**BEARINGS-ONLY TRACKING USING DATA FUSION
AND INSTRUMENTAL VARIABLES**

**MESURES DU GISEMENT-SEUL UTILISANT LA
FUSION DES DONNÉES ET LES VARIABLES
INSTRUMENTALES**

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to the Faculty of the Royal Military College of Canada

by

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For Chantal, Jeremy and Olivier.

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ABSTRACT

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Modern land fighting vehicles are equipped with passive sensors, laser range finders and active radar, all of which can be used to localize a target. Unfortunately, radar and laser warning receivers will also easily reveal and locate a transmitting vehicle. Thus, the replacement of active tracking techniques by more passive means is needed to increase battlefield survivability.

This thesis proposes a recursive Measurement Instrumental Variables Bearings-Only Tracking (MIV-BOT) method for a stationary observer. A smoothing operation directly fuses multi-sensor bearing measurements by exchanging sensor measurements as the instruments in a pseudo linear estimator. The MIV-BOT formulation produces a smoothed velocity estimate parameterized to any position along the target trajectory, which is found from a single laser range finder measurement. Target range predictions, derived from the smoothed two-state velocity estimate, are then used as range measurements in two parallel Kalman filters. The result is a recursive, passive and unbiased fusion scheme. The theoretical development is corroborated by Monte Carlo simulation, in short tracking scenarios. The experimental results show that the fusion scheme produces reliable estimates for non-maneuvring targets. It is shown, with real target data, that the MIV-BOT fusion scheme can reliably estimate the state of the target using a residual-based track quality indicator.

RÉSUMÉ

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Superviseur: Dr. Yiu-Tong Chan

Les véhicules de combat modernes terrestres sont mis en service équipés de capteurs passifs, de détecteurs d'intervalle à laser et d'un radar actif lesquels peuvent être utilisés afin de localiser une cible. Malheureusement, les récepteurs d'avertissement de radar et de laser révèlent facilement la localisation du véhicule de transmission. Ainsi, le remplacement des techniques de poursuites actives par des moyens plus passifs est nécessaire afin d'augmenter les chances de survie au champ de bataille.

Cette thèse propose une méthode récursive de Mesures Instrumentales des Variables de Mesures du Gisement Seul (MIV-MGS) pour un observateur stationnaire. Une opération de lissage met directement en fusion les mesures de gisement des multi-capteurs en utilisant ces mesures de capteur comme instruments dans un estimateur pseudo-linéaire. Le procédé de MIV-MGS produit une lisse évaluation de la vitesse paramétrisée à n'importe quelle position le long de la trajectoire de la cible, laquelle est déterminée à partir d'une simple mesure d'un détecteur d'intervalle à laser. Les prévisions de la portée de la cible, lesquelles sont dérivées des deux-états de la lisse évaluation de la vitesse, sont alors utilisées comme mesures d'intervalles dans deux filtres parallèles de Kalman. Le résultat est une méthode de fusion récursive, passive et non-biaisée. La formulation théorique est examinée par la méthode de Monte Carlo en utilisant des séquences de courtes poursuites. Les résultats expérimentaux prouvent que la méthode de fusion produit des évaluations fiables pour les cibles quiescentes.

Il a été démontré, avec de vraies données, que la méthode de fusion de MIV-MGS peut fiablement estimer l'état de la cible en utilisant un indicateur basé sur la qualité résiduelle de la piste.

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LIST OF PRINCIPLE SYMBOLS

i, j, k, l	- discrete time index
$\bar{\beta}$	- true target bearing, in radians
β	- target bearing, in radians
β_k	- target bearing measurement at time k , in radians
$\beta_{vis_k}, \beta_{ir_k}$	- visible and infrared sensor bearing measurement, at time k , in radians
$\tilde{\beta}_k$	- independent target instrumental variable bearing measurement, at time k , in radians
x_k, y_k	- target Cartesian co-ordinate position, in metres, at time k
x'_k, y'_k	- triangulated target Cartesian co-ordinate position, in metres, at time k
$\bar{x}_k, \bar{y}_k$	- true Cartesian co-ordinate position, in metres, at time k
$\tilde{x}_k, \tilde{y}_k$	- measured target Cartesian co-ordinate position, in metres, at time k
$\tilde{\tilde{x}}_k, \tilde{\tilde{y}}_k$	- predicted target Cartesian co-ordinate position, in metres, at time k
$\dot{x}, \dot{y}$	- target x and y co-ordinate velocity, in metres/second
$\hat{\dot{x}}_k, \hat{\dot{y}}_k$	- filtered target velocity estimate, in metres/second, at time k
$\tilde{\tilde{\dot{x}}}_k, \tilde{\tilde{\dot{y}}}_k$	- smoothed target velocity estimate, in metres/second, at time k
$\tilde{\tilde{\tilde{\dot{x}}}}_k, \tilde{\tilde{\tilde{\dot{y}}}}_k$	- measurement instrumental variable smoothed target velocity estimate, in metres/second, at time k
e_b	- sensor bearing error

e_r	- laser range finder ranging error, in metres
η_k^x, η_k^y	- Cartesian measurement noise, in metres
R_k	- target range measurement, at time k , in metres
$\tilde{R}_k$	- target range prediction, at time k , in metres
$\tilde{R}_{vis_k}, \tilde{R}_{ir_k}$	- visible and infrared sensor target range prediction, at time k , in metres
$\tilde{\tilde{R}}_k$	- exchanged target range prediction, at time k , in metres
T	- the sample period, in seconds
A_k	- filtering observation matrix, at time k
X_k	- Kalman filter state vector in Cartesian co-ordinates, at time k
$\hat{X}_{k k}$	- Kalman filter <i>a posteriori</i> state vector in Cartesian co-ordinates, at time k given k
$\hat{X}_{k+1 k}$	- Kalman filter <i>a priori</i> state vector in Cartesian co-ordinates, for time $k+1$ given k
$\hat{X}_{vis_k}, \hat{X}_{ir_k}$	- visible and infrared sensor Kalman filter state vector in Cartesian co-ordinates, at time k
D_f	- filtering matrix
D_s	- smoothing matrix
Φ_k	- measurement matrix, at time k
G_k	- smoothing observation matrix, at time k
H	- Kalman filter measurement matrix, relating the target state vector, X_k , to the measurement vector, M_k
K_k	- Kalman filter gain, at time k

ν_k	- Kalman filter innovation, at time k
$\mathbf{M}_k$	- Kalman filter measurement vector, at time k
$\mathbf{Q}_k$	- error matrix, at time k
θ	- two-state vector of target's velocity
$\hat{\theta}_k$	- filtered least squares estimate of the target's velocity, at time k
$\tilde{\theta}_k$	- smoothed least squares estimate of the target's velocity, at time k
$\tilde{\theta}_k$	- measurement instrumental variable smoothed least squares estimate of the target's velocity, at time k
$\tilde{\theta}_{vis_k}, \tilde{\theta}_{ir_k}$	- visible and infrared sensor measurement instrumental variable smoothed least squares estimate of the target's velocity, at time k
$\mathbf{W}_f$	- filter weighting matrix
$\mathbf{W}_s$	- smoother weighing matrix
$\mathbf{P}_{k+1 k}$	- Kalman filter prediction error covariance matrix
$\mathbf{P}_{k k}$	- Kalman filter corrector error covariance matrix
Σ_k	- Kalman filter measurement noise covariance matrix, at time k
Ψ	- Kalman filter state transition matrix, relating the target state at time k to the state at time $k+1$
$\mathbf{Z}_k$	- smoother instrumental variable matrix, at time k

LIST OF ABBREVIATIONS

BOT	Bearings-Only Tracking
CRLB	Cramér-Rao Lower Bound
DREV	Defence Research Establishment Valcartier
FIM	Fisher Information Matrix
FOV	Field-of-View
GPS	Global Positioning System
H/V	Horizontal/Vertical
IFOV	Instantaneous Field-of-View
IR	Infrared
IV	Instrumental Variable
IV-BOT	Instrumental Variable Bearings-Only Tracking
JDL	Joint Directors of Laboratories
LAV	Light Armoured Vehicle
LOS	Line-of-Sight
LRF	Laser Range Finder
LS	Least Squares
MIV	Measurement Instrumental Variables
MIV-BOT	Measurement Instrumental Variable Bearings-Only Tracking
PLE	Pseudo Linear Estimator/Estimate
RMS	Root Mean Square

RMSE	Root Mean Square Error
TMA	Target Motion Analysis
UTM	Universal Transverse Mercator

CHAPTER 1 : INTRODUCTION

1.1 Foreword

In the Land Force, fighting vehicles are being fielded with infrared (IR) and visible imaging sensors, laser range finders (LRF) and radar. The imaging sensors provide angular, or bearings-only, information, while the laser and radar provide both bearing and range information. With the information from these combined sensors, the weapon's operator can readily localize a stationary target, and has the potential to obtain highly reliable tracks of a moving target.

Active tracking methods, using signals originating at the tracker, generally yield more accurate estimates of the target states (position and velocity), but at the risk of giving away the observer's position. Unfortunately, as an effective countermeasure, the fighting vehicles of many countries are being equipped with radar and laser warning receivers that will easily reveal and localize a transmitting system.

Target Motion Analysis (TMA) is the localization and tracking of targets using passive techniques [1]. TMA uses acoustic or electromagnetic signals originating from the target. However, TMA is less accurate than active systems. It usually assumes a non-maneuvring target motion and requires optimal observer manoeuvres to obtain converging estimates [1,2]. For some surveillance applications, where precise target

position and velocity information may not be essential, conventional TMA may be an acceptable solution. Nonetheless, in the dynamic land battlefield engagement, operator survival depends on obtaining highly reliable estimates of the target states in a very short timeframe. For the stationary observer, this reliability is not offered by conventional TMA.

Therefore, to increase a vehicle's battlefield survivability, there is a requirement to replace active ranging and target tracking methods. These tracking methods must be passive, accurate and reliable, and should allow the observer to remain stationary.

1.2 Target Motion Analysis and Observability

Classical Bearings-Only Tracking (BOT) is a form of TMA, wherein the target states are estimated from bearings-only observations. For the maritime environment, BOT has been extensively investigated and BOT fundamentals developed [1,2,3], which require an observer manoeuvre for tracking solution convergence. Figure 1.1 shows a typical observer manoeuvre required for accurate estimation in classical BOT. This requirement is not a viable tactical alternative in the land environment.

Figure 1.1 Traditional BOT target-observer geometry.

Constraining the observer to be stationary presents an even more challenging BOT problem. From Figure 1.2 it can be seen that the target bearing, β , is the ratio of the range in the X co-ordinate, x and the Y co-ordinate, y . Thus,

$$\beta = \tan^{-1}\left(\frac{x}{y}\right) \quad (1.1)$$

Observability is the ability of the observer to determine uniquely the target position. It can be seen, in Figure 1.2, that for a stationary observer, the target's X-Y position is not observable [4,5] because the parallel tracks all have the same bearings but different range.

Figure 1.2 Possible track solutions to stationary BOT.

Therefore, the target track is not unique unless the observer moves, as in traditional BOT, or makes at least one position measurement. While only the 2-dimensional (2-D)

case is shown, this requirement also extends to 3-dimensional (3-D) tracking [5,6]. Moreover, Nardone and Aidala [4] showed that, for observability, there must be a sufficient bearing rate or observer baseline such that bearing changes are larger than the bearing measurement noise [4]. Thus, other examples of non-observable systems are approaching/receding radial targets, or stationary targets, that present the same bearing measurement to a stationary observer. Together, the target's course, observer manoeuvre and bearing rate requirements comprise the classical BOT system observability criteria.

For tactical reasons, in the land environment, it is desirable that the observer's location remains fixed in many situations. Thus, to ensure observability, the target range at some point must be measured either by active radar ranging, passive triangulation with another observer, or by LRF. In this thesis, a LRF makes at least a single range measurement. For the purpose of estimator evaluation, periodic range measurements are also provided by triangulation in a completely passive, distributed multi-sensor framework.

1.3 Survey of BOT Literature

When the BOT-TMA problem is formulated as a Kalman filter in Cartesian coordinates, premature collapse of the covariance matrix leads to estimation divergence [2]. In response, a variety of BOT methods have been proposed, including the modified polar filter, which uses bearing rate, range rate divided by range, bearing and reciprocal range [2]. Pseudolinear estimator (PLE) methods have been explored [1,3] which linearize the state equations by incorporating bearing measurements in the observation matrix.

However, the PLE information matrix contains elements that have large magnitudes because of the additional, correlated measurement noise. Consequently, PLE filters provide a false indication of the filter performance and yield biased state estimates [1,3]. This bias manifests itself in an under-estimated target range, which can be very severe at long ranges.

To eliminate the PLE bias, an Instrumental Variable (IV) method replaces the measurements in the PLE observation matrix by generated instrumental variables. Chan and Rudnicki [7] investigated the passive tracking problem by formulating the unknown target states as a constant vector of target initial position and velocity. The target followed a constant speed and course, while the observer executed a series of manoeuvres to satisfy the observability criteria of the system. The target position was then estimated as a linear combination of the past positional estimates and current velocity estimate. It was shown that the bias inherent in a PLE BOT scheme could be significantly reduced through the use of IVs. Although the IV method was shown to be unbiased and consistent, it may diverge under moderate noise levels [1,7]. Conventional BOT with PLE and IV schemes lend themselves to sequential estimation and the systems are passive.

Van Huyssteen [8], Romine and Kamen [9], and others have investigated the fusion of IR imaging sensor data and radar range measurements via a partially decentralized, sequential tracking algorithm. By fusing imager bearing with periodic radar range and bearing measurements, such systems can produce converging position and velocity estimates; however, it was found that the estimates diverged when the target was outside

the imager frame [8,9]. The estimates also diverged when radar range measurements were fused with imager bearing measurements [9]. The hybrid scheme allows sequential imaging sensor measurements to be fused with the asynchronous, but periodic, radar measurements. These schemes are active and allow the observer to remain stationary.

TMA with multi-path time delay [10] aims to eliminate the observer manoeuvre requirement. In addition to classic BOT, multi-path time delay TMA in the maritime environment relies on BOT and time-delay signal measurements reflected off the ocean floor, which are collected by a towed-array sonar [10]. While such schemes can operate with a stationary observer, they are impracticable for many applications on land.

Passive multi-sensor tracking using a distributed network of imaging sensors presents one potential solution for TMA with a stationary observer. Target localization can be performed from simple trigonometric relationships [11]. The impact of registration errors has been investigated [11], by examining the effects of positional and orientation uncertainty. It has been shown that registration errors can significantly affect target position estimates, but not the velocity or acceleration estimates [11]. However, in the dynamic and rugged land environment, distributed implementations may be practical only in limited situations where time and location permit correct registration and synchronization.

In most cases, work in passive BOT estimation appears devoted to the maritime tracking environment where long observation times, non-manoeuving targets and optimal observer manoeuvres are taken for granted [1,2,3,7,12,13]. These requirements

cannot be met in many tactical land situations. Therefore, the stationary observer requires a multi-sensor tracking method that can passively localize a land target.

1.4 Prediction, Filtering and Smoothing

Wiener [14] solved the optimum-filtering problem in the 1940s. Figure 1.3 illustrates the Wiener filter showing an input signal, a transfer function and an output. In particular, the formulation of the Wiener filter consists of:

- a) an input containing signal, $s(t)$, plus independent noise $n(t)$;
- b) a filter transfer function, $G(s)$, which is linear and time-invariant. No other assumption is made as to its form; and
- c) a Least Squares (LS) error performance criterion, where the error is defined as $e(t) = s(t+\delta) - x(t)$.

As a generalization, the Wiener filter output, $x(t)$, is an estimation of $s(t+\delta)$ rather than just $s(t)$. If δ is positive, it is classified as a prediction problem. If $\delta = 0$, it is a filtering, or tracking problem; and, if δ is negative, it is a smoothing problem [14].

Figure 1.3 The Wiener filter.

When δ is positive, the estimator uses all input signals ($t=0$ to k) to predict $s(t+\delta)$. The performance of the prediction estimator is the poorest, since it is trying to estimate the signal at some point in the future without the benefit of the future measurements.

When $\delta = 0$, the filter uses all past signal inputs ($t=0$ to k); and, as new measurements arrive, the filter produces the LS estimate at each time k . This estimator performs better than the prediction estimator because of the additional information available in the measurements.

When δ is negative, all measurements up to, and including, time $t=k$ are used to estimate $s(t)$, $t < k$. Among all δ , the smoothing estimator gives the best estimate of the input signal, since it makes use of all measurements up to $t = k$, to estimate input at $t < k$.

1.5 Data Fusion

Hall and Llinas [15] describe the fusion of multi-sensor data as “the use of multiple types of sensors to increase the accuracy with which a target can be observed.” For example, radar provides the ability to accurately determine a target's range, but has limited angular resolution. In contrast, a single IR imaging sensor can accurately determine a target's angular direction but is unable to measure range. Correct association of the two sensor measurements provides an improved determination of the target's position than could otherwise be obtained by either sensor independently [15].

The Joint Directors of Laboratories (JDL) Data Fusion Working Group Process Model [15] identifies the process, functions, and categories of fusion techniques. The

JDL model consists of a two-layer hierarchy and four levels of processing. This thesis is concerned with the first level of processing, as detailed below.

Level 1 processing is considered the more mature area of data fusion, with roots dating back to Gauss's study of the motion of astronomical bodies in the 18th century [16]. Today, Level 1 Processing, or Object Refinement, combines positional, parametric and identify information to achieve refined representation of objects. This level of processing performs four key functions:

- a) transforms sensor data into consistent sets of units and co-ordinates;
- b) refines and extends in time estimates of an object's position and kinematics;
- c) assigns data to objects to allow the application of statistical estimation techniques; and
- d) refines the estimate of an object's identity or classification

Current Level 1 research focuses on solving the correlation and manoeuvring target problem for multi-sensor, multi-target situations [15].

Centralized, or measurement fusion methods, combine independent observations into a global quantity which is then processed. The combination is usually done through a weighted-average of independent measurements. Some [18,19] claim that centralized fusion methods yield better target tracks, since there is no loss of information between the sensors and the central track processor. Provided adequate communication bandwidth is

available, the central processor has all available information with which to create an optimal estimate of the target states. It is evident that, although centralized methods are optimal, they suffer from increased data transfer and processing requirements and lower survivability due to single point of failure likelihood [15,19].

Decentralized, or track output, fusion methods combine local sensor state estimates into a single quantity. However, distributed fusion methods are less developed than centralized methods [15]. Technical design consideration must be given to the system architecture and how the processing nodes should share the fusion responsibility. This means deciding which sensor should report data to each processing node. Consideration must also be given to how the processing nodes communicate. In particular, the issue of bandwidth requirements, and the communication of raw or processed data, must be considered. And finally, how should the nodes fuse the data for the best performance [17]? Decentralized methods provide reduced databus loading, reduced computational burden in any single processor (compared to a centralized processor), and higher survivability due to the distributed tracking capabilities [19].

It has been noted [18,20] that centralized and decentralized fusion methods assume independent measurement errors. However, in decentralized tracking of manoeuvring targets, the assumption that the estimation errors are independent is not correct because the same process noise enters into the estimation equations. The measurement noise power is insufficient to overcome the process noise effects and yield independent estimation errors [20]. Thus, while decentralized fusion with manoeuvring targets can reduce the estimation errors (over a single sensor), mean error due to target manoeuvre

cannot be averaged out because the effects of process noise correlation in the estimators [18,19,20].

The land environment is a challenging situation for which traditional BOT, with a manoeuvring observer, is not a viable solution. The dynamic and rugged nature of the environment make proper registration of distributed sensors practical only in limited situations. The stationary observer constraint, coupled with the need to passively, yet accurately, localize a target, demonstrates the requirement for real-time (online) stationary BOT. It is also evident that such a system must be robust, and not prone to sensor registration errors. Finally, to obtain the optimal estimate, there is a requirement for a tracking system combining a highly survivable, decentralized fusion architecture that mitigates the loss of measurement information between sensors.

1.6 Thesis Objective and Scope

The objective of this thesis is to present an online solution to the stationary observer tracking problem. The solution is obtained by directly fusing passive imaging sensor bearing measurements with one or more, active target range measurements in a decentralized architecture. The target velocity is modeled as a two-state constant velocity vector, parameterized to a single target position. The target velocity is estimated by smoothing, rather than the traditional filtering methods, and the smoothed velocity estimates are obtained by using independent sensor measurements as the IVs in a smoothing algorithm; hence, each smoothed velocity estimate is computed from all available measurement information. After a single range measurement is taken, the target

position is predicted from the smoothed velocity estimates. The independent range predictions are exchanged as range measurements in a parallel Kalman filter architecture, thus eliminating the correlation between range estimation errors.

1.7 Thesis Organization

This thesis contains 5 chapters and 2 appendices. Chapter 2 develops a smoothing approach to BOT and proposes a measurement instrumental variable bearings-only tracking (MIV-BOT) method. The MIV-BOT method is extended into a decentralized architecture. By predicting range from smoothed velocity estimates at the sensor, and exchanging the predictions, the decentralized architecture eliminates the requirement for active range measurement when the target is non-maneuvring. A recursive fast smoothing method is presented along with linearized signal and measurement models for parallel discrete-time Kalman filters.

Chapter 3 outlines the experimental set-up for a series of Monte-Carlo simulations. Using constant course and speed targets, the simulations are designed to evaluate the filtering and smoothing schemes, and the decentralized fusion architecture. The final experiment investigates the decentralized architecture performance on real target data obtained from visible spectrum and IR imaging sensors. Chapter 4 discusses the experimental results. Finally, Chapter 5 presents the conclusions and recommends areas for further research.

CHAPTER 2 : BEARINGS-ONLY TRACKING THEORY

2.1 Chapter Introduction

This chapter presents the theory and system geometry for a stationary observer BOT method. It first states the simplifying assumptions, and develops non-observable and observable BOT filtering methods. This is followed by the development of batch and recursive smoothing methods to estimate the target velocity. It then reviews the concept of instrumental variables, which are used for bias elimination. The chapter then outlines how multi-sensor measurements can be used as instrumental variables in the smoothing algorithm. Finally, using range prediction and parallel Kalman filters, a complete decentralized data fusion architecture is formulated for BOT.

2.2 Statement of Assumptions

The development of the stationary observer, multi-sensor BOT algorithm is based on the following assumptions:

- a) the observer is stationary and the target remains in the imaging sensor field-of-view (FOV);
- b) the observer's platform has two co-located imaging sensors with similar FOV.

The sensors are synchronized with the same sampling rate and they are correctly registered;

- c) at least a single range measurement is available from a laser range finder (LRF).

The LRF has a small pointing error;

- d) the target is following a constant course and speed trajectory; and
- e) the sensor measurement errors are statistically independent, zero mean random variables.

2.3 System Geometry for a Least Squares Estimation Technique

Let the observer be at the origin in X-Y co-ordinates. The target passes through the FOV and the target bearing, β_i , is measured by each sensor for $i = 1, 2, \dots, k$ samples. At some instant, $i=l$, the LRF makes a single range measurement, R_l . Figure 2.1 illustrates the BOT system geometry for a stationary observer.

The system geometry and tracking scheme proposed in this thesis uses periodic bearing measurements from independent sensors, plus at least a single range measurement, R_l from the LRF. The bearing can be measured passively by any means; however, for precise angular measurements, visible and IR imaging sensors are used. The observer can be stationary and a single active LRF measurement is needed to obtain the estimate of the target velocity. Prior to the LRF measurement, only non-observable estimate of the target velocity can be obtained from bearings-only measurements, as described in the next section.

Figure 2.1 Stationary observer BOT system geometry.

2.4 Non-Observable Estimation of Velocity

The two-state vector of target velocity is $\theta = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$, to be estimated from bearing measurements $\beta_i = \bar{\beta}_i + e_i$, at $t = iT$, where $\bar{\beta}_i$ is the true target bearing, e_i is a zero-mean Gaussian random variable and T is the constant sampling period. Let $\begin{bmatrix} x_i \\ y_i \end{bmatrix}$ be the target position at time i . When there are no position measurements, a stationary observer can only provide a non-observable estimation of velocity from bearings-only measurements. That is, an estimate that is a function of some unknown target position. The formulation of the estimation equations is as follows.

From

$$\begin{aligned} x_i &= x_o + \dot{x} iT \\ y_i &= y_o + \dot{y} iT \end{aligned} \quad (2.1)$$

we have for target range, R_i , and bearing $\bar{\beta}_i$ at i ,

$$\frac{x_i}{y_i} = \frac{R_i \sin \bar{\beta}_i}{R_i \cos \bar{\beta}_i}$$

Substituting $\bar{\beta}_i = \beta_i - e_i$ and cross-multiplying in (2.1) and re-arranging yields

$$y_i [\sin \beta_i \cos e_i - \cos \beta_i \sin e_i] = x_i [\cos \beta_i \cos e_i + \sin \beta_i \sin e_i]$$

and

$$y_i \sin \beta_i - x_i \cos \beta_i = [y_i \cos \beta_i + x_i \sin \beta_i] \tan e_i$$

Since $y_i = R_i \cos \beta_i$ and $x_i = R_i \sin \beta_i$, it follows that

$$y_i \sin \beta_i - x_i \cos \beta_i = R_i \tan e_i = \varepsilon_i \quad (2.2)$$

Putting (2.1) into (2.2) gives

$$-\dot{x}iT \cos \beta_i + \dot{y}iT \sin \beta_i = x_o \cos \beta_i - y_o \sin \beta_i + \varepsilon_i \quad (2.3)$$

Without loss of generality, let $T=1$. For $i = 1, 2, \dots, k$, (2.3) gives

$$\mathbf{A}_k \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \mathbf{\Phi}_k \begin{bmatrix} x_o \\ y_o \end{bmatrix} + \mathbf{q}_k \quad (2.4)$$

where

$$\mathbf{q}_k = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_k \end{bmatrix} \quad (2.5)$$

$$\mathbf{A}_k = \begin{bmatrix} -\cos \beta_1 & \sin \beta_1 \\ -2 \cos \beta_2 & 2 \sin \beta_2 \\ -3 \cos \beta_3 & 3 \sin \beta_3 \\ \vdots & \vdots \\ -k \cos \beta_k & k \sin \beta_k \end{bmatrix} \quad \text{and} \quad (2.6)$$

$$\Phi_k = \begin{bmatrix} \cos \beta_1 & -\sin \beta_1 \\ \cos \beta_2 & -\sin \beta_2 \\ \cos \beta_3 & -\sin \beta_3 \\ \vdots & \vdots \\ \cos \beta_k & -\sin \beta_k \end{bmatrix} \quad (2.7)$$

The pseudo-inverse estimate for $\theta = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$ parameterized to (x_o, y_o) is, at $i = k$,

$$\hat{\theta}_k = \begin{bmatrix} \hat{\dot{x}}_k \\ \hat{\dot{y}}_k \end{bmatrix} = (\mathbf{A}_k^T \mathbf{A}_k)^{-1} \mathbf{A}_k^T \Phi_k \begin{bmatrix} x_o \\ y_o \end{bmatrix} \quad (2.8)$$

We cannot evaluate $\hat{\theta}_k$ unless (x_o, y_o) is known, or where there is available a

measurement $\begin{bmatrix} \tilde{x}_l \\ \tilde{y}_l \end{bmatrix}$, at some sample $i=l$.

2.5 Observable Estimation of Velocity

Suppose there is a noisy position measurement at $i = l$, giving $\begin{bmatrix} \tilde{x}_l \\ \tilde{y}_l \end{bmatrix}$. To relate $\hat{\theta}_l$ to this measurement, let

$$(\mathbf{A}_l^T \mathbf{A}_l)^{-1} \mathbf{A}_l^T \Phi_l = \begin{bmatrix} a_l & b_l \\ c_l & d_l \end{bmatrix}. \quad (2.9)$$

From (2.1) and (2.8) we have

$$\begin{bmatrix} x_l \\ y_l \end{bmatrix} = \mathbf{F}_l \begin{bmatrix} x_o \\ y_o \end{bmatrix} \quad (2.10)$$

where

$$\mathbf{F}_l = \begin{bmatrix} a_l l + 1 & b_l l \\ c_l l & d_l l + 1 \end{bmatrix} \begin{bmatrix} x_o \\ y_o \end{bmatrix} \quad (2.11)$$

Hence, from (2.10), an estimate of $\begin{bmatrix} x_o \\ y_o \end{bmatrix}$ is

$$\begin{bmatrix} \hat{x}_o \\ \hat{y}_o \end{bmatrix} = \mathbf{F}_l^{-1} \begin{bmatrix} \tilde{x}_l \\ \tilde{y}_l \end{bmatrix} \quad (2.12)$$

and the estimate of θ at l , having just received the position measurement $\begin{bmatrix} \tilde{x}_l \\ \tilde{y}_l \end{bmatrix}$ is

$$\hat{\theta}_l = (\mathbf{A}_l^T \mathbf{A}_l)^{-1} \mathbf{A}_l^T \Phi_l \mathbf{F}_l^{-1} \begin{bmatrix} \tilde{x}_l \\ \tilde{y}_l \end{bmatrix} \quad (2.13)$$

2.6 Smoothing the Velocity Estimate

The parameterized filtering BOT algorithm presented above estimates θ with respect to $\begin{bmatrix} \hat{x}_o \\ \hat{y}_o \end{bmatrix}$. However, this is sub-optimal because $\begin{bmatrix} \hat{x}_o \\ \hat{y}_o \end{bmatrix}$ may have large errors, especially in short tracking scenarios. This initial position error will propagate forward into all future target velocity estimates at (2.13). It is, however, possible to obtain a smoothed BOT velocity estimate parameterized to any measured position on the target track. Hence, to obtain the most accurate velocity estimate, the reference position to use should be the one that is more reliable than the initial position estimate from (2.12).

At time l , the relationship between $\begin{bmatrix} x_i \\ y_i \end{bmatrix}, i \leq l$ and $\begin{bmatrix} x_l \\ y_l \end{bmatrix}$ is, from (2.1)

$$\frac{\sin \beta_i}{\cos \beta_i} = \frac{x_l - \dot{x}(l-i)}{y_l - \dot{y}(l-i)} \quad (2.14)$$

Cross multiplying in (2.14) and re-arranging terms yields

$$\dot{x}(l-i) \cos \beta_i - \dot{y}(l-i) \sin \beta_i = x_l \cos \beta_i - y_l \sin \beta_i \quad (2.15)$$

For $i = 1, 2, \dots, k$ samples, (2.15) in matrix form becomes

$$\mathbf{G}_k \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \mathbf{\Phi}_k \begin{bmatrix} x_l \\ y_l \end{bmatrix} \quad (2.16)$$

where

$$\mathbf{G}_k = \begin{bmatrix} (l-1) \cos \beta_1 & -(l-1) \sin \beta_1 \\ (l-2) \cos \beta_2 & -(l-2) \sin \beta_2 \\ (l-3) \cos \beta_3 & -(l-3) \sin \beta_3 \\ \vdots & \vdots \\ (l-k) \cos \beta_k & -(l-k) \sin \beta_k \end{bmatrix} \quad (2.17)$$

Therefore, the smoothed LS velocity estimate $\tilde{\boldsymbol{\theta}}_l = \begin{bmatrix} \tilde{x}_l \\ \tilde{y}_l \end{bmatrix}$, at $k=l$, is

$$\tilde{\boldsymbol{\theta}}_l = \begin{bmatrix} \tilde{x}_l \\ \tilde{y}_l \end{bmatrix} = (\mathbf{G}_l^T \mathbf{G}_l)^{-1} \mathbf{G}_l^T \boldsymbol{\Phi}_l \begin{bmatrix} \tilde{x}_l \\ \tilde{y}_l \end{bmatrix} \quad (2.18)$$

It is shown, in Chapter 4, that the smoothed estimate $\tilde{\boldsymbol{\theta}}_l$ has less error than the filtered estimate $\hat{\boldsymbol{\theta}}_l$.

2.7 Fast Smoothing

For online smoothing, the instant l when a range measurement is made is not known *a priori*. One solution is to store all bearing measurements up to the l^{th} sample and then batch smooth the LS estimate according to (2.18). This requires a large memory; therefore, it is necessary to have a recursive smoothing scheme that allows the on-line estimation of the target state at any given l .

Note that in the smoothing process, each measurement in $\mathbf{G}_k$ is weighted by the factor $(l-k)$. To develop a recursive fast smoothing estimator,

Define a filtering matrix

$$\mathbf{D}_f = \begin{bmatrix} 1 & 0 & \dots & 0 \\ \vdots & 2 & 0 & \vdots \\ & 0 & 3 & 0 \\ & & 0 & \ddots & 0 \\ 0 & & & & k \end{bmatrix} \quad (2.19)$$

and a smoothing matrix

$$\mathbf{D}_s = \begin{bmatrix} (l-1) & 0 & \dots & 0 \\ \vdots & (l-2) & 0 & \vdots \\ & 0 & (l-3) & 0 \\ & & 0 & \ddots & 0 \\ 0 & & & & (l-k) \end{bmatrix} \quad (2.20)$$

Then the matrix operation $\mathbf{G}_k^T \mathbf{G}_k$ in (2.18), for $l=k$, can be re-formulated as

$$(\mathbf{D}_s \mathbf{\Phi}_k)^T \mathbf{D}_s \mathbf{\Phi}_k = \mathbf{\Phi}_k^T \mathbf{D}_s^T \mathbf{D}_s \mathbf{\Phi}_k = \mathbf{\Phi}_k^T \mathbf{W}_s \mathbf{\Phi}_k \quad (2.21)$$

where

$$\mathbf{W}_s = \begin{bmatrix} (l-1)^2 & 0 & \dots & 0 \\ 0 & (l-2)^2 & & \vdots \\ \vdots & & (l-3)^2 & \\ & & & \ddots \\ 0 & & & & (l-k)^2 \end{bmatrix} = l^2 \mathbf{I} - 2\mathbf{D}_f + \mathbf{W}_f \quad (2.22)$$

with

$$\mathbf{W}_f = \begin{bmatrix} 1^2 & 0 & \dots & 0 \\ 0 & 2^2 & & \vdots \\ \vdots & & 3^2 & \\ 0 & & & \ddots \\ & & & & k^2 \end{bmatrix} \quad (2.23)$$

Hence,

$$\Phi_k^T \mathbf{W}_s \Phi_k = l^2 \Phi_k^T \Phi_k - 2l \Phi_k^T \mathbf{D}_f \Phi_k + \Phi_k^T \mathbf{W}_f \Phi_k \quad (2.24)$$

We can now compute recursively at each instant three 2x2 matrices:

$$\Phi_k^T \mathbf{D}_f \Phi_k, \quad \Phi_k^T \Phi_k, \quad \text{and} \quad \Phi_k^T \mathbf{W}_f \Phi_k. \quad (2.25)$$

If, at sample $k=l$ the system receives a range measurement and becomes observable, the smoothing operation $\Phi_l^T \mathbf{W}_s \Phi_l$ at (2.24) can then be found using the three quantities at (2.25). Similarly, the matrix operation $\mathbf{G}_l^T \Phi_l$ can be computed recursively as follows.

Using (2.20) in (2.16)

$$\mathbf{G}_k = \mathbf{D}_s \Phi_k \quad (2.26)$$

So that

$$\mathbf{G}_k^T = \Phi_k^T \mathbf{D}_s = \Phi_k^T (\mathbf{I} - \mathbf{D}_f) \quad (2.27)$$

and,

$$\mathbf{G}_l^T \Phi_l = \Phi_l^T \mathbf{D}_s \Phi_l \equiv l \Phi_l^T \Phi_l - \Phi_l^T \mathbf{D}_f \Phi_l \quad (2.28)$$

We have already computed and stored $\Phi_k^T \Phi_k$ and $\Phi_k^T \mathbf{D}_f \Phi_k$ for (2.25); hence, we easily obtain $\mathbf{G}_l^T \Phi_l$ from (2.28).

Thus, even without the *a priori* knowledge of the instant when $k=l$, the smoothed estimate $\tilde{\theta}_l$ of the target's velocity can be obtained recursively without increasing memory or storage requirements. Of course, this online smoothing estimator scheme yields identical results to the batch smoothing process. The smoothed $\tilde{\theta}_l$ should yield better estimates than the filtered $\hat{\theta}_l$, because the target states are estimated with respect to a measured position, rather than an estimated initial position.

2.8 Instrumental Variables to Eliminate Bias

The smoothed estimate $\tilde{\theta}_k$ is biased if its expected value deviates from the true value θ . That is, $E\{\tilde{\theta}_k\} \neq \theta$. $E\{\tilde{\theta}_k\} - \theta$ is the bias. An estimator is consistent if $\tilde{\theta}_k \rightarrow \theta$ as $k \rightarrow \infty$ [21]. However, in (2.8) and (2.18), $E\{\mathbf{A}_l^T \Phi_l\} \neq 0$ and $E\{\mathbf{G}_l^T \Phi_l\} \neq 0$ because $\mathbf{A}_l$, $\mathbf{G}_l$ and Φ_l contain elements of the noisy measurements β_k . In least squares terminology the regressor and regressand are correlated and the estimates in (2.8) and (2.18) are biased.

One means to eliminate the bias is the method of Instrumental Variables (IV) [21]. IV is a fundamental feature of the fusion scheme presented in this thesis; thus, the IV method is described briefly.

Since the matrices $\mathbf{G}_k$ and $\mathbf{\Phi}_k$ of the smoothing process are functions of the same measured bearings, they are correlated. Let another matrix be available which contains independent estimates of the measurement matrix and call this matrix $\mathbf{Z}_k$.

Then,

$$E\{\mathbf{Z}_k^T \mathbf{q}_k\} = 0, E\{\mathbf{Z}_k^T \mathbf{\Phi}_k\} = 0 \text{ and } E\{\mathbf{Z}_k^T \mathbf{G}_k\} = 0 \quad (2.29)$$

It is necessary that $(\mathbf{Z}_k^T \mathbf{G}_k)^{-1}$ exists for any sample k . The elements of the $\mathbf{Z}_k$ matrix are called the instruments. The batch pseudo-inverse solution for the IV smoothing estimator, for $k=l$, is given by modifying (2.18) to

$$\tilde{\boldsymbol{\theta}}_l = \begin{bmatrix} \tilde{x}_l \\ \tilde{y}_l \end{bmatrix} = (\mathbf{Z}_l^T \mathbf{G}_l)^{-1} \mathbf{Z}_l^T \mathbf{\Phi}_l \begin{bmatrix} \tilde{x}_l \\ \tilde{y}_l \end{bmatrix} \quad (2.30)$$

Since the elements of $\mathbf{Z}_k$ are independent of $\mathbf{G}_k$, the estimator is consistent [21].

Chan and Rudnicki [7] generated sub-optimal instruments based on previous estimates of the target state, using $k-1$ measurements. While the computed instruments are clearly statistically independent of the bearing measurement at time k , they are sub-optimal and estimates may diverge under high levels of corrupting noise [7].

Multiple sensors are available on many modern observation platforms, providing independent target bearing measurements. Since the measurements from different sensors are clearly independent, they make an ideal choice as instruments.

This thesis proposes a fusion method that uses independent sensor measurements as the instruments in the smoothing estimator. In (2.6) and (2.17), the MIV-BOT method simply replaces each β_i with a second, independent sensor bearing measurement, $\tilde{\beta}_i$, to produce the IV matrix $\mathbf{Z}_k$.

2.9 Kalman Filter

The Kalman filter was first proposed at [22], and its ability to produce a recursive solution to the Wiener problem was quickly realized; moreover, unlike the Wiener filter, the Kalman filter can also be modified to cover non-linear time-varying systems. Kalman's solution [22] uses recursive, state variable methods and models the dynamic system as linear system with both deterministic and random inputs.

The process signal model (for m measurements and n state variables) is defined by

$$\mathbf{x}_{k+1} = \boldsymbol{\psi}\mathbf{x}_k + \mathbf{w}_k \quad (2.31)$$

where $\mathbf{x}_k$ is the $(n \times 1)$ process state vector at time k , $\boldsymbol{\psi}$ is the $(n \times n)$ state transition matrix relating $\mathbf{x}_k$ to $\mathbf{x}_{k+1}$, and $\mathbf{w}_k$ is $(n \times 1)$ white process noise with a known covariance. In (2.31), the deterministic input is zero.

The process measurement model is a linear relationship, defined by

$$\zeta_k = C\chi_k + \varphi_k \quad (2.32)$$

where ζ_k is the (m x 1) measurement vector at time k , C is the noiseless (m x n) observation matrix, relating ζ_k to χ_k , and where φ_k is (m x 1) zero mean independent Gaussian measurement noise. φ_k is assumed to be uncorrelated with w_k [14,22].

The covariance matrices for φ_k and w_k are given by

$$E\{w_k w_i^T\} = \begin{cases} Q_k, & i = k \\ 0, & i \neq k \end{cases} \quad E\{\varphi_k \varphi_i^T\} = \begin{cases} \Sigma_k, & i = k \\ 0, & i \neq k \end{cases}$$

Define the *a priori* estimate of χ_k , given $k-1$ measurements, as $\hat{\chi}_{k|k-1}$. Similarly, define the *a posteriori* estimate of χ_k , given k measurements, as $\hat{\chi}_{k|k}$. $\hat{\chi}_{k|k}$ is an estimate of χ_k using all available measurements, with an error

$$\xi_k = \chi_k - \hat{\chi}_{k|k} \quad (2.33)$$

The Kalman filter uses the measurement, ζ_k , to refine $\hat{\chi}_{k|k}$. The new information from the measurement is the innovation, given by

$$\nu_k = \zeta_k - C\hat{\chi}_{k|k-1} \quad (2.34)$$

The *a posteriori* estimate, $\hat{\chi}_{k|k}$, is a linear combination of ν_k and $\hat{\chi}_{k|k-1}$, given by

$$\hat{\chi}_{k|k} = \hat{\chi}_{k|k-1} + G_k \nu_k \quad (2.35)$$

where G_k is the gain. G_k is selected such that the estimation square error is minimized. This is an optimization problem, and it is clearly shown at [14] and [22] that an optimal gain can be calculated recursively by the expression

$$G_k = \Gamma_{k|k-1} C^T (C \Gamma_{k|k-1} C^T + \Sigma_k)^{-1} \quad (2.36)$$

where the *a posteriori* estimate error covariance is

$$\Gamma_k = E\{\xi_k \xi_k^T\} = (I - G_k C) \Gamma_{k|k-1} \quad (2.37)$$

Hence, from (2.31) and since w_k has zero mean, the *a priori* estimate of the process is

$$\hat{\chi}_{k+1|k} = \psi \hat{\chi}_{k|k} \quad (2.38)$$

From (2.33), the *a priori* estimation error is simply

$$\xi_{k+1|k} = \chi_{k+1} - \hat{\chi}_{k+1|k} \quad (2.39)$$

Hence

$$\xi_{k+1|k} = \psi \xi_{k|k} + w_k \quad (2.40)$$

from which follows the *a priori* error covariance

$$\Gamma_{k+1|k} = E\{\xi_{k+1|k} \xi_{k+1|k}^T\} = \psi \Gamma_k \psi^T + Q_k \quad (2.41)$$

Thus, (2.35), (2.36), (2.37), (2.38) and (2.41) are the Kalman filter equations. The recursive implementation is shown graphically in Figure 2.2 as a four step process [14].

From (2.36) it can be seen that if the error covariance estimate at (2.37) does not reflect the true estimation errors, the gain may be insufficient to accurately update the estimate at (2.35), given subsequent measurements. This is typical in low observability BOT tracking, where range measurements are not available.

Figure 2.2 Kalman filter flow chart

However, given at least a single range measurement, and with the availability of independent imaging sensor centroid bearing measurements, we can formulate a recursive on-line process that estimates the target position and velocity.

Once a single range measurement is made, a smoothed estimate of the target velocity is computed by (2.30). For all $k > 1$, the MIV-BOT smoothed LS estimate

$\tilde{\theta}_l = \begin{bmatrix} \tilde{x}_l \\ \tilde{y}_l \end{bmatrix}$ can now be used to predict target range. Given a ranging measurement

vector $\begin{bmatrix} \tilde{x}_l \\ \tilde{y}_l \end{bmatrix}$, the predicted target position, $\begin{bmatrix} \tilde{x}_k \\ \tilde{y}_k \end{bmatrix}$, for any $k > l$ is simply

$$\begin{bmatrix} \tilde{x}_k \\ \tilde{y}_k \end{bmatrix} = \begin{bmatrix} \tilde{x}_l + (k-l)\tilde{\dot{x}}_l \\ \tilde{y}_l + (k-l)\tilde{\dot{y}}_l \end{bmatrix} \quad (2.42)$$

where the range prediction, $\tilde{R}_k$ from the smoothed MIV-BOT estimate is

$$\tilde{R}_k = \sqrt{\tilde{x}_k^2 + \tilde{y}_k^2} \quad (2.43)$$

There are available two independent bearing measurements, β_i and $\tilde{\beta}_i$. It has also been shown at (2.31) and (2.32) how the independent sensors also produce two smoothed velocity estimates, and hence two range predictions. Thus, the bearing measurements and the range predictions can be used as inputs in two parallel Kalman filters.

To define the recursive 2-D Kalman filter solution, model the target states as a 4x1 vector

$$\mathbf{X}_k = \begin{bmatrix} x_k \\ \dot{x} \\ y_k \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \text{x position at time k} \\ \text{x velocity (constant)} \\ \text{y position at time k} \\ \text{y velocity (constant)} \end{bmatrix} \quad (2.44)$$

Let the process measurement vector, $\mathbf{M}_k$, be

$$\mathbf{M}_k = \begin{bmatrix} x_k \\ y_k \end{bmatrix} = \begin{bmatrix} R_k \sin(\beta_k) \\ R_k \cos(\beta_k) \end{bmatrix} + \begin{bmatrix} \eta_k^x \\ \eta_k^y \end{bmatrix} \quad (2.45)$$

where $\begin{bmatrix} \eta_k^x \\ \eta_k^y \end{bmatrix}$ is the measurement noise vector.

Using range prediction, the measurement vector for either sensor then becomes

$$\mathbf{M}_k = \begin{bmatrix} x_k \\ y_k \end{bmatrix} = \begin{bmatrix} \tilde{R}_k \sin(\beta_k) \\ \tilde{R}_k \cos(\beta_k) \end{bmatrix} + \begin{bmatrix} \eta_k^x \\ \eta_k^y \end{bmatrix} \quad (2.46)$$

where $\tilde{R}_k$ is the range prediction taken from the other sensor, and *vice versa*. The measurement matrix, which relates the state vector to the measurements, is

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (2.47)$$

The Kalman filter state transition matrix is

$$\mathbf{\Psi} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.48)$$

and the filter predictor is given as

$$\hat{\mathbf{X}}_{k+1|k} = \mathbf{\Psi} \hat{\mathbf{X}}_{k|k} \quad (2.49)$$

The prediction error covariance is

$$\mathbf{P}_{k+1|k} = \Psi \mathbf{P}_k \Psi^T + \mathbf{Q} \quad (2.50)$$

$\mathbf{Q}$ is assumed zero for the constant course and speed target. We will model any manoeuvre as a Gaussian process, at which point the diagonal elements in the $\mathbf{Q}$ matrix becomes non-zero. The Kalman gain at each iteration is calculated online by

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}^T (\mathbf{H} \mathbf{P}_{k|k-1} \mathbf{H}^T + \Sigma_k)^{-1} \quad (2.51)$$

The filter innovation sequence is

$$V_k = \mathbf{M}_k - \mathbf{H} \hat{\mathbf{X}}_{k|k-1} \quad (2.52)$$

The *a posteriori* estimate update is then

$$\hat{\mathbf{X}}_{k|k} = \hat{\mathbf{X}}_{k|k-1} + \mathbf{K}_k V_k \quad (2.53)$$

with a corresponding error covariance update

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K} \mathbf{H}) \mathbf{P}_{k|k-1} \quad (2.54)$$

The state vector, $\mathbf{X}_{k=l}$, is initialized with the Cartesian LRF measurement. Although the Kalman filter is initialized at $k=l$, the sequence of measurements has been pre-processed by smoothing; hence, the state vector velocity variables are initialized to the smoothed velocity estimate at $k=l$. The error covariance is initialized to diagonal elements of 10^6 .

2.10 Multi-Sensor Data Fusion

Data fusion schemes were reviewed in Chapter 1, where it was shown that centralized schemes combine the measurements to form a weighted average based on the measurement noise statistics. The fused measurement is then used as an input to a single tracking processor. Less developed decentralized architectures process the local tracks at each sensor. The tracks are then sent to a master filter, that combines them into a global estimate. The previous sections in this chapter developed smoothing and Kalman filtering equations, where a single range measurement and independent bearing measurements are available. This section shows how MIV-BOT produces estimates that are a fusion of all available measurement information.

Using visible and IR sensors, the following definitions are used to describe the decentralized fusion architecture. Recall that $\bar{\beta}_i$ is the true target bearing at the co-located visible and IR sensors, for $i = 1, 2, \dots, k$ samples. Then the corresponding imaging sensor bearing measurements are:

$$\beta_{vis_i} = \bar{\beta}_i + e_{vis}$$

$$\beta_{ir_i} = \bar{\beta}_i + e_{ir}$$

where e_{vis} and e_{ir} are the respective independent, zero mean bearing errors.

The LRF range measurement, at $i=1$, is $R_1 = \bar{R}_1 + e_r$, where $\bar{R}_1$ is the true target range and e_r is the ranging error.

At each sensor the observable smoothed MIV-BOT velocity estimates, at $k=l$, are $\tilde{\theta}_{vis_l}$ and $\tilde{\theta}_{ir_l}$. The visible and IR range predictions at each sensor are $\tilde{R}_{vis_k}$ and $\tilde{R}_{ir_k}$, for $k \geq l$.

$\hat{\mathbf{X}}_{vis_{k|k}}$ and $\hat{\mathbf{X}}_{ir_{k|k}}$ are the respective Kalman filter *a posteriori* target state vectors, while $\hat{\mathbf{X}}_{vis_{k+1|k}}$ and $\hat{\mathbf{X}}_{ir_{k+1|k}}$ are the respective Kalman filter *a priori* target state vectors, for $k \geq l$.

2.10.1. Measurement IV Fusion. Equation (2.30) gives the MIV-BOT algorithm for two independent bearing measurements. When using visible and IR imaging sensor bearing measurements, Figure 2.3 shows how a single smoother directly fuses the measurement and the IV into the smoothed velocity estimate. The parameterized smoother calculates, at each sample i , the 2x2 matrices defined at (2.25). The observable smoothed MIV-BOT velocity estimates for each sensor, $\tilde{\theta}_{ir_i}$ and $\tilde{\theta}_{vis_i}$, are computed upon receipt of a ranging measurement vector $\begin{bmatrix} \tilde{x}_i \\ \tilde{y}_i \end{bmatrix}$, at $i=l$. It is evident that the smoothed estimates, $\tilde{\theta}_{ir_i}$ and $\tilde{\theta}_{vis_i}$, are a fusion of all available measurement information.

2.10.2. System Tracking Algorithm. The flow chart in Figure 2.4 shows the algorithm for the visible sensor. The algorithm is identical for the IR sensor. Prior to the range measurement, the unobservable parameterized velocity is calculated by recursively updating the smoothing matrices.

Figure 2.3 Measurement IV fusion by smoothing.

Upon receipt of a single position measurement $\begin{bmatrix} \tilde{x}_l \\ \tilde{y}_l \end{bmatrix}$, at $i=l$, the observable smoothed velocity, $\tilde{\theta}_{vis,l}$, is calculated according to (2.30). The predicted range, $\tilde{R}_{vis,k}$, is calculated from $\begin{bmatrix} \tilde{x}_l \\ \tilde{y}_l \end{bmatrix}$ and $\tilde{\theta}_{vis,l}$, according to (2.33), for all $k \geq l$. At the end of each iteration, the measurement, $\beta_{vis,k}$, and the range prediction, $\tilde{R}_{vis,k}$, are now available for Kalman filtering. Of course, the *a priori* and the *a posteriori* state vectors are fed back to the Kalman filter, to be used in the next iteration. However, the *a posteriori* position estimate is also used to smooth the next velocity estimate; and, if another LRF measurement, R_l , is available, the measurement is used in place of $\tilde{R}_{vis,k}$.

2.10.3. Decentralized Fusion Architecture. Figure 2.5 shows the complete MIV-BOT fusion architecture. The system is decentralized but not fully autonomous, in that there is direct sharing of bearing and range measurements in the local smoothers. However, there is no direct sharing of estimates during the parallel MIV-BOT smoothing operations. After a single range measurement is obtained, the range predictions, $\tilde{R}_{vis_k}$ and $\tilde{R}_{ir_k}$, are fed forward to, and exchanged with, the second sensor. Note that the Kalman filter inputs are the measured bearings, β_{vis_k} and β_{ir_k} , and the exchanged range predictions, $\tilde{R}_{ir_k}$ and $\tilde{R}_{vis_k}$. These are filtered recursively according to (2.31) to (2.36). The *a posteriori* target state vectors are fed back and the *a posteriori* target positional estimates are used to smooth the next bearing measurement. Clearly, in this system, the bearing IVs are contemporaneously uncorrelated with the bearing measurements. They are also assumed to be Gaussian.

The derivation of the Kalman filter [14,22] assumes that the measurements are corrupted by white, independent Gaussian random variables. As R.E. Kalman wrote in his seminal paper [22]: “A *random function of time may be thought of as the output of a dynamic system excited by an independent Gaussian process.*” However, the exchanged range predictions are generated from smoothed velocity estimates and contain correlated errors. Therefore, each range prediction that is used as input into the Kalman filter must be de-correlated. The sub-optimal solution is achieved by adding 1% percent range independent, zero mean, white Gaussian noise.

Figure 2.4 Flow Chart for MIV-BOT Data Fusion.

Figure 2.5 Decentralized MIV-BOT fusion architecture.

2.11 Chapter Summary

This chapter first stated the underlying assumptions for the development of an MIV-BOT estimator. It then outlined the system geometry for stationary observer tracking, when at least a single range measurement is available. Next, non-observable and observable batch filtering and smoothing estimation methods were developed. The large memory requirement for batch smoothing led to the development of a fast, recursive smoothing algorithm which can be implemented without *a priori* knowledge of the ranging instant. A simple method for computing smoothed, independent range predictions was outlined. The stationary MIV-BOT smoothing algorithm and range predictions were combined into a multi-sensor, data fusion architecture for passive tracking.

CHAPTER 3 : EXPERIMENTAL SET-UP

3.1 Chapter Introduction

This chapter outlines the experimental set-up for a series of Monte Carlo simulations, to investigate the performance of the BOT filters, smoothers and data fusion algorithms. In the first experiment, filtered IV-BOT and filtered MIV-BOT velocity estimates are compared with smoothed MIV-BOT estimates, using noiseless reference positions. The second experiment compares the MIV filtering and fast smoothing estimation methods, given a single range measurement. The third set of experiments investigates the decentralized MIV fusion and range prediction scheme for non-maneuvring targets. Finally, in Experiment 4, the decentralized MIV fusion method is tested on 32.5 seconds of real target data from the Defence Research Establishment Valcartier (DREV). The observer is stationary in all experiments.

In the first three experiments a target travels along the X-Y plane, and noiseless bearing measurements are projected onto a X-Z imaging plane. Horizontal and vertical pixel measurements are then corrupted with zero-mean, independent Gaussian noise. The measure of performance is the ensemble root-mean-square (RMS) error, and these are plotted for 100 independent runs in each scenario. All simulations were performed using MATLAB 5.2 on a Pentium III-500 personal computer.

3.1.1. Sensor Models. A visible sensor, an IR sensor and a LRF are modeled in these experiments. These generic models are derived from typical sensor specifications (See Table I) that are available in the literature.

Table I: Sensor Models

	Visible	Infrared
FOV H/V	20/13.3 degrees	20/13.3 degrees
Detector Array (H/V)	768/493	798/400
Frame Rate	30 Hz	30 Hz

H/V = horizontal/vertical

The sensor image frame is formed on the X-Z plane, while 2-D target trajectories lay on the X-Y plane. Each image frame consists of 640 x 480 pixels (H/V), where the imaging sensor field-of-view (FOV) is the number of degrees spanned in azimuth and elevation. In azimuth, this is denoted by Ψ_h , and in elevation by Ψ_v . Figure 3.1 shows the relationship between the 2-D image pixel co-ordinates and the 3-D sensor line-of-sight (LOS) co-ordinates.

Figure 3.1 Imager LOS and image plane co-ordinates.

The horizontal instantaneous FOV (IFOV), α , is

$$\alpha = (\pi / 180)(\Psi_h / 640) \quad (3.1)$$

and the vertical IFOV is

$$\gamma = (\pi / 180)(\Psi_v / 480) \quad (3.2)$$

In the image frame, the measured centroid location is (c_h, c_v) . Since we are using the imaging sensor LOS as the reference frame, and filtering target position and velocity with respect to the LOS, the geometry defined in Figure 3.1 relates the centroid location to

sensor LOS angular co-ordinates. Hence, the centroid horizontal bearing (azimuth) in each frame is

$$\Delta \theta = c_h \alpha \quad (3.3)$$

and the vertical bearing (elevation) is

$$\Delta \phi = c_v \gamma \quad (3.4)$$

Centroid estimation errors, for small targets (around 10 pixels) without clutter, have been modelled as $\sigma_c < 1$ [21]. Practical field research [26,27] using larger targets (80 or more pixels) indicates that the actual measurement noise is substantially higher in moderate clutter. Thus, assuming segmentation in the target extraction algorithm, it is reasonable to model measurement noise as a Gaussian independent zero-mean random variable, where $4 < \sigma_c < 12$.

3.1.2. Laser Range Finder Noise Model. The LRF is co-located with the imaging sensor and returns a range-only measurement. However, it has a range measurement error, and a pointing error. The LRF bearing, b_i , and range measurement, R_i are then

$$b_i = \bar{\beta}_i + e_b \quad (3.5)$$

$$R_i = \bar{R}_i + e_r \quad (3.6)$$

where $\bar{R}_i$ and $\bar{\beta}_i$ are the true target range and bearing, for $i = 1, 2, \dots, k$. The LRF range and pointing error are, e_r and e_b , respectively. Both are assumed to be Gaussian, independent

zero-mean random variables. Relating the LRF noise to the X-Y measurement frame, we note that the Cartesian LRF positional measurement is

$$x_i = R_i \sin b_i = \bar{x}_i + e_x \quad (3.7)$$

$$y_i = R_i \cos b_i = \bar{y}_i + e_y \quad (3.8)$$

where the bar denotes the true target position; and, e_x and e_y are the respective Cartesian measurement errors. Using (3.5) and (3.6) in (3.7) and (3.8) gives

$$e_x = R_i \sin b_i - \bar{R}_i \sin \bar{\beta}_i \approx \bar{R}_i e_b \cos \bar{\beta}_i + e_r \sin \bar{\beta}_i \quad (3.9)$$

$$e_y = R_i \cos b_i - \bar{R}_i \cos \bar{\beta}_i \approx -\bar{R}_i e_b \sin \bar{\beta}_i + e_r \cos \bar{\beta}_i \quad (3.10)$$

with the expected values

$$E \left\{ \begin{bmatrix} e_x \\ e_y \end{bmatrix} \begin{bmatrix} e_x \\ e_y \end{bmatrix}^T \right\} = \begin{bmatrix} \sigma_r^2 \sin^2 \bar{\beta}_i + \bar{R}_i^2 \sigma_b^2 \cos^2 \bar{\beta}_i & (\sigma_r^2 - \sigma_b^2 \bar{R}_i^2) \sin \bar{\beta}_i \cos \bar{\beta}_i \\ (\sigma_r^2 - \sigma_b^2 \bar{R}_i^2) \sin \bar{\beta}_i \cos \bar{\beta}_i & \bar{R}_i^2 \sigma_b^2 \sin^2 \bar{\beta}_i + \sigma_r^2 \cos^2 \bar{\beta}_i \end{bmatrix} \quad (3.11)$$

Where $\sigma_b = E \{e_b^2\}$ and $\sigma_r = E \{e_r^2\}$ is the ranging error standard deviation.

3.2 Experiment 1 - MIV-BOT Validation

3.2.1 Experiment 1a - IV-BOT, MIV-BOT Filtering and Smoothing. The purpose of the first experiment is to determine the relative performance of the unbiased IV-BOT PLE filter in [7], the MIV-BOT filter, and the fast MIV-BOT smoother developed in Chapter 2.

In this scenario, the target position and velocity are $\bar{x}_o = -450\text{m}$, $\bar{y}_o = 3000\text{m}$, $\dot{x} = 5 \text{ m/s}$, $\dot{y} = -2.5 \text{ m/s}$. Bearing measurements are made over a 40 second observation window, with a measurement period $T = 1/30$ seconds. The visible and infrared sensor measurement noise is 4 pixels H/V, with a zero mean Gaussian distribution. The IV-BOT and MIV-BOT filtered X and Y co-ordinate velocity estimates are calculated at each sample, for $i = 1$ to 600, parameterized to the true reference position $(\bar{x}_o, \bar{y}_o)$. These filtered target velocity estimates are obtained from the IV implementation of (2.13). The MIV-BOT smoothed velocity estimates are calculated with respect to the true position, $(\bar{x}_l, \bar{y}_l)$, for each $k \leq l$. Equations (2.24) (2.25) and (2.28) are used to obtain the fast smoothed velocity estimates. Ensemble velocity RMS errors are plotted for the three estimators.

3.2.1. Experiment 1b - IV-BOT, MIV-BOT and Smoothing Under Noise. In the previous experiment, the single estimator performance was investigated when the initial position and the position at l were known exactly. The second simulation in this experiment is similar, except the IV-BOT and MIV-BOT filters estimate the initial position at each interval. The filtered estimate of the initial position is determined by

(2.12). The smoothed estimate of $\begin{bmatrix} x_o \\ y_o \end{bmatrix}$ is calculated at each i , for $i=1,2,\dots,600$ by

$$\tilde{x}_o = \tilde{x}_i - iT\tilde{\dot{x}}_i \quad (3.12)$$

$$\tilde{y}_o = \tilde{y}_i - iT\tilde{\dot{y}}_i \quad (3.13)$$

where $\begin{bmatrix} \tilde{x}_i \\ \tilde{y}_i \end{bmatrix}$ are the fast smoothed velocity estimates produced at each sample i , and $\begin{bmatrix} \tilde{x}_i \\ \tilde{y}_i \end{bmatrix}$ is the measured position. IR bearing measurements are again used as the IVs in the visible sensor and the IR pixel noise is increased to 8 pixels. Smoothed and filtered ensemble velocity RMS errors are plotted for all three estimators.

3.3 Experiment 2 - Filtering and Smoothing Schemes.

Two non-maneuvring target scenarios are simulated in this experiment, to determine the relative performance of two velocity estimation schemes given a single noisy range measurement.

3.3.1. Experiment 2a. Filter/Filter and Smooth/Filter Schemes with a Single Range Measurement. In this first scenario, the target approaches the observer at $\bar{x}_o = -450\text{m}$, $\bar{y}_o = 2500\text{m}$, $\dot{x} = 10 \text{ m/s}$, $\dot{y} = -5 \text{ m/s}$ and the bearing measurement period is $T=1/30\text{s}$. A single, stationary platform with two co-located imaging sensors is simulated for a 40-second track. Two velocity estimation schemes are investigated, and in both schemes, a LRF takes only a single noisy range measurement.

The first scheme filters the target velocity for $k < l$, estimates $\begin{bmatrix} x_o \\ y_o \end{bmatrix}$ according to (2.12.) at $l=600$, and then filters forward for $k \geq l$. All subsequent bearing measurements are filtered with respect to the filtered initial position estimate. This is the known as the filter/filter scheme.

The second scheme first smoothes the target velocity at $k=l$, parameterized to the single LRF position measurement at $l=600$. It then estimates $\begin{bmatrix} x_o \\ y_o \end{bmatrix}$ from the smoothed velocity, according to (3.12) and (3.13). All bearing measurements, for $k \geq l$, are then filtered with respect to the smoothed initial position estimate. This is known as the smooth/filter scheme.

Standard deviations of the filtered and smoothed initial position estimates are computed. Ensemble velocity RMS errors are plotted, for $k=600$ to 1200, for both schemes.

3.3.2. Experiment 2b. Filtering and Smoothing with Passive Ranging. The second part to this experiment investigates the relative performance of the filtering and smoothing schemes when periodic, synchronous and passive range measurements are made. Again, the observer is stationary with target position and velocity $\bar{x}_o = -450\text{m}$, $\bar{y}_o = 2500\text{m}$, $\dot{x} = 10 \text{ m/s}$, $\dot{y} = -5 \text{ m/s}$. The target approaches over a shorter, 30-second observation window. The imaging sensors record bearing measurements at a sampling period $T=1/10\text{s}$. The IR sensor standard deviation is increased to 12 pixels. Figure 3.2 shows a third visible imaging sensor on a separate platform located at 300 metres from the observer. This sensor begins periodic bearing measurements at $k=100$. The Kalman filter equations at (2.33) to (2.46) are used to pre-filter the target centroid in the imager plane. The visible sensor noise covariance matrix, Σ_k , is $\begin{bmatrix} 4^2 & 0 \\ 0 & 4^2 \end{bmatrix}$, while the

infrared covariance matrix is $\begin{bmatrix} 12^2 & 0 \\ 0 & 12^2 \end{bmatrix}$. The X-Z image frame equations are identical to the X-Y frame, except target position and velocity are in pixels and pixels/second. All sensor centroid measurements are tracked by the Kalman filter from $i=1$, at $T=0.1$ seconds. These pre-filtered centroid measurements give the LS centroid estimate at all three sensors. From the pre-filtered centroid, a target bearing at the two observers, β_1 and β_2 , is obtained according to (3.3). Then, the triangulated estimate $\begin{bmatrix} x'_i \\ y'_i \end{bmatrix}$ is obtained according to (3.14) and (3.15) below.

Figure 3.2 Passive ranging geometry.

$$y'_i = \frac{D}{\tan(\beta_1) + \tan(\beta_2)} \quad (3.14)$$

$$x'_i = y'_i \tan(\beta_1) \quad (3.15)$$

For the filtering system, the estimate of $\begin{bmatrix} x_o \\ y_o \end{bmatrix}$ is obtained from the smoothed velocity estimate at $k=l$, according to (3.12) and (3.13). For $k>l$ filtered velocity estimates are calculated from the estimate of $\begin{bmatrix} x_o \\ y_o \end{bmatrix}$, and the centroid bearings.

MIV-BOT smoothed velocity estimates for $k>l$ are calculated parameterized to the periodic, triangulated, and pre-filtered position estimates at (3.14) and (3.15). Velocity and position RMS errors are plotted for both schemes.

3.4 Experiment 3 - Fusing Smooth Track and Measurement Data.

This experiment is divided into 3 scenarios, the purpose of which is to investigate the performance of the complete decentralized fusion estimator developed in Chapter 2. The scenarios use short, realistic tracking times and range prediction. Non-manoeuving targets are simulated and the LRF makes a single range measurement mid-way through the target track.

3.4.1. Experiment 3a - Receding target. Two co-located imaging sensors observe a close range, receding target whose initial position and velocity are $\bar{x}_o = -150\text{m}$, $\bar{y}_o = 1500\text{m}$, $\dot{x} = 10 \text{ m/s}$, $\dot{y} = 10 \text{ m/s}$. The visible and infrared sensor noise

levels are $\sigma=4$ pixels H/V. The sensor measurement period is $T=1/30$ seconds, over a total observation time of 30 seconds. The MIV-BOT scheme smoothes bearing measurements from $i=1$ to $i=449$, to give a velocity estimate parameterized to the position measurement, $\begin{bmatrix} \tilde{x}_i \\ \tilde{y}_i \end{bmatrix}$, taken at $i=450$. The fast MIV-BOT smoothed velocity estimate is obtained according to (2.24) (2.25) and (2.28). From $i=451$ to $i=900$ the target position is predicted from the MIV-BOT smoothed velocity estimate and the measured position. To decorrelate the Kalman filter range inputs, the position prediction is whitened with 1% range independent, zero-mean Gaussian random noise. At each sample, the noisy range prediction is then exchanged as the range measurement in the other Kalman filter, and *vice versa*. The *a posteriori* Kalman filter position estimate is fed back to smooth the next bearing measurement. The ensemble velocity and position RMS errors are plotted for the decentralized fusion scheme.

3.4.2. Experiment 3b - Approaching target. The second scenario investigates the performance of the MIV-BOT decentralized fusion algorithm for approaching target conditions, with a single range measurement. In this experiment a high-angle target approaches the observer at $\bar{x}_o = -150\text{m}$, $\bar{y}_o = 1500\text{m}$, $\dot{x} = 5 \text{ m/s}$, $\dot{y} = -10 \text{ m/s}$. The remainder of the experimental setup is identical to the previous one.

3.4.3. Experiment 3c - Radial target. In this third scenario, a high-angle, high-speed target approaches the observer at $\bar{x}_o = -300\text{m}$, $\bar{y}_o = 3500\text{m}$, $\dot{x} = 0 \text{ m/s}$, $\dot{y} = -27.8 \text{ m/s}$. The remainder of the experimental setup is identical to the previous two scenarios.

3.5 Experiment 4 - DREV Sequence

Experiment 4 is divided into 2 parts, with the purpose of investigating the MIV-BOT fusion algorithm using real data. The experiment uses visible and IR imagery, and Global Positioning System (GPS) data obtained from the Defence Research Establishment Valcartier (DREV). Imagery samples from the sequence are shown at Appendix B. The sequence was recorded on 28 October 1998 and contains 40 seconds of data. In the sequence, a Canadian Cougar Light Armoured Vehicle (LAV) approaches at a high-angle towards the sensor suite. Co-located visible and IR sensor images were captured at a period of $T_i = 1/30$ s and recorded on super VHS with a time code. DREV staff then manually extracted the centroid pixel location for each frame in both spectral bands. This experiment uses a 32.5 second sequence, from time code 03:02:11:00 to 03:02:43:30.

The track begins slightly above the sensor LOS and passes near the image frame origin. The track ends slightly below the LOS. Target GPS position is recorded only in the X-Y co-ordinates and the exact angular offset between the GPS co-ordinate frame and the imager LOS is not known; therefore, estimation is performed for X-Y target position and velocity with respect to the imager LOS co-ordinates.

The target trajectory changes moderately over the course, but the target accelerates and executes a sharp turn at time code 03:02:43:30. The target speed also varies throughout, thus presenting a realistic manoeuvring target. Figure 3.3 shows the manually extracted target position in the image plane, along the visible sensor LOS. The IR data is offset by 25 pixels in the vertical, but is similar in all other respects.

Figure 3.3 Target position as measured in visible image frame.

The vehicle's Universal Transverse Mercator (UTM) position was measured by an on-board GPS at a measurement period of $T_g=1$ second. A second GPS at the sensor site provides a measured target range. The GPS measurement error is approximately 10 metres. Figure 3.4 shows the target range as a function of time, as measured by GPS.

Figure 3.4 Target range as a function of time.

Figure 3.5 Target velocity as a function of time.

The target speed was averaged at each position measurement, and then interpolated at 1/30 seconds over the entire observation window. Figure 3.5 shows the interpolated target speed over the observation period.

Thus, the interpolated range and manually extracted centroid data provide a complete target range and bearing data set. Figure 3.6 shows the X-Y interpolated target trajectory in the visible sensor LOS co-ordinate frame.

Figure 3.6 Target trajectory in visible sensor LOS co-ordinates.

3.5.1. Experiment 4a, DREV Sequence without Manoeuvre Detection. This first experiment uses the MIV-BOT fusion algorithm and a single range measurement during the 32.5 second target course. The visible and infrared centroid measurements are

corrupted with 4 pixels H/V Gaussian random variables. Equation (3.2) and (3.3) are used to relate the noisy centroid location to imager LOS angular measurements. The noisy bearing measurements are MIV-BOT smoothed for 0 to 22 seconds, or samples $i=1$ to $i=659$. A single range measurement is corrupted with an independent Gaussian 1% range random variable at $l=660$. For each sensor, the smoothed target velocity estimate is then obtained from the parameterized MIV-BOT estimate. For $k=661$ to $k=975$, the smoothed velocity estimate is used to predict target range, according to (2.31) and (2.32). The exchanged range predictions, and measured bearings, are then Kalman filtered in the same manner as Experiment 3. RMS position, bearing and velocity errors are plotted.

3.5.2. Experiment 4b. DREV Sequence with Manoeuvre Detection. This experiment is identical to the previous experiment, except it incorporates a track-quality indicator, based on the filter residuals. For any sensor at each $k > 660$, the estimated target

bearing, $\hat{\beta}_k$, is found from the elements of the four-state vector $\mathbf{X}_{k/k} = \begin{bmatrix} \mathbf{X}(1) \\ \mathbf{X}(2) \\ \mathbf{X}(3) \\ \mathbf{X}(4) \end{bmatrix}$ where

$$\hat{\beta}_k = \tan^{-1} \left(\frac{\mathbf{X}(1)}{\mathbf{X}(3)} \right) \quad (3.16)$$

The time-averaged residuals, over a moving window of length, W , are

$$g_k = \left(\frac{\sum_{j=-W}^0 (\hat{\mathbf{B}}_{k+j} - B_{k+j})^2}{k+1} \right)^{1/2} \quad (3.17)$$

If $g_k \leq Th$, where Th is some threshold, then the system is deemed to have a reliable track. Otherwise, g_k indicates that either the target has manoeuvred or the initial track was not reliable. In either case, another range measurement is required and the velocity estimates are smoothed and parameterized to the last measured position. For this experiment, the track quality threshold is set to the noise statistics of the centroid measurement, or 0.135 degrees. W is set to 15 samples, or 0.5 seconds. To prevent over sampling, the LRF is limited to 1 range measurement every 2 seconds.

In the tracking literature, it is common to model a target manoeuvre as a Gaussian noise process [14,18,24,25,26], and use it as an input into the constant course and speed signal model. This assumption was made for the Kalman filter equations in Chapter 2. Thus, during a good track, the target is not manoeuvring and the noise process matrix Q is zero. Once the target is deemed to have manoeuvred, the Q matrix is increased to diagonal elements of 0.1. When $g_k \leq Th$, the manoeuvre is complete and the Q matrix is reset to zero.

The ensemble average number of additional range measurements is recorded over 100 independent runs, along with the ensemble RMS error in target speed, range and bearing.

3.6 Chapter Summary

This chapter has outlined a series of experiments, to investigate the performance of stationary observer target tracking schemes. Using noiseless reference positions, Experiment 1 was designed to compare filtered and smoothed MIV-BOT velocity

estimates with filtered IV-BOT estimates. Experiment 2 compares the MIV filtering and fast smoothing methods, with a single range measurement. Experiment 3 was designed to investigate the decentralized MIV fusion and range prediction scheme for non-maneuvring targets. Finally, the decentralized MIV architecture is tested on real target data from DREV.

The next chapter presents the simulation results for each of the four experimental sets.

CHAPTER 4 : EXPERIMENTAL RESULTS

4.1 Experiment 1 - MIV-BOT Validation

4.1.1.Experiment 1a - IV-BOT, MIV-BOT and Smoothing. The purpose of this experiment was to investigate the relative performance of the IV-BOT filter, the MIV-BOT filter, and the fast MIV-BOT smoother. The IV-BOT and MIV-BOT filtered velocity estimates were parameterized to the true initial position, and the smoothed estimates were parameterized to the true position at t .

Figure 4.1 shows the X co-ordinate velocity errors when centroid observations are corrupted with 4 pixels Gaussian noise. The Y co-ordinate results are not shown, as they are similar. In the short tracking timeframe, the IV-BOT velocity estimates diverge. Early in the track, the IVs are generated from unreliable velocity estimates, because the IV-BOT filter has not reached steady-state. These poor velocity estimates then produce poor IVs.

In contrast, the MIV-BOT filter and smoother have RMS errors that quickly approach the Cramér-Rao Lower Bound (CRLB) (See Appendix A). The errors exceed the BOT CRLB since a single noiseless range measurement is used. The MIV-BOT filter and smoother performance is nearly identical when the true reference position is known. Since both the MIV-BOT filter and smoother reach the CRLB, they are efficient.

With a noiseless reference position and a short tracking scenario, this experiment shows the improvement of MIV-BOT over generated IVs.

Figure 4.1 X co-ordinate velocity RMS error for known reference point.

4.1.2. Experiment 1b - IV-BOT, MIV-BOT and Smoothing Comparison Under Noise. Figure 4.2 shows the IV-BOT and MIV-BOT filter Y co-ordinate errors when noise is present in the bearing measurement and errors exist in the initial position estimate. The X co-ordinate RMS error is very similar to the previous experiment; however, with noise in the initial position estimate, the Y co-ordinate velocity RMS error increases significantly for the MIV-BOT filter.

The smoother performance is similar to the previous experiment, because the smoother's reference position was noiseless in both experiments. It can also be seen in Figure 4.2 that the IV-BOT filter RMS error is also considerably larger. It degrades

significantly when errors in the initial position estimate propagate forward into the velocity estimate.

Figure 4.2 Filter and smoother Y co-ordinate velocity RMS error while estimating initial target position.

These two simulations show that the pseudo linear MIV-BOT filter and smoothing algorithms are efficient when the reference point is known exactly, as the MIV-BOT filter and MIV-BOT smoother achieve the CRLB. The performance of the MIV-BOT filtering and smoothing estimators is superior to the IV-BOT filter, in both test cases. The smoothing algorithm is efficient, and is superior to both filters given an exact range measurement.

4.2 Experiment 2 - Filtering and Smoothing Schemes.

Two non-maneuvring target scenarios were simulated in this experiment, to examine the MIV-BOT filter and the MIV-BOT fast smoother developed in Chapter 2. Two schemes were examined. The first scheme filtered the target velocity for $k < l$, and then filtered forward for $k > l$. This is known as the filter/filter scheme. The second scheme smoothed the target velocity at $k=l$, estimated the target initial position from the smoothed velocity, and then filtered forward for $k > l$. This is known as the smooth/filter scheme. A LRF takes a single noisy range measurement in both schemes.

The second part in the experiment investigated the MIV-BOT filter and smoother when multiple range measurements were available.

4.2.1. Experiment 2a. Filter/Filter and Smooth/Filter Schemes with a Single Range Measurement. The X and Y initial position estimate standard deviations, $\sigma_{\hat{x}_0}$ and $\sigma_{\hat{y}_0}$, are shown below in Table II, for 100 independent runs. It is evident that smoothing significantly reduces the initial position estimation errors.

Table II: Initial Position Estimation Error (m) for Filtering and Smoothing

	Filtering	Smoothing
$\sigma_{\hat{x}_0}$	17.28 m	8.32 m
$\sigma_{\hat{y}_0}$	477.77 m	159.63 m

The X co-ordinate velocity RMS error, for 100 independent runs, is shown in Figure 4.3. It can be seen that since the initial position estimation error in the filter/filter scheme propagates forward, subsequent filtered velocity estimates are poor. The smooth/filter scheme does not depend on the initial position estimate, and the position measurement noise is significantly less than the filtered initial position estimate errors. Consequently, the smoothing algorithm yields lower RMS velocity errors. However, there is error in the target position measurement, and the smoothed initial position estimate. Consequently, the smoother/filter scheme is unable to reach the CRLB with a single range measurement, as this error propagates into the estimates for $k \geq 1$. Both the filter/filter and smooth/filter schemes reached steady-state at 20 seconds, and further reduction in either RMS error is not achieved.

Figure 4.3 X co-ordinate velocity RMS error for filter/filter and smoother/filter schemes, and a single range measurement.

4.2.2. Experiment 2b - Filtering and Smoothing with Passive Ranging. The second part to this experiment simulated a realistic tracking scenario with passive periodic ranging. The stationary observer computed periodic position measurements, by combining pre-filtered bearing measurements from two sensors at separate locations. The RMS pixel error and a typical centroid track are shown below in Figure 4.4 and 4.5, respectively.

Figure 4.4 Visible sensor ensemble pixel RMS error.

From Figure 4.6, it is evident that pre-filtering the centroid at the imaging sensor reduces the sensor bearing RMS error, and of course, reduces the position estimation error. Figure 4.7 shows the X co-ordinate RMS velocity error for the MIV-BOT filtering and smoothing estimators. The plots begin at $k=1$, since the problem is not observable prior to this instant. It is evident that subsequent range measurements permit the smoothed velocity estimate to approach the CRLB. While the same bearing

measurements improve the filter performance, smoothing the velocity estimate at each range measurement yields superior performance. It can also be seen that, although the IR sensor noise is three times larger than the visible sensor noise, both the visible and IR smoothers yield reliable velocity estimates.

Figure 4.5 Typical Kalman filter centroid estimate in the image X-Z plane.

Figure 4.6 Visible sensor bearing RMS error in azimuth and elevation, using triangulation and Kalman filtered centroid tracks.

Figure 4.7 X co-ordinate visible sensor filtered and smoothed velocity RMS error and IR sensor smoothed velocity RMS error, with periodic range measurements.

4.3 Experiment 3 - Fusing Smooth Track and Measurement Data.

4.3.1. Experiment 3a - Receding Target. This experiment investigated the performance of the complete fusion architecture, using only a single range measurement. Figure 4.8 shows the X co-ordinate velocity RMS error and Figure 4.9 shows the decentralized fusion scheme X co-ordinate position RMS error, for the visible sensor. The Y co-ordinate RMS errors are not shown, as they are similar. The plots begin at 15 seconds, since the system is determinant only after the LRF makes a range measurement. It is evident that the MIV-BOT decentralized fusion scheme produces converging velocity and positional estimates, given a single LRF position measurement. It can also be seen that the fusion scheme produces velocity and position estimates with minimal bias.

Figure 4.8 Decentralized MIV-BOT fusion architecture visible sensor ensemble X co-ordinate velocity RMS error and bias, for a receding target.

Figure 4.9 Decentralized MIV-BOT fusion architecture visible sensor ensemble X coordinate position RMS error and bias, for a receding target.

4.3.3. Experiment 3b - Approaching Target. Figure 4.10 shows the velocity RMS errors and estimator bias, and Figure 4.11 shows the corresponding position performance. As can be seen in the two figures, both position and velocity results are similar to the receding target; namely, the velocity and position estimates converge, with minimal bias. The CRLB for velocity estimates is attained quickly. It can be seen that, with a single range measurement, the position estimates are reliable; however, they take considerably more time to attain the CRLB.

Figure 4.10 Decentralized MIV-BOT fusion architecture visible sensor ensemble X co-ordinate velocity RMS error and bias, for an approaching target.

Figure 4.11 Decentralized MIV-BOT fusion architecture visible sensor ensemble X co-ordinate position RMS error and bias, for an approaching target.

4.3.4. Experiment 3c -High-Angle Approaching Target. Figures 4.12 and 4.13 show the decentralized MIV-BOT fusion system RMS errors for a high-speed, high-angle approaching target. The results are consistent with the previous two scenarios. Both velocity and position estimates converge with minimal bias. In Figure 4.13 it is evident that, in this difficult target trajectory, the estimator is unable to attain the CRLB with a single range measurement. However, the position estimates do not diverge and can be considered reliable.

Figure 4.12 Decentralized MIV-BOT fusion architecture visible sensor ensemble X co-ordinate velocity RMS error and bias, for a high-angle target.

Figure 4.13 Decentralized MIV-BOT fusion architecture visible sensor ensemble X co-ordinate position RMS error and bias, for a high-angle target.

The results of Experiment 3 show that the MIV-BOT fusion architecture is robust. Smoothing the target velocity upon receipt of a single range measurement, and then predicting velocity and range, yields converging position estimates. While these three scenarios used a short tracking time, sufficient tracking baseline was available to achieve system observability. This is particularly true in the case of the high-angle approaching target scenario, as a truly radial target will be unobservable.

4.4 Experiment 4 - DREV Sequence

4.4.1. - DREV Sequence without Manoeuvre Detection. Figures 4.14, 4.15 and 4.16 show the DREV sequence RMS errors in velocity and range, when a single range

measurement is used. Note that the plotted range and velocity errors are not relative in this experiment.

It is evident, from these figures, that a manoeuvring target without a track quality/manoeuvre detection scheme will yield diverging range estimates. Since the target is accelerating, the smoothed velocity estimate is not accurate and the predicted range diverges rapidly. Because the diverging range is then used to smooth the next bearing observation, the velocity and range errors also diverge.

Figure 4.14 DREV sequence bearing RMS error for visible sensor without manoeuvre detection.

Figure 4.15 DREV sequence range RMS error for visible sensor without manoeuvre detection.

Figure 4.16 DREV sequence velocity RMS error for visible sensor without manoeuvre detection.

4.4.2. - DREV Sequence with Manoeuvre Detection. Figures 4.17, 4.18, and 4.19 show the MIV-BOT decentralized fusion scheme performance with a residual-based manoeuvre detection scheme (track quality indicator). Table III shows the total tracking time and the average number of additional range measurements initiated by the track quality indicator.

Table III: DREV Sequence Average Range Measurements using a Track Quality Indicator.

Runs	Tracking Time	Additional Range Measurements (ensemble average)
100	32.5	3.8

It is evident in Figures 4.17 and 4.18 that, while the initial velocity estimate was poor, the manoeuvre detection scheme triggers range measurement within two seconds, causing the velocity estimate to converge. Converging velocity estimates are able to give converging range estimates.

Figure 4.17 DREV sequence range RMS error for visible sensor, using manoeuvre detection.

Figure 4.18 DREV sequence bearing RMS error for visible sensor, using manoeuvre detection.

Figure 4.19 DREV sequence measured and RMS velocity error for visible sensor, using manoeuvre detection.

4.5 Summary of Experiments

Four separate experiment sets were conducted, to investigate the relative performance of IV-BOT, MIV-BOT filtering and MIV-BOT smoothing.

Experiment 1 showed that the IV-BOT filter diverges under moderate noise levels, with short tracking scenarios. It was also shown that the MIV-BOT filter produced estimates comparable to the MIV-BOT smoother only when the reference position is known exactly. Given noisy bearing measurements, the MIV-BOT smoother is robust and yields better velocity estimates than the MIV-BOT filter.

It was shown in Experiment 2 that the smooth/filter scheme produces better velocity estimates than the filter/filter scheme. Smoothed initial position estimates are

more accurate than filtered estimates. Consequently, the smooth/filter scheme yields more reliable velocity estimates, due to the reduced error in the initial position. However, with a single range measurement, the smooth/filter scheme estimation RMS errors do not reach the CRLB, as the initial position estimation errors propagate forward into the velocity estimates. The second part of Experiment 2 showed that the smoothing scheme yields the best velocity estimates, when multiple position measurements are available.

Experiment 3 investigated the performance of a complete MIV-BOT fusion architecture. The results were consistent in all test cases. Smoothing the velocity with exchanged range predictions can provide reliable tracks of a constant course and speed target, given an adequate observation baseline. Range and velocity estimates converged in all test cases.

Finally, Experiment 4 investigated the MIV-BOT fusion architecture's performance using real data. With a manoeuvring target, it was shown that the fusion scheme position and velocity estimates diverged when track quality indication is not used. The incorporation of a residual-based, track quality indicator allowed the system to detect and correct poor velocity estimates. Consequently, converging MIV-BOT range and velocity estimates were obtained even with a manoeuvring target.

CHAPTER 5: CONCLUSION AND RECOMMENDATIONS

5.1 Conclusions

This thesis investigated the stationary observer BOT problem, using a set of discrete-time bearing measurements from multiple imaging sensors, and at least a single range measurement. A system identification solution was developed.

Chapter 1 outlined the requirement for a stationary observer, multi-sensor tracking scheme that provides an accurate estimation of the target states. The system observability criteria for a stationary observer were delineated, and a literature survey of BOT was conducted. The benefits of estimating the target states by smoothing, rather than filtering, were outlined; and, the relative merits of centralized and decentralized sensor fusion methods were explained.

Chapter 2 stated the assumptions for a stationary observer BOT system. The non-observable and observable velocity estimation methods, for stationary tracking, were developed. A batch smoothing process, which estimates the target velocity parameterized to any position along the target course, was developed. Because of the memory requirements for real-time implementation, a recursive smoothing BOT algorithm for discrete-time bearing measurements was developed. The smoothing algorithm was then modified to directly fuse bearing measurements as MIVs, eliminating the inherent bias of

the PLE. The fusion scheme was developed into a decentralized system architecture, that predicts target range from MIV-BOT smoothed velocity estimates. Parallel Kalman filters then estimate the target states from the exchanged range predictions, and subsequent bearing measurements. It was shown how each velocity estimate is a fusion of all available measurement information.

Chapter 3 outlined the experimental set-up for investigating the MIV-BOT filtering and smoothing schemes. It also detailed a method to determine the performance of the decentralized fusion architecture developed in Chapter 2.

The experimental results shown in Chapter 4 demonstrate that the MIV-BOT method is efficient, in that the CRLB is approached in short tracking scenarios, and reasonable noise levels. The experiments demonstrate that the MIV-BOT smoothing method is less susceptible to divergence in short-tracking situations, even under high levels of corrupting noise. The incorporation of range predictions from smoothed velocity estimates showed that the MIV-BOT data fusion scheme could reliably estimate the states of a constant course and speed target. By incorporating a residual-based track quality indicator, it was shown that the MIV-BOT data fusion scheme can be used to track a manoeuvring target, with a few additional range measurements. This was confirmed in a test using real data.

5.2 Recommendations for Future Research

This thesis investigated a relatively small area of the stationary observer BOT problem. Six areas for further research are described below:

- 1) The Kalman filter is derived with the Gaussian input assumption. To decorrelate the Kalman filter input measurements, 1% independent zero-mean random range error was added to the range predictions. While this creates uncorrelated Kalman filter range inputs, it is sub-optimal. Therefore, an optimization method that takes into account the correlated range predictions, without pre-whitening the filter inputs, would be very useful.
- 2) The fusion architecture produces a target track for each sensor. Each track represents the fusion of all available passive and active measurement information. It is possible to fuse each track output into a single, global estimate. However, a sound argument can be made that fusing the two tracks will not improve the reliability of the estimate, as no new measurement information would be introduced in the fusion process. Nonetheless, the performance of a single MIV-BOT track should be compared with a global estimate, to determine if additional performance gain can be achieved.
- 3) The MIV-BOT tests were performed on a small set of 2-D target trajectories. A study of 3-D trajectories, combined with a system observability study, will be useful. The goals should be to define the stationary observability criteria for various target trajectories, at realistic noise levels.
- 4) The investigated target tracks were short of duration, and no method was in place to determine the optimum instant when the LRF should make a measurement. Consequently, if the LRF takes a range measurement too early in the track, or the track quality is poor, velocity and position estimates diverge and a subsequent range measurement is required. This has significant observer survivability implications.

Therefore, to assure reliable range predictions following a LRF measurement, the MIV-BOT system requires a track-quality indicator scheme for the non-determinant portion of the target track.

5) Target trajectories were assumed constant course and speed, and target manoeuvres were modeled as a Gaussian process noise. A study of stationary observer MIV-BOT, with manoeuvring target trajectories and input estimation, would be useful.

6) In all experiments, the sensors were co-located and the simulations did not investigate the estimation errors induced by sensor registration errors. It would be useful to investigate the impact of sensor registration errors when using MIV-BOT, comparing co-located and distributed sensors.

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APPENDICES

APPENDIX A

CRAMÉR-RAO LOWER BOUND

The Cramér-Rao lower bound (CRLB) denotes the theoretical lower limit for the covariance matrix of an unbiased estimator [21].

To derive this bound for BOT, let the target state vector be

$$\mathbf{X}_k = \begin{bmatrix} x(j) \\ \dot{x} \\ y(j) \\ \dot{y} \end{bmatrix} \quad (\text{A.1})$$

which is estimated from bearings-only measurements

$$\mathbf{b}^T = [\beta_o \quad \beta_1 \quad \cdots \quad \beta_i \cdots \quad \beta_j]^T \quad (\text{A.2})$$

with

$$\beta_i = \bar{\beta}_i + e_i \quad (\text{A.3})$$

where $\bar{\beta}_i$ is the true target bearing and e_i are zero mean independent Gaussian random variables of variance σ^2 . The Fisher information matrix (FIM) [21] is

$$FIM = \frac{1}{\sigma^2} \left(\frac{\partial \mathbf{b}}{\partial \mathbf{x}} \right)^T \left(\frac{\partial \mathbf{b}}{\partial \mathbf{x}} \right) \quad (\text{A.4})$$

where the Jacobian of the measurement vector is

$$\left(\frac{\partial \mathbf{b}}{\partial \mathbf{x}}\right) = \begin{bmatrix} \frac{\partial b_o}{\partial x(j)} & \frac{\partial b_o}{\partial \dot{x}} & \frac{\partial b_o}{\partial y(j)} & \frac{\partial b_o}{\partial \dot{y}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial b_j}{\partial x(j)} & \frac{\partial b_j}{\partial \dot{x}} & \frac{\partial b_j}{\partial y(j)} & \frac{\partial b_j}{\partial \dot{y}} \end{bmatrix} \quad (\text{A.5})$$

The relationship between j^{th} state and the i^{th} bearing for, $j \geq i$, is

$$\beta_i = \tan^{-1} \left[\frac{x(j) - (j-i)T\dot{x}}{y(j) - (j-i)T\dot{y}} \right] \quad (\text{A.6})$$

The partial derivatives in the Jacobian are

$$\left(\frac{\partial b_i}{\partial x(j)}\right) = \frac{y(j)}{r_i^2} \quad (\text{A.7})$$

$$\left(\frac{\partial b_i}{\partial \dot{x}}\right) = -(j-i)T \left(\frac{\partial b_i}{\partial x(j)}\right) \quad (\text{A.8})$$

$$\left(\frac{\partial b_i}{\partial y(j)}\right) = \frac{-x(j)}{r_i^2} \quad (\text{A.9})$$

$$\left(\frac{\partial b_i}{\partial \dot{y}}\right) = -(j-i)T \left(\frac{\partial b_i}{\partial y(j)}\right) \quad (\text{A.10})$$

with

$$r_i^2 = x(i)^2 + y(i)^2 \quad (\text{A.11})$$

The CRLB covariance matrix, for each state variable in (A.1), is the diagonal of $(FIM)^{-1}$ [21].

APPENDIX B**Sample Frames from the DREV Infrared and Visible Imagery Sequence**

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