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Gravity Wave Diagnosis Using Empirical Normal Modes

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in partial fulfillment of the requirements for the degree of
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avec la pauvreté natale de ma pensée rocheuse

j'avance en poésie comme un cheval de trait

—*Gaston Miron*

Abstract

We adapt the theory of Empirical Normal Modes (ENMs) to diagnose gravity waves generated by a relatively high resolution numerical model solving the primitive equations. The ENM approach is based on the Principal Component Analysis (which consists of finding the most efficient basis explaining the variance of a time series), except that it takes advantage of wave-activity conservation laws. In the present work, the small-amplitude version of the pseudoenergy is used to extract from data quasi-monochromatic three-dimensional empirical modes that describe atmospheric wave activity. The spatial distributions of these quasi-monochromatic modes are identical to the normal modes of the linearized primitive equations when the underlying dynamics can be described with a stochastic linear and forced model, thus establishing a bridge between statistics and dynamics. We use this diagnostic method to study inertia-gravity wave generation, propagation, transience, and breaking over the Rockies, the North Pacific, and Central America in the troposphere-stratosphere-mesosphere GFDL SKYHI general circulation model at a resolution of 1° of latitude by 1.2° of longitude. Besides the action of mountains in exciting orographic waves, inertia-gravity wave activity has been found to be generated at the jet stream level as a possible consequence of a sustained nonlinear and ageostrophic flow. In the Tropical region of the model, the “obstacle effect” has been found to be the major source of inertia-gravity waves. A significant proportion of these inertia-gravity waves was able to reach the model mesosphere without much dissipation and absorption.

Résumé

La théorie des modes normaux empiriques (MNE) est adaptée et utilisée pour procéder au diagnostic d'ondes de gravité simulées par un modèle de circulation générale de l'atmosphère moyenne à résolution relativement élevée. La méthode des MNE s'appuie sur l'analyse en composantes principales (ACP) (qui consiste à trouver une base reproduisant le plus efficacement possible la variance d'un signal) tout en se servant des lois de conservation de l'activité ondulatoire. Le présent travail vise à utiliser l'approximation au premier ordre de la pseudo-énergie pour identifier des modes empiriques tri-dimensionnels et quasi-monochromatiques pouvant être identifiés comme étant des ondes atmosphériques. Les distributions spatiales de ces modes quasi-monochromatiques sont identiques aux modes normaux des équations primitives si la dynamique peut être décrite par un modèle stochastique linéaire et forcé. Il en résulte que cette méthode statistique s'approche du caractère dynamique de l'évolution du système. Elle a été ici employée pour étudier la génération, la propagation, les effets transitoires et le déferlement des ondes de gravité près des Rocheuses, dans la région du Pacifique nord ainsi qu'en Amérique centrale tels que simulés par le modèle de circulation troposphérique-stratosphérique-mésosphérique GFDL SKYHI à la résolution de 1° de latitude par 1.2° de longitude. En plus de l'effet des montagnes générant des ondes de relief, le jet-stream semble agir comme une source d'ondes de gravité inertielles par la non-linéarité et le caractère agéostrophique de l'écoulement. Dans la région tropicale du modèle, "l'effet obstacle" apparaît comme étant la principale source d'ondes de gravité inertielles. Une partie significative de ces ondes a pu atteindre la mésosphère du modèle sans absorption ou dissipation.

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Statement of Originality

The contributions to original knowledge are as follows:

- The theory of Empirical Normal Mode has been developed and applied for the first time to the primitive equations for diagnosing gravity waves using the bilinear form of the pseudoenergy.
- It has been shown that the Principal Oscillation Patterns (POPs) are the Fourier modes when the time sampling of a time series is much more reduced than the space sampling.
- The contribution of the lower boundary to the pseudoenergy in isentropic coordinates has been obtained for the case of a zonally symmetric basic state.
- The impact of stochastic forcing and Rayleigh damping in the dynamics on the empirical modes has been found to affect only the temporal part (the principal components) of the modes.
- A quantitative estimation of the importance of the GFDL SKYHI middle atmospheric forcing in winter by propagating perturbations has been calculated over restricted areas for waves with specific frequencies.
- Different locations of gravity wave sources in the GFDL SKYHI model has been analyzed, and the generated wave forcing has been quantified and discussed.

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Rocky Mountains, the mid-latitude Pacific Ocean, and Central America, respectively. Figures b, d, f, h, j, l, n, p, r, and t are the estimated time tendencies. The arrows pointing to the right at the bottom left corner of the graphs indicate the length an arrow would have if the period of the pair were: b) 60 hours; d) 4.1 hours; f) 3.5 hours; h) 20 hours; j) 3.5 hours; l) 60 hours; n) 12 hours; p) 4 hours; r) 12 hours; and t) 12 hours. Note that the tendencies are estimated with the help of the graph located immediately to its left.

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Table 1a. The first two columns represent the mode number (ordered by their variance), T is the period of the peak of the principal components, α is their relative phase lag, and $K_{ij}/\bar{K}$ is a measure of the fluctuation of the amplitude of the propagating pair (see text). These numbers are obtained from an analysis of the region of the Rockies from 618 hPa to 3.9 hPa.

Table 1b. The same as in Table 1a, except that the analysis is made from 62 hPa to 0.03 hPa.

Table 1c. The same as in Table 1a, except that the analysis is made over the mid-latitude Pacific Ocean from 618 hPa to 3.9 hPa.

Table 1d. The same as in Table 1c, except that the analysis is made from 62 hPa to 0.03 hPa.

Table 1e. The same as in Table 1a, except that the analysis is made over Central America from 618 hPa to 3.9 hPa.

Table 1f. The same as in Table 1e, except that the analysis is made from 62 hPa to 0.03 hPa.

Chapter 1

General Introduction and Thesis Purpose

The increasing concern about ozone depletion and its impact on life stimulated the study of the dynamical, radiative, and photochemical behavior of the stratosphere and mesosphere (the middle atmosphere, see Fig. 1). The stratospheric layer is the locus of the maximum ozone concentration and mixing ratio (Figs. 2a,b), while the mesosphere and the troposphere interact strongly with the stratosphere through dynamical, radiative, and chemical processes: waves originating from the troposphere can propagate to the stratosphere and mesosphere, pollutants and aerosols can be transported from the troposphere into the stratosphere and influence its radiative budget, dynamical processes occurring in the mesosphere can greatly influence the state of the stratosphere, and so on. An adequate understanding of the main forcings on ozone concentrations in the stratosphere must consequently incorporate all three layers, and the state and evolution (in a dynamical, radiative, and chemical sense) of these layers must be well characterized. More fundamentally, atmospheric scientists are trying to establish the mechanisms (and their climatology) responsible for the actual mean state of the middle atmosphere and for the occurrence of intermittent and quasi-periodic phenomena (e.g., stratospheric warmings and the quasi-biennial oscillation). A common practice when studying the atmospheric behavior is to separate the variables of interest into mean and eddy (or wave) components. The study of atmospheric waves of horizontal scales ranging from a few kilometers up to many thousand kilometers and their impact on the mean state through nonlinear interactions is a wide and challenging topic in which a fundamental breakthrough has been made in the seventies when generalized theorems were established. More details on these couplings and a more precise picture of the climatological state of the middle

Figure 1. Mid-latitude temperature profile. Based on the *U.S. Standard Atmosphere* (1976).

[From Andrews *et al.* (1987).]

Figure 2a. Mid-latitude standard ozone concentration profile (molecules m^{-3}). Horizontal bars show the standard deviation about the mean for observed profiles. [From Andrews *et al.* (1987).]

Figure 2b. The standard ozone profile of Fig. 2a plotted in terms of the mass mixing ratio. [From Andrews *et al.* (1987).]

atmosphere will be given in the next chapter.

A class of atmospheric waves which has shown to be of primary importance in the middle atmosphere is the one produced by the vertical density stratification due to gravity. These (inertia-) gravity waves are mainly initiated in the tropospheric region and they propagate mostly upward into the stratosphere and mesosphere where they break and/or are absorbed. The breaking and absorption of gravity waves in the middle atmosphere is a major forcing on the mean flow, and a precise knowledge of the breaking and absorption dynamics would be of great interest. Moreover, some gravity wave generation mechanisms and their relative strength are still ambiguous. The gravity (or buoyancy) wave field in the middle atmosphere being of relatively small horizontal scales (a few kilometers up to ~ 1000 km), their characteristics (in particular their horizontal wavelengths) are difficult to measure directly in the real atmosphere.

In order to anticipate the response of the ozone layer (or any other trace gases in the middle atmosphere) to anthropogenic forcing and/or to describe and explain the natural evolution and climatological state of the middle atmosphere, numerical models of various complexity have been built to reproduce its main features. A global general circulation model (GCM) with grid spacing of a few hundred kilometers can hardly resolve the peak of the gravity wave spectrum, and some gravity wave parameterization is often needed in order to reproduce their impact on the middle atmospheric mean state. A better knowledge of the generation, propagation, absorption, and breaking mechanisms would help in establishing a gravity wave parameterization in numerical models.

This thesis will deal with the problem of isolating gravity wave processes in time evolving and three-dimensional meteorological field datasets without pre-filtering, and of extracting gravity wave modes that are, as much as possible, solutions of the dynamical equations governing the evolution of these fields. Since real and global atmospheric data are too sparse and unavailable at scales of interest, one has to rely on relatively high resolution numerical models based on the primitive

equations on a rotating sphere in order to simulate the gravity wave field. A wave analysis technique based on conservation laws, which will be described in chapter 3, will be applied in order to characterize the gravity wave modes and their role in the atmospheric circulation.

The thesis is organized as follows: chapter 2 presents different concepts that will be used throughout the rest of the work, such as polarization relations, critical levels, and gravity wave excitation mechanisms; chapter 3 exposes the theory of Empirical Normal Modes (including the impact of stochasticity and damping); and in chapter 4, results of the application of the method on the output of the GFDL SKYHI model are presented and analyzed.

Chapter 2

Fundamentals

2.1 Some Properties of Inertia-Gravity Waves

This section presents the basic concepts and results about inertia-gravity waves that will be used throughout this work. It briefly summarizes what is known theoretically, experimentally, and numerically about inertia-gravity waves and their interactions with the mean flow.

2.1.1 A reduced set of equations and the dispersion relation

When a fluid is subject to the force of gravity, a density stratification is generated. The buoyancy and Coriolis forces are the restoring mechanisms responsible for the existence of internal inertia-gravity (or inertia-buoyancy) waves. Consider the hydrostatic primitive equations in log-pressure coordinates ($z \equiv -H \ln(p/p_*)$, where H is the constant scale height of the atmosphere, and p_* is a constant reference pressure level) reduced using the f -plane approximation. If the equations are linearized in the following way: $u = U + u'$, $v = v'$, $w = w'$, $\phi = \Phi(z) + \phi'$, and $\theta = \Theta(z) + \theta'$, we get

$$\begin{aligned}
 Du' - f_0 v' &= -\frac{\partial \phi'}{\partial x} \\
 Dv' + f_0 u' &= -\frac{\partial \phi'}{\partial y} \\
 \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{1}{\rho_0} \frac{\partial}{\partial z} (\rho_0 w') &= 0 \\
 D\theta' + w' \frac{\partial \Theta}{\partial z} &= 0 \\
 \frac{\partial \phi'}{\partial z} &= \frac{R}{H} \theta' e^{-\kappa z/H},
 \end{aligned} \tag{2.1}$$

where (u, v, w) are the velocities in the (x, y, z) direction, ϕ is the geopotential, θ is the potential temperature, $D \equiv \frac{\partial}{\partial t} + U \frac{\partial}{\partial x}$, $R = 287 \text{ (J/K)/kg}$ is the gas constant, $H \sim 7 \text{ km}$

is the scale height of the atmosphere, $\kappa = 2/7$, and $\rho_0 = \rho_s e^{-z/H}$ (ρ_s is the constant surface density). If the buoyancy frequency N is defined by

$$N^2 = \frac{R}{H} \frac{\partial \Theta}{\partial z} e^{-\kappa z/H}, \quad (2.2)$$

and if we solve for w' , we easily get the following equation:

$$(D^2 + f_0^2) \frac{\partial}{\partial z} \left(\frac{1}{\rho_0} \frac{\partial}{\partial z} (\rho_0 w') \right) + N^2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) w' = 0. \quad (2.3)$$

We will seek for a solution of the form $w' = \hat{w} e^{z/2H} e^{i(kx + ly + mz - \omega t)}$. A dispersion relation is thus obtained

$$\hat{\omega}^2 \equiv (\omega - kU)^2 = f_0^2 + \frac{N^2(k^2 + l^2)}{m^2 + 1/4H^2}. \quad (2.4)$$

Internal inertia-gravity waves can thus propagate in the horizontal as well as in the vertical direction. It can be verified that the phase propagation is perpendicular to the energy propagation or

$$(k, l, m) \cdot \left(\frac{\partial \hat{\omega}}{\partial k}, \frac{\partial \hat{\omega}}{\partial l}, \frac{\partial \hat{\omega}}{\partial m} \right) \equiv (k, l, m) \cdot (c_g^{(x)}, c_g^{(y)}, c_g^{(z)}) = 0$$

when the Boussinesq approximation ($m^2 \gg 1/4H^2$) is used, and that when energy propagates upwards and eastwards ($c_g^{(z)} > 0$ and $c_g^{(x)} > 0$), the phase line must tilt eastward in a vertical cut. Gravity waves thus have the property that phase and group propagation are in opposite directions (resp. the same direction) when projected on the vertical (resp. horizontal).

Note also that Lamb waves are solutions of Eq. 2.1 with the additional condition of no vertical motion in geometrical space:

$$D\phi' + w' \frac{R}{H} \Theta e^{-\kappa z/H} = 0.$$

The phase (and group) velocity of Lamb waves is calculated to be $c_s = 7HN/\sqrt{10}$ when $f_0 = 0$.

2.1.2 Polarization relations and hodographs

In order to be able to detect internal inertia-gravity waves in the atmosphere or the ocean when data are available at a single location (but with many points in the vertical direction), a knowledge of the polarization relations (i.e., the way different variables are related to each other) is necessary. These are simply obtained by taking a solution of the form of a progressive plane wave. As an example, let us consider the relation between u' and v' after a rotation has been applied to the coordinate system so that the positive x -axis is oriented along the direction of propagation of the wave (this means that $\hat{\omega} > 0$ and $l = 0$). Eq. 2.1 then gives

$$i\hat{\omega}v' = f_0u'. \quad (2.5)$$

Assuming a specific form for u' and v' :

$$\begin{aligned} u' &= e^{z/2H} |\hat{u}| e^{i\delta} e^{i(kz+mz-\omega t)} \\ v' &= e^{z/2H} |\hat{v}| e^{i(kz+mz-\omega t)}, \end{aligned} \quad (2.6)$$

and using the real and imaginary component of Eq. 2.5, we get $\delta = \pi/2$ ($-\pi/2$) when $f_0 > 0$ (< 0). If we assume that $f_0 > 0$, and if we choose a specific location ($x = 0$) and time ($t = 0$), taking the real part of Eq. 2.6 will give

$$\begin{aligned} u' &= -e^{z/2H} |\hat{u}| \sin(mz) \\ v' &= \frac{f_0}{\hat{\omega}} e^{z/2H} |\hat{u}| \cos(mz). \end{aligned} \quad (2.7)$$

Once the axes have been put to their original angle, a plot of (u', v') (with z as a parameter) will produce a spiral for which the ratio of the major and minor axes is $\hat{\omega}/f_0$. If energy propagates upwards ($m < 0$), the rotation will be clockwise as z increases (see Fig. 3). This is the most common technique when one is interested in measuring the intrinsic frequency and the direction of propagation of internal inertia-gravity waves.

Other polarization relations which are of interest are the following (the Boussi-

Figure 3. The hodograph of an idealized upward propagating monochromatic gravity wave.

nesq approximation has been used):

$$\begin{aligned}
 \frac{u'}{v'} &= \frac{\hat{\omega}k + if_0l}{\hat{\omega}l - if_0k} \\
 \frac{u'}{\phi'} &= \frac{\hat{\omega}k + if_0l}{\hat{\omega}^2 - f_0^2} \\
 \frac{u'}{T'_z} &= \frac{-R(\hat{\omega}k + if_0l)}{Hm^2(\hat{\omega}^2 - f_0^2)} \\
 \frac{u'}{w'} &= \frac{-m(\hat{\omega}k + if_0l)}{\hat{\omega}(k^2 + l^2)}.
 \end{aligned} \tag{2.8}$$

The last one will especially be illuminating when we will consider the problem of gravity wave-mean flow interactions.

2.1.3 Gravity wave generation mechanisms

In the atmosphere and the ocean, gravity waves are ubiquitous. So far, four generation mechanisms have been identified as sources of gravity and inertia-gravity waves in geophysical fluids. The most obvious generating process is the effect of irregular topography. A force is applied by the ground on the fluid (and vice versa) when the flow crosses orographic irregularities and energy is exchanged in the form of gravity wave emission. Another source of gravity waves is attributed to Richardson instabilities of the basic flow. Numerical models have shown that in some favorable conditions, such instabilities can lead to gravity wave emission into the far field (i.e., away from the locus of emission) (Lott *et al.*, 1992; Sutherland *et al.*, 1994). It is thought that an unstable shear layer can sometimes excite and radiate neutral (in terms of stability) and propagating gravity modes. A third mechanism, attributed to convective instabilities, is thought to be prevailing in Tropical regions. Two different dynamical processes are known to radiate gravity waves due to convection: squall-lines and individual ascending convective cells. Away from the Tropics, geostrophic adjustment can also generate inertia-gravity waves when a geostrophically unbalanced atmosphere tends to attain geostrophy by radiating energy. Also, at relatively high Rossby numbers, a nonlinear transfer from rotational (Rossby) modes to smaller wave (gravity) modes can occur without any geostrophic adjustment. In what follows, each one of these mechanisms will be briefly discussed and results obtained from theoretical and numerical models, and from measurements will be presented.

2.1.3.1 Mountain waves

When air flows over a mountain, a pressure gradient is created across the mountain, and momentum (and energy) must be exchanged between the overlying fluid and the ground. Usually, the pressure is greater on the upwind side and lower on the downwind side of the mountain. Thus, air exerts a net force on the ground, and conversely, a drag is felt by the flow. We will consider the problem of evaluating the height at which this drag is felt in a subsequent section. For now, suffice to say that the effect is nonlocal due to the ability of wave to transport horizontal momentum to great heights.

Queney (1948) studied the response of the air flow to the presence of an isolated mountain. Bretherton (1969) evaluated that the vertical wave momentum flux produced by mountains in North Wales amounted to 0.4 N m^{-2} when the crossing basic wind was 19 m s^{-1} . This wave momentum was dissipated at levels higher than 20 km. The details of the theory of mountain waves can be found in many textbooks (e.g., Gill, 1982; Baines, 1995) and we shall not discuss it here.

2.1.3.2 Instabilities as a source of gravity waves

General circulation models of the middle atmosphere have a general tendency to overestimate the wind velocity in the stratosphere and mesosphere. The problem is especially obvious in the Southern Hemisphere where, incidentally, mountain waves are rare. In the late seventies, it was proposed that gravity waves are responsible for most of the eddy forcing in the mesosphere (Houghton, 1978) and that the lack of gravity waves in these models was responsible for the too strong middle atmospheric winds. The effect of subgrid scale mountain waves has been first parameterized in models, but it was soon recognized that other sources should be included. The Tropics and the Southern Hemisphere are poor in mountain ridges, and new gravity wave sources are needed. Numerical models show that Richardson and convective instabilities can act as sources of gravity waves and that taking into account these sources

could help in producing a more realistic parameterized gravity wave effect on the large scale flow in numerical models.

2.1.3.2.1 Richardson or shear instability

We start from the following hydrostatic equations in pressure coordinates which are the linearization about a z -dependent basic velocity $u_0(z)$, taking $f_0 = 0$ and $v = 0$:

$$\begin{aligned} Du' + w' \frac{du_0}{dz} &= -\frac{\partial \phi'}{\partial x} \\ \frac{\partial u'}{\partial x} + \frac{1}{\rho_0} \frac{\partial}{\partial z} (\rho_0 w') &= 0 \\ D\theta' + w' \frac{\partial \theta}{\partial z} &= 0 \\ \frac{\partial \phi'}{\partial z} &= \frac{R}{H} \theta' e^{-\kappa z/H}, \end{aligned} \quad (2.9)$$

A streamfunction can be defined as

$$\rho_0 u' = \frac{\partial \psi}{\partial z} \quad \rho_0 w' = -\frac{\partial \psi}{\partial x}, \quad (2.10)$$

and the four previous equations can be reduced to one (here, $\Psi = \rho_0^{-1/2} \psi$):

$$\left(\frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x} \right)^2 \left(\frac{\partial^2 \Psi}{\partial z^2} - \frac{1}{4H^2} \Psi \right) + N^2 \frac{\partial^2 \Psi}{\partial x^2} - \left(\frac{d^2 u_0}{dz^2} + \frac{1}{H} \frac{du_0}{dz} \right) \left(\frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x} \right) \frac{\partial \Psi}{\partial x} = 0, \quad (2.11)$$

where N^2 is as defined earlier. A solution of the form $\Psi(x, z, t) = \hat{\psi}(z) e^{ik(x-ct)}$ can be expected, and the previous equation reduces to

$$\left[\frac{d^2}{dz^2} + \frac{N^2(z)}{(u_0(z) - c)^2} - \frac{d^2 u_0(z)/dz^2 + H^{-1} du_0/dz}{(u_0(z) - c)} - \frac{1}{4H^2} \right] \hat{\psi}(z) = 0. \quad (2.12)$$

A linear stability criterion can be established by defining $\phi(z) = (u_0 - c)^{-1/2} \hat{\psi}(z)$, where $\hat{\psi}$ and $c = c_r + ic_i$ can be complex. Multiplying Eq. 2.12 by ϕ^* , integrating on the vertical depth of the fluid, and taking the imaginary part of the resulting equation, it is straightforward to show that

$$c_i \int_{z_1}^{z_2} \left\{ \left| \frac{d\phi}{dz} \right|^2 + \frac{1}{4H^2} |\phi|^2 + \left[N^2 - \frac{1}{4} \left(\frac{du_0}{dz} \right)^2 \right] \frac{|\phi|^2}{|u_0 - c|^2} \right\} dz = 0. \quad (2.13)$$

The only way for this equation to be zero is to have $c_i = 0$ (the wave is stable) or (when $c_i \neq 0$)

$$Ri \equiv \frac{N^2}{\left(\frac{du_0}{dz} \right)^2} < \frac{1}{4}. \quad (2.14)$$

The necessary condition for linear instability is thus given by the preceding equation. Growing modes are expected only when the Richardson number Ri is lower than $1/4$. When the basic flow is not unidirectional, the necessary condition for instability becomes

$$Ri \equiv \frac{N^2}{\left(\frac{du_0}{dz}\right)^2 + \left(\frac{dv_0}{dz}\right)^2} < \frac{1}{4}. \quad (2.15)$$

Many authors (Lott *et al.*, 1992; Sutherland *et al.*, 1994; Sutherland and Peltier, 1995; among others) have studied gravity wave emission emerging from an unstable stratified shear layer. Using linear analytic results and nonlinear numerical models, it is now believed that shear excitation of gravity waves in the far field (i.e., away from the shear zone) is at least possible in principle, but the relevance of this source in the real atmosphere is still to be demonstrated since the atmospheric conditions necessary (a very low Richardson number) for wave emission are probably rarely attained for significant periods of time (see Vaughan *et al.*, 1996).

2.1.3.2.2 Convective instability and squall lines

The role of convection in generating gravity waves is thought to be as important as orography at mid-latitudes, and it is believed that it is the primary gravity wave source in the Tropics and maybe in the Southern Hemisphere. At least three different mechanisms leading to gravity wave emission due to convection are now known. The simplest can be described as follows: when a cell of statically unstable air ascends and impinges on an overlying stable layer, this layer is perturbed and some oscillation can take place and propagate vertically with no preferred vertical tilt (this is often referred to as “thermal forcing”). A second mechanism, which requires vertical wind shear, has been proposed by Newton (1960): when an ascending unstable cell penetrates into a shear layer, the cell can obstruct the ambient horizontal flow and act as an obstacle (this is the so-called “obstacle effect”). This effect is somewhat similar to orographic forcing, and the produced gravity waves generally tilt downstream. A third mechanism described in Fovell *et al.* (1992) was observed in a numerical simulation of a squall line. This “mechanical oscillator effect” can be

seen as a succession of ascending cells impinging on an overlying stable layer (e.g., at the tropopause level) and moving backward relative to the mean storm velocity. This backward movement of the ascending cells tends to create gravity waves with phases tilting backward (relative to the direction of the storm) in the vertical.

These mechanisms, although very significant as gravity wave sources (at least in numerical experiments), are not yet incorporated in most gravity wave drag schemes used by general circulation models.

2.1.3.3 Geostrophic adjustment

Away from low-latitude regions (and from the boundary layer), a state of geostrophic balance dominates at large scales. When significant departure from geostrophy is observed, inertia-gravity wave radiation is expected in order to re-establish geostrophy. Moreover, it is now accepted that no invariant slow manifold exists in the real atmosphere, i.e., a state of perfect geostrophy will always evolve in time to give rise to high-frequency wave motions (inertia-gravity waves, see, e.g., Warn and Ménard, 1986).

The way this mechanism operates to produce significant inertia-gravity wave activity on a day-to-day basis is still far from being evaluated. Analytic solutions of a simple linear system simulating adjustment around an initially unbalanced jet stream were studied by Fritts and Luo (1992) and Luo and Fritts (1993). Their results indicate that wave propagation at early times during the adjustment process was perpendicular to the jet axis, but that gravity wave propagation is more parallel to the jet axis at later times. Also, O'Sullivan and Dunkerton (1995) studied the inertia-gravity wave generation by geostrophic adjustment of a baroclinically unstable perturbation in a nonlinear three-dimensional model. In their simulation, they found that the (breaking) inertia-gravity waves so produced were mostly causing isentropic and cross-isentropic mixing of constituents, and that only a few waves were able to penetrate in the middle stratosphere.

2.1.3.4 Nonlinear transfer from Rossby to gravity modes

A weakly nonlinear approach to wave-wave interactions shows that energy can be transferred from slow (Rossby) to fast (gravity) normal modes in a triad interaction. It can be shown that at small Rossby numbers, the triad formed by one slow mode and two fast modes are easy to resonate, and that the triad is in fact catalytic. This means that the slow mode will not transfer net energy to the fast modes, and that the energy exchange among the two fast modes is favoured (Bartello, 1995).

Moreover, the triad formed by two slow modes and a fast mode will never resonate as the Rossby and Froude numbers tend to zero. However, as the Rossby number increases to $O(1)$, a nonlinear energy transfer from the slow modes to the fast mode can be important (Lelong and Riley, 1991). In this process, geostrophic adjustment may not take place, and the Rossby number may remain relatively high.

2.1.4 Eliassen-Palm flux and nonacceleration theorem

Eliassen and Palm (1961) initiated a fundamental work in the field of the interaction of the zonal mean flow with waves. They found that as long as waves are linear, unforced, inviscid, and stationary, a specific flux form (now referred to as the Eliassen-Palm flux) is divergenceless and that no zonal mean flow acceleration can occur. The proof is fairly straightforward and the result can be obtained from the linearized, unforced, inviscid, and steady primitive equations in log-pressure coordinates (the basic state, denoted by subscripts 0, must be zonally symmetric):

$$\begin{aligned}
 \frac{u_0}{a \cos \phi} u'_\lambda + \left[\frac{(u_0 \cos \phi)_\phi}{a \cos \phi} - f \right] v' + u_{0z} w' + \frac{\phi'_\lambda}{a \cos \phi} &= 0 \\
 \frac{u_0}{a \cos \phi} v'_\lambda + \left[\frac{2 u_0}{a} \tan \phi + f \right] u' + \frac{\phi'_\phi}{a} &= 0 \\
 \phi'_z &= \frac{R}{H} \theta' e^{-\kappa z/H} \\
 \frac{u'_\lambda}{a \cos \phi} + \frac{(v' \cos \phi)_\phi}{a \cos \phi} + \frac{1}{\rho_0} (\rho_0 w')_z &= 0 \\
 \frac{u_0}{a \cos \phi} \theta'_\lambda + \frac{\theta_{0\phi}}{a} v' + \theta_{0z} w' &= 0.
 \end{aligned} \tag{2.16}$$

Given Eq. 2.16, the components of the Eliassen-Palm flux can be written

$$\begin{aligned} F^{(\phi)} &= \rho_0 a \cos \phi (u_{0z} \overline{v'\theta'}/\theta_{0z} - \overline{v'u'}) \\ F^{(z)} &= \rho_0 a \cos \phi \{ [f - (a \cos \phi)^{-1} (u_0 \cos \phi)_\phi] \overline{v'\theta'}/\theta_{0z} - \overline{w'u'} \}, \end{aligned} \quad (2.17)$$

(where the overbar denotes a zonal average) which leads to

$$\nabla \cdot \vec{F} \equiv (a \cos \phi)^{-1} \frac{\partial}{\partial \phi} (F^{(\phi)} \cos \phi) + \frac{\partial F^{(z)}}{\partial z} = 0. \quad (2.18)$$

It can be easily shown that the form of the Eliassen-Palm flux for a monochromatic inertia-gravity wave on an f -plane is the same as that of a pure monochromatic gravity wave in Cartesian geometry when the basic state is uniform in space and zonal:

$$\begin{aligned} F^{(y)} &= -\frac{1}{2} \rho_s \frac{kl(m^2 + 1/4H^2)}{N^2(k^2 + l^2)} |\hat{\phi}|^2 \\ F^{(z)} &= \frac{1}{2} \rho_s \frac{km}{N^2} |\hat{\phi}|^2. \end{aligned} \quad (2.19)$$

On the other hand, inertia-gravity waves have longer wavelengths than gravity waves, and the vertical component of the Eliassen-Palm flux associated with IGW is smaller than that of pure gravity waves, given that the other parameters are unchanged. Note that $F^{(z)}$ is negative (positive) when the wave is vertically tilting eastward (westward). This will have important consequences when we will consider wave-mean flow interactions.

A generalized Eliassen-Palm theorem has also been obtained by Andrews and McIntyre (1976, 1978a) where they showed that the Eliassen-Palm flux can be interpreted as a “wave-activity” (pseudomomentum) flux in the form of a conservation law:

$$\frac{\partial WA}{\partial t} + \nabla \cdot \vec{F}_{WA} = S + O(\alpha^3), \quad (2.20)$$

where the wave-activity WA and its flux $\vec{F}_{WA}$ are quadratic in wave amplitude, S denotes nonconservative effects, and α represents the amplitude of wave motion. It is easily seen that for linear, inviscid, unforced, and steady situations, the result obtained by Eliassen and Palm is recovered. There is also another quantity often called the pseudoenergy that is conserved according to Eq. 2.20. In pressure coordinates, the explicit form of A (for the pseudoenergy) is given in the Appendix A.

More interestingly, the forcing on the zonal mean flow by eddies can be expressed in terms of the Eliassen-Palm flux divergence when transformed Eulerian mean (TEM) variables are used. When the zonal momentum equation is zonally averaged and when a zonally symmetric basic state is chosen (i.e., $\bar{u}_0 = u_0(\phi, z) \bar{\lambda}$, where $\bar{\lambda}$ is a unit vector pointing in the zonal direction), it is obtained after some algebra that

$$\frac{\partial \bar{u}}{\partial t} + \bar{v}^* \left(\frac{(u_0 \cos \phi)_\phi}{a \cos \phi} - f \right) + \bar{w}^* \frac{\partial u_0}{\partial z} = \frac{1}{\rho_0 a \cos \phi} \nabla \cdot \bar{F}_{NL}, \quad (2.21)$$

where $\bar{F}_{NL} = \bar{F} + O(\alpha^3)$ and has the same form as in Eq. 2.17, except that the fields u' , v' , w' , and θ' must be interpreted as being solutions of a fully nonlinear dynamical system. The eddy forcing is expressed in terms of a quantity which has a clear dynamical interpretation, at least to order α^2 . Higher order terms on the right-hand side of Eq. 2.21 are needed only when the flux $\bar{F}_{NL}$ is interpreted as the flux of the small-amplitude pseudomomentum. $(\bar{v}^*, \bar{w}^*)$ defines what is called a residual circulation and is written

$$\begin{aligned} \bar{v}^* &= \bar{v} - \rho_0^{-1} (\rho_0 \overline{v' \theta'} / \theta_{0z})_z \\ \bar{w}^* &= \bar{w} + (a \cos \phi)^{-1} (\cos \phi \overline{w' \theta'} / \theta_{0z})_\phi. \end{aligned} \quad (2.22)$$

Note that we introduced a higher order term $O(\alpha^3)$ in Eq. 2.20 since the Eliassen-Palm flux was derived for small-amplitude waves. So far, there is no finite-amplitude theorem of wave-mean flow interaction in an Eulerian framework. Andrews and McIntyre (1978b,c) established an exact theory using generalized Lagrangian-mean (GLM) quantities, but this formalism is very difficult to apply in practice.

The residual circulation defined by Eq. 2.22 can be linked to the GLM circulation under some conditions. Dunkerton (1978) showed that when waves are linear, steady, and conservative, the mean residual circulation is equivalent to the GLM circulation. Hence, a justification for the change of variables made in Eq. 2.22 can be found. This GLM circulation can be defined as follows: Suppose that an unperturbed thin tube of fluid lies at a given latitude circle and at a given altitude. As time evolves, this tube is perturbed and its center of mass calculated along a latitude circle moves in the meridional plane. The velocity of the center of mass is defined to be the GLM circulation.

When the nonacceleration condition is not met ($\nabla \cdot \vec{F}_{NL} \neq 0$), the mean flow acceleration and residual circulation are activated by eddies. We now consider situations in which wave-mean flow interaction occurs as well as some mechanisms responsible for this interaction.

2.1.5 Critical levels

When the background flow is not uniform, gravity wave propagation is affected by this non-uniformity, and wave characteristics change according to the background state. We will here study the effect of a vertically varying basic flow on a propagating gravity wave in two dimensions ($x - z$). This problem was first examined by Booker and Bretherton (1966) and their analysis is presented here.

Eq. 2.12 has a singularity when the wave phase speed matches the background wind velocity. A solution near the singularity (called the critical line) can be determined using the Frobenius method. It has been shown for the linear, nonrotating, and stable ($Ri > 1/4$) case, that absorption occurs at the critical line when a wave approaches it, with an attenuating factor of

$$\exp(-\pi[Ri - 1/4]^{1/2}).$$

For a Richardson number of 1, the wave is attenuated by a factor of about 15. From a linear point of view, wave momentum is almost completely absorbed at the critical line.

The presence of rotation modifies this situation in some ways. If the effect of rotation is incorporated in Eqs. 2.9 (f -plane geometry, taking $u_0 = \gamma_0 z$, where γ_0 is considered constant, and using the Boussinesq approximation), the singular levels are displaced, as can be seen from the following equation for the vertical velocity structure function (here $w' = \hat{w}(z)e^{ikx}$, and $c = 0$):

$$\left[z^2 - \left(\frac{f_0}{k\gamma_0} \right)^2 \right] \hat{w}_{zz} + \left(\frac{2f_0^2}{k^2\gamma_0^2 z} \right) \hat{w}_z + Ri\hat{w} = 0. \quad (2.23)$$

In this particular case, the level of zero wind is at $z = 0$, but it does not lead to any discontinuity (Grimshaw, 1975). Jones (1967) was the first to recognize that singular levels were located where $\hat{\omega} = \omega - ku_0 = \pm f_0$, and he solved Eq. 2.23 using Frobenius method to find that waves were still absorbed at these levels. Kitchen and McIntyre (1980) studied the impact of the variation of f with latitude on the absorption property of Jones' critical levels using β -plane geometry and the ray-tracing technique. Linearly, it turns out that a wave packet is in principle reflected from a Jones' critical level when $\beta \neq 0$ and when there is no dissipation in the system. However, their analysis shows that the wave packet is in practice absorbed when β and the dissipation are small. More recently, Wurtele *et al.* (1996) investigated the nonlinear behavior of inertia-gravity waves under a Jones' critical level using a numerical model. They found that: 1) for a monochromatic wave, the initial development near the critical line is similar to that of the linear theory, except that as time increases, nonlinear reflection occurs producing harmonics of the initial wavelength, so that both absorption and reflection is simulated; 2) when the wave field is initially formed by a broad spectrum (continuous), absorption is observed with no nonlinear breakdown of the flow caused by singular levels.

All these studies suggest that singular levels violate the nonacceleration theorem and that eddy forcing tends to be strong in their vicinity.

2.1.6 Gravity wave breaking

The first to recognize that internal gravity waves can break and generate turbulence in the mesosphere (and higher) due to some instability mechanism was Hodges (1967, 1969). He noted that due to the decrease of air density with height, the gravity wave amplitude could reach large values and produce convective and Richardson instabilities of the total flow in some regions (the wave amplitude growth is due to the tendency of eddy motions to conserve its wave-activity when propagating upwards). Fig. 4 shows a typical temperature profile in the mesosphere. It can be seen that unstable lapse rates appear sporadically ($dT/dz < -g/c_p$). Lindzen (1981) used this idea to construct a parameterization for gravity wave drag (GWD). His calculations will be presented in some detail here, since most of the GWD schemes used in GCMs are based on Lindzen's approach.

In the weak shear limit, and for relatively long waves, the dominant term in front of $\hat{\psi}$ is $N^2/(u_0 - c)^2$. A WKBJ solution which involves only this term can be found (see Gill, 1982) when the vertical scale of the wave is much shorter than the scale of variation of $\lambda \equiv \frac{N}{(u_0 - c)}$:

$$\hat{\psi} = A\lambda^{-1/2} e^{i \int^z \lambda dz'}. \quad (2.24)$$

In the case of a monochromatic wave, $\psi' = A\lambda^{-1/2} e^{iz/2H} e^{i(kz + \int^z \lambda dz' - kct)}$. The continuity equation allows us to write $ku' = -\lambda w'$ when λ varies slowly. It was then hypothesized that turbulence is produced when convective instability occurs. This condition is simply given by $(\theta_0 + \theta')_z < 0$. From the linearized equation of conservation of potential temperature, it is straightforward to find a relation between θ' and u' , keeping in mind that the vertical variations of the medium are weak compared to those associated with the perturbed quantities:

$$\theta'_z = \frac{\theta_0 z}{u_0 - c} u'. \quad (2.25)$$

In other words, it can be verified that the breaking condition is equivalent to

$$u' \geq |c - u_0|. \quad (2.26)$$

Figure 4. Rocket grenade temperature profiles obtained at Wallops Island (38°N) during summer and winter from 1962 to 1965. Note the wavelike structures and superadiabatic lapse rates. [From Fritts (1984b).]

Equality holds at the breaking level z_s . Note incidentally that u' is the linear solution, and that Eq. 2.26 is not equivalent to saying that the breaking level is near a critical level.

We are now in a position to calculate the vertical momentum flux divergence $(\rho_0 \overline{u'w'})_z / \rho_0$ which causes a drag force on the zonal mean flow. Remember that this quantity is zero for steady and conservative waves. For a monochromatic wave at the saturation level z_s , the Reynolds stress is $\rho_0 \overline{u'_s w'_s} = \rho_0 u'_s w'_s / 2 = -\rho_0 k (u_0 - c)^3 / 2N$, and the acceleration of the mean zonal flow is

$$\frac{\partial \bar{u}}{\partial t} = \frac{-k(u_0 - c)^2}{2N} \left(\frac{(u_0 - c)}{H} - 3u_{0z} \right) \quad (2.27)$$

where there is wave breaking. Note that one must know the horizontal wavenumber k and the wave phase speed c *a priori* in order to use this scheme. The altitude at which wave breaking occurs according to this model can be evaluated from Eq. 2.26 by taking the equal sign when $z = z_s$. If the wave amplitude is written at the breaking level z_s ,

$$\max[u'(z_s)] = \hat{u}(z_0) e^{(z_s - z_0)/2H} = |c - u_0|, \quad (2.28)$$

where z_0 is some level under the breaking level, the breaking altitude is

$$z_s = z_0 + 2H \ln \left(\frac{|c - u_0(z_s)|}{\hat{u}(z_0)} \right). \quad (2.29)$$

This monochromatic linear instability model can only give an approximate picture of breaking gravity wave events. Obvious refinements would be to include the effect of a broad wave spectrum, and more general instability conditions (e.g., Richardson instability). These factors have been considered in Hines (1991) where a probabilistic point of view is adopted in establishing an instability criterion (more precisely, the Central Limit Theorem is used to establish the probability of instability of a signal formed by a superposition of gravity waves with independent and uniformly distributed phases). Another questionable aspect of the linear instability model is that it assumes that linear dynamics is valid up to the breaking level, and that nonlinearity is apparent only after breaking occurred. Hines (1991) argued that the

gravity waves with relatively large vertical wavenumbers are those which contribute the most to a shear destabilization of the flow. It turns out that the phase speeds of these waves are relatively small, in fact smaller than the standard deviation of the wind (formed by the broad spectrum of waves) in which they propagate. This implies that nonlinear advection and wave-wave interactions could be very important in the breaking dynamics.

2.2 Conservation Laws in Isentropic Coordinates

The main reason for using the potential temperature as the vertical coordinate is that the flow becomes “two-dimensional” under adiabatic conditions. There are of course some drawbacks: 1) a convectively neutral or unstable flow can lead to singularities since the vertical coordinate may not be monotonically increasing; 2) the lower boundary condition is complicated by the fact that the ground is not an isentrope (note incidentally that the same problem arises in pressure coordinates); 3) at small scales, the vertical dimension becomes very distorted as viewed in physical space because of strong filamentations. On the other hand, the use of isentropic coordinates allows to express the wave-activity densities with Eulerian fields in a relatively simple form. The complexity of the wave-activity expressions can be viewed as hidden in the variable transformation.

The hydrostatic, unforced, conservative primitive equations in isentropic coordinates can be obtained from the equations in log-pressure coordinates by applying the chain rule for spatial and time derivatives on constant θ -surfaces. After some algebra, it is obtained that the primitive equations are written:

$$\begin{aligned}
 a \cos \phi \frac{\partial u}{\partial t} + \left(\frac{1}{2} u^2 + \frac{1}{2} v^2 + M \right)_\lambda - a \cos \phi v \xi &= 0 \\
 a \frac{\partial v}{\partial t} + \left(\frac{1}{2} u^2 + \frac{1}{2} v^2 + M \right)_\phi + a u \xi &= 0 \\
 a \cos \phi \frac{\partial \sigma}{\partial t} + (\sigma u)_\lambda + (\sigma v \cos \phi)_\phi &= 0 \\
 \frac{\partial M}{\partial \theta} &= c_p \left(\frac{p}{p_r} \right)^\kappa \\
 \frac{\partial p}{\partial \theta} &= -g\sigma,
 \end{aligned} \tag{2.30}$$

where σ is the isentropic density, $M = c_p T + \phi$ is the Montgomery function, θ is the potential temperature and also the vertical coordinate, a is the constant Earth radius, p_r is a constant reference pressure, and

$$\xi = f + \frac{v_\lambda}{a \cos \phi} - \frac{(u \cos \phi)_\phi}{a \cos \phi}$$

is the vertical component of absolute isentropic vorticity (see Andrews *et al.* 1987 for more details).

In what follows, we will consider the linearized version of these equations about two different basic states. Firstly, a time independent zonally symmetric basic state will be used to establish local pseudomomentum density conservation. Secondly, the linearization will be done about a time independent asymmetric basic state and local pseudoenergy density conservation will be derived. These conservation laws of local quantities are direct consequences of symmetries of the basic state and they can be derived elegantly from a noncanonical Hamiltonian formalism using Noether's theorem (e.g., Shepherd, 1990). In fact, nonlocal wave-activity densities can be found even when the basic state has no zonal or time symmetries (T. Warn, personal communication). Restriction to small-amplitude waves is not necessary and finite-amplitude conservation laws exist, but only the small-amplitude version will be presented given that this version is the one used in our diagnostic method since the concept of normal modes is only valid in linear dynamics.

2.2.1 Zonally symmetric basic flow

If a zonally symmetric and time independent basic flow is chosen to be a solution of the nonlinear primitive equations, we must have

$$\begin{aligned}
 -\frac{1}{a} \frac{\partial M_0}{\partial \phi} &= f u_0 + \frac{u_0^2 \tan \phi}{a} \\
 v_0 &= 0 \\
 \frac{\partial M_0}{\partial \theta} &= c_p \left(\frac{p_0}{p_r} \right)^\kappa \\
 \frac{\partial p_0}{\partial \theta} &= -g \sigma_0.
 \end{aligned} \tag{2.31}$$

The first of these equations simply states that the basic state is in gradient wind balance. Note that there are four unknowns and three equations, meaning that one can fix, e.g., u_0 and find the other basic fields from u_0 . Now, the linearized equations can be written

$$\begin{aligned}
 a \cos \phi \frac{\partial u'}{\partial t} + u_0 u'_\lambda + M'_\lambda - a \cos \phi v' \xi_0 &= 0 \\
 a \frac{\partial v'}{\partial t} + (u_0 u' + M')_\phi + a u_0 \xi' + a u' \xi_0 &= 0 \\
 a \cos \phi \frac{\partial \sigma'}{\partial t} + \sigma_0 u'_\lambda + u_0 \sigma'_\lambda + (\sigma_0 v' \cos \phi)_\phi &= 0 \\
 \frac{\partial M'}{\partial \theta} &= c_p \kappa \left(\frac{p_0}{p_r} \right)^\kappa \frac{p'}{p_0} \\
 \frac{\partial p'}{\partial \theta} &= -g \sigma',
 \end{aligned} \tag{2.32}$$

and two quadratic quantities obeying a conservation law of the form

$$\frac{\partial (WA)}{\partial t} + \nabla \cdot \vec{F}_{(WA)} = 0$$

called “wave-activities” can be derived. These quantities are the pseudomomentum J where

$$J = -\sigma' u' \cos \phi + \frac{\sigma_0^2 \cos \phi P'^2}{\frac{2}{a} \frac{\partial P_0}{\partial \phi}}, \tag{2.33}$$

where $P = \xi/\sigma$ and

$$a \frac{\partial J}{\partial t} + \left(\frac{u_0 J}{\cos \phi} - \frac{\sigma_0}{2} (u'^2 - v'^2) - \frac{c_p \kappa}{2g p_0} \left(\frac{p_0}{p_r} \right)^\kappa p'^2 \right)_\lambda - \frac{1}{\cos \phi} (\sigma_0 u' v' \cos^2 \phi)_\phi + \left(\frac{p'}{g} M'_\lambda \right)_\theta = 0 \tag{2.34}$$

which is conserved because of the basic state zonal symmetry, and the pseudoenergy A where

$$A = \frac{\sigma_0}{2}(u'^2 + v'^2) + u_0 \sigma' u' - \frac{u_0 \sigma_0^2 P'^2}{\frac{2}{a} \frac{\partial P_0}{\partial \phi}} + \frac{c_p \kappa}{2g p_0} \left(\frac{p_0}{p_r} \right)^\kappa p'^2, \quad (2.35)$$

and

$$a \cos \phi \frac{\partial A}{\partial t} + \frac{\partial F^{(\lambda)}}{\partial \lambda} + \frac{\partial (F^{(\phi)} \cos \phi)}{\partial \phi} + \frac{\partial F^{(\theta)}}{\partial \theta} = 0 \quad (2.36)$$

with

$$\begin{aligned} F^{(\lambda)} &= (u_0 u' + M')(\sigma_0 u' + \sigma' u_0) - \frac{u_0^2 \sigma_0^2 P'^2}{\frac{2}{a} \frac{\partial P_0}{\partial \phi}} \\ F^{(\phi)} &= (u_0 u' + M') \sigma_0 v' \\ F^{(\theta)} &= - \frac{a \cos \phi}{g} M' \frac{\partial p'}{\partial t} \end{aligned} \quad (2.37)$$

is conserved due the steadiness of the basic state.

2.2.2 Zonally asymmetric basic flow

When the time independent basic state is zonally asymmetric, the only globally conserved local quadratic quantity is the pseudoenergy. Its form is slightly different from the zonally symmetric case due to the presence of a nonvanishing meridional basic state velocity. In this case, the basic state must obey the steady state nonlinear primitive equations:

$$\begin{aligned} \left(\frac{1}{2} u_0^2 + \frac{1}{2} v_0^2 + M_0 \right)_\lambda - a \cos \phi v_0 \xi_0 &= 0 \\ \left(\frac{1}{2} u_0^2 + \frac{1}{2} v_0^2 + M_0 \right)_\phi + a u_0 \xi_0 &= 0 \\ (\sigma_0 u_0)_\lambda + (\sigma_0 v_0 \cos \phi)_\phi &= 0 \\ \frac{\partial M_0}{\partial \theta} &= c_p \left(\frac{p_0}{p_r} \right)^\kappa \\ \frac{\partial p_0}{\partial \theta} &= -g \sigma_0. \end{aligned} \quad (2.38)$$

Finding an analytical solution to this set of equations is of course nontrivial since there are as many equations as unknowns. The third equation in Eqs. 2.38 allows to define a basic state mass streamfunction ψ_0 such that

$$\begin{aligned} u_0 \sigma_0 &= - \frac{1}{a} \frac{\partial \psi_0}{\partial \phi} \\ v_0 \sigma_0 \cos \phi &= \frac{1}{a} \frac{\partial \psi_0}{\partial \lambda}. \end{aligned} \quad (2.39)$$

The conservation (following a parcel) of the basic state potential vorticity P_0 multiplied by σ_0

$$u_0 \sigma_0 \frac{\partial P_0}{\partial \lambda} + v_0 \sigma_0 \cos \phi \frac{\partial P_0}{\partial \phi} = -\frac{1}{a} \frac{\partial \psi_0}{\partial \phi} \frac{\partial P_0}{\partial \lambda} + \frac{1}{a} \frac{\partial \psi_0}{\partial \lambda} \frac{\partial P_0}{\partial \phi} = 0 \quad (2.40)$$

allows one to write $\psi_0 = \psi_0(P_0)$. The pseudoenergy A and its flux $\vec{F} = (F^{(\lambda)}, F^{(\phi)}, F^{(\theta)})$ take the form

$$\begin{aligned} A &= \frac{\sigma_0}{2}(u'^2 + v'^2) + \sigma'(u_0 u' + v_0 v') + \frac{\sigma_0}{2} \frac{d\psi_0}{dP_0} P'^2 + \frac{c_p \kappa}{2g p_0} \left(\frac{p_0}{p_r}\right)^\kappa p'^2 \\ F^{(\lambda)} &= (u_0 u' + v_0 v' + M')(\sigma_0 u' + \sigma' u_0) + \frac{u_0 \sigma_0}{2} \frac{d\psi_0}{dP_0} P'^2 \\ F^{(\phi)} &= (u_0 u' + v_0 v' + M')(\sigma_0 v' + \sigma' v_0) + \frac{v_0 \sigma_0}{2} \frac{d\psi_0}{dP_0} P'^2 \\ F^{(\theta)} &= -\frac{a \cos \phi}{g} M' \frac{\partial p'}{\partial t}. \end{aligned} \quad (2.41)$$

The reader is referred to Haynes (1988) for the finite-amplitude version of these wave-activity densities. Haynes treated both zonally symmetric and zonally asymmetric basic states.

The given pseudoenergy is not properly conserved when the bottom boundary is not an isentrope, as it can be seen from Eq. 2.36. The contribution from the bottom boundary to the pseudoenergy should be taken into account using the conservation of potential temperature at the surface. Here, we will consider a basic flow that is zonally symmetric ($v_0 = 0$). In linear form, this gives

$$\frac{\partial \theta'_s}{\partial t} + \frac{u_{0s}}{a \cos \phi} \frac{\partial \theta'_s}{\partial \lambda} + \frac{v'_s}{a} \frac{\partial \theta_{0s}}{\partial \phi} = 0. \quad (2.42)$$

The subscript “s” means that the fields are evaluated at the surface. In θ -coordinates, the linearization allows us to evaluate the fields at $\theta = \theta_{0s}$. After some algebra, and using the Lorenz relationship (Lorenz 1955) $p' = g\sigma_0\theta'$ at the surface, it can be shown that the globally conserved pseudoenergy should be written

$$E = A - 2\beta\delta(\theta - \theta_{0s}), \quad (2.43)$$

when the basic flow is zonally symmetric. Here $\delta(\theta - \theta_{0s})$ is the Dirac distribution and

$$\beta = \sigma_0 u_0 u' \theta'_s + \frac{a P_0 \sigma_0^2 u_0 \theta'^2_s}{2 \frac{\partial \theta_{0s}}{\partial \phi}}.$$

2.2.3 Eddy forcing on the mean flow and the residual circulation

In isentropic coordinates, the eddy forcing on the mean flow can be expressed in terms of the Eliassen-Palm flux divergence as in log-pressure coordinates. An equation for the evolution of the mean momentum can be obtained after some algebra:

$$\frac{\partial}{\partial t}(\overline{\sigma u'}) + \overline{\sigma v'} \left(\frac{(u_0 \cos \phi)_\phi}{a \cos \phi} - f \right) = \frac{\nabla \cdot \overline{\mathbf{F}}}{\cos \phi} + O(\alpha^3). \quad (2.44)$$

The divergence term on the right-hand side is simply the zonally averaged pseudo-momentum flux (or Eliassen-Palm flux in isentropic coordinates). There is no cross isentropic flow and no “vertical velocity” in the preceding equation, but the addition of diabatic heating can be easily incorporated. Also, the similarity between this equation and the TEM equation should be noted.

2.3 Applications to the Middle Atmosphere

As we saw in the preceding sections, the impact of eddies on the general circulation of the atmosphere can be of fundamental importance when the nonacceleration theorem is violated. We will now consider the specific role of wave forcing on the mean zonal flow and meridional circulation of the middle atmosphere. Wave breaking and wave dissipation climatology can, in large part, be assessed with the help of numerical models of the middle atmosphere. All the general circulation models of the middle atmosphere suffer from a too low level of wave activity. Even the simulated planetary waves in the stratosphere have a too weak amplitude (see Andrews *et al.*, 1987, chapter 11). These deficiencies lead to three major problems in numerical simulations of the middle atmosphere: 1) the mean zonal winds are too large throughout the middle atmosphere, and the problem is particularly acute in the mesosphere; 2) the residual (or meridional) circulation, and the meridional mass transport, are inhibited; and 3) the stratospheric winter pole temperature is too cold by at least 10 K in the best models (see, e.g., Hamilton 1995). As we will see, it is believed that the source of these three problems is related in large part to a too low level of gravity wave activity in the middle atmosphere, and to a lack of gravity wave breaking and dissipation in the mesosphere.

2.3.1 The seasonal mean state with and without eddy forcing

In order to specify the role of eddy forcing on the seasonal mean state of the middle atmosphere, Fels (1985) proposed to establish what would have been the mean middle atmosphere temperature distribution without any dynamical forcing effects. He calculated the zonal mean equilibrium temperature based on the actual spatial distribution of the atmospheric gases, allowing for some convection when required by statically unstable conditions. This allows to separate the effect of radiation from that of dynamics in establishing the temperature distribution. However, feedbacks between radiation and dynamics are neglected since the actual gas distribution is partly determined by the dynamics. Fig. 5 shows this equilibrium temperature T_r for January 15 as calculated by Fels, while Fig. 6 represents the actual mean temperature T in solstice conditions. Almost everywhere throughout the middle atmosphere, T_r increases from the winter to the summer hemisphere, while the actual temperature has a more complex distribution. The actual mean winter polar stratospheric temperature is about 40 K warmer than the radiatively determined one, and the actual summer polar stratopause temperature is 10 K colder than T_r at the same location. In the mesosphere, the real atmospheric temperature increases from the summer to the winter hemisphere, while the radiatively determined one decreases. This shows how important dynamical processes are in the determination of the actual mesospheric temperature.

These discrepancies are now thought to be due to forcing effects produced by waves that violate nonacceleration conditions. If the atmospheric flow were composed of waves that do not violate the nonacceleration conditions, the temperature would be close to T_r (given the actual gases distribution).

Geostrophy is the main dynamical balance in the middle atmosphere. Given some lower boundary condition at the tropopause (usually the actual climatological wind), geostrophic winds can be obtained from the thermal wind balance. Fig. 7 represents the geostrophic wind as obtained from T_r . This can be contrasted with the geostrophic wind calculated from the actual temperature distribution (Fig. 8). It is

Figure 5. Time-dependent "radiatively determined" temperature T_r for January 15 from a radiative-convective-photochemical model. [From Andrews *et al.* (1987) and Fels (1985).]

Figure 6. Schematic latitude-height section of zonal mean temperatures ($^{\circ}\text{C}$) for solstice conditions. Dashed lines indicate tropopause, stratopause, and mesopause levels. [From Andrews *et al.* (1987).]

Figure 7. Geostrophic mean zonal winds calculated from the radiative equilibrium temperatures shown in Fig. 4. No values are shown near the equator because of the inapplicability of the geostrophic formula there. Units are m s^{-1} , and westerly winds are positive while easterly winds are negative. [From Geller (1983).]

Figure 8. Schematic latitude-height section of zonal mean zonal wind (m s^{-1}) for solstice conditions; W and E designate centers of westerly (from the west) and easterly (from the east) winds, respectively. [From Andrews *et al.* (1987).]

obvious that almost throughout the middle atmosphere, winds depicted in Fig. 7 are too strong, and that there is no visible “closing off” of the polar jets in the mesosphere. In the actual atmosphere, zonal winds at the mesopause are relatively weak in both hemispheres while they are strongest when calculated from the radiatively determined temperature. This strongly suggests that some kind of drag is needed in order to account for this discrepancy.

2.3.2 Andrews experiment with the GFDL SKYHI model

In order to assess and demonstrate the role of Eliassen-Palm flux divergence on the zonal mean state of the middle atmosphere, Andrews *et al.* (1983) used the GFDL SKYHI general circulation in annual mean insolation conditions. The idea was to compare the eddy forcing as expressed in the transformed Eulerian-mean (TEM) set of equations (when the wave forcing is given by the Eliassen-Palm flux divergence) with the “conventional” Eulerian-mean wave forcing. In their experiment, they compare the middle atmospheric zonal and time mean state with and without eddy forcing (the latter being obtained by imposing a strong Newtonian or Rayleigh damping coefficient on wave motion in the lower stratosphere). It turns out that mean stratospheric and mesospheric winds at mid-latitudes are significantly stronger (by an order of 30 m s^{-1}) in the “no wave” experiment, indicating that middle atmospheric waves tend to drag the mean flow. Of course, this is suggested by Eliassen-Palm flux convergence, whereas the “conventional” eddy momentum flux divergence forcing seems to produce a positive mean flow acceleration in many places. This indicates the physical and dynamical interpretation that the Eliassen-Palm flux (and the pseudomomentum) must have. Moreover, in the quasi-geostrophic (Q-G) framework, one can obtain an elliptic equation relating the zonal mean wind tendency and the eddy forcing in the form of the Eliassen-Palm flux (Andrews *et al.* 1987):

$$\left[\rho_0 \frac{\partial^2}{\partial y^2} + \frac{\partial}{\partial z} \left(\frac{\rho_0 f_0^2}{N^2} \frac{\partial}{\partial z} \right) \right] \bar{u}_t = (\nabla \cdot \bar{F}_{NL})_{yy}.$$

This equation stresses the fundamental importance of the Eliassen-Palm flux on the zonal mean wind tendency (at least in the Q-G case).

2.4 Diagnostic Tools and Gravity Wave Climatology

Different methods have been proposed in order to analyze the waves present in meteorological fields. The most widely used methods (and results obtained from them when available) are presented here. Both real data and model-produced analyses mostly concerned with gravity wave processes will be presented.

2.4.1 Hodograph analyses

The theoretical basis of the hodograph technique has been presented in section 2.1.2 for a monochromatic inertia-gravity wave. When it is applied on a broad spectrum of stratospheric and mesospheric perturbations, it can give rise to results that are not distinguishable from random incoherent fluctuations (Eckermann and Hocking, 1989). This problem can be overcome by pre-identifying and filtering coherent waves in the data before applying the hodograph method. Measurements in the real atmosphere are mainly made using rocketsondes and middle and upper (MU) atmosphere radars. Gavrilov *et al.* (1996) analyzed data obtained from the MU radar located at Shigaraki, Japan (35°N and 139°E) in 1987-88 for an altitude range going from about 67 km to about 83 km (13 winter days and 19 summer days were compiled). In winter (summer), the mean vertical wavelength was found to be near 9.3 km (9.6 km), the mean horizontal wavelength was near 180 km (180 km), and the ratio f/ω was near 0.26 (0.28). The mean line of propagation in the horizontal plane (North being 90°) was 26° (0°). Also, it was found that 20 to 30% of the signal could be attributed to turbulence or instrumental noise.

Hirota and Niki (1985) used the hodograph technique to characterize the climatology of gravity waves in the upper stratosphere and lower mesosphere (30 to 60 km) using rocketsonde data during 1977 to 1982 and covering a wide range of latitudes. These authors found that 1) the vast majority of the waves with a vertical wavelength of about 10 km are propagating upwards with no preferred horizontal direction; 2) wave energy density decays with altitude, suggesting breaking and dissipation; and 3) the most predominant values of f/ω are in a range of 0.2 to 0.4

regardless of latitude and season. For topographic waves generated by a mean wind, $\hat{\omega} = -kU_0$. However, despite the seasonal variation of the mean wind, no related variation of $\hat{\omega}$ is observed. This suggests that a large non-zero phase velocity spectrum exists in the real atmosphere.

Hamilton (1991) analyzed ten years of rocketsonde data (actually, the extended data of Hirota and Niki, 1985) and obtained the direction of propagation of the waves employing the correlation between u' and T'_z (see Eq. 2.8). At mid-latitudes, he found clear eastward propagation in summer and more isotropic propagation in winter. At high-latitudes, a dominance of northward propagation is apparent with almost no seasonal variation. Everywhere in the Northern Hemisphere, clockwise rotation of the hodographs is dominant in summer (90 to 98% are clockwise rotating with altitude), but with somewhat less dominance in winter. Most of the energy is concentrated in the smallest vertical wavelength resolved by the profiles (about 10 km). Hamilton (1988) used the same technique to analyze the gravity wave field of the GFDL SKYHI model at $3^\circ \times 3.6^\circ$ latitude-longitude resolution, and he compared the model wave field at the same geographical locations as in Hamilton (1991) and Hirota and Niki (1985) with an analysis obtained from observations. The two main results are that: 1) propagation is more zonal in the observations than in the model; and 2) the model has stronger wind fluctuations at mid- and low-latitudes than the observations.

Sato (1994) analyzed data obtained from the MU radar (Japan) for waves with periods longer than 10 hours and vertical wavelength shorter than 4 km. It was found that the zonal component of the vertical momentum flux is negative in winter, but almost zero in summer. Among other results, the direction of wave propagation at altitudes between 12 and 22 km was found to be toward the subtropical jet axis, thus excluding the hypothesis that the jet could be the source of these waves.

In summary, these studies show that atmospheric fluctuations with characteristics consistent with those of inertia-gravity waves are observed and measured, and that the sources of these fluctuations could be of many types. Although it is very difficult to assure that these fluctuations are necessarily gravity waves, the fact that

the amplitude of the fluctuations increases with altitude gives a strong argument in favor of an explanation in terms of vertically propagating waves. Pure random and incoherent fluctuations seem unlikely.

2.4.2 Space-time spectral analysis

Hayashi (1982) presented a method for atmospheric wave analysis in which the waves are Fourier transformed in space and time. If a signal $g(x, t)$ is decomposed as

$$g(x, t) = \text{Re} \sum_{k, \omega} [\hat{g}_{k, \omega} e^{i(kx + \omega t)} + \hat{g}_{k, -\omega} e^{i(kx - \omega t)}], \quad (2.45)$$

where k is positive and ω is negative and positive, the space-time power spectrum is defined as

$$P_{k, \omega} = \frac{1}{2} |\hat{g}_{k, \omega}|^2, \quad (2.46)$$

and an ensemble (or frequency band) mean can be applied on the right-hand side when the time series is relatively short. Moreover, the signal can also be written

$$g(x, t) = \sum_k [C_k(t) - iS_k(t)] e^{ikx}, \quad (2.47)$$

where C_k and S_k are the coefficients of the cosine and sine space harmonics. If the time Fourier transform of C_k and S_k is written $\hat{C}_{k, \omega}$ and $\hat{S}_{k, \omega}$, respectively, it can easily be obtained that

$$4P_{k, \pm\omega} = \frac{1}{2} |\hat{C}_{k, \omega}|^2 + \frac{1}{2} |\hat{S}_{k, \omega}|^2 \pm \text{Re}(i\hat{C}_{k, \omega} \hat{S}_{k, \omega}^*). \quad (2.48)$$

It turns out that the positive (negative) sign must be chosen in the right-hand side of the previous equation when the wave travels westward (eastward). Thus it is obtained that

$$4P_{k, \pm\omega} = P_\omega(C_k) + P_\omega(S_k) \pm 2Q_\omega(C_k, S_k), \quad (2.49)$$

where $P_\omega(C_k) = \frac{1}{2} |\hat{C}_{k, \omega}|^2$, $P_\omega(S_k) = \frac{1}{2} |\hat{S}_{k, \omega}|^2$ and $2Q_\omega(C_k, S_k) = \text{Re}(i\hat{C}_{k, \omega} \hat{S}_{k, \omega}^*)$. The waves can thus be separated according to the direction of their phase progression.

Cross spectra of two time series (g and g') can be treated analogously. The cospectra $K_{k, \omega}(g, g') (\equiv \frac{1}{2} \hat{g}_{k, \omega} \hat{g}'_{k, \omega}^*)$ can be written

$$K_{k, \pm\omega}(g, g') = K_\omega(C_k, C'_k) + K_\omega(S_k, S'_k) \pm Q_\omega(C_k, S'_k) \mp Q_\omega(S_k, C'_k). \quad (2.50)$$

Hayashi *et al.* (1989) used this method to study the impact of higher horizontal resolution of the GFDL SKYHI model on the gravity wave field. Usually, a latitude circle and an altitude are selected and the analysis is done on a longitude and time dependent signal. These authors compared the $3^\circ \times 3.6^\circ$ with the $1^\circ \times 1.2^\circ$ latitude-longitude resolutions and showed that the Eliassen-Palm flux divergence is increased in the mesosphere of the higher resolution run. It is claimed that this enhanced divergence is due to the increased of gravity wave activity.

2.4.3 Empirical formulation of a linear tangent operator

Suppose that a linear dynamical system can be written

$$\frac{\partial f}{\partial t} = i T f, \quad (2.51)$$

where T is a linear operator. The eigenvectors of T are the normal modes of the system since

$$\omega_n f_n = T f_n. \quad (2.52)$$

Multiplying by $f^\dagger$ on the right-hand side in Eq. 2.51 and taking the time average (denoted by $\langle \bullet \rangle$), one gets the following equations:

$$\langle \frac{\partial f}{\partial t} f^\dagger \rangle \equiv A = i T \langle f f^\dagger \rangle \equiv i T C. \quad (2.53)$$

If we define the inverse C^{-1} of the covariance matrix C by

$$C C^{-1} = 1, \quad (2.54)$$

it can then be seen that the eigenvector problem of Eq. 2.52 can be rewritten

$$-i A C^{-1} f_n = \omega_n f_n. \quad (2.55)$$

The empirical modes so obtained are the normal modes when the dynamics is linear, but the operator on the left-hand side of Eq. 2.55 is not exactly Hermitian when the time series is not of infinite length, and the empirical modes are not exactly orthogonal under the norm C^{-1} . Moreover, it can be shown that the eigenmodes

obtained from Eq. 2.55 may be identical to the Fourier modes generated from a time Fourier decomposition of a signal f under the condition that the time sampling is small compared with the space sampling and even when Eq. 2.51 is not valid (see the Appendix C). In other words, f could be the solution of a fully nonlinear dynamical system, the decomposition in terms of the solutions of Eq. 2.55 can be nothing more than a Fourier analysis in some special circumstances.

2.4.4 Empirical orthogonal functions

The diagnostic study we will perform in this work is based on methods similar to Empirical Orthogonal Functions (EOFs). These functions form by definition the most efficient basis to describe the variance of a time series. In other words, the EOFs form the spatio-temporal orthogonal basis which can best represent a time series for a given truncation. They are obtained by a simple minimization procedure. Suppose that any field $f(t)$ which has zero time mean can be written

$$f(t) = c a(t) \phi. \quad (2.56)$$

Note that this separation of variables is mostly used when $f(t)$ is the solution of a linear system. $a(t)$ and ϕ can be found when the mean error E defined by

$$E = \|f(t) - c a(t) \phi\|^2, \quad (2.57)$$

where $\|g\|^2 = \langle g^\dagger g \rangle$ (for any g) is minimized. Without loss of generality, $a(t)$ and ϕ are taken to have norm unity. The minimization procedure leads to $\delta E = 0$ and from varying $a(t)$ and ϕ , it follows that

$$\begin{aligned} \phi^\dagger f(t) &= c a(t) \\ \langle f^\dagger(t) a(t) \rangle &= c^* \phi^\dagger. \end{aligned} \quad (2.58)$$

These equations can be put in the form of an eigenvector problem by a simple rearrangement

$$\begin{aligned} \langle R(s, t) a(t) \rangle &= |c|^2 a(s) \\ C \phi &= |c|^2 \phi, \end{aligned} \quad (2.59)$$

where $R(s, t) = f^\dagger(s)f(t)$ and $C = \langle f(t)f^\dagger(t) \rangle$. The covariance matrices R and C are easily shown to be Hermitian so that the sets of eigenvectors $\{a_i(t)\}$ and $\{\phi_i\}$ are bi-orthonormal. When the eigenvalues are ordered so that $|c_1| > |c_2| > \dots$, the field f is best represented (for a truncation N) by

$$f(t) \approx \sum_{i=1}^N c_i a_i(t) \phi_i. \quad (2.60)$$

This expansion is valid for any field f , but when f is a solution of a dynamical system, the physical or dynamical interpretation of the a 's and ϕ 's is ambiguous. They might not bare any meaning other than a statistical one. In fact, this lack of dynamical interpretability is a direct consequence of the choice of the norm. We will even see in the following chapters that a more general norm of the form

$$\|g\|^2 \equiv \langle g^\dagger B g \rangle,$$

with B positive definite, is not appropriate in order to identify normal modes since the bilinear form defining the normal mode orthogonal product is not sign definite. This suggests that the normal modes of a dynamical system can hardly be obtained from a minimization procedure.

Chapter 3

Theory of Empirical Normal Modes

A diagnostic tool developed by Brunet (1994) that uses conservation of wave-activity will be presented and adapted for the purpose of gravity wave diagnosis. This method allows one to statistically extract the normal modes of a linear dynamical system. At the end of this section, we will extend this idea to stochastically forced processes.

3.1 Basic Description

In general, a dynamical system can be written as

$$\frac{\partial f}{\partial t} = G(f),$$

where G can be a nonlinear operator.

If the system is linearized about a time-independent basic state f_0 (where $f(t) = f_0 + f'(t)$) which is a solution of $G(f_0) = 0$, and if the linearized evolution equation is

$$\frac{\partial f'}{\partial t} = iHBf', \quad (3.1)$$

where B and H are time-independent Hermitian operators and f' is assumed to be a complex vector, it is then easily seen that a conserved quantity of the form (f', Bf') is generated. The notation $(f, g) = f^\dagger g$ is used. The converse is true, if B is non-singular, we can always write Eq. 3.1 if (f', Bf') is conserved. In fact, (f', Bf') is an integrated wave-activity. There are as many quadratic globally conserved quantities as there are non-trivial ways of rewriting HB , keeping each term Hermitian. Hence, if HB equals $H'B'$, where H' and B' are Hermitian, then $(f', B'f')$ will also be globally conserved

(here $(f', B'f')$ may be the pseudomomentum, and (f', Bf') the pseudoenergy). The evolution equation for f' in Eq. 3.1 is written as the product of two distinct Hermitian operators (H and B) since the linearized primitive equations describing atmospheric motions can be put into this generalized Hermitian form (the fact that conserved small-amplitude wave-activities exist confirms that Eq. 3.1 is the correct form). A normal mode f'_n will be a solution of Eq. 3.1 with a single frequency of oscillation ω_n :

$$\omega_n f'_n = H B f'_n. \quad (3.2)$$

The implication is that as long as B is non-singular, ω_n will be real and the normal modes will be stable. Unstable modes can arise only when B is singular. Following Held (1985), two normal modes are orthogonal under the bilinear form defined by B . Hence, $(f'_m, B f'_n) = \lambda_n \delta_{mn}$, where λ_n is not necessarily positive since B may not be sign-definite. If the basic state is time-independent and has other symmetries (e.g., a zonal symmetry), degeneracy can occur (in the sense that modes with different wavenumbers and phase speeds can have the same frequency). The remedy is to decompose f'_n into zonal Fourier modes before solving Eq. 3.2.

The basic idea of the Empirical Normal Mode formulation is to take advantage of the orthogonality relationship in the definition of a modified EOF methodology. If the EOF covariance matrix is replaced by

$$K = \langle f'(t) f'^{\dagger}(t) \rangle B \equiv C B, \quad (3.3)$$

then its eigenvectors f'_n , where

$$K f'_n = \lambda_n f'_n, \quad (3.4)$$

are the normal modes. An equivalent but different formulation described in the Appendix B can also be shown to hold for a modified temporal covariance matrix. The expansion $f'(t) = \sum a_n(t) f'_n$ and $\langle a_m a_n^* \rangle = \delta_{mn}$ (in the limit $T \rightarrow \infty$) have been used. The time series is defined on the time interval $[-T, T]$ and we assume that all modes have been excited initially. Note that this time series could be replaced by an ensemble of independent realizations which span all the possible excitations of normal modes

without changing the results that follow. The summation in the expansion must be replaced by an integral when dealing with a continuous spectrum. But in practice, the fields are discretized, and the integrals are replaced by summations that simplify (and avoid) the difficulty of the singular mode continuous spectrum. We will then, in general, consider the operator G to operate from R_N to R_N where N is finite and is the dimension of the space used to represent the dataset.

Another simple way to interpret Eq. 3.4 is to note that the operators HB and $K \equiv CB$ commute ($HBCB - CBHB = 0$), and thus have the same set of eigenvectors (the normal modes) if they are non-degenerate operators. The above commutation relation is simply obtained by multiplying Eq. 3.1 by $f^\dagger$ on the right, and taking the time average (denoted by $\langle \bullet \rangle$):

$$\langle \frac{\partial f'}{\partial t} f^\dagger \rangle = iHBC. \quad (3.5)$$

Integrating by parts, the left-hand side of Eq. 3.5 can also be written $-\langle f' \frac{\partial f'^\dagger}{\partial t} \rangle = iCBH$. This equality is obtained by taking the complex conjugate of Eq. 3.1 and multiplying on the left by f' . This shows that HB and CB commute, since $HBC = CBH$. This does not mean that the eigenvalues of HB (i.e., ω_n) are the same as those of CB (i.e., λ_n) since these two operators are distinct. A similar proof was obtained by Breuer and Sirovitch (1991), but here we capitalize on the conservative properties of the dynamical system to simplify the analysis, since an explicit orthogonal relationship is available.

If the small-amplitude wave-activity is reasonably conserved, then the normal modes of a linear system can be estimated from data. The quality of this estimation depends on how long the time series is relative to the time scale of the dynamical phenomenon under study. Even in the weakly nonlinear limit, this approach is still useful since the ENMs are more physically balanced than ordinary EOFs (e.g., when the covariance matrix is constructed from the geopotential). Thus, each ENM found is nearly a solution of the dynamical equation. In other words, the ENMs generated using the wave-activity are dynamically balanced, whereas the physical interpretation

of ordinary EOFs is difficult to find. Note that for a fixed truncation, the ENMs do not form the most efficient basis in order to explain the variance of a field $f'(t)$. The presence of B in the definition of the matrix K spoils this interpretation.

3.2 The Impact of Noise and Dissipation

Realistic atmospheric time series are in general nonlinear, dissipative and randomly forced. A way to accommodate the ENM analysis with randomness and dissipation is to add damping and random forcing to the dynamics. A simple stochastic modeling of Eq. 3.1 can be obtained by adding these to the evolution equation:

$$\frac{\partial f'}{\partial t} = iHBf' - \gamma f' + \varepsilon, \quad (3.6)$$

where H and B are Hermitian and time-independent, ε and γ are uncorrelated (in time) random forcing (e.g., a Wiener process) and a Rayleigh damping coefficient, respectively. Note that topographic forcing is not in general a Wiener process, and that a special treatment is needed in order to incorporate its effects (see Eq. 2.43). The normal modes, f'_n , of Eq. 3.2 are the eigenvectors of HB and form a complete basis. Now, expanding a solution of Eq. 3.6 in terms of the normal modes $f'(t) = \sum a_n(t)f'_n$, keeping in mind that the principal components a_n are not monochromatic any more, and projecting Eq. 3.6 on a normal mode, one gets an equation governing the principal components:

$$\frac{da_n}{dt} = (i\omega_n - \gamma)a_n + \varepsilon_n, \quad (3.7)$$

where $\lambda_n \varepsilon_n \equiv (f'_n, B \varepsilon)$. In Fourier space, $\tilde{a}_n = (2\pi)^{-1} \int a_n e^{-i\omega t} dt$ and the previous equation reduces to

$$\tilde{a}_n = \frac{\tilde{\varepsilon}_n}{i(\omega - \omega_n) + \gamma}. \quad (3.8)$$

The orthogonality of the principal components is recovered because

$$\int a_n a_m^* dt = 2\pi \int \tilde{a}_n \tilde{a}_m^* d\omega = 2\pi \int \frac{\tilde{\varepsilon}_n \tilde{\varepsilon}_m^*}{(i\omega - i\omega_n + \gamma)(-i\omega + i\omega_m + \gamma)} d\omega = 0 \quad (3.9)$$

when $m \neq n$. The last equality stems from the fact that the noise components, $\tilde{\varepsilon}_n$, are uncorrelated. It is then straightforward to show that the same eigenvalue

problem as in Eq. 3.4 must be solved in order to obtain the ENMs. This means that the presence of the random forcing and Rayleigh dissipation does not alter the spatial distribution of the ENMs and that the noisy part of the signal is felt only in the principal components. This asymmetry between time and space representation suggests that the ENM method could be a real advantage when studying noisy and dissipative dynamical systems. A straightforward time Fourier analysis is not very efficient in extracting noisy non-monochromatic normal modes, especially when the level of dissipation and the overlapping between normal mode spectra are important. Note that a method which depends on the temporal sequence of a time series for extracting normal modes (like Fourier analysis) is subject to failure under a random reordering of the same time series. The ENM approach is unaffected by a random reordering, since Eq. 3.3 is invariant under this operation, hence showing how well the dynamics is already embedded in it.

3.3 ENMs Using Pseudoenergy

The model equations used to construct the bilinear form of the wave-activity are the unforced, conservative, primitive equations in isentropic coordinates as presented in Eqs. 2.30.

In order to linearize the equations of a dynamical system, the choice of a basic state is crucial. Usually, when a flow is forced by topography, lee waves appear, and zonal symmetry is not observed at scales of the order of the orography. Thus, time independence of the basic state is the only symmetry that we will assume. Pseudoenergy will be used to construct the K matrix, and the B matrix is found from Eq. 2.41. The pseudoenergy density may be written

$$A = (f', B f') \quad (3.10)$$

where

$$f' = \begin{pmatrix} u' \\ v' \\ \sigma' \\ P' \\ p' \end{pmatrix} \quad (3.11)$$

and

$$B = \begin{pmatrix} \frac{\sigma_0}{2} & 0 & \frac{u_0}{2} & 0 & 0 \\ 0 & \frac{\sigma_0}{2} & \frac{v_0}{2} & 0 & 0 \\ \frac{u_0}{2} & \frac{v_0}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sigma_0}{2} \frac{d\psi_0}{dP_0} & 0 \\ 0 & 0 & 0 & 0 & \frac{\kappa c_p}{2gP_0} \left(\frac{p_0}{p_r} \right)^\kappa \end{pmatrix} \quad (3.12)$$

This system is over-specified since we should need only three independent components in the definition of f' due to the fact that the primitive equations in hydrostatic form comprise three prognostic equations and two diagnostic equations. This over-specification does not cause any problem if consistency among variables is verified. Furthermore, it allows the matrix B to be written in a very simple form without any differential or nonlocal operators. Note that B is singular when $d\psi_0/dP_0$ changes sign within the domain, hence the possibility of having modal instability as expected from linear theory.

In practice, data are usually not available on isentropic levels but rather in pressure coordinates. Modifications described in Andrews (1987) for the expression of the wave-activity must then be employed. The transformations are given in the Appendix A.

Chapter 4

Results from the GFDL SKYHI Model

4.1 The Dataset

The output of the troposphere-stratosphere-mesosphere GFDL SKYHI general circulation model (Fels *et al.* 1980; Mahlman and Umscheid 1984; Miyahara *et al.* 1986; Hayashi *et al.* 1989; Hamilton 1995; Hamilton *et al.* 1995) was analyzed using the method described in chapter 3. At a horizontal resolution of 1° of latitude by 1.2° of longitude, with 40 vertical levels (from the surface up to about 80 km), and an explicit leapfrog time differencing with time steps of 60 seconds, five days of data sampled hourly were diagnosed for the virtual period December 25, 1983 to December 29, 1983, inclusively (model calendar). Given the model grid resolution, one must be aware that for characteristic wind speeds of the order of 10 m/s, wave motions with time scales less than $\sim 10^4$ seconds cannot be resolved. Thus, inertia-gravity waves occur at the fast limit of the temporal resolution of the model. We have limited our analysis to three spatial windows: one located over the Rocky Mountains in the Western part of North America with area of 4800 km by 2400 km (the center of the grid is at 46.4°N and 114.6°W), another over the Pacific Ocean at mid-latitudes, also with area of 4800 km by 2400 km (and the center of the grid at 46.4°N and 174.6°W), and a third one in a tropical region over Central America with area of 6900 km by 3450 km (center of the grid at 20.0°N and 90.0°W). This was done in order to contrast mountainous, non-mountainous and tropical locations. The analyses were performed in two parts: a first analysis between 618 hPa and 3.9 hPa dealing with the troposphere up to the mid-stratosphere, and a second analysis between 62 hPa and 0.03 hPa dealing with the upper part of the middle atmosphere (stratosphere and mesosphere).

The fact that we restrict the analyses to limited domains with open boundaries may violate wave-activity conservation laws inside the limited domains even when dissipation and forcing are absent. This is due to possible non-vanishing Eliassen-Palm (or more generally wave-activity) fluxes across the boundaries. If it is supposed that the net wave-activity flux across the boundaries is zero, wave-activity densities are conserved in the limited domain, and the theory of Empirical Normal Modes applies as in the case of a closed, periodic, or infinite domain. In practice however, this does not seem to cause serious problems since a wave with a sufficient number of oscillations inside the domain will contribute significantly to the statistics and its modal structure will be represented quite well even though a related non-vanishing wave-activity flux may exist across the boundaries.

4.2 The Analysis

In the present study, the basic state is taken to be the time mean. This means that waves with periods greater than five days (including waves with zero phase speed) will contribute to define the basic state. In order to assess the relative importance of stationary gravity waves (mainly excited by orography and not described by the present ENM analysis) versus transient waves, the standard deviation of the horizontal divergence of the velocity field is calculated over the Rockies. The horizontal divergence is utilized in order to filter out rotational modes. We will then proceed to the ENM analysis.

Note that by taking the basic state to be a time average of the actual flow over five days, it is not necessarily a solution of the steady state nonlinear primitive equations. However, this time average is found in the present work to be fairly zonally symmetric and in approximate gradient wind balance. Hence, it is expected that the chosen basic state will be a good approximate solution. Moreover, the approximate zonal symmetry implies that a nearly degenerate (in the frequencies) eigenvalue problem will be solved when the ENM analysis will be done since no zonal Fourier decomposition is performed on the data.

4.2.1 The horizontal divergence of the velocity field over the Rockies

At mid-latitudes, the horizontal divergence of the velocity field can reveal dominating inertia-gravity modes. It appears that these are most easily seen using this field since rotational modes are then filtered out. Figs. 9a, and 9b show the time average of $\nabla_H \cdot \vec{v}$, and Fig. 9c shows a vertical profile of its standard deviation at a point over the Canadian Rockies (50°N and 120°W). The standard deviation SD is obtained using

$$SD = \langle (\nabla_H \cdot \vec{v} - \langle \nabla_H \cdot \vec{v} \rangle)^2 \rangle^{1/2}.$$

This indicates that the transient component of the gravity wave signal is dominant in the upper stratosphere and in the mesosphere, and that the steady component is more important in the troposphere. In the lower stratosphere (from 200 to 10 hPa), the transient and steady components are more or less of the same magnitude. Here, the relative magnitude of the steady and transient components of the gravity wave signal is measured by the ratio of the time mean with the standard deviation at 50°N and 120°W. Part of the stationary wave packet energy is found to be absorbed in the vicinity of the critical level $ku_0 = f$ (the so-called “Jones’ critical level”, Jones 1967) near 20 hPa, where f is the Coriolis parameter, k is the dominant horizontal wave number ($\sim 2\pi/600$ km, or $\sim 2\pi/6\Delta x$) measured directly from the divergence field, and u_0 is the background wind at 20 hPa (~ 10 m s⁻¹). Interestingly, the vast majority of gravity wave absorption events observed in this study occurs near critical levels. Fig. 9c also indicates that part of the transient component is absorbed near 20 hPa at 50°N and 120°W. This is due to the absorption of nonstationary mountain waves generated by the temporal fluctuations of the large scale flow (see section 4.2.2).

It is interesting to understand what are the sources of the unsteady gravity wave signal (the gravity waves that are not steady mountain waves). The unsteadiness of the surface wind blowing across the mountain range can explain part of the gravity wave spectrum, but as it will be seen later, the jet stream is also thought to be a source of inertia-gravity waves (IGWs). It was not possible to identify IGWs emerging from the jet stream using the horizontal divergence of the velocity field, even though the

Time mean of the horizontal divergence ($\times 10^{-6} \text{ s}^{-1}$) at 211 hPa

Figure 9a. Time mean of the horizontal divergence of the velocity field obtained from the GFDL SKYHI model for the virtual time Dec. 25 1983 (00h00) to Dec. 29 1983 (23h00) at 211 hPa. The arrow indicates the line where a vertical cut has been taken (see Fig. 9b). The center of the depicted region is at 46.4°N and 114.6°W.

Time mean of the horizontal divergence($\times 10^{-6} \text{ s}^{-1}$) over the Rockies

Figure 9b. Vertical cut of the wave shown in Fig. 9a taken from 45°N and 130°W to 55°N and 110°W.

Figure 9c. Vertical profile of the standard deviation of the velocity horizontal divergence at 50°N and 120°W.

Empirical Normal Mode analysis revealed that gravity wave activity was initiated at the jet stream level (see the sections on the ENM analyses and on the Eliassen-Palm flux divergence that follow). The signature of IGWs generated by the jet stream, when using $\nabla_H \cdot \vec{v}$, could be undetectable because those waves are very weak compared with the orographic gravity waves near the tropopause.

4.2.2 The ENM analysis

As a first attempt to extract inertia-gravity modes from the simulated dynamical system, we used a simplified version of the ENM theory described above: the basic wind in the matrix $B_{(p)}$ (see the Appendix A) has been put to zero, which reduces the pseudoenergy to the wave energy. This simplification is based on the previous analysis of Brunet and Vautard (1996) showing that for large wave numbers ($s > 4$, where s is the zonal wave number), the empirical modes obtained from the full ENM analysis are very similar to those obtained by defining the basic wind to be a solid body rotation. In this study, we will then neglect shear processes in the matrix $B_{(p)}$ when establishing the dynamical basis. Despite this simplification, we expect that this approximation is quite reasonable for the high-wavenumber structures obtained in this work. Moreover, this reduction was also adopted in the present work because of the potential instability of the chosen basic state (the time mean of the flow variables). In Eulerian and isentropic coordinates, the term

$$d\psi_0/dP_0 \equiv -\frac{au_0\sigma_0}{\partial P_0/\partial\phi}$$

is obviously problematic when the meridional gradient of the basic state potential vorticity changes sign somewhere in the domain (potentially unstable flow). When the basic state is defined using a relatively long time series, this problem may be eliminated if the meridional derivative of the (long term) mean Ertel potential vorticity is monotonic, and this seems to be the case (e.g., Brunet 1994). In the situation presented here, a five day average was not sufficient to remove the non-monotonic character of the time average potential vorticity. It is also important to understand

that neglecting spatial variations of the basic flow in the matrix B_p does not mean that the empirical modes will not “feel” the presence of the actual background flow (for example, the impact of critical levels), it simply means that the method may not be as accurate in isolating or separating normal modes when their length scales are of the same order as the length scale of the background spatial variation. Also, if critical levels only influence wave propagation at their vicinity (and not all over the analyzed domain), it is expected that wave energy (instead of pseudoenergy) could be used in order to define the modes orthogonal relationship since the “volume” of the critical layers can be considered small as compared to the entire volume of the analyzed domain.

4.2.2.1 The principal components

In the present analysis, the monochromatic character of the PCs is pronounced as can be seen in Figs. 10a,b, and one would be tempted to conclude that the linear nature of the dynamics is important. However, amplitude modulation is still present in many of the PCs. This seems to indicate that if inertia-gravity waves are mainly linear in their propagation properties, some of their excitation processes which depend on the large-scale flow are not necessarily linear.

Figs. 11 shows the distribution of absolute wave periods weighted by the relative norm

$$N_n = \frac{(f'_n, f'_n)}{\sum_m (f'_m, f'_m)}$$

for all the modes. From now on, and for the rest of the text, this norm will be called the “variance”. Note incidentally that f'_n must be non-dimensionalized for the preceding equation to make sense:

$$u' \rightarrow a\Omega u'$$

$$v' \rightarrow a\Omega v'$$

$$\theta' \rightarrow \left(\frac{\Omega a^2 (PVU) p_r}{g \kappa c_p} \right)^{1/2} \theta'$$

Figure 10a. Some principal components for the analysis over the Rockies from 618 to 3.9 hPa. Modes 5 and 6 describe Rossby wave propagation, and modes 41 and 42 describe inertia-gravity modes.

Figure 10b. Some principal components of the GFDL SKYHI model for the analysis made over the Tropics from 62 to 0.03 hPa. Note that the PCs are less monochromatic than in Figure 2a.

Figure 11a. Distribution of absolute wave periods weighted by the variance explained by each mode (from 618 hPa to 3.9 hPa) over the Rockies.

Figure 11b. The same as Fig. 11a, but from 62 hPa to 0.03 hPa.

Figure 11c. The same as Fig. 11a, but over the Pacific Ocean.

Figure 11d. The same as Fig. 11c, but from 62 hPa to 0.03 hPa.

Figure 11e. The same as Fig. 11a, but over the Tropics.

Figure 11f. The same as Fig. 11e, but from 62 hPa to 0.03 hPa.

$$\frac{\partial \theta_0}{\partial p} \rightarrow \frac{(PVU)}{g\Omega} \frac{\partial \theta_0}{\partial p},$$

where a is the radius of the Earth (6.37×10^6 m), Ω is its rotation rate (7.29×10^{-5} s $^{-1}$), and PVU stands for Potential Vorticity Units (10^{-6} K m 2 kg $^{-1}$ s $^{-1}$). The wave energy equation and f'_n are thus non-dimensionalized. Note that under this norm the ENM are not orthogonal. Here, the period of a mode is taken to be the period at which the spectrum of its associated PC is maximum. In Figs. 11, it is seen that the high-frequency wave spectrum is relatively greater in the Tropics than at mid-latitudes, and that the portion of the signal explained by high-frequency waves is similar for oceanic and mountainous regions at mid-latitudes. The Tropics are the only analyzed region where a dominance of high-frequency waves over low-frequency waves is observed. Note that peaks with periods greater than about 20 hours are not representative since the time series is only 120 hours long.

4.2.2.2 The 3-Dimensional wave structures

4.2.2.2.1 Mountainous region

Over the Rockies in the troposphere, the modes explaining the most variance are evidently those associated with Rossby waves. Figs. 12a,b show the horizontal and vertical structure of the second mode in importance out of 120. It represents a vertically evanescent Rossby wave with zonal wave number of about 7. In the stratosphere and mesosphere analysis, Rossby waves of zonal wave number equal or greater than 4 are almost absent due to their absorption in the lower stratosphere (Charney and Drazin 1961). The first modes observed in the middle atmosphere have large-scale structures that show no evidence of coherence and propagation. This is certainly due to the fact that the window size is too small to resolve stratospheric planetary Rossby waves. However in the stratosphere, localized structures with a much finer scale can be observed in the leading empirical normal modes. These structures seem to be associated with westward propagating gravity waves relative to the mean flow (see Fig. 13 for an example) with relatively high vertical wave

V Component of the wind at 537 hPa (mode 2) in m s^{-1}

Figure 12a. Horizontal cut at 537 hPa of the V component of the wind (on a polar stereographic grid) for the second mode in importance (out of 120) explaining 22% of the variance for the analysis of the GFDL SKYHI model. The wavelength is about 4000 km and the absolute period is around 60 hours.

V Component of the wind (vertical cut) for mode 2 in m s^{-1}

Figure 12b. The same as in Fig. 12a except that a vertical cut of the mode is shown. This wave corresponds to a zonal wave number of about 7, and does not propagate into the stratosphere. The cut is taken from 42°N and 160°W to 42°N and 90°W.

U Component of the wind (m s^{-1}) for mode 1

Figure 13. Vertical cut of an inertia-gravity wave moving westward relative to the mean flow and observed in the leading empirical normal modes. The level of absorption corresponds to a critical level $\hat{\omega} = f$. The cut is taken from 45°N and 130°W to 55°N and 110°W .

numbers. These wavy structures could be related to a slow temporal modulation of the quasi-stationary mountain waves described earlier and depicted in Figs. 9a and 9b.

An unsteady incident wind on a mountain ridge can generate propagating gravity waves (Lott and Teitelbaum 1993; Laprise 1993). In the present case, even if the basic wind is taken to be time-independent, the large-scale flow is still time-dependent and can excite phase propagating inertia-gravity waves. Fig. 14 shows the time evolution of the wind just west of the Rockies (at 45°N) near the surface. Propagating mountain waves are thus expected in the present simulation. Fig. 15 shows such a mode which is eastward propagating (this mode has been obtained by doing a separate analysis ranging from 973 to 81 hPa). It is absorbed in the upper troposphere where it encounters a Jones' critical level. Actually, most of the propagating mountain waves diagnosed are absorbed in the upper troposphere and do not propagate into the middle atmosphere. It was observed in this analysis that most of the westward (eastward) propagating tropospheric inertia-gravity waves were located on the west (east) side of the Rockies.

An interesting feature of the 3-D modes is that the origin of the propagating waves can sometimes be traced back and thus can give useful information about moving gravity wave sources, if the length of the analyzed time series is of the same order as the mechanism producing the waves. Fig. 16 shows inertia-gravity wave generation with horizontal wavelength of about 600 km and absolute period of about 3.5 hours near the jet stream. This mode is the 70th out of 120. The necessary condition for instability is met near the jet (i.e., $d\psi_0/dP_0$ changes sign in the jet core). Vertical profiles of the time mean buoyancy frequency and zonal wind are shown in Figs. 17a and 17b respectively. Sutherland *et al.* (1994) and Sutherland and Peltier (1995) studied gravity wave emission caused by hydrodynamic instability with similar profiles. Their analyses based on two dimensional models show that linear processes can lead to spontaneous gravity wave generation under certain conditions. Other nonlinear mechanisms are thought to emit gravity waves (Davis and Peltier 1979;

Figure 14. The zonal wind at 922 hPa just west of the Rockies. This shows that propagating mountain waves are expected. The profile is taken at 45°N and 127°W .

U component of the wind (m s^{-1})

Figure 15. Vertical cut of an eastward propagating inertia-gravity wave in the troposphere with wavelength of 620 km and absolute period of 7.2 hours and absorbed near the tropopause.

U component of the wind (m s^{-1})

Figure 16. Inertia-gravity wave excitation, with wavelength of 600 km and absolute period of 3.5 hours, at the jet stream level. This mode is the 70th in importance out of 120. The cut is taken from 42°N and 114°W to 44°N and 90°W.

Figure 17a. Time mean (over five days) of the square of the buoyancy frequency calculated in a column where the wave presented in Fig. 16 is excited.

Figure 17b. The same as in Fig. 17a, except that the mean zonal wind is shown.

Figure 17c. The Richardson number for profiles shown in Figs. 17a and 17b.

Time mean of the Rossby number ($\times 10^{-2}$)

Figure 17d. Vertical cut of the time mean Rossby number taken from 42°W and 114°W to 44°N and 90°W.

Fritts 1982 and 1984a), and determining a single instability mechanism responsible for the emission of the mode described in Fig. 16 is not straightforward. The minimum Richardson number of the basic flow calculated in a column where inertia-gravity waves are generated is of the order of 8 (see Fig. 17c). Gravity wave generation by Richardson instabilities is thus unlikely. Fritts and Luo (1992) and Luo and Fritts (1993) investigated inertia-gravity wave emission by geostrophic adjustment of an idealized jet stream and studied, using linearized equations, the spontaneous radiation of inertia-gravity waves for an unbalanced initial state. O'Sullivan and Dunkerton (1995) examined the possibility of inertia-gravity wave emission near the jet stream due to an unstable baroclinic development causing geostrophic adjustment at the jet stream level. In our case, inertia-gravity wave emission near the jet stream occurs throughout the five days of analysis. Fig. 17d shows a vertical cut of the time mean Rossby number Ro ($\equiv |\vec{v} \cdot \nabla \vec{v}|/|f\vec{v}|$) just East of the Rockies near 43°N.

Peaks of $Ro \sim 0.8$ to 1.2 are frequently observed on the upper and lower flanks of the SKYHI jet stream and geostrophic adjustment could be expected if balanced ageostrophic motion is not prevailing. Another possibility could be that these waves are generated by a sustained nonlinear and ageostrophic jet stream flow through triad interactions. Bartello (1995) showed using analytic arguments and numerical results that at Rossby numbers of order one, slow modes can interact to generate a fast mode and that geostrophic adjustment may not take place. In order to establish whether geostrophic adjustment or a sustained nonlinear and ageostrophic flow is responsible for this wave emission, a correlation between the Rossby number and the time evolving wave energy of the modes describing waves emerging from the jet stream has been calculated. It turns out that near the jet stream, there is no clear correlation between the Rossby number and the inertia-gravity wave energy. This is an indication that geostrophic adjustment may not be responsible for the presence of these inertia-gravity waves over the jet. Since the Rossby number is relatively large throughout the five days, nonlinear downscale energy transfer as a gravity wave source cannot be excluded. In fact, a longer time series with a succession of linear and nonlinear regimes would be needed in order to resolve this issue. Note that other

modes generated at the jet stream level were observed in the middle atmospheric window and showed in some cases well defined wave propagation and absorption at higher levels in the mesosphere. Fig. 18 shows such a wave absorption near the top of the model probably caused by a Jones' critical level.

4.2.2.2.2 Oceanic region

Over the Pacific Ocean, inertia-gravity waves generated near the surface can be observed (see Fig. 19 for an example). The background flow and the wave characteristics are such that part of the wave packet reaches a Jones' critical level near 700 hPa and are absorbed there. The dynamical source of these waves could be attributed to the boundary layer instability since Richardson numbers less than $1/4$ (and negative) are frequently measured in the model boundary layer over the ocean.

The jet stream over the Pacific has a mean velocity that reaches a maximum value of the order of 55 m s^{-1} , see Fig. 20, and a lot of wave absorption due to singular levels ($\bar{\omega} = \pm f$ and 0) was observed just above. This jet strength is to be compared to the $\sim 30 \text{ m s}^{-1}$ over the Rockies. For an inertia-gravity wave described by an absolute period of 5 hours and a horizontal wavelength of 520 km, the three singular levels are approximately located between 100 and 60 hPa above the jet stream (Fig. 21). Note that a wavy pattern is located above the critical levels and amplifies with height due to non-Boussinesq effects. In this case, the critical levels in the lower stratosphere and in the troposphere make it difficult to verify if the jet stream is the source of these IGWs.

Modes 7 and 8 of the stratospheric and mesospheric analysis were observed to oscillate at the semidiurnal frequency. Their horizontal structure is too large to be fully described by the empirical modes obtained from this limited area analysis. Their vertical wavelength is about 20 km, and the amplitude of the potential temperature and wind velocity at 0.1 hPa associated with these modes are 10 K and 4 m s^{-1} , respectively. These modes probably describe thermal tides. A clear monochromatic diurnal signal was not observed in our analysis.

U Component of the wind (m s^{-1})

Figure 18. Wave absorption in the mesosphere of a mode with the same characteristics as the one shown in Fig. 16 and at the same horizontal location.

U Component of the wind (m s^{-1}) for mode 24

Figure 19. Wave absorption by a Jones' critical level in the lower troposphere near 700 hPa. The horizontal wavelength is ~ 550 km and the absolute period is ~ 6 hours. The vertical cut is taken from 43°N and 160°E to 43°N and 175°E . This mode was obtained from an analysis ranging from 973 to 81 hPa.

Figure 20. Typical profile of the mean zonal wind over the Pacific at mid-latitudes modeled by the GFDL SKYHI. Profile taken at 43°N and 168°E.

U Component of the wind (m s^{-1}) for mode 50

Figure 21. Vertical cut along a latitude band of the zonal component of the wind for mode 50 showing the effect of critical levels just above the jet stream on a gravity wave with absolute period of 5 hours and horizontal wavelength of 520 km. The vertical cut is taken from 40°N and 175°E to 40°N and 170°W.

4.2.2.2.3 Tropical region

Over Central America, inertia-gravity wave activity was high during the five days of analysis. Empirical modes with IGW characteristics were found to explain at least 30% of the signal in the upper troposphere and lower stratosphere. Much of the IGWs were generated in the upper troposphere (the Richardson number was lower than 0.25 around 200 hPa on many occasions during the five days of analysis), and a significant portion of the eastward propagating inertia-gravity waves at 10°N was found to be absorbed near the tropopause due to the presence of persistent westerlies around 200 hPa. One striking feature of the analysis made below 3.9 hPa is the monochromaticity of the modes. The first 40 modes (out of 120), when they are ordered according to their variance, are almost free of amplitude modulation, indicating an almost pure oscillatory regime.

The most important IGW modes with wavelength greater than 1000 km were propagating westward relative to the time mean flow. Eastward propagating IGW modes were not predominant and were not found before mode 26, but the eastward IGW with phase speed greater than about 30 m s^{-1} were able to reach the stratosphere. Also, it was often observed that for relatively monochromatic and high-frequency ENMs, the structures consisted of the superposition of a pattern tilting eastward and another tilting westward. In Fig. 22a we see such a pattern composed of an eastward and a westward wave with approximately the same amplitude. Fig. 22b is another example of some symmetry that could occur in the generation process of IGWs by convection. The total momentum transported upward by the modes shown in Figs. 22a,b is almost zero since the contribution of the component with westward tilt (transporting momentum downward) is almost exactly balanced by the component with eastward tilt (transporting momentum upward). Since symmetric patterns are observed mainly in relatively high-frequency modes (with relatively low variance), the impact on the mean flow by breaking IGWs will come from relatively low frequency IGWs. The nature of the source of these IGWs can be examined by looking at the vertical velocity structure in the upper troposphere. Standing (non-traveling horizontally) convective cells are numerous in the upper tropical troposphere

U Component of the wind (m s^{-1}) for mode 44

Figure 22a. Standing pattern with a period of 5.5 hours and horizontal wavelength of 820 km. The vertical cut is taken from 10°N and 100°W to 10°N and 80°W.

U Component of the wind (m s^{-1}) for mode 32

Figure 22b. Symmetric pattern with a period of 7.5 hours and a horizontal wavelength of 550 km. The vertical cut is taken from 10°N and 120°W to 10°N and 90°W.

of the SKYHI model. These standing and ascending cells can act as obstacles by impinging on the flow of stratified air at levels just above them (this is the so-called “obstacle effect”, Clark *et al.* 1986). The positive vertical shear in the flow observed just over the tropopause near 10°N is consistent with the predominance of IGWs with easterly phase speeds relative to the basic flow in the stratosphere, as is shown in Fig. 23. In Fovell *et al.* (1992), it is argued using a numerical model that squall lines are more efficient than the “obstacle effect” in generating gravity waves that propagate to the stratosphere. The basic mechanism responsible for the gravity wave emission by squall lines is shown to be due to oscillatory updrafts within the storm. But the present resolution is of course too low to see them.

In the tropical upper stratosphere and mesosphere, a broadening of the principal component spectra is observed for the first 20 modes (see Fig. 10b for some examples). This indicates that variations of the background flow may be of primary importance in the propagation of these waves, or that nonlinearities are strong, since we expect in this wave energy analysis these factors to widen the overall PC spectra. Modes 5 to 10 (not shown) represent somewhat disturbed patterns with mixed eastward and westward tilt, and an horizontal and vertical wavelength of about 2000 to 3000 km and ~15 km, respectively. The origin of the waves represented by these patterns is certainly located outside the analyzed window. The subsequent modes (number 20 and higher) with horizontal wavelength ranging from 600 to 1000 km are almost monochromatic (in their time series) and an almost pure oscillatory regime is observed. As can be seen in Figs. 11e,f, the high-frequency component of the flow is relatively more important in the upper region of the middle atmosphere.

U component of the wind (m s^{-1}) for mode 20

Figure 23. Westward propagating inertia-gravity wave with an absolute period of 12 hours and a horizontal wavelength of 750 km. The vertical cut is taken from 10°N and 110°W to 10°N and 80°W.

4.2.3 Pair identification

Phase propagating waves (or wave packets) must necessarily be described by at least two modes, whereas it is entirely sufficient to characterize a standing pattern by a single mode. This is easily seen when considering a simple monochromatic wave propagation in one dimension:

$$\cos(kx - \omega t) = \cos \omega t \cos kx + \sin \omega t \sin kx.$$

On the other hand, a propagating pulse with arbitrary shape $f(x - ct)$ (non-dispersive) is generally described by an infinity of independent modes. The mode frequencies being $\omega = ck$, where c is the constant phase speed. This means that when the spatial structures of some of the Empirical Normal Modes are found to be almost monochromatic (which is not necessarily the case since no *a priori* zonal Fourier decomposition is performed in the present analysis), two modes should be approximately sufficient in order to describe a propagating wave structure. When spatial modes have a broad Fourier spectrum, it is not possible to identify pairs of modes that describe moving wave fronts due to the inherent dispersive character of most atmospheric disturbances. In the present situation, many modes have a monochromatic structure in some specified location inside the domains of analysis. WKB assumptions are used to describe phase propagation at these locations. Note that the approximate zonal symmetry of the basic state implies a nearly degenerate eigenvalue problem, and that this contributes to broaden the spatial Fourier spectrum of the modes.

Two modes describing an advancing wave must necessarily be in quadrature (the phase between the two modes must, spatially and temporally, be near $\pi/2 \approx 1.57$), must have the same frequency and wavenumber, and ideally, the modulus $a_i^2(t) + a_j^2(t) = K_{ij}(t)$ must be constant in time. Table 1 shows some possible pairs, the period associated with the maximum of their spectrum, their relative phase angle α , and the variance of $K_{ij}(t)$ divided by its mean (in this Table, the principal components are normalized so that the time mean of $K_{ij}(t)$ is two). The relative phase angle α has been obtained by calculating the lag-correlation

$$C(\alpha) = \langle a_i(t) a_j(t + \alpha/\omega) \rangle,$$

where ω is the frequency of the pair (i, j) . The phase angle is set by finding the maximum of $|C(\alpha)|$.

Table 1a. Rockies (618 to 3.9 hPa)				
PC 1	PC 2	Period (T)	Phase lag (α)	$K_{ij}/\langle K \rangle$
		(hours)	(radians)	
1	2	120.0	3.142	.678
3	4	60.0	1.676	.601
5	6	40.0	1.571	.677
7	8	30.0	3.142	.774
9	10	24.0	1.571	.754
11	12	20.0	1.571	.696
13	14	17.1	3.299	.614
15	16	15.0	1.676	.793
15	17	15.0	3.351	.791
16	17	15.0	3.351	.576
18	22	12.0	3.142	.769
19	20	13.3	2.356	.671
23	24	10.0	1.885	.664
23	25	10.0	3.142	.842
24	25	10.0	1.257	.669
26	27	9.2	1.361	.556
29	30	8.0	1.571	.497
31	32	7.5	1.676	.733
31	33	7.5	1.676	.816
32	33	7.5	1.676	.863
34	35	7.1	1.780	.920
36	37	6.7	1.885	.677
38	39	6.3	1.990	.611
40	41	5.7	1.100	.718
42	43	6.0	1.047	.886
44	45	5.5	1.152	.528
46	49	5.0	3.770	.750
47	48	5.2	1.204	.596
50	51	4.8	1.309	.634
52	53	4.6	1.361	.710
52	54	4.6	2.723	.672
53	54	4.6	1.361	.682
56	57	4.3	1.466	.624
56	58	4.3	1.466	.944
57	58	4.3	1.466	.820
59	60	4.1	1.518	.568
61	62	4.0	1.571	.774
64	65	3.8	1.676	.653
64	66	3.8	1.676	.993
65	66	3.8	1.676	.734
67	68	3.6	1.728	.551
69	70	3.5	1.780	.706

Table 1b. Rockies (62 to 0.03 hPa)

PC 1	PC 2	Period (T) (hours)	Phase lag (α) (radians)	$K_{ij}/\langle K \rangle$
1	2	120.0	1.937	.549
3	4	60.0	3.142	.865
3	8	60.0	3.142	.792
4	8	60.0	1.466	.734
5	6	17.1	1.466	1.017
9	10	13.3	1.414	.839
11	12	15.0	3.351	.915
13	14	20.0	1.571	.672
15	18	12.0	1.571	.962
16	17	24.0	3.142	.730
19	20	10.9	1.728	.700
21	22	30.0	2.304	.852
23	24	10.0	1.885	.636
25	26	9.2	1.361	.547
25	27	9.2	3.403	.745
26	27	9.2	1.361	.599
28	29	8.6	1.466	.572
31	32	7.5	1.676	.555
33	34	7.1	1.780	.706
35	36	6.7	1.885	.756
37	38	6.3	1.990	.546
39	40	6.0	2.094	.600
39	41	6.0	3.142	.773
40	41	6.0	2.094	.758
42	45	5.7	1.100	.596
43	44	5.5	1.152	.639
46	47	5.2	1.204	.689
48	49	5.0	3.770	1.067
50	51	4.8	1.309	.545
52	53	4.6	1.361	.601
54	55	4.4	1.414	.786
54	56	4.4	1.414	.802
55	56	4.4	1.414	.802
58	59	4.1	1.518	.720
58	62	4.1	3.037	.880
58	63	4.1	1.518	.735
59	62	4.1	3.037	.819
59	63	4.1	3.037	.706
60	61	3.9	1.623	1.051
60	65	3.9	1.623	.746
61	65	3.9	1.623	.918
62	63	4.1	1.518	.669
67	68	3.6	1.728	.653
67	69	3.6	1.728	.705
68	69	3.6	1.728	.706
70	71	3.5	1.780	.757

Table 1c. Mid-latitude Pacific Ocean (618 to 3.9 hPa)				
PC 1	PC 2	Period (T)	Phase lag (α)	$K_{ij}/\langle K \rangle$
		(hours)	(radians)	
1	2	60.0	1.571	.711
3	4	120.0	3.142	.593
5	6	30.0	3.142	.758
7	8	40.0	2.356	.706
9	12	20.0	3.142	.671
10	11	24.0	1.833	.545
13	14	17.1	3.299	.689
15	16	15.0	1.676	.711
18	19	12.0	1.571	.642
18	20	12.0	3.142	.940
19	20	12.0	3.142	1.008
21	22	10.9	1.728	1.213
23	24	10.0	1.885	.884
25	26	8.6	1.466	.901
27	28	9.2	1.361	.875
29	30	8.0	1.571	.604
31	32	7.1	3.560	1.041
31	36	7.1	1.780	.815
32	36	7.1	1.780	.897
33	40	6.3	1.990	.738
34	35	7.5	1.676	.587
37	38	6.7	1.885	.603
39	43	5.7	3.299	.646
39	44	5.7	2.199	.695
41	42	6.0	1.047	.682
43	44	5.7	1.100	.464
47	48	5.0	1.257	.622
49	50	4.8	1.309	.671
49	52	4.8	1.309	.661
50	52	4.8	1.309	.631
51	54	4.6	1.361	.635
51	55	4.6	2.723	.713
53	56	4.4	1.414	.693
54	55	4.6	1.361	.643
57	58	4.3	1.466	.768
57	60	4.3	1.466	.565
58	60	4.3	1.466	.648
61	62	4.0	1.571	.494
61	64	4.0	1.571	.578
62	64	4.0	3.142	.563
63	66	3.9	1.623	.567
65	68	3.8	1.676	.480
67	70	3.6	1.728	.558
69	72	3.5	1.780	.588

Table 1d. Mid-latitude Pacific Ocean (62 to 0.03 hPa)				
PC 1	PC 2	Period (T)	Phase lag (α)	$K_{ij}/\langle K \rangle$
		(hours)	(radians)	
1	2	120.0	2.042	.726
3	4	60.0	1.780	.503
5	6	40.0	3.142	.772
7	12	15.0	3.351	.978
8	16	13.3	2.356	.927
9	10	20.0	1.571	.788
9	15	20.0	2.827	.998
10	15	20.0	3.142	.867
11	14	24.0	3.142	.914
17	18	30.0	3.142	.891
19	20	10.9	1.728	.883
21	22	12.0	1.571	.882
23	24	10.0	1.885	.731
25	26	9.2	1.361	.671
27	28	8.6	1.466	.655
29	30	8.0	1.571	.597
31	32	7.5	1.676	.712
31	33	7.5	1.676	.691
32	33	7.5	1.676	.588
35	36	6.7	1.885	.664
37	38	6.3	1.990	.630
39	40	6.0	2.094	.627
41	42	5.7	1.100	.731
43	44	5.5	1.152	.722
45	46	5.2	1.204	.738
47	48	5.0	3.770	.672
49	50	4.8	1.309	1.072
51	54	4.4	1.414	.762
51	57	4.4	1.414	.781
52	53	4.6	1.361	.741
54	57	4.4	1.414	1.109
55	56	4.3	1.466	.744
58	59	4.0	1.571	1.196
58	62	4.0	1.571	.894
59	62	4.0	3.142	1.274
60	61	4.1	1.518	.677
63	64	3.9	1.623	.899
66	67	3.5	1.780	.659
68	69	3.6	1.728	.718
68	70	3.6	1.728	.996
69	70	3.6	1.728	.900

Table 1e. Central America (618 to 3.9 hPa)				
PC 1	PC 2	Period (T) (hours)	Phase lag (α) (radians)	$K_{ij}/(K)$
		120.0	1.833	.311
1	2	60.0	1.676	.545
3	4	40.0	1.728	.532
5	6	40.0	2.042	.535
5	7	40.0	3.142	.495
6	7	24.0	1.833	.491
9	10	24.0	3.142	.549
9	11	24.0	3.142	.494
10	11	17.1	1.466	.471
13	14	15.0	1.676	.538
15	16	13.3	1.885	.543
17	18	13.3	3.299	.618
17	19	13.3	3.299	.541
18	19	12.0	1.571	.527
20	21	10.9	1.728	.478
22	23	10.0	1.885	.563
24	25	9.2	3.403	.638
26	27	8.6	1.466	.624
28	29	8.0	1.571	.648
30	31	7.5	1.676	.482
32	33	7.1	3.560	.531
34	35	6.7	1.885	.613
36	37	6.7	2.827	.630
36	38	6.7	1.885	.562
37	38	6.3	1.990	.474
39	40	6.0	1.047	.603
41	42	5.7	1.100	.674
43	44	5.5	1.152	.508
45	46	5.2	1.204	.597
47	48	5.0	3.770	.502
49	50	5.0	2.513	.486
49	51	5.0	3.770	.605
50	51	4.8	1.309	.523
52	53	4.4	1.414	.637
55	56	4.4	2.827	.594
55	57	4.4	1.414	.532
56	57	4.3	1.466	.555
58	59	3.9	1.623	.547
61	64	4.0	1.571	.603
62	63	3.6	1.728	.618
65	68	3.6	3.456	.669
65	69	3.5	1.780	.541
67	70	3.5	1.780	.535
67	71	3.6	1.728	.492
68	69	3.5	1.780	.531
70	71			

Table 1f. Central America (62 to 0.03 hPa)				
PC 1	PC 2	Period (T)	Phase lag (α)	$K_{ij}/\langle K \rangle$
		(hours)	(radians)	
1	2	120.0	1.990	.407
3	4	60.0	1.571	.543
5	8	24.0	1.309	1.015
6	7	17.1	3.299	.704
6	9	17.1	3.299	.817
7	9	17.1	2.199	.720
10	11	40.0	2.199	.894
12	13	30.0	3.142	.860
12	15	30.0	3.142	.976
13	15	30.0	2.932	.707
17	18	13.3	1.885	.787
19	20	12.0	1.571	.837
21	22	10.9	1.728	.638
23	24	10.0	1.885	.639
25	26	9.2	1.361	.593
27	28	8.6	1.466	.516
29	30	8.0	1.571	.686
31	32	7.5	1.676	.583
33	34	7.1	1.780	.569
33	35	7.1	2.670	.496
34	35	7.1	3.560	.599
36	37	6.7	1.885	.594
38	39	6.3	1.990	.575
40	41	6.0	1.047	.674
42	43	5.7	1.100	.591
44	45	5.5	1.152	.555
46	47	5.2	1.204	.659
48	49	5.0	3.770	.651
50	51	4.8	1.309	.558
52	53	4.6	1.361	.598
54	55	4.4	1.414	.625
56	57	4.3	1.466	.653
58	59	4.1	1.518	.543
60	61	4.0	1.571	.530
62	63	3.9	1.623	.486
64	65	3.8	1.676	.536
66	67	3.6	1.728	.640
68	69	3.5	1.780	.547
70	71	3.4	1.833	.596

In a scatter diagram of a pair of modes in phase space (a plot of a_i versus a_j with time as the parameter), a perfect coupling would give a circle of constant radius. Noise, external forcing mechanisms, and nonlinear interactions will usually disturb this idealized picture, and a diagram of two modes which seem to form a pair can

be unreadable. A statistical treatment is thus necessary through an estimation of a joint probability distribution function. The optimal density technique (see Silverman, 1986; Brunet, 1994; and the Appendix D of the present work) has been used with a Gaussian kernel to study phase propagation in a statistical sense. Moreover, each component of the tendency vector $d\vec{a}_{ij}/dt \equiv (da_i/dt, da_j/dt)$ can be estimated with the help of the kernel estimator. Note that for a perfect coupling of two stable modes, the tendency vectors are everywhere tangent to the scattered diagram with equal lengths, which is the angular frequency of the wave. For more realistic cases, this tendency is estimated using

$$\frac{f(d\vec{a}_{ij}/dt, \vec{a}_{ij})}{f(1, \vec{a}_{ij})},$$

see the Appendix D.

Figs. 24 show estimated joint probability distribution functions (PDFs) and tendencies for some selected coupled ENMs. In fact, only pairs having the same frequency, a relative phase lag near $\pi/2$, and a relatively “low” value of the K_{ij} variance are shown. In Figs. 24, the mean vector lengths of the slowest waves correspond well with the wave frequencies obtained from the Fourier spectrum of the principal components. However, the tendency values of the fastest waves are underestimated due to the inaccuracy of the time derivative (which is calculated, in the latter case, on a signal with fewer points per wavelength). Note that the sense of rotation is arbitrary and does not indicate the direction of propagation of the waves. Also, when two principal components do not form a pair, the tendency diagram shows no consistency and no definite rotation (not shown).

Figure 24. Joint probability distribution function and time tendencies for some pairs of principal components. Figures a, c, e, g, i, k, m, o, q, and s are the estimated phase space densities. The horizontal and vertical axes are the mode principal components. "Low" ("High") indicates that the analysis has been done from 618 to 3.9 hPa (62 to

.03 hPa). The outer circles represent 10% of the maximum of the density functions. Rockies, Ocean, and Tropic indicate that the analyzed region is located over the Rocky Mountains, the mid-latitude Pacific Ocean, and Central America, respectively. Figures b, d, f, h, j, l, n, p, r, and t are the estimated time tendencies. The arrows

pointing to the right at the bottom left corner of the graphs indicate the length an arrow would have if the period of the pair were: b) 60 hours; d) 4.1 hours; f) 3.5 hours; h) 20 hours; j) 3.5 hours; l) 60 hours; n) 12 hours; p) 4 hours; r) 12 hours;

and t) 12 hours. Note that the tendencies are estimated with the help of the graph located immediately to its left.

Figure 24 (continued)

4.2.4 Polarization relations and hodographs

Ideally, the spatial distribution of an Empirical Normal Mode describing a linear, unforced, and conservative wave phenomenon should reproduce the correct polarization relations among the dynamical fields. Hence, the relative phase and the relative amplitudes of the velocities, the temperature (or the potential temperature), and so on, must obey well defined relations that have been described in Chapter 2. Since no *a priori* zonal wavenumber Fourier decomposition has been done for these ENM analyses, most of the empirical mode spatial distributions will end up being non-monochromatic and a spiral-like hodograph will not be obtained.

The hodograph technique has been used here instead of a more global (in space) statistical evaluation of the ENMs polarization relations because of the overall non-monochromaticity of the modes: it happened that for a given mode, a certain region in space is occupied by a more monochromatic wave, whereas in some other region, waves are non-monochromatic (or there may be no wave at all). Thus, a global evaluation of the polarization relations for a given mode seems useless if one is interested in verifying the relations among the dynamical variables.

Regions occupied by fairly monochromatic waves have been chosen for some empirical modes, and plots of the zonal velocity u_n versus the meridional velocity v_n for some mode number n have been made at a single horizontal location using points at different altitudes. The first thing to note in Figs. 25 is that all the hodographs except one show clear clockwise rotation with increasing altitude (and a greater amplitude of the wind) when rotation is apparent, indicating upward energy propagation. In Figs. 25, the wave amplitudes are generally increasing with altitude. Wave breaking, absorption, or deflection appear as the spirals break down. The only hodograph shown in Fig. 25c which tends to indicate a downward energy propagation has been obtained over the mid-latitude Pacific Ocean in the lower stratosphere where critical levels are present (suggesting that upward propagating waves could sometimes be reflected by them in the model, or that a gravity source region could be located deep in the stratosphere, but this latter scenario is unlikely).

Figure 25a. Some hodographs of the Empirical Normal Mode spatial distributions near the Rocky Mountains. Graph a) goes from 132 to 3.9 hPa at 48.5°N and 106°W ; graph b) goes from 62 to 0.03 hPa at 40.5°N and 107°W ; graph c) goes from 62 to 0.03 hPa at 41.5°N and 105°W ; and graph d) goes from 62 to 0.03 hPa at 35.5°N and 128°W .

Figure 25b. Some hodographs of the Empirical Normal Mode spatial distributions over Central America. Graph a) goes from 132 to 3.9 hPa at 17.5°N and 108°W; graph b) goes from 132 to 3.9 hPa at 14.5°N and 104°W; graph c) goes from 62 to 0.03 hPa at 13.5°N and 115°W; and graph d) goes from 62 to 0.03 hPa at 13.0°N and 94°W.

Figure 25c. Some hodographs of the Empirical Normal Mode spatial distributions over the mid-latitude Pacific Ocean. Graph a) goes from 132 to 3.9 hPa at 46.0°N and 155°W; and graph b) goes from 132 to 3.9 hPa at 36.0°N and 169°E.

The validity of the hodographs can be assessed by calculating the gravity wave intrinsic frequency in three different manners: 1) a direct calculation of $\hat{\omega} = \omega - \vec{k} \cdot \vec{u}_0$; 2) using the dispersion relation

$$\hat{\omega}^2 = f_0^2 + \frac{N^2(k^2 + l^2)}{m^2 + 1/4H^2};$$

and 3) measuring the ratio of the major axis to the minor axis on the hodographs. This calculation has been done for some gravity wave modes. As an example, Fig. 25a (a) shows the hodograph of a wave with the following characteristics (as obtained from the principal component and the spatial distribution of the mode): absolute period of 3.5 hours, horizontal wavelength of 600 km, vertical wavelength of 7 km, basic wind of about 20 m s^{-1} at 7° North of East in the lower stratosphere, angle of propagation of about 25° North of East. The dispersion relation gives an intrinsic wave period of 6.2 hours, the hodograph gives about 6.5 hours, and a direct calculation of $\hat{\omega} = \omega - \vec{k} \cdot \vec{u}_0$ gives 5.8 hours. Such verifications have been done for other inertia-gravity waves with a similar degree of accuracy.

4.2.5 Eliassen-Palm flux divergence

The impact of individual modes on the zonal mean zonal flow and on the residual circulation can be estimated by evaluating the divergence of the Eliassen-Palm flux. In log-pressure coordinates ($z \equiv -H \ln(p/p_r)$), the zonally averaged zonal momentum equation can be written using transformed Eulerian-mean (TEM) variables as in Eq. 2.21. In the present work, the departure quantities have been defined with respect to a time-mean state, but this state is sufficiently zonally symmetric to consider Eq. 2.21 to be relevant (this is especially true in the upper stratosphere and in the mesosphere, where the orographic effects are less prevailing).

In order to quantify the role of each mode on the zonal mean flow and on the residual circulation, the departure quantities in Eq. 2.17 have been replaced by a chosen mode (or a sum of chosen modes) obtained from the simplified analysis described

earlier. The modes relate to their vertical velocity by projecting the departure from the time mean vertical velocity on the principal components:

$$w'_n(\vec{x}) = \langle a_n^*(t) w'(\vec{x}, t) \rangle. \quad (4.1)$$

The eddy forcing of the TEM equations is given by the Eliassen-Palm flux divergence which is schematically written

$$\nabla \cdot \vec{F}_{NL} = \nabla \cdot \left(\sum_{m,n} g_n \vec{G} g_m \right) = \sum_n \nabla \cdot g_n \vec{G} g_n + \nabla \cdot \left(\sum_{m,n (m \neq n)} g_n \vec{G} g_m \right). \quad (4.2)$$

$g_n \vec{G} g_m$ denotes the quadratic form of the Eliassen-Palm flux. The last term on the right-hand side of Eq. 4.2 represents the cross terms and it vanishes when the flux divergence is time averaged, due to the orthogonality of the principal components. The time averaged eddy forcing is thus the sum of the forcing produced by each mode, and it represents non-conservative and nonlinear effects since

$$\left\langle \frac{\partial J}{\partial t} \right\rangle = 0 = -\langle \nabla \cdot \vec{F} \rangle + \langle \vec{S} \rangle + O(\alpha^3), \quad (4.3)$$

where $\langle \bullet \rangle$ denotes a time average, J is a wave-activity, α is the amplitude of wave motion, and S represents an external forcing and nonconservative effects. Note that the nonacceleration theorem states that the divergence of the Eliassen-Palm flux vanishes for unforced linear waves, so they cannot transfer momentum to the mean state (Andrews *et al.* 1987). The calculated budget of the Eliassen-Palm flux divergence can locate the momentum sources and sinks of the flow under study.

We have calculated the time averaged eddy forcing inside the three windows described earlier. Over the Rockies and the Pacific Ocean, it was also averaged over a latitude band ranging from 43°N to 47°N. Over Central America, the zonal average was performed over a latitude band ranging from 8°N to 12°N. Fig. 26 shows the total eddy forcing at each location (in fact, the time average over five days of the right-hand side of Eq. 2.21 is plotted). This was calculated from 262 to 0.07 hPa. Note that the plotted forcing is due solely to transient waves since exactly stationary waves are not described by these empirical modes. Over Western North America (Fig. 26a),

the eddy forcing is somewhat weak in the lower mesosphere ($\sim -7 \text{ m s}^{-1} \text{ day}^{-1}$), and almost no eddy forcing is produced by propagating waves in the lower stratosphere. The most important contribution to the westward drag in the lower mesosphere near 0.3 hPa comes from the first few modes (with absolute period greater than ~ 40 hours). Thus, this negative forcing could be created by large-scale mesospheric perturbations and quasi-stationary mountain waves that reached this level. Mode numbers greater than 20 (which correspond to an absolute period smaller than ~ 11 hours) are almost exclusively tilting eastward (transporting positive momentum upward relative to the basic flow) and exert a positive forcing on the zonal flow near the 0.1 hPa level. If the eddy component of the flow consisted exclusively of a sum of modes ranging from 21 to 120 (representing one third of the total variance), the forcing on the mean zonal flow would reach more than $+5 \text{ m s}^{-1} \text{ day}^{-1}$ in the highest model levels, as is visible in Fig. 27a, showing that these modes are the major contributors to the mean flow acceleration at 0.1 hPa. These modes begin to dissipate (or are partly absorbed) just below the 1 hPa level. Fig. 27b shows the eddy forcing produced by the modes ranging from 21 to 120 in the upper troposphere and lower stratosphere. The source of these waves is seen to be near the jet stream, as the negative value of $-0.27 \text{ m s}^{-1} \text{ day}^{-1}$ indicates. This negative value suggests that waves transporting positive momentum relative to the mean flow are generated (since the production of waves with positive momentum should produce a negative forcing at the locus of emission, and a positive forcing somewhere else in the atmosphere, as is apparent in Fig. 27a). Unfortunately, the vertical velocity field below 321 hPa was not available, and it is impossible to evaluate the deepness of this tropospheric source. Figs. 26a,b also indicate a negative eddy forcing in the upper tropospheric mid-latitude region. This behavior has already been observed in the GFDL SKYHI model by Andrews *et al.* (1983) and is attributed to growing and decaying unstable baroclinic disturbances.

Over the mid-latitude Pacific Ocean, the total eddy forcing exerted by propagating waves is almost everywhere positive (or zero) throughout the middle atmosphere, reaching a maximum of about $30 \text{ m s}^{-1} \text{ day}^{-1}$ at 0.5 hPa ($\sim 53 \text{ km}$). A small eddy forcing of about $10 \text{ m s}^{-1} \text{ day}^{-1}$ is also observed around 10 hPa (32 km) (see Fig

Figure 26. Time and zonally averaged total eddy forcing by propagating waves over: a) Western North America; b) Mid-latitude Pacific Ocean; and c) Central America.

Figure 27. Time and zonally averaged eddy forcing by: a) and b) modes with absolute period smaller than 11 hours over Western North America; c) modes with absolute period smaller than 50 hours over the Pacific Ocean; and d) modes with absolute periods ranging from 4 to 13 hours over Central America.

26b). The time mean Rossby number was still relatively high (reaching maxima of the order of 0.5 to 0.8 in the oceanic jet stream) and inertia-gravity wave emission could be expected. However, the impact of these IGWs on the zonal mean flow and the residual circulation seems marginal since the time mean eddy forcing in the stratosphere and mesosphere is almost exclusively produced by the first two modes (with periods greater than ~ 40 hours) and the contribution from the modes with higher frequencies is weak, as can be seen in Fig. 27c. This could be due to the presence of absorbing critical levels just above the jet stream that could partially inhibit upward inertia-gravity wave propagation, as discussed earlier. Another possibility is that the inertia-gravity wave emission process generated westward and eastward propagating waves at an equal rate.

The eddy forcing over Central America near 10°N is negative almost throughout the model middle atmosphere (Fig. 26c) and it tends to drag the westerly mesospheric winds. This drag is essentially produced by westward tilting inertia-gravity waves, and the main forcing comes from modes with absolute periods ranging from 4 to 13 hours (see Figs. 26c and 27d). This means that the impact of inertia-gravity waves on the mean flow was much more important over Central America than over the Pacific Ocean, as can be seen by comparing Figs. 27c and 27d. Fig. 27c depicts the eddy forcing over the Pacific Ocean at mid-latitudes produced by waves other than the first four modes (which describe large-scale structures), and this forcing is less than $1.5 \text{ m s}^{-1} \text{ day}^{-1}$. In other words, inertia-gravity waves excited by the jet stream at mid-latitudes has a weaker effect on the mean flow and the residual circulation than convectively generated inertia-gravity waves in the Tropics.

Chapter 5

Conclusions

The theory of Empirical Normal Modes of Brunet (1994) was adapted using pseudoenergy for diagnosing atmospheric waves. An Eulerian approach based on the hydrostatic primitive equations in isentropic coordinates was described in order to extract monochromatic and dynamically balanced modes from atmospheric datasets. Moreover, it was shown that the presence of Rayleigh damping and stochastic forcing does not alter the Empirical Normal Mode spatial structures, and that damping and noise are felt only through the broadening of the principal components. The approach was applied in diagnosing relatively small-scale wave phenomena as simulated by the troposphere-stratosphere-mesosphere GFDL SKYHI model. A norm calculated from the wave energy was effectively used in accordance with a WKBJ approximation. This means that spatial variations of the basic state were considered to produce a negligible impact on the empirical basis. The characteristic length scales of the time mean of the fields were much greater than the wave length scale in the model middle atmosphere, giving credibility to the approximation used here.

Three specific locations (the Rocky Mountain area, the mid-latitude Pacific Ocean, and Tropical Central America) were analyzed using a simplified version of the theory. The simplification consists of using the wave energy instead of the pseudoenergy. Despite the fact that variations of the basic state in establishing the matrix B have been neglected, quasi-monochromatic modes with some amplitude modulation were extracted from the dataset. The only empirical normal modes which could not be clearly classified as quasi-monochromatic in this study were the 15 leading mesospheric modes. The small-scale wave phenomena (~ 1000 km or less) were relatively monochromatic (although with some amplitude modulation).

Over the Rockies, steady mountain waves are dominant in the troposphere. In the lower stratosphere (200 to 20 hPa), steady and unsteady gravity waves are of the same order of magnitude, the unsteady component being probably in large part due to time variation of the wind near the surface. The mesospheric gravity wave field is largely dominated by propagating inertia-gravity waves arising from the 200 hPa level, or lower. Jet stream excitation is thought to play a significant role in the inertia-gravity wave generation at mid-latitudes since the time mean Rossby number can reach values of 0.5 to 0.6 on the upper and lower flank of the jet stream. The Eliassen-Palm flux divergence shows that propagating waves tend to exert a positive drag of $\sim 8 \text{ m s}^{-1} \text{ day}^{-1}$ on the highest model levels (at around 0.1 hPa and higher), while a negative drag of $\sim -7 \text{ m s}^{-1} \text{ day}^{-1}$ is felt in the lower part of the mesosphere. The high- (low-) frequency component of the mesospheric flow with absolute period of about 10 hours and lower (higher) exert the positive (negative) drag of 5 (-7) $\text{m s}^{-1} \text{ day}^{-1}$ around 0.07 hPa (0.3 hPa).

Over the mid-latitude Pacific Ocean, inertia-gravity waves were observed in the model boundary layer. These waves were mostly absorbed between 800 and 600 hPa at Jones' critical levels. In the model middle atmosphere over this oceanic region, the inertia-gravity wave field had a very weak impact on the zonal mean flow and residual circulation and the Eliassen-Palm flux divergence was essentially due to the large-scale flow. A positive drag of $30 \text{ m s}^{-1} \text{ day}^{-1}$ was calculated in the lower part of the mesosphere. The mean Rossby number just above and below the jet stream was near unity at some locations, and jet stream IGW excitation was expected to occur. A large portion of the inertia-gravity waves generated near the jet stream was absorbed just above it at Jones' critical layers, and these waves could not penetrate deeper in the middle atmosphere.

Of the three analyzed regions, Central America was the only one where the inertia-gravity wave field (excluding IGWs excited by orography) exerted a negative drag on the zonally averaged zonal momentum equation. A drag of $\sim -9 \text{ m s}^{-1} \text{ day}^{-1}$ in the mesosphere was mainly produced by waves with absolute periods smaller than 12

hours. The mechanism responsible for the generation of these gravity wave emissions is thought to be the "obstacle effect" of tropical convective cells.

The same analyses were also done using a zonally symmetric basic state, and no significant changes were observed, i.e., the monochromatic character of most of the principal components was preserved, and the eddy forcing caused in the zonal mean momentum equation was just slightly modified by about 5-10 % .

No Richardson instability ($Ri < 1/4$) was observed throughout the mid-latitude middle atmosphere (actually, Ri is significantly larger than one) even when wave breaking (more precisely, E-P flux divergence or convergence) occurs. Over Central America, $Ri < 1/4$ was observed at the tropical tropopause level, but not higher in the middle atmosphere (where gravity wave breaking occurs).

An interesting question that has not been addressed here would be to know the effects of shear processes of the basic flow on the empirical modes. In that case, it seems that a good interpolation of the data on isentropic surfaces could be preferable than working in pressure coordinates since the bilinear form of the wave-activity is simpler in the former case.

Appendix A

Equivalence in Pressure Coordinates

In pressure coordinates, the perturbation fields are defined relative to a basic state which is explicitly dependent on pressure. This means that the terms in the definition of the pseudoenergy appearing in Eqs. 3.11 and 3.12 must be transformed according to

$$\begin{aligned}
 u' &\rightarrow u' - \theta' u_{0p} / \theta_{0p} \\
 u_0 &\rightarrow u_0 \\
 v' &\rightarrow v' - \theta' v_{0p} / \theta_{0p} \\
 v_0 &\rightarrow v_0 \\
 \sigma' &\rightarrow \frac{1}{g\theta_{0p}} \frac{\partial}{\partial p} \left(\frac{\theta'}{\theta_{0p}} \right) \\
 \sigma_0 &\rightarrow -(g\theta_{0p})^{-1} \\
 P' &\rightarrow P' - \theta' P_{0p} / \theta_{0p} \\
 p' &\rightarrow -\theta' / \theta_{0p} \\
 p_0 &\rightarrow p_r \left(\frac{T_0}{\theta_0} \right)^{1/\kappa} \\
 \frac{\partial P_0}{\partial \phi} &\rightarrow \frac{\partial P_0}{\partial \phi} - \left(\frac{\theta_{0\phi}}{\theta_{0p}} \right) \frac{\partial P_0}{\partial p}.
 \end{aligned} \tag{A.1}$$

Note that

$$\frac{d\psi_0}{dP_0} = -\frac{au_0\sigma_0}{\partial P_0/\partial \phi} = \frac{av_0\sigma_0 \cos \phi}{\partial P_0/\partial \lambda},$$

so every term in the definition of the pseudoenergy in isentropic coordinates has an equivalent in pressure coordinates. The Ertel potential vorticity P is given in pressure coordinates by

$$P = -g \theta_p \left[f - \frac{(u \cos \phi)_{\phi}}{a \cos \phi} + \frac{v_{\lambda}}{a \cos \phi} \right] + \frac{g \theta_{\lambda} v_p}{a \cos \phi} - \frac{g \theta_{\phi} u_p}{a}, \tag{A.2}$$

and this expression must be used in the right hand side of (A.1). A volume element in θ -coordinates $a^2 \cos \phi \, d\phi \, d\lambda \, d\theta$ becomes in pressure coordinates $a^2 \theta_{0p} \cos \phi \, d\phi \, d\lambda \, dp$. The bilinear form of the pseudoenergy B defined in Eq. 3.12 and the wave vector defined by Eq. 3.11 must be modified according to the correspondences of Eq. A.1 and the new volume element. If the integrated pseudoenergy is written

$$\int a^2 \cos \phi \, d\phi \, d\lambda \, d\theta \, A = - \int g^{-1} a^2 \cos \phi \, d\phi \, d\lambda \, dp \, A_{(p)}, \quad (\text{A.3})$$

the bilinear form $B_{(p)}$ and the wave vector $f'_{(p)}$ become

$$B_{(p)} = \begin{pmatrix} \frac{1}{2} & 0 & -\frac{u_0}{2} & 0 & b_{15} \\ 0 & \frac{1}{2} & -\frac{v_0}{2} & 0 & b_{25} \\ -\frac{u_0}{2} & -\frac{v_0}{2} & 0 & 0 & b_{35} \\ 0 & 0 & 0 & b_{44} & b_{45} \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} \end{pmatrix} \quad (\text{A.4})$$

where

$$\begin{aligned} b_{15} &= b_{51} = -u_{0p}/(2\theta_{0p}) \\ b_{25} &= b_{52} = -v_{0p}/(2\theta_{0p}) \\ b_{35} &= b_{53} = (u_0 u_{0p} + v_0 v_{0p})/(2\theta_{0p}) \\ b_{45} &= b_{54} = -au_0 P_{0p} (2g\theta_{0p}^2)^{-1} (P_{0\phi} - \theta_{0\phi} P_{0p}/\theta_{0p})^{-1} \\ b_{44} &= au_0 (2g\theta_{0p})^{-1} (P_{0\phi} - \theta_{0\phi} P_{0p}/\theta_{0p})^{-1} \\ b_{55} &= -\frac{\kappa c_p}{2p_r \theta_{0p}} \left(\frac{T_0}{\theta_0} \right)^{\frac{\kappa-1}{\kappa}} + \frac{u_{0p}^2 + v_{0p}^2}{2\theta_{0p}^2} + au_0 P_{0p}^2 (2g\theta_{0p}^3)^{-1} (P_{0\phi} - \theta_{0\phi} P_{0p}/\theta_{0p})^{-1} \end{aligned} \quad (\text{A.5})$$

and

$$f'_{(p)} = \begin{pmatrix} u' \\ \frac{\partial}{\partial p} \left(\frac{v'}{\theta_{0p}} \right) \\ P' \\ \theta' \end{pmatrix}. \quad (\text{A.6})$$

Again, we use a five dimensional wave vector in order to avoid the presence of differential operators in $B_{(p)}$.

Appendix B

The Snapshot Method

For a given signal, when the spatial sampling has more degrees of freedom than the time sampling, it is usually preferable to use what is referred to as the snapshot method in order to deal with a covariance matrix of smaller dimension. Since the number N of basic functions necessary to represent a discretized space and time dependent signal is given by the minimum between the number of time sampling and the number of space sampling, the covariance matrix should not be larger than $N \times N$. For example, a signal sampled hourly for five days (120 samples) with a three-dimensional spatial grid of $40 \times 20 \times 22$ should be represented with 120 basic functions instead of 17600 ($= 40 \times 20 \times 22$). Thus, the eigenvalue problem exposed in Chapter 3 could be reformulated with the help of a modified temporal covariance matrix.

It is easy to show that

$$\langle \tau(s, t) a_n^*(t) \rangle = \lambda_n a_n^*(s), \quad (\text{B.1})$$

where

$$\tau(s, t) = (f'^{\dagger}(s), B f'(t)). \quad (\text{B.2})$$

For the discrete case, the dimension of τ is $N \times N$, where N is the number of time sampling. The normal modes are recovered by projecting the wave vector on the principal components

$$f'_n = \langle a_n^*(t) f'(t) \rangle. \quad (\text{B.3})$$

The snapshot method was the one used throughout this work.

Appendix C

Fourier Modes and Eigenvalue Problems

Suppose that a vector f has finite norm M , i.e. $(f, f) = M^2$, and that this vector is defined in the time interval $[-T, T]$ (here, T may tend to infinity, or we can choose $f(T) = f(-T) = 0$). An operator A of the form

$$A = \left\langle \frac{\partial f}{\partial t} f^\dagger \right\rangle = - \left\langle f \frac{\partial f^\dagger}{\partial t} \right\rangle, \quad (\text{C.1})$$

where the bracket denotes a time average, is anti-Hermitian, and a time mean covariance operator $C = \langle f f^\dagger \rangle$ is Hermitian. It will be demonstrated that Fourier modes f_n are the eigenvectors of the operator $-iAC^{-1}$ under certain conditions.

If the vector f is Fourier decomposed as

$$f = \sum_n e^{i\omega_n t} f_n, \quad (\text{C.2})$$

the covariance operator is given by

$$C = \sum_n f_n f_n^\dagger, \quad (\text{C.3})$$

and multiplying successively by C^{-1} and f_m , one gets

$$f_m = \sum_n (f_n, C^{-1} f_m) f_n. \quad (\text{C.4})$$

If $(f_n, C^{-1} f_m) = \delta_{mn}$, in other words, if the Fourier modes are linearly independent, and using the preceding Fourier decomposition, the operator $-iA$ can be written

$$-iA = \sum_n \omega_n f_n f_n^\dagger, \quad (\text{C.5})$$

and the eigenvalue problem

$$-iAC^{-1} f_m = \sum_n \omega_n f_n (f_n, C^{-1} f_m) = \omega_m f_m \quad (\text{C.6})$$

is recovered. This is exactly the same form as Eq. 2.55, which shows that the Fourier modes can be solutions without considering the dynamical operator governing the evolution of f . The condition of linear independence of the Fourier modes is not general, but its probability of occurrence increases when the spatial degrees of freedom of f is much greater than the temporal degrees of freedom in the discretized case.

Appendix D

Estimation of the Joint PDFs

A realistic time series is usually not free of noise. This renders a direct interpretation of the scatter diagrams of two principal components discussed in Chapter 4 more difficult to make. In order to overcome the problem of interpreting noisy phase space structures (see Chapter 3 on the treatment of noise by ENMs), a statistical approach is adopted. The kernel estimator technique (with a Gaussian kernel) has been applied to scatter diagrams of pairs of principal components.

In our case, the kernel estimator is a mapping from a discrete number of events to a continuous distribution. Since we seek for pairs of modes, the dimension d of the space is two, and the number of events N_0 is given by the number of time samplings (120 in our case). By definition, an estimated density is given by

$$f(\alpha, \vec{a}) = \frac{1}{N_0 h^d} \sum_{i=1}^{N_0} \alpha_i K\left(\frac{\vec{a} - \vec{a}_i}{h}\right), \quad (\text{D.1})$$

where α is any weighted quantity (e.g., the principal component tendencies), $\vec{a}_i$ represents the position of the i th member of the dataset (i.e., for a pair of principal components $b(t)$ and $c(t)$, $\vec{a}_i = (b(t_i), c(t_i))$), and h is a smoothing parameter (usually obtained with the help of the standard deviation of the dataset). The kernel K is here chosen to be a Gaussian function

$$K(\vec{a}) = \frac{1}{(2\pi)^{d/2}} e^{-\frac{1}{2}\vec{a} \cdot \vec{a}}. \quad (\text{D.2})$$

The best choice for the smoothing parameter $h = h_{opt}$ is set by minimizing the error between the true kernel and the Gaussian kernel. When N_0 is large, an asymptotic form gives

$$h_{opt} = h_0 \left[\frac{(2d+1)N_0}{4} \right]^{-1/(d+4)}, \quad (\text{D.3})$$

where h_0^2 is the variance of the dataset given by

$$h_0^2 = \frac{1}{N_0} \sum_{i=1}^{N_0} (\vec{a}_i - \bar{\vec{a}}) \cdot (\vec{a}_i - \bar{\vec{a}}).$$

The time tendency estimation is made by using the weighted form

$$T\vec{a} = \frac{f(\frac{d\vec{a}}{dt}, \vec{a})}{f(1, \vec{a})}. \quad (\text{D.4})$$

Appendix E

Some Principal Components

As an example of the character (monochromaticity and amplitude modulation) of the principal components, the first 80 principal components obtained from the analysis made over the Rockies from 618 to 3.9 hPa are presented.

SKYHI.ROCHE3.BAS.VRAI 1-4

SKYHI.ROCHE3.BAS.VRAI 5-8

SKYHI.ROCHE3.BAS.VRAI 9-12

SKYHI.ROCHE3.BAS.VRAI 13-16

SKYHI.ROCHE3.BAS.VRAI 17-20

SKYHI.ROCHE3.BAS.VRAI 21-24

SKYHI.ROCHE3.BAS.VRAI 25-28

SKYHI.ROCHE3.BAS.VRAI 29-32

SKYHI.ROCHE3.BAS.VRAI 33-36

PC

TIME

SKYHI.ROCHE3.BAS.VRAI 37-40

PC

TIME

SKYHI.ROCHE3.BAS.VRAI 41-44

PC

TIME

SKYHI.ROCHE3.BAS.VRAI 45-48

PC

TIME

SKYHI.ROCHE3.BAS.VRAI 49-52

SKYHI.ROCHE3.BAS.VRAI 53-56

SKYHI.ROCHE3.BAS.VRAI 57-60

SKYHI.ROCHE3.BAS.VRAI 61-64

SKYHI.ROCHE3.BAS.VRAI 65-68

SKYHI.ROCHE3.BAS.VRAI 69-72

SKYHI.ROCHE3.BAS.VRAI 73-76

SKYHI.ROCHE3.BAS.VRAI 77-80

References

- Andrews, D. G., 1987: On the interpretation of the Eliassen-Palm flux divergence. *Quart. J. Roy. Meteor. Soc.*, **113**, 323–338.
- , J. R. Holton, and C. B. Leovy, 1987: *Middle Atmosphere Dynamics*. Academic Press, 489 pp.
- , J. D. Mahlman, and R. W. Sinclair, 1983: Eliassen-Palm diagnostics of wave-mean flow interaction in the GFDL “SKYHI” general circulation model. *J. Atmos. Sci.*, **40**, 2768–2784.
- , and M. E. McIntyre, 1976: Planetary waves in horizontal and vertical shear: The generalized Eliassen-Palm relation and the mean zonal acceleration. *J. Atmos. Sci.*, **33**, 2031–2048.
- , and ———, 1978a: Generalized Eliassen-Palm and Charney-Drazin theorems for waves on axisymmetric mean flows in compressible atmospheres. *J. Atmos. Sci.*, **35**, 175–185.
- , and ———, 1978b: An exact theory of nonlinear waves on a Lagrangian-mean flow. *J. Fluid Mech.*, **89**, 609–646.
- , and ———, 1978c: On wave-action and its relatives. *J. Fluid Mech.*, **89**, 647–664.
- Baines, P. G., 1995: *Topographic effects in stratified flows*, Cambridge University Press, 482 pp.
- Bartello, P., 1995: Geostrophic adjustment and inverse cascades in rotating stratified turbulence. *J. Atmos. Sci.*, **52**, 4410–4428.
- Booker, J. R., and F. P. Bretherton, 1966: The critical layer for internal gravity waves in a shear flow. *J. Fluid Mech.*, **27**, 513–539.

- Bretherton, F. P., 1969: Momentum transport by gravity waves. *Quart. J. Roy. Meteor. Soc.*, **95**, 215–243.
- Breuer, K. S., and L. Sirovitch, 1991: The use of the Kharhunen-Loève procedure for the calculation of linear eigenfunctions. *J. Comput. Phys.*, **96**, 277–296.
- Brunet, G., 1994: Empirical normal mode analysis of atmospheric data. *J. Atmos. Sci.*, **51**, 932–952.
- , and R. Vautard, 1996: Empirical normal modes versus empirical orthogonal functions for statistical prediction. *J. Atmos. Sci.*, **53**, 3468–3489.
- Charney, J. G., and P. G. Drazin, 1961: Propagation of planetary-scale disturbances from the lower into the upper atmosphere. *J. Geophys. Res.*, **66**, 83–109.
- Clark, T. L., T. Hauf, and J. P. Kuettner, 1986: Convectively forced internal gravity waves: Results from two-dimensional numerical experiments. *Quart. J. Roy. Meteor. Soc.*, **112**, 899–925.
- Davis, P. A., and W. R. Peltier, 1979: Some characteristic of the Kelvin-Helmholtz and resonant overreflection modes of shear flow instability and of their interaction through vortex pairing. *J. Atmos. Sci.*, **36**, 2394–2412.
- Dunkerton, T., 1978: On the mean meridional mass motions of the stratosphere and mesosphere. *J. Atmos. Sci.*, **35**, 2325–2333.
- Eckermann, S. D., and W. K. Hocking, 1989: Effect of superposition on measurements of atmospheric gravity waves: A cautionary note and some reinterpretations. *J. Geophys. Res.*, **94**, 6333–6339.
- Eliassen, A., and E. Palm, 1961: On the transfer of energy in stationary mountain waves. *Geofys. Publ.*, **22**(3), 1–23.
- Fels, S. B., 1985: Radiative-dynamical interactions in the middle atmosphere. *Adv. Geophys.*, **28A**, 277–300.
- , J. D. Mahlman, M. D. Schwarzkopf, and R. W. Sinclair, 1980: Stratospheric sensitivity to perturbations in ozone and carbon dioxide: radiative and dynamical responses.

J. Atmos. Sci., **37**, 2265–2297.

Fovell, R., D. Durran, and J. R. Holton, 1992: Numerical simulations of convectively generated stratospheric gravity waves. *J. Atmos. Sci.*, **49**, 1427–1442.

Fritts, D. C., 1982: Shear excitation of atmospheric gravity waves. *J. Atmos. Sci.*, **39**, 1936–1952.

———, 1984a: Shear excitation of atmospheric gravity waves. Part II: Nonlinear radiation from a free shear layer. *J. Atmos. Sci.*, **41**, 524–537.

———, 1984b: Gravity wave saturation in the middle atmosphere: A review of theory and observations. *Rev. Geophys. Space Phys.* **22**, 275–308.

———, and Z. Luo, 1992: Gravity-wave excitation by geostrophic adjustment of the jet stream. Part I: Two-dimensional forcing. *J. Atmos. Sci.*, **49**, 681–697.

Gavrilov, N. M., S. Fukao, T. Nakamura, T. Tsuda, M. D. Yamanaka, and M. Yamamoto, 1996: Statistical analysis of gravity waves observed with the middle and upper atmosphere radar in the middle atmosphere. 1. Method and general characteristics. *J. Geophys. Res.*, **101**, 29 511–29 521.

Geller, M. A., 1983: Dynamics of the middle atmosphere. *Space Sci. Rev.*, **34**, 359–375.

Gill, A. E., 1982: *Atmosphere-Ocean Dynamics*, Academic Press, 662 pp.

Grimshaw, R., 1975: Internal gravity waves: Critical layer absorption in a rotating fluid. *J. Fluid Mech.*, **70**, 287–304.

Hamilton, K., 1988: Evaluation of the gravity wave field in the middle atmosphere of the GFDL “SKYHI” general circulation model. *Tech. Doc. 273*, pp. 264–271, World Meteorol. Org., Switzerland.

———, 1991: Climatological statistics of stratospheric inertia-gravity waves deduced from historical rocketsonde wind and temperature data. *J. Geophys. Res.*, **96**, 20 831–20 839.

———, 1995: Comprehensive simulation of the middle atmosphere climate: some recent results. *Clim. Dyn.*, **11**, 223–241.

- , R. J. Wilson, J. D. Mahlman, and L. J. Umscheid, 1995: Climatology of the SKYHI troposphere-stratosphere-mesosphere general circulation model. *J. Atmos. Sci.*, **52**, 5–43.
- Hayashi, Y., 1982: Space-time spectral analysis and its applications to atmospheric waves. *J. Meteor. Soc. Japan*, **60**, 156–171.
- , D. G. Golder, J. D. Mahlman, and S. Miyahara, 1989: The effect of horizontal resolution on gravity waves simulated by the GFDL “SKYHI” general circulation model. *Pure Appl. Geophys.*, **130**, 421–443.
- Haynes, P. H., 1988: Forced, dissipative generalizations of finite-amplitude wave-activity conservation relations for zonal and nonzonal basic flows. *J. Atmos. Sci.*, **45**, 2352–2362.
- Held, I. M., 1985: Pseudomomentum and the orthogonality of modes in shear flows. *J. Atmos. Sci.*, **42**, 2280–2288.
- Hines, C. O., 1991: The saturation of gravity waves in the middle atmosphere. Part I: Critique of linear-instability theory. *J. Atmos. Sci.*, **48**, 1348–1359.
- Hirota, I., and T. Hirooka, 1984: Normal mode Rossby waves observed in the upper stratosphere. Part I: First symmetric modes of zonal wavenumbers 1 and 2. *J. Atmos. Sci.*, **41**, 1253–1267.
- , and T. Niki, 1985: A statistical study of inertia-gravity waves in the stratosphere. *J. Meteor. Soc. Japan*, **63**, 1055–1066.
- Hodges, R. R., Jr., 1967: Generation of turbulence in the upper atmosphere by internal gravity waves. *J. Geophys. Res.*, **72**, 3455–3458.
- , 1969: Eddy diffusion coefficients due to instabilities in internal gravity waves. *J. Geophys. Res.*, **74**, 4087–4090.
- Houghton, J. T., 1978: The stratosphere and mesosphere. *Quart. J. Roy. Meteor. Soc.*, **104**, 1–29.

- Jones, W. L., 1967: Propagation of internal gravity waves in fluids with shear flow and rotation. *J. Fluid Mech.*, **30**, 439–448.
- Kitchen, E. H., and M. E. McIntyre, 1980: On whether inertio-gravity waves are absorbed or reflected when their intrinsic frequency is Doppler-shifted towards f . *J. Meteor. Soc. Japan*, **58**, 118–125.
- Laprise, J. P. R., 1993: An assessment of the WKB approximation to the vertical structure of linear mountain waves: Implication for gravity-wave drag parameterization. *J. Atmos. Sci.*, **50**, 1469–1487.
- Lelong, M.-P., and J. J. Riley, 1991: Internal wave-vortical mode interactions in strongly stratified flows. *J. Fluid Mech.*, **232**, 1–19.
- Lindzen, R. S., 1981: Turbulence and stress owing to gravity wave and tidal breakdown. *J. Geophys. Res.*, **86**, 9707–9714.
- Lorenz, E. N., 1955: Available potential energy and the maintenance of the general circulation. *Tellus*, **7**, 157–167.
- Lott, F., H. Kelder, and H. Teitelbaum, 1992: A transition from Kelvin-Helmholtz instabilities to propagating wave instabilities. *Phys. Fluids A*, **4**, 1990–1997.
- , and H. Teitelbaum, 1993: Linear unsteady mountain waves. *Tellus*, **45A**, 201–220.
- Luo, Z., and D. C. Fritts, 1993: Gravity-wave excitation by geostrophic adjustment of the jet stream. Part II: Three-dimensional forcing. *J. Atmos. Sci.*, **50**, 104–115.
- Mahlman, J. D., and L. J. Umscheid, 1984: Dynamics of the middle atmosphere: successes and problems of the GFDL “SKYHI” general circulation model. *Dynamics of the Middle Atmosphere*, J. R. Holton and T. Matsuno, Eds., Terrapub, Tokyo, 501–525.
- Miyahara, S., Y. Hayashi, and J. D. Mahlman, 1986: Interactions between gravity waves and planetary-scale flow simulated by the GFDL “SKYHI” general circulation model. *J. Atmos. Sci.*, **43**, 1844–1861.
- Newton, C. W., 1960: Hydrodynamic interactions with ambient wind field as a factor in

cumulus development. *Cumulus Dynamics*, C. E. Anderson Ed., Pergamon Press, New York, ??-??.

O'Sullivan, D., and T. J. Dunkerton, 1995: Generation of inertia-gravity waves in a simulated life cycle of baroclinic instability. *J. Atmos. Sci.*, **52**, 3695–3716.

Queney, P., 1948: The problem of air flow over mountains: A summary of theoretical studies. *Bull. Am. Meteorol. Soc.*, **29**, 16–26.

Sato, K., 1994: A statistical study of the structure, saturation and sources of inertio-gravity waves in the lower stratosphere observed with the MU radar. *J. Atmos. Terr. Phys.*, **56**, 755–774.

Shepherd, T. G., 1990: Symmetries, conservation laws, and Hamiltonian structure in geophysical fluid dynamics. *Adv. Geophys.*, **32**, 287–338.

Silverman, B., 1986: *Density Estimation for Statistics and Data Analysis*. Chapman and Hall, 175 pp.

Sutherland, B. R., C. P. Caulfield, and W. R. Peltier, 1994: Internal gravity wave generation and hydrodynamic instability. *J. Atmos. Sci.*, **51**, 3261–3280.

———, and W. R. Peltier, 1995: Internal gravity wave emission into the middle atmosphere from a model tropospheric jet. *J. Atmos. Sci.*, **52**, 3214–3235.

Vaughan, G., S. J. Pepler, and D. S. McKenna, 1996: Aircraft measurements of potential vorticity and Richardson number in baroclinic zones. *Quart. J. Roy. Meteor. Soc.*, **122**, 721–735.

Warn, T., and R. Ménard, 1986: Nonlinear balance and gravity-inertial wave saturation in a simple atmospheric model. *Tellus*, **38A**, 285–294.

Wurtele, M. G., A. Datta, and R. D. Sharman, 1996: The propagation of gravity-inertia waves and lee waves under a critical level. *J. Atmos. Sci.*, **53**, 1505–1523.