in which students could improve their intellectual skills by forcing them to think clearly and concisely in a limited period of time. In comparison, the mini-write for this study was primarily used for assessment purposes. It allowed me to get a perspective on students' constructs and either correct their errors or move on to the next stage.

Journal writing accurately assessed a child's conceptual versus procedural knowledge. Norwood and Carter's (1994) view that a student's basic constructs, in a test situation, could be overlooked completely was certainly proven to be true in this study. In traditional math programmes, a child could fake her way through math by memorizing algorithms and regurgitating rules that had just been merely memorized with little to no knowledge of the conceptual reasoning behind the rule. Cognitive journal writing allowed me to discover just how much conceptual understanding the children had in comparison to her memorized procedural knowledge. This was proven in most of the journal entries of this study.

However, as a result of lesson six, this study showed what could happen when a cognitive-type journal entry was not available for assessment purposes. Without the student's written journal response to their scientific notation, it was difficult to discern whether or not the students had fully understood the concepts behind the questions. Oral questioning of the students was the only way to determine their conceptual understanding in a subsequent lesson.

Journal entries allowed for teacher identification/confirmation of the student's learning styles. Carly was a perfect example of this. Her work during the small group or large group discussions was excellent particularly when manipulatives were involved — which occurred on an almost daily basis. However, when she wrote out her journal entries, it became clear through her examples and explanations, which were frequently visual in nature, that she was a strong visual and tactile learner. Without the journal entries, this would not have necessarily been recognized.

Allowing students to write their entries using a multimodal approach was extremely helpful for all of the participants in this study at different times. As Stix (1994) pointed out in her article, since students worked from the concrete to the pictorial or symbolic and finally to the abstract level of learning in their thought, why shouldn't their journal entries reflect this transition? When Tamar had difficulty explaining the process of adding decimals, she felt comfortable explaining the process as a picture. Even after Carly gave a wonderful written explanation of how to add decimals in linear measurement, she crossed this out and felt more comfortable describing her construct in the form of a picture.

Journal Writing as a tool for students

Journal writing provided children with an environment conducive to the consolidation of their own thoughts. Sometimes the mere writing out of a concept helped a child to better understand a set of ideas. Carly's journal entry in lesson #8 was an example of this. She demonstrated less than a solid understanding of a changing decimal during the course of the full group discussion, however, on paper — in the form of a journal entry — she demonstrated, in detail a correct conceptual and abstract understanding of the process. In this case, journal writing and not oral discussion seemed to be more appropriate.

McIntosh (1991) described this consolidation process succinctly when she stated that, "...the type of writing done in learning logs may help students "own" knowledge instead of just "rent" it." (page 430)

Carly and the other children very definitely took ownership of their knowledge in a number of different journal entries.

From a social/emotional perspective, free-write journal entries allowed students to openly communicate with the teacher, yet privately and away from the other members of the class, thus eliminating fear of peer criticism. This in turn allowed students to express ideas that they might otherwise have felt uncomfortable expressing because they weren't risk-takers. Tamar's example of her writing out, in full, her frustrations over the decimals as used in the time notation was an excellent example of this. She found it difficult to express her views in class for fear of being thought stupid; however, she felt comfortable venting her frustrations and outlining her difficulties in her journal entry.

As a result of this study, it became clear that the free-write was particularly helpful for those students that suffered from math anxiety. They expressed their anxieties on paper and felt comfortable that they wouldn't feel judged for their concerns. Again, Tamar's concerns about time fit this general description.

As well, the free-write journal entries were helpful to the children because they allowed the students to take ownership of their own learning environment. Carly, Alice and Lynn indicated in their free-writes, that they were particularly interested in working with manipulatives. As a result of this comment, programming was altered and most lessons had some activity that involved manipulatives. They also requested, specifically, that more work be done with money and base ten blocks. Again, with this comment in mind, future programming was changed in order to accommodate this wish from the students.

Stewart and Chance (1995) made a distinction in their article, suggesting two different types of free-writes. One type was utilized when requesting their high school-aged students to focus on curriculum items. The second type was utilized when requesting their students to write about anything they wanted to as long as it related to math. At the elementary level, as a result of this study, it became clear that the students required some direction in their free-writes and subsequently were more interested in writing a curriculum-based entry. Even with this modification, both the students and I felt as though it was a worthy endeavor because the students could have more input into their learning environment!

Open ended journal entry questions, such as the one given in lesson nine, was helpful in expanding the students' level of understanding no matter what level they were at. By encouraging the students to think about a math concept from a different approach, they were given the opportunity to discover ideas that they might not have come up with otherwise. This also allowed them to apply already learned knowledge in differing contexts. In terms of assessment, this kind of higher order thinking question allowed me to see how far the brightest student in the class could take a concept since, with such a question, there was a wider scope of answers.

While looking at students at the secondary level, Nahrgang and Petersen (1986) asked their students very open ended questions. Their students responded with some outstanding answers. Adapting this kind of open ended type of question to students at the elementary level was useful and productive for both teacher and student. In this study, students were asked to compare decimals as used in measurement and money. They were also required to analyze each individual piece of decimal knowledge covered throughout the study, evaluate and then translate the information into a comparison. In essence, they were required to put the smaller conceptual pieces into a larger, more abstract puzzle.

It is important to note here that there were difficulties with this type of question within the context of this study. Since the students had never had this type of entry before, they were unsure as to how to approach it. However, it became clear that, with repeated exposure to this type of writing, this type of questioning could become a valuable solidification tool for the students.

Conclusion

The original purpose of this study was to discover primarily if and how journal writing could be beneficial to educator and pupil alike in the learning process. Having completed this study, it has become abundantly clear that journal writing is not merely a good idea, but it is a critically important one as well. As has been seen throughout this study, journal writing helped the educator to: assess understanding; determine the learning styles; discover individual constructs; establish base lines for each of her students. From a student's perspective, journal writing assisted children by allowing her/him to: solidify concepts; provide input into programming; think about a math concept from a greater perspective; and vent frustrations and math anxieties on paper in a therapeutic way. In essence, journal writing was a critically important learning tool for both educator and student.

I believe that journal writing in mathematics should be an integral component of any math programme. Listening to the needs of my students as indicated through journal entries and acting upon that information either for assessment or programming purposes has made me a more effective educator. The process of journal writing helped to better meet the needs of the individual student on a more consistent basis.

As a result of this study, I have decided to alter my journal writing procedures for next year. During the current school year, as mentioned earlier by Lynn, the children had on-going math journals/notebook. After a concept had been taught, I usually asked the children to describe the concept to me and I, in turn, copied their answers onto the board. The children would then write out these answers in their math notebooks. As Lynn pointed out, this helped her to review and study for upcoming tests. This activity was valuable unto itself, but it was clearly not an individualized, interactive journal. Next year, I have decided to continue to have the children, at the end of a mini-unit, go through this process, but I will have them rename the books "math notebooks". In between each of these notebook entries, I will have the children write out different kinds of math journal entries not only for personal assessment purposes, but to allow them time to solidify their understanding of a given concept.

Because of this study, it became evident that the journal entry questions themselves were critically important. Over a period of time, I would like to develop more questions such as the one given in lesson nine. It was open-ended and broad enough in scope that a child could be able to draw on a number of different elements from the concrete/tactile, the pictorial/visual and the abstract in order to answer it. Finding questions such as these would be useful not merely at the end of a unit, but once or twice throughout the unit as well.

The critical importance of finding as many different real life environments or situations to which a concept could be related became evident as well. When I next teach the concept of decimals, as well as money and linear measurement, I will include elements of mass, capacity and volume in the unit.

Another important conclusion was the fact that students needed time to absorb knowledge and new concepts. In spite of my best efforts to provide the perfect constructivist environment where students actively learned and explored, they inevitably needed time to consolidate and solidify new concepts before another set or series of new concepts could be introduced. I rediscovered the necessity of slowing down and allowing the students to absorb and ponder their newly acquired knowledge before forging ahead with a whole new set of concepts!

As a result of having encouraged children in this study to discover their own answers rather than relying on me for a quick-fix response, this action research study took almost 50% longer than initially anticipated. However, as a result of the student's explorations, they had the opportunity to discover conceptual math ideas that they would have missed if I had given them the right answer all the time! As well, their discovered understanding was so entrenched that these same students were able to voluntarily apply these ideas to a number of different academic areas. In a subsequent Science Unit on nutrition, because of their earlier decimal work, they were able to quickly and easily understand units of minuscule decimal measurements as they related to nutritional values in vitamins, minerals, etc. In a subsequent unit on fractions, because they had gained such a solid understanding of decimals, they worked through the entire fraction unit in less than one week!

In conclusion, I realized that if I was going to meet the needs of my students effectively and realistically, journal writing had to be an integral component of my math programme. It was important to meet the needs of the individual student on a more consistent basis by reading and listening to their constructs and views expressed through their journal entries and acting upon that information either for assessment or programming purposes. Similarly, as students explored math concepts through journal writing, they could become more aware of their own understanding, gain confidence and ultimately take greater ownership for their learning.

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Appendix

Lesson #1 — Student Work

Decimals ALICE

A decimal seperats the hundreds, tens and ones column from the tenths, hundreths and thousnth column. When you go shopping a decimal seperates the dollars and the change. Decimals are very important because without then you couldn't divide some columns from other columns and cash from change. Decimals have to do with place value. Without decimals you wouldn't have any change. Place Value is important because it tells you where to put hundreds, tens and ones.

CARLY

What is a decimal

A decimal is an important dot that seperates change from dollars. A decimal is used in place value, anything before a decimal is millions, thousands, hundreds, tens or ones. Anything after a decimal is tenths hundredths or thousandths. With a decimal it is easy to tell how much change you have and how much cash you have. With out a decimal it would all be put put together. If you just want to read change, you just look on the left side of the decimal. If you wanted to say 50 it could be .50 or .5, they both stand for 50. If you wanted to say 5 you would put a zero after the decimal then put 5.

LYNN

I think decimals are if you are saying a percentage of something and it is below 1 than you put zero, point what the numbers is, or if were being marked on a test and we got half a mark than it would be your mark then point 5. and if your measuring somthing you may write 2m.02cm, and if you want to find the square root of somthing on a calculator and it a number that can't go in 2 than on a calculato it will show a number and then a decimal and then you will see other numbers and if you need to find what percentage of you calculator and you got somthing and a half you percentage will have a number decimal and then more numbers.

TAMAR

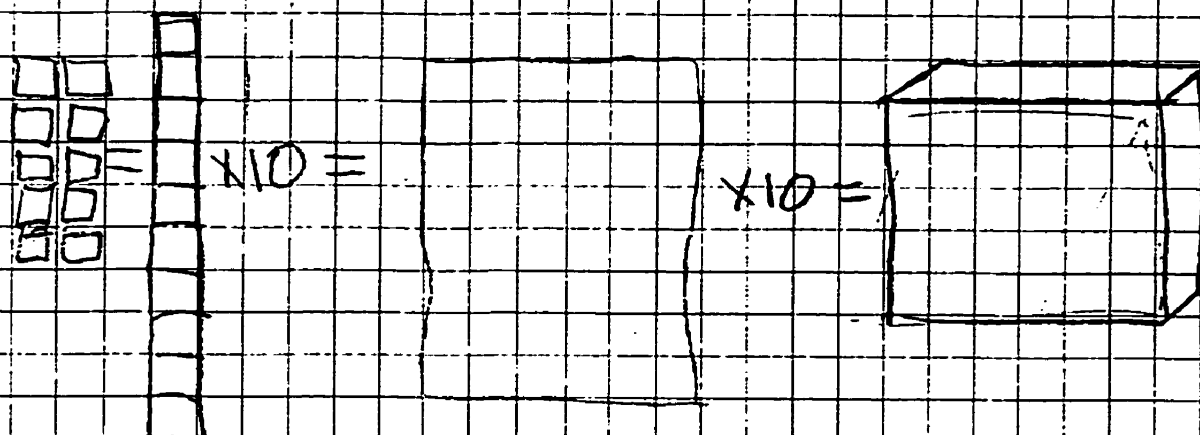
A desimal is a dot that comes after three numbers or in an amount of money like \$10.39, or if its like 100,000.0. If there weren't any decimals you wouldn't know how to say 100,000.0 one billion or something like that or wether you say one tousand and thirty nine dollers or ten dollers thirty nine cents. So decimals are extreemely important. They also tell you sort of what because 2:30 could be 23 oclock. So they put two decimals like this : . A decimal tells you how to read time read money amounts how to read high numbers like, 336.443.2 2.246. What ever number that would be! So decimals are really

Lesson #3 — Student Work

ALICE

Decimals

When you have 10 centicubes it equals 1 ten stick, when you have 10 ten sticks it equals 1 flat and when you have 10 flats it equals a thousands cube.



When your using decimals the first column to the right is the column where you put dimes or tens sticks and the column beside that is the column that you put pennies or centacubes. The columns to the lift of the decemal is where you put the hundreds flat which is like a dollar.

CARLY

What is A Decimal

A decimal is change, On the right there is 2 colums the tens colum and a ones colum. .1 is bigger than .01 because .1 is in the tens colum, .01 is in the ones colum. To have a dollar before the decimal you will have to ten dimes and to get the dimes you will need 100 pennies. Ten pennies = a dime, 10 dimes = a dollar. It is just like 10 cm cube = a ten stick and 10 ten stick s = a flat.

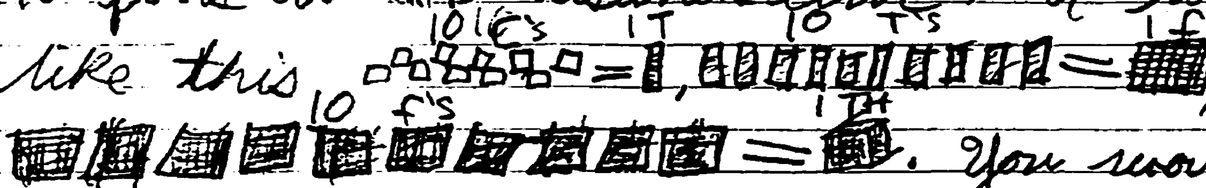
LYNN

A decimal is the part after the wholes like in money the doller is the whole and the cent is the decimal. ~~It~~ With base ten blocks ~~you~~ the centy cubes, ten of them equal a ten stick a ten ten sticks equals a flat, a flat is like a dollar and a ten stick is like ten cents and centy cubes are like cents a decimal is the ones, in money. The decimal says how much you have leftover. A decimal is always after the dot. You can tell what the number is after the dot and before. Before the dot if it is a one, than you put it with a zero before the number and if its a ten it comes before the zero and same with the hunbreds only you have to put more zeros after the number same with the thousan

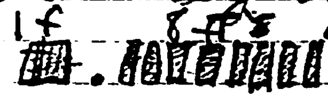
TAMAR

C=centicube
T=Ten stick
F=Flat
TH=Thousands cube

A decimal is ~~a~~ a dot like this . tha is used mostly in money amounts or like 10 centicubes is 1 ten stick and 10 ten sticks are 1 flat and 10 flats are a 1 thousands cube. It looks like this



. You would read one dollar and eighty cents like this 1.80 or 1.8 or



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Lesson #4 — Student Work

ALICE

142

For the question 4.34 + 2.96 you start with 434 and you put the number 4 farthest to the right and put it in the hundredths column then you take the 3 and put it in the tenths column then you take the other 4 and put it in the ones column with the 2.96 you take the 6 and put it in the hundredths column, then you take the 9 and put it in the tenths column and put the 2 in the ones column then you add them together by adding up the hundredths column first then so on.. and you answer is 7.30

ONES	TENTHS	HUNDREDTHS
2	9	6
4	3	4
7	3	0

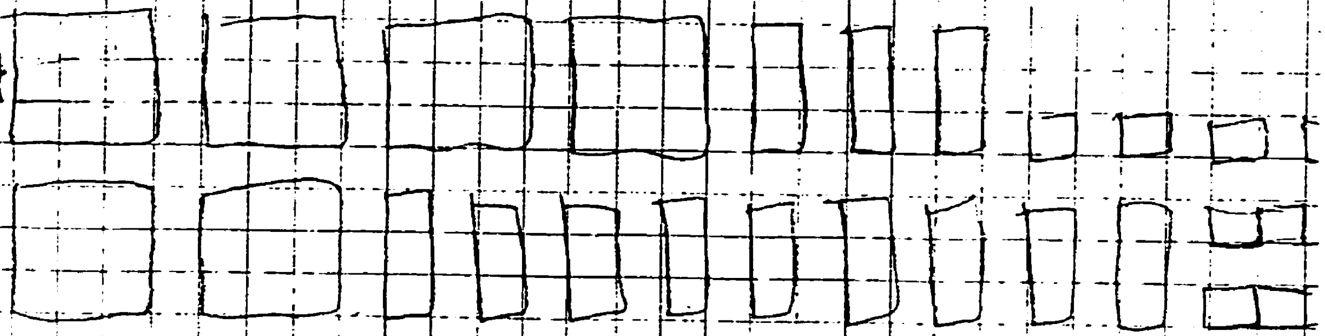
You take 4 centicube and 6 centicube then add them together the you Take 3 tens stick and 4 tens stick then add them together then you take 4 flats and 2 flats then add them together.

CARLY:

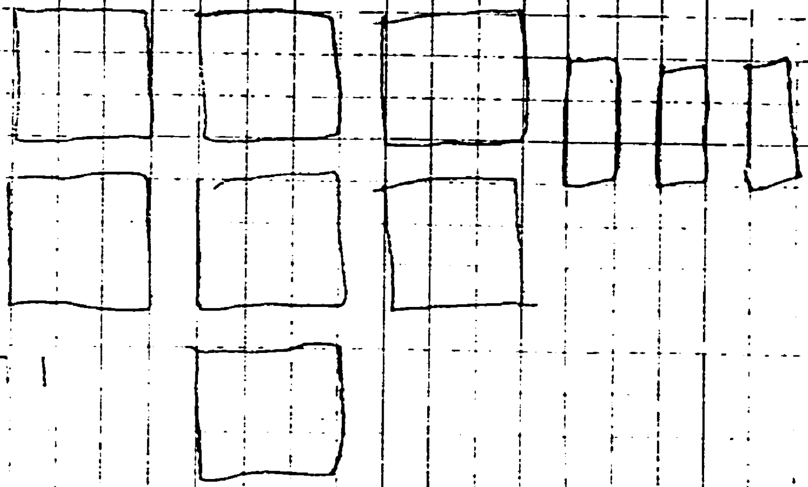
~~To add a decimal you have to add up colu… For example 4.34 + 2.96 … you add up the hundredths column … then you add up the tenths column … Then you add up the ones column 4+2 6 Then you … it all together.~~

4.34

2.96



You exchang 10 cm cubes for a 10 stick
Then you exchang 10 ten sticks for a flat
Then you have your answer But Before
your answer you have to add the blocks together



Your answer is 7.30.

LYNN

If you were to do 4.34 + 2.96 you would add up the hundreths(4, 6) witch is ten and you can make a tens stick with that because a tens stick is made out of ten centy cubes. then you add up the tens colum, (9+3) witch equals 12, plus the ten stick that was carried over from the ones, so you will have 13 tens sticks and you take ten of the tens sticks and make a flat, witch is made out of ten tens sticks and a hundred centy cubes, so you have 3 tens sticks and no centy cubes. Then you add the ones up witch is 4 and 2 plus the flat you carried so it's 4 flats plus 2 flats plus 1 flat and that is 7 flats and the 3 tens sticks from before so the answer is 7.30. You put the decimal always before the tens colum becaus you can't hundreds into 1 hundred. The centy cubes are called the hundreds colum because you fit a hundred ones in in a hundred.

TAMAR

When you add a decimal first you make the question like this 4.34 plus 2.96 would be . + . = so

9 tens

= so the answer is

this, the extra two so its instead of .

In words, seven point ~~three~~ 0 or 7.30. To see if you're right you do

$$\begin{array}{r} 4.34 \\ +\,2.96 \\ \hline 7.30 \end{array}$$

mean that they are turned into either a or but not a because there isn't a smaller thing than a.

Lesson #5 — Student Work

ALICE

Sometimes I get a little frustarated about the interactions book because some of the questions are hard. I would like to use the other Math books a little I like doing a unit on decimals but could we do a unit on fractions later in the year. Math is fun. I like it when we get to work with partners. I like it better this year because theres less people.

I get frustarated because some of the questions are hard and nobody understands them.

Could we use the paper money more because we cut all of it out.

CARLY

I enjoy doing math on the carpet. One thing I like about it is working with based ten blocks and money. I would like to work more with the money. I like working in partners or as a group. One of the things I don't like is the interactions book. Sometimes it is ok but I personaly like numbers. I get frustrated when we do math in our heads and I forget so I enjoy doing math on paper or with base ten blocks or with money. I like to add and subtart and multiply small numbers. I am not good at dividing.

LYNN

I feel about math that it is one of my favourite subjects because I liked doing multiply and decimals. I didn't like investigating your school as much because it was a different thing we were doing every day. In decimals I thought it was helpful useing the base ten blocks because it makes sense and is fun. I don't think it was helpful when in multiplycation when we were doing point less maths because for instance I will not use a multiplycation chart when I get older because I would do multiplication the normal way. I like useing the interactions because the other book of math is all practicly question answers not problem solving answers. I didn't like math last year or the year before or the year before or the year before. I like writing in my journal because I think it helps me study for tests and it refreshens my memory.

TAMAR

The part where we use the blocks in a question like 3:26 minus 3:38, I get the part like you do 8 minus 6 is 2 and then you do 2 minus 3 but you can't do that so you carry the 3 and make it a 2 and you put a 1 beside the 2 and it's 12 so it's 12 minus 3 which is 9 and then you do 2 minus 3 which is impossible and you don't have anything to carry so how do you do that? I'm so confused just because of that. It's confus because there is nothing else to carry and you can't add another colum or take from the last colum or second last colum.

Lesson #6 — Student Work

Alice

Grade 5 Measurement/Decimal Worksheet

Using a ruler, a measuring tape, or a metre stick, measure the following items and write down how long each item is:

Right hand thumb from tip to second knuckle: [illegible] 5 mil

Right hand index finger from tip to top of webbing between 1st & 2nd fingers: 6 cm

Right hand middle finger from tip to top of webbing between 2nd & 3rd fingers: 7 cm

Right hand fourth finger from tip to top of webbing between 3rd & 4th fingers: 7 cm

Your nose — from bridge to tip: 4 cm [illegible] mil

Your arm span from 2nd finger tip to 2nd finger tip: 143 cm

Answer the following questions ensuring that all calculations are done directly on this sheet.

Using the information above, how much longer is your index finger than your thumb and by how much? [illegible] because

Thumb 5.50 [illegible] mil bigger

Index 6.00

What's the difference between your longest finger and your shortest finger? [illegible] 5 mil.

Which is bigger, your nose or your thumb and by how much? My thumb is bigger by 1 cm.

Come up with your own question and write it out on the space below.

What's bigger your index finger or your 4th finger?

Write out the answer to your question in the space below.

My 4th finger is 1 cm bigger than my Index finger.

CARLY

Grade 5 Measurement/Decimal Worksheet

Using a ruler, a measuring tape, or a metre stick, measure the following items and write down how long each item is:

Right hand thumb from tip to second knuckle: 5 cm 5 mm

Right hand index finger from tip to top of webbing between 1st & 2nd fingers: 6 cm

Right hand middle finger from tip to top of webbing between 2nd & 3rd fingers: 6 cm 5 mm

Right hand fourth finger from tip to top of webbing between 3rd & 4th fingers: 6 cm

Your nose — from bridge to tip: 4 cm 5 mm

Your arm span from 2nd finger tip to 2nd finger tip: 139 cm 4 . 5 m.

Answer the following questions ensuring that all calculations are done directly on this sheet.

Using the information above, how much longer is your index finger than your thumb and by how much? the difference is 5 mm
I got the answer by finding out the diff[illegible] between 5 cm 5 mm and 6 cm 6 cm is the index

What's the difference between your longest finger and your shortest finger? 2 cm 5 mm
4 cm 5 mm is my shortest finger. the 6 cm 5 mm is the biggest finger I did the ques[illegible] on the same [illegible]

Which is bigger, your nose or your thumb and by how much?
My thumb is bigger than my nose by 1 cm.

Come up with your own question and write it out on the space below.

by how much is your middle finger bigger than your index finger. ~~My middle finger is bigger by 5 mm~~

Write out the answer to your question in the space below.

My midde finger is bigger by 5 mm.

Lynn

Grade 5 Measurement/Decimal Worksheet

Using a ruler, a measuring tape, or a metre stick, measure the following items and write down how long each item is:

Right hand thumb from tip to second knuckle: 5cm

Right hand index finger from tip to top of webbing between 1st & 2nd fingers: 5.5cm

Right hand middle finger from tip to top of webbing between 2nd & 3rd fingers: 6.5cm

Right hand fourth finger from tip to top of webbing between 3rd & 4th fingers: 6.5

Your nose — from bridge to tip: 3.5cm

Your arm span from 2nd finger tip to 2nd finger tip: 137 cm

Answer the following questions ensuring that all calculations are done directly on this sheet.

Using the information above, how much longer is your index finger than your thumb and by how much? 5.5cm - 5cm = .5mm

What's the difference between your longest finger and your shortest finger? 6.5 - 5cm = 1.5cm

Which is bigger, your nose or your thumb and by how much? nose, 5cm - 3.5 = 1.5cm

Come up with your own question and write it out on the space below.

how much different is your longest finger and your nose?

Write out the answer to your question in the space below.

The difference between my longest finger and my nose is 6.5cm - 3.5cm = 3cm

Grade 5 Measurement/Decimal Worksheet

Using a ruler, a measuring tape, or a metre stick, measure the following items and write down how long each item is:

Right hand thumb from tip to second knuckle: 5 cm's

Right hand index finger from tip to top of webbing between 1st & 2nd fingers: 6.1 cm's

Right hand middle finger from tip to top of webbing between 2nd & 3rd fingers: 8.5 cm's

Right hand fourth finger from tip to top of webbing between 3rd & 4th fingers: 5 cm's

Your nose — from bridge to tip: 4 cm's

Your arm span from 2nd finger tip to 2nd finger tip: 139 cm's

Answer the following questions ensuring that all calculations are done directly on this sheet.

Using the information above, how much longer is your index finger than your thumb and by how much? 6.1 − 5 = 1.1 cm's

What's the difference between your longest finger and your shortest finger? 8.5 − 5 = 3.5 cm's

Which is bigger, your nose or your thumb and by how much? my thumb by 1 cm

Come up with your own question and write it out on the space below.

Which is the biggest finger and which is your smallest out of your thumb first and second by how much

Write out the answer to your question in the space below.

my second finger is the biggest and my thumb is my smallest my second finger is bigger by 3.5 cm's.

$$\begin{array}{r} 8.5 \\ -5 \\ \hline 3.5 \end{array}$$

Lesson #7 — Student Work

ALICE
1. .57cm

1. CARLY
There is 35 cm left.

1. Lynn There is .7 left

1 TAMAR 64 cm's

2 ALICE

.5
.1
.1
.1
.1
+ .1
1.00

2 CARLY

$$\begin{array}{r} .5 \\ +.5 \\ \hline .80 \end{array} \qquad \begin{array}{r} 5/8 \\ +5/8 \\ \hline 80 \end{array}$$

2. Lynn 8+8+8+8+8+8+8+[illegible]=50

2 TAMAR

$$\begin{array}{r} .5 \\ .1 \\ .2 \\ +.2 \\ \hline 1.Wh \end{array}$$

3 ALICE

[illegible] .1 into 10 p[illegible]es.

3. CARLY

You could show point .01 by cutting the .1 into ten equal pieces and take one of those pieces and [illegible] point .01.

3. LYNN You would cut each .1 up into 8 equal parts

3 TAMAR To make .01 with a fruit roll up you do

.01 is this one

4 ALICE

$$\begin{array}{r} .5 \\ -.2 \\ \hline .3 \end{array}$$

4. CARLY

First it started off as .5 but then she ~~took~~ took away .2 so there was .3 left

4. .5-.2=.3
LYNN

4 TAMAR

.5-.2=.3

ALICE

5. CARLY

$$\begin{array}{r} 24 \\ \times 20 \\ \hline 00 \\ 480 \\ \hline 480 \end{array}$$

5. .2 + .25 = .4.5

5 TAMAR

.25 − .2 = .05

LYNN

6 ALICE

$$\begin{array}{r} .3 \\ -.15 \\ \hline .15 \end{array}$$

6 CARLY

She started off with point .3 but took away .15.

6. LYNN $.3 - .15 = .15$

6 TAMAR

$.3 - .15 = .25$

Lesson #8 — Student Work

Math

ALICE

A changing decimal is when every time you change the name to a decimal, a millimetre or a centimeter you change the name to the right of the decimal. For example when you measure from your 3 finger to the palm of your hand its 16 centimetres 1 decimetre and 6 centimetres which 1.6 decimetres.

You change it by, say you have 15 centimetres to get it to decimet you just have to put the decimal betwee the 1 and the 6.

From my third finger to my rist is 15 cm you could also say 1 dm and 5 cm because in 15 cm there is at least ten cm witch makes a dm. Then there is 5 cm left. If you say it in dm it will be 1.5. The whole is the dm and decimal is cm. Both ways you do this you have the same amou of messersment. [redacted]. For 15 cm another way to say it is 150 mm. When I do this I look for the amount of cm or mm or dm that could fit into the whole.

LYNN

What a changing decimal can do as it relates to measurement
A changing decimal can change in three different ways
It changes the whole, it changes the decimal and it changes the measurement. If you were to take a measuring tape and measure the length from the tip of your finger to were your wrist begins (mine is 14 cm) you would 14 cm as centimeters and 140 mm as millimeters and 1.4 as decimeters and if it was something even longer like from one tip of the finger to the other tip of the finger with your arms spread out (mine is 137 cm) you have 4 different types of measurement, the millimeters the centimeters the decimeters and the meters witch a whole meter is made up of a thousand mm a hundred cm and ten decimeters. If you were to right down how long it is from one tip of the finger to the other using meters you would wright 1.37 meters.

TAMAR

A changing decimal is when you have frome your middle finger to ~~the~~ bottom of your hand which mine is 16 cm's you can also say 1.6 decimeters which is 1 decim and 6 centemeters. Or you could say 160 milimeters.

I changed the centimeters to decimeters by looking how many centimeters I had (16) and ~~thaught~~ 10 centimeters is 1 dec. and 6 centimeters is the same. The way I changed the 1.6 dec's. ~~to~~ mills is I well, if I have 1 dec that would be 100 mills and 6 centis would be 60 mili's so thats how I got 160 mills.

Like in 110 you can say 11 dec's. or 110 centis. or 1010 mills. Thats alot!

So thats how I change a decimal

Lesson #9 — Student Work

Math Journal Alice

The differences between decimals, money and measurement are that you call them different things like in money there's the tens column and ones column and in decimals there called the tenths and hundredths column and in measurement there called the decimeter column and the centimetre column.

The similarities are that once you get one hole it goes in the ones column and all of them use the numbers in the same way like if in the hundredths column once it reaches ten it goes into the next column.

A difference of decimals in money and in measurement is in measurement you use the words meters and decimeters, cm and mm. In money you use the that have to do with money. A simalarity is that in mesurement and money you can count by tens there are 10 mm in a cm and 10 cm in a dm and 10 dm in a meter For money there is 10 pennis in a dime and 10 dims in a dollar. A difference is that for measurement there are four words that you use and for money there is three words you could use to get up to a dollar. Another simularoty is 1.3 could be used for money or mesurment because you could change it from 1.3 dm to 1.3 dollars You could use any number

L37NN

~~What is the difference ... and simutaratiex~~

Describe the differences and simularities of decimals as they relate to measurement and money

I think they are the same because the have a whole and the have decimals. They are different because in money the use different names for stuff like pennys, dimes, [illegible] [illegible] to millimeters centimeters, decimeters and meters. But they are the same because the may [illegible] the decimal like centimeters [illegible] in the [illegible] [illegible] a cent would be the same as dimes and [illegible] meters and [illegible] like

cent

5.41 centimeter

meter decimeter

They are different because money doesn't have anything smaller than [illegible] (in decimals I don't think you use quarters)

TAMAR

One similaritie for measurement and money using decimals is in measurement you for example: My foot lenght is 26 cm's you could say 2.6 decs. or 260 milis but that's not with the decimal. You could say alot of stuff and also for money you could say for example: 2.6 dollars. Dollars are like decs, and cents are like milis.