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**THE STATISTICAL VALIDITY OF USING RATIO VARIABLES
IN HUMAN KINETICS RESEARCH**

by

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ABSTRACT

There were two main purposes of this investigation. The first was to examine the validity and reliability of commonly used ratio variables in human kinetics research, and to evaluate four ratio score models used to deflate the effect of the denominator. The second was to use computer simulation procedures to investigate the effect of using ratio variables on the circularity assumption of the covariance matrix and type I error rates in RM ANOVA tests. It is shown that a suitable common deflation model for all ratio variables may not exist, and different models should be used to derive an appropriate deflation model in empirical research. The results indicate that high reliability of the component variables does not necessarily result in high reliability of the transformed ratio variable. Thus, when a ratio variable is used the reliability should be examined based on the ratio variable data. It is recommended that five criteria be used to evaluate and compare the validity of deflation models: (a) zero correlation between a derived ratio variable and the denominator variable, (b) no curvilinear relationship between the derived ratio and the denominator in the scatterplots, (c) equality of the estimated expected value of the model and calculated mean of the derived ratio data, (d) high R^2 , and (e) high reliability of the derived ratio data. Simulation results show that the characteristics (ϵ_{x1} , ϵ_{x2} , V_{x1}/V_{x2} , and ρ_{x1x2}) of the two component variables strongly affect $\epsilon_{x1/x2}$ and the type I error rate. In the condition $\epsilon_{x1}=\epsilon_{x2}$, the magnitude of $\epsilon_{x1/x2}$ is virtually the same as that of ϵ_{x1} and ϵ_{x2} , regardless the level of ρ_{x1x2} and V_{x1}/V_{x2} . When $\epsilon_{x1}<1.0$ and $\epsilon_{x2}=1.0$, $\epsilon_{x1/x2}$ tends to have smaller magnitudes when ρ_{x1x2} and V_{x1}/V_{x2} are high, and greater magnitudes when ρ_{x1x2} and V_{x1}/V_{x2} are low. $\epsilon_{x1/x2}$ exhibited the greatest bias and the largest standard deviation, resulting in a serious inflation of type I error rate in the

condition $V_{x1}/V_{x2}=0.5$, regardless of the conditions ϵ_{x1} , ϵ_{x2} , and ρ_{x1x2} . If homogeneity of the denominator variable (small V_{x2}) and large sample size are present, it may reduce the likelihood of bias in $\hat{\epsilon}_{x1/x2}$ and protect the type I error rate.

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CHAPTER I. INTRODUCTION

Introduction

Ratio variables have a long history of use in different disciplines related to human kinetics research (e.g., biomechanics, exercise epidemiology, physical education, sport physiology, and sport psychology). A ratio variable is a composite variable that consists of a numerator variable and a denominator variable. For example, the moment of force in biomechanics is usually expressed as a ratio variable which consists of the measured moment of force divided by body mass, and the Vo₂max in physiology is usually expressed as a ratio variable which consists of the measured Vo₂max divided by body mass. Per capita income and other variables in sociology are ratios with population size as the denominator to compare different countries. Waist and hip girth and nondominant and dominant strength ratio are popularly used in the epidemiology and health fields, and many sport statistics are expressed as a ratio of successful attempts divided by the total number of attempts (e.g., basketball shooting percentage).

Ratio variables are popularly used in different areas of human kinetics research, and they can be classified by the purpose of use. Ratio variables are typically used for scaling data, with the variable of primary interest being the numerator variable. Dividing by the denominator is an attempt to express the numerator on a per-unit of the denominator basis in order to remove or deflate the effects of the denominator's variation from the numerator's variation, and full deflation would yield a zero correlation between the ratio variable and its denominator variable. For example, heavier people can lift heavier weights partially because they have greater body mass. Often it is desirable to compare individuals with respect to strength independent of (or at least minimizing the effect

of) body mass. In Olympic weightlifting competitions, this is done by forming homogeneous groups with respect to body mass, and individuals are compared only to others of similar body mass. In other situations the formation of homogeneous groups is either not possible or not desirable. For example, in schoolboy weightlifting competitions there often are not enough competitors to form different weight classes. In this situation, a ratio variable, weight lifted divided by body mass, is computed and used to declare an overall champion (Keller, 1977). In many research studies, it is desirable to examine changes in strength, or between-group differences in strength, independent of body mass changes on group differences. The ratio variable, strength/body mass, is used in such situations to “deflate” the effect of body mass.

The term “scaling”, “eliminating”, “removing” and “deflating” are equivalent terms used in the literature, but the term “deflating” is used through this dissertation for consistency and familiarity in the field. Ratio variables derived for deflation purposes (attempting to remove the effect of the denominator variable from the variation of the numerator variable) serve as dependent variables in many studies in different disciplines.

Ratio variables are also used as measurement variables where the purpose is not to derive a denominator-free variable. In this case, a ratio variable has some special aspect not accounted for by the numerator or denominator variables, either individually or additively. For example, the ratio of waist and hip girths is treated as a combined measurement variable, called the waist-to-hip ratio (WHR). It is used to describe body shape, and has been shown to be very useful in epidemiology and obesity studies to examine morbidity, stroke, heart disease, breast cancer, and other diseases (Darroch & Mosimann, 1985; Sonnenschein, Kim, Pasternack, & Toniolo, 1993).

Since the statistical consequences of using ratios were first mentioned by Pearson (1897), there has been a recognition of both the theoretical and practical limitations of using ratio variables (e.g., Anderson & Lydic, 1977a, 1977b; Albrecht, 1978; Albrecht, Gelvin, & Hartman, 1993; Atchley, Gaskins, & Anderson, 1976; Firebaugh & Gibbs, 1985; Kronmal, 1993; Long, 1980; MacMillan & Daft, 1980; Packard & Boardman, 1988; Schuessler, 1974). For deflation purposes, some researchers have indicated that the transformation of data into ratios may produce misleading interpretations due to the residual correlation between the ratio variable and its denominator (Atchley et al., 1976; Firebaugh & Gibbs, 1985; Kronmal, 1993; Long, 1980; Schuessler, 1974). Others have argued that a correctly used ratio variable does not create "spurious" correlations and misleading results (Corruccini, 1995; Kasarda & Nolan, 1979; Kritzer, 1990; MacMillan & Daft, 1980).

The focus on ratio issues has mostly been on the correlation between a ratio variable and its component variables, and the choice of the most valid deflation models (Albrecht et al., 1993). If the purpose for using a ratio variable is to deflate the effects of the denominator's variation from the numerator, the main issue is whether a simple ratio variable can fully deflate the denominator effect, as evidenced by a zero correlation between a ratio variable and its denominator variable. Albrecht and his colleagues (1978, 1993), and others, examined the statistical characteristics of ratios and the methods to fully control the denominator effect under different regression models. The results indicated that correctly formulated adjusted ratios do control for the correlation between a ratio variable and its denominator, but could introduce denominator-related changes in variances of the ratio variable (i.e., the variance of a ratio variable decreases as the value

of the denominator variable increases). If the hypothesis to be tested has initially been formulated in terms of a ratio variable, some researchers (Corruccini, 1995; Kuh & Meyer, 1955) have argued that the question of spurious correlation between a ratio and its component variables does not arise. Actually, for those ratio variables (e.g., waist/hip ratio), although a zero correlation is not an issue, the statistical consequences of using such variables should be examined because the reliability of the ratios is affected by variation of the two component variables. However, the reliability issue of using ratio variables is usually neglected by researchers.

Although in practice there may be two different purposes for using ratio variables, the consequences of using ratio variables in statistical inferential analyses are the same. It can be shown that the variation and covariation of the component variables strongly affect the statistical characteristics of a derived ratio variable in descriptive and inferential analyses.

Some of the issues regarding the validity of using ratio variables have been noted in the literature. For example, the issues of using ratio variables in correlation and linear regression analyses have been discussed for a long time. Friedlander (1980), and many others, pointed out that a linear regression model with a ratio as the dependent variable or the independent variable would not appropriately control for the effect of the denominator, and therefore could yield misleading results. It has been indicated that a reciprocal component of the denominator should be included in the regression model. However, some other issues concerning the use of ratio variables have not been resolved.

In human kinetics research, ratio variables are commonly used in both descriptive and inferential statistics. When a ratio variable is derived for deflation purposes, a

residual correlation between the ratio and its denominator variable may seriously affect the validity of the ratio variable. However, it appears that this is rarely examined, and the impact of the denominator on related ratio variables should be investigated empirically in all studies. In addition to the problem of a non-zero correlation between a ratio and its denominator, the use of a ratio variable could introduce denominator-related distortions of variances. In some studies in human kinetics research, the denominator related variance distortion in a ratio variable may not be an issue because of the small range of the denominator variable in the data, while in others it may be a serious issue because of the large range of the denominator variable. For non-deflation type ratio variables, given that the ratio variable has content validity, information about the reliability of the ratio variable and variation effect of the component variables in the inferential analysis needs to be known. However, such information is usually missing in human kinetics research publications.

Researchers in human kinetics often use ratio variables in inferential analyses, especially in the analysis of variance with repeated measures (RM ANOVA). It is apparent that the characteristics of a ratio variable in the repeated measures will be strongly influenced by the characteristics of the numerator and denominator variables. How the characteristics of the component variables affect the type I error rate in RM ANOVA is still not known. An investigation of these issues has not yet been done for the commonly used ratio variables in human kinetics research.

Purpose of the Study

A primary purpose of this study was to investigate whether ratio variables are appropriately used in the human kinetics research domain. For ratio variables used for deflation purposes, the validity of the ratio variable was a main concern. A simple ratio variable and adjusted ratio variables were investigated and compared using variables commonly used in our field. The validity of deflation models for each ratio variable was evaluated.

Because the reliability of a measurement variable is one of the most important considerations in any empirical research, this study examined the reliability properties of ratio variables. The reliability of a ratio variable is more complex than the reliability of a raw score variable because it is affected by several factors, specifically, the reliability of the numerator and denominator variables, the correlation between the two component variables, and the relative variation of the two variables. These effects are rarely addressed in the literature. The reliability of common ratio measures in human kinetics research needs to be examined for possible bias as a result of these effects.

The second purpose was to investigate how the RM ANOVA test using a ratio variable violates the assumptions of this statistical procedure. Since a ratio variable $Y_i = X_{1i}/X_{2i}$ is a combined variable which contains numerator and denominator components, the covariance structure of ratio variables for trial i and j , $Y_i = X_{1i}/X_{2i}$ and $Y_j = X_{1j}/X_{2j}$ say, is different from that of the raw scores. The covariance structure of ratio variables in RM ANOVA is affected by not only the covariances of the component variables across trials, but also the covariance between the numerator and denominator variables within each trial. These effects may cause serious violations of some of the assumptions (e.g.,

circularity), and the violation of the circularity assumption may affect type I error rates. Although the impact of the violation of the circularity assumption has been discussed extensively when a raw score is used, there is little discussion on the effects of the component variables in RM ANOVA using ratio variables, and it remains an unresolved issue.

In summary, there is insufficient information in the related literature to provide appropriate guidance for human kinetics researchers to resolve the following issues:

1. The validity and reliability of a number of commonly used ratio variables based on different deflation models in human kinetics research.
2. The effect of using ratio variables on the structure of the covariance matrix and type I error rates in RM ANOVA analysis.

Therefore, the purpose of this investigation was to seek answers to the above questions, and to provide a guide to the correct use of ratio variables. In doing so, the most popularly used ratio variables were selected from a survey of the field of human kinetics. Then, the issues addressed in the previous section were investigated. First, the deflation models were compared and the best deflation model was identified for the selected variables. Second, the reliability and validity of the ratio variables were evaluated. Third, the impact of using ratio variables in RM ANOVA was investigated under different conditions of the variation and correlation between the numerator and denominator variables. The results were intended to provide guidance for more appropriate use of ratio variables in human kinetics research.

CHAPTER II. LITERATURE REVIEW

Statistical Characteristics of Ratios

The Distribution of Ratio Variables

One of the basic assumptions of the statistical models commonly used in human kinetics is that, in the population, the dependent variable is distributed normally. A question of interest in the literature is, given the distribution characteristics of the component variables that are used to form a ratio dependent variable, what is the distribution of the ratio variable?

It is noted that the normality assumption in most statistical procedures is not critical unless sample sizes are small (e.g., $n < 15$ in RM ANOVA procedures). The general statistical properties of ratios of random normal variables have been examined by a number of statisticians (Fieller, 1932; Kendall & Stuart, 1969; Marsaglia, 1965; Schneeberger & Fleischer, 1993). For example, it can be shown that the distribution of the ratio of two independent standard normal random variables is a Cauchy distribution (Kendall & Stuart, 1969). The density function is

$$df(y) = \frac{dy}{\pi(1 + y^2)} \quad -\infty \leq y \leq \infty \quad (1)$$

where $Y = Z_1/Z_2$. In this function the ratio of Z_1 and Z_2 is treated as a new independent variable and the effect of Z_1 and Z_2 on the ratio cannot be seen.

The general form of the density function of the ratio of two random variables can be derived. If $f(X_1, X_2)$ is the joint density of (X_1, X_2) , then $\psi(y)$, the density of the ratio $Y = X_1/X_2$ is given by:

$$\psi(y) = \int_{-\infty}^{\infty} f(yx_2, x_2) |x_2| dx_2 \quad -\infty < y < +\infty \quad (2)$$

Ratio distributions under bivariate normal conditions have examined analytically (Fieller, 1932; Schneeberger & Fleischer, 1993), and also through the use of simulation procedures (Friedlander, 1980). The density function of the bivariate normal distribution for X_1 and X_2 is given by Schneeberger and Fleischer (1993):

$$f(x_1, x_2) = \frac{1}{2\pi\sigma_{x_1}\sigma_{x_2}\sqrt{1-\rho_{12}^2}} \exp\left\{-\frac{1}{2(1-\rho_{12}^2)}\left[\left(\frac{x_2 - \mu_{x_2}}{\sigma_{x_2}}\right)^2 - 2\rho_{12}\frac{x_2 - \mu_{x_2}}{\sigma_{x_2}} \times \frac{x_1 - \mu_{x_1}}{\sigma_{x_1}} + \left(\frac{x_1 - \mu_{x_1}}{\sigma_{x_1}}\right)^2\right]\right\} \quad (3)$$

As we can see from (2) and (3), the distribution of the ratio of continuous variables X_1 and X_2 is affected by the following factors:

μ_{x_1}, μ_{x_2} : the population means,

$\sigma_{x_1}, \sigma_{x_2}$: the population standard deviations,

ρ_{12} : the population correlation of X_1 and X_2 .

The distribution of the ratio variable derived from (2) and (3) has infinite moments when the component variables can take negative values, but this is usually not the case in human kinetics because the component variables used are usually positive and fall within a certain range. Fieller (1932) pointed out that the limitation of the component variables to the positive quadrant may change the moments of the ratio distribution to finite values without having a visible effect on the appearance of the distribution. Using the aforementioned functions and a computer simulation, Friedlander (1980) demonstrated that when the correlation of the component variables is high and the coefficients of variation are large, there could be a considerable deviation of the ratio distribution from normality. The simulation also showed that when the correlation

between the numerator and denominator increases, the mean and standard deviation of the ratio variable tend to decrease.

Because of the complex relationship between a ratio and its component variables, the effect of the characteristics of the component variables (e.g., μ_{x1} , μ_{x2} , σ_{x1} , σ_{x2} , and ρ_{x1x2}) on the distribution characteristics of a ratio variable has attracted the attention of many researchers in different disciplines. For example, Schneeberger and Fleischer (1993) showed different ratio distribution shapes under various conditions on X_1 and X_2 . Hinkley (1969) discussed the distribution of the ratio of two correlated normal random variables, and Korhonen and Narula (1989) proposed a method to compute the distribution of the ratio of the absolute values of two correlated normal random variables. Issues relating to the normality of ratio variables were also discussed by Samuel and Norman (1985, 1986), and Arnold and Brockett (1992).

Summarizing these studies, the distribution of a ratio variable is affected by the covariation and the distribution characteristics of the numerator and denominator variables. When the numerator variable is approximately normal and the denominator is less variable or non-stochastic, the ratio variable is still approximately normal. When the variation of the component variables is large, and the correlation between the component variables is high, considerable deviation of the ratio distribution from normality may occur.

For some ratio variables used in human kinetics, the denominator may have large variation and the component variables may be strongly skewed. When a ratio variable is derived from the component variables which have large variation (e.g., $V=\sigma/\mu>0.2$, V refers to the coefficient of variation defined as the standard deviation divided by the

mean) or are strongly skewed, the ratio variable may be less skewed but the distribution remains non-normal (Friedlander, 1980). A log transformation is a method to make the ratio data more normally distributed. however, this option may lack appeal because of the difficulty of interpretation in some situations.

Using a computer simulation, Atchley et al. (1976) demonstrated how changes in the coefficients of variation of two bivariate normally distributed variables ($V_{x1}=\sigma_{x1}/\mu_1$, $V_{x2}=\sigma_{x2}/\mu_2$) affect the skewness and the kurtosis of a ratio variable. The results showed that the distribution of the ratio variable was strongly skewed to the right when the variation of the denominator was relatively greater than that of the numerator. For example, the coefficient of skewness reached 2.06 when $V_{x1}/V_{x2}=0.1$, and as V_{x1}/V_{x2} approached 1.0 the skewness of the distribution of the ratio variable decreased to 0.1. Additionally, when the variation of the denominator was relatively greater than that of the numerator, the kurtosis of the ratio variable was very high (e.g., the coefficient of the kurtosis >10.0 when $V_{x1}/V_{x2}<0.2$). When V_{x1}/V_{x2} approached 1.0 the kurtosis of the ratio variable decreased more quickly than skewness. The kurtosis also became slight when $V_{x1}/V_{x2}>0.3$. Both skewness and kurtosis values were unaffected (i.e., remained near zero) for conditions in which $V_{x1}/V_{x2}>1.0$.

In summary, the distribution of a ratio variable is strongly affected by the relative variation of its component variables. Although the component variables may be normally distributed, as the coefficient of variation of the denominator variable (V_{x2}) increases relative to that of the numerator, the distribution of the ratio variable becomes skewed and leptokurtic. Obviously, it is not appropriate to assume that ratio variables meet the normality assumption based on knowledge of the normality of the component variables.

Variance, Covariance, and Correlation of Ratios

It is more complex to evaluate the variances, covariances, and correlations of ratio variables than that of raw score variables because a ratio variable is a combination of two component variables. The variances and covariances of the component variables strongly affect the statistical properties of ratio variables. Pearson (1897) first addressed this issue, and developed a formula to approximate the correlation between ratios using classic measurement theory and Taylor's approximation method (Formula (7)). The variance and covariance of a ratio variable can also be approximated based on Pearson's procedure (Friedlander, 1980). The following are the formulas for the approximations to the mean, variance, covariance, and correlation:

$$\mu_{x_1/x_2} = E\left(\frac{X_1}{X_2}\right) \approx \frac{\mu_{x_1}^2}{\mu_{x_2}^2} (1 - \rho_{x_1x_2} V_{x_1} V_{x_2} + V_{x_2}^2) \quad (4)$$

$$\sigma_{x_1/x_2}^2 = Var\left(\frac{X_1}{X_2}\right) \approx \frac{\mu_{x_1}^2}{\mu_{x_2}^2} (V_{x_1}^2 + V_{x_2}^2 - 2\rho_{x_1x_2} V_{x_1} V_{x_2}) \quad (5)$$

$$\sigma_{(x_1/x_2)(x_3/x_4)} = Cov\left(\frac{X_1}{X_2}, \frac{X_3}{X_4}\right) \approx \frac{\mu_1}{\mu_2} \frac{\mu_3}{\mu_4} (\rho_{x_2x_4} V_{x_2} V_{x_4} + \rho_{x_1x_3} V_{x_1} V_{x_3} - \rho_{x_2x_3} V_{x_2} V_{x_3} - \rho_{x_1x_4} V_{x_1} V_{x_4}) \quad (6)$$

$$\rho_{(x_1/x_2)(x_3/x_4)} = Corr\left(\frac{X_1}{X_2}, \frac{X_3}{X_4}\right) \approx \frac{\rho_{x_1x_3} V_{x_1} V_{x_3} - \rho_{x_1x_4} V_{x_1} V_{x_4} - \rho_{x_2x_3} V_{x_2} V_{x_3} + \rho_{x_2x_4} V_{x_2} V_{x_4}}{\sqrt{V_{x_1}^2 + V_{x_2}^2 - 2\rho_{x_1x_2} V_{x_1} V_{x_2}} \sqrt{V_{x_3}^2 + V_{x_4}^2 - 2\rho_{x_3x_4} V_{x_3} V_{x_4}}} \quad (7)$$

Here V_{x1} , V_{x2} , V_{x3} , and V_{x4} are coefficients of variation (standard deviation divided by mean) for variables X_1 , X_2 , X_3 , and X_4 , and ρ_{x1x2} , ρ_{x1x3} , ρ_{x1x4} , ρ_{x2x3} , ρ_{x2x4} , and ρ_{x3x4} are correlation coefficients between the component variables. By replacing a component with unity in the formula, various ratio correlations can be derived. For example, the

correlation between X_1/X_2 and X_2 can be estimated by taking $X_3=X_2$ and $X_4=1$. It should be noted that the formulas are approximations, and they are accurate only when the coefficients of variation for all component variables are small enough to render cubic and higher order variation terms of the Taylor series negligible. Chayes (1971) showed that Pearson's approximate formula became increasingly inaccurate when the coefficient of variation of the component variables in the formula was greater than 0.15. Therefore, the accuracy of using the formulas for human kinetics measures with greater variation becomes an issue.

Formulas (4) to (7) show that the statistical characteristics of a ratio variable can be very different from that of the component variables. First, Formulas (4) to (7) show that the statistical characteristics of a derived ratio variable are complex functions of the characteristics of the component variables. Second, by knowing the characteristics of the component variables, researchers can approximate the characteristics of a ratio variable using these formulas. Third, Formulas (4) to (7) are approximations and the accuracy of these formulas depends upon the properties of the component variables. In some situations these approximations may be acceptable, while in other situations they may not be acceptable due to large variation in the component variables.

Reliability of Ratio Variables

Whenever a measurement is administered to subjects in a study, researchers would like to have some assurance that the measurement scores could be replicated if the same individuals were measured again under similar circumstances. One aspect of this replication is the degree of consistency in the inter-individual variability, often referred to the reliability of a measure. Based on classical measurement theory (observed score =

true score + error), the degree of reliability in a two trial situation is often expressed by a Pearson correlation coefficient, ranging from 0.00 to 1.00. For the situations with three or more trials, the intraclass correlation provides a more valid approximation of the reliability. The intraclass correlation for the within-subject factor in the RM ANOVA model, assuming compound symmetry of the covariance matrix, is defined as $r_{int} = (\sigma_{\pi}^2) / (\sigma_{\pi}^2 + \sigma_e^2)$. The estimate of the intraclass correlation from data can be obtained as $\hat{r}_{int} = (MS_s - MS_{res}) / MS_s$, where MS_s is the mean square for subjects and MS_{res} is the mean square for error (details of the intraclass correlation are shown in the chapter on methods and procedures).

Because the correlation and variation of the numerator and denominator variables affect the reliability of a ratio variable, the evaluation of reliability for a ratio variable in terms of the properties of the components is more complex than for a raw score variable. Holzinger claimed that the reliability of a ratio variable could be approximated from Pearson's correlation estimation Formula (7) (Cronbach, 1941):

$$\rho_{\frac{u_1}{v_1} \times \frac{u_2}{v_2}} \approx \frac{\rho_{u_1 u_2} V_{u_1}^2 - 2\rho_{u_1 v_2} V_{u_1} V_{v_2} + \rho_{v_1 v_2} V_{v_1}^2}{V_{u_1}^2 - 2\rho_{u_1 v_2} V_{u_1} V_{v_2} + V_{v_1}^2} \quad (8)$$

where $\rho_{u_1 u_2}$ and $\rho_{v_1 v_2}$ are the reliability coefficients of the numerator and denominator variables, and V_{u_1} and V_{v_1} are the coefficients of variation of the component variables. This formula is derived by substituting X_1 for X_3 and X_2 for X_4 in Formula (7). However, Formula (8) has never been used in the literature. The validity of Formula (8) for the approximation of reliability is suspect, because substituting X_1 for X_3 and X_2 for X_4 in Formula (7) and assuming that the variable $Y = X_1/X_2$ correlates with itself would

result in $\rho_{yy} \equiv 1.0$. Thus, this formula may not be appropriate to estimate the reliability of a ratio variable.

Evaluating the reliability of ratio variables is important in any study, and it is affected by both the numerator and denominator variables. However, the reliability of ratio variables in the literature is rarely examined.

Ratio Variable Issues In the Literature

The controversy over the validity of using ratio variables has a long history. The issue has been commented on by statisticians, sociologists, economists, political scientists, geologists, zoologists, and others (e.g., Albrecht, 1978; Chayes, 1949; Chayes, 1971; Firebaugh & Gibbs, 1985; Kronmal, 1993; MacMillan & Daft, 1980; Pearson, 1897, 1910; Schuessler, 1974; Yule, 1910). The discussion in this dissertation focuses on the validity of using ratio variables for deflation purposes, and the validity of using ratio variables in RM ANOVA.

Using Ratio Variables for the Purpose of Deflation

The literature indicates that a simple ratio in which a measurement variable is divided by another measurement variable may not always adequately deflate the denominator effect, as evidenced by the residual correlation between the ratio and its denominator variable. For example, Pearson (1897), Chayes (1949), Atchley (1976), and Albrecht (1978) warned that a simple ratio may be inadequate for the purpose of deflation. Pearson showed that the correlation between a ratio variable and its denominator is usually not zero, even when there is no correlation between the component variables. Based on Pearson's formula (Formula (7)), Albrecht (1978)

derived the pattern of correlation between the ratio ($Y=X_1/X_2$) and its denominator (X_2) under different combinations of the coefficient of variation (V_{X_1}/V_{X_2}). In general, the value of ρ_{y,x_2} is always less than ρ_{x_1,x_2} and only approaches ρ_{x_1,x_2} for high values of V_{X_1}/V_{X_2} . When $V_{X_1}/V_{X_2} \leq 1.0$, ρ_{y,x_2} is negative, and when ρ_{x_1,x_2} approaches zero, ρ_{y,x_2} rapidly approaches -1.0. Therefore, one should not assume that a simple ratio would have zero correlation with its denominator.

The more interesting question is how to interpret the correlation ρ_{y,x_2} given the difference between ρ_{y,x_2} and ρ_{x_1,x_2} . MacMillan and Daft (1979, 1980) gave a reasonable explanation for the case $\rho_{y,x_2}=0$. They showed that ρ_{y,x_2} should be zero, not when X_1 and X_2 are unrelated, but when X_1 and X_2 are related in a special way (a linear relationship between X_1 and X_2 with a zero intercept). ρ_{y,x_2} is sometimes recommended as a measure of "nonproportionality" (Firebaugh & Gibbs, 1985; Kasards & Nolan, 1979). If ρ_{y,x_2} equals zero, it suggests that there is a linear relationship between X_1 and X_2 with a zero intercept, however, if ρ_{y,x_2} is not zero, one cannot evaluate the degree of nonproportionality by ρ_{y,x_2} alone. Sources of nonproportionality include a non-zero y-intercept, non-linearity, or both.

To overcome the inadequacy of using a simple ratio as a deflation method, various adjusted deflation models have been recommended (Albrecht et al., 1993; Atchley, 1978; Gelvin & Albrecht, 1985). In order to get full deflation, it is necessary to derive a new scaled variable in which the variation is not affected by the variation of the denominator variable. The main statistical criterion for full deflation is that the adjusted ratio Y_{adj} should have a zero correlation with its control variable (the denominator variable in a simple ratio). Albrecht et al. pointed out that deficiencies in simple ratios

can be alleviated by incorporating regression coefficients describing the bivariate relationship between the numerator and denominator variables (they called these two variables measurement and control variables). Four deflation models were suggested by Albrecht to determine the best ratio variable that can fully remove the effect of the denominator: (a) a simple ratio model: $Y=X_1/X_2$, (b) an intercept adjusted ratio model, $Y=(X_1 - a)/X_2$, where a is a constant, (c) an allometrically adjusted ratio model, $Y=X_1/X_2^k$, where k is a constant, and (d) a fully adjusted ratio model, $Y=(X_1 - a)/X_2^k$, which includes both an intercept and exponent parameter. A full description of these models is provided in Chapter 3. Albrecht et al. (1993) indicated that correctly using the adjusted models does control for the correlation between a ratio variable and its denominator variable. However, the major drawback of using these adjusted ratio models is that it could introduce denominator-related distortions of variances (i.e., the variance of a ratio variable Y or Y_{adj} decreases as the value of the denominator X_2 increases). The reduction of variance in Y_{adj} as the denominator X_2 increases is most marked for allometrically adjusted and fully adjusted ratios. The degree of the distortion of variance also depends on the range of the denominator variable in the empirical data, and the distortion may be serious for some data sets and trivial for others. Therefore, the validation of a ratio variable in a study should include not only a ratio model selection to achieve $\rho_{y,x_2}=0$, but also a ratio model evaluation to examine the degree of distortion of variance.

Ratio Variables in Regression Analysis

The validity of using ratio variables in linear regression and correlation analysis has been extensively discussed in the literature (Albrecht et al., 1993; Belsley, 1972;

Bollen, 1979; Chilton, 1982; Corruccini, 1995; Firebaugh et al., 1985; Friedlander, 1980; Kronmal, 1993; MacMillan et al., 1984, 1980; Pendleton, 1984, Schuessler, 1974). The validity issue has been summarized and clarified by Firebaugh and Gibbs' research (1985). Firebaugh and Gibbs discussed ratio issues for the cases in which all variables appearing in the regression equation are divided by a common variable, or one of the independent variables also appears as the denominator of the dependent variable. A summary of their results follows.

Given that the raw score regression model is the true model, a ratio score regression model in which both the dependent variable and one of the independent variables are divided by another variable would generally be appropriate only if the reciprocal of the denominator variable is included as an independent variable. The only exception is the case where the numerator is directly proportional to the denominator (Friedlander, 1980). It is also indicated that even if a regression equation does include the reciprocal term, it may produce inefficient estimates if the deflation causes a heteroscedastic error term. Friedlander indicated that the partial correlation of two ratio variables with a common denominator variable, adjusting for the reciprocal of the denominator, is the same as the partial correlation of the numerator variables, adjusting for the denominator.

Based on a linear relationship between the original dependent variable, the common deflator variable and the independent variables in the regression equation, three regression models involving ratio variables were considered in Kronmal's research (1993): (a) both independent and dependent variables are divided by a common variable, (b) only the dependent variable is a ratio, and (c) only one of the independent variables is

a ratio. It was confirmed that including the reciprocal of the denominator variable in the case (a) could solve the problem of possible biases, given the error term in the equation is not heteroscedastic. Using Neyman's example (1952), Kronmal (1993) illustrated the serious spurious correlation between two ratios when both the independent and dependent variables in the regression equation are divided by a common variable, but the original independent variable and dependent variable do not have a linear relation. For the case (b), analytic results and an empirical example suggested that a problem would occur when only the dependent variable is a ratio. Even though the numerator of the ratio is uncorrelated with the original independent variable, the ratio dependent variable could be significantly correlated with the independent variable through its relationship to the denominator of the ratio. For the case (c), using a ratio as an independent variable in a regression equation often results in a better predictor than using only the numerator or the denominator variable in the equation, but is usually not a better model than one which includes both components.

Firebaugh and Gibbs (1985) also concluded that ratio variables can be used in regression analyses without apology in sociology if the following rules governing the use of ratio variables are followed: (a) use regression analysis rather than correlation analysis when analyzing ratio variables, except when assessing whether X_1 and X_2 are proportional, (b) avoid mixed methods (i.e., part ratio, part component), (c) strive for a reliable measure of the denominator variable, because measurement error in the denominator is likely to cause considerable bias, (d) include $1/X_2$ as a regressor (X_2 is the denominator in the ratio variable), unless it can be safely assumed that the Y-intercept is zero.

Ratio Variables in Analysis of Variance

As mentioned in previous sections, a ratio variable is often used in inferential analyses (often RM ANOVA) in human kinetics research. Therefore, we are especially interested in the statistical consequences of using ratio variables in such an analysis. Unfortunately, there appears to be no publications discussing the issue of using ratio variables in RM ANOVA, although, Anderson and Lydic did look at ratio variables in the randomized groups ANOVA model (Anderson et al. 1977a, 1977b). Bivariate normal observations X and Y were generated by computer, and a Monte Carlo simulation procedure was designed to investigate whether the statistical properties induced by ratio transformations can produce a loss of sensitivity (power) in a two-group random-subject and fixed-treatment ANOVA (although the comparing power for two different model may not be valid, see discussion in the following section). They also tried to characterize the circumstances under which the statistical analysis of ratio data is appropriate. Three different models were used in their simulations. First, an ANOVA model ignoring the covariate:

$$x_{1ij} = \mu + \tau_j + e_{ij} \quad (9)$$

Second, an ANCOVA model:

$$x_{1ij} = \mu' + \tau_j' + \beta x_{2ij} + e_{ij}' \quad (10)$$

Third, for the ratios, an ANOVA model using the measured dependent variable divided by the covariate as the new dependent variable:

$$x_{1ij} / x_{2ij} = \mu'' + \tau_j'' + e_{ij}'' \quad (11)$$

x_{1ij} : numerator variable;

x_{2ij} : denominator variable;

μ : the grand mean;

τ_j : a constant associated with the j th treatment;

e_{ij} : the experimental error associated with the i th subject under the j th treatment.

The data were generated based on following conditions:

1. The correlations between the numerator and denominator variables ($\rho=0.0, 0.25, 0.50, 0.75, 0.90, 0.99$).
2. Treatment effects ($\tau=0.1, 0.2, 0.3, 0.4, 0.5, 0.75, 1.0$).
3. Mean conditions ($\mu_{x1}=10$ and $\mu_{x2}=5, \mu_{x1}=10$ and $\mu_{x2}=10, \mu_{x1}=5$ and $\mu_{x2}=10$).
4. Combinations of the coefficient of variation between the numerator and denominator variables ($V_{x1}/V_{x2}=0.5, 1.0, 2.0$). The variances of the component variables were fixed for all the conditions ($\sigma_{x1}^2=\sigma_{x2}^2=1$). Two groups were used and the sample size was 10 observations per group.

The useful finding of this study was that the power of the ratio ANOVA model was highly dependent upon the ratio of the coefficients of variation of the component variables. However, comparisons among the three proposed models are not meaningful, because separate ANOVA models with different hypotheses were compared in Anderson and Lydic's study. The raw score model, the ANCOVA model, and the ratio model actually tested different hypotheses. In their two-group ANOVA design, the null statistical hypothesis of the raw score model would be that no difference exists between the raw score population means in the two groups. The null hypothesis for the ANCOVA model is that no difference exists between the two population means after these group means have been adjusted for differences between the groups on the relevant covariate variable. In general, ANCOVA is more powerful than ANOVA because of the smaller

error variance in ANCOVA (this statement is true only when a good covariate variable is used and the assumptions are met). Therefore, it is obvious that ANCOVA has greater power than ANOVA in Anderson and Lydic's study.

The power of the F-tests in these three ANOVA models in this study are not comparable, but this study did show that the power within the ratio ANOVA model is affected by the coefficients of variation of the component variables. The study indicated that the ratio model performed poorly when the coefficient of variation of the denominator variable was greater than the coefficient of variation of the numerator variable.

Although there are intensive discussions on the issue of violating the assumptions of RM ANOVA for raw score variables, there appears to be no publication investigating ratio issues in RM ANOVA. When a ratio variable is used in RM ANOVA, the circularity assumption of the variance and covariance matrix for the within-subject factor is affected not only by the variation and covariation of the numerator variable over trials but also by that of the denominator variable. When a ratio variable is used in RM ANOVA, the variance and covariance of the denominator (including the common denominator condition) over trials may seriously affect the population characteristics of the covariance matrix (i.e., affect the circularity of the population covariance matrix in the within-subject factor).

Ratio Variables and Unsolved Issues in Human Kinetics Research

Ratio variables are commonly used in human kinetics research, and are used in almost all of the areas in the human kinetics domain (e.g., from physical education to

physiology). Table 2-1 shows how ratio variables are used in a number of different disciplines. In these studies, some ratio variables are used for deflation purposes and some are not, and usually serve as measurement variables in inferential analyses, especially in ANOVA. The analysis of ratio variables with ANOVA procedures is common in human kinetics research, for example, between 1992 and 1995, 62% of the articles published in the *Research Quarterly for Exercise and Sport* used ANOVA or MANOVA, and 39% of these articles used ratio variables. Although ratio variables are widely used, the validity and reliability issues of using these ratio variables are rarely acknowledged and investigated in human kinetics research. The following are the most important issues regarding the use of ratio variables in our field.

Deflation Issues

In human kinetics research, as shown above, many ratio variables are used for deflation purposes. That is, using ratio variables is an attempt to express the numerator on a per-unit of the denominator basis in order to remove the effects of the denominator's variation from the numerator. When the initial purpose of using a ratio variable is deflation, it has been shown that two assumptions (homogeneous variances of a ratio variable over the range of the denominator variable, and zero correlation between the ratio variable and its denominator) should be examined. If the assumptions are violated by the simple ratio variable, other alternative deflation models should be considered. Recently, the validity of the simple ratio Vo2max score ($\text{Vo2max}/\text{body mass}$) has been examined and compared with alternative deflation ratios (Heil, 1997; Nevill, 1994; Vanderburgh & Katch, 1996; Welsman, Armstrong, Nevill, Winter, and Kirby, 1996). The model $Y = X_1/X_2^k$ was preferred for Vo2max in some of these studies and the magnitudes of k reported varied from 0.41-0.80. However, a systematic investigation of

Table 2-1
Ratio Variables in Human Kinetics Research

Discipline	Variable Definition	Examples
Pedagogy	Total energy expenditure of lesson is defined by the energy expenditure in a class divided by the body weight (kcal/kg).	McKenzie, Y. L. et al. (1995). A randomized group ANOVA was used for two groups.
Exercise Epidemiology	Weight (kg) divided by height squared (m).	Hofstetter, C. R. et al. (1991).
	Total cholesterol divided by high-density lipoprotein cholesterol (a stronger indicator of potential coronary risk).	Rimmer, J. H. et al. (1994). t-test
Psychology	The Weber Ratio - just noticeable difference (JND) divided by the point of subjective equality (PSE: the location or distance perceived as equal to the standard on 50% of the trials).	Meeuwssen, H.J. et al. (1992). MANOVA was used.
Physiology	Ventilatory Threshold (ml/kg*min). Relative Fat (%).	Hughes, R. A. et al. (1991).
	Mean power output/body weight. Work in 30s/body weight.	Hill, D. W. et al. (1992).
Education	Defining intelligence quotient as $IQ = 100 * \text{mental age (MA)} / \text{chronological age (CA)}$ and educational quotient as $EQ = 100 * \text{educational age (EA)} / \text{CA}$.	Friedlander, L.J. (1980).
Biomechanics	The vertical, anteroposterior, and mediolateral ground reaction force variables divided by body mass.	Schot, P.K. et al. (1992). RM MANOVA
	Antero-postero ground reaction force and vertical ground reaction force.	Sanderson, D. J. et al. (1993). RM ANOVA
Growth & Development	Waist/Hip ratio.	Sonnenschein, E. G. et al. (1993).
Management	The administration size divided by the total organization size.	Friedlander, L.J. (1980).
Sociology	The urban population divided by the total population. The number of deaths divided by the total population. The births divided by the population.	Bollen, K. A. et al. (1979). Schuessler, K. (1974).

the deflation issue has not been done, and the homogeneous variance assumption was not examined in these studies. As shown in Table 2-1, simple ratio data are still commonly used in analyses without any validation.

Reliability Issues

Because data are virtually always measured with error, the consideration of the reliability of a measurement variable is crucial for any study. When a ratio variable is used in an analysis, the reliability issue becomes more important due to the complexity of combining two variables in one. Surprisingly, the reliability issue is rarely addressed in our field and needs to be investigated.

Issues in RM ANOVA with Ratio Data

As mentioned in previous sections, RM ANOVA with ratio data is a commonly used technique in human kinetics research. Issues of using raw scores as a measurement variable in RM ANOVA have been discussed extensively in the literature. Schutz and Gessaroli (1987) have given a comprehensive review on this topic with empirical examples in our field. For the one-way RM ANOVA, the assumptions of circularity, a normal distribution of the measurement variable, and homoscedasticity of the error variances are required for the exact F statistics of the treatment effect to be valid. In practice, ANOVA procedures are very robust with respect to violations of the normality assumption, but very sensitive to violation of the assumptions of circularity and homoscedasticity of the error variances. Eom and Schutz (1993) showed that the inflation in Type I error rates in the naive F tests when $\epsilon < 1.0$ is not, as commonly assumed, merely a function of the population epsilon value. Rather, it is a function of the nature of the relative magnitudes of the population values and the sample estimate of

epsilon. That is, a large Type I error rate is associated with an overestimated epsilon. On the other hand, when the population epsilon is underestimated, the Type I error rate is considerably smaller than the corresponding nominal level.

However, the effects of using ratio score variables on the assumptions in RM ANOVA are hardly noticed and discussed in our field. As shown in Formula (4), (5), (6) and (7), the variances and covariances of ratio variables are affected by characteristics of the numerator and denominator variables. Even though there are the formulas available to approximate the variance and covariance of ratio variables, there are few publications that investigate the impact of using ratio variables in RM ANOVA. The following are some of the issues that need to be investigated.

First of all, we do not know to what extent the ratio transformation affects the circularity assumption of the ratio populations in RM ANOVA. For example, if the raw score populations meet the circularity assumption, we do not know how changes in the variance and covariance of the common denominator affect the circularity property in RM ANOVA using ratio variables. Second, when an inference test is conducted from a sample of ratio data based on the RM ANOVA, it is not clear how changes in the variances and covariances of the component variables affects the sampling characteristics of the variance and covariance matrix of the ratio variable.

In summary, when ratio variables are used in human kinetics research, the following questions should be addressed:

1. What is the purpose of using ratio variables in the study? If it is for deflation purposes, has an appropriate deflation model been considered?
2. Does the ratio variable meet the reliability criterion in the study?

3. How does the ratio variable affect the assumptions if a RM ANOVA procedure is to be used?

CHAPTER III. METHODS AND PROCEDURES

Variable Selection

As shown in previous chapters, ratio variables are commonly used in almost all areas of human kinetics research. For the purposes of this study, four of the most frequently used ratio variables from several disciplines within the field were selected and studied, but the principles developed based on these four variables are applicable to all ratio variables. Publication and research databases common to the field were searched for relevant studies that used ratio variables in repeated measurement situations, and letters were sent to researchers requesting their data. There were three criteria used to choose appropriate studies and data sets for this research. First, the study had to be in the human kinetics research domain, and the ratio variable used in the study had to be a commonly used variable. Second, the data sets included the ratio variable and one of its component variables, or both the numerator variable and the denominator variable, so that the ratio variable could be derived. Third, the ratio variable data (both the numerator and denominator variables) were repeatedly measured.

Data sets utilizing four commonly used ratio variables meeting the above criteria were obtained and analyzed. There were two data sets in which the ratio variable was used for deflation purposes. The first data set was obtained from a study conducted at the University of British Columbia (Taunton, 1992). The ratio variable used in this study was Vo2max/kg and the sample size was 52. The second data set was from Bacon's research (1997). The diffusing capacity of the lungs for carbon monoxide (DLco) divided

by alveolar ventilation volume (V_A) was the ratio variable (DL_{co}/V_A) used for deflation purposes, and 13 subjects were in Bacon's study. The other two sets of ratio data selected contained ratio variables that were not used for the purpose of deflation. One of these two data sets was from a project conducted by Kramer, Nusca, Bisbee, MacDermid, Kemp, and Boley (1994). The nondominant/dominant forearm strength ratio was the ratio variable used, and 43 subjects were in the project. The other ratio data set was the waist and hip girth ratios obtained from the study conducted by Sonnenschein, Kim, Pasternack, and Toniolo (1993), and the sample size was 1851. These four data sets served as a basis for the investigation in this dissertation.

Deflation Model Evaluation

The Models

In human kinetics research, a simple ratio of two component variables is commonly used for the purpose of deflation. As shown in the literature review chapter, a simple ratio may not adequately eliminate the denominator effect, and alternative models need to be considered. The validity of the four deflation models used in literature was evaluated for each of the ratio variables chosen for the purpose of deflation in this study.

Simple ratio model. Based on classic measurement theory, a variable can be expressed as a population mean with a error term. A simple ratio $Y = X_1/X_2$ can be presented as $Y_i = X_{1i}/X_{2i} = \mu + e_i$ (where μ is the population mean for the ratio variable Y , and e_i is the error term for subject i : $i=1, 2, \dots, N$). In the literature, a linear regression function omitting the intercept is commonly utilized to represent the relationship between the expected mean of the simple ratio and its component variables (Albrecht, et al., 1993:

Allison, Paultre, Goran, Poehlman, & Heymsfield, 1995; Nevill, Ramsbottom, & William, 1992), i.e.:

$$X_{1it} = \beta_t X_{2it} + e_{it} \quad (i=1, 2, \dots, n; t=1, 2, \dots, k) \quad (12)$$

Where X_{1it} is the numerator variable, X_{2it} is the denominator variable. The simple ratio score in trial t for a subject i based on this model can thus be presented as

$Y_{it} = X_{1it}/X_{2it} = \beta_t + e_{it}$, where the slope β_t in Formula (12) serves as the expected population mean μ_t , and e_{it} is the error term ($e_{it} = e_{it}/X_{2it}$). Occasionally, ordinary least-squares linear regression based on Equation (12) has been used to fit the component variable data (separately for each trial t) and obtain the estimated slope ($\hat{\beta}_t$) of the model, and $\hat{\beta}_t$ used as the estimate of the expected value of the simple ratio (e.g., Nevill et al., 1992). The weakness of using this model is that the error term e_{it} is related to the denominator variable $e_{it} = e_{it}/X_{2it}$. This implies that the expected value of e_{it} given X_{2it} is still zero but its variance varies as a function of X_{2it} (Allison, et al., 1995). Although using this model could alter the error distribution, it has been used by researchers to estimate the expected mean of the simple ratio variable. Allison stated: "having been alerted to the fact that taking ratios alters the error distribution, the case for carefully checking the residuals for homoscedasticity becomes even more compelling". Therefore, a graphical criterion was included in this study to identify the visible distortion of the variation of the ratio variable.

Linear regression model (LRM). This model is an intercept adjusted ratio model:

$$X_{1it} = \beta_{(adj)t} X_{2it} + a_t + e_{it} \quad (13)$$

Where a_t is the intercept and $\beta_{(adj)t}$ is the slope of the regression equation for trial t , the other terms are the same as defined in previous section. Based on the model, the ratio for each subject is represented by the transformed form $Y_{(adj)it} = (X_{1it} - a_t) / X_{2it} = \beta_{(adj)t} + e_{it}$, where e_{it} is the error term ($e_{it} = e_{it} / X_{2it}$). This linear regression model was used to fit the component variable data (separately for each trial t) to obtain the estimates of the slope, $\hat{\beta}_{(adj)t}$ and $\hat{a}_t$ in Equation (13). $\hat{\beta}_{(adj)t}$ serves as the estimate of the expected value of the adjusted ratio variable. The central issue hinges on the desire for a simple ratio (or an adjusted ratio variable based on the LRM model) which is independent of (i.e., uncorrelated with) the denominator. Allison (1995) indicated that under bivariate normality and a linear relationship between the component variables, a population intercept (a_t) equal to zero is a necessary and sufficient condition for the population correlation ($\rho_{y,x2}$) to be zero. Therefore, in the model LRM, subtracting the regression intercept a_t was necessary in order to meet the zero correlation criterion. As with the simple ratio model, the weakness of the LRM model is that the error distribution is related to the denominator variable due to $e_{it} = e_{it} / X_{2it}$.

Nonlinear regression model 1 (NLRM1). This model has the form of an exponent adjusted ratio:

$$X_{1it} = \beta_{(adj)t} X_{2it}^k e_{it} \quad (14)$$

Nevill, Ramsbottom, and Williams (1992) indicated that the multiplicative error term is a particularly attractive feature of this model. It accommodates a spread in the subjects' X_{1it} and X_{2it} values when plotted against each other, provided these scores diverge at a constant proportion to each other--and most component variables used for deflation purposes appear to exhibit this pattern. This model has been used by numerous

researchers in our field (e.g., Nevill, 1994; Vanderburgh & Katch, 1996; Welsman, Armstrong, Nevill, Winter, & Kirby, 1996).

In this study, Equation (14) was fitted to the component variable data to obtain the estimated parameters in the model and $\hat{\beta}_{(adj)it}$ serves as the estimate of the expected value of the adjusted ratio variable. The nonlinear regression option with Levenberg-Marquardt algorithm in SPSS was used for the fitting process (Norusis, M. J., 1997). The adjusted ratio for each subject is presented as $Y_{(adj)it} = X_{1it}/X_{2it}^k = \beta_{(adj)it}e_{it}$.

Nonlinear regression model 2 (NLRM2). This model is a fully adjusted ratio model of the form:

$$X_{1it} = \beta_{(adj)it} X_{2it}^k e_{it} + a_t \quad (15)$$

This model, which was suggested by Huxley (1972) and used by Albrecht et al. (1993), has both the intercept a_t and constant k . The model fitting procedure for Equation (15) was similar to that used for the NLRM1 using the nonlinear regression option with Levenberg-Marquardt algorithm in SPSS. Equation (15) was fitted to the component variable data to derive the estimates of the three parameters (a , k , and $\beta_{(adj)it}$) in the model. and $\hat{\beta}_{(adj)it}$ serves as the estimate of the expected value of the adjusted ratio variable. The ratio $\beta_{(adj)it}$ for subject i in trial t is presented by the equation

$$Y_{(adj)it} = (X_{1it} - a) / X_{2it}^k = \beta_{(adj)it} e_{it}.$$

Controlling for the Denominator Effect

The empirical data were fitted to the different deflation models mentioned above, then the simple ratio variable and the adjusted ratio variables derived from the deflation models were evaluated with the following criteria:

1. Statistical criterion. The linear relationship between the derived ratio variable (the simple ratio variable Y or the adjusted ratio variable Y_{adj}) and the denominator variable (X_2) is zero or nearly so. The Pearson correlation coefficient between the derived ratio variable and X_2 was calculated to ascertain the strength of the linear relationship.

2. Graphical criterion. The relationship between Y_t and X_{2t} can be plotted as a horizontal line (i.e., the least-squares regression line has a slope of zero).

3. Algebraic criterion. The estimate of the expected value $\hat{\beta}_t$ is equal to the empirical mean of the ratio variable. For example, in a simple ratio model ($X_{1it} = \beta_t X_{2it} + e_{it}$), the expected mean in the ratio variable data $\bar{Y}_t = \sum(X_{1it}/X_{2it})/n$ should be equal to the estimated parameter $\hat{\beta}_t$. It is noted that in the simple ratio model the expected value would be $E(Y_{it}) = E[\beta_t + e_{it}/X_{2it}] = \beta_t + E[e_{it}/X_{2it}]$. Given that the error term e_{it} has mean zero ($E(e_{it}) = 0$), the expected value $E[e_{it}/X_{2it}] = 0$ provided e_{it} and X_{2it} are uncorrelated (in this case, $E(e_{it}/X_{2it}) = E(e_{it})E(1/X_{2it}) = 0$). Therefore, the expected value of Y_{it} is equal to β_t . In practice, the estimate of the expected value $\hat{\beta}_t$ should be equal to the empirical mean $\bar{Y}_t$.

4. R^2 . High R^2 in the deflation model.

5. Reliability. The reliability of the derived ratio data was assessed by using an intraclass correlation approach, the details of which are shown in next section.

When evaluating the second criterion, by graphing Y_t or $Y_{(adj)t}$ and X_{2t} in a scatterplot, we can examine the impact of denominator related distortions of variances of a ratio variable. A distortion of the variances as the value of the denominator increases may affect the validity of the deflation models. Bivariate plots and the algebraic criterion

are simple but powerful supplements that reveal the distortion of the variances of ratio variables. The plots and algebraic criterion safeguard against situations where a correlation is low despite a strong curvilinear relationship between X_{2t} and Y_t or $Y_{(adj)t}$. A valid deflation model for the ratio variable should meet all of these criteria.

The traditional ANCOVA model (based on an assumed linear relationship among the raw score variables) could also be used to test the differences between levels of the independent variable after dependent variable scores are adjusted for differences associated with one or more covariates. However, the ANCOVA model may not be a good model for deflation purposes. Vanderburgh (1998) has shown that the traditional ANCOVA model is not a most parsimonious and plausible model for scaling (deflation) purposes and nonlinear regression is better than the ANCOVA model. That is, in traditional ANCOVA using non-transformed data, the group difference is assumed to be additive, not multiplicative, thus for group comparisons, the difference between groups must be an additive constant. In a nonlinear regression model, however, a more plausible multiplicative difference is assumed which allows for an increasing group difference as the "covariate" becomes larger (Nevill et al., 1992). The other weakness which makes the ANCOVA method lack appeal is that if the assumption of parallel slopes is violated, a more complex method (Johnson-Neyman follow-up method, Johnson & Neyman, 1936) has to be used for the group comparisons. Therefore, the ANCOVA model was not included in the examination of the deflation models in this study.

Based on the criteria mentioned above, the simple ratio variable and other adjusted ratio variables were computed and compared with each of the empirical data sets. By comparing the different ratio variables, I was able to find an appropriate ratio for

each of the chosen data sets. How the best deflation model should be developed was discussed.

Reliability of a Ratio

When a ratio variable is used, the reliability of the ratio variable is crucial for the validity of the study. Unreliable repeated measurements of a ratio variable in an inferential analysis could seriously affect the validity of the study. Therefore, the reliability of the four ratio variables is investigated in this research.

As argued in previous sections, the reliability issues are more complex for ratio variables because a ratio variable is a combined variable which consists of two component variables. It was shown in Formula (8) that the reliability of a simple ratio variable is affected not only by the reliability of the component variables but also by the variances, covariances, and the correlation of the component variables. Even when the component variables are reliable, it does not automatically guarantee that the derived ratio variable has appropriate reliability. It is expected that a ratio variable is less reliable than its component variables in repeated measurements because both the numerator and denominator variables are sources of lack of reliability.

A researcher may ask how the reliability of ratio data can be approximated from reliability of the component variable data. If the assumptions of the formula are met, the reliability of simple ratio variables with test-retest measurement data can be evaluated by Pearson's approach (Formula (7)) based on the correlation of the component variables. The most important assumption for using this formula is that the coefficients of variation for all the component variables are small enough ($V < 0.15$) so that the cubic and higher

order variation terms in the Taylor series are negligible. The variation and the correlation of the component variables may also strongly affect the validity of using Formula (7).

With repeated measurement data involving more than two repeated observations (e.g., trials), the intraclass correlation coefficient can be used to compute the reliability of the observed scores. According to classic measurement theory, reliability is defined as the ratio of true score variance (σ_{π}^2) to observed score variance ($\sigma_{\pi}^2 + \sigma_{\epsilon}^2$). Using the additive model with the assumption that subjects are a random variable and the time (trial) factor is a fixed variable (the most common situation in human kinetics research), the intraclass correlation coefficient can be estimated from the mean squares in a one-way RM ANOVA test (Winer, 1991):

$$\hat{r}_{\text{int}} = \frac{MS_s - MS_{\epsilon}}{MS_s + (k - 1)MS_{\epsilon}}, \quad (16)$$

where MS_s is the mean square due to subjects, and MS_{ϵ} is the mean square due to error. k is the number of trials. In our field, the measured scores for several trials are usually averaged to increase the reliability. The intraclass correlation for the mean of measured scores in several trials is computed as:

$$\hat{r}_{\text{int}} = \frac{MS_s - MS_{\epsilon}}{MS_s} \quad (17)$$

These formulas have not included the variance due to the treatment, because it is not a part of the true variance or the total variance in the reliability evaluations. Both the two formulas appear in publications to report the reliability.

In summary, using the above procedures (i.e., ratio variable selection, deflation model evaluation, and reliability evaluation), we examined the empirical ratio variable data and investigated issues related to the validity of ratio variables used in our field. For

the selected ratio variables, this study evaluated the validity of the four deflation models. A most reliable and valid deflation model was suggested for each of the selected empirical data sets.

Computer Simulation Investigation of Ratio Issues

The validity issues of some ratio variables used in our field were addressed by examining four empirical data sets according to the criteria shown in the above section. However, the generalizability of those findings is limited because the procedures were based on specific empirical samples. For example, how the variation and correlation of the component variables affect the validity of using ratio variables in RM ANOVA was not examined. In fact, we cannot know how different combinations of variation of the component variables affect the validity of a ratio variable in RM ANOVA by investigating limited empirical studies. Because RM ANOVA is a commonly used technique in our field, the validity of using ratio variables in RM ANOVA requires further investigation.

Simulated population data of ratio score variables were used to investigate characteristics of ratio variables and the extent to which they may lead to a violation of assumptions in the one-way RM ANOVA. The effects of the relative variation, covariation, and correlation between the numerator and denominator variables were examined under different conditions. The parameters of the ratio variables for generating the population data were based on the statistical characteristics of the selected component variables. By varying these controlled conditions for a broad range of the parameters, we

investigated how certain conditions of the component variable distributions lead to violation of the assumptions of RM ANOVA when a ratio variable was used.

The effects of using ratio variables on the type I error rate in the one-way RM ANOVA model (treatment fixed and subjects random) were investigated using Monte Carlo simulation procedures. It is well known that violation of the circularity assumption results in inflation in the type I error rate, and that the inflation can be alleviated by using the epsilon correction procedures (Greenhouse & Geisser, 1959; Huynh & Feldt, 1976). In this study, the effect of the ratio transformation on the circularity property of the population covariance matrix and the consequences of using sample data of ratio variables in a RM ANOVA test were examined. Furthermore, the effect of using ratio variables on the type I error rate in one-way RM ANOVA test was investigated.

In general, by using simulation procedures, it was expected that the following questions could be answered:

1. If the component variables produce a covariance matrix that meets the circularity assumption in RM ANOVA, does the covariance matrix of the ratio variable comprised of these components also exhibit circularity?
2. If the component variables produce a covariance matrix that violates the circularity assumption, under what conditions do the ratio variables produce a covariance matrix which leads to less or greater violation of the assumption?
3. What is the sampling distribution of the ϵ estimate (a measure of the degree to which the circularity assumption is violated; see below for a complete definition) when a ratio variable is used in RM ANOVA, and how does this affect the type I error rate of the F test?

Overview of the Simulation Conditions and Investigations

To investigate the issues addressed above, a single group and five trials ($g=1$, $k=5$) RM ANOVA in different controlled conditions was designed for the investigation. Raw score populations of the component variables were first generated based upon controlled conditions. Because the raw score variables in our field have positive values and lie within a certain finite range (e.g., Vo2, Height, Weight, and waist and hip girths), a generated raw score population was a large positive population. Table 3-1 shows the design and layout of the population data of the component variables. X_{1ij} is defined as the numerator variable and X_{2ij} as the denominator variable, i is the subject number and j is the trial number (X_1 and X_2 are used through this dissertation to represent the numerator and denominator variables). The ratio score variables ($Y=X_1/X_2$) were derived by transformations from the raw score populations. The following sections describe the conditions under which the simulations were conducted.

Box's Epsilon (ϵ). Box's ϵ , a measure of the degree of departure from circularity in a population covariance matrix, is a function of the variances and covariances in the population matrix. Box (1954) defined ϵ as:

$$\epsilon = \frac{k^2(\bar{\sigma}_{jj} - \bar{\sigma})^2}{(k-1)(\sum \sum \sigma_{jj} - 2k \sum \bar{\sigma}_{jj}^2 + k^2 \bar{\sigma}^2)} \quad (18)$$

Where

k : the order of the covariance matrix (number of trials),

$\bar{\sigma}_{jj}$: the mean of the variances (diagonal elements),

Table 3-1
Data Matrix of the Component Variables

<i>Trials</i>	<i>Numerator</i>					<i>Denominator</i>				
	T_1	T_2	T_3	T_4	T_5	T_1	T_2	T_3	T_4	T_5
<i>Subject</i>										
S_1	X_{111}	X_{112}	X_{113}	X_{114}	X_{115}	X_{211}	X_{212}	X_{213}	X_{214}	X_{215}
S_2	X_{121}	X_{122}	X_{123}	X_{124}	X_{125}	X_{221}	X_{222}	X_{223}	X_{224}	X_{225}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_i	X_{i11}	X_{i12}	X_{i13}	X_{i14}	X_{i15}	X_{2i1}	X_{2i2}	X_{2i3}	X_{2i4}	X_{2i5}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_N	X_{1N1}	X_{1N2}	X_{1N3}	X_{1N4}	X_{1N5}	X_{2N1}	X_{2N2}	X_{2N3}	X_{2N4}	X_{2N5}

$\bar{\sigma}$: the grand mean of all elements in the covariance matrix.

$\bar{\sigma}_j$: the mean of jth row of the covariance matrix.

σ_{jj} : an individual entry of the covariance matrix ($j, j'=1,2,\dots,k$).

When the population data matrix meets the circularity assumption, $\epsilon = 1.0$; otherwise, $\epsilon < 1.0$, with a minimum of $1/(k-1)$, where k is the number of repeated trials in this study (the order of the covariance matrix).

Epsilon is used to correct a positive bias in the naive F test by adjusting the degrees of freedom by an amount proportional to ϵ . That is, the computed F statistic for the repeated measures factor in a repeated measures design is distributed as $F[(k-1)\epsilon, (n-1)(k-1)\epsilon]$. In general, the characteristics of a population covariance matrix are unknown and are estimated by a sample covariance matrix. Collier, Baker, Mandeville, and Hays (1967) showed that replacing the components in Formula (18) with those estimated from a sample covariance matrix and computing an estimate of epsilon ($\hat{\epsilon}$) to adjust the degree of freedom results in a relatively robust F-test for reasonable sample sizes of 15 or larger. That is, the naive F statistic is approximately distributed as $F[(k-1)\hat{\epsilon}, (n-1)(k-1)\hat{\epsilon}]$. We will refer to this procedure for testing as the $\hat{\epsilon}$ -adjusted F test. Huynh and Feldt (1976, 1978) subsequently showed that the $\hat{\epsilon}$ -adjusted test is negatively biased and would make the F test too conservative. The bias is most serious when a population ϵ is greater than 0.75, especially when the sample size is small. They recommended a less biased estimate $\tilde{\epsilon}$:

$$\tilde{\epsilon} = \frac{n(k-1)\hat{\epsilon} - 2}{(k-1)[n-1-(k-1)\hat{\epsilon}]} \quad (19)$$

For any values of n and k , $\tilde{\epsilon} \geq \hat{\epsilon}$, and the equality holds when $\hat{\epsilon} = 1/(k-1)$. The upper bound of $\tilde{\epsilon}$ was set to unity, though it could be greater than unity.

In this study, there were two ϵ conditions for the two component variables generated. The values of ϵ were set to provide violation ($\epsilon=0.70$), and no violation ($\epsilon=1.0$) of the circularity assumption (e.g., Hertzog & Rovine, 1985; Huynh, 1978). The two ϵ conditions for the component variables resulted in four combinations for the ratio populations (i.e., $\epsilon_{x1}=1.0, \epsilon_{x2}=1.0$; $\epsilon_{x1}=1.0, \epsilon_{x2}=0.7$; $\epsilon_{x1}=0.7, \epsilon_{x2}=1.0$; $\epsilon_{x1}=0.7, \epsilon_{x2}=0.7$).

Knowing the ϵ value in a raw score population and systematically varying the correlation and relative variances of the component variables allowed us to investigate the impact of using the ratio variable on the circularity assumption of the covariance matrices. For $\epsilon=1.0$ of the component variable population, the population variances and the correlation between the repeated trials were constant, and the values were set based upon the waist and hip girth variables in Sommenschein, Kim, Pasternack, and Toniolo's study (1993). For the $\epsilon=0.70$ condition, in each population data of the component variables, the correlation between the repeated trials followed a simplex pattern in which the adjacent trials were more highly correlated than the trials which were farther apart (the details of correlation conditions are discussed later). Given the characteristics of the component variables, the effect of using ratio variables on the population ϵ was evaluated by comparing the ϵ values of the ratio score populations ($\epsilon_{x1/x2}$). The effect of using ratio variables on the sampling characteristics of $\hat{\epsilon}_{x1/x2}$ was evaluated by comparing the means and standard deviations of the $\hat{\epsilon}_{x1/x2}$ in different simulation conditions. It can be

speculated that a sample covariance matrix is always expected to violate the circularity assumption to some degree, even when the population matrix does not. By investigating the ϵ estimates of samples drawn from the ratio population under different conditions, we could examine the effect of using ratio variables on the sampling behavior of the estimated ϵ coefficient ($\hat{\epsilon}_{x1/x2}$).

Population means of the component variables. The population means of the component variables were set to $\mu_{x1j}=75.0$ and $\mu_{x2j}=100.0$ for all treatment trials based on the waist and hip girths in Sonnenschein, Kim, Pasternack, and Toniolo's study (1993). Here j denotes the j th trial of the repeated measurement experiment. By setting the means to be constant over trials, we were able to investigate how ratio variables would affect the type I error rates of the RM ANOVA test in different combination conditions of the variances and covariances of the component variables.

ρ_{x1x2} , V_{x1} , and V_{x2} . As shown in Formulas (4), (5), (6), and (7), both the correlations of the component variables within trials and over trials, and the coefficients of variation of the component variables play an important role in determining the ratio variable means, variances, covariances, and correlations. Among the four types of correlations between the numerator and denominator variables (i.e., $\rho_{x1i,x2i}$, $\rho_{x1i,x2j}$, $\rho_{x1i,x1j}$, $\rho_{x2i,x2j}$, $i, j=1,2,\dots,5$), the most meaningful correlations in practical research are $\rho_{x1i,x2i}$, $\rho_{x1i,x1j}$, and $\rho_{x2i,x2j}$. Here $\rho_{x1i,x2i}$ is the correlation between the numerator and denominator variables in the same trial (i), $\rho_{x1i,x1j}$ is the correlation between the numerator variables in different trials (i, j), and $\rho_{x2i,x2j}$ is the correlation between the denominator variables in different trials. The correlation $\rho_{x1i,x2j}$ ($i \neq j$) is the correlation between the numerator variable in trial i and the denominator variable in trial j . Its magnitude was based on the

other correlation components in the correlation matrix (the details on computing $\rho_{x1i,x2j}$ are shown in a subsequent section).

The initial correlation among the five trials was also based on the empirical data (Sonnenschein et al., 1993). The main objective of the investigation was to examine how the relative variation and the correlation pattern over trials between the numerator and denominator variables affect the characteristics of the covariance matrix of a ratio population. Therefore, to clearly show the main effect in the investigation, the magnitudes of $\rho_{x1i,x1j}$ and $\rho_{x2i,x2j}$ were set to fixed values for the condition of $\epsilon_{x1}=\epsilon_{x2}=1.0$. That is, for the $\epsilon_{x1}=\epsilon_{x2}=1.0$ condition, the correlation between the two adjacent trials in Sonnenschein's data set ($\rho_{x1i,x1j}=0.87$ and $\rho_{x2i,x2j}=0.82$, $i \neq j$) was employed to form the covariance matrix for the raw score population data with equal variances and equal covariances over the five trials. These two values also served as a baseline for other epsilon conditions (i.e., $\rho_{x11,x12}=0.87$ and $\rho_{x21,x22}=0.82$ in the conditions $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=0.7$). For the $\epsilon=0.70$ condition, the pattern of the correlations $\rho_{x1i,x1j}$ and $\rho_{x2i,x2j}$ followed a simplex pattern showing the correlation between the first trial and subsequent trials decreasing exponentially (i.e., ρ , ρ^2 , ρ^3 , ρ^4 ...). To investigate how the correlation between the numerator and denominator affect the result of using ratio variables in RM ANOVA, the three levels of the correlation between the numerator and denominator in each trial (ρ_{x1ix2i}) were chosen to represent a range of low (.50), medium (.70), and high (.90) relationships (e.g., Atchley, 1976). The value of ρ_{x1ix2i} was fixed in all five trials in each population to reflect a fact that many ratios in human kinetics have a near constant correlation between the numerator and denominator variables in a repeated measurement

experiment (waist/hip, weight/height, and nondominant/dominant strength). Therefore, this investigation was limited to a fixed $\rho_{x_1x_2i}$ over trials.

V_{x_1} and V_{x_2} are the coefficients of variation of the numerator X_1 and denominator X_2 , respectively, where the coefficient of variation is $V_t = \sigma_t / \mu_t$. Three relative variation situations were simulated ($V_{x_1}/V_{x_2}=2.0, 1.0, 0.5$). The three combinations of the coefficient of variation between the numerator and denominator variables were designed to investigate the ratio effect and to cover different relative variations between the numerator and denominator variables.

Preparing Population Covariance Matrices.

The first step in generating the population data was to prepare the population covariance matrices of interest. Table 3-2 shows the format of the population covariance matrix with five repeated measurement trials. Because this covariance matrix is symmetrical, only the upper right cells are shown in Table 3-2.

The two shadowed parts are the sub-covariance matrices for the numerator and denominator variables, respectively. The elements of the two shadowed sub-covariance matrices were derived according to the defined population epsilon (ϵ_{xi} , $i=1, 2$), correlations of the two component variables over the repeated trials, and coefficient of variation (V_{x_1} , and V_{x_2}) for each of the simulation conditions.

The unshadowed part of the matrix contains the covariances between the two component variables in the five measurement trials. The elements in the diagonal of the unshadowed sub-matrix are the covariances between the component variables in the same trial. These covariances were calculated by using the following equation:

$$\sigma_{x_1i,x_2i} = Cov(X_{1i}, X_{2i}) = \rho_{x_1,x_2} \sqrt{\sigma_{x_1i} \sigma_{x_2i}} \quad i = 1, 2, \dots, 5 \quad (20)$$

where $\rho_{x1i,x2i}$ denotes the correlation between the numerator and denominator variables in the same trial. Since the variances and the correlation between the numerator and denominator variables in the same trial were defined from the controlled conditions, the $Cov(X_{1i}, X_{2i})$ in the five trials was easily derived. The off-diagonal elements of the unshadowed sub-matrix are the covariances between the numerator and denominator variables in different trials (trial i and j):

$$\sigma_{x1i,x2j} = Cov(X_{1i}, X_{2j}) = \rho_{x1i,x2j} \sqrt{\sigma_{x1i} \sigma_{x2j}} \quad i, j = 1, 2, \dots, 5 \quad i \neq j \quad (21)$$

where $\rho_{x1i,x2j}$ is the correlation between the numerator variable in the i th trial and the denominator variable in the j th trial. This correlation can not be obtained directly from the defined control conditions, and thus must be estimated.

It is expected that the correlation between X_1 and X_2 in different trials ($\rho_{x1i,x2j}$, $i \neq j$) will be lower than that between X_1 and X_2 in the same trial. The rationale for this is twofold. First, if X_1 and X_2 were perfectly reliable, it might be expected that the correlation between X_1 and X_2 in different trials would be identical to that between X_1 and X_2 in the same trial. However, the reliabilities of X_1 and X_2 are considerably less than unity, and this trial-to-trial unreliability of both X_1 and X_2 will affect (reduce) the correlations between X_1 and X_2 in different trials. It could be hypothesized that this reduction would be a function of the reliability of X_1 and X_2 . Second, the correlation matrices of the empirical data sets (e.g., Kramer's data, 1994, and Sonnenschein's data, 1993) do show that $\rho_{x1i,x2j}$ is less under the condition $i \neq j$ than in the $i = j$ condition. The relationships between $\rho_{x1i,x2i}$ and $\rho_{x1i,x2j}$ ($i \neq j$) were examined using two empirical data sets (Kramer et al., 1994, Sonnenschein et al., 1993), and the examination suggested that an

appropriate approximation for the correlation between the numerator in trial i and the denominator in trial j could be presented as:

$$\rho_{x_{1i}, x_{2j}} = \rho_{x_{1i}, x_{1i}} \sqrt{\rho_{x_{1i}, x_{1i}}} \sqrt{\rho_{x_{2j}, x_{2j}}} \quad i \neq j \quad (22)$$

The estimated values of the correlation $\rho_{x_{1i}, x_{2j}}$ ($i \neq j$) using Formula (22) were very close to the empirical correlation values in the correlation matrices in these two data sets (the largest deviation was only 0.02). An analytic approach based on the attenuation correlation formula (Nunnally, 1978, pp215-221) also supported the validity of Formula (22). The details of the support based on the attenuation correlation formula are presented in Appendix A.

To derive the covariances between the numerator and denominator in different trials, we replaced $\rho_{x_{1i}, x_{2j}}$ in Formula (21) with Formula (22):

$$\sigma_{x_{1i}, x_{2j}} = \text{Cov}(X_{1i}, X_{2j}) = \rho_{x_{1i}, x_{2j}} \sqrt{\sigma_{x_{1i}, x_{1i}}} \sqrt{\sigma_{x_{2j}, x_{2j}}} \quad i, j = 1, 2, \dots, 5 \quad i \neq j \quad (23)$$

Based upon the defined conditions, Formula (23) was used to calculate the off-diagonal elements in the unshadowed part of the covariance matrix.

In order to generate the raw score populations for this investigation, each of the elements in the covariance matrix in Table 3-2 was defined or calculated based upon the controlled conditions and the procedures shown above. A Turbo PASCAL computer program developed by Eom (1993) was employed to construct the covariance matrices for X_1 and X_2 (i.e., shadowed parts of the population matrix in Table 3-2). With the specified population parameters (i.e., variances, correlations, and means), the computer program first constructed a population covariance matrix with an epsilon equal to unity ($\epsilon=1$) for the component variables. Second, each of the matrices was further manipulated

by varying the correlations between repeated trials for each of the component variables to reflect a simplex pattern, resulting in a population matrix possessing a desired epsilon value ($\epsilon=0.70$). Third, the covariances between X_1 and X_2 (i.e., unshadowed part of the population matrix in Table 3-2) were calculated individually following the procedures shown earlier. The covariance matrices based on the defined conditions for generating the raw score populations are shown in Appendix B.

Data Generation

A FORTRAN program, which incorporated the Random Number Generator in IMSL (IMSL, 1991), was developed to conduct the data generation and the simulation. The Multivariate Random Number Generator (DRNMVN) routine in IMSL was used to generate raw score population data matrices according to the defined population covariance matrix in each of the experimental conditions shown in Table 3-3. Each raw score data matrix contained $N \times 2k$ data points (see Table 3-1), where N refers to the population size and k refers the number of treatment trials in the experiment (i.e., $N=90,000$ and $k=5$). The first five variables were the numerator variable (X_1) over five treatment trials, and the last five variables were the denominator variable (X_2) over five treatment trials. Therefore, there were $2k=10$ measurements for each subject in each condition (five measurements for the numerator variable and five measurements for the denominator variable). The ratio population data were derived by dividing each value in the first five columns by the corresponding value of the denominator variable in the last five columns in the raw score data matrices.

Table 3-3
Computer Simulation Conditions

ε	μ	$\rho_{x_1x_2}$	v
$\varepsilon_{x_1} = \varepsilon_{x_2} = 1.0$		0.9	$V_{x_1}=0.2 \ V_{x_2}=0.1$
$\varepsilon_{x_1} = \varepsilon_{x_2} = 0.7$	$\mu_{x_1} = 75 \ \mu_{x_2} = 100$	0.7	$V_{x_1}=V_{x_2}=1.55$
$\varepsilon_{x_1} = 0.7 < \varepsilon_{x_2} = 1.0$		0.5	$V_{x_1}=0.1 \ V_{x_2}=0.2$
$\varepsilon_{x_1} = 1.0 > \varepsilon_{x_2} = 0.7$			

Overview of Methods Used to Answer Questions

I first investigated the characteristics of the ratio variable populations under different variation and correlation conditions of the component variables. This process provided evidence to examine how characteristics of the component variables affected the circularity assumption of the covariance matrices for the derived ratio variable population. Box's ϵ values of the ratio score populations in different conditions were computed to provide information regarding the degree of non-circularity of the covariance matrices of the ratio variables.

Second, the behavior of the sample estimates of ϵ ($\hat{\epsilon}$) for ratio data across the simulated conditions was examined. The FORTRAN Monte Carlo simulation program was used for the purpose of investigating the effect of using ratio variables on the sampling characteristics of $\hat{\epsilon}$. After drawing samples of n subjects from the ratio score populations ($n=15, 30, 45$), the FORTRAN program conducted 2000 one-way RM ANOVA tests in each condition for the ratio score data. The following sample estimates were computed: mean squares in the F test, standard deviation of the mean squares, mean of $\hat{\epsilon}$, standard deviation of $\hat{\epsilon}$, and type I error rate. By comparing these sample estimates in different conditions, we were able to investigate how the relative variance and correlation changes between the numerator and denominator variables affected the type I error rate of the F test with ratio variable data.

CHAPTER IV. RESULTS AND DISCUSSION: CHARACTERISTICS OF RATIO VARIABLES

In this chapter, the results emanating from each of the four empirical data sets are presented. First, I present detailed evaluation results for each of the two data sets in which the ratio variables were used for the purpose of deflation. The four ratio deflation models are compared and evaluated based on the criteria listed in the methods and procedures chapter. The five criteria: statistical, graphical, algebraic, R^2 , and reliability, are examined for each of the deflation models. Then, the best deflation model is identified for each of the two data sets, and general issues related to the development of an appropriate deflation model for ratio variables in human kinetics research are discussed. Second, the statistical characteristics of the other two sets of ratio data (in which the ratio variables were not used for the purpose of deflation) are evaluated, and the issue of reliability of using ratio variables is discussed.

Ratio Variables Used for Deflation Purposes

Empirical Data Set 1: Vo2max/Body Mass

These data were obtained from a study conducted at the Vancouver General Hospital and the University of British Columbia (Taunton, 1992), which investigated the effectiveness of exercise for healthy women aged 65 to 75 years. Seventy-six subjects were randomly assigned into a water-based exercise group, a land-based aerobic group or a classroom health promotion group (control group). Each group met for one hour, three times a week, over a twelve weeks period. The variables of interest were measured at

three time points (trials) during the twelve weeks (pre-test, mid-program test, and post-test). The measured variables included maximal fatigue, strength, body composition, flexibility, balance, and Vo2max, with the latter measure being one of the most important variables measured in the study. The measured Vo2max was divided by the body mass in order to express the Vo2max on a per-unit of body mass basis, thus deflating the effects of the body mass variation from the measured Vo2max variable. The derived ratio Vo2max variable, Vo2max/kg, then served as a measurement variable in a 3 x 3 (group x time) RM ANOVA to answer the research questions. The researchers' primary interest was to investigate whether cardiovascular endurance, as measured by Vo2max per-unit of body mass, was affected by the different training techniques.

To evaluate the validity of the Vo2max/kg ratio variable and the alternative deflation models in these data, the initial plan was to evaluate the three groups separately (G_1 =water, n_1 =27; G_2 =land, n_2 =25, G_3 =control, n_3 =23; k =3). However, as shown in Table 4-1, the land and control groups had a considerable amount of missing data, especially in the second and third measurement points, due to subject drop out. Because the F tests in the 3 x 3 (group x time) mixed model ANOVA showed no significant differences in Vo2max/kg among the groups and no significant groups by time interaction, the complete cases in the three groups were combined and analyzed together (n =52).

Simple Ratio Model

The results reported in Table 4-2 are based on the simple ratio data. As shown in Table 4-2, the correlation between Vo2max (X_1 , raw score) and body mass (X_2) ranged from 0.46 to 0.54 in the three repeated trials. This result indicates that the variation of

Table 4-1

Means and Standard Deviations of Body Mass, Raw Score Vo2max, and Vo2max/kg: Data Set I

	Body Mass			Raw Score Vo2max			Vo2max/kg		
	T1	T2	T3	T1	T2	T3	T1	T2	T3
Group1 n=27	65.83 (10.18) ^a n=27	65.60 (9.89) 25	65.70 (10.21) 23	1232.06 (249.01) 27	1346.79 (203.88) 25	1365.39 (206.71) 23	18.94 (3.87) 27	20.79 (3.42) 25	21.05 (3.31) 23
Group2 n=26	67.69 (14.63) n=26	67.25 (14.72) 20	67.59 (14.89) 19	1205.09 (196.28) 26	1332.08 (220.72) 20	1400.10 (217.61) 19	18.24 (3.34) 26	20.21 (3.11) 20	21.23 (3.80) 19
Group3 n=23	67.76 (10.24) n=23	64.15 (8.71) 16	66.33 (7.70) 12	1221.16 (235.64) 23	1201.45 (190.21) 16	1359.82 (187.13) 12	18.22 (3.52) 23	18.95 (3.48) 16	20.85 (4.31) 12
All Groups n=52 ^c	66.29 (11.97) V=0.18 ^b	66.31 (11.67) 0.18	66.48 (11.52) 0.17	1231.18 (230.69) 0.19	1319.19 (190.74) 0.14	1373.39 (205.16) 0.15	18.85 (3.49) 0.19	20.22 (3.10) 0.15	21.02 (3.56) 0.17

^a Standard deviations are in brackets. ^b V is coefficient of variation. ^c No missing values.

Table 4-2

Evaluation of the Simple Ratio Model: Data Set 1

Ratio Model	r X_1 vs X_2	r Y vs X_2	Estimated Expected Value $\hat{\beta}$	Empirical Mean $\bar{Y}$
$Y = X_1/X_2$				
Trial 1	0.48	-0.45	18.28	18.85
Trial 2	0.54	-0.61	19.57	20.22
Trial 3	0.46	-0.58	20.30	21.02
Reliability Coefficient				
	X_1	X_2	Y	
Intraclass r				
Average Trials	0.88	0.99	0.89	
Single Trial	0.72	0.99	0.72	

body mass does affect the variation of the raw score Vo2max variable. The heavier subjects tended to have greater Vo2max raw scores. An appropriate ratio variable could be used to fully remove the effect of body mass from the Vo2max variable.

The question of interest with these data is whether the simple ratio (Vo2max/kg) can appropriately deflate the effect of the body mass from the Vo2max. As shown in the methods and procedures chapter, one important criterion is that the correlation between the derived Vo2max/kg and body mass should be zero or nearly so if the simple ratio can appropriately deflate the body mass from the Vo2max. This criterion is not met, as the results ($r_{y|x_2i}$ in Table 4-2) show substantial negative correlations (0.45-0.61) between the Vo2max/kg (Y) and body mass (X_2) in all three trials. Given a positive correlation between the two component variables, using Pearson's approximation function (Formula (7)). Albrecht (1978) showed that the correlation between a ratio and its denominator variable is negative when the coefficient of variation of the denominator variable is equal to or greater than that of the numerator variable. As can be seen in Table 4-1, the coefficients of variation of the body mass are almost equal to or greater than that of the raw score Vo2max in all three trials, and the raw score Vo2max and body mass have positive correlation. Thus, negative correlations between Vo2max/kg and body mass in these data are to be expected.

The graphical criterion explained in the previous chapter was designed to identify any visible pattern of a relationship between a ratio variable and its denominator variable, and any denominator related distortion of the variances for the ratio variables. If the simple deflation model is valid, Vo2max/kg should have not only a zero linear correlation but also zero curvilinear relation with the body mass. As shown in the three scatterplots

in the first panel of Figure 4-1, a negative relationship pattern is clearly shown in all three trials. It confirms that the simple ratio variable favors the lighter subjects and disadvantages the heavier subjects. That is, the lighter subjects show larger Vo2max/kg than the heavier subjects. There is no suggestion of a curvilinear relationship between the Vo2max/kg and the body mass in any of the three trials. If there were curvilinear relationship between Vo2max/kg and body mass, the ratio Vo2max/kg would not be valid for the deflation purpose. However, this is not the case in these data. The scatterplots, and the negative Pearson correlation coefficients, indicate that the commonly used ratio, Vo2max/kg , over-deflates the denominator effect and may not be valid for deflation purposes based on these two criteria.

As stated in the previous chapter, a correctly formulated ratio variable should result in virtual equality of the estimate of the expected value of the ratio model and the calculated mean average of the empirical data. In the simple ratio model, the estimate of the expected value ($\hat{\mu}_y$) of the Vo2max/kg was computed from the β coefficient in the linear regression equation without an intercept (i.e., $X_{1it} = \beta_1 X_{2it} + e_{it}$). The empirical means in the three trials were computed from the ratio Vo2max/kg data. The calculations show that the empirical means are consistently greater than the estimate of the expected values in the simple ratio model (see Table 4-2). The over-estimation in the three trials is between 0.57 to 0.72, about 4% error relative to the actual mean of the empirical data.

The last criterion for evaluating the validity of a ratio data is its reliability. The reliability of these data was calculated based on Formulas (16) and (17) shown in the methods and procedures chapter. The mean squares used in Formulas (16) and (17) were derived from a three trials RM ANOVA test performed on each of the three dependent

Figure 4-1. The scatterplots of the simple ratio and the LRM ratio: Data Set 1.

variables, the raw score Vo2max, body mass, and simple ratio Vo2max/kg. Then, the intraclass correlation coefficients (r_{x1x1} , r_{x2x2} , r_{yy}) were calculated based on the derived mean squares of the RM ANOVA tests. The intraclass correlations are included in reporting of results. The discussion of reliability is based on the intraclass correlation for the mean of all trials (Formula (17)), but the single trial reliability (Formula (16)) is also reported as both forms are often presented in the literature. Based on the definition, the single trial reliability reported is the same as the average pairwise correlation of the trials in the data.

The results (see Table 4-2) show that the body mass data are very reliable ($r_{x2x2}=0.99$), and the reliability of the raw score Vo2max data is also high ($r_{x1x1}=0.88$). The high reliability of body mass in these data is expected because body mass should not be substantially affected in the twelve week experiment period. The simple ratio did not reduce the reliability of the ratio data, with the reliability of the simple ratio data being close to that of the measured Vo2max variables ($r_{y1y1}=0.89$). Because the reliability of the denominator variable is so high, the reliability of the derived simple ratio variable is very close to the reliability of the numerator variable. This confirms Cronbach's deduction (1941). That is, when the denominator variable is highly reliable (e.g., r_{x2x2} close to unity), it can be proved that the reliability of the ratio variable is close to the reliability of the numerator variable.

In summary, for the evaluation of the simple ratio in Data Set 1, the ratio Vo2max/kg does not appropriately deflate the effect of body mass from the Vo2max variable. Although the simple ratio has acceptable reliability, the simple ratio does not fully deflate the effect of body mass. It only reverses the relationship between the

measured Vo2max and the body mass from positive to negative. Based on the results of examining the criteria for deflation purposes in these data, the simple ratio does not meet the important criteria of full deflation, especially zero correlation between the ratio variable and the denominator variable, and equality of calculated empirical mean and the estimate of the expected value. Therefore, it is concluded that the simple ratio does not appropriately deflate the effect of the body mass for the Vo2max variable in these data. An appropriate alternate deflation model may be needed for this commonly used measure.

Alternative Ratio Models

These data were fitted to the three alternative deflation models (LRM, NLRM1, and NLRM2, see p31 for definitions). The results are shown in Table 4-3.

As shown in Table 4-3, the adjusted ratio Vo2max variables have a near-zero correlation with the denominator variable (body mass) in all three alternative models. For the models LRM and NLRM2, the correlations between the adjusted ratio Vo2max and the body mass in the three trials were not greater than 0.02. When NLRM1 is used, the correlations were as small as 0.01 in all three trials. Thus, in this data it is concluded that all three models meet the criterion of zero correlation.

Some researchers (e.g., Albrecht, 1993) claimed that adjusted ratios especially with the nonlinear models (NLRM1 and NLMR2) bring about a serious denominator-related distortion of variances. This distortion of variance would be indicated in scatterplots of the adjusted ratio variable versus the denominator variable. That is, the adjusted ratio variable has less variance for greater values of the denominator and higher variance for smaller values of the denominator. As shown in Figures 4-1

Table 4-3
Evaluation of the Alternate Models: Data Set 1

	r Y vs X ₂	Reliability Intraclass r		R ²	Empirical Mean Y	Estimated Expected Value $\hat{\beta}$	Parameters	
Y=(X ₁ -a)/X ₂ (LRM)		all Trials	Single Trial				a	k
Trial 1	0.02			0.229	9.22	9.23	619.53	
Trial 2	-0.01	0.83	0.62	0.293	8.84	8.84	732.98	
Trial 3	-0.01			0.214	8.24	8.23	826.05	
Y=X ₁ /X ₂ ^k (NLRM1)								
Trial 1	0.01			0.233	148.63	148.67		0.50
Trial 2	-0.01	0.83	0.62	0.289	198.41	198.38		0.45
Trial 3	-0.01			0.210	251.51	251.47		0.41
Y=(X ₁ -a)/X ₂ ^k (NLRM2)								
Trial 1	0.01			0.233	312.64	309.47	-367.80	0.39
Trial 2	-0.02	0.82	0.60	0.291	56.20	55.42	426.96	0.66
Trial 3	-0.02			0.212	44.88	45.73	563.89	0.69

(panel 2) and 4-2, the scatterplots of the three ratios in the three trials do not indicate visible variance decreases when the value of the denominator variable (body mass) increases (although the sample size might be too small to visually reveal any such effects). Therefore, body mass related distortion of variance does not seem to be an issue for these three models in this data set.

The third criterion for the evaluation of a deflation model is the equality of the estimated expected value and the empirical means. The calculations (see Table 4-3) indicate that the expected means are accurately estimated (within +/- 1% difference) by all three models.

The reliability of the adjusted ratio variables is examined and shown in Table 4-3. The reliability of the adjusted ratio Vo2max variables was lower than the reliability of the two component variables (e.g., 0.83 compared to 0.88 and 0.99). There was no substantial difference in the reliability among these three models for these data. The relative lower reliability for the adjusted ratio variables was expected, and for the following reasons. The LRM ratio is based on a simple linear regression relationship $X_{1it} = a_t + \beta_t X_{2it} + e_{it}$, where e_{it} is the error term and β_t serves as the expected value of the adjusted ratio data. The adjusted ratio variable (LRM) can be expressed as $Y_{it} = (X_{1it} - a_t) / X_{2it} = \beta_t + e_{it} / X_{2it}$. Of interest in the LRM model is the error term e_{it} / X_{2it} . Heteroscedasticity occurs when the variance of Y_{it} is not constant for all values of X_{2it} , and the level of heteroscedasticity would directly affect the reliability of the data. The larger the range of the variable X_{2it} , the greater the heteroscedasticity of the data, and the lower the reliability of Y_{it} . The heteroscedasticity of the LRM model is very sensitive to the range of the denominator variable (Pedhazur, 1982). Thus, the heteroscedasticity that is introduced by

Figure 4-2. The scatterplots of the NLRM1 ratio and NLRM2 ratio: Data Set 1.

the ratio transformation is the main factor to reduce the reliability of the adjusted ratio variable in the LRM model. Because the range of the body mass in these data is relatively small, the reliability of the adjusted ratio variable is reduced, but only slightly, and the reliability of Y under both the LRM and NLRM1 models is still reasonably strong (0.83).

The NLRM1 ratio is based on the relationship $X_{1it} = Y_i X_{2it}^k \cdot e_{it}$. Allison (1995) claimed that this approach might be quite desirable and appropriate. However, when the adjusted ratio variable is used in group comparisons or RM ANOVA, it is valid only when the exponent parameters of the adjusted ratio variable in the different trials are identical. Because the exponents are different in the different trials, the units of the adjusted ratio variable are different. For example, when calculating the reliability of the adjusted ratio variable, one could not calculate the reliability coefficient by using the NLRM1 model with different parameters such as $k_1=0.50$, $k_2=0.45$, and $k_3=0.41$ in the three trials. Therefore, a common exponent ($k=0.45$) was derived by averaging the three exponent parameters (i.e., $k=(k_1+k_2+k_3)/3$). The common $k=0.45$ was used to derive the NLRM1 ratio for each trial, and the reliability calculations were based on those data. When the common exponent k is used in the different trials, it increases the variances of the adjusted ratio variable. Because a common k is used, the inflation of the variances is more serious in the first and third trials. Therefore, it reduces the reliability of the adjusted ratio variable in the NLRM1 model. For the same reason, a common k has to be used in the NLRM2 model, thus reducing the reliability of the adjusted ratio data.

As explained in the methods and procedures chapter, the ordinary least squares linear regression option in SPSS was used to fit the LRM model. The nonlinear

regression option with a Levenberg-Marquardt algorithm in SPSS was used to fit the NLRM1 and NLRM2 models. It has been shown that the simple ratio over-adjusts the effect of the denominator and is not an appropriate deflation model for these data. To compare the relative merits of the three alternative models, R^2 , the proportion of variance accounted for by the model, was examined. It indicates that the differences of R^2 among the three adjusted ratios are trivial (see Table 4-3). The magnitudes of the R^2 obtained using the NLRM2 model are slightly higher than those for the NLRM1 model, and the LRM model has slightly higher R^2 than the other two nonlinear models in these data but difference is not substantial. Results also show that the reliability of the three adjusted ratio data was virtually the same. These results suggest that there is no substantial difference among the three models in their ability to provide a valid ratio score for these data.

It may seem contradictory that the three deflation models with different parameters fit these data about equally well. The explanation for this phenomenon is that the regression lines diverge outside the range of the observed data in accordance with the very different regression models. For example, as shown in Figure 4-3, the fitted curvilinear regression lines of the second trial in Data Set 1 are very close to linear in the relatively narrow range of these observed data. However, the nonlinear models do show the better fit than the linear model in some other situations (Albrecht et al., 1993).

Given the similar goodness of fit in the models, the LRM and NLRM1 models may be more preferable than the NLRM2 model for these data because the LRM and NLRM1 models are easier to use. If the LRM model is used, the linear regression approach (which is available in most commercial statistical software) can be used to fit

Figure 4-3. Comparison of the fit of the three deflation models: DataSet 1.

the data.

As shown above, LRM and NLRM1 have successfully met the criteria and are simpler than NLRM2 for deflation purposes in Data Set 1, although with some reduction in the reliability. The simple ratio model failed to meet these criteria. Therefore, if the reliability of the two models is acceptable, it may be suggested that LRM and NLRM1 are statistically valid and appropriate for the purpose of deflation in Data Set 1.

Even though the LRM and NLRM1 ratios have similar validity based on statistical criteria, physiologists may have reason to choose one over the other. In recent years, NLRM1 has been used in some human kinetics research (e.g., Heil, 1997; Nevill, Ramsbottom, & Williams, 1992; Vanderburgh & Katch, 1996). Based on the theory of geometric similarity, a common value of 0.67 for the k parameter was developed for $\text{Vo}_{2\text{max}}$ (Astrand, & Rodahl, 1986; Heil, 1997). Heil (1997) pointed out that the 0.67 exponent appears to be more appropriate for samples that are very similar with respect to age, training background, body height, etc. However, a sample that is homogeneous on so many measurement variables is hard to find in an empirical study. This study supports that of other researchers who have shown that a common magnitude of the k parameter in the NLRM1 model for $\text{Vo}_{2\text{max}}$ data is questionable (e.g., Bergh, Sjodin, Forsberg, & Svedenhag, 1991; Nevill, 1994; Welsman, Armstrong, Nevill, Winter, & Kirby, 1996). The derived magnitudes of k for $\text{Vo}_{2\text{max}}$ reported in the literature range from 0.41 to 0.80, indicating that it is a data dependent parameter, and therefore it is not appropriate to use a common parameter k in the NLRM1 model for $\text{Vo}_{2\text{max}}$. It is suggested here that researchers should fit a NLRM1 model to their data and derive a sample-dependent k value for the adjusted ratio variable $\text{Vo}_{2\text{max}}$ in each study. Although the NLRM2 model

is not suggested for this specific data set, it should be considered by researchers as a possible deflation model.

When alternative deflation models are used, special caution must be taken when comparing the means of the adjusted ratio variables among repeated trials or different groups, and when interpreting the meaning of the actual values of the adjusted ratio variables. First, when the nonlinear regression models are used, group or trial comparisons are appropriate only when the exponents (k) for different groups or trials are identical. For example, in Data Set 1, because the magnitudes of the parameter k of the NLRM1 model in the three trials are slightly different (i.e., $k_1=0.50$, $k_2=0.45$, and $k_3=0.41$), the scale of the adjusted Vo2max of NLRM1 in the three trials are different. That is, the scales of the adjusted Vo2max in the three trials are $\text{ml}\cdot\text{kg}^{-0.50}\cdot\text{min}^{-1}$, $\text{ml}\cdot\text{kg}^{-0.45}\cdot\text{min}^{-1}$, and $\text{ml}\cdot\text{kg}^{-0.41}\cdot\text{min}^{-1}$, respectively. Thus, one could not compare the three trial means of the adjusted Vo2max derived from the NLRM1 model under the different scales. To compare the three group means, the scale of the adjusted ratio variable in all trials or groups should be identical. In these data, a common k parameter of NLRM1 can be obtained by the average of the three k parameters over the three trials (i.e., $k=(k_1+k_2+k_3)/3$).

Averaging different values of the k parameter is not the only method for getting the same unit scale in different trials or groups. If different groups are compared (not the same group in different trials), the data could also be pooled and a common k can be derived by fitting the NLRM1 model to the pooled data. However, when the NLRM1 model is used to fit the data in repeated measurement data like Data Set 1, the pooled data may not be valid for this purpose because it would violate the assumption of

independence of the error variance over subjects. That is, if the repeatedly measured data were pooled, there would be a between-subject error correlation because the repeated measurements for the same subject were treated as different subjects in the pooled data.

A second caution is that nonlinear regression modeling yields adjusted ratio variables that may not be as easy to interpret as the raw score component variables due to the complex scale (e.g., $\text{ml}\cdot\text{kg}^{-0.45}\cdot\text{min}^{-1}$ in Data Set 1). However, the relative differences among groups or trials are comparable. The actual group or trial differences can be described dimensionlessly and expressed as a percentage difference. For example, the sample means of the adjusted Vo2max using NLRM1 with common $k=0.45$ in the first and second trials are 184.72 and 198.41, respectively. This difference can be expressed by saying that based on the NLRM1 model the first trial had 93% of the body mass adjusted Vo2max of the second trial. It should also be noted that the percentage difference comparison depends on which model is being employed.

Empirical Data Set 2: D_{LCO}/V_A

This data set was from a research project conducted in the Sports Medicine Center at the University of British Columbia (Bacon, 1997). One of the purposes of the research was to investigate whether there was a change in “diffusing capacity of the lungs for carbon monoxide” (D_{LCO}) within an ovulatory menstrual cycle in regularly menstruating women. In order to deflate the variation of the inspired volumes, which was measured as alveolar ventilation volume (V_A), the ratio variable D_{LCO}/V_A was used in the study. This procedure of deflation is frequently used and D_{LCO}/V_A is a common variable in similar studies in the related literature (e.g., Crapo & Forster, 1989). However, unlike the

Vo2max/kg ratio measure, the statistical and validity issues of this ratio have never been addressed in the literature. There were 13 subjects in the study, and D_{LCO} and V_A were measured at five points during the menstrual cycle. The five test points were early menstrual, late menstrual, early follicular, midcycle, and mid luteal. A one-way RM ANOVA ($n=13$, $k=5$) was used to analyze these data. Table 4-4 shows the descriptive characteristics of the data.

Simple Ratio Model

The criteria to evaluate the validity of the model with these data are the same as in Data Set 1. As shown in Table 4-5, the correlations between the measured D_{LCO} and V_A vary from 0.29 to 0.59 in the five measurement trials. This indicates the positive relationship between V_A and D_{LCO} , and thus an appropriate deflation model could be used to deflate the effect of V_A from the D_{LCO} data.

The results show negative correlations between the simple ratio D_{LCO}/V_A and V_A in the five trials, indicating that the simple ratio model over-adjusts the effect of V_A from the D_{LCO} data. The scatterplots of Figure 4-4 also show that the subjects who have greater V_A tend to have smaller D_{LCO}/V_A (this trend was not clearly shown in the first trial), but there is no suggestion of a possible curvilinear relationship between the simple ratio D_{LCO}/V_A and V_A . The estimated expected value and the empirical means are not the same over the five trials. The intraclass correlation shows that the simple ratio D_{LCO}/V_A has a reliability ($r_{yy}=0.90$) which is approximately the same as its components.

In summary, although the simple ratio model meets some of the criteria, the ratio D_{LCO}/V_A may not be an appropriate model for deflation in these data because it over-deflates the effect of V_A from D_{LCO} . An alternative deflation model should be considered.

Table 4-4

Means and Standard Deviations of Va, Raw Score DLco, and DLco/Va: Data Set 2

	Va					DLco					DLco/Va				
	T ₁	T ₂	T ₃	T ₄	T ₅	T ₁	T ₂	T ₃	T ₄	T ₅	T ₁	T ₂	T ₃	T ₄	T ₅
Mean	5.82	5.76	5.95	5.89	5.93	22.20	22.17	22.13	21.32	21.69	3.82	3.91	3.74	3.66	3.70
Sd	0.55	0.85	0.85	0.90	0.85	3.28	3.34	3.41	3.42	2.23	0.48	0.72	0.49	0.56	0.48
V	0.09	0.15	0.14	0.15	0.14	0.15	0.15	0.15	0.16	0.10	0.13	0.18	0.13	0.15	0.13
n	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13

Table 4-5

Evaluation of the Simple Ratio Model: Data Set 2

Ratio Model	r X_1 vs X_2	r Y vs X_2	Estimated Expected Value $\hat{\beta}$	Empirical Mean $\bar{Y}$
$Y = X_1/X_2$				
Trial 1	0.58	-0.08	3.81	3.82
Trial 2	0.29	-0.59	3.80	3.91
Trial 3	0.59	-0.35	3.69	3.74
Trial 4	0.49	-0.49	3.58	3.66
Trial 5	0.50	-0.68	3.61	3.70
Reliability Coefficient				
	X_1	X_2	Y	
Intraclass r				
All Trials	0.90	0.93	0.90	
Single Trial	0.62	0.71	0.64	

Figure 4-4. The scatterplots of the simple ratio, LRM, NLRM1, and NLRM2 ratios: Data Set 2.

Alternative Ratio Models

Three alternative deflation models were evaluated using the same procedures used with Data Set 1. As shown in Table 4-6, when the alternative deflation models (LRM, NLRM1, and NLRM2) are used, the adjusted ratio D_{LCO} variables have a near-zero correlation with V_A . The scatterplots in Figure 4-4 do not show a curvilinear relationship between the adjusted ratio variable and its denominator variable, and the estimated expected value and the empirical mean of the data are virtually the same. When the NLRM2 model was fitted to the data for the second, third, and fifth trials, the estimate of the k parameters were similar to those for the NLRM1 model in these three trials. However, when the R^2 was used to evaluate goodness of fit for the models with these data, as shown in Table 4-6, the NLRM2 does not have a better fit than the NLRM1 model in the three trials. When the NLRM2 model was fitted to the data for the first trial and fourth trial, the estimate of the k parameter was unity, and the NLRM2 model is the same as the LRM model in the two trials. It suggests that the NLRM2 model does not have a better fit than the LRM model in the two trials. The R^2 for the NLRM1 and NLRM2 models in the other three trials did not substantially differ. The LRM and NLRM1 models could be suggested for these data because they are more parsimonious than the NLRM2 model.

It is noted that the sample size is small in Data Set 2 ($n=13$). If the sample size is small, it may result in very different magnitudes of the parameters in the fitting model (especially for the nonlinear regression models) for similar data in different trials. For example, the magnitude of the estimated parameter k in the NLRM1 model ranged from 0.30 to 0.93 over the five trials in Data Set 2. The serious inconsistency of the estimated

Table 4-6
Evaluation of the Alternate Models: Data Set 2

	r Y vs X ₂	Reliability Intraclass r		R ²	Estimated Expected Value $\hat{\beta}$	Empirical Mean $\bar{Y}$	a	k
<hr/>								
Y=(X ₁ -a)/X ₂ (LRM)		All Trials	Single Trial					
Trial 1	-0.010			0.34	3.51	3.51	1.76	
Trial 2	0.003			0.10	1.16	1.16	15.52	
Trial 3	0.040	0.90	0.62	0.35	2.37	2.37	8.00	
Trial 4	-0.010			0.24	1.88	1.89	10.22	
Trial 5	0.005			0.25	1.30	1.30	13.97	
<hr/>								
Y=X ₁ /X ₂ ^k (NLRM1)								
Trial 1	-0.0200			0.34	4.34	4.34		0.93
Trial 2	-0.0003			0.10	13.23	13.23		0.30
Trial 3	0.0300	0.89	0.62	0.35	7.13	7.12		0.64
Trial 4	-0.0100			0.24	8.46	8.46		0.52
Trial 5	0.0003			0.25	11.39	11.39		0.36
<hr/>								
Y=(X ₁ -a)/X ₂ ^k (NLRM2)								
Trial 1	-0.010			0.35	3.51	3.51	1.76	1.00
Trial 2	-0.001			0.09	13.03	13.02	0.22	0.30
Trial 3	0.050	0.88	0.60	0.36	7.81	8.01	-1.00	0.61
Trial 4	-0.010			0.24	1.88	1.88	10.22	1.00
Trial 5	0.040			0.25	12.60	13.02	-1.00	0.35

parameters could reduce the reliability of the adjusted ratio variable data because a common k has to be used to obtain the ratio scores in different trials, and the low reliability will affect the validity of using the deflation model. A larger sample size may be needed to reduce the inconsistency of the parameters in these data.

A Summary of the Evaluation of the Deflation Models

First, one should not assume that a simple ratio model is valid for deflation purposes, but rather that the procedures developed in this study could be used to determine a best deflation model. Second, the validity of a deflation model depends on the sample size and the statistical characteristics of the particular component variables used, and an optimal deflation model for all variables may not exist. Different deflation models should be used to fit each empirical data set for determining the best deflation model. Third, to derive a best deflation model, the following criteria could be considered: (a) zero correlation between the ratio variable and the denominator variable, (b) no curvilinear relationship between the ratio variable and the denominator variable, (c) equality of the estimated expected value of the model and the empirical mean of the ratio variable data, (d) high R^2 in the deflation model, and (e) high reliability of the ratio variable data.

Ratio Variables Used Not for Deflation Purposes

As previously mentioned, in human kinetics research, ratio variables are also used as a measurement variable of direct interest, without any intent to derive a denominator-free variable. In this case, a ratio variable has some special aspect not accounted for by the numerator or denominator variables individually. The main concern

when using this kind of ratio variables as a measurement variable is the reliability of the ratio variable and the effect of covariation of the component variables in a statistical analysis. In this section, two empirical studies were examined and the reliability issue of using this type of ratio variable was addressed. The effect of covariation of the component variables in a statistical analysis was investigated, using simulation procedures, in Chapter V.

Empirical Data Set 3: Nondominant Torque/Dominant Torque

These data are from a research project conducted by Kramer, Nusca, Bisbee, MacDermid, Kemp, and Boley (1994). The study examined the reliability of forearm pronation and supination isometric strength (torques) in absolute units and nondominant/dominant ratios. Twenty-one men and 22 women were tested with two repetitions in two occasions on each of two machines, a BTE work simulator and a Cybex dynamometer. The data we received from the authors were collected from the BTE work simulator. To reduce the variability of the measurement, the measured strengths of the two repetitions in each occasion were averaged. Then, the absolute torques of the dominant arm and nondominant arm, and the nondominant/dominant ratio on the two occasions were calculated. The reliability of the component variables and the nondominant/dominant ratio was evaluated.

Tables 4-7 and 4-8 show the characteristics of the ratio variable and the component variables for the forearm pronation and supination, respectively. The results show that the absolute torques exhibited large between subject variability ($V > 0.40$), and high reliabilities (0.94-0.98). However, with the nondominant/dominant ratio variable

Table 4-7

Means, Standard Deviations, and Reliability Coefficients of the Torques and Nondominant/Dominant Ratio (Pronation)

Pronation Torques	ND1 ^a	ND2	D1	D2	RATIO-1	RATIO-2
Mean	8.01	8.24	8.95	8.98	0.91	0.94
Sd	3.71	3.58	4.12	3.97	0.22	0.16
V	0.46	0.43	0.46	0.44	0.24	0.17
Reliability All Trials	0.97		0.94		0.68	
Single Trial	0.94		0.89		0.51	

^a ND1: The torque of the non-dominant arm in the first trial; ND2: The torque of the non-dominant arm in the second trial; D1: The torque of the dominant arm in the first trial; D2: The torque of the dominant arm in the second trial; RATIO-1: The non-dominant/dominant ratio in the first trial; RATIO-2: The nondominant/dominant ratio in the second trial.

Table 4-8

Means, Standard Deviations, and Reliability Coefficients of the Torques and Nondominant/Dominant Ratio (Supination)

Supination Torques	ND1	ND2	D1	D2	RATIO-1	RATIO-2
Mean	7.49	7.67	8.17	8.19	0.93	0.97
Sd	3.31	3.27	3.47	3.67	0.18	0.25
V	0.44	0.43	0.42	0.45	0.19	0.26
Reliability All Trials	0.95		0.98		0.75	
Single Trial	0.90		0.96		0.61	

the coefficients of variation were reduced considerably ($0.26 \geq V \geq 0.17$), as were the reliabilities (0.68 and 0.75 for the average of the two trials, and 0.51 and 0.61 for a single trial). The results show that the ratio transformation substantially reduces the reliability in these data even though the reliability of the component variables is high. As shown in Formula (7) in the literature review chapter, the reliability of ratio variables in a single trial is affected by not only the coefficient of variation but also by the within and between trial correlations of the component variables. Therefore, although the component variables are very reliable in these data, the reliability of the ratio variable reduces substantially because of the effect of the within and between trial correlations of the component variables. The details of the discussion on how these factors affect the reliability of the ratio variable are presented in a following section. The results in the Data Set 3 suggest that one should not assume that ratio data are reliable merely because the component variables are reliable. The reliability of the ratio data should be computed whenever the ratio transformation is conducted.

Empirical Data Set 4: Waist Girth/Hip Girth

These data were obtained from a study conducted in the New York University Medical Center (Sonnenschein, Kim, Pasternack, & Toniolo, 1993). The subjects were 35-65 year old women who visited a breast screening clinic and enrolled in the New York University Women's Health Study between 1985 and 1991 ($n=14,290$). The purpose of the original study was to investigate the relationship between endogenous hormones and cancer. The longitudinal data included the following information: A self-administered questionnaire which gathered demographic, medical, lifestyle, and reproductive

information, including current menopausal status and self-reported height and weight; a self-administered semi-quantitative food frequency questionnaire; and waist and hip girths measured for 1,851 cohort members at each clinic visit.

The waist/hip ratio has recently become a common measure in health science research, as it purportedly reflects fat distribution in the body. High ratio scores are indicative of health risks associated with obesity. Therefore, the data used in our study were the waist and hip girth measures. All 1,851 subjects had the first three longitudinal measurements of waist and hip girths, 693 subjects had the first four measurements, and only 109 subjects had all five measurements. To maximize the number of subjects in the examination of reliability, only the measurement data in the first three trials were included in this investigation.

Table 4-9 shows the descriptive statistics for the hip, waist and ratio, for all three trials. The coefficients of variation for the waist and hip girths exhibited very little change over the three trials, ranging from 0.15-0.16 for the waist girth and 0.10-0.11 for the hip girth. It indicates that waist girth is more variable than hip girth in these data ($V_{\text{waist}} > V_{\text{hip}}$). The coefficients of variation of the ratio variable (waist/hip) were somewhat lower, ranging from 0.09-0.10 over the three trials. The reliability of the ratio variable (0.89 for all trials and 0.73 for a single trial) was still quite high although it was lower than that of the component variables.

A Discussion on the Reliability of the Ratio Variable: Data Sets 3 and 4

The results of this study show that the waist/hip ratio variable in Data Set 4 has much higher reliability than the nondominant/dominant ratio in Data Set 3, although the reliability of the component variables are high in both data sets. As shown in Tables 4-7,

Table 4-9

Means, Standard Deviations, and Reliability Coefficients of the Waist and Hip Girth, and Waist /Hip Ratio

	Waist			Hip			W/H Ratio		
	T1	T2	T3	T1	T2	T3	T1	T2	T3
Mean	74.93	76.27	76.74	97.71	100.83	100.96	0.77	0.76	0.76
Sd	11.64	11.97	11.80	10.77	10.00	9.86	0.07	0.07	0.07
V	0.16	0.16	0.15	0.11	0.10	0.10	0.09	0.10	0.10
Reliability									
All Trials		0.96			0.94			0.89	
Single Trial		0.89			0.84			0.73	

4-8, and 4-9, the coefficients of variation of the component variables in Data Set 4 were much smaller than in Data Set 3. However, as shown in Formula (7), the reliability of a ratio variable has a complex relationship with the coefficients of variation and the within and between trial correlation of the component variables. It is not immediately obvious why the reliability of the ratio in Data Set 4 is so much higher than that of the ratio (pronation) in Data Set 3. Tables 4-10 and 4-11 presents the comparative values for all the statistics that could contribute to this effect. It can be seen that all correlations are higher in Data Set 3, and yet the reliability of the ratio is lower. It was speculated that the higher reliability in Data Set 4 could be due to either the lower coefficients of variation, or the much larger difference of the coefficients of variation between the numerator and denominator variables.

To explore these possibilities, Data Set 3 (the pronation variable only) was used to create three additional data conditions by varying the magnitudes of the coefficients of variation. The reliability of the ratio variable was then approximated for each condition by applying Formula (7) (a validation of the accuracy of Formulas (4), (5), and (7) is given in Appendix C). The within and between trial correlations were set to the original values in Data Set 3 (as in Table 4-10) for all the three additional conditions. The three sets of the coefficients of variation were set based on the values in Data Set 4 to reflect the following situations: (a) smaller, but all equal, values, (b) numerator value larger than denominator value, and (c) numerator value smaller than denominator value. Table 4-12 shows these values.

In Condition 1, although the coefficient of variation for both the numerator and the denominator variables was reduced from approximately 0.45 to 0.16, the results show

Table 4-10
Correlations in Data Set 3 and Data Set 4

Correlation Components		Correlation		
		Data 3	Data 4	Difference
X ₁ and X ₂ trial 1	r_{x11x21}	0.85	0.83	0.02
X ₁ and X ₂ trial 2	r_{x12x22}	0.92	0.80	0.12
X ₁ trial 1 and trial 2	r_{x11x12}	0.94	0.89	0.05
X ₂ trial 1 and trial 2	r_{x21x22}	0.89	0.84	0.05
X ₁ trial 1 and X ₂ trial 2	r_{x11x22}	0.88	0.71	0.17
X ₁ trial 2 and X ₂ trial 1	r_{x12x21}	0.84	0.79	0.05

Table 4-11
Coefficients of Variation in Data Set 3 and Data Set 4

		Coefficient of Variation		
		Data 3	Data 4	Difference
Numerator				
trial 1	V_{x11}	0.46	0.16	0.30
trial 2	V_{x12}	0.43	0.16	0.27
Denominator				
trial 1	V_{x21}	0.46	0.11	0.35
trial 2	V_{x22}	0.44	0.10	0.34
Ratio				
trial 1	V_{y1}	0.24	0.09	0.15
trial 2	V_{y2}	0.17	0.10	0.07
Ratio	Reliability			
	r_{single}	0.51	0.73	-0.22

Table 4-12
Reliability of the Ratio Variable in Data Set 3 under Different Coefficient of Variation
Conditions

		Data Set 3	Condition 1 $V_{x1}=V_{x2}$	Condition 2 $V_{x1}>V_{x2}$	Condition 3 $V_{x1}<V_{x2}$
Numerator					
trial 1	V_{x11}	0.46	0.16	0.16	0.11
trial 2	V_{x12}	0.43	0.16	0.16	0.11
Denominator					
trial 1	V_{x21}	0.46	0.16	0.11	0.16
trial 2	V_{x22}	0.44	0.16	0.10	0.16
Ratio	Reliability				
	r_{single}	0.51	0.50	0.72	0.60

Note. All correlation values are identical to those listed under Data Set 3 in Table 4-10.

that the reliability of the derived ratio variable did not increase ($r_{\text{single}}=0.50$). This result is expected if we set $V_{x11}=V_{x21}=V_{x12}=V_{x22}$, Formula (7) is simplified to:

$$\rho_{\left(\frac{x_{11}}{x_{21}} \text{ and } \frac{x_{12}}{x_{22}}\right)} = \frac{(\rho_{x_{11}x_{12}} + \rho_{x_{21}x_{22}}) - (\rho_{x_{11}x_{22}} + \rho_{x_{21}x_{12}})}{2\sqrt{(1 - \rho_{x_{11}x_{21}})(1 - \rho_{x_{12}x_{22}})}} \quad (24)$$

In Formula (24), the coefficients of variation are cancelled out, and the reliability of the ratio variable becomes only a function of the within and between trial correlations of the numerator and denominator variables. The four coefficients of variation in Data Set 3 were approximately equal ($0.43 \leq V \leq 0.46$), and thus the similar value ($r_{\text{single}}=0.51$) for the reliability of the ratio variable with those data. This indicates that the reliability of the ratio variable is not affected by the coefficient of variation if the coefficients of variation of the component variables are the same.

In Condition 2, the coefficient of variation of the numerator variable was higher than that of the denominator variable, the values being set equal to those of trials 1 and 2 of Data Set 4. As shown in Table 4-12, the reliability of the ratio variable increased from 0.51 to 0.72, a value very close to the observed ratio reliability of Data Set 4 (0.73). In Condition 3, the coefficient of variation of the numerator variable was lower than that of the denominator variable, and the reliability of the ratio variable still increased beyond the 0.51 value of Data Set 3, but only to 0.60. Given that the within and between trial correlations of the component variables did not change, the results of these two conditions suggest that the reliability of a ratio variable is high if the coefficients of variation of the component variables are different, and the effect is most pronounced when the numerator variable has the higher coefficient of variation.

A comparison of Data Set 4 and C_2 is of interest. C_2 and Data Set 4 have identical

coefficients of variation, but differ in the values of the correlations (C_2 has correlations equal to those of Data Set 3). As can be seen in Table 4-10, all of the correlations of Data Set 3 are greater than those of Data Set 4, some by as much as 0.17. However, this has very little effect on the reliability of the ratios, as they are almost identical (0.72 in Condition 2 and 0.73 in Data Set 4).

The results indicate that the unequal coefficients of variation of the numerator and denominator variables result in the higher reliability of the ratio variable in Data Set 4 than in Data Set 3.

A Summary of the Evaluation of the Ratio Variables not for Deflation

In human kinetics research, ratio variables similar to those in Data Set 3 and Data Set 4 are commonly used, however little or no consideration is given to the reliability of these derived measures. The results of this study have clearly shown that even if the component variables are highly reliable the derived ratio variable data may have low reliability. The consequences of such reduction in reliability could seriously affect the validity of empirical research, because unless the repeated measurements are reliable research decisions based on these data may be questionable. For example, low reliability results in an increased error term (for the RM effect) in a RM ANOVA, thus reducing statistical power. As was shown with Data Sets 3 and 4, the reliability of the ratio variables derived from component measures are affected by the coefficient of variation, within and between trial correlations between the numerator and denominator variables (see Tables 4-10 and 4-11). Therefore, whenever a ratio variable is used, researchers should compute the reliability of the derived ratio scores, and not assume that strong reliabilities in the component measures automatically lead to high reliability in the ratio

measures. If the reliability of the ratio variable is too low, the validity of the research may be compromised and an alternative approach should be considered. For example, in the case of nondominant/dominant ratio, the strength of the nondominant and dominant arms may have to be analyzed separately.

CHAPTER V. CHARACTERISTICS OF RATIO VARIABLES AS A FUNCTION OF THE COMPONENT VARIABLES: A SIMULATION STUDY

This chapter reports on how the characteristics of the component variables affect the circularity condition of the covariance matrix and type I error rates when a ratio variable is used in one-way RM ANOVA. As mentioned in the methods and procedures chapter, I first generated the populations of the numerator and denominator variables based on the controlled characteristics of the two component variables (see Table 3-3). The data for each of the 90,000 cases within a population consisted of a numerator and denominator variable for each of the five repeated observations (hereafter referred to as trials). The ratio value, X_{1i}/X_{2i} ($i=1, 90000, i=1, \dots, 5$), was then computed for each of the five paired X_{1i} and X_{2i} values for each case. The effects of the characteristics of the component variables on the magnitude of epsilon (ϵ), the measure of the extent to which the covariance matrix of the ratio variable departs from circularity, were examined for each of the transformed ratio score populations. Then, a Monte Carlo simulation procedure was used to investigate the sampling characteristics of $\hat{\epsilon}_{x1/x2}$ under different sample sizes. Finally, the effect of using ratio variables on the type I error rate was investigated.

Population Characteristics of $\epsilon_{x1/x2}$

The Effect of ϵ_{x1} , V_{x1}/V_{x2} and ρ_{x1x2} on the Magnitude of $\epsilon_{x1/x2}$

Table 5-1 presents the values of $\epsilon_{x1/x2}$ for each of the 33 simulated ratio variable populations. The values of $\epsilon_{x1/x2}$ for the covariance matrix of the ratio population data are

Table 5-1

Population Values of $\varepsilon_{X1/X2}$ for the Ratio Variable X_1/X_2 under Different Component Variable Conditions

	$V_{X1}/V_{X2}=2.0$	$V_{X1}/V_{X2}=1.0$	$V_{X1}/V_{X2}=0.5$
$\varepsilon_{X1}=\varepsilon_{X2}=1.0$			
$\rho_{X1X2}=0.9$	1.00	1.00	1.00
$\rho_{X1X2}=0.7$	1.00	1.00	1.00
$\rho_{X1X2}=0.5$	1.00	1.00	1.00
$\varepsilon_{X1}=\varepsilon_{X2}=0.7$			
$\rho_{X1X2}=0.9$	0.70	0.73	0.72
$\rho_{X1X2}=0.7$	0.70	0.71	0.72
$\rho_{X1X2}=0.5$	0.70	0.71	0.72
$\varepsilon_{X1}=0.7 \ \varepsilon_{X2}=1.0$			
$\rho_{X1X2}=0.9$	0.59	0.87	0.86
$\rho_{X1X2}=0.7$	0.64	0.91	0.96
$\rho_{X1X2}=0.5$	0.70	0.91	0.99
$\varepsilon_{X1}=1.0 \ \varepsilon_{X2}=0.7$			
$\rho_{X1X2}=0.7$	0.78	0.81	0.71
$\rho_{X1X2}=0.5$	0.99	0.84	0.72

presented for each of the three conditions of equality/inequality of the coefficients of variation of the component variables, four epsilon (ϵ_{x1} , ϵ_{x2}) conditions, and three levels of correlation between X_1 and X_2 (ρ_{x1x2}).

Magnitude of $\epsilon_{x1/x2}$ when the Ratio Components Meet the Assumption of Circularity

The results in Table 5-1 show that the data in the transformed ratio score population meet the assumption of circularity when the populations of the numerator and denominator variables meet the assumption (i.e., the population ϵ of the ratio variable equals unity $\epsilon_{x1/x2} = 1.0$, when $\epsilon_{x1} = \epsilon_{x2} = 1.0$). This indicates that when both the population ϵ values of the component variables equal unity (i.e., perfect circularity), the ratio transformation does not affect the circularity of the covariance matrix, and the homogeneity of covariance condition still holds in the population of a ratio variable. This homogeneity holds regardless of the levels of correlation and the relative variation between the numerator and denominator variables (i.e., ρ_{x1x2} and V_{x1}/V_{x2}).

Magnitude of $\epsilon_{x1/x2}$ when the Ratio Components Violate the Assumption of Circularity

As shown in Table 5-1, when the population covariance matrices of the numerator and denominator variables have the same degree of violation of circularity ($\epsilon_{x1} = \epsilon_{x2} = 0.7$), the $\epsilon_{x1/x2}$ of the ratio population covariance matrix does not deviate substantially from the ϵ of the component variable population covariance matrices. The value of $\epsilon_{x1/x2}$ increased slightly (from 0.70 to 0.72) when the coefficient of variation of the numerator variable decreased relative to the denominator variable (i.e., V_{x1}/V_{x2} changed from 2.0 to

0.5), regardless of the levels of the correlation ($\rho_{x_1x_2}$). The correlation between the numerator and denominator variables, $\rho_{x_1x_2}$, has no substantial effect on ϵ_{x_1/x_2} (except ϵ_{x_1/x_2} was slightly higher when $V_{x_1}/V_{x_2}=1.0$, $\rho_{x_1x_2}=0.9$) in the condition $\epsilon_{x_1} = \epsilon_{x_2} = 0.7$.

In the $\epsilon_{x_1}=0.7$ and $\epsilon_{x_2}=1.0$ condition, the value of ϵ_{x_1/x_2} increased considerably as the coefficient of variation of the numerator variable decreased relative to the denominator variable. For example, when $\rho_{x_1x_2}=0.9$ and V_{x_1}/V_{x_2} decreased from 2.0 to 0.5, the value of ϵ_{x_1/x_2} increased from 0.59 to 0.86. In the other two $\rho_{x_1x_2}$ conditions, ϵ_{x_1/x_2} exhibited a similar pattern of change. Additionally, the population ϵ_{x_1/x_2} is affected by the correlation between the numerator and denominator variables. As shown in Table 5-1, when the $\rho_{x_1x_2}$ decreased from 0.9 to 0.5, the value of ϵ_{x_1/x_2} increased, especially when the coefficients of variation of the component variables were not equal.

To systematically examine the characteristics of ϵ_{x_1/x_2} , and to understand how the two factors, $\rho_{x_1x_2}$ and V_{x_1}/V_{x_2} , affect ϵ_{x_1/x_2} , the condition $\epsilon_{x_1}=1.0$ and $\epsilon_{x_2}=0.7$ was also included in the ϵ_{x_1/x_2} investigation. When this condition was included in this study, it was acknowledged that the denominator variable usually does not show a greater violation of the assumption of circularity than the numerator variable with human kinetics research data. For example, when a ratio variable is used for deflation in our field, only the numerator variable is expected to be affected by the treatment, and the variation of the denominator is usually not affected (e.g., Vo2max/kg). For those ratio variables not used for deflation purposes, the numerator and denominator variable usually have a similar variance pattern (e.g., waist/hip girth, nondominant/dominant strength). In addition, the simulation program was unable to generate the population data when $\rho_{x_1x_2}=0.9$, because

the derived covariance matrix for the ratio variable was not positive definite. Therefore, the subsequent investigation on the sampling characteristics of $\epsilon_{x1/x2}$ will not include this condition, and the results reported for the condition $\epsilon_{x1}=1.0$ and $\epsilon_{x2}=0.7$ are based on only two correlation conditions ($\rho_{x1x2}=0.7$ and $\rho_{x1x2}=0.5$).

In the condition $\epsilon_{x1}=1.0$ and $\epsilon_{x2}=0.7$, the results show that $\epsilon_{x1/x2}$ decreased when the ratio of the coefficients of variation of the component variables decreased from 2.0 and 1.0 to 0.5 when $\rho_{x1x2}=0.7$, and decreased constantly from $V_{x1}/V_{x2}=2.0$ to 0.5 for $\rho_{x1x2}=0.5$. The pattern of $\epsilon_{x1/x2}$ over the three variation conditions is the opposite from the pattern in the $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$ condition. The results also show that $\epsilon_{x1/x2}$ increased when ρ_{x1x2} decreased in all three variation conditions. This effect of ρ_{x1x2} on $\epsilon_{x1/x2}$ was the same in both ϵ conditions ($\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$, and $\epsilon_{x1}=1.0$ and $\epsilon_{x2}=0.7$).

Given the covariance structure and the interrelationship of the component variables shown in the methods and procedures chapter, the results indicate that the population ϵ of a ratio variable ($\epsilon_{x1/x2}$) is a function of ϵ_{x1} and ϵ_{x2} , the correlation between the component variables (ρ_{x1x2}), and the relative variation of the component variables (V_{x1}/V_{x2}). If the matrices of the component variables have no violation, or the same degree of violation, of the assumption of circularity ($\epsilon_{x1}=\epsilon_{x2}$), the results indicate that the magnitude of the $\epsilon_{x1/x2}$ is virtually the same as that of ϵ_{x1} and ϵ_{x2} , regardless the level of ρ_{x1x2} and V_{x1}/V_{x2} . However, when the component variables have different degrees of violation of the assumption of circularity, one can not assume that the structure of the covariance matrix of the ratio variable is a simple function of the component variables. The results indicate that $\epsilon_{x1/x2}$ has a negative relationship with ρ_{x1x2} and V_{x1}/V_{x2} , when

only the covariance matrix of the numerator variable violates the assumption of circularity. That is, $\epsilon_{x1/x2}$ tends to have smaller magnitudes when ρ_{x1x2} and V_{x1}/V_{x2} are high, and greater magnitudes when ρ_{x1x2} and V_{x1}/V_{x2} are low. If only the covariance matrix of the denominator variable violates the assumption of circularity, $\epsilon_{x1/x2}$ has a negative relationship with ρ_{x1x2} but a positive relationship with V_{x1}/V_{x2} .

The Effect of ϵ_{xi} , V_{x1}/V_{x2} and ρ_{x1x2} on the Magnitude of $\rho_{(x1/x2)ij}$

Researchers in human kinetics usually measure raw score variables in repeated measurement experiments and have some knowledge of the magnitudes of the correlation between variables in repeated trials. However, the correlation between two ratio variables in a repeated measures design (i.e., $\rho_{(x1/x2)ij}$, $i \neq j$) may not be the same as the correlation between raw score variables (i.e., $\rho_{(x1)ij}$ or $\rho_{(x2)ij}$, $i \neq j$). In this section, I report how certain factors affect the correlation between ratio variables in repeated measurement situations.

Given the designed covariance structure and the interrelationship of the component variables, Table 5-2 presents the inter-trial correlations of the ratio variable as a function of ϵ_{x1} and ϵ_{x2} , the correlation between the component variables (ρ_{x1x2}), and the relative variation of the component variables (V_{x1}/V_{x2}). The unshaded panels of Table 5-2 show the assigned inter-trial correlations of the component variables for the different ϵ_{x1} and ϵ_{x2} , and V_{x1}/V_{x2} conditions. In the conditions $\epsilon_{x1}=\epsilon_{x2}=0.7$, $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$, and $\epsilon_{x1}=1.0$ and $\epsilon_{x2}=0.7$, the correlations for the variable(s) for which $\epsilon_{xi}=0.7$ varied from high for adjacent trials correlation to low for the most distant trials (i.e., $\rho_{xi1..xi5}$). The

Table 5-2

Intertrial Correlations of Ratio Variables for Specific Conditions of the Component Variables

	$V_{x1}/V_{x2}=2.0$	$V_{x1}/V_{x2}=1.0$	$V_{x1}/V_{x2}=0.5$
Input: $\epsilon_{x1}=\epsilon_{x2}=1.0$			
ρ_{ij} : Numerator	0.87	0.87	0.85
Denominator	0.82	0.81	0.83
Ratio:			
$\rho_{x1i,x2i}=0.9$	0.89	0.84	0.79
0.7	0.87	0.83	0.80
0.5	0.87	0.83	0.80
Input: $\epsilon_{x2}=\epsilon_{x2}=0.7$			
ρ_{ij} : Numerator	0.87-0.66	0.87-0.60	0.85-0.60
Denominator	0.82-0.49	0.79-0.45	0.83-0.45
Ratio:			
$\rho_{x1i,x2i}=0.7$	0.88-0.70	0.82-0.52	0.80-0.42
Input: $\epsilon_{x1}=0.7, \epsilon_{x2}=1.0$			
ρ_{ij} : Numerator	0.87-0.66	0.87-0.60	0.85-0.60
Denominator	0.82	0.81	0.83
Ratio:			
$\rho_{x1i,x2i}=0.7$	0.87-0.64	0.83-0.73	0.80-0.86
Input: $\epsilon_{x1}=1.0, \epsilon_{x2}=0.7$			
ρ_{ij} : Numerator	0.87	0.87	0.85
Denominator	0.82-0.49	0.79-0.45	0.83-0.45
Ratio:			
$\rho_{x1i,x2i}=0.7$	0.88-0.94	0.82-0.56	0.80-0.40

correlation matrix for these conditions approximated a simplex pattern. The shaded panels are the calculated inter-trial correlations of the derived ratio variables for the data in a five trials design population for varying conditions of ϵ_{x1} and ϵ_{x2} , and the relative magnitudes of the coefficients of variation of the component variables. The full correlation matrices for the condition $\epsilon_{x1}=\epsilon_{x2}=1.0$ are presented in Table 5-3 and the correlation matrices for the condition $\rho_{x1x2}=0.7$ in the other epsilon conditions are presented in Table 5-4. The other correlation matrices are in Appendix D.

In general, as shown in shaded panels of Table 5-2, the relative variation of the numerator and denominator variables affects the correlations among the ratio variables in the five trials (ρ_{yiyj} , where $y_i=x_{1i}/x_{2i}$ and $i \neq j$). That is, ρ_{yiyj} decreased when V_{x1}/V_{x2} decreased in the condition $\epsilon_{x1}=\epsilon_{x2}=1.0$. Varying ρ_{x1x2} from 0.9 to 0.5 had no substantial effect on the magnitude of the correlations in the ρ_{yiyj} matrices. This was true for all three ϵ conditions ($\epsilon_{x1}=\epsilon_{x2}=0.7$, $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$, and $\epsilon_{x1}=1.0$ and $\epsilon_{x2}=0.7$), thus only the results for the $\rho_{x1}=0.7$ condition are presented in Table 5-4 and summarized in the shaded panels of Table 5-2.

When the component variables have the same degree of violation of the assumption of circularity (i.e., $\epsilon_{x1}=\epsilon_{x2}=0.7$), and the correlations follow a simplex pattern, the correlation matrices of the ratio variable exhibit a decreasing pattern but do not follow the simplex pattern exactly for all V_{x1}/V_{x2} conditions. For example, if the correlations follow the simplex pattern (given $V_{x1}/V_{x2}=2.0$ and $\rho_{y1y2}=0.88$), the magnitudes of the correlations should follow the pattern: $\rho_{y1y3}=\rho_{y1y2}^2=0.77$, $\rho_{y1y4}=\rho_{y1y2}^3=0.68$, and $\rho_{y1y5}=\rho_{y1y2}^4=0.60$. However, in the condition $\epsilon_{x1}=\epsilon_{x2}=0.7$, the magnitude of the

Table 5-3

The Correlation Matrices of The Component and Ratio Variable Populations ($\epsilon_{x1}=\epsilon_{x2}=1.0$)

$V_{x1}/V_{x2}=2.0$					$\rho(x_{11}, x_{1j})$					$\rho(x_{21}, x_{2j})$				
1.00	0.87	0.87	0.87	0.87	1.00	0.82	0.82	0.82	0.82	1.00	0.82	0.82	0.82	0.82
	1.00	0.87	0.87	0.87		1.00	0.82	0.82	0.82		1.00	0.82	0.82	0.82
		1.00	0.87	0.87			1.00	0.82	0.82		1.00	0.82	0.82	0.82
			1.00	0.87				1.00	0.82		1.00	0.82	0.82	0.82
				1.00					1.00		1.00	0.82	0.82	0.82
$\rho_{x1x2}=.9$					$\rho(y_1, y_j)$					$\rho_{x1x2}=.7$				
1.00	0.89	0.89	0.89	0.89	1.00	0.88	0.87	0.87	0.87	1.00	0.87	0.87	0.87	0.87
	1.00	0.89	0.89	0.89		1.00	0.87	0.87	0.87		1.00	0.87	0.87	0.87
		1.00	0.89	0.89			1.00	0.87	0.87		1.00	0.87	0.87	0.87
			1.00	0.89				1.00	0.87		1.00	0.87	0.87	0.87
				1.00					1.00		1.00	0.87	0.87	0.87
											1.00	0.87	0.87	0.87
												1.00	0.87	0.87
													1.00	0.87
														1.00
$V_{x1}/V_{x2}=1.0$					$\rho(x_{11}, x_{1j})$					$\rho(x_{21}, x_{2j})$				
1.00	0.87	0.87	0.87	0.87	1.00	0.81	0.81	0.81	0.81	1.00	0.81	0.81	0.81	0.81
	1.00	0.87	0.87	0.87		1.00	0.81	0.81	0.81		1.00	0.81	0.81	0.81
		1.00	0.87	0.87			1.00	0.81	0.81		1.00	0.81	0.81	0.81
			1.00	0.87				1.00	0.81		1.00	0.81	0.81	0.81
				1.00					1.00		1.00	0.81	0.81	0.81
$\rho_{x1x2}=.9$					$\rho(y_1, y_j)$					$\rho_{x1x2}=.7$				
1.00	0.83	0.83	0.84	0.84	1.00	0.83	0.83	0.83	0.83	1.00	0.83	0.83	0.83	0.83
	1.00	0.84	0.84	0.84		1.00	0.83	0.83	0.83		1.00	0.83	0.83	0.83
		1.00	0.84	0.84			1.00	0.83	0.83		1.00	0.83	0.83	0.83
			1.00	0.84				1.00	0.83		1.00	0.83	0.83	0.83
				1.00					1.00		1.00	0.83	0.83	0.83
											1.00	0.83	0.83	0.83
												1.00	0.83	0.83
													1.00	0.83
														1.00
$V_{x1}/V_{x2}=0.5$					$\rho(x_{11}, x_{1j})$					$\rho(x_{21}, x_{2j})$				
1.00	0.85	0.85	0.85	0.85	1.00	0.83	0.83	0.83	0.83	1.00	0.83	0.83	0.83	0.83
	1.00	0.85	0.85	0.85		1.00	0.83	0.83	0.83		1.00	0.83	0.83	0.83
		1.00	0.85	0.85			1.00	0.83	0.83		1.00	0.83	0.83	0.83
			1.00	0.85				1.00	0.83		1.00	0.83	0.83	0.83
				1.00					1.00		1.00	0.83	0.83	0.83
$\rho_{x1x2}=.9$					$\rho(y_1, y_j)$					$\rho_{x1x2}=.7$				
1.00	0.79	0.79	0.79	0.79	1.00	0.80	0.80	0.80	0.80	1.00	0.80	0.80	0.80	0.80
	1.00	0.79	0.79	0.79		1.00	0.80	0.80	0.79		1.00	0.80	0.80	0.80
		1.00	0.79	0.78			1.00	0.80	0.79		1.00	0.80	0.80	0.80
			1.00	0.79				1.00	0.80		1.00	0.80	0.80	0.80
				1.00					1.00		1.00	0.80	0.80	0.80
											1.00	0.80	0.80	0.80
												1.00	0.80	0.80
													1.00	0.80
														1.00

Table 5-4

The Correlation Matrices of Ratio Variable Populations ($\rho_{x_1x_2}=0.7$)

$\varepsilon_{x_1}=\varepsilon_{x_2}=1.0$	$\varepsilon_{x_1}=\varepsilon_{x_2}=0.7$	$\varepsilon_{x_1}=0.7, \varepsilon_{x_2}=1.0$	$\varepsilon_{x_1}=1.0, \varepsilon_{x_2}=0.7$
$V_{x_1}/V_{x_2}=2.0$			
1.00 0.88 0.87 0.87 0.87	1.00 0.88 0.77 0.74 0.70	1.00 0.87 0.74 0.69 0.64	1.00 0.88 0.91 0.94 0.94
1.00 0.87 0.87 0.87	1.00 0.88 0.78 0.74	1.00 0.88 0.74 0.69	1.00 0.88 0.92 0.92
1.00 0.87 0.87	1.00 0.88 0.77	1.00 0.87 0.74	1.00 0.88 0.89
1.00 0.87	1.00 0.87	1.00 0.86	1.00 0.89
1.00	1.00	1.00	1.00
$V_{x_1}/V_{x_2}=1.0$			
1.00 0.83 0.83 0.83 0.83	1.00 0.82 0.71 0.62 0.52	1.00 0.83 0.79 0.76 0.73	1.00 0.82 0.76 0.73 0.56
1.00 0.83 0.83 0.83	1.00 0.82 0.71 0.63	1.00 0.83 0.79 0.76	1.00 0.82 0.76 0.58
1.00 0.83 0.83	1.00 0.82 0.71	1.00 0.83 0.78	1.00 0.82 0.61
1.00 0.83	1.00 0.80	1.00 0.81	1.00 0.65
1.00	1.00	1.00	1.00
$V_{x_1}/V_{x_2}=0.5$			
1.00 0.80 0.80 0.80 0.80	1.00 0.80 0.60 0.50 0.42	1.00 0.80 0.82 0.84 0.86	1.00 0.80 0.58 0.47 0.40
1.00 0.80 0.80 0.79	1.00 0.74 0.61 0.50	1.00 0.80 0.82 0.84	1.00 0.73 0.59 0.47
1.00 0.80 0.79	1.00 0.77 0.57	1.00 0.80 0.82	1.00 0.77 0.56
1.00 0.80	1.00 0.71	1.00 0.80	1.00 0.71
1.00	1.00	1.00	1.00

correlations ($\rho_{y_1y_2}=0.88$, $\rho_{y_1y_3}=0.77$, $\rho_{y_1y_4}=0.74$, and $\rho_{y_1y_5}=0.70$) decreased slower than would be the case in a simplex pattern.

The results show two different patterns in the correlation matrices of the ratio variables for the two conditions $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$, and $\epsilon_{x1}=1.0$ and $\epsilon_{x2}=0.7$ (see the two shaded panels from the bottom of Table 5-2). In the condition $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$, the decreasing correlation pattern in the correlation matrix disappeared when the denominator variable had the higher relative variation ($V_{x1}/V_{x2}=0.5$). In this case the correlations among the ratio variables actually increased slightly as the distance between trials increased: $\rho_{y_1y_2}=0.80$, $\rho_{y_1y_3}=0.82$, $\rho_{y_1y_4}=0.84$, and $\rho_{y_1y_5}=0.86$. In the condition $\epsilon_{x1}=1.0$ and $\epsilon_{x2}=0.7$ the decreasing correlation pattern also disappeared, but when the numerator variable (not the denominator variable) had the higher relative variation ($V_{x1}/V_{x2}=2.0$).

This study provides some details of the effect of the characteristics of components on the correlation between ratio variables. The results indicate that both the relative variation and the relative ϵ level of the two component variables affect the correlations among ratio variables in a repeated trials design. When the variation of the denominator variable increases relative to the numerator variable (i.e., V_{x1}/V_{x2} decreases), the correlation among the ratio variables decreases in the conditions $\epsilon_{x1}=\epsilon_{x2}=1.0$ and $\epsilon_{x1}=\epsilon_{x2}=0.7$. If both the numerator and denominator variables meet the assumption of circularity ($\epsilon_{x1}=\epsilon_{x2}=1.0$), a constant correlation pattern is shown in the correlation matrix of the ratio variables. If both the correlation matrices of the numerator and denominator variables are designed to follow the simplex pattern and have the same level of violation of the assumption of circularity ($\epsilon_{x1}=\epsilon_{x2}=0.7$), a decreasing correlation pattern, similar but

not identical to a simplex pattern, is shown in the correlation matrix of the ratio variables.

In the other two ϵ conditions, the pattern of the correlation matrix of the ratio variable is affected by the combined factor of relative level of variation and ϵ level of the component variables. If only the numerator variable violates the assumption of circularity, a pattern of increasing correlations is shown when $V_{x1}/V_{x2}=0.5$. If only the denominator variable violates the assumption of circularity, a pattern of increasing correlations is shown when $V_{x1}/V_{x2}=2.0$.

To examine how the characteristics of the component variables affect the correlation pattern of the ratio variable, the comparative values for all the statistics and the correlation matrices of the component variables that could contribute to the correlation pattern of the ratio variable for five related conditions are presented in Table 5-5. The first two columns show the correlation matrices of the two component variables and the third column shows the correlation matrix for the transformed ratio variable, and the $\epsilon_{x1/x2}$ value associated with that matrix.

In Conditions 1 and 2, the two component variables have the same correlation pattern, and the numerator has a higher variation relative to the denominator ($V_{x1}/V_{x2}=2.0$). As shown in Table 5-5, when ρ_{x1x2} decreases from 0.9 to 0.5, the correlation matrix of the ratio variable still exhibits a decreasing pattern, but to a lesser degree. The comparison between Conditions 2 and 3 shows that when the denominator has the constant correlation pattern and large variation ($V_{x1}/V_{x2}=0.5$) the decreasing pattern of the correlation disappears for the ratio variable. The comparison of the correlation matrices in Conditions 1, 2 and 3 indicates that the correlation pattern of the ratio variable is close to that of the component variable which has larger relative

Table 5-5

Comparison of the Correlation Matrices of the Ratio Variable under Selected Conditions

Correlation X_{1ij}					Correlation X_{2ij}					Correlation Y_{ij}				
Condition1: $\varepsilon_{x1}=0.7, \varepsilon_{x1}=1.0, \rho_{x1x2}=0.9, V_{x1}/V_{x2}=2.0$										$\varepsilon_{x1/x2}=0.59$				
1.00	0.87	0.76	0.71	0.66	1.00	0.82	0.82	0.82	0.82	1.00	0.89	0.72	0.64	0.58
	1.00	0.87	0.76	0.71		1.00	0.82	0.82	0.82		1.00	0.90	0.71	0.64
		1.00	0.87	0.75			1.00	0.82	0.82			1.00	0.87	0.71
			1.00	0.85				1.00	0.82				1.00	0.82
				1.00					1.00					1.00
<hr/>														
Condition2: $\varepsilon_{x1}=0.7, \varepsilon_{x1}=1.0, \rho_{x1x2}=0.5, V_{x1}/V_{x2}=2.0$										$\varepsilon_{x1/x2}=0.70$				
1.00	0.87	0.76	0.71	0.66	1.00	0.82	0.82	0.82	0.82	1.00	0.87	0.75	0.71	0.66
	1.00	0.87	0.76	0.71		1.00	0.82	0.82	0.82		1.00	0.87	0.75	0.71
		1.00	0.87	0.75			1.00	0.82	0.82			1.00	0.87	0.75
			1.00	0.85				1.00	0.82				1.00	0.85
				1.00					1.00					1.00
<hr/>														
Condition3: $\varepsilon_{x1}=0.7, \varepsilon_{x1}=1.0, \rho_{x1x2}=0.5, V_{x1}/V_{x2}=0.5$										$\varepsilon_{x1/x2}=0.99$				
1.00	0.85	0.76	0.66	0.60	1.00	0.83	0.83	0.83	0.83	1.00	0.80	0.80	0.81	0.82
	1.00	0.85	0.76	0.68		1.00	0.83	0.83	0.83		1.00	0.80	0.81	0.81
		1.00	0.85	0.75			1.00	0.83	0.83			1.00	0.81	0.81
			1.00	0.83				1.00	0.83				1.00	0.80
				1.00					1.00					1.00
<hr/>														
Condition4: $\varepsilon_{x1}=1.0, \varepsilon_{x1}=0.7, \rho_{x1x2}=0.5, V_{x1}/V_{x2}=2.0$										$\varepsilon_{x1/x2}=0.99$				
1.00	0.87	0.87	0.87	0.87	1.00	0.82	0.65	0.57	0.49	1.00	0.87	0.87	0.88	0.88
	1.00	0.87	0.87	0.87		1.00	0.80	0.63	0.57		1.00	0.87	0.87	0.88
		1.00	0.87	0.87			1.00	0.80	0.66			1.00	0.87	0.87
			1.00	0.87				1.00	0.76				1.00	0.87
				1.00					1.00					1.00
<hr/>														
Condition5: $\varepsilon_{x1}=1.0, \varepsilon_{x1}=0.7, \rho_{x1x2}=0.5, V_{x1}/V_{x2}=0.5$										$\varepsilon_{x1/x2}=0.72$				
1.00	0.85	0.85	0.85	0.85	1.00	0.83	0.65	0.55	0.47	1.00	0.80	0.61	0.51	0.45
	1.00	0.85	0.85	0.85		1.00	0.78	0.65	0.55		1.00	0.75	0.62	0.52
		1.00	0.85	0.85			1.00	0.80	0.63			1.00	0.78	0.60
			1.00	0.85				1.00	0.75				1.00	0.72
				1.00					1.00					1.00

variation. That is, in Conditions 1 and 2, the numerator has the larger variation, and the correlation pattern of the ratio variable is closer to the decreasing pattern of the numerator. In Condition 3 the correlation pattern of the ratio variable is more similar to the denominator due to the larger variation of the denominator variable. These results seem to indicate that the correlation pattern of the ratio variable is mostly dependent on the relative variation of the component variables (V_{x1}/V_{x2}). To confirm that the correlation pattern of the ratio variable is more similar to that of the component variable which has a large coefficient of variation, the two conditions under the condition $\epsilon_{x1}=1.0$ and $\epsilon_{x2}=0.7$ were also examined (Conditions 4 and 5 in Table 5-5). In these two conditions, the correlation matrices of the component variables have the opposite pattern from Condition 3. Condition 4 results show that the decreasing pattern of the ratio variable does not appear when the numerator has a constant correlation pattern with large variation. In Condition 5, when the denominator has larger variation and a decreasing correlation pattern, the results show that the ratio variable also exhibits a decreasing correlation pattern which is similar to that of the denominator. These results confirm that the correlation pattern of the ratio variable is mostly affected by the component variable which has the larger coefficient of variation.

As shown above, the correlation among the derived ratio variables over time is more complex than the component variables. The ϵ level and relative variation of the component variables play important role in the correlation of ratio variables in a repeated measurement design. Therefore, one should not automatically assume that the ratio variable would show the same correlation pattern as the component variables. Especially, in the situation $\epsilon_{x1}<1.0$ and $\epsilon_{x2}=1.0$ (a common case in human kinetics research) and the

denominator variable has higher variability, the derived ratio variable would not exhibit a decreasing correlation pattern over time.

Sample Characteristics of $\hat{\epsilon}_{x1/x2}$

Since ϵ was introduced by Box (1954), it has been used as a correction factor in a RM ANOVA to control for probable inflation in type I error rates brought about by heterogeneity of covariance. For any covariance matrix, the upper and lower limits of ϵ are 1.0 and $1/(k-1)$, where k is the number of trials. Because the population ϵ is unknown in practice, it is usually estimated from a sample covariance matrix (using the same equation used to compute ϵ , but applying it to a sample covariance matrix). It can be shown that given $\epsilon=1.0$ this estimate of ϵ ($\hat{\epsilon}$) always exhibits some degree of downward bias, and this bias becomes larger with increasing k . Eom (1993) examined the distribution characteristics of $\hat{\epsilon}$. Given normality, $\epsilon=1.0$, and $k=5$ in the population, a Monte Carlo simulation showed that the means of $\hat{\epsilon}$ (2,000 samples) with $n=15, 30, 45$ were 0.77 (Sd=0.08), 0.87(0.05), and 0.91(0.04), respectively. For $\epsilon=0.7$ and $k=5$, the means of $\hat{\epsilon}$ with $n=15, 30, 45$ were 0.61(Sd=0.09), 0.66(0.08), and 0.67(0.06), respectively (Eom, 1993).

Ratio variables are commonly used in RM ANOVA in our field, however, the sampling characteristics of $\hat{\epsilon}$ for a covariance matrix derived from ratio scores, $\hat{\epsilon}_{x1/x2}$, are not well understood. How the relative variation and correlation of the numerator and denominator variables affect $\hat{\epsilon}_{x1/x2}$ is unknown. Therefore, one of the purposes in this study was to systematically examine the sampling characteristics of $\hat{\epsilon}_{x1/x2}$. With the

aforementioned characteristics of ϵ of the component variables and the characteristics of the ratio score variable populations in mind, using Monte Carlo simulation procedures, we now investigate the sampling characteristics of $\hat{\epsilon}_{x1/x2}$ for different conditions.

The Monte Carlo simulation was conducted using a five-trial ($k=5$) repeated measures design with three sample sizes ($n=15, 30, 45$) and 2,000 repetitions from each of the simulated population data sets ($N=90,000$). The simulation results are summarized in Tables 5-6, 5-7, and 5-8. These three tables show the distribution characteristics of $\hat{\epsilon}_{x1/x2}$ for each of the populations.

Condition 1 ($\epsilon_{x1}=\epsilon_{x2}=1.0$)

Table 5-6 clearly shows that $\hat{\epsilon}_{x1/x2}$ is a downward biased estimator and the degree of bias is virtually unaffected by the correlation between the numerator and denominator variables. For example, for the condition $n=15$ and $V_{x1}/V_{x2}=2.0$, the mean of $\hat{\epsilon}_{x1/x2}$ was 0.76-0.77 under all three correlation conditions. However, $\hat{\epsilon}_{x1/x2}$ is affected by the coefficients of variation of the numerator and denominator variables. The mean of $\hat{\epsilon}_{x1/x2}$ decreases when the coefficient of variation of the denominator variable increases relative to the numerator variable. For example, when the sample size was 15 and V_{x1}/V_{x2} decreased from 2.0 to 0.5, the mean of $\hat{\epsilon}_{x1/x2}$ decreased from 0.76 to 0.68. Additionally, the estimator became more variable, as evidenced by the increase in the standard deviation of $\hat{\epsilon}_{x1/x2}$ from 0.08 when $V_{x1}/V_{x2}=2.0$ to 0.12 when $V_{x1}/V_{x2}=0.5$. The effect of this, and of the characteristics of the component variables, on type I error rates in a RM ANOVA is discussed in a later section.

Table 5-6

Distribution Characteristics of $\hat{\epsilon}_{x1/x2}$ ($\epsilon_{x1}=\epsilon_{x2}=1.0$, $\mu_{x1}=75$, $\mu_{x2}=100$)

$\rho_{x1/x2}$	Sample Size	$V_{x1}/V_{x2}=2.0$			$V_{x1}/V_{x2}=1.0$			$V_{x1}/V_{x2}=0.5$		
		$\hat{\epsilon}_{x1/x2}$ mean	Sd	$\epsilon_{x1/x2} - \hat{\epsilon}_{x1/x2}$	$\hat{\epsilon}_{x1/x2}$ mean	Sd	$\epsilon_{x1/x2} - \hat{\epsilon}_{x1/x2}$	$\hat{\epsilon}_{x1/x2}$ mean	Sd	$\epsilon_{x1/x2} - \hat{\epsilon}_{x1/x2}$
0.9	15	.76	(.08)	.24	.75	(.08)	.25	.68	(.12)	.32
	30	.86	(.06)	.14	.85	(.06)	.15	.78	(.11)	.22
	45	.90	(.04)	.10	.90	(.04)	.10	.83	(.10)	.17
0.7	15	.76	(.08)	.24	.75	(.08)	.25	.69	(.11)	.31
	30	.86	(.05)	.14	.85	(.06)	.15	.78	(.11)	.22
	45	.90	(.04)	.10	.89	(.05)	.11	.83	(.10)	.17
0.5	15	.77	(.08)	.23	.74	(.09)	.26	.69	(.11)	.31
	30	.87	(.05)	.13	.84	(.06)	.16	.79	(.10)	.21
	45	.91	(.04)	.09	.89	(.05)	.11	.83	(.09)	.17

Note. The population value of $\epsilon_{x1/x2}$ is 1.0 for all conditions.

The sample size effect on $\hat{\epsilon}_{x1/x2}$ is obvious: when the sample size increases, the bias of $\hat{\epsilon}_{x1/x2}$ decreases. This reduction is approximately 0.10 when n increases from 15 to 30, and 0.04 when n increases from 30 to 45. In general, as expected, the sample size effect on $\hat{\epsilon}_{x1/x2}$ of the ratio variable data is the same as for the component variables (raw score data) reported in literature.

Condition 2 ($\epsilon_{x1}=\epsilon_{x2}=0.7$)

In this condition, the covariance matrices of both the numerator and denominator variables exhibited moderate violations of the assumption of circularity and the intertrial correlation followed the simplex pattern for both the numerator and denominator variables. Table 5-7 summarizes the simulation results for this condition, and reveals the downward bias in $\hat{\epsilon}_{x1/x2}$. The $\hat{\epsilon}_{x1/x2}$ had the greatest bias when $V_{x1}/V_{x2}=0.5$, but the level of V_{x1}/V_{x2} had less effect here than in the $\epsilon_{x1}=\epsilon_{x2}=1.0$ condition (see Figure 5-1).

The

standard deviation of $\hat{\epsilon}_{x1/x2}$ increased slightly when V_{x2} increased, but also less than the increase shown in the $\epsilon_{x1}=\epsilon_{x2}=1.0$ condition. Given $\epsilon_{x1}=\epsilon_{x2}=0.7$ and the simplex pattern of the intertrial correlation for the component variables, the correlation between the numerator and denominator variables (ρ_{x1x2}) does not seem to have an effect on the sampling characteristics of $\hat{\epsilon}_{x1/x2}$.

Condition 3 ($\epsilon_{x1}=0.7, \epsilon_{x2}=1.0$)

In this condition, only the covariance matrix of the numerator component violated

Table 5-7
Distribution Characteristics of $\hat{\epsilon}_{X1/\lambda_2}$ ($\epsilon_{X1}=\epsilon_{X2}=0.7$, $\mu_{X1}=75$, $\mu_{X2}=100$)

$\rho_{X1,2}$	Sample Size	$V_{X1}/V_{X2}=2.0$				$V_{X1}=V_{X2}=1.0$				$V_{X1}/V_{X2}=0.5$			
		ϵ_{X1/λ_2}	$\hat{\epsilon}_{X1/\lambda_2}$ mean	Sd	$\epsilon_{X1/\lambda_2} - \hat{\epsilon}_{X1/\lambda_2}$	ϵ_{X1/λ_2}	$\hat{\epsilon}_{X1/\lambda_2}$ mean	Sd	$\epsilon_{X1/\lambda_2} - \hat{\epsilon}_{X1/\lambda_2}$	ϵ_{X1/λ_2}	$\hat{\epsilon}_{X1/\lambda_2}$ mean	Sd	$\epsilon_{X1/\lambda_2} - \hat{\epsilon}_{X1/\lambda_2}$
0.9	15	.70	.60	(.09)	.10	.73	.63	(.10)	.10	.72	.58	(.11)	.14
	30		.65	(.07)	.05		.68	(.09)	.05		.63	(.09)	.09
	45		.67	(.06)	.03		.70	(.08)	.03		.65	(.09)	.07
0.7	15	.70	.61	(.09)	.09	.71	.62	(.10)	.09	.72	.58	(.11)	.14
	30		.65	(.08)	.05		.66	(.09)	.05		.63	(.10)	.09
	45		.67	(.06)	.03		.68	(.08)	.03		.65	(.09)	.07
0.5	15	.70	.61	(.09)	.09	.71	.61	(.10)	.10	.72	.58	(.11)	.14
	30		.65	(.08)	.05		.65	(.09)	.06		.63	(.10)	.09
	45		.67	(.07)	.03		.67	(.08)	.04		.65	(.09)	.07

Figure 5-1. $\hat{\epsilon}_{x1/x2}$ in the conditions $\epsilon_{x1}=\epsilon_{x2}=1.0$, and $\epsilon_{x1}=\epsilon_{x2}=0.7$.

the assumption of circularity. That is, the intertrial correlations of the numerator variable followed the simplex pattern, while those for the denominator were constant for all pairs of trials. In a previous section, it was shown that both $\rho_{x_1x_2}$ and V_{x_1}/V_{x_2} affect the population ϵ of the ratio variable when $\epsilon_{x_1}=0.7$ and $\epsilon_{x_2}=1.0$. In this condition, ϵ_{x_1/x_2} tends to have a smaller magnitude when $\rho_{x_1x_2}$ and V_{x_1}/V_{x_2} are high, and a greater magnitude when $\rho_{x_1x_2}$ and V_{x_1}/V_{x_2} are low. With the aforementioned characteristics of the population ϵ_{x_1/x_2} in mind, we now examine the characteristics of $\hat{\epsilon}_{x_1/x_2}$ in this condition.

As can be seen in Table 5-8, the bias of $\hat{\epsilon}_{x_1/x_2}$ becomes more pronounced when the coefficient of variation of the denominator variable is high ($V_{x_1}/V_{x_2}=0.5$). For example, the mean of $\hat{\epsilon}_{x_1/x_2}$ was lowest (0.53-0.61) and the deviation $\epsilon_{x_1/x_2}-\hat{\epsilon}_{x_1/x_2}$ was only 0.06-0.09 for the $V_{x_1}/V_{x_2}=2.0$ condition when $n=15$, whereas the mean of $\hat{\epsilon}_{x_1/x_2}$ was slightly larger (0.60-0.69) but the deviation was much larger (0.26-0.30) for the $V_{x_1}/V_{x_2}=0.5$ condition. Figure 5-2 illustrates this trend.

In contrast to Conditions 1 and 2, the magnitude of the correlation between the numerator variable and the denominator variable affects the magnitude of $\hat{\epsilon}_{x_1/x_2}$ in Condition 3. As shown in Table 5-8, when the correlation $\rho_{x_1x_2}$ decreases, the difference between $\hat{\epsilon}_{x_1/x_2}$ and ϵ_{x_1/x_2} tends to increase, although by only a small amount (a difference of 0.03-0.04 between the $\rho_{x_1x_2}=0.9$ and $\rho_{x_1x_2}=0.5$ conditions when $n=15$). There appears to be no V_{x_1}/V_{x_2} by $\rho_{x_1x_2}$ interaction effect, as the bias of $\hat{\epsilon}_{x_1/x_2}$ increases when the V_{x_2} increases, regardless the level of the correlation $\rho_{x_1x_2}$.

Table 5-8

Distribution Characteristics of $\hat{\epsilon}_{x1/x2}$ ($\epsilon_{x1}=0.7$, $\epsilon_{x2}=1.0$, $\mu_{x1}=75$, $\mu_{x2}=100$)

ρ_{x1x2}	Sample Size	$V_{x1}/V_{x2}=2.0$				$V_{x1}=V_{x2}=1.0$				$V_{x1}/V_{x2}=0.5$			
		$\epsilon_{x1/x2}$	$\hat{\epsilon}_{x1/x2}$ mean	Sd	$\epsilon_{x1/x2} - \hat{\epsilon}_{x1/x2}$	$\epsilon_{x1/x2}$	$\hat{\epsilon}_{x1/x2}$ mean	Sd	$\epsilon_{x1/x2} - \hat{\epsilon}_{x1/x2}$	$\epsilon_{x1/x2}$	$\hat{\epsilon}_{x1/x2}$ mean	Sd	$\epsilon_{x1/x2} - \hat{\epsilon}_{x1/x2}$
0.9	15	.59	.53	(.06)	.06	.87	.70	(.09)	.17	.86	.60	(.10)	.26
	30		.56	(.07)	.03		.78	(.07)	.09		.67	(.09)	.19
	45		.57	(.06)	.03		.81	(.07)	.06		.70	(.08)	.16
0.7	15	.64	.57	(.09)	.07	.91	.71	(.09)	.20	.96	.67	(.11)	.29
	30		.60	(.07)	.04		.80	(.07)	.11		.76	(.11)	.20
	45		.62	(.06)	.02		.84	(.06)	.07		.80	(.10)	.16
0.5	15	.70	.61	(.09)	.09	.91	.71	(.09)	.20	.99	.69	(.11)	.30
	30		.65	(.08)	.05		.79	(.08)	.12		.78	(.10)	.21
	45		.67	(.07)	.03		.83	(.06)	.08		.83	(.09)	.16

Figure 5-2. $\hat{\epsilon}_{x1/x2}$ in the condition $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$.

Summary of Characteristics of $\hat{\epsilon}_{x1/x2}$ and $\epsilon_{x1/x2}$

Monte Carlo simulation procedures were used to investigate how the characteristics of the numerator and denominator variables affect the circularity of the ratio score population covariance matrix (the effect on the population $\epsilon_{x1/x2}$) and of the sampling characteristics of $\hat{\epsilon}_{x1/x2}$ in the different conditions. As indicated in the methods and procedures chapter, the investigation was based on a ratio variable with a specific covariance structure that is similar to that observed in human kinetics research. The ratio variable population was constructed from populations of the component variables with specific distribution characteristics. That is, the numerator and denominator variables were restricted to a bivariate normal distribution, constant within-trial correlation between the numerator and denominator variables ($\rho_{x1x2}=0.9, 0.7, 0.5$), and constant intertrial correlation ($\epsilon_{xi}=1.0, i=1,2$) or a simplex pattern of the intertrial correlations ($\epsilon_{xi}<1.0$). Following summary of the findings are based on the above conditions.

1. This study indicates that if the component variables meet the assumption of circularity ($\epsilon_{x1}=\epsilon_{x2}=1.0$), the covariance matrix of a ratio variable also meets the assumption ($\epsilon_{x1/x2}=1.0$), and the downward bias of $\hat{\epsilon}_{x1/x2}$ has a pattern similar to what is known to hold for raw score variables. In the condition $\epsilon_{x1}=\epsilon_{x2}=0.7$, $\epsilon_{x1/x2}$ is virtually the same as that of the component variables, and the downward bias of $\hat{\epsilon}_{x1/x2}$ is less than in the condition $\epsilon_{x1}=\epsilon_{x2}=1.0$. The population ϵ value of a ratio variable is affected by the relative variation of the component variables (V_{x1}/V_{x2}) when only the numerator variable has a moderate violation of the assumption of circularity (e.g., $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$). The covariance matrix of a ratio score population tends to have a more homogeneous

covariance structure and higher bias level of $\hat{\epsilon}$ when the coefficient of variation of the numerator variable decreases.

2. The correlation between the numerator and denominator variables does not affect the population ϵ nor the sampling characteristics of $\hat{\epsilon}_{x_1/x_2}$ if the two component variables do not violate the assumption of circularity (i.e., $\epsilon_{x_1}=\epsilon_{x_2}=1.0$), or if they have the same moderate degree of violation $\epsilon_{x_1}=\epsilon_{x_2}=0.7$. However, if the numerator variable violates the assumption of circularity and the denominator variable does not (e.g., $\epsilon_{x_1}=0.7$ and $\epsilon_{x_2}=1.0$), ϵ_{x_1/x_2} increases when the correlation between the numerator and denominator variable ($\rho_{x_1x_2}$) decreases, especially when $V_{x_1}\neq V_{x_2}$. The bias of $\hat{\epsilon}_{x_1/x_2}$ is more serious in the condition $\epsilon_{x_1}=0.7$ and $\epsilon_{x_2}=1.0$ than in the conditions $\epsilon_{x_1}=\epsilon_{x_2}=1.0$ and $\epsilon_{x_1}=\epsilon_{x_2}=0.7$, especially when $\rho_{x_1x_2}$ is low.

As shown above, $\hat{\epsilon}_{x_1/x_2}$ is affected by a combination of the characteristics of the component variables (V_{x_1}/V_{x_2} , $\rho_{x_1x_2}$, ϵ_{x_1} , and ϵ_{x_2}). If a ratio variable is used, given certain mean differences, and magnitudes of ϵ_{x_1} and ϵ_{x_2} of the component variables, the results indicate that the magnitude of $\hat{\epsilon}_{x_1/x_2}$ would be substantially affected by V_{x_1}/V_{x_2} and $\rho_{x_1x_2}$. Thus, it would affect the type I error rates in the RM ANOVA. The details of these effects are shown in the following sections.

Type I Error Rates

Expected Effects of Characteristics of X_1 and X_2 on Type I Error Rates for X_1/X_2

In the previous two sections, the effect of using ratio variables on the magnitude of ϵ_{x_1/x_2} has been presented. Because noncircularity ($\epsilon<1.0$) affects the type I error rate in

a one-way RM ANOVA (Winer, 1991), the implications of using ratio variables on type I error rates can be indicated by looking at the values of $\epsilon_{x1/x2}$ in the different conditions.

Based on the characteristics of $\epsilon_{x1/x2}$, this section discusses how the correlation and relative variation between the numerator and denominator variables affect the type I error rate in a one-way RM ANOVA.

Given that the numerator and denominator variables have no violation, or the same level of violation, of the assumption of circularity (e.g., $\epsilon_{x1}=\epsilon_{x2}=1.0$, or $\epsilon_{x1}=\epsilon_{x2}=0.7$), bivariate normal distribution, and the restricted covariance structure in this study, it seems that using a ratio variable results in no change in the population value of $\epsilon_{x1/x2}$. That is $\epsilon_{x1/x2}=1.0$ when $\epsilon_{x1}=\epsilon_{x2}=1.0$, and $\epsilon_{x1/x2}=0.7$ when $\epsilon_{x1}=\epsilon_{x2}=0.7$, regardless the levels of ρ_{x1x2} and V_{x1}/V_{x2} (see Table 5-1). Thus, if the covariance matrices of both the component variables do not violate the assumption of circularity, we may conclude that the empirical type I error rate of a ratio variable in a one-way RM ANOVA would be close to the nominal level. No substantial effect on the type I error rate is expected from the correlation between the numerator and denominator variables (ρ_{x1x2}) and the relative variation of the component variables (V_{x1}/V_{x2}). If the component variables have violated the assumption of circularity at the same level (e.g., $\epsilon_{x1}=\epsilon_{x2}=0.7$), the inflation of the type I error rate using a ratio variable is expected to be equal to that of its components, but ρ_{x1x2} and V_{x1}/V_{x2} would have little additional effect. The results of this study suggest that if the numerator and denominator variables have the same level of violation on circularity ($\epsilon_{x1}=\epsilon_{x2}$), ρ_{x1x2} and V_{x1}/V_{x2} would have no effect on the type I error rate in RM ANOVA test with the ratio variable data.

However, if only one of the component variables violates the assumption of

circularity (e.g., $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$, or $\epsilon_{x1}=1.0$ and $\epsilon_{x2}=0.7$), ρ_{x1x2} and V_{x1}/V_{x2} of the component variables affect the level of violation of the assumption of circularity for the ratio variable (see Table 5-1). The following statements are based on the condition $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$. The results indicate that $\epsilon_{x1/x2}$ has low values when ρ_{x1x2} is high and $V_{x1}/V_{x2}=2.0$, and high values when ρ_{x1x2} is low and $V_{x1}/V_{x2}=0.5$. Therefore, if only the numerator variable violates the assumption of circularity in a ratio RM ANOVA, it is expected that there is a greater empirical type I error rate when $V_{x1}>V_{x2}$, and a smaller empirical type I error rate when $V_{x1}<V_{x2}$. Additionally, the higher the correlation between the numerator and denominator variables, the greater the empirical type I error rate.

Monte Carlo Simulation Investigation on Type I error Rates

The inflation of the type I error rate for the F test on the trials effect in a one-way RM ANOVA is related to the level of the violation of circularity in a population covariance matrix which is measured by ϵ . In practice, the population epsilon ϵ is usually not known and it has to be estimated by the sample epsilon $\hat{\epsilon}$. However, the epsilon estimate $\hat{\epsilon}$ based on a sample is a biased estimator of population ϵ . Therefore, the characteristics of the distribution of the sample $\hat{\epsilon}$ need to be investigated to show the impact of $\hat{\epsilon}$ on the type I error rate in a RM ANOVA.

As shown in previous sections, the $\epsilon_{x1/x2}$ estimate $\hat{\epsilon}_{x1/x2}$, is a biased estimator and the bias is affected by ρ_{x1x2} and V_{x1}/V_{x2} , especially when only one of the component variables violates the assumption of circularity. In this Monte Carlo study, I investigated

how the interactive characteristics of the numerator and denominator variables affect the empirical type I error rates when using ratio variables in a one-way RM ANOVA design ($k=5$). Table 5-9 summarizes the general pattern of type I error rates for the three ϵ conditions across the levels of α (.01, .05, .10), V_{x1} and V_{x2} , and ρ_{x1x2} . The values in each block in Table 5-9 were the observed type I error rates based on the average of the observed type I error rates of the three sample sizes ($n=15, 30, 45$) and 2,000 repetitions from each of the simulated population data sets ($N=90,000$).

The general pattern of the empirical type I error rates was repetitively demonstrated across the three correlation conditions ($\rho_{x1x2}=0.9, .7, .5$) and relative variation conditions when $\epsilon_{x1}=\epsilon_{x2}=1.0$ (see Table 5-9). The correlation ρ_{x1x2} does not have substantial effect on the type I error rate in the condition $\epsilon_{x1}=\epsilon_{x2}=1.0$, with the differences between the observed type I error rates over the three ρ_{x1x2} conditions never exceeding 0.003. Additionally, there is no consistent pattern of the difference between the three correlation conditions. There is a slight decrease in the type I error rate when the variation of the numerator variable decreases. The largest discrepancy occurs in the conditions $\rho_{x1x2}=0.9, \alpha=0.05$ and $\rho_{x1x2}=0.5, \alpha=0.10$, where the type I error rate under condition $V_{x1}/V_{x2}=0.5$ is 0.008 less than it is under $V_{x1}/V_{x2}=2.0$. This difference represents a 15% and 8% difference, respectively. However, these differences are small and indicate that there is no substantial deviation of the empirical type I error rate from the nominal level when a ratio variable is used if the component variables do not violate the assumption of circularity.

As shown in Table 5-9, the type I error rate is inflated when there are heterogeneous covariances of the component variables (i.e., $\epsilon_{x1}=\epsilon_{x2}=0.7, \epsilon_{x1}=0.7$ and

Table 5-9
Empirical Type I Error Rates

ϵ	$V_{x1}/V_{x2}=2.0$			$V_{x1}/V_{x2}=1.0$			$V_{x1}/V_{x2}=0.5$			
	$\alpha=$	.01	.05	.10	.01	.05	.10	.01	.05	.10
$\epsilon_{x1}=\epsilon_{x2}=1.0$										
$\rho_{x1x2}=.9$		.010	.053	.097	.012	.048	.099	.008	.045	.094
$\rho_{x1x2}=.7$		.010	.051	.099	.011	.051	.098	.009	.044	.093
$\rho_{x1x2}=.5$		.010	.051	.100	.010	.049	.099	.010	.044	.092
$\epsilon_{x1}=\epsilon_{x2}=0.7$										
$\rho_{x1x2}=.9$		.020	.064	.116	.021	.067	.114	.019	.060	.113
$\rho_{x1x2}=.7$		.021	.066	.116	.023	.067	.112	.020	.066	.115
$\rho_{x1x2}=.5$		.022	.066	.115	.023	.065	.114	.019	.066	.112
$\epsilon_{x1}=0.7 \quad \epsilon_{x2}=1.0$										
$\rho_{x1x2}=.9$		.030	.073	.123	.015	.059	.113	.014	.060	.111
$\rho_{x1x2}=.7$		.024	.072	.118	.013	.057	.105	.010	.049	.096
$\rho_{x1x2}=.5$		.022	.067	.112	.014	.055	.104	.010	.045	.093

$\epsilon_{x2}=1.0$), and is affected by ρ_{x1x2} and V_{x1}/V_{x2} in the latter condition. To clearly show these effects and their relationship to $\epsilon_{x1/x2}$ and $\hat{\epsilon}_{x1/x2}$, detailed results based on $\alpha=0.05$ with $n=45$ are presented in Table 5-10.

The results in Table 5-10 indicate that the type I error rate is consistently inflated when the component variables have the same degree of violation on the assumption of circularity (i.e., $\epsilon_{x1}=\epsilon_{x2}=0.7$). For example, the mean of $\hat{\epsilon}_{x1/x2}$ ranged between 0.65 and 0.70 ($n=45$), and the actual type I error rate ranged between 0.061 and 0.068 ($\alpha=0.05$). There is no clear effect pattern of ρ_{x1x2} and V_{x1}/V_{x2} on the type I error rate, which is to be expected based on the findings of the population $\epsilon_{x1/x2}$ in the previous section (it was shown that $\epsilon_{x1/x2}$ seems independent of ρ_{x1x2} and V_{x1}/V_{x2}).

The lowest panel of Table 5-10 shows the effect of ρ_{x1x2} and V_{x1}/V_{x2} on the empirical type I rate in the condition $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$. The results suggest that when only the numerator variable violates the assumption of circularity, the empirical type I error rate has a positive relationship with ρ_{x1x2} and V_{x1}/V_{x2} . For example, based on $\alpha=0.05$ and $n=45$ in the condition $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$, the greatest type I error rate in this study was 0.073 when $\rho_{x1x2}=0.9$ and $V_{x1}/V_{x2}=2.0$, and the smallest type I error rate was 0.041 when $\rho_{x1x2}=0.5$ and $V_{x1}/V_{x2}=0.5$. These results are expected, based on the differences in the population value of $\epsilon_{x1/x2}$ for the different conditions of V_{x1}/V_{x2} and ρ_{x1x2} . That is, the higher the value of V_{x1}/V_{x2} and the lower the value of ρ_{x1x2} , the lower the magnitude of $\epsilon_{x1/x2}$ and the higher the type I error rate.

Table 5-10

Effect of $\epsilon_{x1/x2}$ and ρ_{x1x2} on Empirical Type I Error Rates

ϵ	$V_{x1}/V_{x2}=2.0$					$V_{x1}/V_{x2}=1.0$					$V_{x1}/V_{x2}=0.5$				
	$\frac{\epsilon_{x1/x2}}{n}$				$\alpha=.05^a$ (45)	$\frac{\epsilon_{x1/x2}}{n}$				$\alpha=.05$ (45)	$\frac{\epsilon_{x1/x2}}{n}$				$\alpha=.05$ (45)
	15	30	45			15	30	45			15	30	45		
$\epsilon_{x1}=\epsilon_{x2}=1.0$															
$\rho_{x1x2}=.9$	1.0	.76	.86	.90	.058	1.0	.75	.85	.90	.049	1.0	.68	.78	.83	.043
$\rho_{x1x2}=.7$	1.0	.76	.86	.90	.053	1.0	.75	.85	.89	.050	1.0	.69	.78	.83	.047
$\rho_{x1x2}=.5$	1.0	.77	.87	.91	.054	1.0	.74	.84	.89	.049	1.0	.69	.79	.83	.044
.....															
$\epsilon_{x1}=\epsilon_{x2}=0.7$															
$\rho_{x1x2}=.9$	0.7	.60	.65	.67	.061	.73	.63	.68	.70	.064	.72	.58	.63	.65	.065
$\rho_{x1x2}=.7$	0.7	.61	.65	.67	.066	.71	.62	.66	.68	.068	.72	.58	.63	.65	.068
$\rho_{x1x2}=.5$	0.7	.61	.65	.67	.066	.71	.61	.65	.67	.068	.72	.58	.63	.65	.066
.....															
$\epsilon_{x1}=0.7$ $\epsilon_{x2}=1.0$															
$\rho_{x1x2}=.9$	.59	.53	.56	.57	.073	.87	.70	.78	.81	.060	.86	.60	.67	.70	.059
$\rho_{x1x2}=.7$	.64	.57	.60	.62	.072	.91	.71	.80	.84	.055	.96	.67	.76	.80	.044
$\rho_{x1x2}=.5$	.70	.61	.65	.67	.063	.91	.71	.79	.83	.052	.99	.69	.78	.83	.041

^a $\alpha=.05$: Nominal type I error rate.

Sample Estimates $\hat{\epsilon}_{x1/x2}$ and Type I Error Rates

In RM ANOVA designs, the Box $\hat{\epsilon}$ and Huynh-Feldt $\tilde{\epsilon}$ are sometimes used to protect against a probable inflation in the type I error rates in the within-subject F test whenever the $\hat{\epsilon}$ of a sample is less than 1.0 (the definitions for $\hat{\epsilon}$ and $\tilde{\epsilon}$ are in the methods and procedures chapter). Collier et al. (1967), Davidson (1972), and many others have shown that, in general, violation of the assumption of circularity ($\epsilon < 1.0$) results in the artificial inflation of the F values for the within-subject main effects, thus inflating the type I error rate. As indicated in the chapter on methods and procedures, in practice, the population ϵ is unknown and the sample $\hat{\epsilon}$ is used to estimate the population ϵ , and an $\hat{\epsilon}$ -adjusted F test may be conducted to correct the inflation of the type I error rate. Huynh and Feldt (1970, 1976) indicated that the $\hat{\epsilon}$ -adjusted F test is negatively biased and would make the F test too conservative. They showed that the bias is most serious when a population ϵ is above 0.75, especially when the sample size is small. Thus a less biased estimate, $\tilde{\epsilon}$, was suggested (see Formula (19)). Huynh and Feldt also indicated that the $\tilde{\epsilon}$ -adjusted F test may result in an inflated type I error rate when $\tilde{\epsilon}$ is less than 0.75, and thus a more conservative approach (e.g., $\hat{\epsilon}$) is needed in such situations. In this study, we investigated how the bias of $\hat{\epsilon}_{x1/x2}$ and the type I error rate are affected by ρ_{x1x2} , V_{x1}/V_{x2} , and ϵ_{x1} and ϵ_{x2} .

To investigate the sampling characteristics of $\hat{\epsilon}_{x1/x2}$, the simulation results shown in Tables 5-6, 5-7, and 5-8 are expanded and graphically presented in Figures 5-3, 5-4, and 5-5. Figures 5-3, 5-4, and 5-5 show the mean of the 2,000 $\hat{\epsilon}_{x1/x2}$ values with plus and minus two standard deviations in each of the simulated conditions.

As shown in Figure 5-3, even though $\epsilon_{x1/x2}=1.0$, the values of $\hat{\epsilon}_{x1/x2}$ were considerably smaller than unity for all the levels of ρ_{x1x2} and V_{x1}/V_{x2} , which indicates biased estimation of $\epsilon_{x1/x2}$. For example, the lower bound of $\hat{\epsilon}_{x1/x2}$ was much lower than 0.75 for $n=15$ in all three V_{x1}/V_{x2} conditions. When the sample size increased, the lower bound increased substantially, especially in the conditions $V_{x1}/V_{x2}=2.0$ and $V_{x1}/V_{x2}=1.0$.

Given $\epsilon_{x1/x2}=1.0$, when a within-subject F test is conducted, there should be no correction. However, because the population epsilon is not known in practice, $\hat{\epsilon}_{x1/x2}$ will result in a “false” correction of the F test in all cases of Figure 5-3, and $\bar{\epsilon}_{x1/x2}$ in many of them. For $n=30$ and 45 $\hat{\epsilon}_{x1/x2}$ is usually greater than 0.75 and thus $\bar{\epsilon}$ adjustment would usually be applied, resulting in only a slight bias. But for $n=15$, because $\hat{\epsilon}_{x1/x2}$ is quite often smaller than 0.75, we would use the Box $\hat{\epsilon}$ adjustment in the F test and that will cause a considerable decrease in type I error rate. In the condition $V_{x1}/V_{x2}=0.5$, because the lower bound is much lower than in the other two conditions, the $\hat{\epsilon}_{x1/x2}$ adjustment would be used for those low $\hat{\epsilon}_{x1/x2}$ values ($\hat{\epsilon}_{x1/x2}<0.75$), and an serious over adjustment in the F test is expected. More subjects in a sample in the condition $V_{x1}/V_{x2}=0.5$ are needed to reduce the bias of the $\hat{\epsilon}_{x1/x2}$, and even with $n=45$ there is still considerable bias and large sampling variability.

Figure 5-4 shows the range of $\hat{\epsilon}_{x1/x2} \pm 2sd$ in the condition $\epsilon_{x1}=\epsilon_{x2}=0.7$. The results show that the variability of $\hat{\epsilon}_{x1/x2}$ increases when the variation of the denominator variable increases. The upper bound of $\hat{\epsilon}_{x1/x2}$ was approximately 0.8 for all the variation conditions, but the lower bound was lower when V_{x2} increased. As shown in a previous section, if $\epsilon_{x1}=\epsilon_{x2}=0.7$, the values of $\epsilon_{x1/x2}$ ranged from 0.70 to 0.73 in different V_{x1}/V_{x2}

Figure 5-3. Mean of epsilon estimates with ± 2 sd for $\epsilon_{x1} = \epsilon_{x2} = 1.0$, $\epsilon_{x1/x2} = 1.0$.

Figure 5-4. Mean of epsilon estimates with ± 2 sd for $\varepsilon_{x1}=\varepsilon_{x2}=0.7$, $\varepsilon_{x1/\lambda^2}=0.7$.

and $\rho_{x_1x_2}$ conditions. The results show that some values of $\hat{\epsilon}_{x_1/x_2}$ were equal or very close to the ϵ_{x_1/x_2} and some values of $\hat{\epsilon}_{x_1/x_2}$ over- or under-estimated ϵ_{x_1/x_2} by a considerable degree. An adjusted F test, using $\hat{\epsilon}$ or $\bar{\epsilon}$, with the $\hat{\epsilon}_{x_1/x_2}$ equal or close to the ϵ_{x_1/x_2} would appropriately estimate the population ϵ_{x_1/x_2} , and appropriately adjust the degrees of freedom in the F test to protect the type I error rate. However, as shown in Figure 5-4, there are more values of $\hat{\epsilon}_{x_1/x_2}$ lower than ϵ_{x_1/x_2} , thus an over-correction is more likely than an under-correction in the condition $\epsilon_{x_1}=\epsilon_{x_2}=0.7$, resulting in an overly conservative F test.

Figure 5-5 shows the range of $\hat{\epsilon}_{x_1/x_2} \pm 2sd$ in the condition $\epsilon_{x_1}=0.7$ and $\epsilon_{x_2}=1.0$. In this condition, it has been shown that the magnitudes of ϵ_{x_1/x_2} are different over the different levels of $\rho_{x_1x_2}$ and V_{x_1}/V_{x_2} . The ϵ_{x_1/x_2} had low values (i.e., lower than 0.7) when $V_{x_1}/V_{x_2}=2.0$ and high values (i.e., higher than 0.85) when $V_{x_1}/V_{x_2}=1.0$ or $V_{x_1}/V_{x_2}=0.5$. In the conditions $V_{x_1}/V_{x_2}=1.0$ and $V_{x_1}/V_{x_2}=0.5$, the magnitudes of ϵ_{x_1/x_2} were high and substantial bias in $\hat{\epsilon}_{x_1/x_2}$ was shown. Therefore, the $\hat{\epsilon}_{x_1/x_2}$ adjustment conducted in most samples (especially when $n=15$ where $\hat{\epsilon}_{x_1/x_2}<0.50$ sometimes) would overcorrect the degrees of freedom associated with the F test, resulting in type I error rate considerably less than the nominal level. In the condition $V_{x_1}/V_{x_2}=2.0$, the over-correction is also more evident than the under-correction because more samples have lower $\hat{\epsilon}_{x_1/x_2}$ values.

In general, given $\epsilon_{x_1/x_2}=1.0$, no adjustment should be needed, however, in practice the adjusted F test would be used in all the samples and result in F tests that were too conservative. If ϵ_{x_1} and/or ϵ_{x_2} are not equal to 1.0, using an adjusted F test in a RM ANOVA may result in over-correction in some conditions and under-correction in other

Figure 5-5. Mean of epsilon estimates with ± 2 sd for $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$. (ϵ_{x1/κ_2} noted by $\star$)

conditions.

Table 5-11 shows the summary for the results of $\epsilon_{x1}=\epsilon_{x2}=0.7$, and $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$ conditions. Because similar patterns were shown in different ρ_{x1x2} conditions, only the results for $\rho_{x1x2}=0.7$ ($n=15$) are presented in Table 5-11. In Table 5-11, an “appropriate correction” is defined as the adjusted F tests based on the magnitudes of $\hat{\epsilon}_{x1/x2}$ in the range $\epsilon_{x1/x2} \pm 0.05$, an “under-correction” as an insufficient correction of the F test occurring because $\hat{\epsilon}_{x1/x2}$ exceeds $\epsilon_{x1/x2}$ by more than 0.05, and an “over-correction” as too conservative an adjustment in the F tests resulting from the conditions where $\hat{\epsilon}_{x1/x2}$ is less than $\epsilon_{x1/x2}$ by more than 0.05. The results of the adjusted F tests with an $\hat{\epsilon}_{x1/x2}$ adjustment in the different conditions was compared to the results with the $\epsilon_{x1/x2}$ adjustment.

Given that the mean of $\hat{\epsilon}_{x1/x2}$ is smaller than $\epsilon_{x1/x2}$ in all cases in Table 5-11, the $\hat{\epsilon}_{x1/x2}$ adjustment is likely to show an over-correction in the F test. When the $\epsilon_{x1/x2}$ adjustment is used, the conservative F tests would result in type I error rate seriously less than the nominal level in the condition $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$. That is, the percent of over-correction was 58.7%, 95.1%, and 98.6% for the three V_{x1}/V_{x2} conditions, respectively. The results also indicate that using an $\hat{\epsilon}_{x1/x2}$ correction can not substantially increase the probability of a correct adjustment in the F test in the two conditions $\epsilon_{x1}=\epsilon_{x2}=0.7$ and $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$. Although it can not increase the percent of appropriate corrections in the F test, as shown in Table 5-11, the $\hat{\epsilon}_{x1/x2}$ adjustment substantially reduces the percent of the over-correction and increases the percent of the under-correction.

Table 5-11

A Summary of the Effect of V_{x1}/V_{x2} on the Type I Error Rates ($\rho_{x1x2}=0.7$, $\alpha=.05$, $n=15$)

ϵ_{x1}	ϵ_{x2}	V_{x1}/V_{x2}	$\epsilon_{x1/x2}$	$\hat{\epsilon}_{x1/x2}$	Type I Error	$\hat{\epsilon}_{x1/x2}$					
						Appropriate Correction ^a		Under-Correction		Over-Correction	
						$\hat{\epsilon}$	$\bar{\epsilon}$	$\hat{\epsilon}$	$\bar{\epsilon}$	$\hat{\epsilon}$	$\bar{\epsilon}$
0.7	0.7	2.0	0.70	0.61 (.09)	.061	27.0	25.6	6.0	49.6	67.0	24.8
		1.0	0.71	0.62 (.10)	.065	26.4	22.2	8.1	50.0	65.5	27.8
		0.5	0.72	0.58 (.11)	.067	16.4	21.9	4.2	34.8	79.4	43.3
0.7	1.0	2.0	0.64	0.57 (.09)	.068	32.1	26.1	9.2	50.0	58.7	23.9
		1.0	0.91	0.71 (.09)	.054	4.6	4.8	0.3	47.6	95.1	47.6
		0.5	0.96	0.67 (.11)	.055	1.3	17.1	0.1	15.9	98.6	67.0

^a Appropriate correction refers to percent of times an adjustment in the F test would be "appropriate" defined as $|\hat{\epsilon}_{x1/x2} - \epsilon_{x1/x2}| \leq 0.05$. Under-correction is defined as insufficient adjustment which occurs when $\hat{\epsilon}_{x1/x2} > \epsilon_{x1/x2} + 0.05$, and over-correction is defined as too conservative adjustment which occurs when $\hat{\epsilon}_{x1/x2} < \epsilon_{x1/x2} - 0.05$.

In general, the over-correction of the F test is more pronounced in the two conditions $\epsilon_{x1}=\epsilon_{x2}=1.0$, and $\epsilon_{x1}=0.7$ and $\epsilon_{x1}=1.0$, than in the condition $\epsilon_{x1}=\epsilon_{x2}=0.7$. The most severely over-corrected F test would be in the condition $V_{x1}/V_{x2}=0.5$ and $\epsilon_{x1}=0.7$ and $\epsilon_{x1}=1.0$. The results indicate that large bias and extreme variability of $\hat{\epsilon}_{x1/x2}$ would be expected when $V_{x1}<V_{x2}$, especially with small sample size (the sample size equal or smaller than 15 which is very common in some fields in human kinetics research). If $V_{x1}<V_{x2}$ in the component variable data, it is quite possible that a researcher could get a very low $\hat{\epsilon}_{x1/x2}$ and the over-corrected F test would make the within-subject F test too conservative. In this case, the $\tilde{\epsilon}_{x1/x2}$ adjustment may be an alternative choice but it may result in an under-correction. The results suggest that, when a ratio variable is used, if the denominator variable has lower variation than the numerator variable and sample size is large, the risk of the over-correction in the RM ANOVA F test could be substantially reduced.

CHAPTER VI: A SUMMARY OF THE INVESTIGATION

The purposes of this dissertation were twofold. The first was to use a practical procedure to evaluate four deflation models for ratio scores, and to examine the validity and reliability of commonly used ratio variables in human kinetics research. The second was to investigate how the statistical characteristics of the component variables affect the assumptions and results of a RM ANOVA test. This chapter presents a summary and implications of the findings of this study.

Empirical Ratio Data Study

In this investigation it was shown that the simple ratio model commonly used for deflation purposes did not appropriately deflate the effect of the denominator for the two empirical data sets. The linear regression model with an intercept (LRM) and the nonlinear regression model without an intercept (NLRM1) seemed equally preferable for deflation purposes in these data.

These results imply that an optimal deflation model useful for all ratio variables may not exist, and different models should be applied to a data set to determine the appropriate deflation model. It is recommended that the procedures developed in this study be used to determine the best deflation model for commonly used ratio variables in our field. To obtain the best ratio variable for deflation purposes, one should fit each of the four deflation models to the data and evaluate the validity of the model using five criteria: (a) zero correlation between a derived ratio variable and the denominator variable, (b) no curvilinear relationship between the derived ratio and the denominator in

the scatterplots, (c) equality of the estimated expected value of the model and calculated empirical mean of the derived ratio data, (d) high R^2 , and (e) high reliability of the derived ratio data.

It was shown that the reliability of a ratio variable is strongly affected by not only within and between trial correlations, but also by the relative variation of the component variables (V_{x1}/V_{x2}). If the coefficients of variation of the component variables in two trials are the same, the results show that the reliability of the ratio variable is a function of only the within and between trial correlation of the component variables, and is not affected by the coefficients of variation. However, given that the within and between trial correlations of the component variables do not change, unequal coefficients of variation of the numerator and denominator variables may result in the high reliability for the ratio variable, and the effect is most pronounced when the denominator variable has the smaller coefficient of variation ($V_{x1} > V_{x2}$). It implies that researchers should compute the reliability of the derived ratio scores, and not assume that strong reliabilities in the component measures automatically lead to strong reliability in the ratio measures.

Simulation Investigation

Characteristics of $\epsilon_{x1/x2}$, ρ_{yiyj} , and $\hat{\epsilon}_{x1/x2}$

The magnitude of epsilon of a ratio variable, $\epsilon_{x1/x2}$, and its sample estimate, $\hat{\epsilon}_{x1/x2}$, were shown to be affected not only by the magnitudes of epsilon of the component variables ϵ_{x1} and ϵ_{x2} , but also by the relative variation V_{x1}/V_{x2} and the correlation (ρ_{x1x2}) between the component variables. The nature of these relationships can be summarized as follows.

Characteristics of $\epsilon_{x1/x2}$. If the component variables have the same level of epsilon ($\epsilon_{x1} = \epsilon_{x2}$), a ratio variable has virtually the same epsilon value as the component variables. However, the magnitude of epsilon of a ratio variable $\epsilon_{x1/x2}$ is strongly affected by the relative variation and the correlation of the component variables if the covariance matrix of the numerator variable or of the denominator variable has a heterogeneous structure (ϵ_{x1} or ϵ_{x2} is smaller than unity).

If the numerator variable violates the assumption of circularity and the denominator does not ($\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$), the results show that $\epsilon_{x1/x2}$ has a negative relationship with V_{x1}/V_{x2} and ρ_{x1x2} . When V_{x1}/V_{x2} and ρ_{x1x2} decrease the population epsilon of the ratio variable $\epsilon_{x1/x2}$ increases. Figure 6-1 shows these findings. When only the denominator variable violates the assumption of circularity ($\epsilon_{x1}=1.0$ and $\epsilon_{x2}=0.7$), the pattern of $\epsilon_{x1/x2}$ over the three variation conditions is the opposite from the condition $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$, but the effect of ρ_{x1x2} on $\epsilon_{x1/x2}$ is the same in the two epsilon conditions. That is, in the condition $\epsilon_{x1}=1.0$ and $\epsilon_{x2}=0.7$, when V_{x1}/V_{x2} increases and ρ_{x1x2} decreases the population epsilon of the ratio variable $\epsilon_{x1/x2}$ increases.

Characteristics of ρ_{yiyj} . The results show that the magnitudes in the correlation matrices of the ratio variable are mainly affected by the between trial correlation of the numerator and of the denominator variables, and the relative variation of the two component variables (V_{x1}/V_{x2}). Varying ρ_{x1x2} has no substantial effect on the magnitude of the correlations in the ρ_{yiyj} matrices.

When $\epsilon_{x1}=\epsilon_{x2}=1.0$, the correlation matrix of the ratio variable has a constant pattern and the magnitudes of ρ_{yiyj} decrease when V_{x1}/V_{x2} decreases from 2.0 to 0.5.

Figure 6-1. The $\epsilon_{x1/x2}$ values and the empirical type I error rates in the condition $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$ ($\alpha=0.05$, $n=45$).

When $\epsilon_{x1}=\epsilon_{x2}=0.7$ and the correlations of the component variables follow a simplex pattern, the correlation matrices of the ratio variable exhibit a slower decreasing pattern than the simplex pattern in all V_{x1}/V_{x2} conditions. In the two conditions $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$, and $\epsilon_{x1}=1.0$ and $\epsilon_{x2}=0.7$, the results indicate that the correlation pattern of the ratio variable is more similar to that of the component variable (numerator or denominator) which has a larger coefficient of variation than the other component.

Characteristics of $\hat{\epsilon}_{x1/x2}$. The value of epsilon estimated from a sample, $\hat{\epsilon}_{x1/x2}$, is a biased estimator of the population epsilon of the ratio variable $\epsilon_{x1/x2}$, and the bias varies according to the conditions of ϵ_{x1} , ϵ_{x2} , V_{x1}/V_{x2} , and ρ_{x1x2} .

When $\epsilon_{x1}=\epsilon_{x2}=1.0$, the bias of $\hat{\epsilon}_{x1/x2}$ is affected by the coefficient of variation of the component variables, but the correlation ρ_{x1x2} does not have substantial effect on the bias in this condition. That is, if $V_{x1}/V_{x2} \geq 1.0$, the mean values of $\hat{\epsilon}_{x1/x2}$ for $n=15$ are between 0.74 and 0.77 (Sd= 0.08-0.09), and the mean bias is between 0.23 to 0.26.

When the variation of the denominator increases ($V_{x1}/V_{x2}=0.5$), the bias of $\hat{\epsilon}_{x1/x2}$ is more serious, and the mean value of $\hat{\epsilon}_{x1/x2}$ is 0.68 (Sd=0.12), and the mean bias is 0.32.

When $\epsilon_{x1}=\epsilon_{x2}=0.7$, the bias of $\hat{\epsilon}_{x1/x2}$ is also more serious when V_{x1}/V_{x2} has a low value ($V_{x1}/V_{x2}=0.5$), but the effect of V_{x1}/V_{x2} is less than in the condition $\epsilon_{x1}=\epsilon_{x2}=1.0$. The correlation ρ_{x1x2} does not have a substantial effect in this condition. The largest mean bias of $\hat{\epsilon}_{x1/x2}$ is 0.14 and the standard deviation of $\hat{\epsilon}_{x1/x2}$ is 0.11 ($V_{x1}/V_{x2}=0.5$, $n=15$).

If the numerator variable violates the assumption of circularity and the denominator does not (e.g., $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$), both V_{x1}/V_{x2} and ρ_{x1x2} affect the bias of

$\hat{\epsilon}_{x1/x2}$. That is, the bias of $\hat{\epsilon}_{x1/x2}$ increases when V_{x1}/V_{x2} and ρ_{x1x2} decrease. The bias of $\hat{\epsilon}_{x1/x2}$ is more pronounced in the condition $\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$ than the other two conditions ($\epsilon_{x1}=\epsilon_{x2}=1.0$ and $\epsilon_{x1}=\epsilon_{x2}=0.7$). Among the cases considered, the largest bias of $\hat{\epsilon}_{x1/x2}$ was 0.3 and the standard deviation of $\hat{\epsilon}_{x1/x2}$ is 0.11 ($V_{x1}/V_{x2}=0.5$, $n=15$).

If a ratio variable is used, given certain mean differences, and magnitudes of ϵ_{x1} and ϵ_{x2} of the two component variables, the results indicate that the magnitudes of $\epsilon_{x1/x2}$ and $\hat{\epsilon}_{x1/x2}$ are substantially affected by V_{x1}/V_{x2} and ρ_{x1x2} . When the magnitudes of $\epsilon_{x1/x2}$ and $\hat{\epsilon}_{x1/x2}$ vary, the type I error rates of the RM ANOVA tests is affected. The impact of using ratio variables on the type I error rate in a RM ANOVA test is summarized in the following sections.

Type I Error When Using Ratio Variables in RM ANOVA

Type I Error Rate. The simulation investigation was designed to examine how the characteristics of the component variables affect the type I error rate of the F test in a one-way RM ANOVA based on the transformed ratio variable. The advantage of this study is that we could investigate the impact of using the ratio variable in a one-way RM ANOVA, given that the characteristics of the ratio variable population and the related characteristics of the component variables were known. In the Monte Carlo simulation investigation, results show that the naïve F tests in a five-trial one-way RM ANOVA on a ratio variable result in a type I error rate that is close to the nominal level in the condition $\epsilon_{x1}=\epsilon_{x2}=1.0$ ($\epsilon_{x1/x2}=1.0$), regardless on the condition of V_{x1}/V_{x2} and ρ_{x1x2} . The type I error rate of the F test exhibits inflation in the condition $\epsilon_{x1}=\epsilon_{x2}=0.7$ ($\epsilon_{x1/x2}\approx 0.7$), but there is no clear effect of V_{x1}/V_{x2} and ρ_{x1x2} . If only the numerator variable violates the assumption of circularity ($\epsilon_{x1}=0.7$ and $\epsilon_{x2}=1.0$), the type I error rate of the F test increases

when V_{x1}/V_{x2} and ρ_{x1x2} increase. Because the value of $\epsilon_{x1/x2}$ indicates the level of violation of circularity in the population covariance matrix of the ratio variable, the high level of the inflation of the type I error rate in the F test should correspond to the low value of $\epsilon_{x1/x2}$. As shown in the first graph in Figure 6-1, $\epsilon_{x1/x2}$ decreases when V_{x1}/V_{x2} and ρ_{x1x2} increase. Therefore, it is expected that the inflation of the type I error rate is more severe when V_{x1}/V_{x2} and ρ_{x1x2} increase (see Figure 6-1).

Sample Estimates $\hat{\epsilon}_{x1/x2}$ and Type I Error Rates. Because the value of $\epsilon_{x1/x2}$ is not known in practice, in RM ANOVA designs, the Box epsilon $\hat{\epsilon}$ and H-F $\tilde{\epsilon}$ are used to protect against a probable inflation in the type I error rates in the within-subject F test whenever $\hat{\epsilon}$ of a sample is less than 1.0. Using Monte Carlo procedures, the sampling characteristics of $\hat{\epsilon}_{x1/x2}$ was investigated and the effects of $\hat{\epsilon}_{x1/x2}$ and H-F $\tilde{\epsilon}_{x1/x2}$ in the RM ANOVA were investigated.

The results indicate that over-adjustment in the $\hat{\epsilon}$ -adjusted F test is more pronounced in the two conditions $\epsilon_{x1} = \epsilon_{x2} = 1.0$, and $\epsilon_{x1} = 0.7$ and $\epsilon_{x2} = 1.0$. The large bias and extreme variability of $\hat{\epsilon}$ -adjusted F tests would be expected when $V_{x1}/V_{x2} = 0.5$ especially with smaller sample size (e.g., $n=15$). If the H-F $\tilde{\epsilon}$ correction is used, it reduces the risk of over-adjustment but increases the risk of under-adjustment.

In summary, this dissertation investigated the validity and reliability of some commonly used ratio variables in human kinetics research, and the effect of using ratio variables on the circularity assumption of the covariance matrix, type I error rates in RM ANOVA tests. It shows that different models should be used to derive an appropriate deflation model in empirical research. A commonly used deflation model for all ratio

variables may not exist. The results indicate that high reliability of the component variables does not necessarily result in high reliability of the transformed ratio variable. Thus, when a ratio variable is used the reliability should be examined based on the ratio variable data. Simulation results show that the characteristics of the two component variables (ϵ_{x1} , ϵ_{x2} , V_{x1}/V_{x2} , and ρ_{x1x2}) strongly affect the circularity of the covariance matrix of the ratio variable, and the type I error rate in RM ANOVA tests. In general, the mean $\hat{\epsilon}_{x1/x2}$ exhibited the greatest bias and the largest standard deviation, resulting in a serious inflation of type I error rate in the condition $V_{x1}/V_{x2}=0.5$, regardless the conditions ϵ_{x1} , ϵ_{x2} , and ρ_{x1x2} . If homogeneity of the denominator variable (small V_{x2}) and large sample size are present, it may reduce likelihood of bias in $\hat{\epsilon}_{x1/x2}$ and protect the type I error rate.

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APPENDICES

Appendix A

Approximating ρ_{x_1, x_2} Using the Attenuation Correlation Function

In the classic measurement model, two variables would be expected to have uncorrelated errors, and errors on either variable would be uncorrelated with true scores in either variable. The observed scores of two variables can be expressed as $X_1 = T_1 + e_1$, $X_2 = T_2 + e_2$, where T_i is a true score and e_i is the error ($i=1,2$). The correlation between the two observed variables can be shown as follows:

$$\begin{aligned}\rho_{x_1 x_2} &= \frac{\sigma_{x_1 x_2}}{\sigma_{x_1} \sigma_{x_2}} = \frac{1/N \sum (T_1 + e_1)(T_2 + e_2)}{\sigma_{x_1} \sigma_{x_2}} \\ &= \frac{1/N (\sum T_1 T_2 + \sum T_1 e_2 + \sum T_2 e_1 + \sum e_1 e_2)}{\sigma_{x_1} \sigma_{x_2}}.\end{aligned}$$

Under the postulated model, all terms but the first in the numerator are zero, so the formula can be written as

$$\rho_{x_1 x_2} = \frac{\sigma_{T_1 T_2}}{\sigma_{x_1} \sigma_{x_2}}.$$

This shows that the covariance of observed scores is equal to the covariance of true scores $\sigma_{T_1 T_2} = \rho_{x_1 x_2} \sigma_{x_1} \sigma_{x_2} = \sigma_{x_1 x_2}$. Because the reliability is defined as true variance divided by observed variance $\rho_{xx} = \sigma_T^2 / \sigma_x^2$ (ρ_{xx} refers to the reliability for a variable X), if there were no errors present, the true correlation between two variables X_1 and X_2 ($\rho_{T_1 T_2}$) would be as follows:

$$\begin{aligned}\rho_{T1T2} &= \frac{\sigma_{T1T2}}{\sigma_{T1}\sigma_{T2}} = \frac{\sigma_{t1t2}}{\sqrt{\rho_{t1t1}}\sigma_{t1}\sqrt{\rho_{t2t2}}\sigma_{t2}} \\ &= \frac{\sigma_{t1t2}/(\sigma_{t1}\sigma_{t2})}{\sqrt{\rho_{t1t1}}\sqrt{\rho_{t2t2}}} = \frac{\rho_{t1t2}}{\sqrt{\rho_{t1t1}}\sqrt{\rho_{t2t2}}}.\end{aligned}$$

This formula is spoken of as the “correction” for attenuation, but it is really an estimate of the magnitude of a correlation if two variables were made perfectly reliable (Nunnally, 1978, p215-221).

In this study, the attenuation formula was employed to estimate the correlation between the numerator and denominator variables in different trials $\rho_{x1i,x2j}$:

$$\rho_{T1iT2j} = \frac{\rho_{x1i,x2j}}{\sqrt{\rho_{x1x1}}\sqrt{\rho_{x2x2}}} \quad i \neq j \quad (\text{A-1})$$

where ρ_{T1iT2j} is true correlation (measured without error) between X_1 in the i th trial and X_2 in the j th trial, $\rho_{x1i,x2j}$ is the observed population correlation between X_1 in the i th trial and X_2 in the j th trial, and ρ_{x1x1} and ρ_{x2x2} are the reliability coefficients of X_1 and X_2 , respectively. The reliability coefficients of X_1 and X_2 were derived from the defined covariance matrix of the component variables in each of the controlled conditions.

As shown above, this investigation assumed that the within trial correlation between the numerator and denominator variables was the same, and the variance of the component variables was constant, over all five trials. Therefore, the best approximation for the true correlation between the numerator in trial i and the denominator in trial j would be the correlation between the two variables in the same trial. That is, $\rho_{T1iT2j} = \rho_{x1i,x2i}$. Therefore, Formula (A-1) can be rewritten to show the observed correlation between the numerator and denominator variables in different trials:

$$\rho_{\epsilon_1, \epsilon_2} = \rho_{\epsilon_1, \epsilon_2} \sqrt{\rho_{\epsilon_1 \epsilon_1}} \sqrt{\rho_{\epsilon_2 \epsilon_2}} \qquad i \neq j. \qquad (A-2)$$

Appendix B

Covariance Matrices Used to Generate the Population Data

$$\varepsilon_{x1}=\varepsilon_{x2}=1.0, V_{x1}/V_{x2}=2.0, \rho_{x1x2}=0.9$$

225.000	195.000	195.000	195.000	195.000	131.582	110.996	110.996	110.996	110.996
195.000	225.000	195.000	195.000	195.000	110.996	131.582	110.996	110.996	110.996
195.000	195.000	225.000	195.000	195.000	110.996	110.996	131.582	110.996	110.996
195.000	195.000	195.000	225.000	195.000	110.996	110.996	110.996	131.582	110.996
195.000	195.000	195.000	195.000	225.000	110.996	110.996	110.996	110.996	131.582
131.582	110.996	110.996	110.996	110.996	95.000	78.000	78.000	78.000	78.000
110.996	131.582	110.996	110.996	110.996	78.000	95.000	78.000	78.000	78.000
110.996	110.996	131.582	110.996	110.996	78.000	78.000	95.000	78.000	78.000
110.996	110.996	110.996	131.582	110.996	78.000	78.000	78.000	95.000	78.000
110.996	110.996	110.996	110.996	131.582	78.000	78.000	78.000	78.000	95.000

$$\varepsilon_{x1}=\varepsilon_{x2}=1.0, V_{x1}/V_{x2}=1.0, \rho_{x1x2}=0.9$$

135.000	117.000	117.000	117.000	117.000	162.000	135.942	135.942	135.942	135.942
117.000	135.000	117.000	117.000	117.000	135.942	162.000	135.942	135.942	135.942
117.000	117.000	135.000	117.000	117.000	135.942	135.942	162.000	135.942	135.942
117.000	117.000	117.000	135.000	117.000	135.942	135.942	135.942	162.000	135.942
117.000	117.000	117.000	117.000	135.000	135.942	135.942	135.942	135.942	162.000
162.000	135.942	135.942	135.942	135.942	240.000	195.000	195.000	195.000	195.000
135.942	162.000	135.942	135.942	135.942	195.000	240.000	195.000	195.000	195.000
135.942	135.942	162.000	135.942	135.942	195.000	195.000	240.000	195.000	195.000
135.942	135.942	135.942	162.000	135.942	195.000	195.000	195.000	240.000	195.000
135.942	135.942	135.942	135.942	162.000	195.000	195.000	195.000	195.000	240.000

$$\varepsilon_{x1}=\varepsilon_{x2}=1.0, V_{x1}/V_{x2}=0.5, \rho_{x1x2}=0.9$$

53.000	45.000	45.000	45.000	45.000	131.042	109.675	109.675	109.675	109.675
45.000	53.000	45.000	45.000	45.000	109.675	131.042	109.675	109.675	109.675
45.000	45.000	53.000	45.000	45.000	109.675	109.675	131.042	109.675	109.675
45.000	45.000	45.000	53.000	45.000	109.675	109.675	109.675	131.042	109.675
45.000	45.000	45.000	45.000	53.000	109.675	109.675	109.675	109.675	131.042
131.042	109.675	109.675	109.675	109.675	400.000	330.000	330.000	330.000	330.000
109.675	131.042	109.675	109.675	109.675	330.000	400.000	330.000	330.000	330.000
109.675	109.675	131.042	109.675	109.675	330.000	330.000	400.000	330.000	330.000
109.675	109.675	109.675	131.042	109.675	330.000	330.000	330.000	400.000	330.000
109.675	109.675	109.675	109.675	131.042	330.000	330.000	330.000	330.000	400.000

$\varepsilon_{x1}=\varepsilon_{x2}=1.0$, $V_x1/V_{x2}=2.0$, $\rho_{x1x2}=0.7$

135.000	135.000	195.000	195.000	195.000	102.341	86.330	86.330	86.330	86.330
135.000	135.000	195.000	195.000	195.000	86.330	102.341	86.330	86.330	86.330
135.000	135.000	225.000	195.000	195.000	86.330	86.330	102.341	86.330	86.330
135.000	135.000	195.000	225.000	195.000	86.330	86.330	86.330	102.341	86.330
135.000	135.000	195.000	195.000	225.000	86.330	86.330	86.330	86.330	102.341
102.341	86.330	86.330	86.330	96.330	95.000	78.000	78.000	78.000	78.000
102.341	86.330	86.330	86.330	86.330	95.000	95.000	78.000	78.000	78.000
86.330	86.330	102.341	86.330	86.330	78.000	78.000	95.000	78.000	78.000
86.330	86.330	86.330	102.341	86.330	78.000	78.000	78.000	95.000	78.000
86.330	86.330	86.330	86.330	102.341	78.000	78.000	78.000	78.000	95.000

$\varepsilon_{x1}=\varepsilon_{x2}=1.0$, $V_x1/V_{x2}=1.0$, $\rho_{x1x2}=0.7$

117.000	117.000	117.000	117.000	117.000	126.000	105.732	105.732	105.732	105.732
117.000	117.000	117.000	117.000	117.000	105.732	126.000	105.732	105.732	105.732
117.000	117.000	135.000	117.000	117.000	105.732	105.732	126.000	105.732	105.732
117.000	117.000	117.000	135.000	117.000	105.732	105.732	105.732	126.000	105.732
117.000	117.000	117.000	117.000	135.000	105.732	105.732	105.732	105.732	126.000
105.732	105.732	105.732	105.732	105.732	240.000	195.000	195.000	195.000	195.000
105.732	105.732	105.732	105.732	105.732	195.000	240.000	195.000	195.000	195.000
105.732	105.732	126.000	105.732	105.732	195.000	195.000	240.000	195.000	195.000
105.732	105.732	105.732	126.000	105.732	195.000	195.000	195.000	240.000	195.000
105.732	105.732	105.732	105.732	126.000	195.000	195.000	195.000	195.000	240.000

$\varepsilon_{x1}=\varepsilon_{x2}=1.0$, $V_x1/V_{x2}=0.5$, $\rho_{x1x2}=0.7$

53.000	45.000	45.000	45.000	45.000	101.922	85.302	85.302	85.302	85.302
45.000	53.000	45.000	45.000	45.000	85.302	101.922	85.302	85.302	85.302
45.000	45.000	53.000	45.000	45.000	85.302	85.302	101.922	85.302	85.302
45.000	45.000	45.000	53.000	45.000	85.302	85.302	85.302	101.922	85.302
45.000	45.000	45.000	45.000	53.000	85.302	85.302	85.302	85.302	101.922
101.922	85.302	85.302	85.302	85.302	400.000	330.000	330.000	330.000	330.000
85.302	101.922	85.302	85.302	85.302	330.000	400.000	330.000	330.000	330.000
85.302	85.302	101.922	85.302	85.302	330.000	330.000	400.000	330.000	330.000
85.302	85.302	85.302	101.922	85.302	330.000	330.000	330.000	400.000	330.000
85.302	85.302	85.302	85.302	101.922	330.000	330.000	330.000	330.000	400.000

$$\varepsilon_{x1}=\varepsilon_{x2}=1.0, V_{x1}/V_{x2}=2.0, \rho_{x1x2}=0.5$$

225.000	195.000	195.000	195.000	195.000	73.101	61.664	61.664	61.664	61.664
195.000	225.000	195.000	195.000	195.000	61.664	73.101	61.664	61.664	61.664
195.000	195.000	225.000	195.000	195.000	61.664	61.664	73.101	61.664	61.664
195.000	195.000	195.000	225.000	195.000	61.664	61.664	61.664	73.101	61.664
195.000	195.000	195.000	195.000	225.000	61.664	61.664	61.664	61.664	73.101
73.101	61.664	61.664	61.664	61.664	95.000	78.000	78.000	78.000	78.000
61.664	73.101	61.664	61.664	61.664	78.000	95.000	78.000	78.000	78.000
61.664	61.664	73.101	61.664	61.664	78.000	78.000	95.000	78.000	78.000
61.664	61.664	61.664	73.101	61.664	78.000	78.000	78.000	95.000	78.000
61.664	61.664	61.664	61.664	73.101	78.000	78.000	78.000	78.000	95.000

$$\varepsilon_{x1}=\varepsilon_{x2}=1.0, V_{x1}/V_{x2}=1.0, \rho_{x1x2}=0.5$$

135.000	117.000	117.000	117.000	117.000	90.000	75.523	75.523	75.523	75.523
117.000	135.000	117.000	117.000	117.000	75.523	90.000	75.523	75.523	75.523
117.000	117.000	135.000	117.000	117.000	75.523	75.523	90.000	75.523	75.523
117.000	117.000	117.000	135.000	117.000	75.523	75.523	75.523	90.000	75.523
117.000	117.000	117.000	117.000	135.000	75.523	75.523	75.523	75.523	90.000
90.000	75.523	75.523	75.523	75.523	240.000	195.000	195.000	195.000	195.000
75.523	90.000	75.523	75.523	75.523	195.000	240.000	195.000	195.000	195.000
75.523	75.523	90.000	75.523	75.523	195.000	195.000	240.000	195.000	195.000
75.523	75.523	75.523	90.000	75.523	195.000	195.000	195.000	240.000	195.000
75.523	75.523	75.523	75.523	90.000	195.000	195.000	195.000	195.000	240.000

$$\varepsilon_{x1}=\varepsilon_{x2}=1.0, V_{x1}/V_{x2}=0.5, \rho_{x1x2}=0.5$$

53.000	45.000	45.000	45.000	45.000	72.801	60.930	60.930	60.930	60.930
45.000	53.000	45.000	45.000	45.000	60.930	72.801	60.930	60.930	60.930
45.000	45.000	53.000	45.000	45.000	60.930	60.930	72.801	60.930	60.930
45.000	45.000	45.000	53.000	45.000	60.930	60.930	60.930	72.801	60.930
45.000	45.000	45.000	45.000	53.000	60.930	60.930	60.930	60.930	72.801
72.801	60.930	60.930	60.930	60.930	400.000	330.000	330.000	330.000	330.000
60.930	72.801	60.930	60.930	60.930	330.000	400.000	330.000	330.000	330.000
60.930	60.930	72.801	60.930	60.930	330.000	330.000	400.000	330.000	330.000
60.930	60.930	60.930	72.801	60.930	330.000	330.000	330.000	400.000	330.000
60.930	60.930	60.930	60.930	72.801	330.000	330.000	330.000	330.000	400.000

$$\varepsilon_{x1}=\varepsilon_{x2}=0.7, V_{x1}/V_{x2}=2.0, \rho_{x1x2}=0.9$$

225.000	195.000	170.000	160.000	150.000	131.582	110.996	92.398	83.656	75.568
195.000	225.000	196.000	170.000	160.000	110.996	131.582	109.844	90.896	83.656
170.000	196.000	225.000	195.000	170.000	92.398	109.844	131.582	109.564	92.398
160.000	170.000	195.000	225.000	192.000	83.656	90.896	109.564	131.582	105.818
150.000	160.000	170.000	192.000	225.000	75.568	83.656	92.398	105.818	131.582
131.582	110.996	92.398	83.656	75.568	95.000	78.000	62.000	54.000	47.000
110.996	131.582	109.844	90.896	83.656	78.000	95.000	76.000	60.000	54.000
92.398	109.844	131.582	109.564	92.398	62.000	76.000	95.000	76.000	62.000
83.656	90.896	109.564	131.582	105.818	54.000	60.000	76.000	95.000	72.000
75.568	83.656	92.398	105.818	131.582	47.000	54.000	62.000	72.000	95.000

$$\varepsilon_{x1}=\varepsilon_{x2}=0.7, V_{x1}/V_{x2}=1.0, \rho_{x1x2}=0.9$$

135.000	117.000	104.000	95.000	81.000	162.000	134.188	116.097	101.923	84.954
117.000	135.000	116.000	104.000	95.000	134.188	162.000	133.613	116.097	101.923
104.000	116.000	135.000	116.000	102.000	116.097	133.613	162.000	133.613	114.975
95.000	104.000	116.000	135.000	111.000	101.923	116.097	133.613	162.000	130.012
81.000	95.000	102.000	111.000	135.000	84.954	101.923	114.975	130.012	162.000
162.000	134.188	116.097	101.923	84.954	240.000	190.000	160.000	135.000	110.000
134.188	162.000	133.613	116.097	101.923	190.000	240.000	190.000	160.000	135.000
116.097	133.613	162.000	133.613	114.975	160.000	190.000	240.000	190.000	160.000
101.923	116.097	133.613	162.000	130.012	135.000	160.000	190.000	240.000	188.000
84.954	101.923	114.975	130.012	162.000	110.000	135.000	160.000	188.000	240.000

$$\varepsilon_{x1}=\varepsilon_{x2}=0.7, V_{x1}/V_{x2}=0.5, \rho_{x1x2}=0.9$$

53.000	45.000	40.000	35.000	32.000	131.042	109.675	91.782	78.975	70.177
45.000	53.000	45.000	40.000	36.000	109.675	131.042	106.299	91.782	80.095
40.000	45.000	53.000	45.000	40.000	91.782	106.299	131.042	108.000	90.000
35.000	40.000	45.000	53.000	44.000	78.975	91.782	108.000	131.042	103.402
32.000	36.000	40.000	44.000	53.000	70.177	80.095	90.000	103.402	131.042
131.042	109.675	91.782	78.975	70.177	400.000	330.000	260.000	220.000	190.000
109.675	131.042	106.299	91.782	80.095	330.000	400.000	310.000	260.000	220.000
91.782	106.299	131.042	108.000	90.000	260.000	310.000	400.000	320.000	250.000
78.975	91.782	108.000	131.042	103.402	220.000	260.000	320.000	400.000	300.000
70.177	80.095	90.000	103.402	131.042	190.000	220.000	250.000	300.000	400.000

$\varepsilon_{x1}=\varepsilon_{x2}=0.7$, $V_{x1}/V_{x2}=2.0$, $\rho_{x1x2}=0.7$

225.000	195.000	170.000	160.000	150.000	102.341	86.330	71.865	65.066	58.775
195.000	225.000	195.000	170.000	160.000	86.330	102.341	85.434	70.697	65.066
170.000	195.000	225.000	195.000	170.000	71.865	85.434	102.341	85.216	71.865
160.000	170.000	195.000	225.000	192.000	65.066	70.697	85.216	102.341	82.303
150.000	160.000	170.000	192.000	225.000	58.775	65.066	71.865	82.303	102.341
102.341	86.330	71.865	65.066	58.775	95.000	78.000	62.000	54.000	47.000
86.330	102.341	85.434	70.697	65.066	78.000	95.000	76.000	60.000	54.000
71.865	85.434	102.341	85.216	71.865	62.000	76.000	95.000	76.000	62.000
65.066	70.697	85.216	102.341	82.303	54.000	60.000	76.000	95.000	72.000
58.775	65.066	71.865	82.303	102.341	47.000	54.000	62.000	72.000	95.000

$\varepsilon_{x1}=\varepsilon_{x2}=0.7$, $V_{x1}/V_{x2}=1.0$, $\rho_{x1x2}=0.7$

135.000	117.000	104.000	95.000	81.000	126.000	104.368	90.297	79.273	66.075
117.000	135.000	116.000	104.000	95.000	104.368	126.000	103.921	90.297	79.273
104.000	116.000	135.000	116.000	102.000	90.297	103.921	126.000	103.921	89.425
95.000	104.000	116.000	135.000	111.000	79.273	90.297	103.921	126.000	101.120
81.000	95.000	102.000	111.000	135.000	66.075	79.273	89.425	101.120	126.000
126.000	104.368	90.297	79.273	66.075	240.000	190.000	160.000	135.000	110.000
104.368	126.000	103.921	90.297	79.273	190.000	240.000	190.000	160.000	135.000
90.297	103.921	126.000	103.921	89.425	160.000	190.000	240.000	190.000	160.000
79.273	90.297	103.921	126.000	101.120	135.000	160.000	190.000	240.000	188.000
66.075	79.273	89.425	101.120	126.000	110.000	135.000	160.000	188.000	240.000

$\varepsilon_{x1}=\varepsilon_{x2}=0.7$, $V_{x1}/V_{x2}=0.5$, $\rho_{x1x2}=0.7$

53.000	45.000	40.000	35.000	32.000	101.922	85.302	71.386	61.425	54.582
45.000	53.000	45.000	40.000	36.000	85.302	101.922	82.677	71.386	62.296
40.000	45.000	53.000	45.000	40.000	71.386	82.677	101.922	84.000	70.000
35.000	40.000	45.000	53.000	44.000	61.425	71.386	84.000	101.922	80.424
32.000	36.000	40.000	44.000	53.000	54.582	62.296	70.000	80.424	101.922
101.922	85.302	71.386	61.425	54.582	400.000	330.000	260.000	220.000	190.000
85.302	101.922	82.677	71.386	62.296	330.000	400.000	310.000	260.000	220.000
71.386	82.677	101.922	84.000	70.000	260.000	310.000	400.000	320.000	250.000
61.425	71.386	84.000	101.922	80.424	220.000	260.000	320.000	400.000	300.000
54.582	62.296	70.000	80.424	101.922	190.000	220.000	250.000	300.000	400.000

$$\varepsilon_{x1}=\varepsilon_{x2}=0.7, V_{x1}/V_{x2}=2.0, \rho_{x1x2}=0.5$$

225.000	195.000	170.000	160.000	150.000	73.101	61.664	51.332	46.476	41.982
195.000	225.000	195.000	170.000	160.000	61.664	73.101	61.025	50.498	46.476
170.000	195.000	225.000	195.000	170.000	51.332	61.025	73.101	60.869	51.332
160.000	170.000	195.000	225.000	192.000	46.476	50.498	60.869	73.101	58.788
150.000	160.000	170.000	192.000	225.000	41.982	46.476	51.332	58.788	73.101
73.101	61.664	51.332	46.476	41.982	95.000	78.000	62.000	54.000	47.000
61.664	73.101	61.025	50.498	46.476	78.000	95.000	76.000	60.000	54.000
51.332	61.025	73.101	60.869	51.332	62.000	76.000	95.000	76.000	62.000
46.476	50.498	60.869	73.101	58.788	54.000	60.000	76.000	95.000	72.000
41.982	46.476	51.332	58.788	73.101	47.000	54.000	62.000	72.000	95.000

$$\varepsilon_{x1}=\varepsilon_{x2}=0.7, V_{x1}/V_{x2}=1.0, \rho_{x1x2}=0.5$$

135.000	117.000	104.000	95.000	81.000	90.000	74.549	64.498	56.624	47.196
117.000	135.000	116.000	104.000	95.000	74.549	90.000	74.229	64.498	56.624
104.000	116.000	135.000	116.000	102.000	64.498	74.229	90.000	74.229	63.875
95.000	104.000	116.000	135.000	111.000	56.624	64.498	74.229	90.000	72.229
81.000	95.000	102.000	111.000	135.000	47.196	56.624	63.875	72.229	90.000
90.000	74.549	64.498	56.624	47.196	240.000	190.000	160.000	135.000	110.000
74.549	90.000	74.229	64.498	56.624	190.000	240.000	190.000	160.000	135.000
64.498	74.229	90.000	74.229	63.875	160.000	190.000	240.000	190.000	160.000
56.624	64.498	74.229	90.000	72.229	135.000	160.000	190.000	240.000	188.000
47.196	56.624	63.875	72.229	90.000	110.000	135.000	160.000	188.000	240.000

$$\varepsilon_{x1}=\varepsilon_{x2}=0.7, V_{x1}/V_{x2}=0.5, \rho_{x1x2}=0.5$$

53.000	45.000	40.000	35.000	32.000	72.801	60.930	50.990	43.875	38.987
45.000	53.000	45.000	40.000	36.000	60.930	72.801	59.055	50.990	44.497
40.000	45.000	53.000	45.000	40.000	50.990	59.055	72.801	60.000	50.000
35.000	40.000	45.000	53.000	44.000	43.875	50.990	60.000	72.801	57.446
32.000	36.000	40.000	44.000	53.000	38.987	44.497	50.000	57.446	72.801
72.801	60.930	50.990	43.875	38.987	400.000	330.000	260.000	220.000	190.000
60.930	72.801	59.055	50.990	44.497	330.000	400.000	310.000	260.000	220.000
50.990	59.055	72.801	60.000	50.000	260.000	310.000	400.000	320.000	250.000
43.875	50.990	60.000	72.801	57.446	220.000	260.000	320.000	400.000	300.000
38.987	44.497	50.000	57.446	72.801	190.000	220.000	250.000	300.000	400.000

$\varepsilon_{x1}=0.7$, $\varepsilon_{x2}=1.0$, $V_{x1}/V_{x2}=2.0$, $\rho_{x1x2}=0.9$

225.000	195.000	170.000	160.000	150.000	131.582	110.996	103.637	100.543	97.350
195.000	225.000	196.000	170.000	160.000	110.996	131.582	111.280	103.637	100.543
170.000	196.000	225.000	195.000	170.000	103.637	111.280	131.582	110.996	103.637
160.000	170.000	195.000	225.000	192.000	100.543	103.637	110.996	131.582	110.139
150.000	160.000	170.000	192.000	225.000	97.350	100.543	103.637	110.139	131.582
131.582	110.996	103.637	100.543	97.350	95.000	78.000	78.000	78.000	78.000
110.996	131.582	111.280	103.637	100.543	78.000	95.000	78.000	78.000	78.000
103.637	111.280	131.582	110.996	103.637	78.000	78.000	95.000	78.000	78.000
100.543	103.637	110.996	131.582	110.139	78.000	78.000	78.000	95.000	78.000
97.350	100.543	103.637	110.139	131.582	78.000	78.000	78.000	78.000	95.000

$\varepsilon_{x1}=0.7$, $\varepsilon_{x2}=1.0$, $V_{x1}/V_{x2}=1.0$, $\rho_{x1x2}=0.9$

135.000	117.000	104.000	95.000	81.000	162.000	135.942	128.167	122.496	113.110
117.000	135.000	116.000	104.000	95.000	135.942	162.000	135.360	128.167	122.496
104.000	116.000	135.000	116.000	102.000	128.167	135.360	162.000	135.360	126.929
95.000	104.000	116.000	135.000	111.000	122.496	128.167	135.360	162.000	132.410
81.000	95.000	102.000	111.000	135.000	113.110	122.496	126.929	132.410	162.000
162.000	135.942	128.167	122.496	113.110	240.000	195.000	195.000	195.000	195.000
135.942	162.000	135.360	128.167	122.496	195.000	240.000	195.000	195.000	195.000
128.167	135.360	162.000	135.360	126.929	195.000	195.000	240.000	195.000	195.000
122.496	128.167	135.360	162.000	132.410	195.000	195.000	195.000	240.000	195.000
113.110	122.496	126.929	132.410	162.000	195.000	195.000	195.000	195.000	240.000

$\varepsilon_{x1}=0.7$, $\varepsilon_{x2}=1.0$, $V_{x1}/V_{x2}=0.5$, $\rho_{x1x2}=0.9$

53.000	45.000	40.000	35.000	32.000	123.762	103.581	97.658	91.350	87.348
45.000	53.000	45.000	40.000	36.000	103.581	123.762	103.581	97.658	92.646
40.000	45.000	53.000	45.000	40.000	97.658	103.581	123.762	103.581	97.658
35.000	40.000	45.000	53.000	44.000	91.350	97.658	103.581	123.762	102.424
32.000	36.000	40.000	44.000	53.000	87.348	92.646	97.658	102.424	123.762
123.762	103.581	97.658	91.350	87.348	400.000	330.000	330.000	330.000	330.000
103.581	123.762	103.581	97.658	92.646	330.000	400.000	330.000	330.000	330.000
97.658	103.581	123.762	103.581	97.658	330.000	330.000	400.000	330.000	330.000
91.350	97.658	103.581	123.762	102.424	330.000	330.000	330.000	400.000	330.000
87.348	92.646	97.658	102.424	123.762	330.000	330.000	330.000	330.000	400.000

$\varepsilon_{x1}=0.7$, $\varepsilon_{x2}=1.0$, $V_{x1}/V_{x2}=2.0$, $\rho_{x1x2}=0.7$

225.000	195.000	170.000	160.000	150.000	102.341	86.330	80.606	78.200	75.717
195.000	225.000	196.000	170.000	160.000	86.330	102.341	86.551	80.606	78.200
170.000	196.000	225.000	195.000	170.000	80.606	86.551	102.341	86.330	80.606
160.000	170.000	195.000	225.000	192.000	78.200	80.606	86.330	102.341	85.664
150.000	160.000	170.000	192.000	225.000	75.717	78.200	80.606	85.664	102.341
102.341	86.330	80.606	78.200	75.717	95.000	78.000	78.000	78.000	78.000
86.330	102.341	86.551	80.606	78.200	78.000	95.000	78.000	78.000	78.000
80.606	86.551	102.341	86.330	80.606	78.000	78.000	95.000	78.000	78.000
78.200	80.606	86.330	102.341	85.664	78.000	78.000	78.000	95.000	78.000
75.717	78.200	80.606	85.664	102.341	78.000	78.000	78.000	78.000	95.000

$\varepsilon_{x1}=0.7$, $\varepsilon_{x2}=1.0$, $V_{x1}/V_{x2}=1.0$, $\rho_{x1x2}=0.7$

135.000	117.000	104.000	95.000	81.000	126.000	105.732	99.686	95.275	87.975
117.000	135.000	116.000	104.000	95.000	105.732	126.000	105.280	99.686	95.275
104.000	116.000	135.000	116.000	102.000	99.686	105.280	126.000	105.280	98.722
95.000	104.000	116.000	135.000	111.000	95.275	99.686	105.280	126.000	102.986
81.000	95.000	102.000	111.000	135.000	87.975	95.275	98.722	102.986	126.000
126.000	105.732	99.686	95.275	87.975	240.000	195.000	195.000	195.000	195.000
105.732	126.000	105.280	99.686	95.275	195.000	240.000	195.000	195.000	195.000
99.686	105.280	126.000	105.280	98.722	195.000	195.000	240.000	195.000	195.000
95.275	99.686	105.280	126.000	102.986	195.000	195.000	195.000	240.000	195.000
87.975	95.275	98.722	102.986	126.000	195.000	195.000	195.000	195.000	240.000

$\varepsilon_{x1}=0.7$, $\varepsilon_{x2}=1.0$, $V_{x1}/V_{x2}=0.5$, $\rho_{x1x2}=0.7$

53.000	45.000	40.000	35.000	32.000	101.922	85.302	80.424	75.230	71.933
45.000	53.000	45.000	40.000	36.000	85.302	101.922	85.302	80.424	76.297
40.000	45.000	53.000	45.000	40.000	80.424	85.302	101.922	85.302	80.424
35.000	40.000	45.000	53.000	44.000	75.230	80.424	85.302	101.922	84.349
32.000	36.000	40.000	44.000	53.000	71.933	76.297	80.424	84.349	101.922
101.922	85.302	80.424	75.230	71.933	400.000	330.000	330.000	330.000	330.000
85.302	101.922	85.302	80.424	76.297	330.000	400.000	330.000	330.000	330.000
80.424	85.302	101.922	85.302	80.424	330.000	330.000	400.000	330.000	330.000
75.230	80.424	85.302	101.922	84.349	330.000	330.000	330.000	400.000	330.000
71.933	76.297	80.424	84.349	101.922	330.000	330.000	330.000	330.000	400.000

$\varepsilon_{x1}=0.7$, $\varepsilon_{x2}=1.0$, $V_{x1}/V_{x2}=2.0$, $\rho_{x1x2}=0.5$

225.000	196.000	170.000	160.000	150.000	73.101	61.664	57.576	55.857	54.083
196.000	225.000	196.000	170.000	150.000	61.664	73.101	61.822	57.576	55.857
170.000	196.000	225.000	195.000	170.000	57.576	61.822	73.101	61.664	57.576
160.000	170.000	195.000	225.000	192.000	55.857	57.576	61.664	73.101	61.188
150.000	160.000	170.000	192.000	225.000	54.083	55.857	57.576	61.188	73.101
73.101	61.664	57.576	55.857	54.083	95.000	78.000	78.000	78.000	78.000
61.664	73.101	61.822	57.576	55.857	78.000	95.000	78.000	78.000	78.000
57.576	61.822	73.101	61.664	57.576	78.000	78.000	95.000	78.000	78.000
55.857	57.576	61.664	73.101	61.188	78.000	78.000	78.000	95.000	78.000
54.083	55.857	57.576	61.188	73.101	78.000	78.000	78.000	78.000	95.000

$\varepsilon_{x1}=0.7$, $\varepsilon_{x2}=1.0$, $V_{x1}/V_{x2}=1.0$, $\rho_{x1x2}=0.5$

135.000	117.000	104.000	95.000	81.000	90.000	75.523	71.204	68.053	62.839
117.000	135.000	116.000	104.000	95.000	75.523	90.000	75.200	71.204	68.053
104.000	116.000	135.000	116.000	102.000	71.204	75.200	90.000	75.200	70.516
95.000	104.000	116.000	135.000	111.000	68.053	71.204	75.200	90.000	73.561
81.000	95.000	102.000	111.000	135.000	62.839	68.053	70.516	73.561	90.000
90.000	75.523	71.204	68.053	62.839	240.000	195.000	195.000	195.000	195.000
75.523	90.000	75.200	71.204	68.053	195.000	240.000	195.000	195.000	195.000
71.204	75.200	90.000	75.200	70.516	195.000	195.000	240.000	195.000	195.000
68.053	71.204	75.200	90.000	73.561	195.000	195.000	195.000	240.000	195.000
62.839	68.053	70.516	73.561	90.000	195.000	195.000	195.000	195.000	240.000

$\varepsilon_{x1}=0.7$, $\varepsilon_{x2}=1.0$, $V_{x1}/V_{x2}=0.5$, $\rho_{x1x2}=0.5$

53.000	45.000	40.000	35.000	32.000	72.801	60.930	57.446	53.735	51.381
45.000	53.000	45.000	40.000	36.000	60.930	72.801	60.930	57.446	54.498
40.000	45.000	53.000	45.000	40.000	57.446	60.930	72.801	60.930	57.446
35.000	40.000	45.000	53.000	44.000	53.735	57.446	60.930	72.801	60.249
32.000	36.000	40.000	44.000	53.000	51.381	54.498	57.446	60.249	72.801
72.801	60.930	57.446	53.735	51.381	400.000	330.000	330.000	330.000	330.000
60.930	72.801	60.930	57.446	54.498	330.000	400.000	330.000	330.000	330.000
57.446	60.930	72.801	60.930	57.446	330.000	330.000	400.000	330.000	330.000
53.735	57.446	60.930	72.801	60.249	330.000	330.000	330.000	400.000	330.000
51.381	54.498	57.446	60.249	72.801	330.000	330.000	330.000	330.000	400.000

$\varepsilon_{x1}=1.0$, $\varepsilon_{x2}=0.7$, $V_{x1}/V_{x2}=2.0$, $\rho_{x1x2}=0.7$

225.000	195.000	195.000	195.000	195.000	100.879	85.097	75.869	70.805	66.056
195.000	225.000	195.000	195.000	195.000	85.097	100.879	83.999	74.635	70.805
195.000	195.000	225.000	195.000	195.000	75.869	83.999	100.879	83.999	75.869
195.000	195.000	195.000	225.000	195.000	70.805	74.635	83.999	100.879	81.758
195.000	195.000	195.000	195.000	225.000	66.056	70.805	75.869	81.758	100.879
100.879	85.097	75.869	70.805	66.056	95.000	78.000	62.000	54.000	47.000
85.097	100.879	83.999	74.635	70.805	78.000	95.000	76.000	60.000	54.000
75.869	83.999	100.879	83.999	75.869	62.000	76.000	95.000	76.000	62.000
70.805	74.635	83.999	100.879	81.758	54.000	60.000	76.000	95.000	72.000
66.056	70.805	75.869	81.758	100.879	47.000	54.000	62.000	72.000	95.000

$\varepsilon_{x1}=1.0$, $\varepsilon_{x2}=0.7$, $V_{x1}/V_{x2}=1.0$, $\rho_{x1x2}=0.7$

135.000	117.000	117.000	117.000	117.000	124.200	102.877	94.407	86.718	78.278
117.000	135.000	117.000	117.000	117.000	102.877	124.200	102.877	94.407	86.718
117.000	117.000	135.000	117.000	117.000	94.407	102.877	124.200	102.877	94.407
117.000	117.000	117.000	135.000	117.000	86.718	94.407	102.877	124.200	102.334
117.000	117.000	117.000	117.000	135.000	78.278	86.718	94.407	102.334	124.200
124.200	102.877	94.407	86.718	78.278	240.000	190.000	160.000	135.000	110.000
102.877	124.200	102.877	94.407	86.718	190.000	240.000	190.000	160.000	135.000
94.407	102.877	124.200	102.877	94.407	160.000	190.000	240.000	190.000	160.000
86.718	94.407	102.877	124.200	102.334	135.000	160.000	190.000	240.000	188.000
78.278	86.718	94.407	102.334	124.200	110.000	135.000	160.000	188.000	240.000

$\varepsilon_{x1}=1.0$, $\varepsilon_{x2}=0.7$, $V_{x1}/V_{x2}=0.5$, $\rho_{x1x2}=0.7$

53.000	45.000	45.000	45.000	45.000	101.922	85.302	75.717	69.649	64.726
45.000	53.000	45.000	45.000	45.000	85.302	101.922	82.677	75.717	69.649
45.000	45.000	53.000	45.000	45.000	75.717	82.677	101.922	84.000	74.246
45.000	45.000	45.000	53.000	45.000	69.649	75.717	84.000	101.922	81.333
45.000	45.000	45.000	45.000	53.000	64.726	69.649	74.246	81.333	101.922
101.922	85.302	75.717	69.649	64.726	400.000	330.000	260.000	220.000	190.000
85.302	101.922	82.677	75.717	69.649	330.000	400.000	310.000	260.000	220.000
75.717	82.677	101.922	84.000	74.246	260.000	310.000	400.000	320.000	250.000
69.649	75.717	84.000	101.922	81.333	220.000	260.000	320.000	400.000	300.000
64.726	69.649	74.246	81.333	101.922	190.000	220.000	250.000	300.000	400.000

$\varepsilon_{x1}=1.0$, $\varepsilon_{x2}=0.7$, $V_{x1}/V_{x2}=2.0$, $\rho_{x1x2}=0.5$

225.000	195.000	195.000	195.000	195.000	73.101	61.664	54.977	51.308	47.867
195.000	225.000	195.000	195.000	195.000	61.664	73.101	60.869	54.083	51.308
195.000	195.000	225.000	195.000	195.000	54.977	60.869	73.101	60.869	54.977
195.000	195.000	195.000	225.000	195.000	51.308	54.083	60.869	73.101	59.245
195.000	195.000	195.000	195.000	225.000	47.867	51.308	54.977	59.245	73.101
73.101	61.664	54.977	51.308	47.867	95.000	76.000	62.000	54.000	47.000
61.664	73.101	60.869	54.083	51.308	76.000	95.000	76.000	60.000	54.000
54.977	60.869	73.101	60.869	54.977	62.000	76.000	95.000	76.000	62.000
51.308	54.083	60.869	73.101	59.245	54.000	60.000	76.000	95.000	72.000
47.867	51.308	54.977	59.245	73.101	47.000	54.000	62.000	72.000	95.000

$\varepsilon_{x1}=1.0$, $\varepsilon_{x2}=0.7$, $V_{x1}/V_{x2}=1.0$, $\rho_{x1x2}=0.5$

135.000	117.000	117.000	117.000	117.000	90.000	74.549	68.411	62.839	56.723
117.000	135.000	117.000	117.000	117.000	74.549	90.000	74.549	68.411	62.839
117.000	117.000	135.000	117.000	117.000	68.411	74.549	90.000	74.549	68.411
117.000	117.000	117.000	135.000	117.000	62.839	68.411	74.549	90.000	74.155
117.000	117.000	117.000	117.000	135.000	56.723	62.839	68.411	74.155	90.000
90.000	74.549	68.411	62.839	56.723	240.000	190.000	160.000	135.000	110.000
74.549	90.000	74.549	68.411	62.839	190.000	240.000	190.000	160.000	135.000
68.411	74.549	90.000	74.549	68.411	160.000	190.000	240.000	190.000	160.000
62.839	68.411	74.549	90.000	74.155	135.000	160.000	190.000	240.000	188.000
56.723	62.839	68.411	74.155	90.000	110.000	135.000	160.000	188.000	240.000

$\varepsilon_{x1}=1.0$, $\varepsilon_{x2}=0.7$, $V_{x1}/V_{x2}=0.5$, $\rho_{x1x2}=0.5$

53.000	45.000	45.000	45.000	45.000	72.801	60.930	54.083	49.749	46.233
45.000	53.000	45.000	45.000	45.000	60.930	72.801	59.055	54.083	49.749
45.000	45.000	53.000	45.000	45.000	54.083	59.055	72.801	60.000	53.033
45.000	45.000	45.000	53.000	45.000	49.749	54.083	60.000	72.801	58.095
45.000	45.000	45.000	45.000	53.000	46.233	49.749	53.033	58.095	72.801
72.801	60.930	54.083	49.749	46.233	400.000	330.000	260.000	220.000	190.000
60.930	72.801	59.055	54.083	49.749	330.000	400.000	310.000	260.000	220.000
54.083	59.055	72.801	60.000	53.033	260.000	310.000	400.000	320.000	250.000
49.749	54.083	60.000	72.801	58.095	220.000	260.000	320.000	400.000	300.000
46.233	49.749	53.033	58.095	72.801	190.000	220.000	250.000	300.000	400.000

Appendix C

Accuracy of the Estimation Formulas (Ratio Mean, Sd., and Correlation)

To examine the accuracy of the Pearson approximations for ratio variables (Formulas (4), (5), and (7)), the computed statistics of the nine simulated ratio variable populations shown in the first panel of Table 5-1 (corresponding to $\epsilon_{x1} = \epsilon_{x2} = 1.0$) are compared with the approximations of the mean, standard deviation, and correlation based on Formulas (4), (5), and (7). Because the covariance can be expressed as a function of the magnitudes of the standard deviation and correlation of the ratio variables, the accuracy of Formula (6) was not separately examined. The means and standard deviations, for both the numerator and denominator variables used to create the ratio variable, are shown in Table D-1 for each of the nine conditions of V_{x1}/V_{x2} and ρ_{x1x2} . Table D-2 presents $\mu_{x1/x2}$ and $\sigma_{x1/x2}$ calculated from the first trial of the population data of the ratio variable, and the approximations $\bar{\mu}_{x1/x2}$ and $\bar{\sigma}_{x1/x2}$ based on Formulas (4) and (5). The approximations are shown in the brackets. The values of the ρ_{y1y2} were calculated between the first trial and second trial of the ratio population data, and the values of the $\bar{\rho}_{y1y2}$ were approximated from Formula (7) based on the characteristics of the component variables. The subscript y_1 refers the ratio variable of the first trial, and y_2 refers the second trial in the population of the ratio variable data.

As can be seen in Table D-2, although a slight under-approximation exists in the conditions $V_{x1}/V_{x2}=2.0$ and $V_{x1}/V_{x2}=1.0$, the ratio variable means are accurately approximated by Formula (4). When the variation of the denominator variable increased from $V_{x1}/V_{x2}=2.0$ to $V_{x1}/V_{x2}=0.5$, the population mean was slightly under-approximated:

Table D-1
The Population Characteristics of the Component Variables (X_1, X_2)

	$V_{x1}/V_{x2}=2.0$		$V_{x1}/V_{x2}=1.0$		$V_{x1}/V_{x2}=0.5$	
	μ_{x1} (σ_{x1})	μ_{x2} (σ_{x2})	μ_{x1} (σ_{x1})	μ_{x2} (σ_{x2})	μ_{x1} (σ_{x1})	μ_{x2} (σ_{x2})
Trial 1						
$\rho_{x1x2}=0.9$	75.04 (14.98)	100.03 (9.74)	75.03 (11.60)	100.05 (15.49)	75.02 (7.27)	100.07 (20.00)
$\rho_{x1x2}=0.7$	75.04 (14.98)	100.04 (9.75)	75.03 (11.60)	100.06 (15.49)	75.02 (7.27)	100.08 (20.01)
$\rho_{x1x2}=0.5$	75.04 (14.98)	100.04 (9.75)	75.03 (11.60)	100.06 (15.49)	75.02 (7.27)	100.07 (20.00)
Trial 2						
$\rho_{x1x2}=0.9$	75.01 (15.01)	100.02 (9.77)	75.00 (11.63)	100.02 (15.52)	75.00 (7.29)	100.04 (20.04)
$\rho_{x1x2}=0.7$	75.01 (15.01)	100.02 (9.78)	75.01 (11.63)	100.04 (15.54)	75.00 (7.29)	100.05 (20.06)
$\rho_{x1x2}=0.5$	75.01 (15.01)	100.03 (9.78)	75.01 (11.63)	100.04 (15.54)	75.00 (7.29)	100.06 (20.07)

Table D-2
Comparison Between Approximations ($\bar{\mu}_{x1/x2}, \bar{\sigma}_{x1/x2}, \bar{\rho}_{y1y2}$) and the Population Characteristics ($\mu_{x1/x2}, \sigma_{x1/x2}, \rho_{y1y2}$)

	$V_{x1}/V_{x2}=2.0$			$V_{x1}/V_{x2}=1.0$			$V_{x1}/V_{x2}=0.5$		
	$\mu_{x1/x2}$ ($\bar{\mu}_{x1/x2}$)	$\sigma_{x1/x2}$ ($\bar{\sigma}_{x1/x2}$)	ρ_{y1y2} ($\bar{\rho}_{y1y2}$)	$\mu_{x1/x2}$ ($\bar{\mu}_{x1/x2}$)	$\sigma_{x1/x2}$ ($\bar{\sigma}_{x1/x2}$)	ρ_{y1y2} ($\bar{\rho}_{y1y2}$)	$\mu_{x1/x2}$ ($\bar{\mu}_{x1/x2}$)	$\sigma_{x1/x2}$ ($\bar{\sigma}_{x1/x2}$)	ρ_{y1y2} ($\bar{\rho}_{y1y2}$)
$\rho_{x1x2}=0.9$	.744 (.744)	.092 (.090)	.891 (.900)	.752 (.752)	.054 (.052)	.834 (.840)	.769 (.767)	.110 (.090)	.792 (.814)
$\rho_{x1x2}=0.7$	.747 (.747)	.113 (.111)	.875 (.876)	.756 (.755)	.094 (.090)	.831 (.838)	.772 (.769)	.133 (.112)	.797 (.820)
$\rho_{x1x2}=0.5$	.750 (.750)	.131 (.130)	.866 (.869)	.760 (.759)	.123 (.116)	.829 (.838)	.776 (.772)	.154 (.130)	.801 (.823)

the greatest difference between the approximated value and true value was 0.004 (0.5% error). The standard deviation also shows an under-approximated pattern. However, the difference between the approximate value and the true value is much higher in the condition $V_{x1}/V_{x2}=0.5$ than in the other two conditions. The approximate standard deviations in the condition $V_{x1}/V_{x2}=0.5$ underestimated the true values by 0.020, 0.021, and 0.024, and the percent error were 18.2%, 15.8%, and 15.6%, respectively. The approximate correlation (Formula (7)) between the first trial and the second trial of the ratio variable result in overestimate when the variation of the denominator variable increased from $V_{x1}/V_{x2}=2.0$ to $V_{x1}/V_{x2}=0.5$. In the condition $V_{x1}/V_{x2}=0.5$, the overestimations were 0.022, 0.023, and 0.022, and the percent errors were 2.7%, 2.9%, and 2.7%, respectively.

The results indicate that the three parameters (the mean and standard deviation in the first trial and the correlation between the two trials) are not well approximated by Formulas (4), (5), and (7) when $V_{x1}/V_{x2}=0.5$. It can be concluded that if the variation of the denominator variable is relatively higher than of the numerator variable ($V_{x1}/V_{x2}=0.5$), the approximation formulas (Formulas (4), (5), and (7)) may not accurately approximate the population mean, standard deviation, and correlation of ratio variables.

Appendix D
Correlation Matrices of the Component and Ratio Variable Populations

D-1

The Correlation Matrixes in The condition $\epsilon_{x1} = \epsilon_{x2} = 0.7$

$V_{x1}/V_{x2}=2.0$		$\rho(x_{11}, x_{13})$		$\rho(x_{21}, x_{23})$		$\rho(y_1, y_3)$		$\rho_{x1x2}=.5$		$\rho(y_1, y_3)$	
1.00	0.87	0.76	0.71	0.66	1.00	0.82	0.65	0.57	0.49	1.00	0.87
	1.00	0.87	0.76	0.71		1.00	0.80	0.63	0.57		1.00
		1.00	0.87	0.76			1.00	0.80	0.55		
			1.00	0.85				1.00	0.76		
				1.00					1.00		
$\rho_{x1x2}=.9$		$\rho(y_1, y_3)$		$\rho_{x1x2}=.7$		$\rho(y_1, y_3)$		$\rho(x_{21}, x_{23})$		$\rho(y_1, y_3)$	
1.00	0.89	0.81	0.79	0.77	1.00	0.88	0.77	0.74	0.70	1.00	0.75
	1.00	0.91	0.83	0.79		1.00	0.88	0.78	0.74		0.87
		1.00	0.90	0.81			1.00	0.88	0.77		1.00
			1.00	0.91				1.00	0.87		1.00
				1.00					1.00		1.00
$V_{x1}/V_{x2}=1.0$		$\rho(x_{11}, x_{13})$		$\rho(x_{21}, x_{23})$		$\rho(y_1, y_3)$		$\rho_{x1x2}=.5$		$\rho(y_1, y_3)$	
1.00	0.87	0.77	0.70	0.60	1.00	0.79	0.67	0.56	0.45	1.00	0.67
	1.00	0.86	0.77	0.70		1.00	0.79	0.67	0.56		0.71
		1.00	0.86	0.76			1.00	0.79	0.67		0.75
			1.00	0.82				1.00	0.78		0.85
				1.00					1.00		1.00
$\rho_{x1x2}=.9$		$\rho(y_1, y_3)$		$\rho_{x1x2}=.7$		$\rho(y_1, y_3)$		$\rho_{x1x2}=.5$		$\rho(y_1, y_3)$	
1.00	0.83	0.72	0.65	0.56	1.00	0.82	0.71	0.62	0.52	1.00	0.62
	1.00	0.82	0.72	0.66		1.00	0.82	0.71	0.63		0.71
		1.00	0.83	0.72			1.00	0.82	0.71		0.82
			1.00	0.80				1.00	0.80		1.00
				1.00					1.00		1.00
$V_{x1}/V_{x2}=0.5$		$\rho(x_{11}, x_{13})$		$\rho(x_{21}, x_{23})$		$\rho(y_1, y_3)$		$\rho_{x1x2}=.5$		$\rho(y_1, y_3)$	
1.00	0.85	0.75	0.66	0.60	1.00	0.83	0.65	0.55	0.45	1.00	0.80
	1.00	0.85	0.75	0.68		1.00	0.78	0.65	0.55		1.00
		1.00	0.85	0.75			1.00	0.80	0.63		1.00
			1.00	0.83				1.00	0.75		1.00
				1.00					1.00		1.00
$\rho_{x1x2}=.9$		$\rho(y_1, y_3)$		$\rho_{x1x2}=.7$		$\rho(y_1, y_3)$		$\rho_{x1x2}=.5$		$\rho(y_1, y_3)$	
1.00	0.79	0.57	0.47	0.38	1.00	0.80	0.60	0.50	0.42	1.00	0.51
	1.00	0.71	0.57	0.46		1.00	0.74	0.61	0.50		0.62
		1.00	0.75	0.53			1.00	0.77	0.57		0.78
			1.00	0.68				1.00	0.71		1.00
				1.00					1.00		1.00

D-3

The Correlation Matrices in the Condition $\varepsilon_{x1}=1.0$ $\varepsilon_{x2}=0.7$

Vx1/Vx2=2.0										
1.00		$\rho(x_{11}, x_{1j})$			$\rho(y_1, y_j)$			$\rho(x_{21}, x_{2j})$		
	0.87	0.87	0.87	0.87	1.00	0.82	0.65	0.57	0.49	
	1.00	0.87	0.87	0.87	1.00	1.00	0.80	0.63	0.57	
		1.00	0.87	0.87		1.00	1.00	0.80	0.65	
			1.00	0.87			1.00	1.00	0.76	
				1.00				1.00	1.00	
Vx1/Vx2=1.0										
1.00		$\rho(x_{11}, x_{1j})$			$\rho(y_1, y_j)$			$\rho(x_{21}, x_{2j})$		
	0.87	0.87	0.87	0.87	1.00	0.88	0.91	0.94	0.94	
	1.00	0.87	0.87	0.87	1.00	1.00	0.88	0.92	0.92	
		1.00	0.87	0.87		1.00	1.00	0.88	0.89	
			1.00	0.87			1.00	1.00	0.89	
				1.00				1.00	1.00	
Vx1/Vx2=0.5										
1.00		$\rho(x_{11}, x_{1j})$			$\rho(y_1, y_j)$			$\rho(x_{21}, x_{2j})$		
	0.85	0.85	0.85	0.85	1.00	0.82	0.76	0.73	0.56	
	1.00	0.85	0.85	0.85	1.00	1.00	0.82	0.76	0.58	
		1.00	0.85	0.85		1.00	1.00	0.82	0.61	
			1.00	0.85			1.00	1.00	0.65	
				1.00				1.00	1.00	
Vx1/Vx2=2.0										
1.00		$\rho(x_{11}, x_{1j})$			$\rho(y_1, y_j)$			$\rho(x_{21}, x_{2j})$		
	0.87	0.87	0.87	0.87	1.00	0.82	0.76	0.73	0.56	
	1.00	0.87	0.87	0.87	1.00	1.00	0.82	0.76	0.58	
		1.00	0.87	0.87		1.00	1.00	0.82	0.61	
			1.00	0.87			1.00	1.00	0.65	
				1.00				1.00	1.00	
Vx1/Vx2=1.0										
1.00		$\rho(x_{11}, x_{1j})$			$\rho(y_1, y_j)$			$\rho(x_{21}, x_{2j})$		
	0.87	0.87	0.87	0.87	1.00	0.88	0.91	0.94	0.94	
	1.00	0.87	0.87	0.87	1.00	1.00	0.88	0.92	0.92	
		1.00	0.87	0.87		1.00	1.00	0.88	0.89	
			1.00	0.87			1.00	1.00	0.89	
				1.00				1.00	1.00	
Vx1/Vx2=0.5										
1.00		$\rho(x_{11}, x_{1j})$			$\rho(y_1, y_j)$			$\rho(x_{21}, x_{2j})$		
	0.85	0.85	0.85	0.85	1.00	0.82	0.76	0.73	0.56	
	1.00	0.85	0.85	0.85	1.00	1.00	0.82	0.76	0.58	
		1.00	0.85	0.85		1.00	1.00	0.82	0.61	
			1.00	0.85			1.00	1.00	0.65	
				1.00				1.00	1.00	
Vx1/Vx2=2.0										
1.00		$\rho(x_{11}, x_{1j})$			$\rho(y_1, y_j)$			$\rho(x_{21}, x_{2j})$		
	0.87	0.87	0.87	0.87	1.00	0.82	0.76	0.73	0.56	
	1.00	0.87	0.87	0.87	1.00	1.00	0.82	0.76	0.58	
		1.00	0.87	0.87		1.00	1.00	0.82	0.61	
			1.00	0.87			1.00	1.00	0.65	
				1.00				1.00	1.00	
Vx1/Vx2=1.0										
1.00		$\rho(x_{11}, x_{1j})$			$\rho(y_1, y_j)$			$\rho(x_{21}, x_{2j})$		
	0.87	0.87	0.87	0.87	1.00	0.88	0.91	0.94	0.94	
	1.00	0.87	0.87	0.87	1.00	1.00	0.88	0.92	0.92	
		1.00	0.87	0.87		1.00	1.00	0.88	0.89	
			1.00	0.87			1.00	1.00	0.89	
				1.00				1.00	1.00	
Vx1/Vx2=0.5										
1.00		$\rho(x_{11}, x_{1j})$			$\rho(y_1, y_j)$			$\rho(x_{21}, x_{2j})$		
	0.85	0.85	0.85	0.85	1.00	0.82	0.76	0.73	0.56	
	1.00	0.85	0.85	0.85	1.00	1.00	0.82	0.76	0.58	
		1.00	0.85	0.85		1.00	1.00	0.82	0.61	
			1.00	0.85			1.00	1.00	0.65	
				1.00				1.00	1.00	
Vx1/Vx2=2.0										
1.00		$\rho(x_{11}, x_{1j})$			$\rho(y_1, y_j)$			$\rho(x_{21}, x_{2j})$		
	0.87	0.87	0.87	0.87	1.00	0.82	0.76	0.73	0.56	
	1.00	0.87	0.87	0.87	1.00	1.00	0.82	0.76	0.58	
		1.00	0.87	0.87		1.00	1.00	0.82	0.61	
			1.00	0.87			1.00	1.00	0.65	
				1.00				1.00	1.00	
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	1.00	0.87	0.87	0.87	1.00	1.00	0.88	0.92	0.92	
		1.00	0.87	0.87		1.00	1.00	0.88	0.89	
			1.00	0.87			1.00	1.00	0.89	
				1.00				1.00	1.00	
Vx1/Vx2=0.5										
1.00		$\rho(x_{11}, x_{1j})$			$\rho(y_1, y_j)$			$\rho(x_{21}, x_{2j})$		
	0.85	0.85	0.85	0.85	1.00	0.82	0.76	0.73	0.56	
	1.00	0.85	0.85	0.85	1.00	1.00	0.82	0.76	0.58	
		1.00	0.85	0.85		1.00	1.00	0.82	0.61	
			1.00	0.85			1.00	1.00	0.65	
				1.00				1.00	1.00	
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1.00		$\rho(x_{11}, x_{1j})$			$\rho(y_1, y_j)$			$\rho(x_{21}, x_{2j})$		
	0.87	0.87	0.87	0.87	1.00	0.82	0.76	0.73	0.56	
	1.00	0.87	0.87	0.87	1.00	1.00	0.82	0.76	0.58	
		1.00	0.87	0.87		1.00	1.00	0.82	0.61	
			1.00	0.87			1.00	1.00	0.65	
				1.00				1.00	1.00	
Vx1/Vx2=1.0										
1.00		$\rho(x_{11}, x_{1j})$			$\rho(y_1, y_j)$			$\rho(x_{21}, x_{2j})$		
	0.87	0.87	0.87	0.87	1.00	0.88	0.91	0.94	0.94	
	1.00	0.87	0.87	0.87	1.00	1.00	0.88	0.92	0.92	
		1.00	0.87	0.87		1.00	1.00	0.88	0.89	
			1.00	0.87			1.00	1.00	0.89	
				1.00				1.00	1.00	
Vx1/Vx2=0.5										
1.00		$\rho(x_{11}, x_{1j})$			$\rho(y_1, y_j)$			$\rho(x_{21}, x_{2j})$		
	0.85	0.85	0.85	0.85	1.00	0.82	0.76	0.73	0.56	
	1.00	0.85	0.85	0.85	1.00	1.00	0.82	0.76	0.58	
		1.00	0.85	0.85		1.00	1.00	0.82	0.61	
			1.00	0.85			1.00	1.00	0.65	
				1.00				1.00	1.00	
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1.00		$\rho(x_{11}, x_{1j})$			$\rho(y_1, y_j)$			$\rho(x_{21}, x_{2j})$		
	0.87	0.87	0.87	0.87	1.00	0.82	0.76	0.73	0.56	
	1.00	0.87	0.87	0.87	1.00	1.00	0.82	0.76	0.58	
		1.00	0.87	0.87		1.00	1.00	0.82	0.61	
			1.00	0.87			1.00	1.00	0.65	
				1.00				1.00	1.00	
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1.00		$\rho(x_{11}, x_{1j})$			$\rho(y_1, y_j)$			$\rho(x_{21}, x_{2j})$		
	0.87	0.87	0.87	0.87	1.00	0.88	0.91	0.94	0.94	
	1.00	0.87	0.87	0.87	1.00	1.00	0.88	0.92	0.92	
		1.00	0.87	0.87		1.00	1.00	0.88	0.89	
			1.00	0.87			1.00	1.00	0.89	
				1.00				1.00	1.00	
Vx1/Vx2=0.5										
1.00		$\rho(x_{11}, x_{1j})$			$\rho(y_1, y_j)$			$\rho(x_{21}, x_{2j})$		
	0.85	0.85	0.85	0.85	1.00	0.82	0.76	0.73	0.56	
	1.00	0.85	0.85	0.85	1.00	1.00	0.82	0.76	0.58	
		1.00	0.85	0.85		1.00	1.00	0.82	0.61	
			1.00	0.85			1.00	1.00	0.65	
				1.00				1.00	1.00	
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	0.87	0.87	0.87	0.87	1.00	0.82	0.76	0.73	0.56	
	1.00	0.87	0.87	0.87	1.00	1.00	0.82	0.76	0.58	
		1.00	0.87	0.87		1.00	1.00	0.82	0.61	
			1.00	0.87			1.00	1.00	0.65	
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	0.87	0.87	0.87	0.87	1.00	0.88	0.91	0.94	0.94	
	1.00	0.87	0.87	0.87	1.00	1.00	0.88	0.92	0.92	
		1.00	0.87	0.87		1.00	1.00	0.88	0.89	
			1.00	0.87			1.00	1.00	0.89	
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	1.00	0.85	0.85	0.85	1.00	1.00	0.82	0.76	0.58	
		1.00	0.85	0.85		1.00	1.00	0.82	0.61	
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	1.00	0.87	0.87	0.87	1.00	1.00	0.82	0.76	0.58	
		1.00	0.87	0.87		1.00	1.00	0.82	0.61	
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	1.00	0.87	0.87	0.87	1.00	1.00	0.88	0.92	0.92	
		1.00	0.87	0.87		1.00	1.00	0.88	0.89	
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	0.85	0.85	0.85	0.85	1.00	0.82	0.76	0.73	0.56	
	1.00	0.85	0.85	0.85	1.00	1.00	0.82	0.76	0.58	
		1.00	0.85	0.85		1.00	1.00	0.82	0.61	
			1.00	0.85			1.00	1.00	0.65	
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	1.00	0.87	0.87	0.87	1.00	1.00	0.82	0.76	0.58	
		1.00	0.87	0.87		1.00	1.00	0.82	0.61	
			1.00	0.87			1.00	1.00	0.65	
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1.00		$\rho(x_{11}, x_{1j})$			$\rho(y_1, y_j)$			$\rho(x_{21}, x_{2j})$		
	0.87	0.87	0.87	0.87	1.00					