UNIVERSITY OF CALGARY

**Experimental Investigation of Multiple Solutions for Liquid Holdup in Upward
Inclined Stratified Flow**

by

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A THESIS

**SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
DEGREE OF MASTER OF ENGINEERING**

DEPARTMENT OF CHEMICAL AND PETROLEUM ENGINEERING

CALGARY, ALBERTA

NOVEMBER, 1999

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0-612-49702-X

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Abstract

The mechanistic model of Taitel and Dukler (1976) has previously been reported to exhibit a region in which three solutions exist for the height of the gas-liquid interface in the stratified/wave flow regime. Several papers have been published which address this problem, and the consensus, based on theory, was that one should always accept the solution that yielded the minimum *in situ* liquid volume fraction.

Most laboratory studies have not dealt with flow rates in the ranges where the multiple solutions occur. An experimental study has been carried out using the University of Calgary multiphase flow loop to obtain such data. Tests have been performed over a range of liquid flow rates and pipe inclination angles typical of the multiple solution region.

This work reviews the relevant theory, investigates situations where theory suggests that multiple solutions should exist, and attempts to show whether or not such solutions do physically exist.

Acknowledgments

The author would like to acknowledge the following:

Dr. Garry Gregory; for the many hats you wear.

Mr. Ed DeSouza, for your support.

Mr. Bryan Leppard, without your help the computer code so essential to this work would not exist.

Dr. Harvey Yarranton, my supervisor, for providing just the right amount of supervision at just the right time.

The financial support of Neotechnology Consultants Ltd. and the moral support of its employees.

The technical staff of the University of Calgary.

And finally, all of you who supported and encouraged me along the way.

Dedication

*This work is dedicated to my family, Shar, Emma, and Noah,
for all that you have sacrificed.
Anything that I achieve in this life is shared by you.*

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Nomenclature

A	=	cross sectional area
C	=	Blasius equation constant
C_{IV}	=	critical wave velocity for inviscid Kelvin-Helmholtz analysis
C_V	=	critical wave velocity for viscid Kelvin-Helmholtz analysis
D	=	diameter
E	=	phase volume fraction
f	=	friction factor
g	=	gravitational acceleration
h_L	=	height of the liquid layer
Δh	=	root mean square wave height
k	=	absolute roughness
L	=	length
m, n	=	Blasius equation exponent
P	=	pressure
Re	=	Reynolds number
S	=	perimeter
t	=	time
V	=	velocity
x	=	distance
X	=	Lockhart-Martinelli parameter
Y	=	Taitel-Dukler inclination parameter

Greek

α	=	pipe inclination angle
μ	=	absolute viscosity
ν	=	kinematic viscosity
ρ	=	density
τ	=	shear stress

Subscripts

G	=	gas
i	=	interfacial
L	=	liquid
S	=	superficial

Superscripts

$\sim$	=	dimensionless variable
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1.0 Introduction

Within the context of this work, multiphase flow will refer to the flow of gas-liquid mixtures. Multiphase flow is of interest to the chemical process, nuclear, and petroleum industries. Multiphase flow occurs in the cooling systems of nuclear reactors, in heat exchangers, reactors, and other chemical processing equipment, and, perhaps most commonly, in the production and transport of hydrocarbon fluids.

The simultaneous presence of gas and liquid in a pipe requires a more complex method of analysis than that applied to single phase flow problems. Consider, for example, the problem of pipeline transport. For a single phase system, this will often reduce to a question of pressure loss. For a multiphase system, questions of pressure loss, liquid holdup, and flow pattern must all be considered.

Two terms used in the preceding paragraph refer to important characteristics of multiphase flow that require further definition. The first, flow pattern, refers to the way in which the two phases are distributed in the pipe. Although numerous terms have been used to describe flow patterns, Figure 1.1 illustrates what the author considers to be the six basic flow patterns for horizontal gas-liquid flow.

The second term that requires further explanation is liquid holdup. This generally refers to the fraction of the pipe cross section that is occupied by liquid (i.e., the *in situ* liquid volume fraction). For the purpose of analysis, the annular-mist and froth flow patterns are treated as homogeneous. In these cases, the velocities of the gas and liquid phases are effectively identical and the *in situ* liquid volume fraction is the same as the

input liquid volume fraction. In any other flow pattern, the analysis is complicated by the fact that the velocity of the gas phase is greater than that of the liquid phase. As a result, the *in situ* liquid volume fraction is greater than the liquid volume fraction based on the input flow rates. That is, the liquid phase will hold up relative to the gas phase; thus the term liquid holdup.

Figure 1.1 Basic Flow Patterns for Horizontal Gas-Liquid Flow

It can be seen from the preceding explanations that multiphase flow is a complex

phenomenon. As such, rigorous theoretical solutions to the problems of multiphase flow have proven elusive. Early works in multiphase flow largely focused on empirical correlations. Many such correlations have proven to be capable of acceptable accuracy and are still in common use today.

Since the mid seventies, mechanistic models have been demonstrated to represent the most promising approach for improving the accuracy of pressure drop and liquid holdup calculations in multiphase flow systems. To date, such models generally involve a combination of rigorous development, classical fluid dynamics, and purely empirical relationships. They can be quite complex but usually account for most important system variables, such as fluid properties, pipe inclination angle, diameter and roughness. They can also be capable of very good accuracy.

Taitel and Dukler (1976) presented a mechanistic model to describe transitions between five basic flow regimes in horizontal and inclined two-phase flow. Central to the flow pattern model is a generalized model for stratified flow. This is a one-dimensional two-fluid model, in which separate steady state momentum balance equations are written for the gas and liquid phases. It has become the foundation for numerous subsequent models.

Some years ago, it was discovered (Baker et. al. (1987, 1988)) that for uphill flow at relatively low liquid flow rates, the Taitel-Dukler model exhibits a region in which three solutions exist for the height of the interface in the stratified/wave flow regime. Each solution represents a different *in situ* liquid volume fraction. Other two-fluid models that follow the ground work laid by Taitel and Dukler exhibit similar behaviour.

One such model, that closely parallels the work of Taitel and Dukler, was the Oliemans (1987) mechanistic model. This model was developed for stratified smooth and wavy flow, and has proven to be capable of good accuracy at flow rates typical of gas-condensate pipelines.

Several papers have been published which address the multiple solution problem, and the consensus was that one should always choose the solution that yields the minimum *in situ* liquid fraction. As a result of such publications and discussions of the phenomenon at both the 1991 and 1993 International Conferences on Multiphase Flow, a commercial software implementation (**PIPEFLO** from Neotechnology Consultants Limited) of the Oliemans (1987) model was revised to always select the smallest root (i.e., the lowest liquid level) as the correct solution. Shortly after the release of the software, the vendor was contacted by a client who claimed that the earlier version of the software, that typically selected one of the larger roots, was in much better agreement with their measured field data. The newer version was predicting liquid holdups that were too low by a factor of twenty or more, which had serious implications with respect to sizing liquid handling facilities. Following many discussions with the client, the vendor concluded that, in this case the larger root gave a better answer.

The vendor, because of this experience, raised the issue of multiple roots at the 1995 International Conference on Multiphase Flow. Most of those present felt that the issue had been dealt with. However, representatives of two major European oil companies agreed that there was still uncertainty, based on their own field data and data from similar mechanistic models. Further, they announced that they were initiating internal projects to review the entire situation. To date, however, the author is not aware

of any published results from either of those proposed studies. Thus, it is currently unclear whether one should always pick any particular root, or whether circumstances should dictate the choice of one solution over another.

Most laboratory studies have not dealt with flow rates in the range where multiple solutions occur. An experimental study has been carried out using the multiphase flow loop at the University of Calgary to obtain such data. Tests have been performed over a range of low liquid flow rates and pipe inclination angles that are typical of the multiple solution region.

It is the intent of this work to review the relevant theory, evaluate situations where the theory would suggest that multiple roots could exist, and attempt to show, based on the experimental investigations, whether or not such solutions do physically exist.

2.0 Literature Review

2.1 Taitel and Dukler Stratified Flow Model

In their paper, Taitel and Dukler (1976) presented one of the classical works of multiphase flow literature. They proposed a mechanistic model to describe transitions between five basic flow patterns (see Figure 1.1): stratified smooth, stratified wavy, intermittent (i.e., slug and elongated bubble), annular dispersed (i.e., annular mist), and dispersed bubble. The model takes into account many of the parameters that can influence the flow regime, including gas and liquid flow rates, fluid properties, pipe diameter, and inclination of the pipe.

Their approach begins with the assumption of stratified flow and then proposes mechanisms by which the flow regime might make a transition away from stratified flow. This means that the basis of their model is a generalized model for stratified flow. It is this model that exhibits multiple roots for the liquid level and is central to the present work. As such, any literature review would be incomplete without a thorough review of the stratified flow model as presented by Taitel and Dukler.

As noted earlier, the stratified flow model presented by Taitel and Dukler (1976) is a one-dimensional, two-fluid model. It assumes steady, isothermal flow with no mass transfer between the gas and liquid phases. Each of these assumptions can be addressed by appropriate application of the model.

For stratified flow as depicted in Figure 2.1, momentum balances on the two phases (liquid and gas) are given by:

$$-A_L \left(\frac{dP}{dx} \right) - \tau_L S_L + \tau_i S_i + \rho_L A_L g \sin \alpha = 0 \quad (1)$$

$$-A_G \left(\frac{dP}{dx} \right) - \tau_G S_G - \tau_i S_i + \rho_G A_G g \sin \alpha = 0 \quad (2)$$

where A is area, D is diameter, h_L is the height of the liquid, S is perimeter, α is the inclination of the pipe, V is actual velocity, (dP/dx) is the axial pressure gradient, τ is shear stress, ρ is density, and g is the acceleration due to gravity. The subscripts L, G, and i refer to the liquid phase, gas phase, and interface respectively.

Figure 2.1 Geometry for Inclined Stratified Flow

Assuming that the pressure gradient is the same in each phase allows these two equations to be combined by equating the pressure drop in the two phases. This yields:

$$\tau_G \frac{S_G}{A_G} - \tau_L \frac{S_L}{A_L} + \tau_i S_i \left(\frac{1}{A_L} + \frac{1}{A_G} \right) + (\rho_L - \rho_G) g \sin \alpha = 0 \quad (3)$$

The shear stresses are given by:

$$\tau_L = f_L \frac{\rho_L V_L^2}{2} \quad (4)$$

$$\tau_G = f_G \frac{\rho_G V_G^2}{2} \quad (5)$$

$$\tau_i = f_i \frac{\rho_G (V_G - V_L)^2}{2} \quad (6)$$

where f is the friction factor.

The liquid and gas phase friction factors are assumed to be given by the classic Blasius (1913) relation as:

$$f_L = C_L \left(\frac{D_L V_L}{\nu_L} \right)^{-n} = C_L (Re_L)^{-n} \quad (7)$$

$$f_G = C_G \left(\frac{D_G V_G}{\nu_G} \right)^{-m} = C_G (Re_G)^{-m} \quad (8)$$

where ν is the kinematic viscosity.

D_L and D_G are the hydraulic diameters and are evaluated as recommended by Agrawal et al. (1973):

$$D_L = \frac{4 A_L}{S_L} \quad (9)$$

$$D_G = \frac{4 A_G}{(S_G + S_i)} \quad (10)$$

That is, the interface does not influence the liquid phase (i.e., similar to open-channel flow), while acting as a stationary surface with respect to the gas phase (i.e., similar to closed-duct flow).

Following Gazley (1949), Taitel and Dukler assume that $f_i \approx f_g$ for smooth stratified flow. They further assume that, at conditions of interest, the gas phase velocity is much greater than the interfacial velocity. Thus, simplifying Equation 6, the interfacial shear is identical to the gas wall shear. The definition of the interfacial shear is, thus, consistent with the definition of the hydraulic diameter of the gas phase; it says, in effect, that the interface functions as a stationary boundary (i.e., a wall) with respect to the gas phase.

The coefficients used in the definitions of the friction factors are:

$C_L = C_G = 0.046$, $n = m = 0.2$ for turbulent flow and $C_L = C_G = 16$, $n = m = 1.0$ for laminar flow. Note that the Blasius relation is independent of the pipe roughness. Thus, a number of Blasius-type equations have been developed, with different turbulent flow coefficients, in an effort to provide the best fit over a range of pipe roughness. The specific coefficients used by Taitel and Dukler are taken from Lockhart and Martinelli (1949). Note also that the laminar flow coefficients are such that the Blasius relation reduces to the traditional definition of the friction factor for laminar flow.

Taitel and Dukler then combined all of the above equations in dimensionless form using reference variables D for length, D^2 for area and the superficial phase velocities for the gas and liquid velocities. With dimensionless variables indicated by a tilde ($\sim$), the resulting equation is:

$$X^2 \left[\left(\tilde{V}_L \tilde{D}_L \right)^{-n} \tilde{V}_L^2 \frac{\tilde{S}_L}{\tilde{A}_L} \right] - \left[\left(\tilde{V}_G \tilde{D}_G \right)^{-m} \tilde{V}_G^2 \left(\frac{\tilde{S}_G}{\tilde{A}_G} + \frac{\tilde{S}_i}{\tilde{A}_L} + \frac{\tilde{S}_i}{\tilde{A}_G} \right) \right] - 4Y = 0 \quad (11)$$

where the Lockhart-Martinelli (1949) parameter (X) is given by:

$$X^2 = \frac{\frac{4C_L}{D} \left(\frac{V_{SL} D}{v_L} \right)^{-n} \frac{\rho_L V_{SL}^2}{2}}{\frac{4C_G}{D} \left(\frac{V_{SG} D}{v_G} \right)^{-m} \frac{\rho_G V_{SG}^2}{2}} = \frac{|(dP/dx)_{SL}|}{|(dP/dx)_{SG}|} \quad (12)$$

and the inclination parameter introduced by Taitel and Dukler (Y) is:

$$Y = \frac{(\rho_L - \rho_G) g \sin \alpha}{\frac{4 C_G}{D} \left(\frac{V_{SG} D}{v_G} \right)^{-m} \frac{\rho_G V_{SG}^2}{2}} = \frac{(\rho_L - \rho_G) g \sin \alpha}{|(dP/dx)_{SG}|} \quad (13)$$

where $|(dP/dx)_S|$ is the pressure drop due to one phase flowing alone in the pipe.

Each of the dimensionless variables depends only on the dimensionless liquid height (see Figure 2.1) given by:

$$\tilde{h}_L = h_L / D \quad (14)$$

From geometry, the dimensionless variables are given by:

$$\tilde{A}_L = 0.25 \left[\pi - \cos^{-1} (2 \tilde{h}_L - 1) + (2 \tilde{h}_L - 1) \sqrt{1 - (2 \tilde{h}_L - 1)^2} \right] \quad (15)$$

$$\tilde{A}_G = 0.25 \left[\cos^{-1} (2 \tilde{h}_L - 1) - (2 \tilde{h}_L - 1) \sqrt{1 - (2 \tilde{h}_L - 1)^2} \right] \quad (16)$$

$$\tilde{A} = 0.25 \pi \quad (17)$$

$$\tilde{S}_L = \pi - \cos^{-1} (2 \tilde{h}_L - 1) \quad (18)$$

$$\tilde{S}_G = \cos^{-1} (2 \tilde{h}_L - 1) \quad (19)$$

$$\tilde{S}_i = \sqrt{1 - (2 \tilde{h}_L - 1)^2} \quad (20)$$

$$\tilde{V}_L = \tilde{A} / \tilde{A}_L \quad (21)$$

$$\tilde{V}_G = \tilde{A} / \tilde{A}_G \quad (22)$$

$$\tilde{D}_L = 4 \tilde{A}_L / \tilde{S}_L \quad (23)$$

$$\tilde{D}_G = 4 \tilde{A}_G / (\tilde{S}_G + \tilde{S}_i) \quad (24)$$

Taitel and Dukler follow this outline of their model with the statement, “Thus, each X-Y pair corresponds to a unique value of h_L/D for all conditions of pipe size, fluid properties, flow rate, and pipe inclinations for which stratified flow exists.”¹ It is now known that this statement is not true for all conditions. There is a region for which multiple solutions for h_L/D exist.

The Taitel and Dukler paper goes on to discuss mechanisms by which transitions away from stratified flow can occur. As these are not directly relevant to this work, they will not be considered here.

¹ Taitel and Dukler (1976), p. 49.

2.2 Multiple Solution Region of Stratified Flow Models

Baker et. al. (Baker and Gravestock (1987), Baker et. al. (1988)) introduced new pressure loss and liquid holdup correlations developed specifically for gas/condensate systems. The liquid holdup correlation introduced closely parallels the development by Taitel and Dukler (1976). As such, some “Notes on the Taitel-Dukler Momentum Balance” were included in the papers.

Since their liquid holdup model demonstrated some erratic behaviour, a closer examination of the Taitel-Dukler momentum balance equations was undertaken. It was found that the equations yielded multiple solutions for the dimensionless liquid level for values of the Taitel-Dukler inclination parameter (Y), defined by Equation (13), less than -3.8. Further, it was found that, of the five multiphase pipelines for which they presented data, four operated, at least partially, within the multiple solution region. To avoid the problem of multiple solutions, the model was adjusted to limit the minimum value of Y to -3.8. The authors offer no justification for this approach beyond the fact that there was no obvious physical explanation for the multiple solutions.

Landman (1991a, 1991b) dealt specifically with the multiple solution region of the Taitel-Dukler model. Landman begins with a review of the Taitel-Dukler model and shows how that model can be applied to flow in a rectangular duct by using modified definitions for the dimensionless parameters. It is noted that where the liquid holdup (E_L) as a function of liquid height for the circular geometry is given by:

$$E_L = \frac{A_L}{A} = 1 - \frac{1}{\pi} \left[\cos^{-1}(2\tilde{h}_L - 1) - (2\tilde{h}_L - 1) \sqrt{1 - (2\tilde{h}_L - 1)^2} \right] \quad (25)$$

the relationship for a rectangular duct is given by the trivial relationship $E_L = \tilde{h}_L$.

To verify the method of Taitel and Dukler (1976), Landman (1991a, 1991b) considers holdup curves (E_L versus X) generated for laminar flow in a square duct. Two curves were generated. The first was developed from a modified set of Taitel-Dukler equations for flow in rectangular ducts. The second was developed from an exact solution for flow in a rectangular duct. Displaying both curves on a single plot allowed the results of the exact solution to be compared with those from the Taitel-Dukler (1976) model. The two show excellent qualitative agreement in that both exhibit the multiple solution region. The quantitative differences “suggest” that the stresses used in the Taitel-Dukler (1976) model could be improved.

Landman (1991a, 1991b) proposes the following physical interpretation of the multiple solutions. For low liquid holdup, the dominant factors in liquid flow are the interfacial and wall stresses, not the force of gravity. As the holdup increases, there is competition between the gravity force and the axial pressure gradient. The gravity force can become so significant (and does in the multiple solution region) that it impedes the liquid flux and the X^2 term begins to decrease. As the liquid holdup increases further, the gas wall stress and the interfacial stress increase due to the increasing gas velocity. This accelerates the liquid phase and at some point overwhelms the gravity force and the liquid wall stress. Thus, there are two folds in the curve of X^2 versus h_L/D . Depending on the interaction of liquid holdup, velocity, and gravity, there may be as many as three liquid

level solutions for a given value of X^2 .

Landman (1991a) also presents a means to define the boundaries of the multiple solution region. For a given value of Y (in the multiple solution region) the holdup curve (E_L versus X^2) may have a pair of folds as shown in Figure 2.2. As Y is varied, the coordinates of these folds can be tracked and plotted on a curve of X^2 versus Y . As Landman (1991a) notes, on the curve of X^2 versus Y , a cusp is observed, which is expected when two folds come together. The cusp occurs at the maximum value of Y (i.e., the critical value of Y) for which multiple solutions exist. Thus, the multiple solution region is bounded in X^2 , Y coordinates.

Figure 2.2 Taitel-Dukler Liquid Level Curves

The critical value of Y , presented by Landman (1991a), was -3.737 for turbulent gas / turbulent liquid flow. For turbulent gas / laminar liquid, the critical value of Y was given as -3.697. Note that this suggests that the critical value is sensitive to the way in which the shear stresses are defined.

A procedure by which the multiple solution region can be identified in superficial velocity coordinates is outlined:

- “
- a) The flow regime (laminar/turbulent for each phase) is guessed at.
 - b) X^2 and Y are calculated.
 - c) Test for multivaluedness; if single-valued, h_L/D is calculated from the Taitel-Dukler relation.
 - d) The local Reynolds numbers are calculated; if these lie in a different flow regime than (a), start again but in the new flow regime.
 - e) Flow pattern transition criteria (e.g. Taitel & Dukler 1976) can be tested once steps (a)-(d) are consistent.”²

Landman (1991a) suggest that step (c) can be carried out by two methods. First, by searching for solutions over 25 subintervals between zero and one. Second, by applying the parameterization outlined in Section 3.1.5.

Landman (1991a, 1991b) considers both modified Kelvin-Helmholtz and linear stability criteria. These stability calculations show that when three roots for the liquid

² Landman (1991a), p. 390.

level are predicted to occur, the root with the lowest holdup is the most likely to be stable, the middle root either stable or unstable, and the upper root unstable. Stability analyses are considered in more detail in Section 2.3.

When multiple solutions are found to be stable, transient simulations demonstrated that hysteresis can occur. In a specific example, variations in the gas rate cause the liquid level to shift from the middle root to the lower root. However, Landman (1991a, 1991b) stresses that it is important that work be done to show whether the predictions contained in his work can be verified in the field or in the laboratory.

Nitheanandan and Soliman (1994) presented a mechanistic model for determining the stratified flow boundary for condensing flows in horizontal and slightly inclined pipes. Two formulations, a complete and a simplified version, are considered for the equilibrium liquid level. The modelling approach followed the work of Taitel and Dukler (1976), with the added complexity that heat transfer along the condensation path (i.e., along the flow path) results in a continuously changing liquid level.

The complete formulation of their model yields a single solution for the liquid level while the simplified version can yield three solutions, similar to the multiple solution region of the Taitel-Dukler model. Nitheanandan and Soliman present no stability criterion, only the fact that the single solution of the complete formulation of their model is always closest to the lowest solution (i.e., the minimum liquid holdup) in the multiple solution region. Based on this and the earlier works of Landman (1991a) and Barnea and Taitel (1992), they conclude that the lowest solution should be accepted while the two higher solutions “may be considered mathematically feasible but physically

impossible.”³

2.3 Stability Analyses for the Multiple Solution Region

In their paper “Structural and Interfacial Stability of Multiple Solutions for Stratified Flow,” Barnea and Taitel (1992) present an extremely thorough treatment of stability analyses as applied to the multiple solutions for liquid level in stratified two-phase flow. A second work, “Interfacial and Structural Stability of Separated Flow,” (Barnea and Taitel (1994)) published in a special supplement to the International Journal of Multiphase Flow, combined the work contained in the 1992 paper and others into a thorough treatment of stability for both stratified and annular flow. Considering the overlap, these two works will be considered together.

Barnea and Taitel (1992, 1994) recommend that a distinction be made between the stability of the interface and the physical stability of the solution itself. They propose that one should first perform a “structural stability analysis” in which it is determined which solution is physically stable; and that only the physically stable solutions be subjected to the Kelvin-Helmholtz (K-H) stability analysis. Alternatively, the K-H stability analysis could be performed, and the stratified flow solutions subjected to the structural stability check.

Although details of the stability criteria will be presented later, a brief summary of

³ Nitheanandan and Soliman (1994), p. 31.

the two stability mechanisms will be presented here. K-H instability, first described by Scottish physicist, Lord William Thomson Kelvin (1824-1907), and German physicist, Hermann Ludwig Ferdinand von Helmholtz (1821-1894), is usually considered to be the source of transitions away from stratified flow. In pipeline flow, K-H instability results from the Bernoulli effect, in which the acceleration of the gas phase over the crest of the wave causes a local pressure decrease. This pressure decrease overcomes the stabilizing influence of gravity and results in uncontrolled wave growth.

Barnea and Taitel (1992, 1994) present two forms of Kelvin-Helmholtz stability criteria, one considers viscous effects while the other neglects them. The meaning of the two K-H stability criteria is best described as follows. Viscid K-H instability indicates a transition from stable stratified flow to a flow regime characterized by the presence of “large-amplitude roll waves.” For low liquid levels, these will persist as roll waves. while for high liquid levels, the roll waves will bridge the pipe resulting in slug flow. Inviscid K-H instability indicates an absolute transition away from stratified flow when the Bernoulli effect overcomes gravity and the roll waves are drawn to the top of the pipe. For low liquid levels, this will lead to a transition to annular flow, while for high liquid levels, this will result in a transition to slug flow.

Structural stability was first considered by Barnea and Taitel (1989, 1990) in relation to the stability of the liquid film in annular flow, which is always unstable by K-H criteria. It considers whether the structure of the flow can be maintained; that is, whether the steady-state solution should be expected to occur physically. In the simplest terms, it is an indication of whether liquid backflow, which would lead to a transition to intermittent flow, will occur. Barnea and Taitel (1992, 1994) show how this approach

can be applied to stratified flow and that it can sometimes eliminate solutions which would be considered stable if only K-H stability were considered.

The K-H stability criteria deal with the stability of the interface. The structural stability approach, previously applied to annular flow by Barnea (1991), can be applied to evaluate the physical stability of the flow. This approach assumes an ideal interface (i.e., a uniform liquid level along the length of the pipe), and then considers the physical stability of the flow with respect to this ideal state.

As noted earlier, the essence of the linear structural stability analysis is that instability will occur when some portion of the liquid flows downhill (i.e., liquid backflow occurs) and accumulates at the upstream end of the pipe until the transition to intermittent flow occurs. It is shown that this is the case for the middle root, while the lower (i.e., the lowest liquid level) and upper roots are stable.

A non-linear structural stability analysis may show that the upper root is unstable to large disturbances. Normally, this kind of non-linear stability is evaluated by performing transient numerical simulations to determine whether conditions exist in which a disturbance would cause the liquid velocity to become negative. A negative liquid velocity has been shown (Barnea and Taitel (1990)) to lead to a transition to slug flow.

It is unreasonable to expect that numerous transient simulations be performed to evaluate the stability of one possible liquid level solution of a steady-state model. However, Barnea and Taitel (1992) offer a more practical solution to the problem of non-

linear stability. They suggest that one use the transient momentum balance to evaluate the liquid velocity over a range of liquid holdup values between the middle and upper roots. If anywhere between the middle and upper roots the liquid velocity is negative, the upper root is unstable due to non-linear structural stability considerations. It is shown that non-linear structural stability occurs over only a very narrow range of conditions. Of the three solutions, the lowest is always structurally stable, the middle is always unstable by linear structural stability, and the upper is usually unstable by non-linear structural stability. It is also noted that the upper solution is usually unstable to K-H stability.

Barnea and Taitel (1992, 1994) recommend that both structural stability and one of the more traditional K-H stability criteria be considered when evaluating the overall stability of stratified flow solutions.

Three conclusions are outlined:

- a) “Multiple solutions for upwards inclined stratified flow occur at low liquid flow rates and a narrow range of gas flow rates.”⁴
- b) “When multiple solutions occur we have three solutions for the liquid level. The thinnest [smallest h_L] solution is the one that is always stable. The middle solution is always linearly structurally unstable and the thickest solution is almost always unstable to both structural and K-H instability, indicating that it would not exist.”⁴

⁴ Barnea and Taitel (1992), p. 829.

- c) “The possibility of hysteresis, namely the possible existence of two liquid levels for the same operating conditions, is very unlikely. For all the examples tested we observed only an extremely narrow region for which this can happen.”⁴

⁴ Barnea and Taitel (1992), p. 829.

3.0 Application of Theory

Much of the theory from the works outlined in the preceding literature review was incorporated into a computer program. At the heart of this computer program is a routine that solves the Taitel-Dukler equations for the liquid level and, where there are multiple solutions, evaluates the stability of each solution. The program allows for detailed comparison of the measured data and the theoretical predictions. While the details of the proprietary computer code must remain confidential, Figure 3.1 shows an outline of the approach taken.

Two main programs were written to reference this routine.

The first program calls the routine repeatedly according to a predefined matrix of superficial gas velocity versus superficial liquid velocity. This allows the multiple solution region to be defined for a specified set of pipe data and fluid properties. This routine was used during the design of the experimental work to ensure that conditions to be examined in the laboratory included points within the multiple solution region.

The second program calls the routine repeatedly according to entries in an input data file. This program was used to evaluate the exact conditions studied in the laboratory. The resulting prediction(s) for the liquid holdup could be compared with the measured liquid holdup. Where multiple solutions exist, it can be determined which solution is closest to the measured liquid holdup.

Figure 3.1 **Flow Chart for Evaluation of Multiple Solutions**

3.1 Application of Theory - Equations

3.1.1 Phase Friction Factors

For turbulent flow, the friction factors for the gas and liquid phases were evaluated using the explicit equation of Chen (1979) as recommended by Gregory and Fogarasi (1985). The Chen equation is given by:

$$\frac{1}{\sqrt{f}} = -4 \log F_1 \quad (26)$$

$$F_1 = 0.2698 \left(\frac{k}{D} \right) - \frac{5.0452}{Re} \log F_2 \quad (27)$$

$$F_2 = 0.3539 \left(\frac{k}{D} \right)^{1.1098} + \frac{5.8506}{Re^{0.8981}} \quad (28)$$

where (k/D) is the relative roughness of the pipe and Re is the Reynolds number.

For laminar flow, the friction factors were determined using the equation:

$$f = \frac{16}{Re} \quad (29)$$

Note that in all of the above equations, the Reynolds number was calculated using the superficial velocity of the phase. This was done because the resulting friction factors were used to determine the superficial phase pressure gradients.

3.1.2 Phase Pressure Gradients

The superficial pressure gradient (i.e., the pressure gradient of one phase flowing alone in the pipe) of each phase was determined from the equation:

$$\left(\frac{dP}{dx} \right)_s = \frac{2 f \rho V_s^2}{g_c D} \quad (30)$$

3.1.3 Taitel-Dukler Inclination Parameter

As previously noted, the inclination parameter introduced by Taitel and Dukler is given by:

$$Y = \frac{(\rho_L - \rho_G) g \sin \alpha}{\frac{4 C_G}{D} \left(\frac{V_{SG} D}{v_G} \right)^{-m} \frac{\rho_G V_{SG}^2}{2}} = \frac{(\rho_L - \rho_G) g \sin \alpha}{|(dP/dx)_{SG}|} \quad (13)$$

3.1.4 Lockhart-Martinelli Parameter

As previously noted, the Lockhart-Martinelli parameter is given by:

$$X^2 = \frac{\frac{4C_L}{D} \left(\frac{V_{SL} D}{v_L} \right)^{-n} \frac{\rho_L V_{SL}^2}{2}}{\frac{4C_G}{D} \left(\frac{V_{SG} D}{v_G} \right)^{-m} \frac{\rho_G V_{SG}^2}{2}} = \frac{|(dP/dx)_{SL}|}{|(dP/dx)_{SG}|} \quad (12)$$

3.1.5 Landman Parameterization

Landman presented a series of equations that define the multiple solution region in X^2 , Y coordinates.

The boundaries of the multiple solution region are given by:

a) Right-hand boundary:

$$Y = k_0 + k_1 X^2 + k_2 X^4, \quad 0 < X^2 \leq X_C^2 \quad (31)$$

ii) Left-hand boundary:

$$X^2 = X_C^2 + b_0 \tanh b_1 (Y_C - Y) + b_2 (Y_C - Y) \exp[-b_3 (Y_C - Y)], \quad |Y_C| < |Y| \leq |Y_1| \quad (32)$$

iii) Left-hand boundary (small X):

$$\log|Y| = a_0 + a_1 \log X, \quad |Y_l| < |Y| \quad (33)$$

The values of the constants fitted to the equations for turbulent gas / turbulent liquid flow and turbulent gas / laminar liquid flow are tabulated below.

Table 3.1 Constants for Landman Parameterization

Constant	Turb. Gas / Turb. Liquid	Turb. Gas / Lam. Liquid
X_c	0.04678	0.08376
Y_c	-3.737	-3.697
Y_l	-17.80	-20.00
k_0	-3.896	-3.896
k_1	48.98	20.28
k_2	10550	1092
b_0	-0.002184	-0.006989
b_1	1.908	1.800
b_2	0.001264	0.004293
b_3	1.147	0.9744
a_0	-0.5851	-0.5259
a_1	-0.6895	-0.8000

3.1.6 Taitel-Dukler Equation for X

Rearranging Equation 11 to solve for X gives:

$$X = \sqrt{\frac{4Y + \left[(\tilde{V}_G \tilde{D}_G)^{-m} \tilde{V}_G^2 \left(\frac{\tilde{S}_G}{\tilde{A}_G} + \frac{\tilde{S}_i}{\tilde{A}_L} + \frac{\tilde{S}_i}{\tilde{A}_G} \right) \right]}{\left[(\tilde{V}_L \tilde{D}_L)^{-n} \tilde{V}_L^2 \frac{\tilde{S}_L}{\tilde{A}_L} \right]}} \quad (34)$$

Values of the dimensionless liquid height (h_L/D) for which this value of X equals the value determined from the equation presented by Lockhart and Martinelli (Equation 32) are solutions to the Taitel-Dukler equation.

3.1.7 Kelvin-Helmholtz Stability Criteria

The analysis presented is based on the one-dimensional, two-fluid model. The continuity and momentum equations for the gas and liquid phase are combined by equating the pressure drop in the two phases. Assuming that the flow is both incompressible and isothermal, and that surface tension can be neglected, this yields:

$$\frac{\partial E_L}{\partial t} + E_L \frac{\partial V_L}{\partial x} + V_L \frac{\partial E_L}{\partial x} = 0 \quad (35)$$

$$\frac{\partial E_G}{\partial t} + E_G \frac{\partial V_G}{\partial x} + V_G \frac{\partial E_G}{\partial x} = 0 \quad (36)$$

$$\rho_L \frac{\partial V_L}{\partial t} - \rho_G \frac{\partial V_G}{\partial t} + \rho_L V_L \frac{\partial V_L}{\partial x} - \rho_G V_G \frac{\partial V_G}{\partial x} + (\rho_L - \rho_G) g \cos \alpha \frac{A}{A'_L} \frac{\partial E_L}{\partial x} = F \quad (37)$$

where

$$F = -\frac{\tau_L S_L}{A E_L} + \frac{\tau_G S_G}{A E_G} + \frac{\tau_i S_i}{A} \left(\frac{1}{E_L} + \frac{1}{E_G} \right) - (\rho_L - \rho_G) g \sin \alpha \quad (38)$$

and

$$A'_L = \frac{dA_L}{dh_L} \quad (39)$$

The shear stresses are defined just as by Taitel and Dukler (1976) except that the absolute value in the definition of the interfacial shear is included as:

$$\tau_i = \frac{f_i \rho_G (V_G - V_L) |V_G - V_L|}{2} \quad (40)$$

The interfacial friction factor was assumed to have a constant value of 0.014 for gas wall friction factors less than 0.014. Otherwise, it was set equal to the gas wall friction factor.

The criterion for K-H stability, including viscous effects (viscid K-H), can be determined from a linear stability analysis on the two-fluid model as:

$$\left(C_v - C_{iv}\right)^2 + \frac{\rho_L \rho_G}{\rho^2 E_L E_G} (V_G - V_L)^2 - \frac{\rho_L - \rho_G}{\rho} g \cos \alpha \frac{A}{A_L'} < 0 \quad (41)$$

where

$$\rho = \frac{\rho_L}{E_L} + \frac{\rho_G}{E_G} \quad (42)$$

The criterion for K-H stability, neglecting viscous effects (inviscid K-H), is identical to that given above with the exception that the first term, which includes the influence of shear stresses, is dropped.

C_v , the phase velocity of a continuity wave (Wallis (1969)), is defined by Barnea and Taitel as the critical wave velocity (for viscous flow) at the onset of instability:

$$C_v = \frac{\left(\frac{\partial F}{\partial E_L}\right)_{V_{SL}, V_{SG}}}{\left[\left(\frac{\partial F}{\partial V_{SG}}\right)_{V_{SL}, E_L} - \left(\frac{\partial F}{\partial V_{SL}}\right)_{V_{SG}, E_L}\right]} \quad (43)$$

C_{IV} , defined by Wallis (1969) as a weighted mean reference velocity, is defined by Barnea and Taitel as the critical velocity for inviscid K-H analysis:

$$C_{IV} = \frac{\rho_L V_L E_G + \rho_G V_G E_L}{\rho_L E_G + \rho_G E_L} \quad (44)$$

Crowley et al. (1992) presented a paper considering the transition from stratified flow to slug flow. The model that they present is based on one-dimensional two-phase wave theory, and includes the calculation of the velocity of continuity waves, which as noted above is identical to the critical wave velocity defined by Barnea and Taitel (1992). Detailed equations for the derivatives included in the definition of the continuity wave velocity are provided. These equations (which have been confirmed by the author of the present work) are given by:

$$\frac{\partial F}{\partial V_{SL}} = \left[\frac{f_L \rho_L \left(\frac{\pi}{4}\right)^2 \tilde{S}_L \tilde{A}_L^{-3} V_{SL}}{D} \right] \pm \left[\frac{f_i \rho_G \left(\frac{\pi}{4}\right)^3 \tilde{S}_i \tilde{A}_L^{-1} \tilde{A}_G^{-1}}{D} \right] (-V_{SL} \tilde{A}_L^{-2} + V_{SG} \tilde{A}_L^{-1} \tilde{A}_G^{-1}) \quad (45)$$

$$\frac{\partial F}{\partial V_{SG}} = \left[\frac{f_G \rho_G \left(\frac{\pi}{4} \right)^2 \tilde{S}_G \tilde{A}_G^{-3} V_{SG}}{D} \right] + \left[\frac{f_i \rho_G \left(\frac{\pi}{4} \right)^3 \tilde{S}_i \tilde{A}_L^{-1} \tilde{A}_G^{-1}}{D} \right] (-V_{SL} \tilde{A}_L^{-1} \tilde{A}_G^{-1} + V_{SG} \tilde{A}_G^{-2}) \quad (46)$$

$$\begin{aligned} \frac{\partial F}{\partial \tilde{A}_L} &= \left[\frac{f_L \rho_L \left(\frac{\pi}{4} \right)^2 V_{SL}^2}{2D} \right] \left[\tilde{A}_L^{-3} \left(\frac{\partial \tilde{S}_L}{\partial \tilde{A}_L} \right) - 3 \tilde{S}_L \tilde{A}_L^{-4} \right] \\ &- \left[\frac{f_G \rho_G \left(\frac{\pi}{4} \right)^2 V_{SG}^2}{2D} \right] \left[\tilde{A}_G^{-3} \left(\frac{\partial \tilde{S}_G}{\partial \tilde{A}_L} \right) - 3 \tilde{S}_G \tilde{A}_G^{-4} \right] \\ &+ \left[\frac{f_i \rho_G \left(\frac{\pi}{4} \right)^3}{2D} \right] \left\{ V_{SG}^2 \left[\tilde{A}_L^{-1} \tilde{A}_G^{-3} \left(\frac{\partial \tilde{S}_i}{\partial \tilde{A}_L} \right) - \tilde{S}_i \tilde{A}_L^{-2} \tilde{A}_G^{-3} + 3 \tilde{S}_i \tilde{A}_L^{-1} \tilde{A}_G^{-4} \right] \right. \\ &\quad - 2 V_{SL} V_{SG} \left[\tilde{A}_L^{-2} \tilde{A}_G^{-2} \left(\frac{\partial \tilde{S}_i}{\partial \tilde{A}_L} \right) - 2 \tilde{S}_i \tilde{A}_L^{-3} \tilde{A}_G^{-2} + 2 \tilde{S}_i \tilde{A}_L^{-2} \tilde{A}_G^{-3} \right] \\ &\quad \left. + V_{SL}^2 \left[\tilde{A}_L^{-3} \tilde{A}_G^{-1} \left(\frac{\partial \tilde{S}_i}{\partial \tilde{A}_L} \right) - 3 \tilde{S}_i \tilde{A}_L^{-4} \tilde{A}_G^{-1} + \tilde{S}_i \tilde{A}_L^{-3} \tilde{A}_G^{-2} \right] \right\} \quad (47) \end{aligned}$$

The additional partial derivatives introduced in the above equation are given by:

$$\frac{\partial \tilde{S}_L}{\partial \tilde{A}_L} = \frac{2}{[1 - (2\tilde{h}_L - 1)^2]} \quad (48)$$

$$\frac{\partial \tilde{S}_G}{\partial \tilde{A}_L} = - \frac{\partial \tilde{S}_L}{\partial \tilde{A}_L} = - \frac{2}{[1 - (2\tilde{h}_L - 1)^2]} \quad (49)$$

$$\frac{\partial \tilde{S}_i}{\partial \tilde{A}_L} = - \frac{2(2\tilde{h}_L - 1)}{[1 - (2\tilde{h}_L - 1)^2]} \quad (50)$$

Each of Equations 47, 48, and 49 contains a double sign (i.e., $\pm$ or $\mp$). The upper sign is used if $V_G > V_L$, and the lower if $V_L > V_G$.

Considering that,

$$\frac{\partial \tilde{A}_L}{\partial E_L} = \frac{\pi}{4} \quad (51)$$

and,

$$\frac{\partial F}{\partial E_L} = \frac{\partial F}{\partial \tilde{A}_L} \frac{\partial \tilde{A}_L}{\partial E_L} \quad (52)$$

all that is needed to define the critical wave velocity used by Barnea and Taitel (1992) is available.

It is critical to note that the sign convention used by Crowley et al. (1992) in the definition of the function, F , is opposite to that used by Barnea and Taitel (1992). Thus, the sign of any value calculated using Equations 47, 48, and 49 must be changed before the value can be used in the stability criteria outlined by Barnea and Taitel (1992).

3.1.8 Linear Structural Stability Criteria

The linear structural stability analysis begins with transient momentum and continuity equations given by:

$$\frac{\partial V_L}{\partial t} = - \frac{V_{SL}}{E_L L} \left(V_L - \frac{V_{SL}}{E_L} \right) - \frac{F}{\rho_L} \quad (53)$$

$$\frac{\partial E_L}{\partial t} = \frac{1}{L} (V_{SL} - V_L E_L) \quad (54)$$

A linear stability analysis on these two equations gives the criterion for the linear structural stability of stratified flow:

$$\left(\frac{\partial F}{\partial E_L} \right)_{V_{SL} - V_{SG}} > 0 \quad (55)$$

Considering the definition of F , it can be seen that it will always increase with increasing gas superficial velocity for constant liquid superficial velocity and liquid volume fraction and decrease with increasing liquid superficial velocity for constant gas superficial velocity and liquid volume fraction. Thus, the above stability criterion is equivalent to saying that continuity waves will have a positive velocity (i.e., $C_v > 0$); that is that continuity waves will move from upstream to downstream. In other words, structural instability will occur when some portion of the liquid flows downhill (i.e., liquid backflow occurs) and accumulates at the upstream end of the pipe until the transition to intermittent flow occurs.

3.1.9 Non-Linear Structural Stability Criteria

Barnea and Taitel (1992) suggest that one use the right hand side of the transient momentum balance (Equation 53) to evaluate the liquid velocity over a range of liquid holdup values between the middle and upper roots. If anywhere between the middle and upper roots the liquid velocity is negative, the upper root is unstable due to non-linear structural stability considerations.

4.0 Experimental Method

The experimental apparatus was made up of two main components: the multiphase flow loop at the University of Calgary and the computerized data acquisition system. Oil and air were the fluids considered.

4.1 Materials

The liquid used in this facility is a low viscosity mineral oil (Shell Pella Oil A). The properties of the oil ($\rho = 823.2 \text{ kg/m}^3$, $\mu = 6.61 \text{ mPa}\cdot\text{s}$) were supplied by the manufacturer and confirmed in the laboratory by the technical staff at the University of Calgary.

Air was supplied to the system from the building compressors. The air stream was passed through a filter to remove any entrained liquids. From the filter, the air stream passed through a pressure regulator which restricted the pressure to 351.6 kPa.

4.2 Apparatus

4.2.1 University of Calgary Multiphase Flow Loop

Figure 4.1 shows a schematic diagram of the University of Calgary multiphase flow loop.

Figure 4.1 University of Calgary Multiphase Flow Loop

The pipeline test section was mounted on a twenty-four meter long triangular truss that could be inclined to a range of angles ($\pm 5^\circ$). The truss was made up of four six meter sections bolted together. A sleeve in the center of the truss allowed the system to rotate about a shaft anchored to the wall. For additional safety and stability the quarter points of the truss were supported by joining them with steel cables via overhead pulleys.

The pipeline test section was made up of 1.22 meter flanged sections of transparent acrylic pipe. The inside diameter of the pipe was 50.8 mm. The pipe was mounted on top of the truss on supports capable of small height adjustments to ensure the pipe was level. Individual pipe sections were bolted together and sealed with O-rings.

The oil was stored in two steel tanks with a combined volume of 2.27 m³. The

storage system was equipped with a shell and tube heat exchanger. The flow of water (used as the coolant) through the heat exchanger was controlled by a temperature sensor on the liquid line near the test section inlet manifold.

Liquid pumping was provided by a 2.24 kW (2.27 m³/h) positive displacement pump. Although a second, higher capacity, 14.91 kW (15.9 m³/h), centrifugal pump was available, it was not required due to the low liquid flow rates investigated in this study. A bypass line with a pressure regulator allowed excess oil, not required for the specific flow rate being investigated, to be returned to the storage tanks.

A secondary loop equipped with a positive displacement pump allowed the oil from the storage tanks to be pumped through a bed of calcium chloride pellets and returned to the storage tanks. This ensured that the oil contained no water, which is critical to ensure the accuracy of the volume fraction measurements. Accumulated water could be drained manually from the settling pot at the bottom.

The liquid flow could be controlled by either an orifice or one of four rotameters. Both control options were tied to the data acquisition system. These links will be described in more detail in the discussion of the data acquisition system.

As noted earlier, air was supplied to the system from the building compressors. Similar to the liquid flow, the gas flow could be controlled by either an orifice or one of two rotameters. As with the liquid flow, both control options were tied to the data acquisition system.

The rotameters for controlling both the gas and liquid phases were mounted on the metering panel. Beside the metering panel was the cabinet containing the electronic components for the measurement and recording of experimental data. It was via the equipment in this cabinet that the computerized data acquisition system was tied to the controls and instrumentation of the multiphase flow loop.

The gas and liquid streams were originally combined upstream of the flexible hose used to introduce the fluids into the test section. At the start of this project it was decided to combine the gas and liquid streams at the upstream end of the test section (downstream of the flexible hose) to minimize the entrance slugging introduced by the previous geometry. Although no system is free of entrance effects, this modification to the inlet geometry did help to minimize entrance slugging.

Three capacitance type volume fraction sensors were built at the University of Calgary and mounted along the length of the test section. These were mounted in such a way that they did not result in any disturbance of the flow. A complete description of the sensors can be found in Gregory and Mattar (1973). Due to a shortage of pressure transducers at the start of this project, pressure measurement was not available in the inclinable test section. Although not ideal, this was acceptable as the focus of this work is liquid holdup prediction. Also note that due to a failure in the truss, the final portion of the test section, including the third volume sensor, is inclined at a different angle than the remainder of the pipeline. This difference is significant enough that volume fraction measurements from the third volume sensor were disregarded.

The discharge of the test section was attached to a separator via a combination of

flexible and fixed pipe. The separator was a 0.4 m diameter pressure vessel, 1.83 m tall, into which the gas-liquid mixture was introduced tangentially. Oil was removed from the bottom of the separator and returned to the oil storage tanks. The pressure in the separator, under flowing conditions, was sufficient to return the oil to the storage tanks via the three inch return line. Air was removed from the top of the vessel and vented via the exhaust gas duct.

4.2.2 Computerized Data Acquisition System

The data acquisition system associated with the University of Calgary multiphase flow laboratory was developed by Dr. F. A. Sobbi (formerly of the University of Calgary) using the LabVIEW™ software (National Instruments (1993)). The “Two Phase Flow Program” is installed on a personal computer that is stored separately from the flow loop. The computer can be linked to the flow loop using three 50 pin ribbon connectors.

The system guides the user through the startup procedure for the multiphase flow loop, incorporates all controls and data output into a single screen displayed while experiments are conducted, and, when a data acquisition session is terminated, guides the user through the facility shut down procedure. Further details of the computerized data acquisition system are presented in Appendix A.

4.3 Experimental Procedure

A general outline of the experimental procedure is as follows:

1. The computer housing the data acquisition program was linked to the University of Calgary multiphase flow loop and the data acquisition program run.
2. The desired inclination angle for the test section was set and confirmed with a multi function angle level.
3. The valves were set to direct flow to and from the inclined test section.
4. The start up procedure (per the data acquisition system) was completed.
5. The desired gas and liquid flow rates were set.
6. The system was allowed to stabilize.
7. Data were gathered over some time period (typically 1 - 2 minutes). The flow pattern was observed and recorded in the data acquisition system.
8. Data collection was terminated.
9. The run number was incremented and Steps 5 - 9 repeated until the desired experiments were complete.
10. The shut down procedure (per the data acquisition system) was completed.

4.4 Scope of Experiments

A total of 209 experiments were performed. The range of variables investigated was:

Superficial gas velocity: 8.43 to 24.75 m/s

Superficial liquid velocity: 0.010 to 0.091 cm/s

Test section inclination: +2 degrees from the horizontal.

A detailed outline of the experimental results are included in Appendix B.

5.0 Results and Discussion

The following sections present comparisons of the measured data (see Appendix B) and the corresponding calculated values (see Appendix C), both within and outside the multiple solution region. The results of the Landman (1991a) parameterization and the rigorous search for solutions to the Taitel-Dukler equation are compared. Within the multiple solution region, the results of the various stability criteria are reviewed. Relevant discussions are presented throughout. Where statistical analyses are presented, the error is defined as:

$$\text{Error} = 100 \frac{E_{L(\text{Calc})} - E_{L(\text{Meas})}}{E_{L(\text{Meas})}} \quad (56)$$

such that, when the calculated value exceeds the measured value, the error is positive.

The absolute error is defined as:

$$\text{Absolute Error} = 100 \frac{|E_{L(\text{Calc})} - E_{L(\text{Meas})}|}{E_{L(\text{Meas})}} \quad (57)$$

5.1 Multiple Solution Region

The Blasius-type friction factors incorporated into the Taitel-Dukler equation mean that there are actually two forms of the equation depending on whether the liquid phase is characterized as being in the laminar or turbulent flow regime. The following sections include a review of criteria defining the boundary between laminar and turbulent

flow, and definitions of the multiple solution region for the experimental work using both characterizations.

5.1.1 Discussion of Laminar/Turbulent Transition Criteria

As noted earlier, the Blasius-type friction factor equations used by Taitel and Dukler had previously been presented by Lockhart and Martinelli (1949). The coefficients to be used in these equations vary depending on whether the flow is laminar or turbulent. As such, Lockhart and Martinelli presented a discussion of the transition criteria that should be applied. Govier and Aziz (1972) also presented a discussion of the laminar/turbulent transition in multiphase flow. As well, Govier and Omer (1962) presented some very interesting observations related to friction factors in multiphase flow.

Lockhart and Martinelli argue that the presence of a second phase in the pipe will increase the effective Reynolds number relative to that of a single phase flowing alone. Consider, for example, single phase gas flowing in a pipe. If the gas superficial velocity is held constant as liquid is introduced into the pipe, the actual gas velocity will increase and the resulting gas phase Reynolds number will increase.

Lockhart and Martinelli further argue that, as a result, the transition Reynolds number (generally accepted as ~ 2000) would be reduced by the presence of a second phase in the pipe. They state that the transition should be defined by the maximum Reynolds number that will allow laminar liquid flow to be maintained in the presence of turbulent gas flow. They propose that a transition Reynolds number, based on the

superficial velocity of the phase, of 1000 should be reasonable.

Govier and Aziz state that deviations from true laminar flow begin to occur at Reynolds numbers of approximately 1225. These take the form of a wavy core which increases in size and magnitude as the Reynolds number increases. Fully turbulent flow evolves over the range of Reynolds numbers between 1225 and 3000.

Govier and Omer (1962) studied the flow of gas-liquid mixtures in horizontal pipes. Their data are presented in terms of the superficial liquid friction factor derived from the measured pressure losses. One component of their analysis is a plot of the superficial liquid friction factor against the superficial liquid Reynolds number; in effect, a Moody diagram in which multiple curves are presented for a range of gas-liquid volume ratios (0 - 200). Among their observations is that, as the gas-liquid ratio increases, the discontinuity due to the transition from laminar to turbulent flow decreases in magnitude (in terms of the friction factor), covers a wider range of Reynolds numbers, and moves steadily to lower values of the Reynolds number. The transition Reynolds number moves from a single phase value of approximately 2000 to a value of approximately 500 for an input gas-liquid volume ratio of 200.

This work considers upward inclined two-phase flow. One would expect that the influence of gravity in this scenario should impact the transition from laminar to turbulent flow differently than in horizontal flow. Consider the case of flow of gas and liquid at fixed superficial velocities in a pipe of fixed diameter. If such a pipe were moved from horizontal to upward inclined flow, the only change to the phase Reynolds numbers would be due to the velocity. Generally, the liquid holdup in upward inclined flow is

greater than that in horizontal flow. If the liquid holdup increases, for fixed superficial velocities, the true liquid velocity will decrease and the true gas velocity will increase. Thus the liquid phase Reynolds number should decrease; that is, the flow of liquid should move toward laminar (or more laminar) flow.

At higher gas velocities, the dimensionless liquid level can be very small. Thus, the flow regime (laminar/turbulent) may be related to the flow of gas across thin films. In such cases, the traditional criteria for the laminar/turbulent transition in pipeline flow may not apply. This possibility was not pursued as such cases should be well beyond the multiple solution region.

It appears that there is no definitive answer to the question of what value of the Reynolds number would constitute the criterion for the transition from laminar to turbulent flow in upward inclined two-phase flow. As will be shown, this question is of considerable significance to the present work.

5.1.2 Definition of the Multiple Solution Region for Turbulent Gas / Turbulent Liquid Flow

Figure 5.1 shows the boundaries of the multiple solution region for the University of Calgary multiphase flow loop when inclined upward two degrees from the horizontal. These boundaries are based on the assumption of turbulent gas and turbulent liquid flow, and were defined using the computer code outlined in Section 3.0. Where the rigorous search and the Landman parameterization did not agree, the results of the search were used to define the boundaries of the multiple solution region.

The gas phase superficial Reynolds number within this region ranges from 63,000 to 112,000. The liquid phase superficial Reynolds number within this region ranges from 0.05 to 10.75; laminar flow by the generally accepted criterion. The corresponding actual liquid Reynolds numbers, based on the solution corresponding to the lowest liquid volume fraction, range from 8 to 145.

Figure 5.1 Multiple Solution Region for Turbulent Gas / Turbulent Liquid Flow

5.1.3 Definition of the Multiple Solution Region for Turbulent Gas / Laminar Liquid Flow

Figure 5.2 shows the boundaries of the turbulent gas / laminar liquid multiple solution region for the University of Calgary multiphase flow loop when inclined upward two degrees from the horizontal. These boundaries were found using the same method as the turbulent/turbulent boundaries except that the laminar liquid coefficients from the Blasius friction factor equation were used with the Taitel-Dukler equation.

Figure 5.2 Multiple Solution Region for Turbulent Gas / Laminar Liquid Flow

The gas phase superficial Reynolds number within this region ranges from 63,000 to 113,000. The liquid phase superficial Reynolds number ranges from 0.38 to 16.45; laminar flow by the generally accepted criterion. The corresponding actual liquid Reynolds numbers, based on the solution corresponding to the lowest liquid volume fraction, range from 50 to 550.

5.1.4 Further Discussion of Laminar/Turbulent Transition Criterion

The data presented in the preceding sections, define the multiple solution region for the University of Calgary multiphase flow loop, inclined upward two degrees from the horizontal, for both the turbulent gas / turbulent liquid and turbulent gas / laminar liquid assumptions. It is clear that, by any of the transition criteria presented in section 5.1.1, calculations in the multiple solution region (for this application) should be defined based on the turbulent gas / laminar liquid characterization. Further, it is apparent that the choice of the laminar or turbulent flow characterization for the liquid phase has a significant impact on the boundaries of the multiple solution region.

Although conventional criteria support the use of the laminar liquid flow assumption, analysis of the data will be carried out using both the laminar and turbulent characterizations for the liquid phase. This will be done for the following reasons:

- There is no definitive criterion for the transition between laminar and turbulent flow in gas-liquid flow.
- The work of Govier and Omer (1962) suggests that the transition Reynolds

number (superficial) decreases with increasing gas-liquid volume ratio. The minimum input gas-liquid ratio investigated in this study was 10,840, approximately 54 times the maximum gas-liquid ratio investigated by Govier and Omer. Thus, if the trend shown in their data continues, the transition Reynolds number for the experimental data in this work may be significantly less than the minimum of approximately 500 observed in their work.

- The influence of gravity on inclined flow presents an added complexity beyond those encountered in horizontal gas-liquid flow.
- The difference between the laminar and turbulent characterization has a significant impact on the definition of the multiple solution region.
- There are, almost certainly, academic and commercial implementations of the Taitel-Dukler model that assume that the turbulent characterization applies at all stratified flow conditions.

5.2 Data From Existing Sources

Data taken from several existing sources were reviewed as part of this study.

Seventy-eight data points from a Tulsa University Fluid Flow Projects study of low liquid loading in near-horizontal two-phase flow (Meng (1999) and Meng et. al.

(1999)) were evaluated. These data were gathered in a 50.2 mm inside diameter acrylic pipe at inclination angles of +1 and +2 degrees from the horizontal. Superficial gas velocities ranged from 6.5 - 27.0 m/s and superficial liquid velocities ranged from 0.1 - 5.4 cm/s.

These data were analyzed using both the laminar and turbulent characterizations for the liquid phase. In spite of being gathered for low liquid flow rates in uphill flow, the general conditions for which multiple solutions might be expected, none of the 78 measurements were found to fall within the multiple solution region when using the turbulent liquid characterization. When the laminar liquid characterization was used, two points were found to fall in the multiple solution region. In both cases, the measured liquid volume fraction was less than that corresponding to the lowest solution of the Taitel-Dukler model. It is interesting note that, for these two points, the single solution based on the turbulent liquid flow characterization corresponds, approximately, to the largest of the three solutions based on the laminar liquid characterization. The superficial liquid Reynolds numbers of these two points are 16.6 and 9.9.

An additional 80 data points taken from the Stanford Multiphase Flow Database (Petalas and Aziz (1994)), which includes the data from the University of Calgary Databank, were evaluated. 52 data points were observed to be in wave flow, the remaining 28 in smooth stratified flow. None of the 52 measurements in wave flow were found to fall in the multiple solution region regardless of the liquid phase characterization. Two of the measurements in smooth stratified flow were found to fall in the multiple solution region when the turbulent liquid characterization was used. This number increased to six when the laminar liquid characterization was used. In each case,

regardless of the liquid phase characterization, the measured liquid volume fraction was closest to that corresponding to the low solution from the Taitel-Dukler model.

The liquid phase superficial Reynolds numbers for the six points range between 42.2 and 232.0. In each of the six cases, the calculated holdup based on the laminar liquid characterization was significantly better than that based on the turbulent liquid characterization.

The lack of data in the multiple solution region demonstrates the requirement for a focused experimental program to gather data for use in evaluating the multiple solution region.

5.3 University of Calgary Data - Turbulent Liquid Characterization

The following sections will review the results of calculations for the data gathered at the University of Calgary multiphase flow loop. All calculations will assume that the turbulent liquid phase characterization applies. The accuracy of calculations both inside and outside the multiple solution region will be evaluated. As well, the results of the Landman (1991a) parameterization and the various stability criteria will be evaluated.

5.3.1 Calculations Within the Multiple Solution Region

Of 209 experimental data points, 37 fell within the predicted multiple solution region. Figures 5.3 - 5.5 show the comparisons of the measured liquid volume fraction

and the low, middle, and high calculated liquid volume fraction, respectively, for these 37 points.

A detailed review of the measured and calculated liquid volume fractions shows that of the 37 data points within the multiple solution region; 24 measured liquid volume fractions fall below the low solution, 12 between the low and middle solutions, and only one between the middle and high solutions. Of the twelve measurements that fall between the low and middle solutions, eleven are clearly closer to the low solution (seven of these are within fifteen percent), while one falls almost exactly between the two solutions. The single measurement that falls between the middle and high solutions falls very close to the middle solution.

Table 5.1 summarizes the results of a statistical analysis on the data within the multiple solution region.

Table 5.1 Accuracy of Liquid Volume Fraction Predictions Within the Multiple Solution Region - Turbulent Liquid

	Low Solution	Middle Solution	High Solution
Mean Error (%)	33.1	342.1	1381.2
Standard Deviation (%)	70.3	179.1	1191.1
Mean Absolute Error (%)	53.5	343.0	1381.2

Figure 5.3 Comparison of the Measured and Calculated Liquid Volume Fraction - Turbulent - Low Solution

Figure 5.4 Comparison of the Measured and Calculated Liquid Volume Fraction - Turbulent - Middle Solution

Figure 5.5 Comparison of the Measured and Calculated Liquid Volume Fraction - Turbulent - High Solution

The statistical analysis clearly suggests that, for these data, the low solution can be considered to be the most reliable. In fact, the positive value of the mean error indicates that even the low solution for the liquid volume fraction is generally greater than the measured value. This lends support to the theoretical works presented to date; all of which suggest that the low solution should be considered the most stable and thereby the acceptable result. However, a detailed review of the data shows that the possibility of the middle solution giving a viable result cannot be ignored.

5.3.2 Calculations Outside the Multiple Solution Region

As previously noted, 37 of 209 experimental data points fell within the multiple solution region. Figure 5.6 shows the comparison of the measured and calculated liquid volume fraction for the remaining 172 data points (i.e., those outside of the multiple solution region). It is clear from the plot that the results of the calculations are separated into two distinct groups. The two groups fall on either side of the multiple solution region.

Referring to Equation 12 it can be seen that X increases with decreasing superficial gas velocity. As such, the two groups can be considered the “Low V_{SG} Points” (i.e., X greater than the boundary of the multiple solution region) and “High V_{SG} Points” (i.e., X less than the boundary of the multiple solution region).

Figure 5.6 Comparison of the Measured and Calculated Liquid Volume Fraction Outside the Multiple Solution Region - Turbulent Liquid Characterization

The form of the Taitel and Dukler curve (h_L/D versus X , see Figure 2.2), is such that for values of X greater than the upper boundary of the multiple solution region, the calculated liquid level will always be greater than the highest liquid level in the multiple solution region. Similarly, for values of X less than the lower boundary of the multiple solution region, the calculated liquid level will always be less than the minimum liquid level in the multiple solution region. For the “Low V_{SG} Points” included in this data set, liquid volume fractions between 0.28 and 0.60 were calculated using the model of Taitel and Dukler.

Table 5.2 summarizes the results of a statistical analysis on the data outside the multiple solution region.

Table 5.2 Accuracy of Liquid Volume Fraction Predictions Outside the Multiple Solution Region - Turbulent Liquid

	All Data Points	“Low V_{SG} Points”	“High V_{SG} Points”
Mean Error (%)	1008.1	1851.4	15.4
Standard Deviation (%)	1160.3	966.3	64.7
Mean Absolute Error (%)	1025.6	1851.4	53.5

Based on these data, the multiple solution region appears to define a boundary beyond which the accuracy of the Taitel-Dukler model is questionable.

5.3.3 The Landman Parameterization

The parameterization presented by Landman (1991a) predicts that 37 of the 209 measured data points should fall within the multiple solution region. Two of these points are in error relative to those found by searching for multiple solutions. In one case, the Landman parameterization indicates that multiple solutions should be expected where only one solution was found. In another, it indicates that only one solution should be anticipated where three were found. In 35 of 37 cases, the Landman parameterization and the search for multiple solutions are in agreement, an accuracy of 94.6 percent. Thus, the Landman technique appears to be a reliable means of defining the multiple solution region where the turbulent liquid phase characterization is assumed to apply.

5.3.4 Stability Criteria

For each of the 37 experimental data points that fell within the multiple solution region, stability was assessed with both the inviscid (IKH) and viscid (VKH) Kelvin-Helmholtz criteria as well as the linear structural criteria. The non-linear structural stability of the upper solution in each case was also evaluated.

In each case, the results of the IKH and VKH stability criteria were the same. The low solution was always stable by both criteria. In 20 of the 37 cases, the middle solution was unstable by both criteria. In every case, the high solution was unstable by both criteria.

Barnea and Taitel (1992, 1994) stated that the linear structural stability criteria should always indicate that the high and low solutions were stable and the middle solution unstable. Of the 37 cases, only the middle solution was predicted to be unstable in 22 cases, both the low and middle solutions were predicted to be unstable in 12 cases, and both the middle and high solutions were predicted to be unstable in 3 cases. It was found that these unexpected results, two unstable solutions, were due to the difference between the interfacial friction factor recommended by Barnea and Taitel and that used by Taitel and Dukler. When the linear structural stability was analyzed using the original interfacial friction factor recommended by Taitel and Dukler, the results were consistent with those of Barnea and Taitel. The middle solution was found to be unstable in every case, while the high and low solutions were predicted to be stable.

The non-linear structural stability criteria indicated that the high solution was

unstable in all 37 cases.

Thus, if the results of the linear structural stability criteria based on the Taitel and Dukler interfacial friction factor are accepted, only the low solution should be stable. The high solution is always unstable due to non-linear structural instability and both VKH and IKH instability. The middle solution is unstable due to both VKH and IKH instability in twenty of thirty-seven cases, and is unstable due to linear structural instability in every case.

5.4 University of Calgary Data - Laminar Liquid Characterization

Like the sub-sections of Section 5.3, the following sections will review the results of calculations for the data gathered at the University of Calgary multiphase flow loop. In this case, all calculations will assume that the laminar liquid phase characterization applies, as conventional theory would suggest. As before, the accuracy of calculations both inside and outside the multiple solution region and the results of the Landman (1991a) parameterization and the various stability criteria will be evaluated.

5.4.1 Calculations Within the Multiple Solution Region

Of 209 experimental data points, 65 fell within the multiple solution region. Figures 5.7 - 5.9 show the comparisons of the measured liquid volume fraction and the low, middle, and high calculated liquid volume fraction, respectively, for these 65 points.

A detailed review of the measured and calculated liquid volume fractions shows that of the 65 data points within the multiple solution region 4 measured liquid volume fractions fall below the low solution, 55 between the low and middle solutions, and 6 between the middle and high solutions. Of the fifty-five measurements that fall between the low and middle solutions, twenty-eight are clearly closer to the low solution, while three are closest to the middle solution. The remainder fall between the two solutions with no clear bias for either the low or middle solution. The six measurements that fall between the middle and high solutions are much closer to the middle solution in every case.

Thus, of a total of sixty-five data points that fall within the multiple solution region, thirty-two are closest to the low solution, nine are closest to the middle solution, and the remaining twenty-four are inconclusive, falling between the low and middle solutions. Unlike the results based on the turbulent liquid characterization, these results suggest that hysteresis, as suggested by Landman (1991a, 1991b) could occur.

The sequence of experiments was designed to allow testing of the theory that hysteresis could occur. The data gathered early in the experimental program (September - October, 1997) were generally gathered in blocks of ten data points. Within each block of experiments, the superficial liquid velocity was fixed, and the superficial gas velocity gradually decreased. The data gathered later in the experimental program (June - July, 1998) were gathered in blocks of twelve data points. Within each block, the superficial liquid velocity was fixed (corresponding to a block from the previous work), and the superficial gas velocity first decreased and then increased in steps. Thus, over the total experimental program, data for similar gas and liquid superficial velocities were

gathered both as the superficial gas velocity was increased, and as it was decreased.

Figure 5.7 Comparison of the Measured and Calculated Liquid Volume Fraction - Laminar - Low Solution

Figure 5.8 Comparison of the Measured and Calculated Liquid Volume Fraction - Laminar - Middle Solution

Figure 5.9 Comparison of the Measured and Calculated Liquid Volume Fraction - Laminar - High Solution

A review of the location in the sequence of runs of the nine cases for which the middle solution was most accurate showed that they occurred both when the superficial gas velocity was increased and when it was decreased. However, the cases were distinguished by the time frame during which the experiment was done. Eight of the nine “middle solution” cases were found to have been done in October, 1997. Similarly, thirty of thirty-two cases for which the low solution was most accurate were done in June and July, 1998. Inconclusive runs were found in both time frames.

To the best of the author’s ability to determine, there should have been no difference between the two groups of runs. No changes were made to either the equipment or the data acquisition system between the two groups of runs. The author carried out all of the runs personally, thus, no operational changes should have influenced the results. Yet, something in the earlier runs allowed results consistent with the middle solution to be found.

This further highlights the importance of the laminar/turbulent transition for the liquid phase. Eight of the nine points for which the measured holdup is closest to the middle solution fall outside the multiple solution region when the turbulent liquid characterization is applied.

The ratio of the liquid holdup corresponding to the middle solution to that corresponding to the low solution ranges from 1.2 to 8.7 for the nine cases for which the measured value is closest to the middle solution. Thus, always accepting the low solution as the correct solution could have significant consequences. For example, if the results were used to design liquid handling facilities, those facilities could be undersized.

Table 5.3 summarizes the results of a statistical analysis on the data within the multiple solution region.

Table 5.3 Accuracy of Liquid Volume Fraction Predictions Within the Multiple Solution Region - Laminar Liquid

	Low Solution	Middle Solution	High Solution
Mean Error (%)	-47.5	265.5	1655.8
Standard Deviation (%)	25.9	218.3	1129.8
Mean Absolute Error (%)	48.5	269.3	1655.8

The statistical analysis suggests that, for these data, the low solution can be considered to be the most reliable. As with the turbulent liquid characterization, this lends support to the theoretical works presented to date; all of which suggest that the low solution should be considered the most stable and thereby the acceptable result. However, a detailed review of the data shows that the possibility of the middle solution giving a viable result may be significant when the laminar liquid characterization is employed.

5.4.2 Calculations Outside the Multiple Solution Region

As previously noted, 65 of 209 experimental data points fell within the multiple solution region. Figure 5.10 shows the comparison of the measured and calculated liquid volume fraction for the remaining 144 data points (i.e., those outside of the multiple solution region). As was seen for the turbulent liquid characterization in Section 5.3.2,

the results of the calculations are separated into two distinct groups by the multiple solution region. A smaller number of points fall in the “Low V_{SG} Points” group as the number of points in the multiple solution region increases when the laminar liquid characterization is applied.

Table 5.4 summarizes the results of a statistical analysis on the data outside the multiple solution region.

Table 5.4 Accuracy of Liquid Volume Fraction Predictions Outside the Multiple Solution Region - Laminar Liquid

	All Data Points	“Low V_{SG} Points”	“High V_{SG} Points”
Mean Error (%)	797.3	1862.7	14.4
Standard Deviation (%)	1102.3	942.5	63.4
Mean Absolute Error (%)	818.8	1862.7	51.6

Based on these data, the multiple solution region appears to define a boundary beyond which the accuracy of the Taitel-Dukler model is questionable.

Figure 5.10 Comparison of the Measured and Calculated Liquid Volume Fraction Outside the Multiple Solution Region - Laminar Liquid Characterization

5.4.3 The Landman Parameterization

The Landman parameterization predicts that 65 of the 209 measured data points should fall within the multiple solution region. Four of these points are in error relative to those found by searching for multiple solutions. In three cases, the Landman parameterization indicates that multiple solutions should be expected where only one solution was found. In one case, it indicates that only one solution should be anticipated where three were found. In 61 of 65 cases, the Landman parameterization and the search for multiple solutions are in agreement, an accuracy of 93.8 percent. Thus, the Landman technique appears to be a reliable means of defining the multiple solution region regardless of the liquid phase characterization that is assumed to apply.

5.4.4 Stability Criteria

For each of the 65 experimental data points that fell within the multiple solution region, stability was assessed with both the inviscid (IKH) and viscid (VKH) Kelvin-Helmholtz criteria as well as the linear structural criteria. The non-linear structural stability of the upper solution in each case was also evaluated.

In each case, the results of the IKH and VKH stability criteria were the same. The low solution was always stable by both criteria. In 16 of the 65 cases, the middle solution was unstable by both criteria. In every case, the high solution was unstable by both criteria.

Barnea and Taitel (1992, 1994) stated that the linear structural stability criteria

should always indicate that the high and low solutions were stable and the middle solution unstable. Of the 65 cases, only the middle solution was predicted to be unstable in 55 cases, both the middle and high solutions were predicted to be unstable in 3 cases, and all three solutions were predicted to be stable in 7 cases. When the linear structural stability was analyzed using the original interfacial friction factor recommended by Taitel and Dukler, the three cases where two solutions were reported to be unstable were eliminated. However, the number of cases where all three solutions were reported to be stable increased from seven to twenty-five.

It is interesting to note that of the nine cases for which the middle solution is the most accurate, three are included in the seven (and seven within the twenty-five) for which linear structural stability reports the middle solution as stable.

The non-linear structural stability criteria indicated that the high solution was unstable in all 65 cases.

Thus, when the laminar liquid characterization is applied, the low solution should always be stable. The middle solution is unstable due to both VKH and IKH instability in sixteen of sixty-five cases. However, the middle solution is reported to be stable by IKH, VKH, and linear structural stability criteria in seven or twenty-five cases, depending on the definition of the interfacial friction factor that is applied. The high solution is always unstable due to non-linear structural instability and both VKH and IKH instability.

5.5 Closing Discussions

The preceding sections raise several interesting points of discussion. First is the matter of which characterization of the liquid phase, laminar or turbulent, is most appropriate. Based on the data, it appears that the turbulent characterization gives better agreement between the measured and calculated values. It is only the presence of additional points in the multiple solution region that causes the mean absolute error of the results based on the laminar liquid characterization to be less than that of the turbulent liquid characterization.

Also, the physical interpretation of the multiple solution region precludes laminar flow. Landman (1991a) offers a physical interpretation of the multiple solution region that attributes the middle solution to liquid backflow. Barnea and Taitel (1992) recommend the use of linear structural stability to eliminate the middle solution due to the presence of liquid backflow. The presence of a net liquid backflow may be inconsistent with laminar flow.

The original intent of this work was to show, based on experimental data, whether or not the multiple solutions predicted for the dimensionless liquid level in upward inclined stratified flow do physically exist. Generally, the data support the solution corresponding to the lowest liquid holdup as the most reliable solution. However, there are some data that support the solution corresponding to the middle value for the liquid holdup. The delineation of these data along time lines could be indicative of some systematic experimental error, or it could be indicative of some physical change that caused a shift from one solution to another within the multiple solution region.

One difficulty with always assuming that the lowest solution is correct is that it leads to a step change in the dimensionless liquid height (i.e., the liquid holdup). For example, consider the scenario for $Y = -4$ illustrated in Figure 2.2. If the lowest solution is assumed to apply in the multiple solution region, the dimensionless liquid level would change abruptly from approximately 0.12 to approximately 0.28 at the transition from the multiple solution region to the single solution region. Generally, phenomena such as liquid holdup do not experience such abrupt changes, but instead, move through continuous transitions. One plausible explanation for the observed liquid holdup values would be that some of the measured data fall within such a transition region.

Another issue to be considered is the systematic over-prediction of the Taitel-Dukler model at low superficial gas velocities. Referring to Figure 2.2, it can be seen that the Taitel-Dukler curve exhibits a rapid transition from low to high liquid levels. If this transition were shifted to occur at lower superficial gas velocities this could improve the overall results of calculated values outside the multiple solution region. Oliemans (1987) showed that one means to do this would be to redefine the interfacial shear.

Crowley and Rathe (1988) demonstrated the impact of assuming that the interfacial friction factor was ten times the gas wall friction factor. This change had a dramatic effect on both the transition from low to high liquid levels as seen in Figure 2.2 and the critical value of the inclination parameter (Y) below which multiple solutions occur.

Since the Oliemans (1987) model was part of the original inspiration for this work, the author considered evaluating that model. The development of this model

closely paralleled the work of Taitel and Dukler (1976). However, Oliemans presented a more general form of Equation (11) in which the Blasius friction factors used by Taitel and Dukler were replaced by friction factors that it was recommended be determined by using the Colebrook equation.

Oliemans (1987) proposed that the interfacial friction factor be calculated in the same way as the gas and liquid phase friction factors with the interfacial velocity equal to the liquid phase velocity and using an interfacial roughness defined by:

$$k_i = \begin{cases} 3\sqrt{2} \Delta h & \text{for } \Delta h < h_L \\ 3\sqrt{2} h_L & \text{for } \Delta h \geq h_L \end{cases} \quad (58)$$

where Δh is the root mean square wave height. This presented a problem in this work (and in almost any industrial application) as the wave height was not measured. The alternative is to use the definition in terms of the liquid level (i.e., assume, in effect, that the wave height is equal to the liquid level) in every circumstance. This is ill-posed in the multiple solution region where the calculated liquid level can vary quite widely. Such an assumption could, particularly in the case of the solution corresponding to the highest liquid holdup, lead to values for the interfacial roughness larger than the pipe diameter. Further, for the calculations based on the measured data, the ratio of the interfacial roughness applied to the upper solution to that applied to the lower solution is as high as seventy-five. Because of these difficulties, the Oliemans model was not evaluated.

The linear structural stability predictions, based on the laminar liquid characterization, that all three solutions were stable were unexpected. However, as noted

above, the linear stability criterion may be inconsistent with laminar flow.

There are two specific concerns with respect to the model of Taitel and Dukler (1976): the presence of multiple solutions for the liquid holdup and the systematic over-prediction of the liquid holdup at low superficial gas velocities. It appears, based on the work of Oliemans (1987) and Crowley and Rathe (1988), that both of these concerns can be addressed by considering the definition of the interfacial friction factor (Taitel and Dukler (1976) assume this is equal to the gas phase friction factor). Each of the other assumptions made by Taitel and Dukler (1976) (i.e., one-dimensional, steady-state, isothermal, no mass transfer between the phases) can be addressed by appropriate application of the model.

Thus, as is often the case in works such as this, the questions that were the original focus of the work have been answered, at least in part, with more questions.

6.0 Conclusions and Recommendations

An experimental program was undertaken at the University of Calgary to gather data spanning the multiple solution region that exists in the Taitel-Dukler model for upward inclined two-phase flow. Generally, these data support the published theoretical works that recommend that one accept the solution yielding the lowest liquid volume fraction.

There are some data that support the possibility that the middle solution may also be viable, particularly if the laminar liquid phase characterization is assumed to apply. The same experiments were repeated, after several months, using the same equipment and, contrary to the first experiments, the solution yielding the lowest liquid volume fraction best fit the data. Further, during the experiments, no hysteresis was observed anywhere in the calculated multiple solution region. In future works, a greater emphasis should be placed on the reproducibility of data to avoid any question of systematic error.

Another issue raised by the comparison of the measured and calculated liquid volume fractions was the systematic under-prediction of the liquid volume fraction at low gas superficial velocities (i.e., V_{SG} below the boundary of the multiple solution region). The work of Taitel and Dukler (1976) involved a number of assumptions (i.e., one-dimensional, steady-state, isothermal, no mass transfer between the phases, $f_l = f_g$). Most of these assumptions can be addressed by appropriate application of the model. The exception would be the assumption that the interfacial friction factor is the same as the gas phase friction factor. The work of Oliemans (1987) and Crowley and Rathe (1988) shows that manipulation of the interfacial friction factor may change the boundaries of

the multiple solution region and lead to improved liquid holdup predictions at low superficial gas velocities.

The choice of the laminar or turbulent flow characterization for the liquid phase has a significant impact on the boundaries of the multiple solution region. Calculations based on the turbulent liquid characterization better match the measured data presented in this study. There remains, however, a need for a definitive criterion for the transition between laminar and turbulent flow in upward inclined gas-liquid flow. The definition of such a criterion would be an excellent topic for further research.

The Landman (1991a) parameterization technique offers a reliable means of defining the multiple solution region. Regardless of the liquid phase characterization that was applied, the accuracy of this technique, when compared with a search for solutions to the Taitel-Dukler equation, was greater than ninety percent.

To some degree, the results of this study were inconclusive. One recommendation for addressing this in future work relates to the quality of the data acquisition. As noted in the discussion of the experimental apparatus, pressure measurement was not available during this study. This was considered acceptable at the time because the focus of this study was liquid holdup; this was a mistake. The addition of pressure measurements would have added flexibility to the analyses that could have been applied to issues such as the value of the interfacial friction factor.

7.0 Bibliography

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Appendix A - Details of the Computerized Data Acquisition System

The data acquisition system associated with the University of Calgary multiphase flow laboratory was developed by Dr. F. A. Sobbi (formerly of the University of Calgary) using the LabVIEW™ software (National Instruments (1993)) developed by National Instruments. LabVIEW™ is a programming application; instead of using a traditional text-based programming language, it uses a graphical programming language, G, to create programs. LabVIEW™ programs, referred to as virtual instruments, can take advantage of numerous specialized function libraries for instrumentation and control, data acquisition, analysis, presentation, and storage.

The “Two Phase Flow Program” is installed on a personal computer that is stored separately from the flow loop. The computer can be linked to the flow loop using three 50 pin ribbon connectors via three National Instruments data acquisition boards. An external analog multiplexer board is used to increase the number of input channels for data acquisition. The details of the boards are as follows:

- i) AT-MIO-16L-9 (Multi-function input/output board)
 - Used with AMUX-64T (Analog multiplexer for AT-MIO-16).
 - 18 active channels dedicated to temperature and pressure measurement.
- ii) PC-TIO-10 (Timer, counter board)
 - 9 active channels dedicated to measurement of frequency output from volume sensors.
- iii) AT-AO-10 (Analog output board)
 - 2 active channels used to control the air and oil control valves.

Data are reported at 1.5 second intervals. Raw data are written to the file generated during data acquisition. Data are processed only to allow for on-screen monitoring of the data acquisition.

The “Two Phase Flow Program” guides the user through the start up procedure. This includes the definition of an output file name, selection of instrumentation, and storage of fixed variables for a given run, including atmospheric pressure and temperature, orifice plate sizes, and details of pipe geometry (i.e., inside diameter, length, inclination, and roughness).

As noted previously, the user can choose between orifice and rotameter controls for both the liquid and gas phases. Where orifice control is selected, as is typically done for the gas phase, the valve opening is controlled remotely, from within the data acquisition system. Where rotameter control is chosen, as is typically done for the liquid phase, the controller in the data acquisition system allows the user to define a rotameter setting and displays the volumetric flow rate and superficial velocity associated with that setting. Once the desired rotameter setting has been displayed, the user must adjust the rotameter manually (i.e., the rotameter setting is NOT controlled remotely).

The system also stores sufficient information during the start up and shut down processes to allow the volume sensors to be uniquely calibrated for each lab session. During the start up procedure the data acquisition system guides the user through the process of taking a volume sensor measurement with the pipe empty of liquid (i.e., single phase gas flow). Similarly, during the shut down procedure, the system guides the user

through the process of taking a volume sensor measurement with the pipe filled with liquid (i.e., single phase liquid flow).

Once all of the relevant data have been specified, the start up process completed, and the equipment selections made, the process of gathering data for multiphase flowing conditions is begun. All of the controls and data outputs for this process are incorporated into a single screen (Figure A.11). The system includes controls for the liquid and gas rotameters (both type and setting) and control valves (% opening). The run number and observed flow pattern can be entered manually. System temperatures, oil and gas flow rates, total pressure loss, average liquid holdup, and separator oil level are displayed both numerically and graphically. Also included are a high level alarm for the separator and an emergency shut down button.

When a series of runs is complete, the system guides the user through the shut down process, including, as previously noted, the full volume sensor reading.

The series of screens seen during a typical laboratory session are shown in Figures A.1 - A.15. These figures also provide a detailed outline of the start up and shut down processes.

Figure A.1 Title Screen - Two Phase Flow Program

S-2

1. Open splash pots isolation valves for the pipe under test (SP1 to SP3) on pannel # 4.
2. Check that valves are leading flow to correct places.
3. Note the atmospheric pressure on the barometer.
4. Check air orifice plate size.
5. check oil orifice plate size.
6. Put the Instrument Power "ON".
7. Open isolate valve 2, null valves 1, 3 and 4, and Common valve on pannel 4.

ENTER FILE PATH AND NAME TO SAVE DATA
 C:\DATA\BANK\DEC.DAT

PRESS TO CONTINUE

Figure A.2 Start Up Screen S-2 - Two Phase Flow Program

S-3

ENTER THE FOLLOWING DATA

Atmospheric pressure : mm Hg

Room Temperature deg C

Air orifice plate inches

Oil orifice plate : inches

Pipe length : feet

Pipe size : inches **AREA: 3.142**

Pipe inside diameter : inches **ROUNDER: 0**

Pipe angle : deg.

Pipe roughness : ft.

PIPE LENGTH:		
1" PIPE	32.104	FEET
2" PIPE	32.125	FEET
3" PIPE	32.250	FEET

AIR ORIFICE SIZE = 0.500 INCHES. 6/4/95

PRESS TO CONTINUE

Figure A.3 Start Up Screen S-3 - Two Phase Flow Program

S-4

MEASURING DEVICES

ENTER NUMBER OF PRESSURE TRANSDUCERS

ENTER NUMBER OF VOLUME SENSORS

PRESS TO CONTINUE

Figure A.4 Start Up Screen S-4 - Two Phase Flow Program

SYSTEM PRESSURE CONTROL

S-5

1. CLOSE EXHAUST CONTROLLER VENT ON PANNEL # 2.
2. CLOSE INSTRUMENT VENT ON PANNEL # 2.
3. OPEN INSTRUMENT AIR VALVE BEHIND OIL ROTAMETER.
4. OPEN INSTRUMENT AIR VALVE ON PANNEL # 2.
5. CLOSE MANUAL CONTROL VALVE ON PANNEL # 2.
6. OPEN AUTOMATIC CONTROL VALVE ON PANNEL # 2.
7. OPEN EXHAUST CONTROL VALVE.
THERE ARE THREE EXHAUST VALVES (SMALL, MEDIUM AND LARGE) ONE OF THESE
MUST BE SELECTED IN RELATION TO THE OUTPUT FLOW.
8. CLOSE THE AIR SUPPLY ORIFICE VENT AND ROTAMETER BYPASS VENT ON PANNEL # 1
9. SLOWLY CRACK OPEN THE SUPPLY ORIFICE AIR VALVE ON PANNEL # 1
ONCE PRESSURE HAS BUILT UP, OPEN FULL.

PRESS TO CONTINUE

Figure A.5 Start Up Screen S-5 - Two Phase Flow Program

SYSTEM PRESSURE CONTROL

S-6

10. THE SYSTEM MUST BE NOW PRESSURIZED BY USING THE COMPUTER OR MANUAL.
11. SYSTEM PRESSURIZING

AUTOMATIC PRESSURIZING

SET AIR CONTROL VALVE TO 3X, SO SYSTEM PRESSURIZE SLOWLY.

WHEN SYSTEM IS PRESSURIZED TO 25 PSIG, SET TO ZERO THIS CLOSES AIR
CONTROL VALVE.

AIR CONTROL VALVE OPENING :

3X

MANUAL PRESSURIZING

USE THE AIR ROTAMETER ON MAIN PANEL TO PRESSURIZE SYSTEM

WHEN SYSTEM PRESSURE IS 25 PSIG, CLOSE THE AIR ROTAMETER.

12. OPEN ISOLATE VALVES 1, 3 AND 4 AND CLOSE NULL 1, 3 AND 4 ON PANNEL 4.
13. OPEN AIR SUPPLY ORIFICE ISOLATE AND CLOSE AIR SUPPLY ORIFICE NULL.

PRESS TO CONTINUE

Figure A.6 Start Up Screen S-6 - Two Phase Flow Program

FLOW RATE CONTROL

AIR FLOW RATE ORIFICE
 OIL FLOW RATE ROTAMETER

PRESS TO CONTINUE

Figure A.7 Flow Rate Control - Two Phase Flow Program

AUTOMATIC AIR FLOW CONTROL

Open Supply Orifice Air. air supply orifice isolate and Measure Orifice Static valves on the main panel. Close air supply orifice vent, measure vent and PH Null valves. Open Air Control Valve in STEPS. This will avoid system shocks.

CONTROL VALVE OPENING %:

Superficial Air Velocity - (ft/sec) 0.0000E+0

Air Flow Rate, SCF/min 0.0000E+0

PRESS TO CONTINUE

Figure A.8 Automatic Air Flow Control - Two Phase Flow Program

OIL FLOW RATE SETTING

1. Check whether the valve for the large pump is open or closed. If it is closed then it is OK to run the small pump. When the large pump is going to be used, this valve must be open.
2. Close all rotameter valves.
3. Put water supply on. Check that the oil temperature controller works.
4. Put the CaCl₂ dryer pump on.
5. Start the oil pump.
6. Depending on the flow rate, use one of the rotameters.
7. Once liquid velocity has been set on rotameter, wait 10-15 minutes for the flow to stabilize.

PRESS TO CONTINUE

Figure A.9 Oil Flow Rate Setting - Two Phase Flow Program

OIL FLOW RATE CONTROL ROTAMETER SETTING

<p>150.0 - 150 - S</p> <p>100.0 - Q, ml/s</p> <p>50.0 - Q, bbl/min.</p> <p>0.0 - Vel, ft/s</p> <p>0.0000E+0</p>	<p>250.0 - 250 - L</p> <p>200.0 - Q, ml/s</p> <p>150.0 - Q, bbl/min.</p> <p>100.0 - Vel, ft/s</p> <p>0.0000E+0</p>	<p>600.0 - 600 - S</p> <p>400.0 - Q, ml/s</p> <p>200.0 - Q, bbl/min.</p> <p>0.0 - Vel, ft/s</p> <p>0.0000E+0</p>	<p>600.0 - 600 - L</p> <p>400.0 - Q, ml/s</p> <p>200.0 - Q, bbl/min.</p> <p>0.0 - Vel, ft/s</p> <p>0.0000E+0</p>
<p>OIL CONTROL VALVE SETTING %</p> <p>0</p>		<p>GAS OIL RATIO, MSCF/STB</p> <p>0.0000E+0</p>	
<p>PRESS TO CONTINUE</p>			

Figure A.10 Oil Flow Rate Control - Two Phase Flow Program

Figure A.11 Primary Data Acquisition - Two Phase Flow Program

SHUTTING DOWN THE SYSTEM

1 VOLUME SENSOR 100 % READING

Before the system is shut down a 100 % volume sensor reading must be taken. This varies slightly each day due to temperature.

2 OPEN NULL VALVES AND CLOSE ISOLATE VALVES ON PANNELS 1 AND 4

3 CLOSE THE CLEANING VALVE (AIR TO OIL TUBE) THIS VALVE IS BEHIND THE OIL ROTAMETER ON LEFT

4 PRESS TO CONTINUE TO CLOSE THE AIR CONTROL VALVE.

PRESS TO CONTINUE

Figure A.12 Shutting Down the System - Two Phase Flow Program

WAIT 5-10 MINUTES. PRESS TO CONTINUE TO GET THE READING FOR
THE 100 % LIQUID FLOW FROM THE VOLUME SENSORS.

PRESS TO CONTINUE

Figure A.13 Volume Sensor Calibration - Two Phase Flow Program

SHUT THE OIL SYSTEM OFF

1. CLOSE OIL ROTAMETER
2. TURN PUMP OFF
3. CLOSE WATER TAP. THIS VALVE IS BEHIND PANNEL #1.
4. SHUT THE CaCl₂ DRYER PUMP.

PRESS TO CONTINUE

Figure A.14 Shutting the Oil System Off - Two Phase Flow Program

CLEAN OUT THE OIL OF PIPE

1. OPEN THE CLEANING VALVE (AIR TO OIL TUBE)
2. PUT LARGE EXHAUST VLV ON LINE
3. SET AIR VELOCITY TO MAXIMUM. INCREASE IN STEPS.

GAS CONTROL VALVE 1: OPENNING

4. LET THE AIR FLOW FOR 10 - 15 MINUTES.
5. SET GAS CONTROL VALVE TO ZERO
6. SHUT SUPPLY ORIFICE AIR OFF ON PANNEL 1
7. OPEN AIR SUPPLY ORIFICE VENT ON PANNEL 1
8. TURN AUTO ON "OFF-POSITION" AND MANUAL ON "ON-POSITION ON PANNEL 2
9. SLOWLY OPEN EXHAUST REGULATOR. THIS DEPRESSURIZE THE WHOLE SYSTEM. ONCE DEPRESSURIZED
10. CLOSE EXHAUST REGULATOR
11. TURN OFF THE INSTRUMENT AIR - ONE ON PANNEL 2 - ONE BEHIND ROTAMETER
12. OPEN INSTRUMENT VENT AND EXHAUST CONTROL VENT ON PANNEL 2.

PRESS TO CONTINUE

Figure A.15 Clean Out Pipe - Two Phase Flow Program

Appendix B - Measured Data

Table B.1 Measured Data (September 30, 1997)

Pipe Inside Diameter = 2.00inches							
No.	V_{SG} (m/s)	V_{SL} (cm/s)	$T(^{\circ}C)$	Holdup			Flow Pattern*
				Upstr	Mid	Dnstr	
1	17.419	0.091	22	0.02245	0.03388	0.00313	16
2	17.313	0.082	22	0.0261	0.03198	0.00283	16
3	17.248	0.072	22	0.01945	0.02949	0.00231	16
4	17.369	0.059	22	0.02112	0.02714	0.00313	16
5	17.249	0.051	22	0.02294	0.02566	0.002	16
6	17.415	0.04	22	0.02139	0.02343	0.00292	16
7	17.267	0.031	22.1	0.01581	0.02302	0.00325	16
8	17.436	0.051	22	0.02022	0.02393	0.00407	16
9	17.29	0.051	22	0.0177	0.02211	0.00436	16
10	17.356	0.091	22	0.02072	0.02406	0.00419	16
11	17.249	0.051	22	0.01368	0.02124	0.00319	16

* Within the "Two Phase Flow Program," Flow Pattern 16 is stratified wavy. In each case, the flow pattern was observed to be stratified with a rippling interface.

Table B.2 Measured Data (October 1, 1997)

Pipe Inside Diameter = 2.00inches							
No.	V_{SG} (m/s)	V_{SL} (cm/s)	$T(^{\circ}C)$	Holdup			Flow Pattern*
				Upstr	Mid	Dnstr	
1	17.596	0.091	22.1	0.03072	0.03826	0.01611	16
2	17.681	0.082	22.1	0.03076	0.03789	0.01739	16
3	17.547	0.072	22.1	0.03346	0.0362	0.01851	16
4	17.721	0.059	22.1	0.02999	0.03511	0.01706	16
5	17.601	0.051	22.1	0.02969	0.03506	0.01763	16
6	17.625	0.04	22.1	0.03148	0.03563	0.01818	16
7	17.67	0.031	22.1	0.02814	0.0351	0.01837	16
8	17.543	0.051	22	0.03433	0.03583	0.01906	16
9	18.133	0.051	22	0.03043	0.03373	0.01702	16
10	18.011	0.091	22	0.03361	0.0353	0.01839	16
11	18.072	0.051	22	0.02751	0.03225	0.01745	16

* Within the "Two Phase Flow Program," Flow Pattern 16 is stratified wavy. In each case, the flow pattern was observed to be stratified with a rippling interface.

Table B.3 Measured Data (October 14, 1997)

Pipe Inside Diameter = 2.00inches							
No.	V _{SG} (m/s)	V _{SL} (cm/s)	T(°C)	Holdup			Flow Pattern*
				Upstr	Mid	Dnstr	
1	23.773	0.01	21.2	0.03333	0.0273	0.01026	16
2	20.908	0.01	21.2	0.03874	0.03076	0.01505	16
3	19.933	0.01	21.3	0.04026	0.03332	0.01763	16
4	18.798	0.01	21.2	0.03849	0.03468	0.01935	16
5	16.964	0.01	21.3	0.04268	0.03633	0.02048	16
6	15.44	0.01	21.3	0.04325	0.03764	0.02124	16
7	13.857	0.01	21.2	0.04146	0.03985	0.02348	16
8	12.228	0.01	21.3	0.0489	0.04271	0.0251	16
9	10.622	0.01	21.4	0.05625	0.04998	0.02773	16
10	9.51	0.01	21.4	0.04715	0.04294	0.02217	16
11	8.433	0.01	21.4	0.06974	0.02564	0.01187	16

* Within the "Two Phase Flow Program," Flow Pattern 16 is stratified wavy. In each case, the flow pattern was observed to be stratified with a rippling interface.

Table B.4 Measured Data (October 15, 1997)

Pipe Inside Diameter = 2.00inches							
No.	V _{SG} (m/s)	V _{SL} (cm/s)	T(°C)	Holdup			Flow Pattern*
				Upstr	Mid	Dnstr	
1	24.67	0.019	21.4	0.01861	0.01824	0.01786	16
2	23.28	0.019	21.3	0.02072	0.02082	0.02237	16
3	21.258	0.019	21.3	0.02533	0.02501	0.02839	16
4	19.455	0.019	21.3	0.03142	0.02826	0.03284	16
5	17.275	0.019	21.3	0.03676	0.03004	0.03627	16
6	15.911	0.019	21.3	0.03079	0.03023	0.03626	16
7	14.385	0.019	21.3	0.03409	0.03268	0.03913	16
8	12.803	0.019	21.3	0.03923	0.0357	0.04163	16
9	11.174	0.019	21.4	0.04701	0.04221	0.04177	16
10	9.864	0.019	21.3	0.04457	0.03514	0.03887	16
11	24.753	0.031	21.4	0.01866	0.0195	0.02955	16
12	23.351	0.031	21.4	0.02317	0.02061	0.02965	16
13	21.274	0.031	21.4	0.02473	0.02271	0.03163	16
14	19.37	0.031	21.3	0.02816	0.02414	0.0341	16
15	17.683	0.031	21.3	0.02651	0.0262	0.03462	16
16	16.09	0.031	21.3	0.03646	0.02734	0.03504	16
17	14.416	0.031	21.4	0.03308	0.02986	0.03657	16
18	12.753	0.031	21.3	0.03266	0.03299	0.03756	16
19	11.084	0.031	21.3	0.0451	0.03964	0.04069	16
20	9.847	0.031	21.3	0.03808	0.03425	0.03563	16

* Within the "Two Phase Flow Program," Flow Pattern 16 is stratified wavy. In each case, the flow pattern was observed to be stratified with a rippling interface.

Table B.5 Measured Data (October 16, 1997)

Pipe Inside Diameter = 2.00inches							
No.	V _{SG} (m/s)	V _{SL} (cm/s)	T(°C)	Holdup			Flow Pattern*
				Upstr	Mid	Dnstr	
1	24.391	0.04	21.5	0.06462	0.02678	0.01448	16
2	23.096	0.04	21.5	0.06105	0.02933	0.01647	16
3	21.013	0.04	21.5	0.05274	0.03248	0.01994	16
4	19.2	0.04	21.4	0.06168	0.0344	0.02222	16
5	17.467	0.04	21.5	0.05488	0.03568	0.02404	16
6	15.902	0.04	21.4	0.05937	0.03652	0.02552	16
7	14.159	0.04	21.4	0.06666	0.03923	0.02716	16
8	12.694	0.04	21.5	0.04892	0.04268	0.02955	16
9	11.09	0.04	21.4	0.06414	0.04378	0.02892	16
10	9.800	0.04	21.4	0.06398	0.04219	0.02591	16
11	24.691	0.051	21.5	0.04643	0.02426	0.01822	16
12	23.127	0.051	21.4	0.05919	0.02546	0.01868	16
13	21.055	0.051	21.5	0.04901	0.02758	0.02003	16
14	19.164	0.051	21.5	0.06077	0.02803	0.02083	16
15	17.431	0.051	21.5	0.06071	0.02983	0.02305	16
16	15.885	0.051	21.4	0.04632	0.03146	0.02379	16
17	14.205	0.051	21.5	0.051	0.03337	0.02481	16
18	12.763	0.051	21.5	0.0563	0.0365	0.02618	16
19	11.124	0.051	21.4	0.06087	0.0389	0.02683	16
20	9.564	0.051	21.5	0.0481	0.03777	0.02728	16

* Within the "Two Phase Flow Program," Flow Pattern 16 is stratified wavy. In each case, the flow pattern was observed to be stratified with a rippling interface.

Table B.6 Measured Data (October 21, 1997)

Pipe Inside Diameter = 2.00inches							
No.	V _{SG} (m/s)	V _{SL} (cm/s)	T(°C)	Holdup			Flow Pattern*
				Upstr	Mid	Dnstr	
1	24.299	0.059	22.2	0.68534	0.02372	0.02971	16
2	23.105	0.059	22.2	0.72623	0.02601	0.03185	16
3	21.056	0.059	22.1	0.85867	0.02763	0.03351	16
4	19.185	0.059	22.1	0.80272	0.0291	0.03569	16
5	17.491	0.059	22.1	0.79184	0.03056	0.03636	16
6	15.875	0.059	22.1	0.65157	0.03212	0.03757	16
7	14.18	0.059	22.1	0.76285	0.03485	0.03947	16
8	12.673	0.059	22	0.89249	0.03867	0.04101	16
9	11.063	0.059	22.1	1.05787	0.03911	0.03922	16
10	9.770	0.059	22.1	1.01904	0.03797	0.03916	16
11	24.47	0.072	22.1	0.73146	0.02037	0.03111	16
12	23.155	0.072	22.1	0.84993	0.02176	0.03156	16
13	21.076	0.072	22.1	0.77984	0.02304	0.03322	16
14	19.184	0.072	22	0.84052	0.02524	0.03509	16
15	17.446	0.072	22.1	0.85892	0.02667	0.03501	16
16	15.894	0.072	22.1	0.81728	0.02861	0.03621	16
17	14.186	0.072	22	0.78694	0.0315	0.03783	16
18	12.7	0.072	22	1.09212	0.03592	0.04026	16
19	11.093	0.072	22.1	1.00107	0.03773	0.03845	16
20	9.774	0.072	21.9	1.07123	0.03465	0.03846	16

* Within the "Two Phase Flow Program," Flow Pattern 16 is stratified wavy. In each case, the flow pattern was observed to be stratified with a rippling interface.

Table B.7 Measured Data (October 23, 1997)

Pipe Inside Diameter = 2.00inches							
No.	V _{SG} (m/s)	V _{SL} (cm/s)	T(°C)	Holdup			Flow Pattern*
				Upstr	Mid	Dnstr	
1	24.298	0.082	21.1	0.03142	0.02949	0.01735	16
2	23.116	0.082	21.1	0.02999	0.0315	0.02084	16
3	21.057	0.082	21	0.02608	0.03386	0.02566	16
4	19.16	0.082	21	0.02834	0.03647	0.03058	16
5	17.432	0.082	21	0.03251	0.03803	0.03318	16
6	15.94	0.082	21	0.03335	0.04043	0.0346	16
7	14.207	0.082	21	0.03654	0.0437	0.03952	16
8	12.717	0.082	21	0.03634	0.04786	0.04522	16
9	11.191	0.082	21	0.04918	0.04793	0.03995	16
10	9.825	0.082	21	0.03804	0.04854	0.0391	16
11	24.748	0.091	21	0.03265	0.02805	0.02661	16
12	23.239	0.091	20.9	0.02506	0.02988	0.02586	16
13	21.107	0.091	21	0.03169	0.0321	0.0292	16
14	19.231	0.091	20.9	0.02925	0.03414	0.03282	16
15	17.519	0.091	20.9	0.03062	0.03561	0.03504	16
16	15.963	0.091	21	0.026	0.03768	0.03484	16
17	14.221	0.091	21	0.0274	0.04059	0.03973	16
18	12.724	0.091	21	0.03496	0.04458	0.04284	16
19	11.147	0.091	21	0.0445	0.04579	0.03926	16
20	9.865	0.091	21	0.0403	0.04505	0.04003	16

* Within the "Two Phase Flow Program," Flow Pattern 16 is stratified wavy. In each case, the flow pattern was observed to be stratified with a rippling interface.

Table B.8 Measured Data (June 17, 1998)

Pipe Inside Diameter = 2.00inches							
No.	V _{SG} (m/s)	V _{SL} (cm/s)	T(°C)	Holdup			Flow Pattern*
				Upstr	Mid	Dnstr	
1	19.466	0.01	21.7	0.01032	0.00348	0.02245	16
2	17.835	0.01	21.7	0.01223	0.00693	0.02591	16
3	15.835	0.01	21.7	0.01278	0.00837	0.00557	16
4	14.132	0.01	21.7	0.01452	0.00929	0.01037	16
5	11.882	0.01	21.7	0.01885	0.00861	0.0054	16
6	10.125	0.01	21.7	0.01783	0.01199	0.01453	16
7	10.198	0.01	21.7	0.0251	0.00906	0.02052	16
8	12.034	0.01	21.8	0.02189	0.01333	0.02817	16
9	13.937	0.01	21.7	0.01671	0.00949	0.01234	16
10	15.761	0.01	21.7	0.01422	0.00757	0.01035	16
11	17.826	0.01	21.7	0.01537	0.00583	0.00374	16
12	19.828	0.01	21.8	0.01423	0.00561	0.0158	16
13	20.094	0.02	21.9	0.02134	0.01162	0.01693	16
14	17.969	0.02	21.8	0.02194	0.01434	0.03138	16
15	15.977	0.02	21.8	0.02381	0.01556	0.02917	16
16	14.205	0.02	21.9	0.02621	0.01656	0.03069	16
17	11.9	0.02	21.8	0.03075	0.01668	0.02797	16
18	10.138	0.02	21.9	0.02559	0.01481	0.0364	16
19	10.277	0.02	21.8	0.033	0.0221	0.02935	16
20	12.078	0.02	21.9	0.02165	0.01686	0.04737	16
21	13.968	0.02	21.9	0.01923	0.01214	0.01991	16
22	15.801	0.02	21.8	0.01876	0.00914	0.0148	16
23	17.853	0.02	21.9	0.01795	0.00801	0.01754	16
24	19.845	0.02	21.9	0.01747	0.00767	0.01448	16

* Within the "Two Phase Flow Program," Flow Pattern 16 is stratified wavy. In each case, the flow pattern was observed to be stratified with a rippling interface.

Table B.9 Measured Data (June 18, 1998)

Pipe Inside Diameter = 2.00inches							
No.	V _{SG} (m/s)	V _{SL} (cm/s)	T(°C)	Holdup			Flow Pattern*
				Upstr	Mid	Dnstr	
1	19.67	0.03	22.4	0.00006	0.00685	0.00548	16
2	18.134	0.03	22.4	0.00453	0.00833	0.00993	16
3	16.138	0.03	22.4	-	0.01049	0.01254	16
4	13.936	0.03	22.5	-	0.01317	0.01552	16
5	12.09	0.03	22.5	-	0.01479	0.0204	16
6	10.009	0.03	22.5	0.0065	0.01273	0.02053	16
7	10.03	0.03	22.5	-	0.02165	0.02466	16
8	11.793	0.03	22.5	-	0.01669	0.03054	16
9	14.092	0.03	22.6	-	0.01148	0.01916	16
10	15.95	0.03	22.5	-	0.00832	0.01601	16
11	18.053	0.03	22.6	0.00267	0.00764	0.01304	16
12	20.083	0.03	22.6	0.00829	0.0067	0.01471	16
13	19.783	0.04	22.8	0.00802	0.01206	0.0192	16
14	18.225	0.04	22.8	0.0174	0.01331	0.01899	16
15	16.213	0.04	22.9	0.00498	0.01448	0.02357	16
16	13.932	0.04	22.9	0.01843	0.0171	0.02365	16
17	12.174	0.04	23	0.01572	0.01806	0.02408	16
18	9.958	0.04	22.9	0.01243	0.01717	0.021	16
19	10.048	0.04	23	0.0147	0.0236	0.035	16
20	11.768	0.04	23	0.0115	0.01912	0.03066	16
21	14.136	0.04	22.9	0.01585	0.01315	0.01974	16
22	16.01	0.04	23	0.01703	0.01129	0.01429	16
23	18.095	0.04	22.9	0.01099	0.00976	0.01367	16
24	20.076	0.04	23	0.01766	0.00826	0.01146	16
25	19.853	0.05	23	0.0134	0.01362	0.01788	16
26	18.178	0.05	23	0.02212	0.01442	0.01631	16
27	16.258	0.05	23	0.02076	0.01576	0.02121	16

28	14.408	0.05	23	0.01921	0.01816	0.02305	16
29	12.193	0.05	22.9	0.0218	0.01958	0.02415	16
30	9.999	0.05	23	0.01054	0.01792	0.0258	16
31	10.039	0.05	22.9	0.01433	0.02454	0.03815	16
32	11.826	0.05	22.9	0.02808	0.02073	0.03323	16
33	14.128	0.05	22.9	0.01487	0.01491	0.02186	16
34	16.027	0.05	22.9	0.01708	0.0123	0.01663	16
35	18.118	0.05	22.8	0.02143	0.01069	0.01083	16
36	20.076	0.05	22.9	0.02626	0.0099	0.01467	16
37	19.877	0.06	22.8	0.01174	0.0152	0.01903	16
38	18.222	0.06	22.8	0.00867	0.01592	0.02145	16
39	16.23	0.06	22.9	0.03743	0.01691	0.02568	16
40	13.951	0.06	22.9	0.03075	0.01985	0.0291	16
41	12.146	0.06	22.9	0.02807	0.02132	0.02339	16
42	9.998	0.06	23	0.04089	0.02273	0.02989	16
43	10.068	0.06	23	0.03598	0.02631	0.03576	16
44	11.865	0.06	23	0.04087	0.02417	0.03757	16
45	14.184	0.06	23	0.03967	0.017	0.02574	16
46	16.015	0.06	23.1	0.04489	0.0155	0.01941	16
47	18.112	0.06	23.1	0.03495	0.0139	0.01638	16
48	20.12	0.06	23.1	0.02007	0.01432	0.01391	16

* Within the "Two Phase Flow Program," Flow Pattern 16 is stratified wavy. In each case, the flow pattern was observed to be stratified with a rippling interface.

Table B.10 Measured Data (July 1, 1998)

Pipe Inside Diameter = 2.00inches							
No.	V _{SG} (m/s)	V _{SL} (cm/s)	T(°C)	Holdup			Flow Pattern*
				Upstr	Mid	Dnstr	
1	19.999	0.07	25.6	0.02277	0.01378	-	16
2	17.923	0.07	25.6	0.02224	0.01554	-	16
3	15.983	0.07	25.7	0.02515	0.01769	0.00018	16
4	14.145	0.07	25.7	0.02729	0.02024	0.00306	16
5	12.347	0.07	25.7	0.02828	0.02303	0.00212	16
6	10.142	0.07	25.7	0.03939	0.02646	0.00232	16
7	10.178	0.07	25.7	0.03922	0.02812	0.00122	16
8	11.962	0.07	25.7	0.03239	0.02545	0.00161	16
9	13.911	0.07	25.8	0.02827	0.01938	0.00357	16
10	15.78	0.07	25.8	0.02754	0.01766	-	16
11	17.932	0.07	25.8	0.02732	0.01604	0.00146	16
12	20.032	0.07	25.9	0.02226	0.01599	0.00501	16
13	20.157	0.08	25.8	0.02745	0.0175	-	16
14	18.011	0.08	25.8	0.02787	0.0182	0.00275	16
15	15.986	0.08	25.8	0.02915	0.01878	0.00212	16
16	14.208	0.08	25.7	0.03039	0.02049	0.00123	16
17	11.913	0.08	25.6	0.02848	0.02294	0.0022	16
18	10.178	0.08	25.6	0.03173	0.02356	0.00331	16
19	10.265	0.08	25.4	0.04285	0.02753	0.00563	16
20	12.039	0.08	25.3	0.02842	0.02326	0.00673	16
21	13.967	0.08	25.3	0.02665	0.01601	0.00286	16
22	15.888	0.08	25.3	0.0265	0.01412	0.00241	16
23	18.008	0.08	25.4	0.02524	0.01274	0.00126	16
24	19.921	0.08	25.3	0.02238	0.01154	0.00149	16

* Within the "Two Phase Flow Program," Flow Pattern 16 is stratified wavy. In each case, the flow pattern was observed to be stratified with a rippling interface.

Appendix C - Calculation Results

Table C.1 Calculation Results - Turbulent Liquid Characterization

Angle (degrees)	V_{SL} (cm/s)	V_{SG} (m/s)	Liquid Holdup			Landman Result
			Low Solution	Middle Solution	High Solution	
-2	0.010	8.434	0.597	-	-	1
-2	0.010	9.510	0.558	-	-	1
-2	0.010	10.125	0.535	-	-	1
-2	0.010	10.198	0.532	-	-	1
-2	0.010	10.622	0.517	-	-	1
-2	0.010	11.882	0.470	-	-	1
-2	0.010	12.034	0.464	-	-	1
-2	0.010	12.228	0.457	-	-	1
-2	0.010	13.857	0.016	0.033	0.392	3
-2	0.010	13.937	0.016	0.035	0.389	3
-2	0.010	14.132	0.015	0.038	0.381	3
-2	0.010	15.440	0.012	0.060	0.323	3
-2	0.010	15.761	0.012	0.066	0.307	3
-2	0.010	15.835	0.012	0.076	0.303	3
-2	0.010	16.964	0.010	0.106	0.233	3
-2	0.010	17.826	0.010	-	-	1
-2	0.010	17.835	0.010	-	-	1
-2	0.010	18.798	0.009	-	-	1
-2	0.010	19.466	0.008	-	-	1
-2	0.010	19.828	0.008	-	-	1
-2	0.010	19.933	0.008	-	-	1
-2	0.010	20.908	0.008	-	-	1
-2	0.010	23.773	0.007	-	-	1
-2	0.019	9.864	0.545	-	-	1
-2	0.020	10.138	0.535	-	-	1
-2	0.020	10.277	0.529	-	-	1
-2	0.019	11.174	0.496	-	-	1
-2	0.020	11.900	0.469	-	-	1
-2	0.020	12.078	0.463	-	-	1
-2	0.019	12.803	0.435	-	-	1
-2	0.020	13.968	0.388	-	-	1
-2	0.020	14.205	0.378	-	-	1
-2	0.019	14.385	0.371	-	-	3
-2	0.020	15.801	0.018	0.064	0.305	3

Angle (degrees)	V_{SL} (cm/s)	V_{SG} (m/s)	Liquid Holdup			Landman Result
			Low Solution	Middle Solution	High Solution	
-2	0.019	15.911	0.017	0.067	0.299	3
-2	0.020	15.977	0.018	0.068	0.296	3
-2	0.019	17.275	0.015	0.124	0.205	3
-2	0.020	17.853	0.014	-	-	1
-2	0.020	17.969	0.014	-	-	1
-2	0.019	19.455	0.012	-	-	1
-2	0.020	19.845	0.012	-	-	1
-2	0.020	20.094	0.012	-	-	1
-2	0.019	21.258	0.011	-	-	1
-2	0.019	23.280	0.010	-	-	1
-2	0.019	24.670	0.009	-	-	1
-2	0.031	9.847	0.545	-	-	1
-2	0.030	10.009	0.539	-	-	1
-2	0.030	10.030	0.539	-	-	1
-2	0.031	11.084	0.500	-	-	1
-2	0.030	11.793	0.473	-	-	1
-2	0.030	12.090	0.462	-	-	1
-2	0.031	12.753	0.436	-	-	1
-2	0.030	13.936	0.389	-	-	1
-2	0.030	14.092	0.383	-	-	1
-2	0.031	14.416	0.369	-	-	1
-2	0.030	15.950	0.024	0.064	0.297	3
-2	0.031	16.090	0.024	0.067	0.290	3
-2	0.030	16.138	0.023	0.069	0.287	3
-2	0.031	17.267	0.020	0.120	0.207	3
-2	0.031	17.670	0.019	-	-	1
-2	0.031	17.683	0.019	-	-	1
-2	0.030	18.053	0.018	-	-	1
-2	0.030	18.134	0.018	-	-	1
-2	0.031	19.370	0.016	-	-	1
-2	0.030	19.670	0.016	-	-	1
-2	0.030	20.083	0.015	-	-	1
-2	0.031	21.274	0.014	-	-	1
-2	0.031	23.351	0.013	-	-	1
-2	0.031	24.753	0.012	-	-	1
-2	0.040	9.801	0.547	-	-	1
-2	0.040	9.958	0.541	-	-	1
-2	0.040	10.048	0.538	-	-	1
-2	0.040	11.090	0.500	-	-	1

Angle (degrees)	V_{SL} (cm/s)	V_{SG} (m/s)	Liquid Holdup			Landman Result
			Low Solution	Middle Solution	High Solution	
-2	0.040	11.768	0.474	-	-	1
-2	0.040	12.174	0.459	-	-	1
-2	0.040	12.694	0.439	-	-	1
-2	0.040	13.932	0.390	-	-	1
-2	0.040	14.136	0.381	-	-	1
-2	0.040	14.159	0.380	-	-	1
-2	0.040	15.902	0.032	0.057	0.300	3
-2	0.040	16.010	0.031	0.060	0.294	3
-2	0.040	16.213	0.029	0.067	0.283	3
-2	0.040	17.415	0.023	0.133	0.188	3
-2	0.040	17.467	0.023	0.142	0.178	3
-2	0.040	17.625	0.023	-	-	1
-2	0.040	18.095	0.021	-	-	1
-2	0.040	18.225	0.021	-	-	1
-2	0.040	19.200	0.019	-	-	1
-2	0.040	19.783	0.018	-	-	1
-2	0.040	20.076	0.018	-	-	1
-2	0.040	21.013	0.017	-	-	1
-2	0.040	23.096	0.015	-	-	1
-2	0.040	24.391	0.014	-	-	1
-2	0.051	9.565	0.555	-	-	1
-2	0.050	9.999	0.539	-	-	1
-2	0.050	10.039	0.538	-	-	1
-2	0.051	11.124	0.498	-	-	1
-2	0.050	11.826	0.472	-	-	1
-2	0.050	12.193	0.458	-	-	1
-2	0.051	12.763	0.436	-	-	1
-2	0.050	14.128	0.381	-	-	1
-2	0.051	14.205	0.378	-	-	1
-2	0.050	14.408	0.370	-	-	1
-2	0.051	15.885	0.301	-	-	1
-2	0.050	16.027	0.041	0.052	0.294	1
-2	0.050	16.258	0.036	0.063	0.281	3
-2	0.051	17.249	0.028	0.112	0.210	3
-2	0.051	17.249	0.028	0.112	0.210	3
-2	0.051	17.290	0.028	0.116	0.206	3
-2	0.051	17.431	0.027	0.131	0.187	3
-2	0.051	17.436	0.027	0.131	0.187	3
-2	0.051	17.543	0.027	-	-	1

Angle (degrees)	V_{SL} (cm/s)	V_{SG} (m/s)	Liquid Holdup			Landman Result
			Low Solution	Middle Solution	High Solution	
-2	0.051	17.601	0.027	-	-	1
-2	0.051	18.072	0.025	-	-	1
-2	0.050	18.118	0.025	-	-	1
-2	0.051	18.133	0.025	-	-	1
-2	0.050	18.178	0.024	-	-	1
-2	0.051	19.164	0.022	-	-	1
-2	0.050	19.853	0.021	-	-	1
-2	0.050	20.076	0.020	-	-	1
-2	0.051	21.055	0.019	-	-	1
-2	0.051	23.127	0.017	-	-	1
-2	0.051	24.691	0.015	-	-	1
-2	0.059	9.771	0.548	-	-	1
-2	0.060	9.998	0.540	-	-	1
-2	0.060	10.068	0.537	-	-	1
-2	0.059	11.063	0.501	-	-	1
-2	0.060	11.865	0.470	-	-	1
-2	0.060	12.146	0.460	-	-	1
-2	0.059	12.673	0.440	-	-	1
-2	0.060	13.951	0.389	-	-	1
-2	0.059	14.180	0.379	-	-	1
-2	0.060	14.184	0.379	-	-	1
-2	0.059	15.875	0.302	-	-	1
-2	0.060	16.015	0.294	-	-	1
-2	0.060	16.230	0.283	-	-	1
-2	0.059	17.369	0.031	0.120	0.198	3
-2	0.059	17.491	0.030	0.137	0.178	3
-2	0.059	17.721	0.029	-	-	1
-2	0.060	18.112	0.028	-	-	1
-2	0.060	18.222	0.027	-	-	1
-2	0.059	19.185	0.024	-	-	1
-2	0.060	19.877	0.023	-	-	1
-2	0.060	20.120	0.022	-	-	1
-2	0.059	21.056	0.021	-	-	1
-2	0.059	23.105	0.018	-	-	1
-2	0.059	24.299	0.017	-	-	1
-2	0.072	9.775	0.548	-	-	1
-2	0.070	10.142	0.535	-	-	1
-2	0.070	10.178	0.533	-	-	1
-2	0.072	11.093	0.499	-	-	1

Angle (degrees)	V_{SL} (cm/s)	V_{SG} (m/s)	Liquid Holdup			Landman Result
			Low Solution	Middle Solution	High Solution	
-2	0.070	11.962	0.467	-	-	1
-2	0.070	12.347	0.452	-	-	1
-2	0.072	12.700	0.439	-	-	1
-2	0.070	13.911	0.391	-	-	1
-2	0.070	14.145	0.381	-	-	1
-2	0.072	14.186	0.379	-	-	1
-2	0.070	15.780	0.307	-	-	1
-2	0.072	15.894	0.301	-	-	1
-2	0.070	15.983	0.296	-	-	1
-2	0.072	17.248	0.037	0.105	0.212	3
-2	0.072	17.446	0.035	0.124	0.189	3
-2	0.072	17.547	0.034	0.144	0.166	3
-2	0.070	17.923	0.032	-	-	1
-2	0.070	17.932	0.032	-	-	1
-2	0.072	19.184	0.027	-	-	1
-2	0.070	19.999	0.025	-	-	1
-2	0.070	20.032	0.025	-	-	1
-2	0.072	21.076	0.023	-	-	1
-2	0.072	23.155	0.020	-	-	1
-2	0.072	24.470	0.019	-	-	1
-2	0.082	9.826	0.546	-	-	1
-2	0.080	10.178	0.533	-	-	1
-2	0.080	10.265	0.530	-	-	1
-2	0.082	11.191	0.496	-	-	1
-2	0.080	11.913	0.469	-	-	1
-2	0.080	12.039	0.464	-	-	1
-2	0.082	12.717	0.438	-	-	1
-2	0.080	13.967	0.388	-	-	1
-2	0.082	14.207	0.378	-	-	1
-2	0.080	14.208	0.378	-	-	1
-2	0.080	15.888	0.301	-	-	1
-2	0.082	15.940	0.298	-	-	1
-2	0.080	15.986	0.296	-	-	1
-2	0.082	17.313	0.040	0.106	0.207	3
-2	0.082	17.432	0.039	0.118	0.192	3
-2	0.082	17.681	0.037	-	-	1
-2	0.080	18.008	0.034	-	-	1
-2	0.080	18.011	0.034	-	-	1
-2	0.082	19.160	0.030	-	-	1

Angle (degrees)	V_{SL} (cm/s)	V_{SG} (m/s)	Liquid Holdup			Landman Result
			Low Solution	Middle Solution	High Solution	
-2	0.080	19.921	0.027	-	-	1
-2	0.080	20.157	0.027	-	-	1
-2	0.082	21.057	0.025	-	-	1
-2	0.082	23.116	0.022	-	-	1
-2	0.082	24.298	0.020	-	-	1
-2	0.091	9.865	0.545	-	-	1
-2	0.091	11.147	0.497	-	-	1
-2	0.091	12.724	0.438	-	-	1
-2	0.091	14.221	0.378	-	-	1
-2	0.091	15.963	0.298	-	-	1
-2	0.091	17.356	0.044	0.107	0.203	3
-2	0.091	17.419	0.043	0.113	0.195	3
-2	0.091	17.519	0.042	0.126	0.180	3
-2	0.091	17.596	0.041	-	-	1
-2	0.091	18.011	0.038	-	-	1
-2	0.091	19.231	0.031	-	-	1
-2	0.091	21.107	0.026	-	-	1
-2	0.091	23.239	0.023	-	-	1
-2	0.091	24.748	0.021	-	-	1

Table C.2 Calculation Results - Laminar Liquid Characterization

Angle (degrees)	V_{SL} (cm/s)	V_{SG} (m/s)	Liquid Holdup			Landman Result
			Low Solution	Middle Solution	High Solution	
-2	0.010	8.434	0.597	-	-	1
-2	0.010	9.510	0.558	-	-	1
-2	0.010	10.125	0.535	-	-	1
-2	0.010	10.198	0.532	-	-	1
-2	0.010	10.622	0.517	-	-	1
-2	0.010	11.882	0.006	0.021	0.470	3
-2	0.010	12.034	0.006	0.022	0.464	3
-2	0.010	12.228	0.006	0.024	0.456	3
-2	0.010	13.857	0.004	0.039	0.392	3
-2	0.010	13.937	0.004	0.040	0.389	3
-2	0.010	14.132	0.004	0.042	0.381	3
-2	0.010	15.440	0.004	0.068	0.322	3
-2	0.010	15.761	0.004	0.076	0.307	3
-2	0.010	15.835	0.004	0.076	0.303	3
-2	0.010	16.964	0.003	0.107	0.233	3
-2	0.010	17.826	0.010	-	-	1
-2	0.010	17.835	0.010	-	-	1
-2	0.010	18.798	0.009	-	-	1
-2	0.010	19.466	0.008	-	-	1
-2	0.010	19.828	0.008	-	-	1
-2	0.010	19.933	0.008	-	-	1
-2	0.010	20.908	0.008	-	-	1
-2	0.010	23.773	0.007	-	-	1
-2	0.019	9.864	0.545	-	-	1
-2	0.020	10.138	0.535	-	-	1
-2	0.020	10.277	0.529	-	-	1
-2	0.019	11.174	0.496	-	-	1
-2	0.020	11.900	0.469	-	-	1
-2	0.020	12.078	0.012	0.018	0.462	1
-2	0.019	12.803	0.009	0.026	0.434	3
-2	0.020	13.968	0.007	0.039	0.388	3
-2	0.020	14.205	0.007	0.042	0.378	3
-2	0.019	14.385	0.007	0.044	0.370	3
-2	0.020	15.801	0.006	0.076	0.305	3
-2	0.019	15.911	0.006	0.071	0.299	3
-2	0.020	15.977	0.006	0.073	0.295	3
-2	0.019	17.275	0.005	0.127	0.203	3

Angle (degrees)	V_{SL} (cm/s)	V_{SG} (m/s)	Liquid Holdup			Landman Result
			Low Solution	Middle Solution	High Solution	
-2	0.020	17.853	0.014	-	-	1
-2	0.020	17.969	0.014	-	-	1
-2	0.019	19.455	0.012	-	-	1
-2	0.020	19.845	0.012	-	-	1
-2	0.020	20.094	0.012	-	-	1
-2	0.019	21.258	0.011	-	-	1
-2	0.019	23.280	0.010	-	-	1
-2	0.019	24.670	0.009	-	-	1
-2	0.031	9.847	0.545	-	-	1
-2	0.030	10.009	0.539	-	-	1
-2	0.030	10.030	0.539	-	-	1
-2	0.031	11.084	0.500	-	-	1
-2	0.030	11.793	0.473	-	-	1
-2	0.030	12.090	0.462	-	-	1
-2	0.031	12.753	0.016	0.019	0.436	3
-2	0.030	13.936	0.010	0.037	0.389	3
-2	0.030	14.092	0.010	0.039	0.383	3
-2	0.031	14.416	0.009	0.043	0.369	3
-2	0.030	15.950	0.008	0.071	0.297	3
-2	0.031	16.090	0.008	0.074	0.289	3
-2	0.030	16.138	0.007	0.076	0.287	3
-2	0.031	17.267	0.007	0.125	0.205	3
-2	0.031	17.670	0.019	-	-	1
-2	0.031	17.683	0.019	-	-	1
-2	0.030	18.053	0.018	-	-	1
-2	0.030	18.134	0.018	-	-	1
-2	0.031	19.370	0.016	-	-	1
-2	0.030	19.670	0.016	-	-	1
-2	0.030	20.083	0.015	-	-	1
-2	0.031	21.274	0.014	-	-	1
-2	0.031	23.351	0.013	-	-	1
-2	0.031	24.753	0.012	-	-	1
-2	0.040	9.801	0.547	-	-	1
-2	0.040	9.958	0.541	-	-	1
-2	0.040	10.048	0.538	-	-	1
-2	0.040	11.090	0.500	-	-	1
-2	0.040	11.768	0.474	-	-	1
-2	0.040	12.174	0.459	-	-	1
-2	0.040	12.694	0.439	-	-	1

Angle (degrees)	V _{SL} (cm/s)	V _{SG} (m/s)	Liquid Holdup			Landman Result
			Low Solution	Middle Solution	High Solution	
-2	0.040	13.932	0.013	0.034	0.389	3
-2	0.040	14.136	0.012	0.037	0.381	3
-2	0.040	14.159	0.012	0.038	0.380	3
-2	0.040	15.902	0.009	0.069	0.300	3
-2	0.040	16.010	0.009	0.072	0.294	3
-2	0.040	16.213	0.009	0.077	0.283	3
-2	0.040	17.415	0.008	0.143	0.183	3
-2	0.040	17.467	0.023	-	-	1
-2	0.040	17.625	0.023	-	-	1
-2	0.040	18.095	0.021	-	-	1
-2	0.040	18.225	0.021	-	-	1
-2	0.040	19.200	0.019	-	-	1
-2	0.040	19.783	0.018	-	-	1
-2	0.040	20.076	0.018	-	-	1
-2	0.040	21.013	0.017	-	-	1
-2	0.040	23.096	0.015	-	-	1
-2	0.040	24.391	0.014	-	-	1
-2	0.051	9.565	0.555	-	-	1
-2	0.050	9.999	0.539	-	-	1
-2	0.050	10.039	0.538	-	-	1
-2	0.051	11.124	0.498	-	-	1
-2	0.050	11.826	0.472	-	-	1
-2	0.050	12.193	0.458	-	-	1
-2	0.051	12.763	0.436	-	-	1
-2	0.050	14.128	0.015	0.035	0.381	3
-2	0.051	14.205	0.015	0.036	0.378	3
-2	0.050	14.408	0.014	0.040	0.369	3
-2	0.051	15.885	0.011	0.067	0.301	3
-2	0.050	16.027	0.011	0.071	0.293	3
-2	0.050	16.258	0.010	0.077	0.280	3
-2	0.051	17.249	0.009	0.122	0.208	3
-2	0.051	17.249	0.009	0.122	0.208	3
-2	0.051	17.290	0.009	0.126	0.203	3
-2	0.051	17.431	0.009	0.144	0.180	3
-2	0.051	17.436	0.009	0.145	0.179	3
-2	0.051	17.543	0.027	-	-	1
-2	0.051	17.601	0.027	-	-	1
-2	0.051	18.072	0.025	-	-	1
-2	0.050	18.118	0.025	-	-	1

Angle (degrees)	V_{SL} (cm/s)	V_{SG} (m/s)	Liquid Holdup			Landman Result
			Low Solution	Middle Solution	High Solution	
-2	0.051	18.133	0.025	-	-	1
-2	0.050	18.178	0.024	-	-	1
-2	0.051	19.164	0.022	-	-	1
-2	0.050	19.853	0.021	-	-	1
-2	0.050	20.076	0.020	-	-	1
-2	0.051	21.055	0.019	-	-	1
-2	0.051	23.127	0.017	-	-	1
-2	0.051	24.691	0.015	-	-	1
-2	0.059	9.771	0.548	-	-	1
-2	0.060	9.998	0.540	-	-	1
-2	0.060	10.068	0.537	-	-	1
-2	0.059	11.063	0.501	-	-	1
-2	0.060	11.865	0.470	-	-	1
-2	0.060	12.146	0.460	-	-	1
-2	0.059	12.673	0.440	-	-	1
-2	0.060	13.951	0.022	0.027	0.389	3
-2	0.059	14.180	0.018	0.033	0.379	3
-2	0.060	14.184	0.018	0.033	0.379	3
-2	0.059	15.875	0.012	0.066	0.301	3
-2	0.060	16.015	0.012	0.070	0.294	3
-2	0.060	16.230	0.012	0.076	0.282	3
-2	0.059	17.369	0.010	0.133	0.193	3
-2	0.059	17.491	0.030	-	-	1
-2	0.059	17.721	0.029	-	-	1
-2	0.060	18.112	0.028	-	-	1
-2	0.060	18.222	0.027	-	-	1
-2	0.059	19.185	0.024	-	-	1
-2	0.060	19.877	0.023	-	-	1
-2	0.060	20.120	0.022	-	-	1
-2	0.059	21.056	0.021	-	-	1
-2	0.059	23.105	0.018	-	-	1
-2	0.059	24.299	0.017	-	-	1
-2	0.072	9.775	0.548	-	-	1
-2	0.070	10.142	0.535	-	-	1
-2	0.070	10.178	0.533	-	-	1
-2	0.072	11.093	0.499	-	-	1
-2	0.070	11.962	0.467	-	-	1
-2	0.070	12.347	0.452	-	-	1
-2	0.072	12.700	0.439	-	-	1

Angle (degrees)	V_{SL} (cm/s)	V_{SG} (m/s)	Liquid Holdup			Landman Result
			Low Solution	Middle Solution	High Solution	
-2	0.070	13.911	0.391	-	-	1
-2	0.070	14.145	0.381	-	-	3
-2	0.072	14.186	0.379	-	-	3
-2	0.070	15.780	0.014	0.063	0.306	3
-2	0.072	15.894	0.014	0.065	0.300	3
-2	0.070	15.983	0.014	0.068	0.296	3
-2	0.072	17.248	0.012	0.120	0.209	3
-2	0.072	17.446	0.012	0.144	0.179	3
-2	0.072	17.547	0.034	-	-	1
-2	0.070	17.923	0.032	-	-	1
-2	0.070	17.932	0.032	-	-	1
-2	0.072	19.184	0.027	-	-	1
-2	0.070	19.999	0.025	-	-	1
-2	0.070	20.032	0.025	-	-	1
-2	0.072	21.076	0.023	-	-	1
-2	0.072	23.155	0.020	-	-	1
-2	0.072	24.470	0.019	-	-	1
-2	0.082	9.826	0.546	-	-	1
-2	0.080	10.178	0.533	-	-	1
-2	0.080	10.265	0.530	-	-	1
-2	0.082	11.191	0.496	-	-	1
-2	0.080	11.913	0.469	-	-	1
-2	0.080	12.039	0.464	-	-	1
-2	0.082	12.717	0.438	-	-	1
-2	0.080	13.967	0.388	-	-	1
-2	0.082	14.207	0.378	-	-	1
-2	0.080	14.208	0.378	-	-	3
-2	0.080	15.888	0.016	0.064	0.301	3
-2	0.082	15.940	0.016	0.066	0.298	3
-2	0.080	15.986	0.015	0.067	0.295	3
-2	0.082	17.313	0.013	0.124	0.202	3
-2	0.082	17.432	0.013	0.139	0.184	3
-2	0.082	17.681	0.037	-	-	1
-2	0.080	18.008	0.034	-	-	1
-2	0.080	18.011	0.034	-	-	1
-2	0.082	19.160	0.030	-	-	1
-2	0.080	19.921	0.027	-	-	1
-2	0.080	20.157	0.027	-	-	1
-2	0.082	21.057	0.025	-	-	1

Angle (degrees)	V_{SL} (cm/s)	V_{SG} (m/s)	Liquid Holdup			Landman Result
			Low Solution	Middle Solution	High Solution	
-2	0.082	23.116	0.022	-	-	1
-2	0.082	24.298	0.020	-	-	1
-2	0.091	9.865	0.545	-	-	1
-2	0.091	11.147	0.497	-	-	1
-2	0.091	12.724	0.438	-	-	1
-2	0.091	14.221	0.378	-	-	1
-2	0.091	15.963	0.017	0.065	0.297	3
-2	0.091	17.356	0.014	0.128	0.197	3
-2	0.091	17.419	0.014	0.136	0.187	3
-2	0.091	17.519	0.042	-	-	1
-2	0.091	17.596	0.041	-	-	1
-2	0.091	18.011	0.038	-	-	1
-2	0.091	19.231	0.031	-	-	1
-2	0.091	21.107	0.026	-	-	1
-2	0.091	23.239	0.023	-	-	1
-2	0.091	24.748	0.021	-	-	1

Table C.3 Kelvin-Helmholtz Stability Results - Turbulent Liquid Characterization

Angle (degrees)	V_{SL} (cm/s)	V_{SG} (m/s)	IKH Stability			VKH Stability		
			Low Solution	Middle Solution	High Solution	Low Solution	Middle Solution	High Solution
-2	0.010	13.857	stable	stable	unstable	stable	stable	unstable
-2	0.010	13.937	stable	stable	unstable	stable	stable	unstable
-2	0.010	14.132	stable	stable	unstable	stable	stable	unstable
-2	0.010	15.440	stable	stable	unstable	stable	stable	unstable
-2	0.010	15.761	stable	stable	unstable	stable	stable	unstable
-2	0.010	15.835	stable	stable	unstable	stable	stable	unstable
-2	0.010	16.964	stable	unstable	unstable	stable	unstable	unstable
-2	0.020	15.801	stable	stable	unstable	stable	stable	unstable
-2	0.019	15.911	stable	stable	unstable	stable	stable	unstable
-2	0.020	15.977	stable	stable	unstable	stable	stable	unstable
-2	0.019	17.275	stable	unstable	unstable	stable	unstable	unstable
-2	0.030	15.950	stable	stable	unstable	stable	stable	unstable
-2	0.031	16.090	stable	stable	unstable	stable	stable	unstable
-2	0.030	16.138	stable	stable	unstable	stable	stable	unstable
-2	0.031	17.267	stable	unstable	unstable	stable	unstable	unstable
-2	0.040	15.902	stable	stable	unstable	stable	stable	unstable
-2	0.040	16.010	stable	stable	unstable	stable	stable	unstable
-2	0.040	16.213	stable	stable	unstable	stable	stable	unstable
-2	0.040	17.415	stable	unstable	unstable	stable	unstable	unstable
-2	0.040	17.467	stable	unstable	unstable	stable	unstable	unstable
-2	0.050	16.027	stable	stable	unstable	stable	stable	unstable
-2	0.050	16.258	stable	stable	unstable	stable	stable	unstable
-2	0.051	17.249	stable	unstable	unstable	stable	unstable	unstable
-2	0.051	17.249	stable	unstable	unstable	stable	unstable	unstable
-2	0.051	17.290	stable	unstable	unstable	stable	unstable	unstable
-2	0.051	17.431	stable	unstable	unstable	stable	unstable	unstable
-2	0.051	17.436	stable	unstable	unstable	stable	unstable	unstable
-2	0.059	17.369	stable	unstable	unstable	stable	unstable	unstable
-2	0.059	17.491	stable	unstable	unstable	stable	unstable	unstable
-2	0.072	17.248	stable	unstable	unstable	stable	unstable	unstable
-2	0.072	17.446	stable	unstable	unstable	stable	unstable	unstable
-2	0.072	17.547	stable	unstable	unstable	stable	unstable	unstable
-2	0.082	17.313	stable	unstable	unstable	stable	unstable	unstable
-2	0.082	17.432	stable	unstable	unstable	stable	unstable	unstable
-2	0.091	17.356	stable	unstable	unstable	stable	unstable	unstable
-2	0.091	17.419	stable	unstable	unstable	stable	unstable	unstable
-2	0.091	17.519	stable	unstable	unstable	stable	unstable	unstable

Table C.4 Kelvin-Helmholtz Stability Results - Laminar Liquid Characterization

Angle (degrees)	V_{SL} (cm/s)	V_{SG} (m/s)	IKH Stability			VKH Stability		
			Low Solution	Middle Solution	High Solution	Low Solution	Middle Solution	High Solution
-2	0.010	11.882	stable	stable	unstable	stable	stable	unstable
-2	0.010	12.034	stable	stable	unstable	stable	stable	unstable
-2	0.010	12.228	stable	stable	unstable	stable	stable	unstable
-2	0.010	13.857	stable	stable	unstable	stable	stable	unstable
-2	0.010	13.937	stable	stable	unstable	stable	stable	unstable
-2	0.010	14.132	stable	stable	unstable	stable	stable	unstable
-2	0.010	15.440	stable	stable	unstable	stable	stable	unstable
-2	0.010	15.761	stable	stable	unstable	stable	stable	unstable
-2	0.010	15.835	stable	stable	unstable	stable	stable	unstable
-2	0.010	16.964	stable	unstable	unstable	stable	unstable	unstable
-2	0.020	12.078	stable	stable	unstable	stable	stable	unstable
-2	0.019	12.803	stable	stable	unstable	stable	stable	unstable
-2	0.020	13.968	stable	stable	unstable	stable	stable	unstable
-2	0.020	14.205	stable	stable	unstable	stable	stable	unstable
-2	0.019	14.385	stable	stable	unstable	stable	stable	unstable
-2	0.020	15.801	stable	stable	unstable	stable	stable	unstable
-2	0.019	15.911	stable	stable	unstable	stable	stable	unstable
-2	0.020	15.977	stable	stable	unstable	stable	stable	unstable
-2	0.019	17.275	stable	unstable	unstable	stable	unstable	unstable
-2	0.031	12.753	stable	stable	unstable	stable	stable	unstable
-2	0.030	13.936	stable	stable	unstable	stable	stable	unstable
-2	0.030	14.092	stable	stable	unstable	stable	stable	unstable
-2	0.031	14.416	stable	stable	unstable	stable	stable	unstable
-2	0.030	15.950	stable	stable	unstable	stable	stable	unstable
-2	0.031	16.090	stable	stable	unstable	stable	stable	unstable
-2	0.030	16.138	stable	stable	unstable	stable	stable	unstable
-2	0.031	17.267	stable	unstable	unstable	stable	unstable	unstable
-2	0.040	13.932	stable	stable	unstable	stable	stable	unstable
-2	0.040	14.136	stable	stable	unstable	stable	stable	unstable
-2	0.040	14.159	stable	stable	unstable	stable	stable	unstable
-2	0.040	15.902	stable	stable	unstable	stable	stable	unstable
-2	0.040	16.010	stable	stable	unstable	stable	stable	unstable
-2	0.040	16.213	stable	stable	unstable	stable	stable	unstable
-2	0.040	17.415	stable	unstable	unstable	stable	unstable	unstable
-2	0.050	14.128	stable	stable	unstable	stable	stable	unstable
-2	0.051	14.205	stable	stable	unstable	stable	stable	unstable
-2	0.050	14.408	stable	stable	unstable	stable	stable	unstable

Angle (degrees)	V_{SL} (cm/s)	V_{SG} (m/s)	IKH Stability			VKH Stability		
			Low Solution	Middle Solution	High Solution	Low Solution	Middle Solution	High Solution
-2	0.051	15.885	stable	stable	unstable	stable	stable	unstable
-2	0.050	16.027	stable	stable	unstable	stable	stable	unstable
-2	0.050	16.258	stable	stable	unstable	stable	stable	unstable
-2	0.051	17.249	stable	unstable	unstable	stable	unstable	unstable
-2	0.051	17.249	stable	unstable	unstable	stable	unstable	unstable
-2	0.051	17.290	stable	unstable	unstable	stable	unstable	unstable
-2	0.051	17.431	stable	unstable	unstable	stable	unstable	unstable
-2	0.051	17.436	stable	unstable	unstable	stable	unstable	unstable
-2	0.060	13.951	stable	stable	unstable	stable	stable	unstable
-2	0.059	14.180	stable	stable	unstable	stable	stable	unstable
-2	0.060	14.184	stable	stable	unstable	stable	stable	unstable
-2	0.059	15.875	stable	stable	unstable	stable	stable	unstable
-2	0.060	16.015	stable	stable	unstable	stable	stable	unstable
-2	0.060	16.230	stable	stable	unstable	stable	stable	unstable
-2	0.059	17.369	stable	unstable	unstable	stable	unstable	unstable
-2	0.070	15.780	stable	stable	unstable	stable	stable	unstable
-2	0.072	15.894	stable	stable	unstable	stable	stable	unstable
-2	0.070	15.983	stable	stable	unstable	stable	stable	unstable
-2	0.072	17.248	stable	unstable	unstable	stable	unstable	unstable
-2	0.072	17.446	stable	unstable	unstable	stable	unstable	unstable
-2	0.080	15.888	stable	stable	unstable	stable	stable	unstable
-2	0.082	15.940	stable	stable	unstable	stable	stable	unstable
-2	0.080	15.986	stable	stable	unstable	stable	stable	unstable
-2	0.082	17.313	stable	unstable	unstable	stable	unstable	unstable
-2	0.082	17.432	stable	unstable	unstable	stable	unstable	unstable
-2	0.091	15.963	stable	stable	unstable	stable	stable	unstable
-2	0.091	17.356	stable	unstable	unstable	stable	unstable	unstable
-2	0.091	17.419	stable	unstable	unstable	stable	unstable	unstable

Table C.5 Structural Stability Results - Turbulent Liquid Characterization

Angle (degrees)	V_{SL} (cm/s)	V_{SG} (m/s)	Linear Stability			Non-Linear Stability
			Low Solution	Middle Solution	High Solution	
-2	0.010	13.857	unstable	unstable	stable	unstable
-2	0.010	13.937	stable	unstable	stable	unstable
-2	0.010	14.132	stable	unstable	stable	unstable
-2	0.010	15.440	stable	unstable	stable	unstable
-2	0.010	15.761	stable	unstable	stable	unstable
-2	0.010	15.835	stable	unstable	stable	unstable
-2	0.010	16.964	stable	unstable	stable	unstable
-2	0.020	15.801	stable	unstable	stable	unstable
-2	0.019	15.911	stable	unstable	stable	unstable
-2	0.020	15.977	stable	unstable	stable	unstable
-2	0.019	17.275	stable	unstable	stable	unstable
-2	0.030	15.950	stable	unstable	stable	unstable
-2	0.031	16.090	stable	unstable	stable	unstable
-2	0.030	16.138	stable	unstable	stable	unstable
-2	0.031	17.267	stable	unstable	stable	unstable
-2	0.040	15.902	unstable	unstable	stable	unstable
-2	0.040	16.010	unstable	unstable	stable	unstable
-2	0.040	16.213	unstable	unstable	stable	unstable
-2	0.040	17.415	stable	unstable	stable	unstable
-2	0.040	17.467	stable	unstable	unstable	unstable
-2	0.050	16.027	unstable	unstable	stable	unstable
-2	0.050	16.258	unstable	unstable	stable	unstable
-2	0.051	17.249	stable	unstable	stable	unstable
-2	0.051	17.249	stable	unstable	stable	unstable
-2	0.051	17.290	stable	unstable	stable	unstable
-2	0.051	17.431	stable	unstable	stable	unstable
-2	0.051	17.436	stable	unstable	stable	unstable
-2	0.059	17.369	stable	unstable	stable	unstable
-2	0.059	17.491	stable	unstable	unstable	unstable
-2	0.072	17.248	unstable	unstable	stable	unstable
-2	0.072	17.446	stable	unstable	stable	unstable
-2	0.072	17.547	stable	unstable	unstable	unstable
-2	0.082	17.313	unstable	unstable	stable	unstable
-2	0.082	17.432	unstable	unstable	stable	unstable
-2	0.091	17.356	unstable	unstable	stable	unstable
-2	0.091	17.419	unstable	unstable	stable	unstable
-2	0.091	17.519	unstable	unstable	stable	unstable

Table C.6 Structural Stability Results - Laminar Liquid Characterization

Angle (degrees)	V_{SL} (cm/s)	V_{SG} (m/s)	Linear Stability			Non-Linear Stability
			Low Solution	Middle Solution	High Solution	
-2	0.010	11.882	stable	unstable	stable	unstable
-2	0.010	12.034	stable	unstable	stable	unstable
-2	0.010	12.228	stable	unstable	stable	unstable
-2	0.010	13.857	stable	unstable	stable	unstable
-2	0.010	13.937	stable	unstable	stable	unstable
-2	0.010	14.132	stable	unstable	stable	unstable
-2	0.010	15.440	stable	unstable	stable	unstable
-2	0.010	15.761	stable	unstable	stable	unstable
-2	0.010	15.835	stable	unstable	stable	unstable
-2	0.010	16.964	stable	unstable	stable	unstable
-2	0.020	12.078	stable	stable	stable	unstable
-2	0.019	12.803	stable	unstable	stable	unstable
-2	0.020	13.968	stable	unstable	stable	unstable
-2	0.020	14.205	stable	unstable	stable	unstable
-2	0.019	14.385	stable	unstable	stable	unstable
-2	0.020	15.801	stable	unstable	stable	unstable
-2	0.019	15.911	stable	unstable	stable	unstable
-2	0.020	15.977	stable	unstable	stable	unstable
-2	0.019	17.275	stable	unstable	stable	unstable
-2	0.031	12.753	stable	stable	stable	unstable
-2	0.030	13.936	stable	unstable	stable	unstable
-2	0.030	14.092	stable	unstable	stable	unstable
-2	0.031	14.416	stable	unstable	stable	unstable
-2	0.030	15.950	stable	unstable	stable	unstable
-2	0.031	16.090	stable	unstable	stable	unstable
-2	0.030	16.138	stable	unstable	stable	unstable
-2	0.031	17.267	stable	unstable	stable	unstable
-2	0.040	13.932	stable	unstable	stable	unstable
-2	0.040	14.136	stable	unstable	stable	unstable
-2	0.040	14.159	stable	unstable	stable	unstable
-2	0.040	15.902	stable	unstable	stable	unstable
-2	0.040	16.010	stable	unstable	stable	unstable
-2	0.040	16.213	stable	unstable	stable	unstable
-2	0.040	17.415	stable	unstable	stable	unstable
-2	0.050	14.128	stable	stable	stable	unstable
-2	0.051	14.205	stable	stable	stable	unstable
-2	0.050	14.408	stable	unstable	stable	unstable

Angle (degrees)	V_{SL} (cm/s)	V_{SG} (m/s)	Linear Stability			Non-Linear Stability
			Low Solution	Middle Solution	High Solution	
-2	0.051	15.885	stable	unstable	stable	unstable
-2	0.050	16.027	stable	unstable	stable	unstable
-2	0.050	16.258	stable	unstable	stable	unstable
-2	0.051	17.249	stable	unstable	stable	unstable
-2	0.051	17.249	stable	unstable	stable	unstable
-2	0.051	17.290	stable	unstable	stable	unstable
-2	0.051	17.431	stable	unstable	unstable	unstable
-2	0.051	17.436	stable	unstable	unstable	unstable
-2	0.060	13.951	stable	stable	stable	unstable
-2	0.059	14.180	stable	stable	stable	unstable
-2	0.060	14.184	stable	stable	stable	unstable
-2	0.059	15.875	stable	unstable	stable	unstable
-2	0.060	16.015	stable	unstable	stable	unstable
-2	0.060	16.230	stable	unstable	stable	unstable
-2	0.059	17.369	stable	unstable	stable	unstable
-2	0.070	15.780	stable	unstable	stable	unstable
-2	0.072	15.894	stable	unstable	stable	unstable
-2	0.070	15.983	stable	unstable	stable	unstable
-2	0.072	17.248	stable	unstable	stable	unstable
-2	0.072	17.446	stable	unstable	unstable	unstable
-2	0.080	15.888	stable	unstable	stable	unstable
-2	0.082	15.940	stable	unstable	stable	unstable
-2	0.080	15.986	stable	unstable	stable	unstable
-2	0.082	17.313	stable	unstable	stable	unstable
-2	0.082	17.432	stable	unstable	stable	unstable
-2	0.091	15.963	stable	unstable	stable	unstable
-2	0.091	17.356	stable	unstable	stable	unstable
-2	0.091	17.419	stable	unstable	stable	unstable